



The Abdus Salam
International Centre for Theoretical Physics



2218-2

Mediterranean School on Nano-Physics
held in Marrakech - MOROCCO

2 - 11 December 2010

Ferroelectrics: Theoretical Concepts and Applications

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Ferroelectrics: Theoretical Concepts and Applications

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Plan du cours

- Dielectric Materials
 - Local Field
 - Dielectric Relaxation
- Ferroelectric Materials
- Landau Theory
- Applications



Dielectric Materials

- Definition of permittivity

$$\vec{P} = \epsilon_0 \chi \vec{E}_{\text{ext}}$$

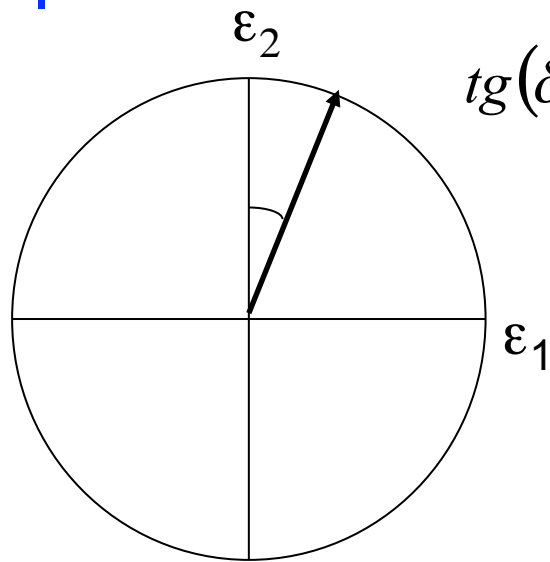
$$\vec{D} = \epsilon_0 \vec{E}_{\text{ext}} + \vec{P}$$

$$\epsilon_r = \epsilon_0 (1 + \chi)$$

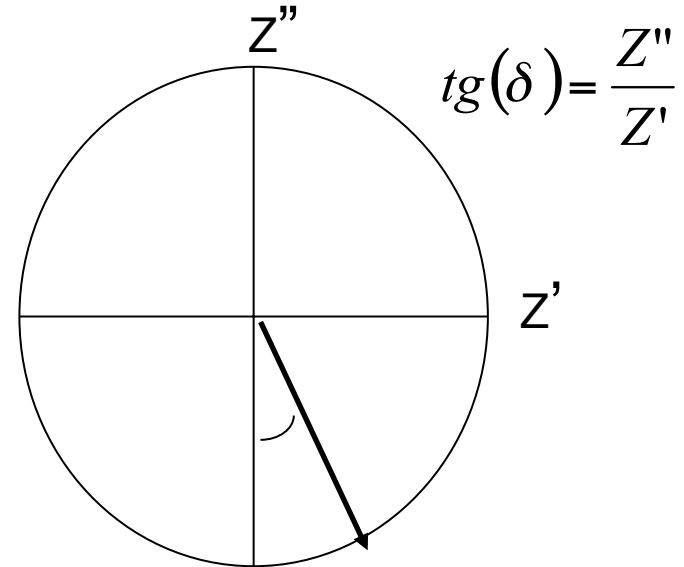
- Complex representation

$$\epsilon_r^*(\omega) = \epsilon_{1r}(\omega) + j\epsilon_{2r}(\omega)$$

Dielectric Materials



$$\operatorname{tg}(\delta) = \frac{\varepsilon_2}{\varepsilon_1}$$



$$\operatorname{tg}(\delta) = \frac{Z''}{Z'}$$

$$\frac{1}{Z} = jC^* \omega \quad \text{avec} \quad C^*(\omega) = \varepsilon^*(\omega) C_0$$

$$\frac{1}{Z} = jC\omega + \frac{1}{R} = j\omega \chi_1(\omega) C_0 + \omega \varepsilon_2(\omega) C_0$$



Dielectric Materials

$$\vec{P} = \frac{d\vec{p}}{d\tau}$$

$$\mathbf{P} = \mathbf{P}_e + \mathbf{P}_a + \mathbf{P}_d + \mathbf{P}_c$$

Electronic Polarization : \mathbf{P}_e

Atomic or ionic polarization: \mathbf{P}_a

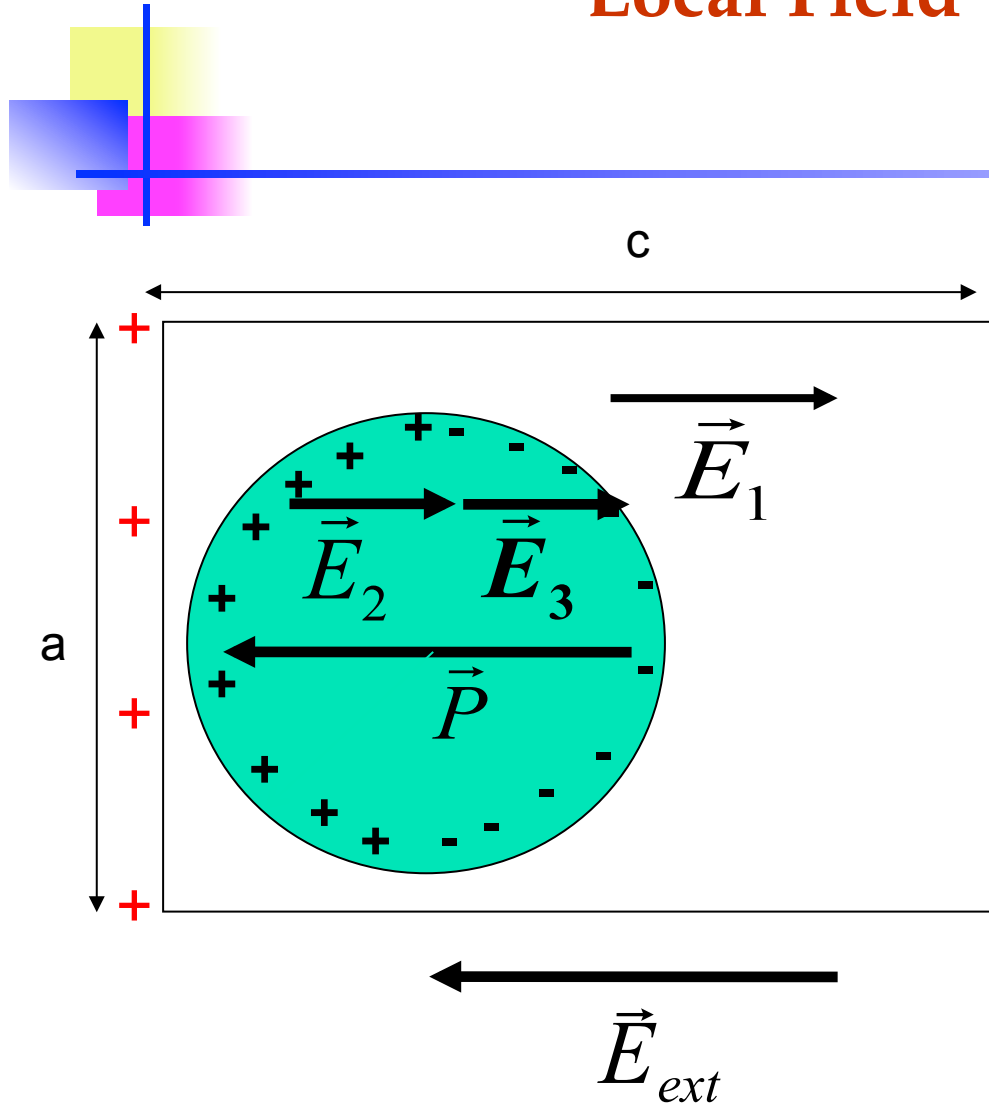
Dipolar polarization: \mathbf{P}_d

Space charge polarization : \mathbf{P}_c



Dielectric Materials

Local Field



$$\vec{E}_{Loc} = \vec{E}_{ext} + \vec{E}_1 + \vec{E}_2 + \vec{E}_3$$

\vec{E}_{ext} = External field

\vec{E}_1 = Depolarizing field due to charge density

\vec{E}_2 = Lorentz field

\vec{E}_3 = field of atoms inside the cavity

Local Field

Determination of E_1

Materials that have an ellipsoidal shape have an important property:

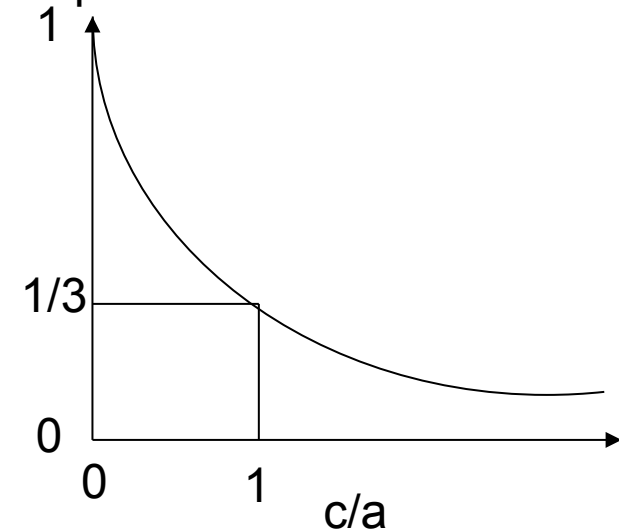
- When the field is uniform, the polarization is uniform.
- Polarization is related to the field by the depolarization factor N :

$$E_{1x} = -\frac{N_x P_x}{\epsilon_0}; E_{1y} = -\frac{N_y P_y}{\epsilon_0}; E_{1z} = -\frac{N_z P_z}{\epsilon_0}$$

$$- N_x + N_y + N_z = 1$$

Rq: N_x , N_y et N_z are the depolarization factors, their values depend on the relationship between the principal axes of the ellipsoid.

Facteur de dépolarisation

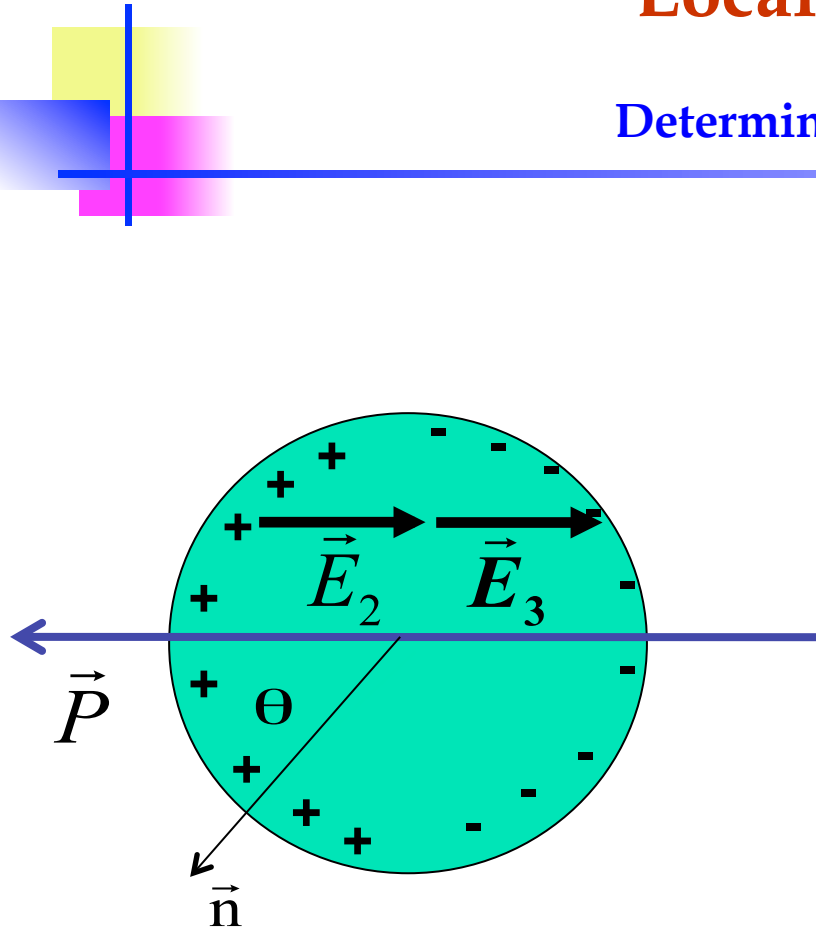


For a sphere $N_x=N_y=N_z = 1/3$

$$E_1 = -\frac{P}{3\epsilon_0}$$

Local Field

Determination of E2



$$\sigma_p = \vec{P} \cdot \vec{n} = P \cos \theta$$

$$E_2 = \frac{1}{4\pi\epsilon_0} \iint \frac{\sigma ds}{r^2} \cos \theta$$

$$E_2 = \frac{1}{4\pi\epsilon_0} \int_0^\pi \frac{P \cos \theta}{a^2} a^2 \cdot 2\pi \cdot \sin \theta \cdot \cos \theta \cdot d\theta$$

$$E_2 = \frac{P}{2\epsilon_0} \int_0^\pi \cos^2 \theta \cdot \sin \theta \cdot d\theta = \frac{P}{3\epsilon_0}$$

Local Field

Determination of E3

Field E3, due to the dipoles inside the cavity is the only term that depend of the crystal structure. It has been shown that for a reference site with a cubic environment into a sphere, that:

$$E_3 = 0$$

$$E_{\text{Loc}} = \underbrace{E_0 + E_1}_{E_{\text{Mac}}} + E_2 + E_3 = E_{\text{Mac}} + \frac{P}{3\epsilon_0}$$

Local Field

Clausius Mossotti law

The polarization is expressed in terms of local field by :

$P = N\alpha E_{loc}$: which is the microscopic representation

α : is the polarizability of material

- In the case of a material with several kind of atom j , the polarization is written:

$$P = \sum_j N_j \alpha_j E_{loc}(j) = \sum_j N_j \alpha_j \left(E_{Mac} + \frac{P}{3\epsilon_0} \right) \Rightarrow$$

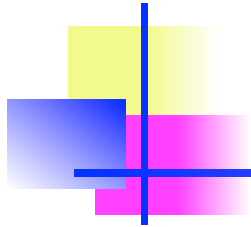
$$\chi = \epsilon_r - 1 = \frac{P}{\epsilon_0 E_{Mac}} = \frac{\sum_j N_j \alpha_j}{\epsilon_0 - \frac{1}{3} \sum_j N_j \alpha_j}$$

$$\text{On a } \chi = \epsilon_r - 1 \Rightarrow \frac{\epsilon_r - 1}{\epsilon_r + 2} = \frac{1}{3\epsilon_0} \sum_j N_j \alpha_j : \text{Clausius Mossotti relation}$$



Catastrophe de Mossotti

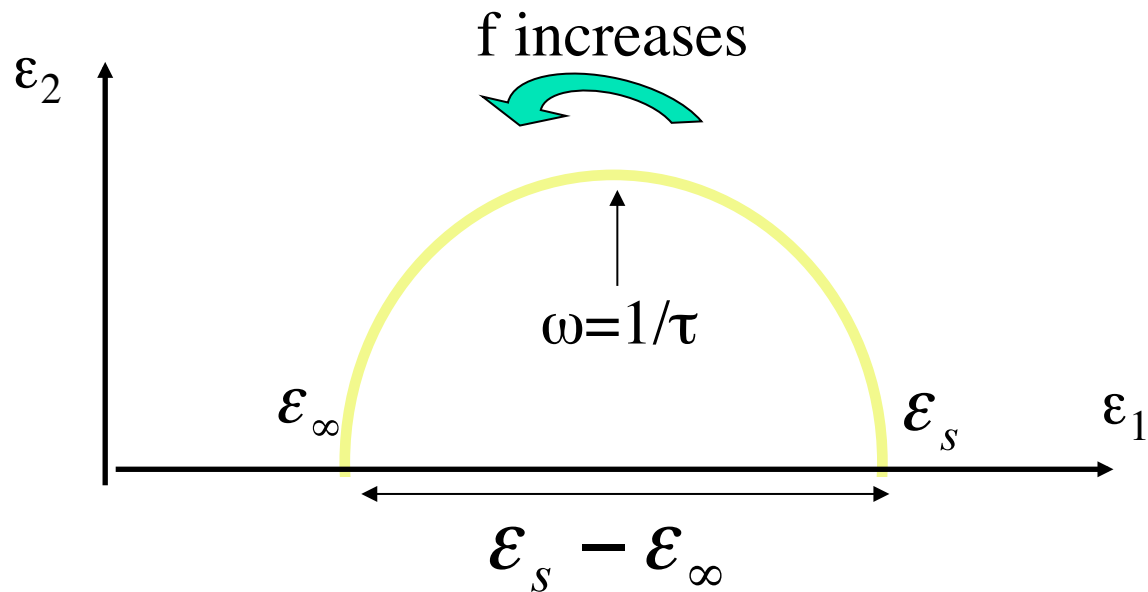
Dielectric Relaxation



Dielectric Relaxation

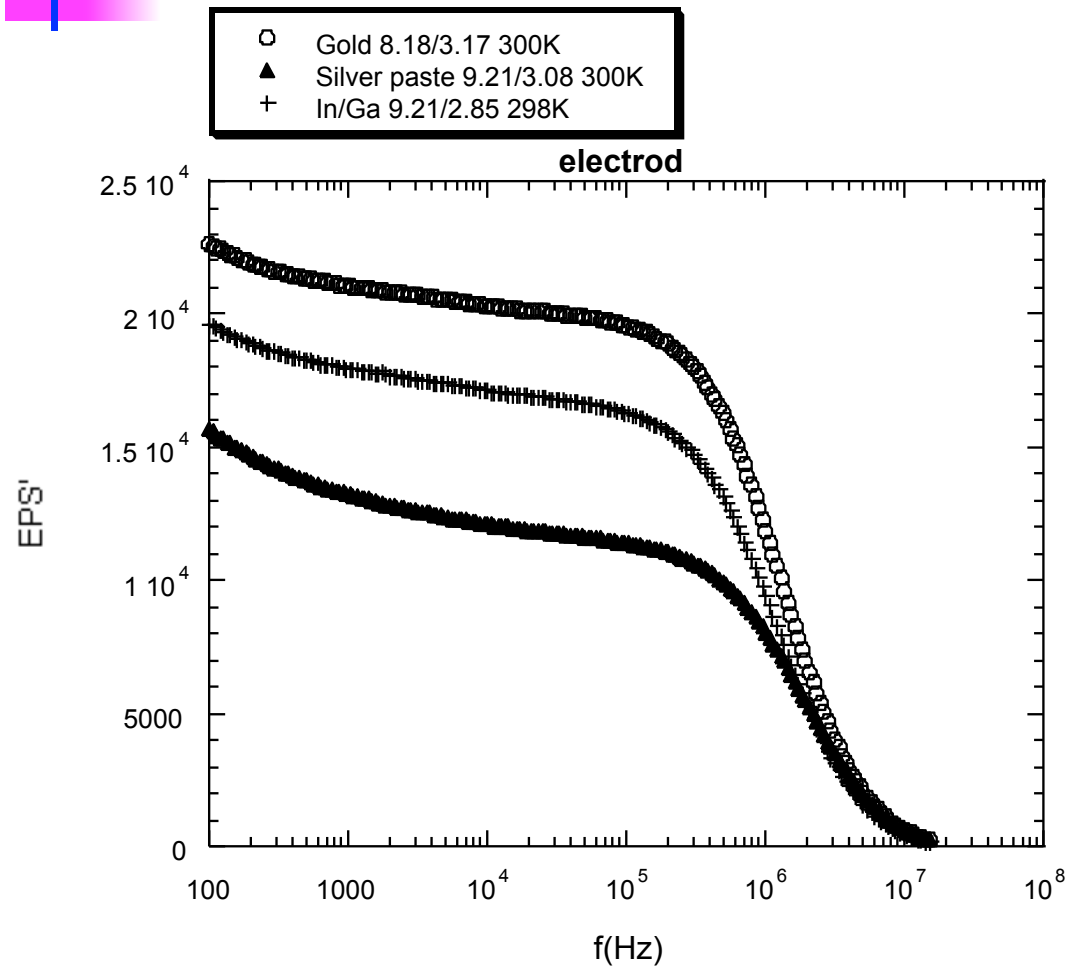
Cole-Cole Diagram

$$\varepsilon^*(\omega) = \varepsilon_1(\omega) - j\varepsilon_2(\omega) = \varepsilon_\infty + \frac{\varepsilon_s - \varepsilon_\infty}{1 + j\omega\tau}$$



Dielectric Relaxation

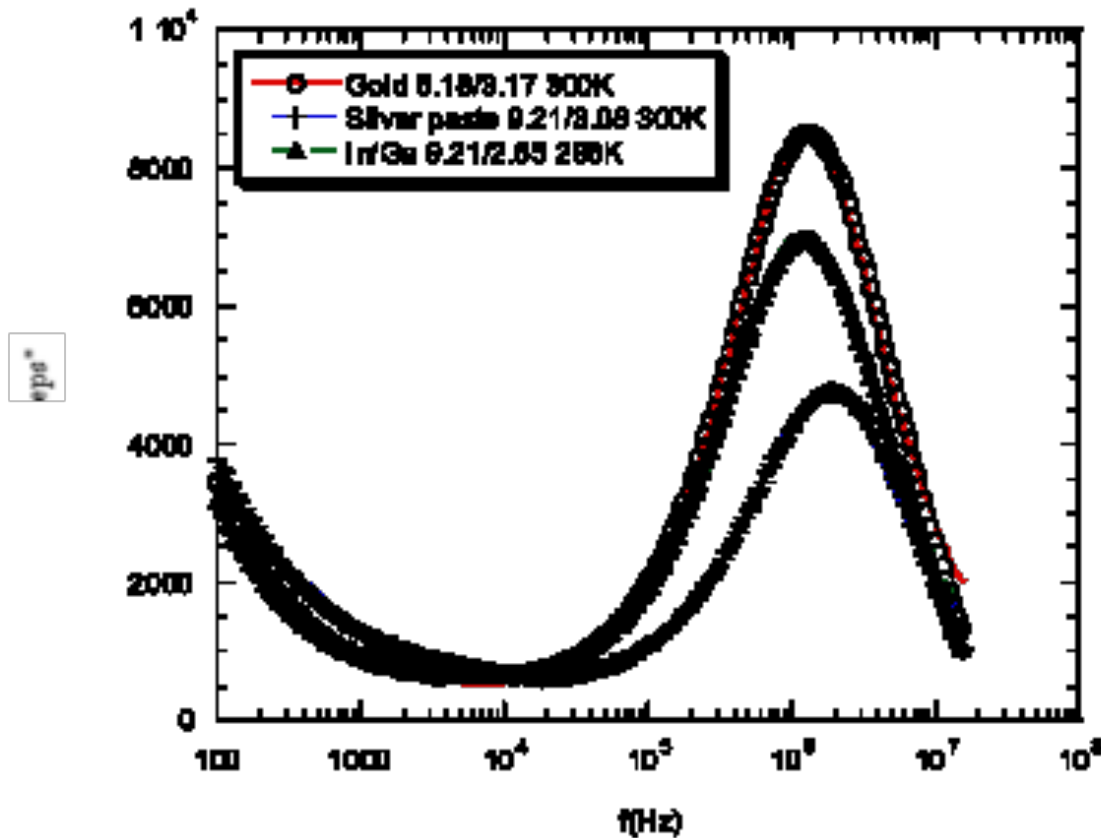
Real part of permittivity



$$\epsilon_1 = \epsilon_\infty + \frac{\epsilon_s - \epsilon_\infty}{1 + \omega^2 \tau^2}$$

Dielectric Relaxation

Imaginary part

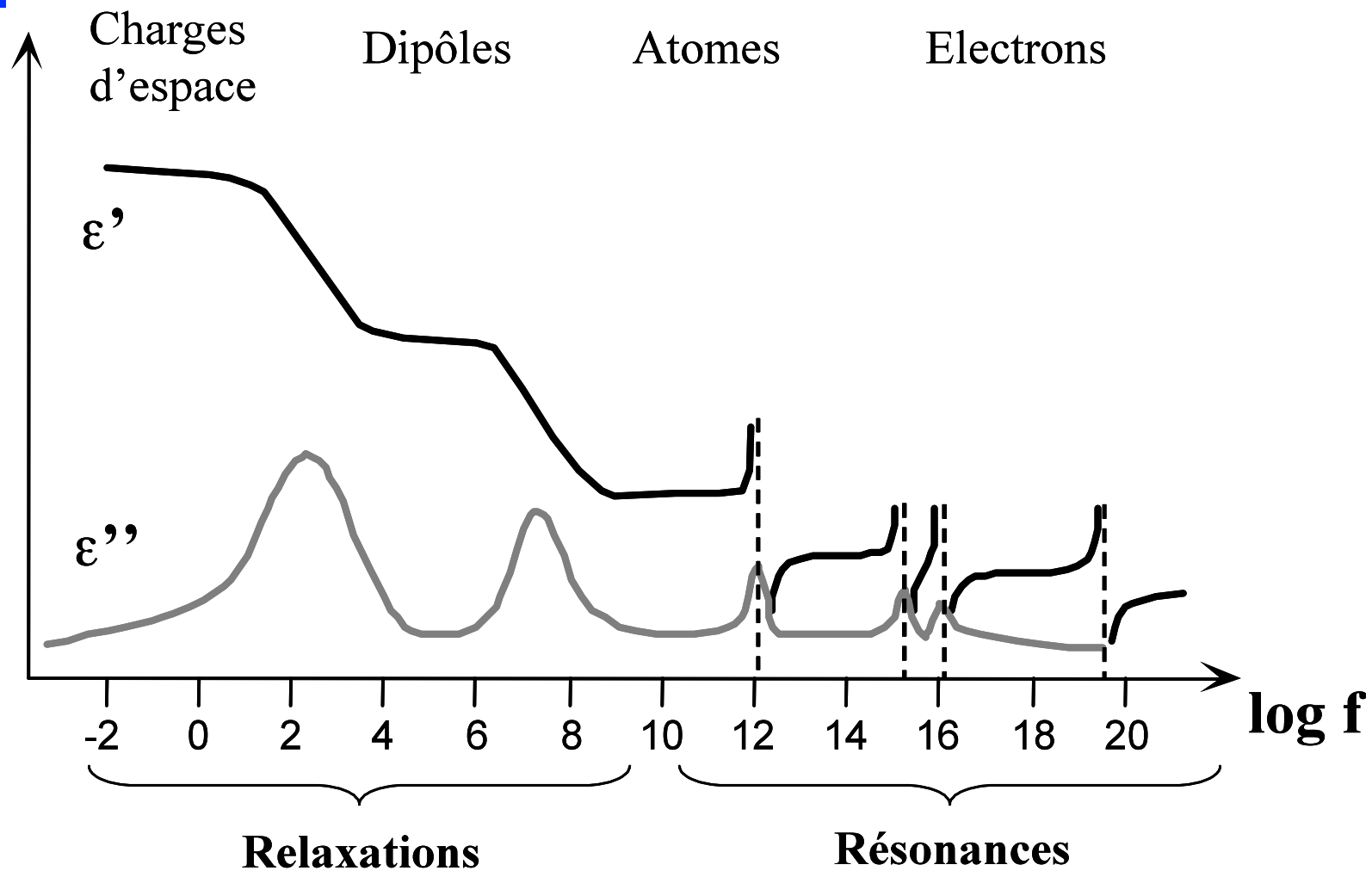


$$\epsilon_1 = \frac{(\epsilon_s - \epsilon_\infty)\omega\tau}{1 + \omega^2\tau^2}$$

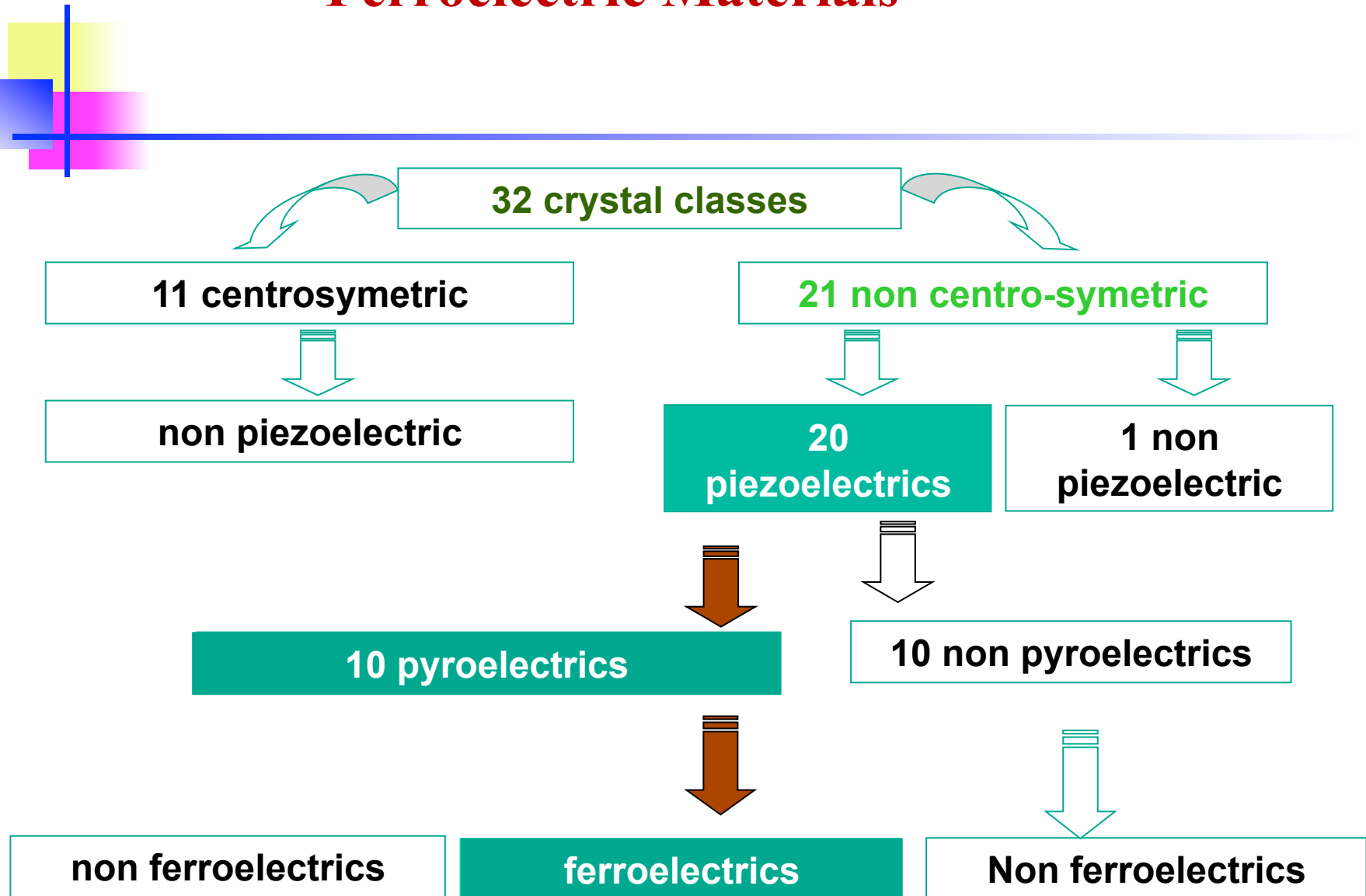
Effet de la température et de la conductivité

Dielectric Relaxation

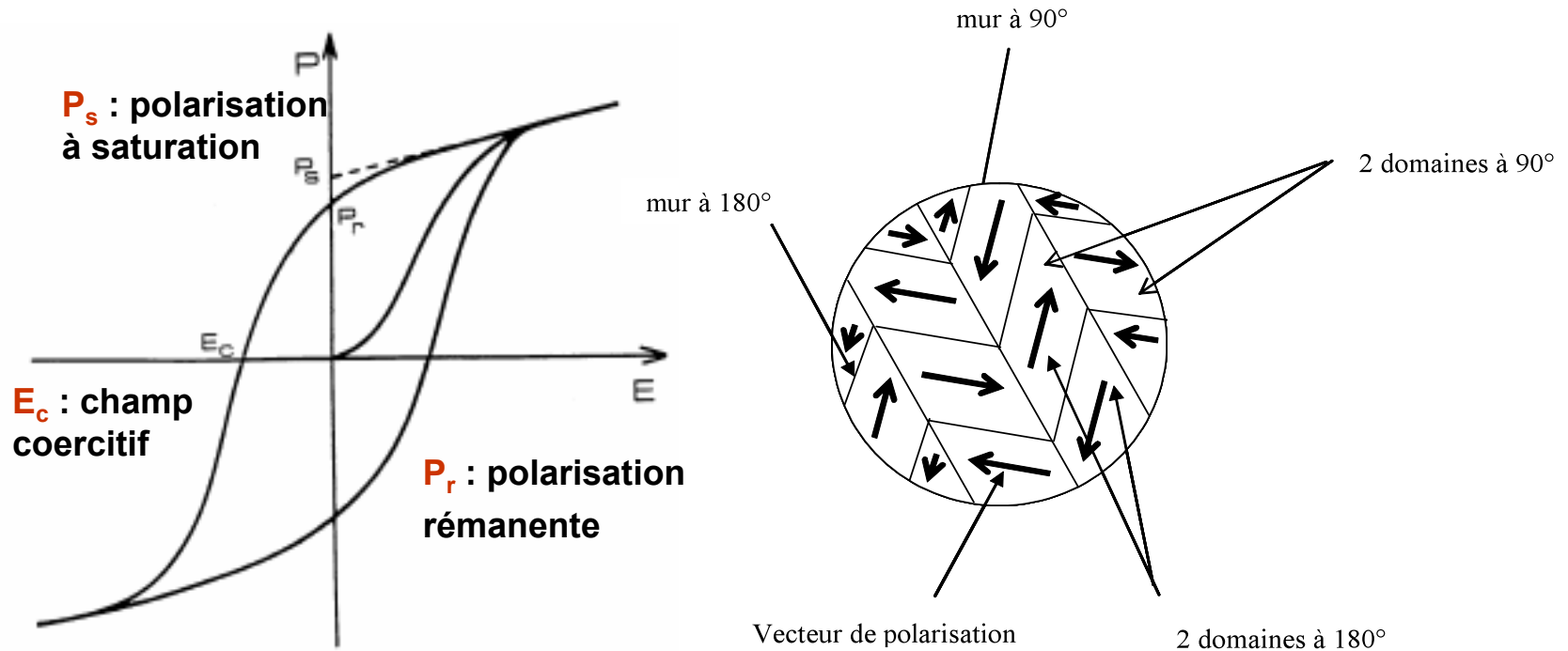
Relaxation and resonance



Ferroelectric Materials

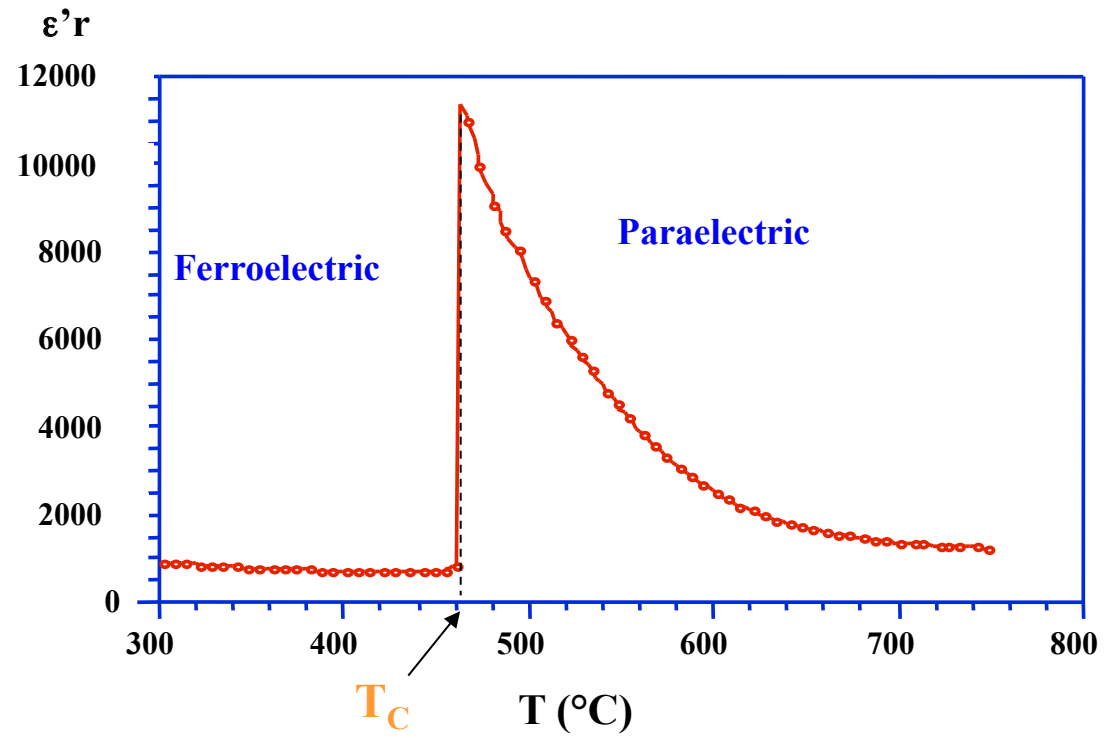


Ferroelectric Materials

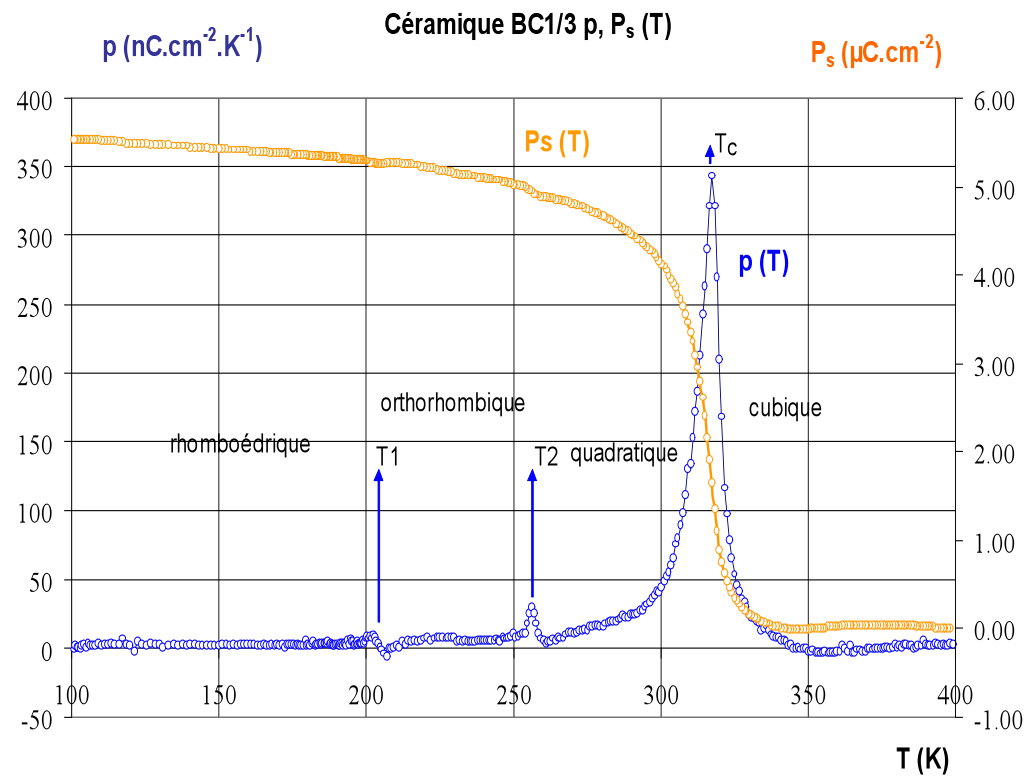


Hysteresis Loop

Ferroelectric Materials



Pyroelectric parameter



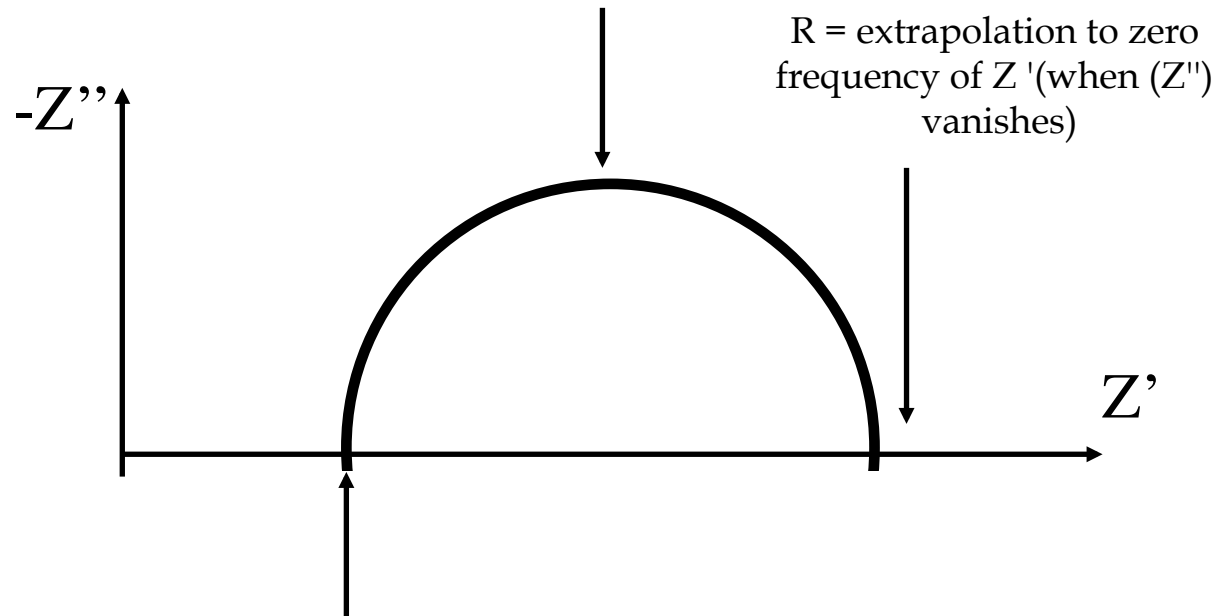
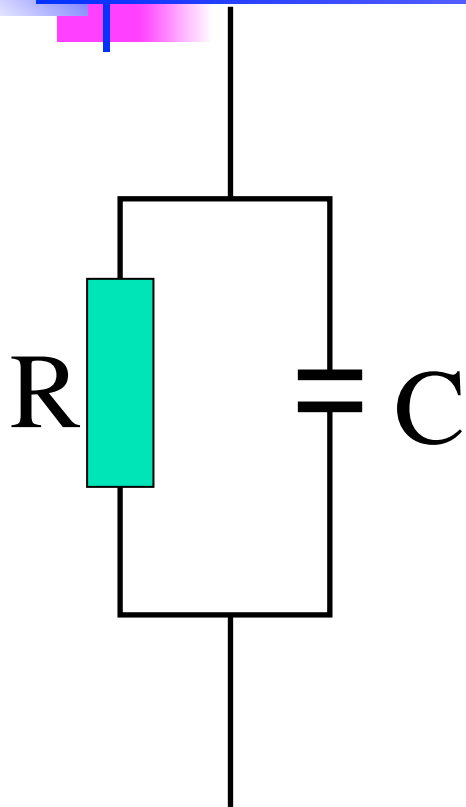
Le courant de polarisation est proportionnel à la vitesse de changement de température et au signe de variation

$$i = \frac{\partial P}{\partial t} = \frac{\partial P}{\partial T} \frac{\partial T}{\partial t} = \frac{\partial P}{\partial T} k$$

Impedance of a circuit R, C parallel

if R and C are independent of frequency,
we write $Z = Z' - jz''$

at the top of the semicircle we have: $RC = 1/\omega$



When the frequency tends towards infinity (Z'') vanishes

Landau Theory

Landau Model for ferroelectric-paraelectric phase transition

$$F = F_0 + \alpha P^2 + \beta P^4 + \gamma P^6 - \vec{P} \cdot \vec{E}$$

$$\alpha = k(T - T_C); \text{ k is a constant}$$

$$\text{at thermodynamic equilibrium: } \frac{\partial F}{\partial P} = 0$$

$$\Rightarrow 2k(T - T_C)P + 4\beta P^3 - E = 0$$

When $E = 0$ and according to the temperature T , the crystal is in a state of stable equilibrium if :

- For $T > T_C$, the polarization is zero ($P = 0$), its the paraelectric state $\Rightarrow \frac{\partial F}{\partial P} = 0 \Rightarrow P = 0$

- For $T < T_C$, the polarization is nonzero ($P \neq 0$), its the ferroelectric state $\Rightarrow \frac{\partial F}{\partial P} = 0$

$$\Rightarrow 2k(T - T_C)P + 4\beta P^3 = 0 \quad (E = 0)$$

$$P = \pm \sqrt{-\frac{k}{2\beta}(T - T_C)} : \text{ This assumption corresponds to a second order transition}$$

Landau Theory

Curie Weiss law

$$\frac{\partial F}{\partial P} = 0 \Rightarrow 2k(T - T_C)P + 4\beta P^3 - E = 0$$

$$E = 2k(T - T_C)P + 4\beta P^3$$

or on a : $P = (\epsilon - \epsilon_0)E \Rightarrow dP = \epsilon dE$

$$\frac{1}{\epsilon} = \frac{\partial E}{\partial P} = 2k(T - T_C) + 12\beta P^2 \text{ (paraelectric phase), the stable state is obtained for } P = 0$$

$$\Rightarrow \frac{1}{\epsilon} = 2k(T - T_C) = C(T - T_C) \text{ with } k_C = 2k$$

$$\epsilon = \frac{C}{T - T_C} \text{ avec } C = \frac{1}{k_C} : \text{ Curie Weiss law}$$

Landau Theory

Curie Weiss law

- in the ferroelectric state, the equilibrium state is obtained for:

$$P = \pm \sqrt{-\frac{k}{2\beta}(T - T_C)}$$

$$\Rightarrow \frac{\partial F}{\partial P} = 0 \Rightarrow \frac{1}{\epsilon} = \left. \frac{\partial E}{\partial P} \right|_T = 2k(T - T_C) + 12\beta P^2$$

$$\frac{1}{\epsilon} = 2k(T - T_C) + 12\beta \frac{k}{2\beta}(T_C - T)$$

$$= 2k(T - T_C) + 6k(T_C - T)$$

$$= 4k(T_C - T) = 2k_C(T_C - T) = C'(T_C - T)$$

$$\text{with } C' = \frac{1}{2k_C} = \frac{1}{2}C$$

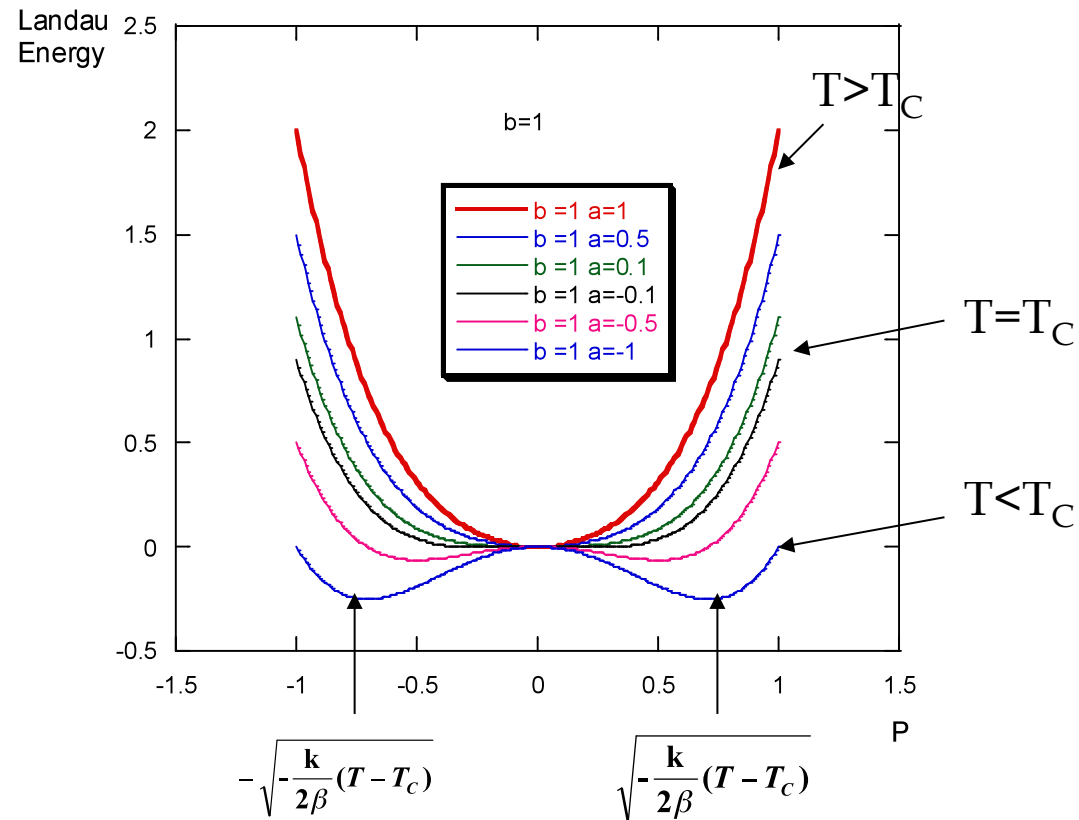
$$\text{so } \epsilon = \frac{\frac{1}{2}C}{T_C - T} \quad \text{et} \quad \epsilon = \frac{C'}{T_C - T}$$

\Rightarrow the variation of $\frac{1}{\epsilon}$ is linear in the ferroelectric phase but

the slope is less than that of the paraelectric phase ($C' = \frac{1}{2}C$)

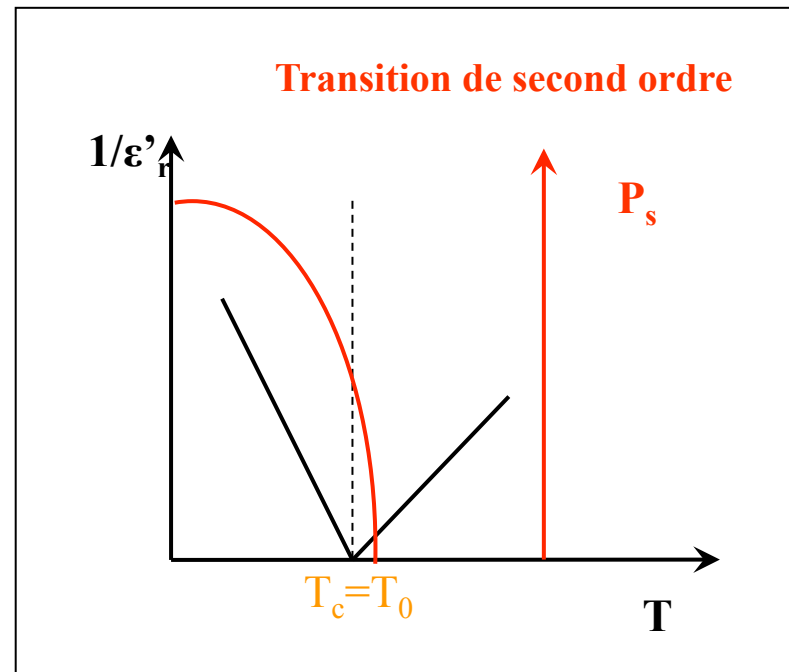
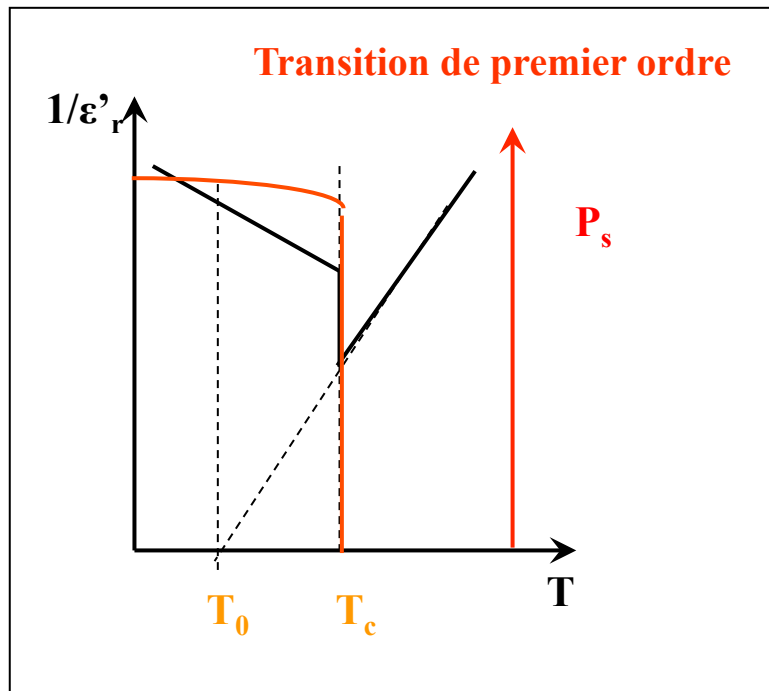
Landau Theory

Free energy : Second order transition



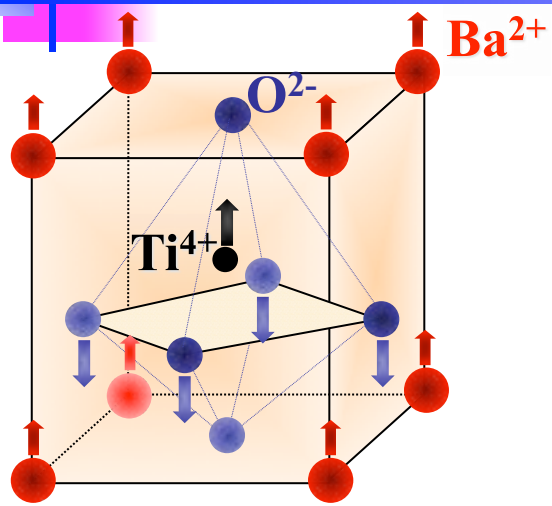
Landau Theory

Variation of permittivity and polarization

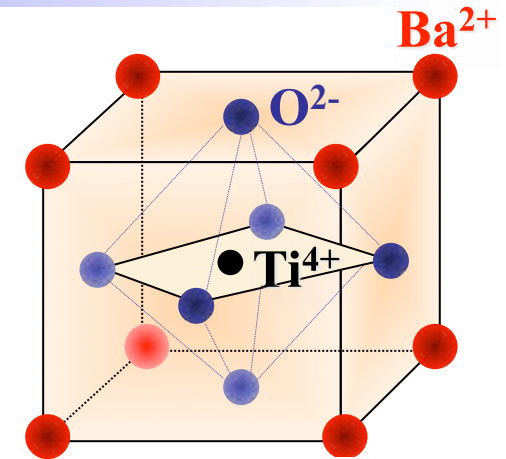


Applications

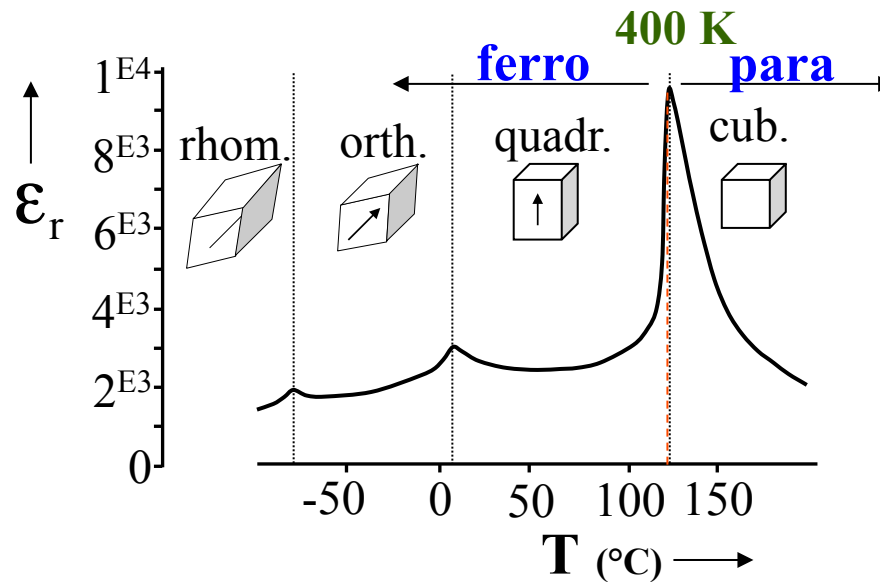
Perovskite structure (ABO₃): BaTiO₃



$T < T_c$: P4mm

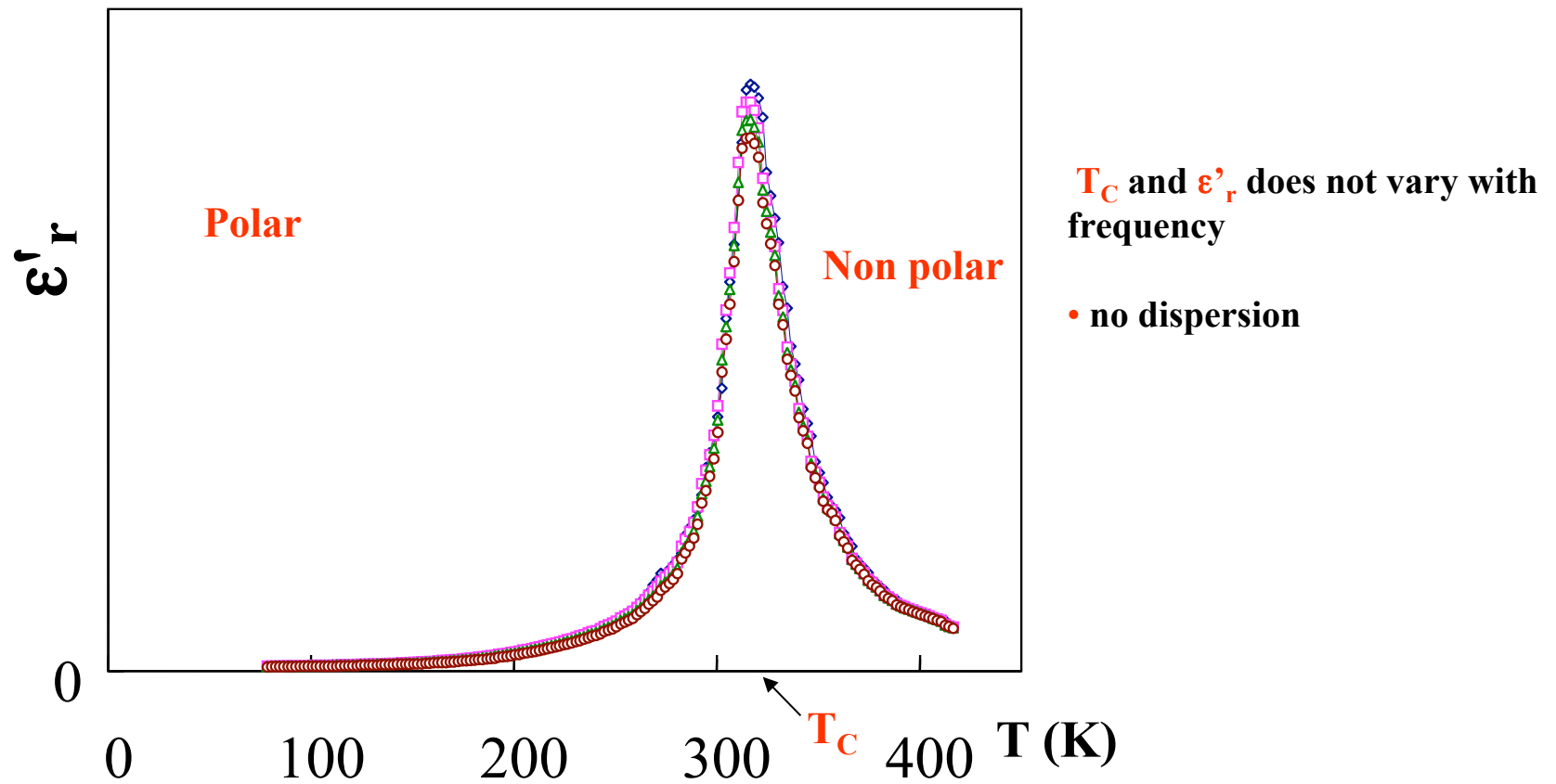


$T > T_c$: Pm3m



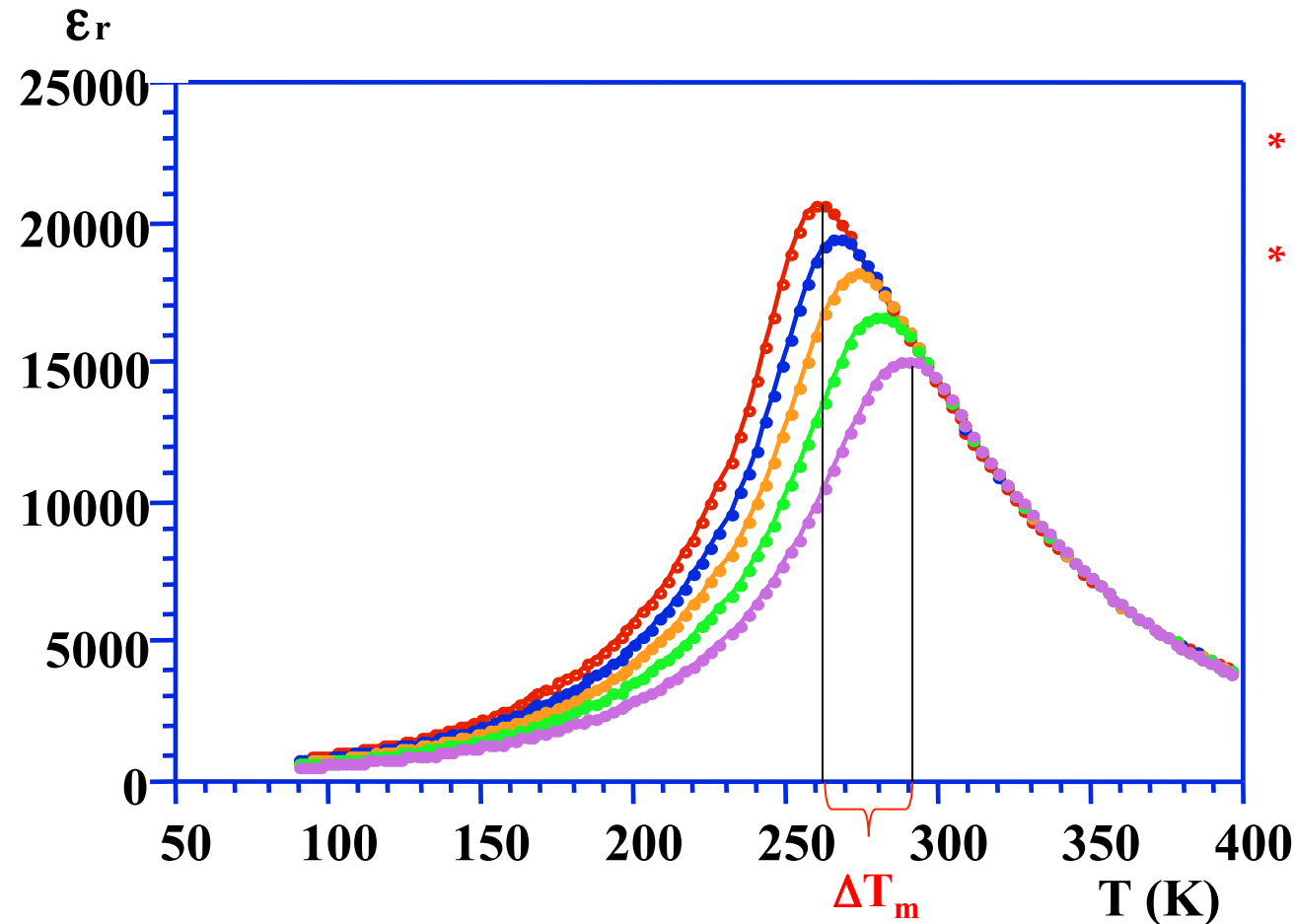
Applications

Classic Ferroelectric



Applications

Relaxor



- * Dispersion in ferroelectric state
- * Variation of T_m with frequency

Applications

Relaxor

Local fluctuation of composition in the A site caused by the substitution



Nanometric areas

