



**The Abdus Salam
International Centre for Theoretical Physics**



2177-19

**ICTP Latin-American Basic Course on FPGA Design for Scientific
Instrumentation**

15 - 31 March 2010

**Digital arithmetic II
(basic arithmetic operations - Shifters)**

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Outline

- Digital CMOS design

- Arithmetic operators

 - Adders

 - Comparators

 - Shifters

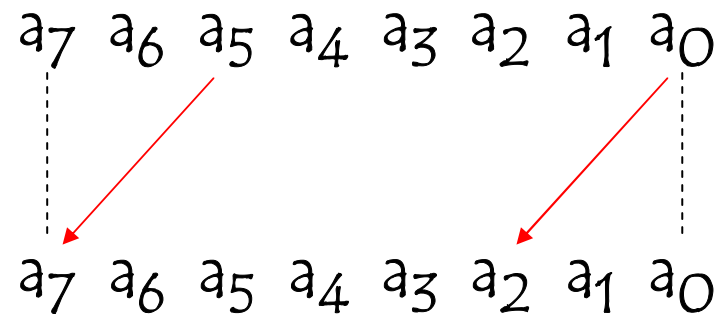
 - Multipliers



Shifters

Shifting a value

Let consider a value a coded on 8 bits
 a can be shifted to the left by n positions ($0 \leq n < 8$)



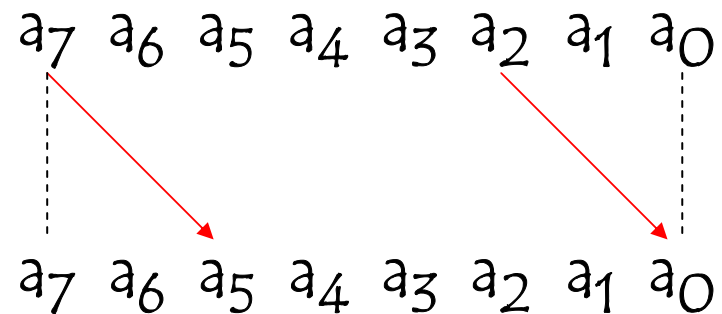
For a natural number, shift left is a multiplication by 2^n

Shifters

Shifting a value

Let consider a value a coded on 8 bits
 a can be shifted to the right by n positions ($0 \leq n < 8$)

logic



For a natural number, shift right is a division by 2^n

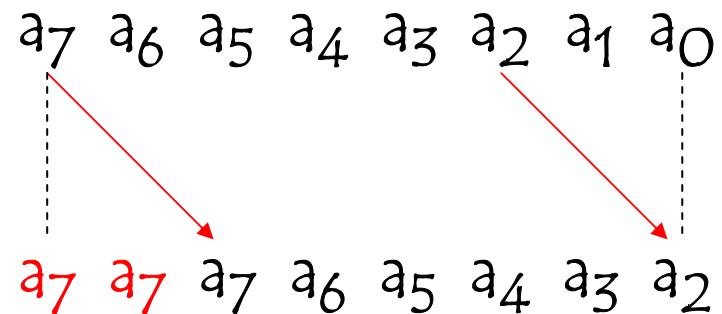


Shifters

Shifting a value

Let consider a value a coded on 8 bits
 a can be shifted to the right by n positions ($0 \leq n < 8$)

arithmetic



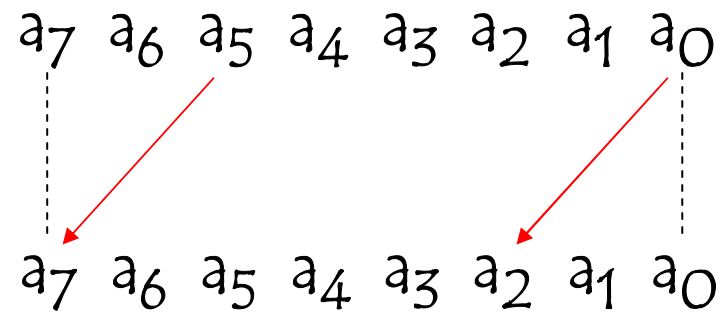
For a **relative** number, shift right is a division by 2^n



Shifters

Shifting a value

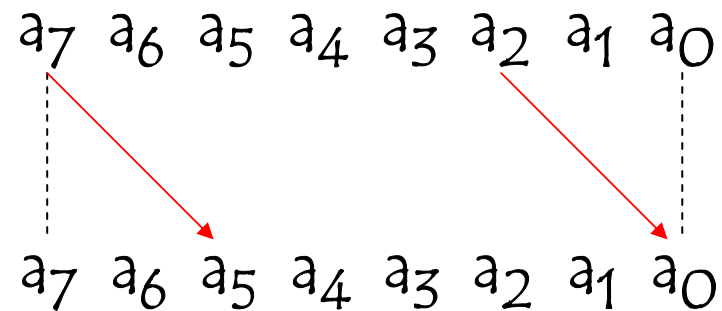
Let consider a value a coded on 8 bits
 a can be rotated to the left by n positions ($0 \leq n < 8$)



Shifters

Shifting a value

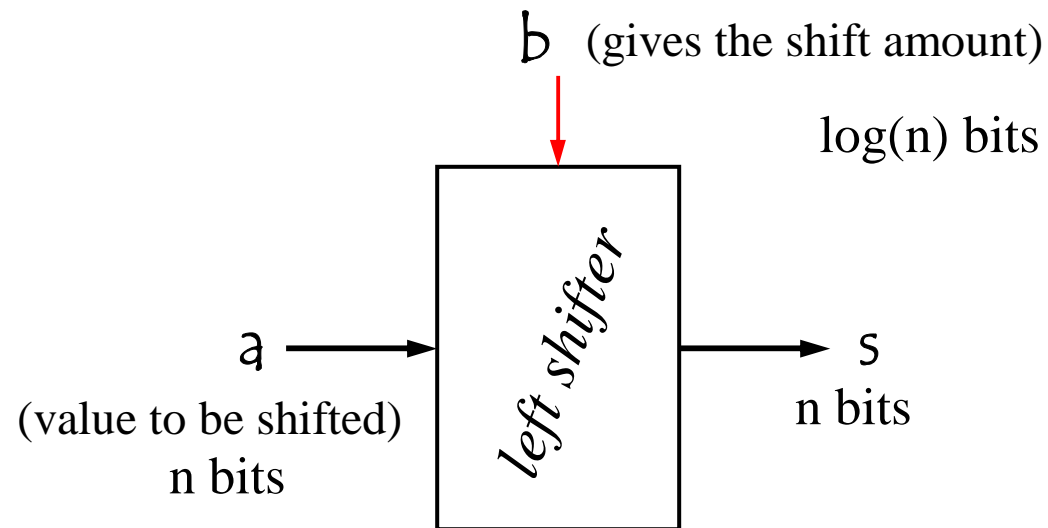
Let consider a value a coded on 8 bits
 a can be rotated to the right by n positions ($0 \leq n < 8$)



Shifters

Shifting a value

Implementation (left shifter)



Shifters

Shifting a value by 1 position to the left

Boolean function

b coded on 1 bit

If $b = 0$

$$s_i = a_i$$

else

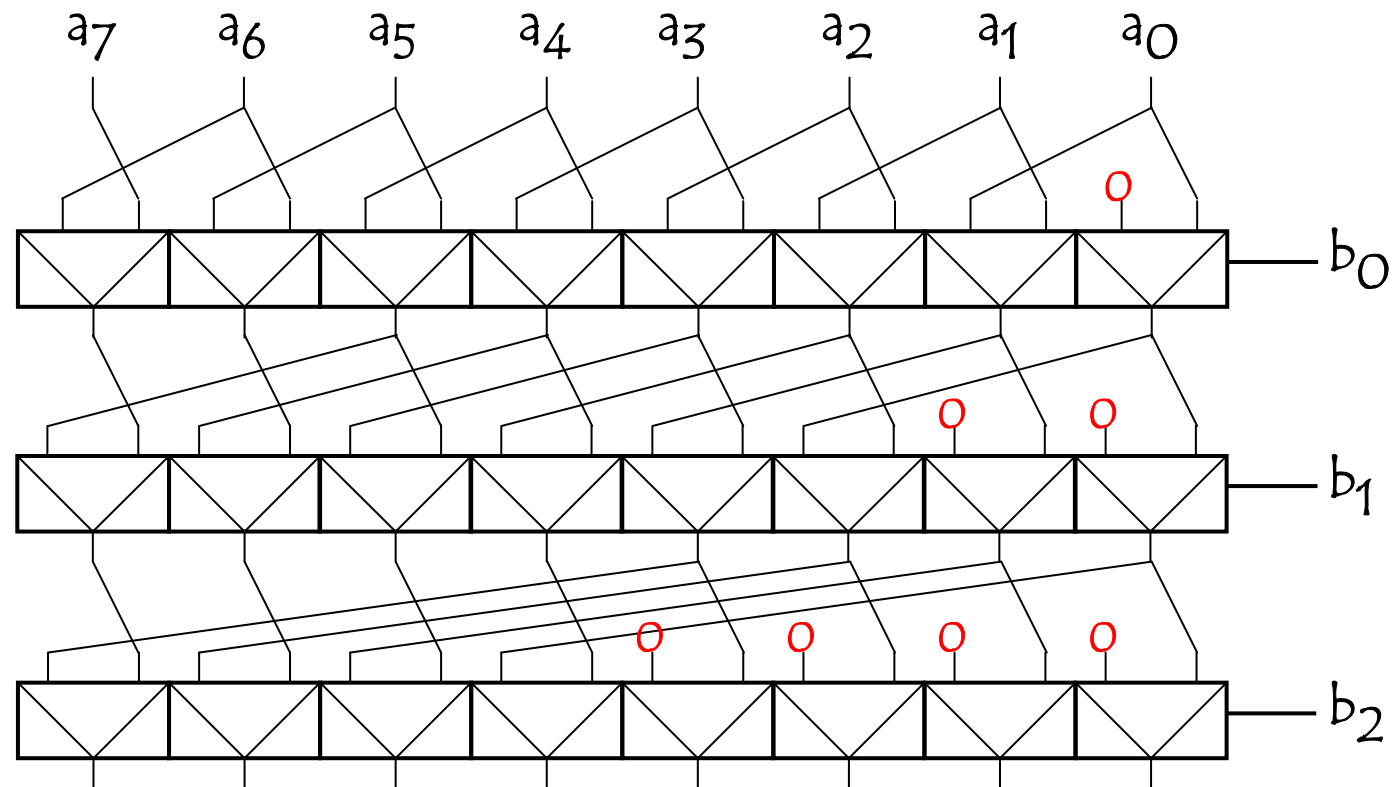
$$s_i = a_{i-1} \quad \text{assuming } a_{-1} = 0$$

$$s_i = b \cdot a_{i-1} + \bar{b} \cdot a_i \quad \Rightarrow \quad \text{2-input Multiplexer}$$



Shifters

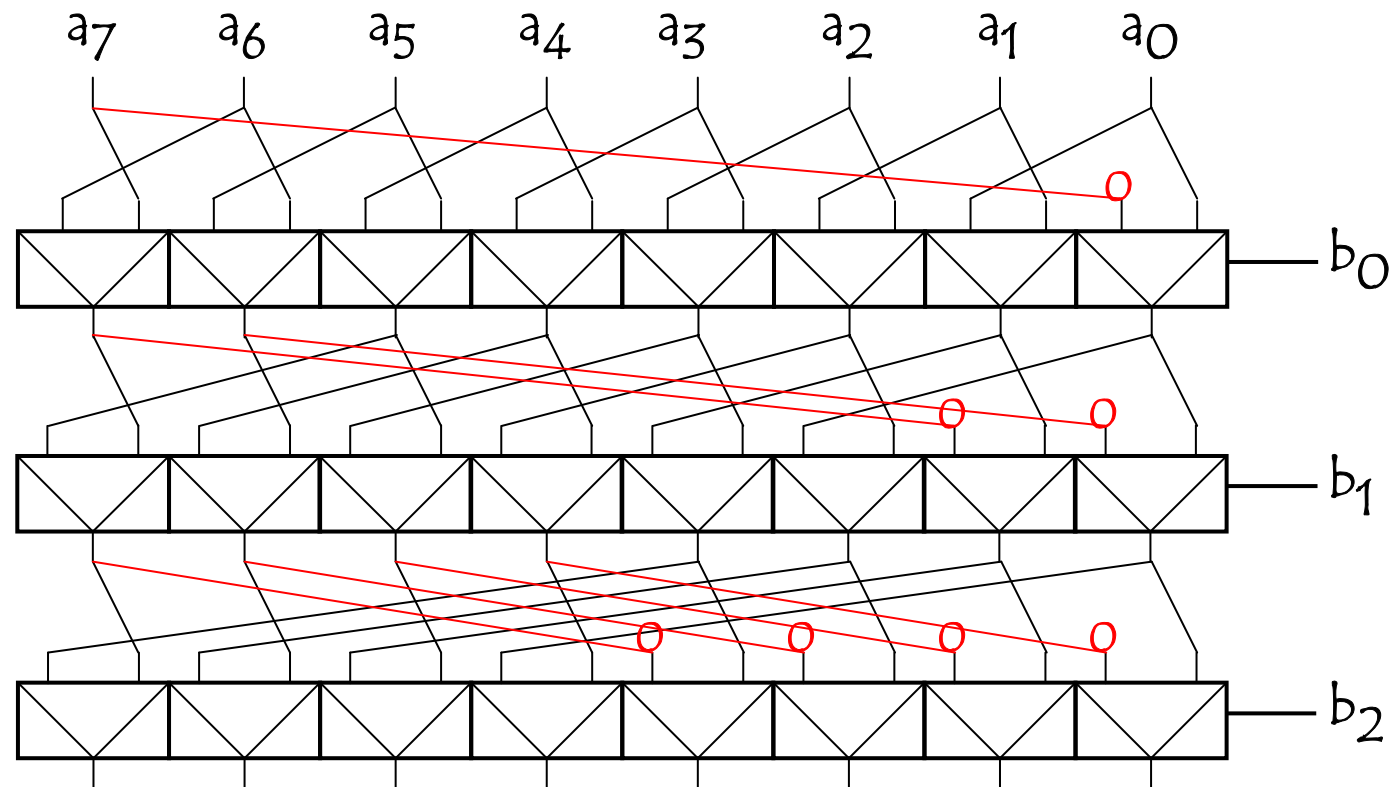
Shifting a value to the left



$\text{Area} \propto n \log(n)$ $\text{Delay} \propto \log(n)$

Shifters

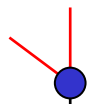
Rotate a value to the left



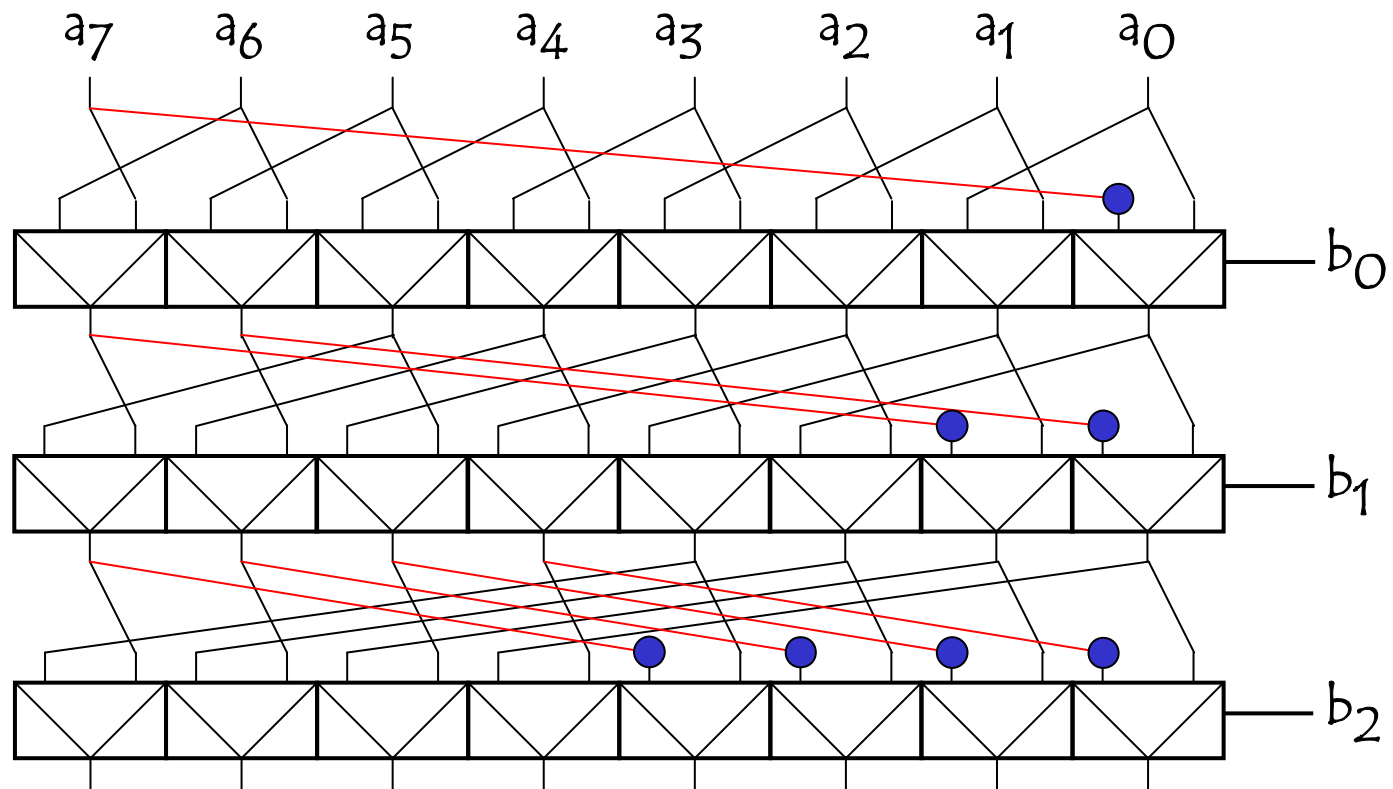
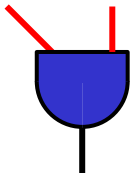
Shifters

Shift / Rotate a value to the left

shift-left



=



Shifters

Shifting a value

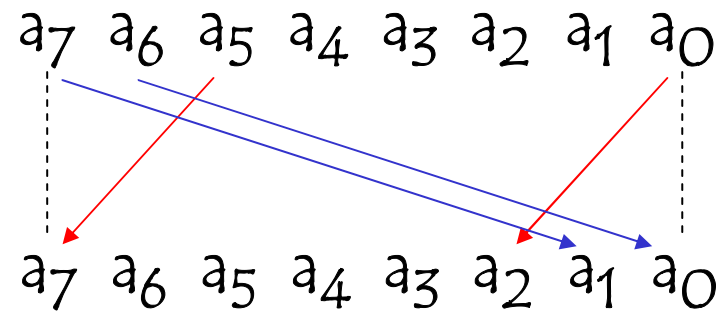
Rotate left by 2

=

Rotate right by 6

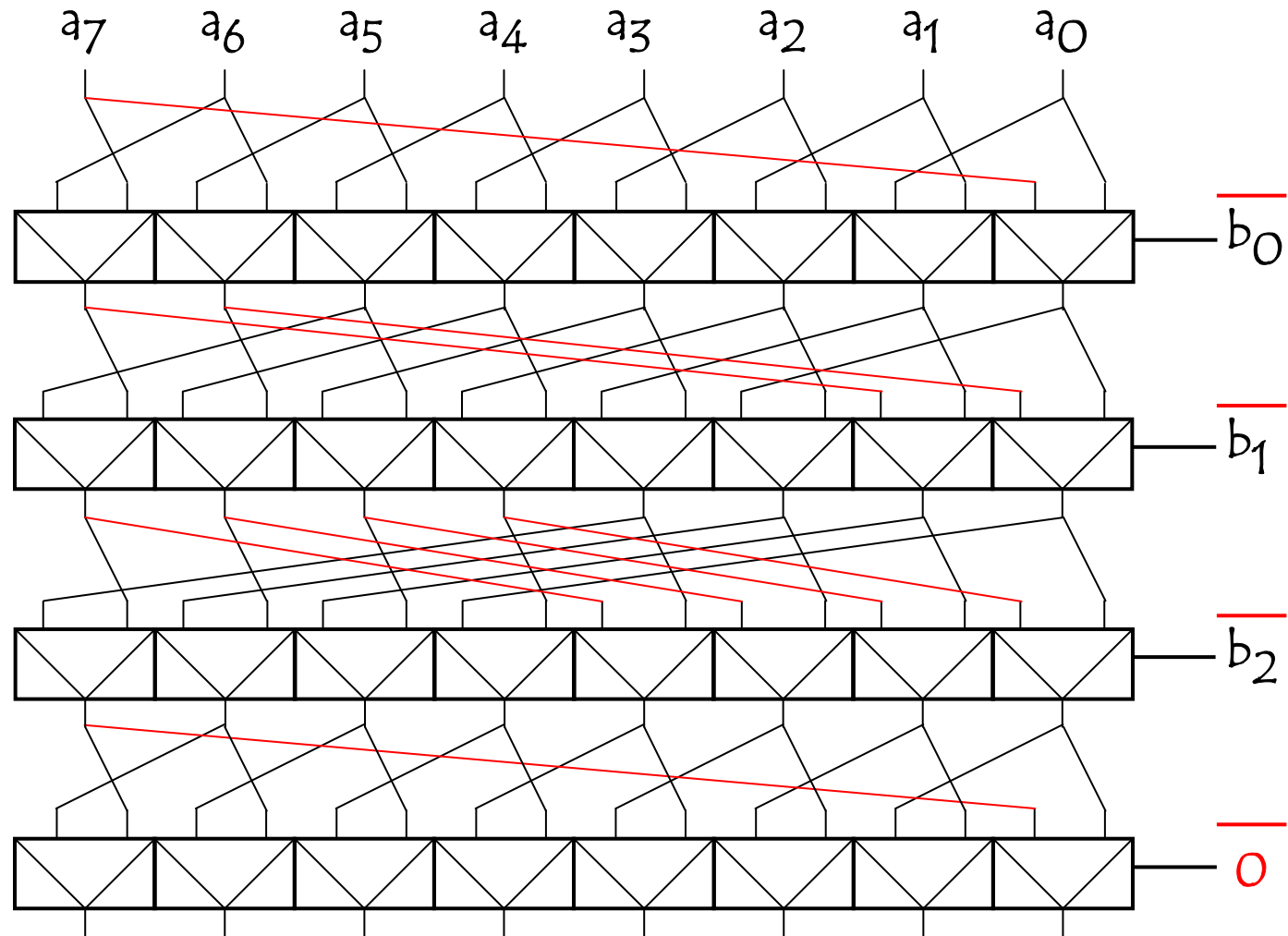
$$6 = -2$$

$$110 = \overline{010} + 1$$



Shifters

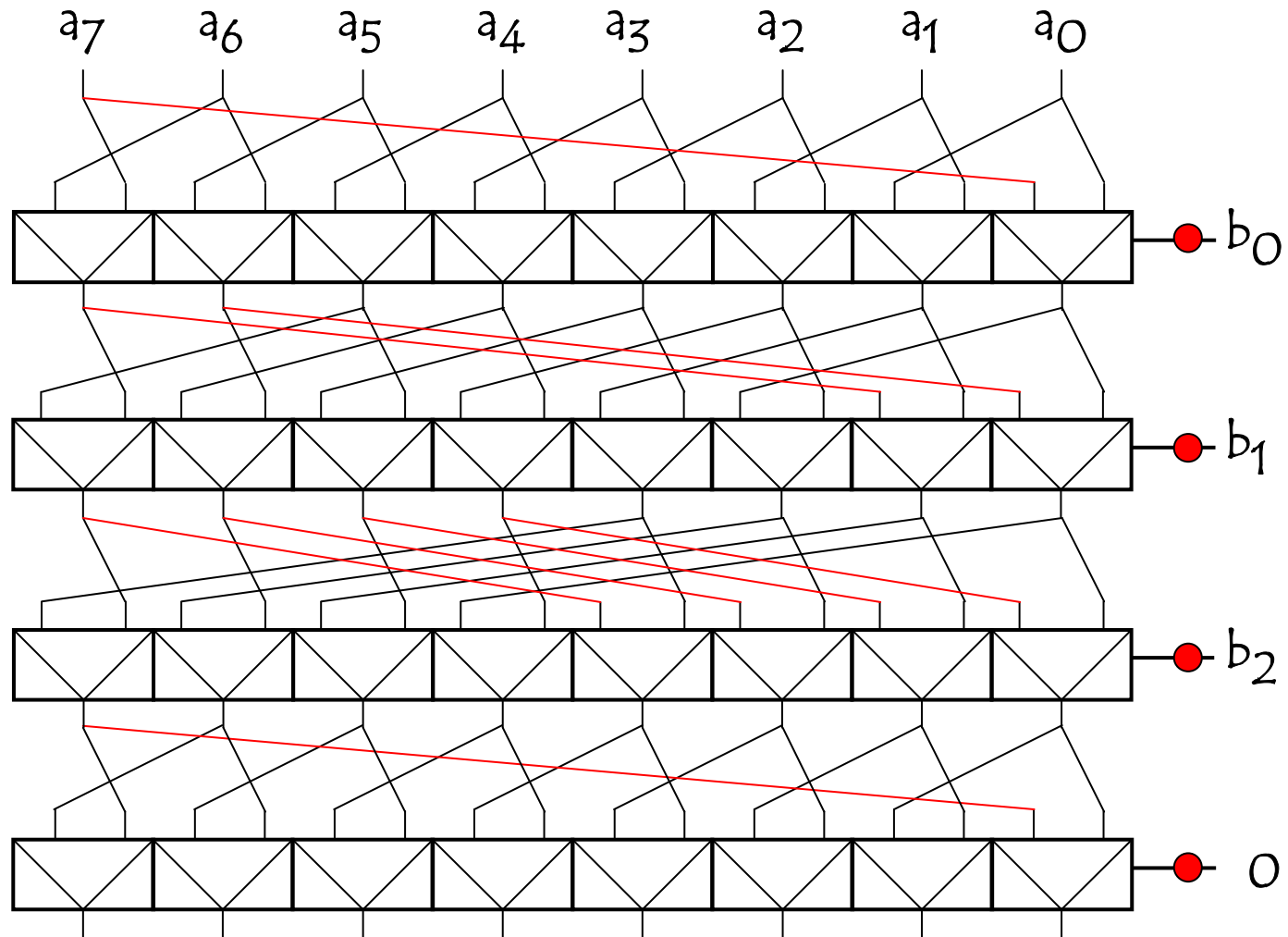
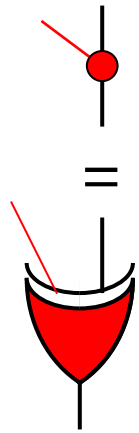
Rotate a value to the **right**



Shifters

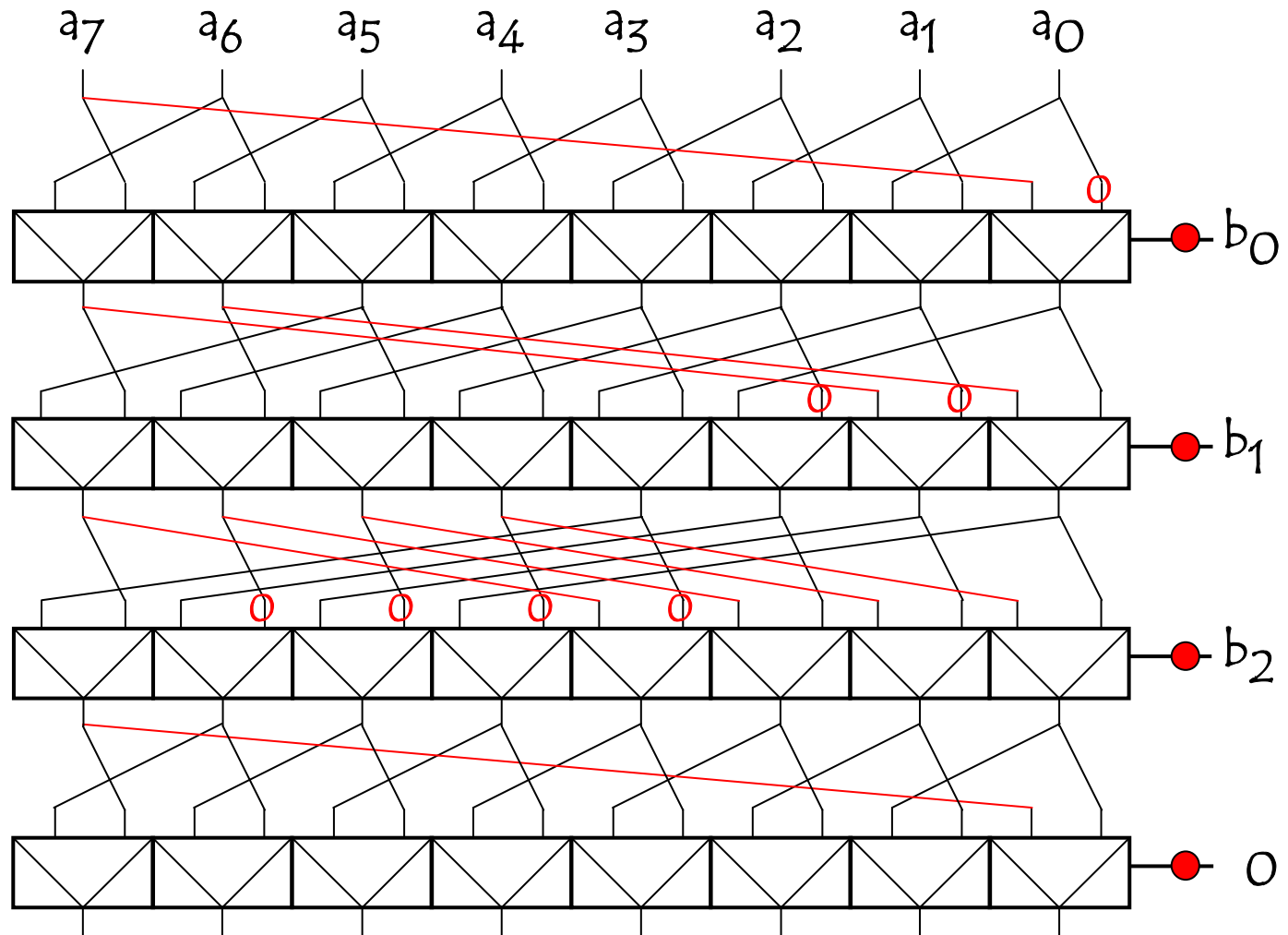
Rotate a value to the **left** / **right**

left



Shifters

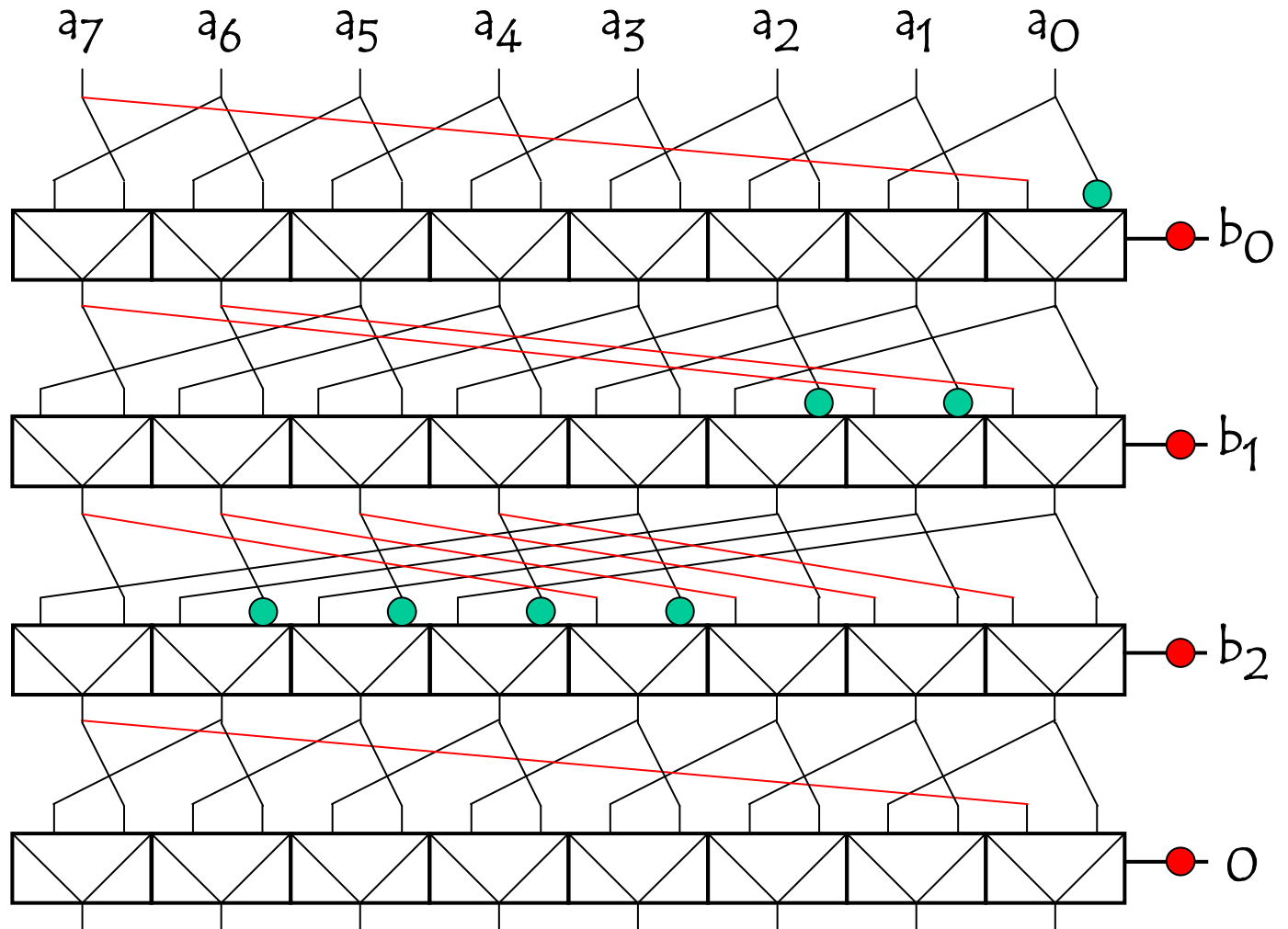
Shift logic a value to the right



Shifters

Shift / Rotate a value to the right

$$\begin{aligned}
 & \text{=} \\
 & \text{shift-r-log.} \\
 & (\text{shift-r-ari. } a_i + \\
 & \text{shift-r-ari. } a_7)
 \end{aligned}$$



Shifters

Shift / Rotate a value to the left / right

