The Abdus Salam
International Centre for Theoretical Physics

United Nations
Educational, Scientific and
Cultural Organization

IAEA
International Atomic Energy Agency

2177-18

ICTP Latin-American Basic Course on FPGA Design for Scientific Instrumentation

15 - 31 March 2010

Digital arithmetic II
(basic arithmetic operations - Comparators)

BAZARGAN SABET Pirouz
LIP6, University Pierre et Matie Curie
Paris
France

Outline

- Digital CMOS design
- Arithmetic operators
  - Adders
  - Comparators
  - Shifters
  - Multipliers

Comparators

Comparing a natural number to a constant : =

Let consider a natural number a coded on 8 bits using Natural Binary Code

$$a_7\ a_6\ a_5\ a_4\ a_3\ a_2\ a_1\ a_0$$
$$= ?$$
$$0\ 0\ 0\ 0\ 0\ 0\ 0\ 0$$

0 / 1

Comparators

Comparing a natural number to zero : =

Boolean function

Null = 1 if

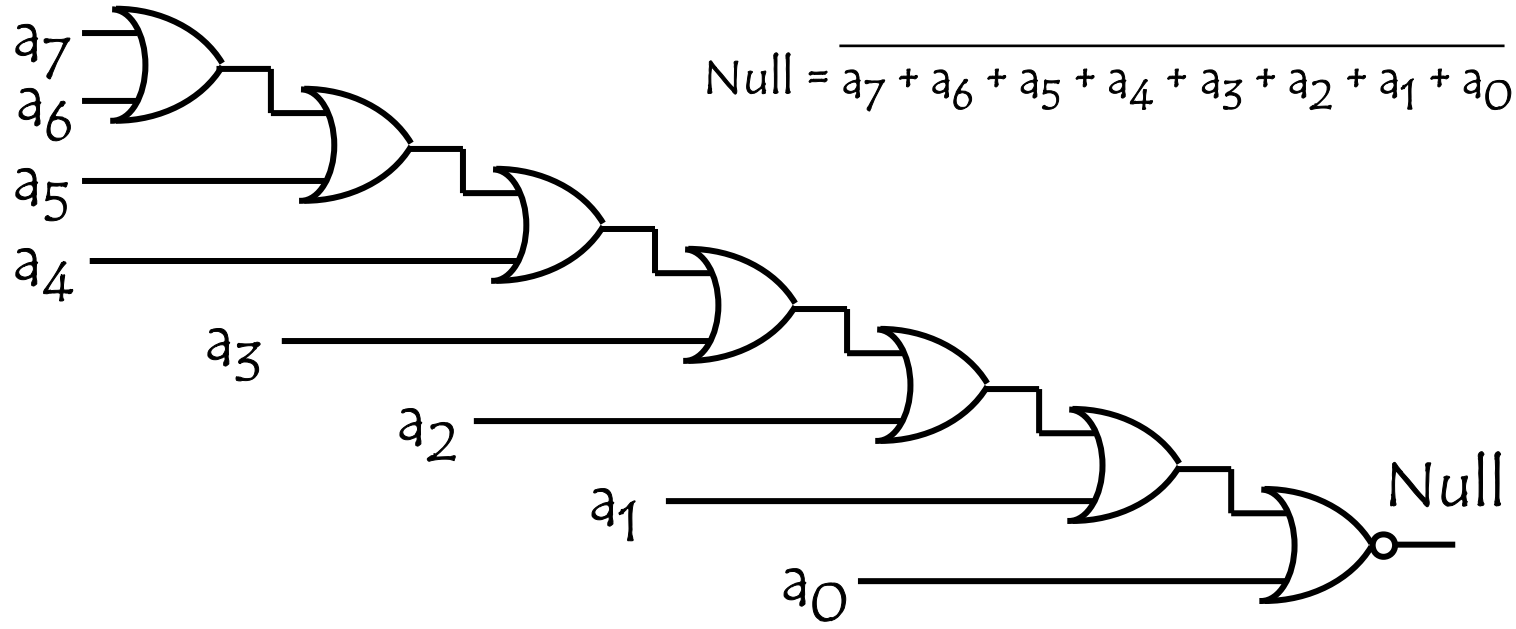
$$\overline{a_7} \cdot \overline{a_6} \cdot \overline{a_5} \cdot \overline{a_4} \cdot \overline{a_3} \cdot \overline{a_2} \cdot \overline{a_1} \cdot \overline{a_0} = 1$$

$$\text{Null} = a_7 + a_6 + a_5 + a_4 + a_3 + a_2 + a_1 + a_0$$

Comparators

Comparing a natural number to zero : =

Implementation

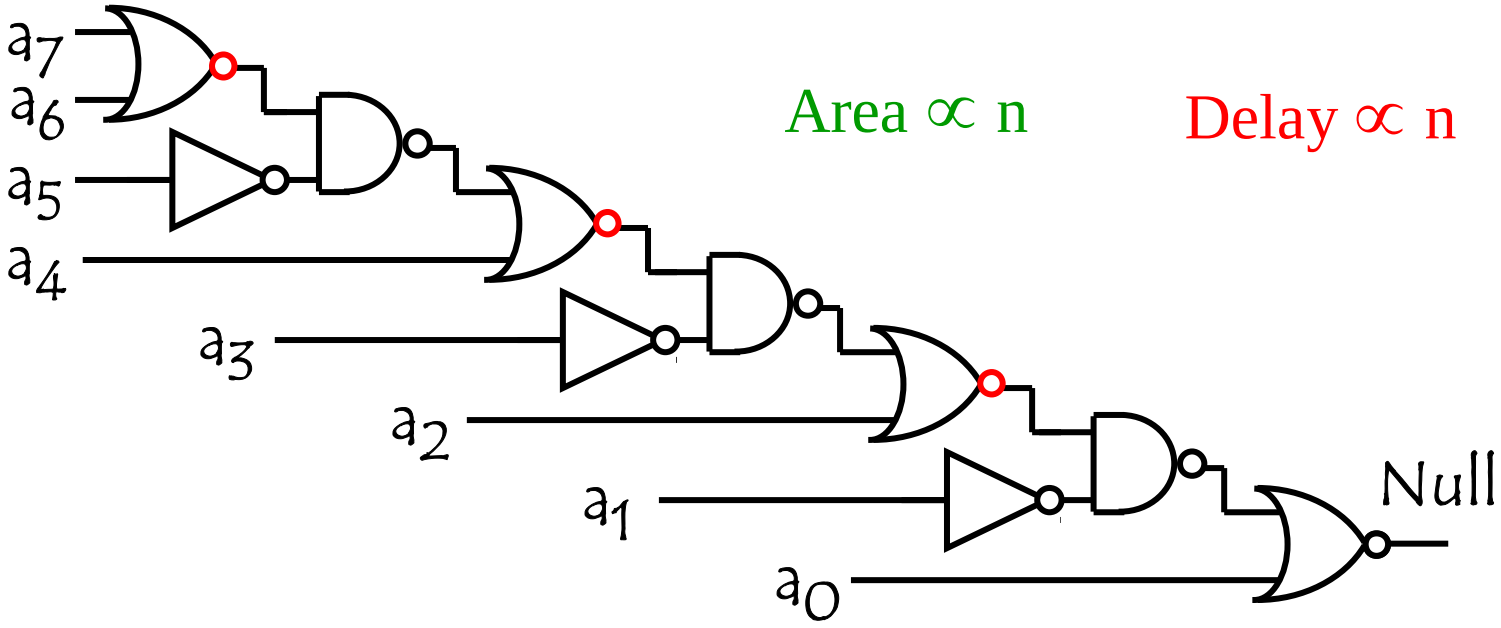


$$\text{Null} = \overline{a_7 + a_6 + a_5 + a_4 + a_3 + a_2 + a_1 + a_0}$$

Comparators

Comparing a natural number to zero : =

Implementation



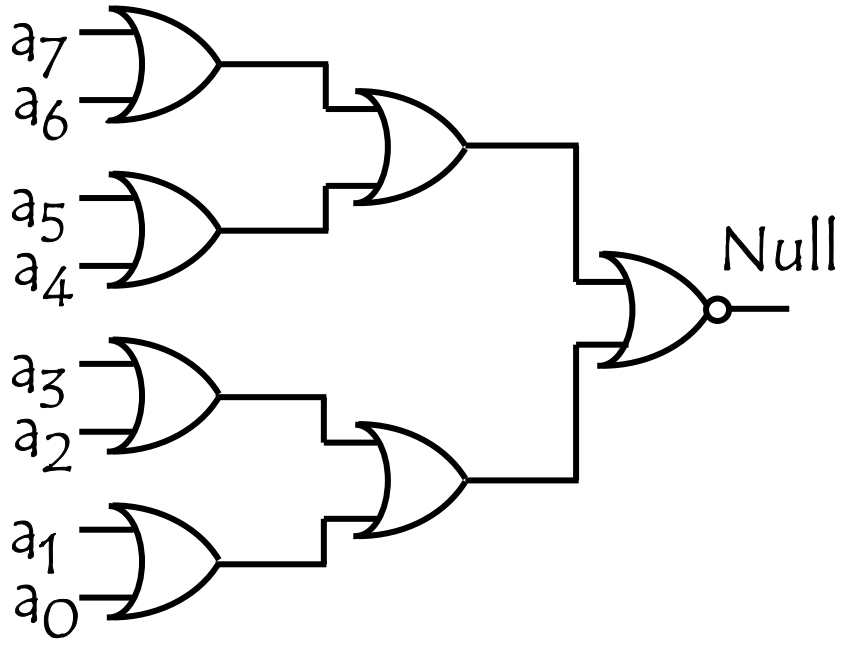
Area ∝ n

Delay ∝ n

Comparators

Comparing a natural number to zero : =

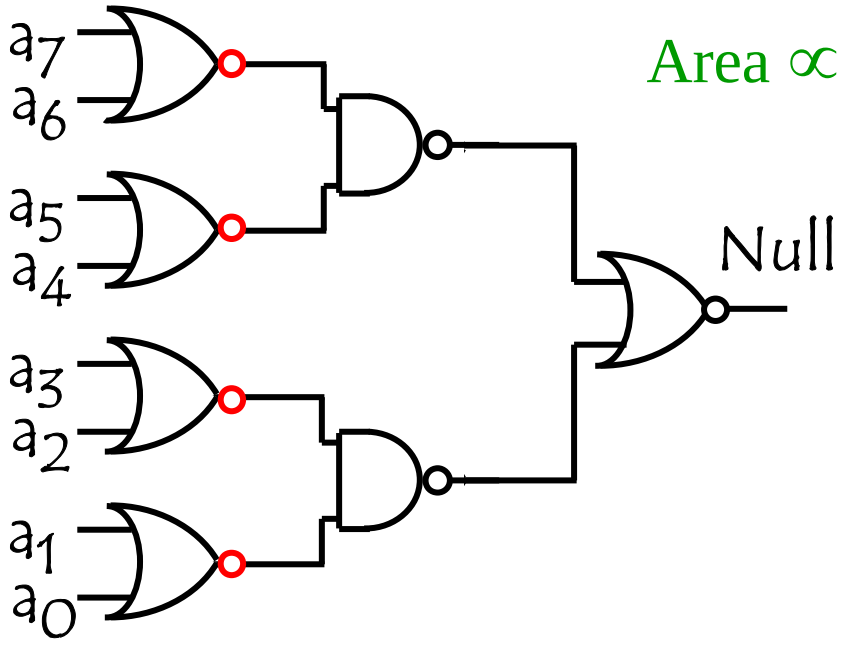
Implementation improvement



Comparators

Comparing a natural number to zero : =

Implementation improvement



Area ∝ n Delay ∝ *log*(n)

Comparators

Comparing two natural numbers : =

Let consider two natural numbers a and b coded on 8 bits using Natural Binary Code

$$a_7\ a_6\ a_5\ a_4\ a_3\ a_2\ a_1\ a_0$$
$$= ?$$
$$b_7\ b_6\ b_5\ b_4\ b_3\ b_2\ b_1\ b_0$$

0 / 1

Comparators

Comparing two natural numbers : =

Boolean function

a Equal b if : $a_7=b_7$ and $a_6=b_6$ and … and $a_0=b_0$

a Equal b if : $\overline{(a_7 \oplus b_7)} \cdot \ \ldots \ \cdot \overline{(a_0 \oplus b_0)} = 1$

$$\text{Equal} = \overline{(a_7 \oplus b_7) + \ \ldots \ + (a_0 \oplus b_0)}$$

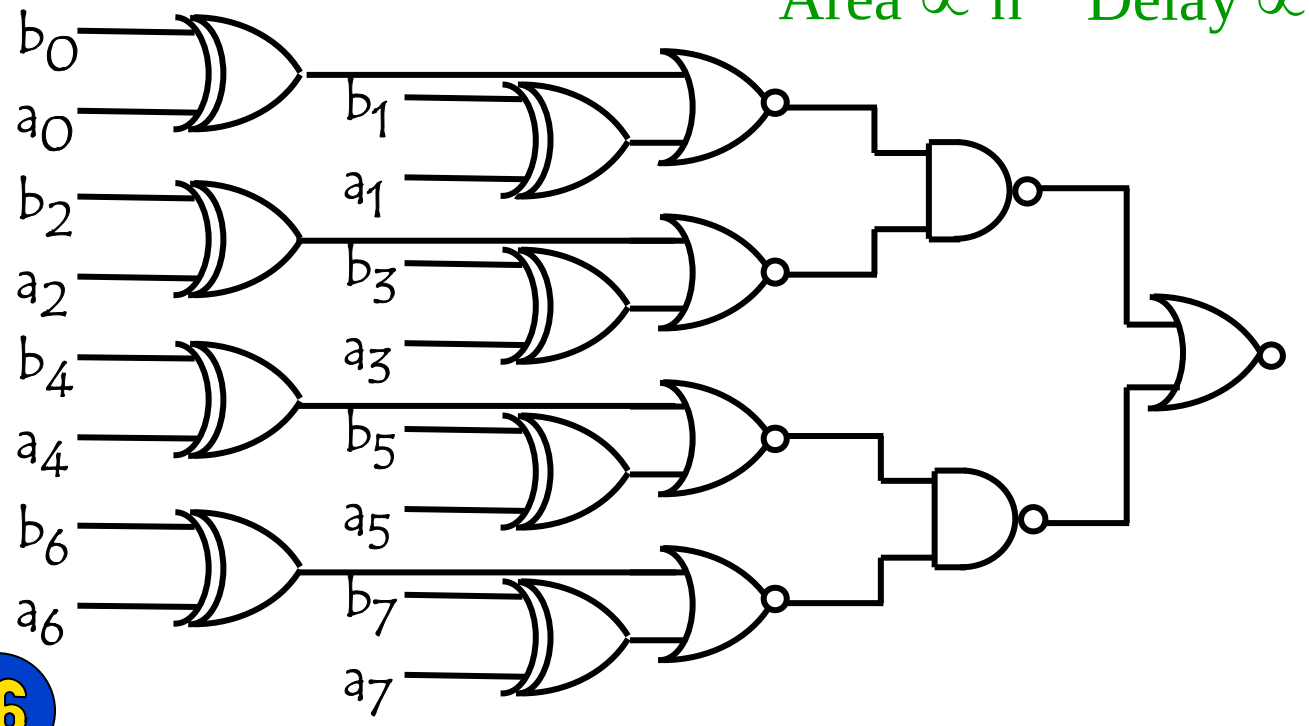
$$\text{Equal} = \overline{(\ e_7 \ ) + \ \ldots \ + (\ e_0 \ )}$$

Comparators

Comparing two natural numbers : =

Implementation

Area ∝ n Delay ∝ *log*(n)



Comparators

Comparing two natural numbers : <

Let consider two natural numbers a and b
coded on 8 bits using Natural Binary Code

$$a_7\ a_6\ a_5\ a_4\ a_3\ a_2\ a_1\ a_0$$
$$<\ ?$$
$$b_7\ b_6\ b_5\ b_4\ b_3\ b_2\ b_1\ b_0$$

0 / 1

Comparators

Comparing two natural numbers : <

Boolean function

$a < b$ if : $a_7 < b_7$ or ($a_7 = b_7$ and ($a_6 < b_6$ or ($a_6 = b_6$ and …)))

$a_7\ a_6\ a_5\ a_4\ a_3\ a_2\ a_1\ a_0$

$<\ ?$

$b_7\ b_6\ b_5\ b_4\ b_3\ b_2\ b_1\ b_0$

0 / 1

Comparators

Comparing two natural numbers : <

Boolean function

$a < b$ if : $a_7 < b_7$ or ($a_7 = b_7$ and ($a_6 < b_6$ or ($a_6 = b_6$ and …)))

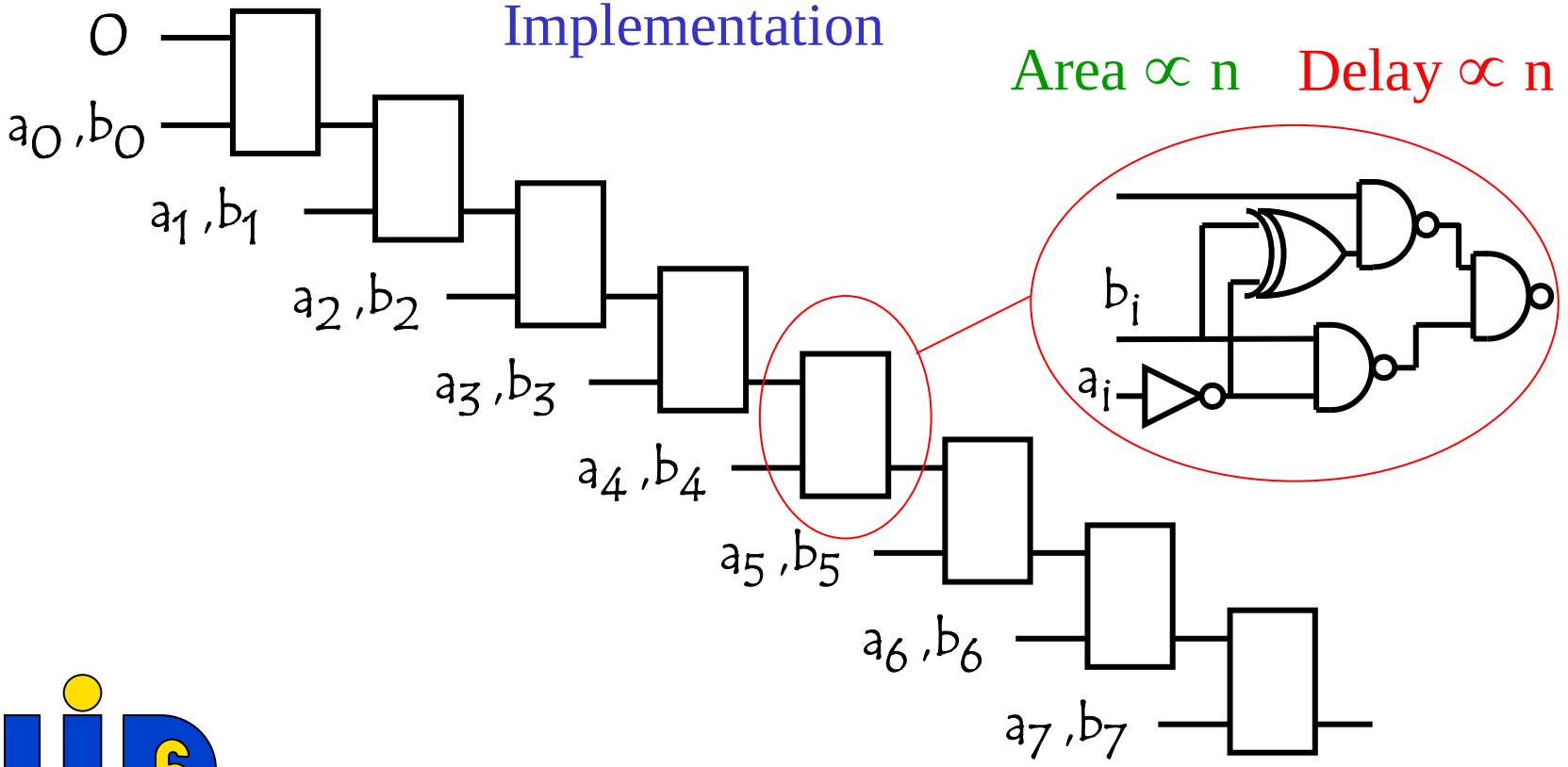
$a < b$ if : $\overline{a_7} b_7 + ((\overline{a_7 \oplus b_7}) \cdot (\overline{a_6} b_6 + ((\overline{a_6 \oplus b_6}) \cdot \ldots)))$

Comparators

Comparing two natural numbers : <

Implementation

Area ∝ n Delay ∝ n



Comparators

Comparing two natural numbers : <

Implementation Improvement

$a < b$ if : $a_7 < b_7$ or ($a_7 = b_7$ and ($a_6 < b_6$ or ($a_6 = b_6$ and …)))

$a < b$ if : $\overline{a_7} b_7 + ((\overline{a_7 \oplus b_7}) \cdot (\overline{a_6} b_6 + ((\overline{a_6 \oplus b_6}) \cdot \ldots)))$

$\overline{a_i} b_i + (\overline{a_i \oplus b_i})$.previous

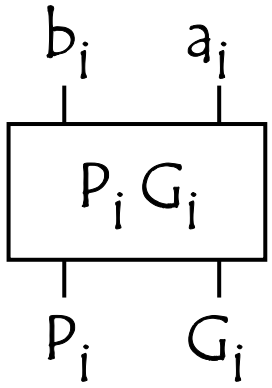
Propagation

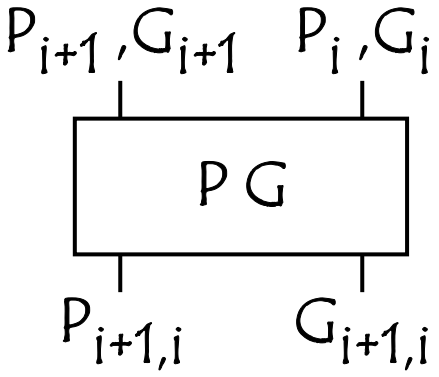
Generation

Comparators

Comparing two natural numbers : <

Implementation Improvement





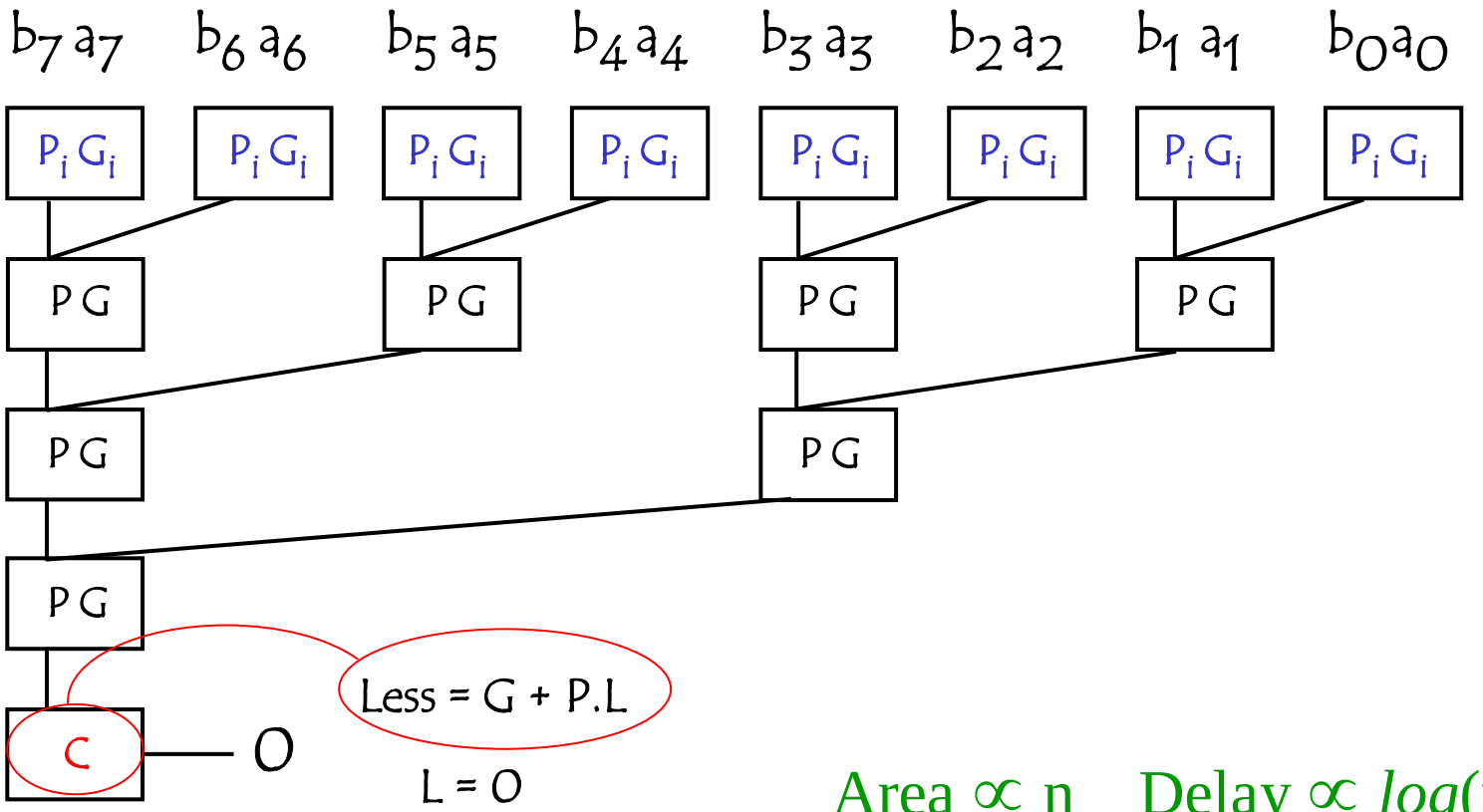
$$G_i = \overline{a_i}\, b_i$$

$$P_i = \overline{a_i} \oplus b_i$$

$$G_{i+1,\,i} = G_{i+1} + G_i . P_{i+1}$$

$$P_{i+1,\,i} = P_i \cdot P_{i+1}$$

Comparators



Area ∝ n Delay ∝ *log*(n)

Comparators