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Slow noise in Landau-Zener theory

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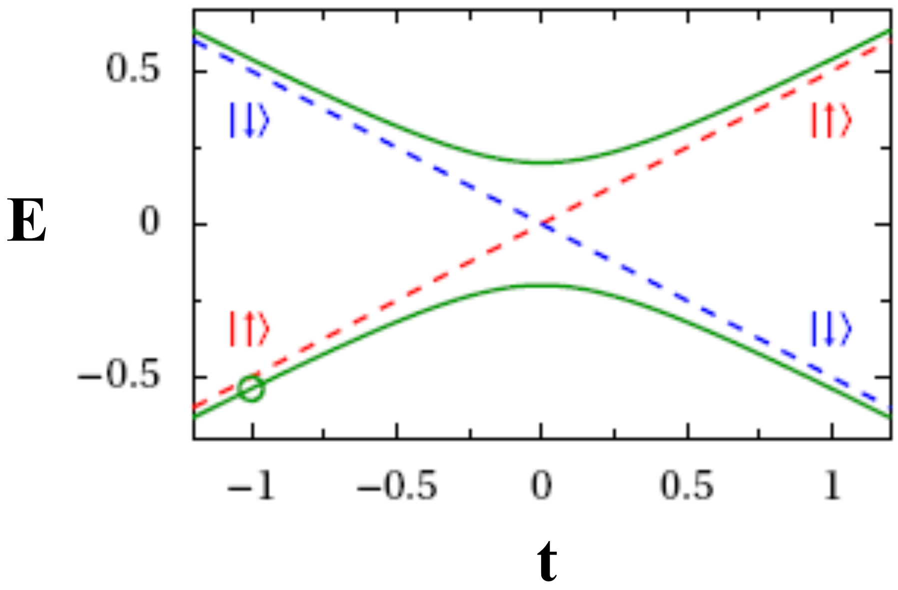
Slow noise in Landau-Zener theory

- classical Gaussian noise
- off-diagonal (transverse noise)
- noise-assisted vs noise- induced LZ transitions

Marrakech, 2 - 11 December

Landau-Zener transition



time-dependent two-level system

$$H = \frac{vt}{2}\sigma^z + J\sigma^x$$

- diabatic states: $|\uparrow\rangle, |\downarrow\rangle$
- adiabatic states

$$P_{\uparrow\to\downarrow} = 1 - \exp\left(-\frac{2\pi J^2}{v}\right)$$

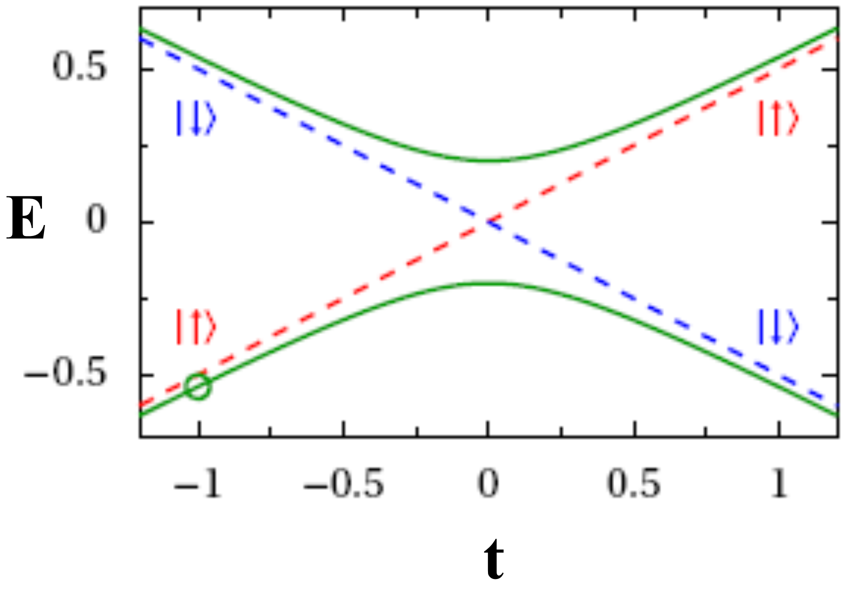
initial state: $|\psi(t=-\infty)\rangle = |\uparrow\rangle$

? time evolution

? spin-flip probability $P_{\uparrow\to\downarrow}$

L.D. Landau, 1932
C. Zener, 1932,
E. Majorana, 1932
E.C.G. Stückelberg, 1932

Times scales of the LZ problem



$$H = \frac{vt}{2}\sigma^z + J\sigma^x$$

Landau-Zener time

$\tau_{LZ} = J/v$ **adiabatic transition**

$\tau_{LZ} = 1/\sqrt{v}$ **sudden transition**

$\tau_c = 1/J$ **"collision" time**

dimensionless parameter

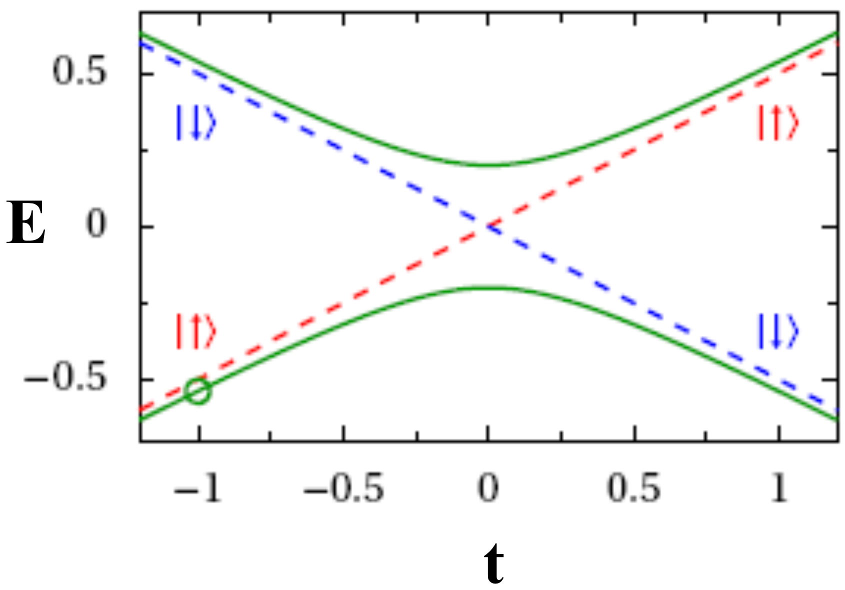
$$\nu = \frac{2\pi J^2}{v}$$

$$P_{\uparrow\rightarrow\uparrow} = \exp\left(-\frac{2\pi J^2}{v}\right)$$

- $\exp(-2\pi\tau_{LZ}/\tau_c)$ **adiabatic**
- $\exp(-2\pi\tau_{LZ}^2/\tau_c^2)$ **sudden**

Mullen, Ben-Jacob, Gefen, Schuss, 1989

LZ transition: fast and slow coloured classical noises



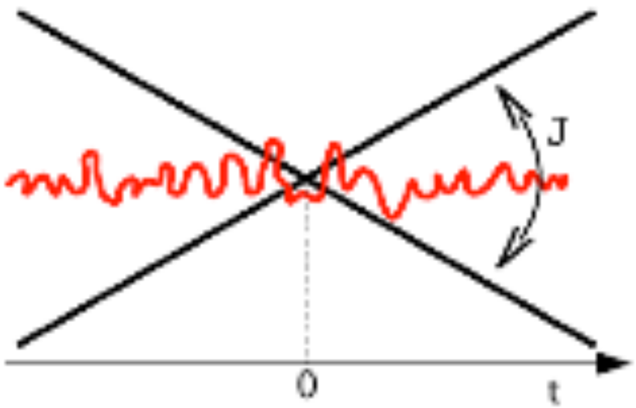
$$H = \frac{vt}{2}\sigma^z + \mathcal{J}^\alpha \sigma^\alpha$$

$\alpha = x$ or $\alpha = x, y$

$$\mathcal{J}^\alpha = J_0^\alpha + f_\alpha(t)$$

Kayanuma, 1984

$$\langle f_\alpha(t) f_{\alpha'}(t+\tau)\rangle = J_\alpha^2 \delta_{\alpha\alpha'} e^{-\gamma|\tau|}$$



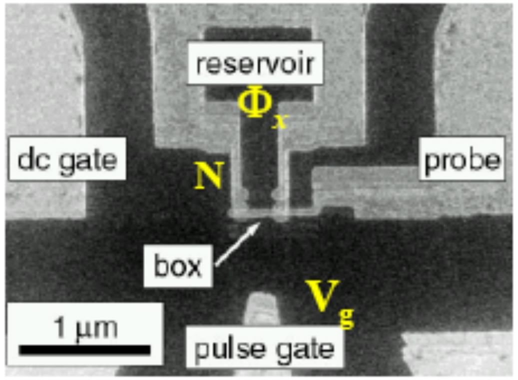
Fast noise: $\tau_{LZ} \gg 1/\gamma$

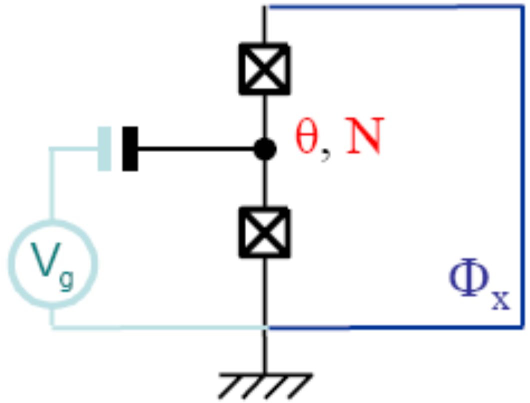
Slow noise: $\tau_{LZ} \ll 1/\gamma$

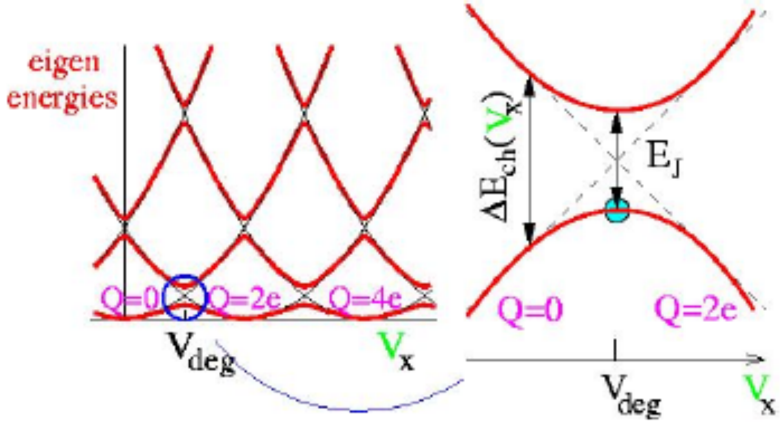
Why to bother about noise?

LZ transition: charge qubits

Josephson charge qubits







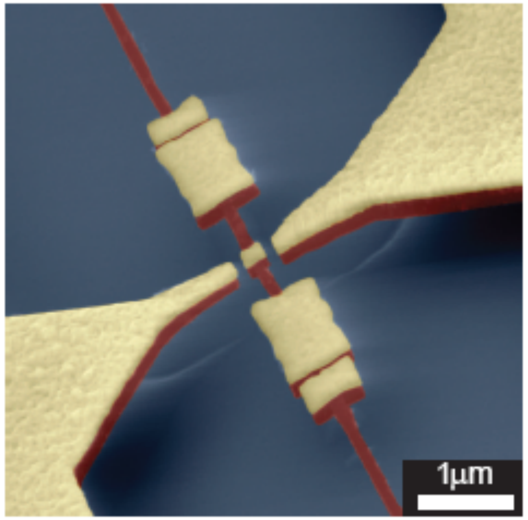
$$H = E_C \left(N - \frac{C_g V_g}{2e}\right)^2 - E_J \cos\left(\pi \frac{\Phi_x}{\Phi_0}\right) \cos\theta$$

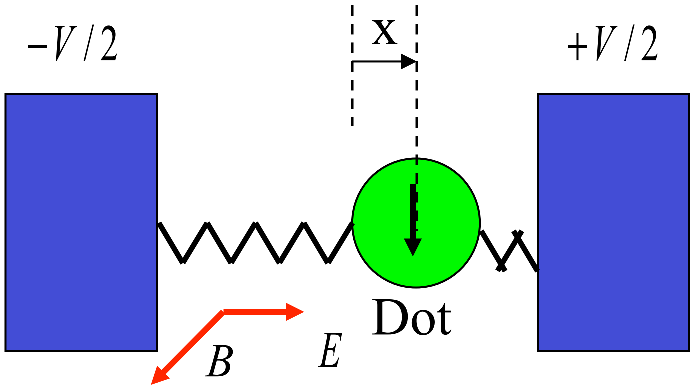
tunable E_J

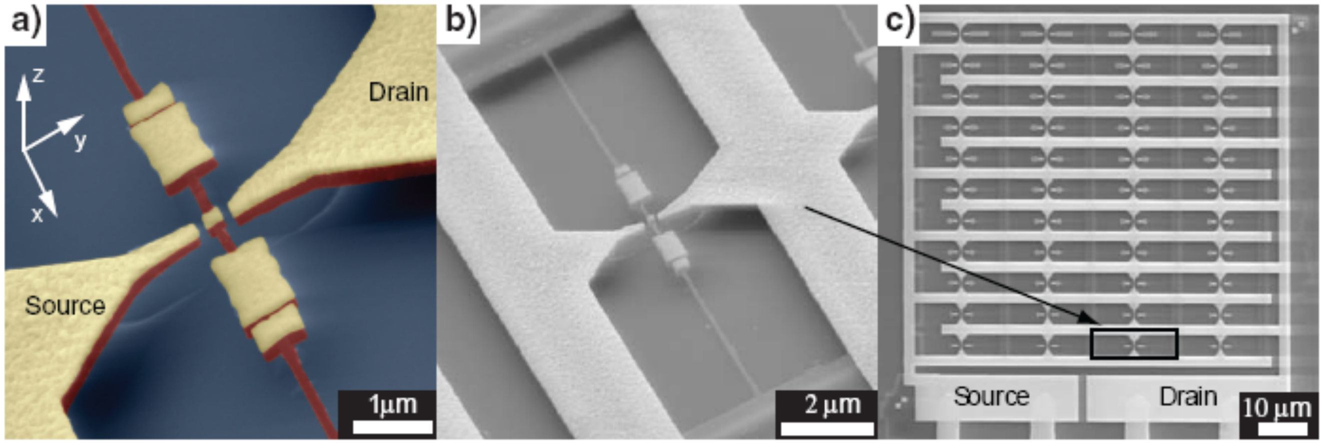
2 states only, e.g. for $E_C \gg E_J$

$$H = -\frac{1}{2} E_{ch}(V_g)\, \sigma_z - \frac{1}{2} E_J(\Phi_x)\, \sigma_x$$

Nano-electro-mechanical shuttling





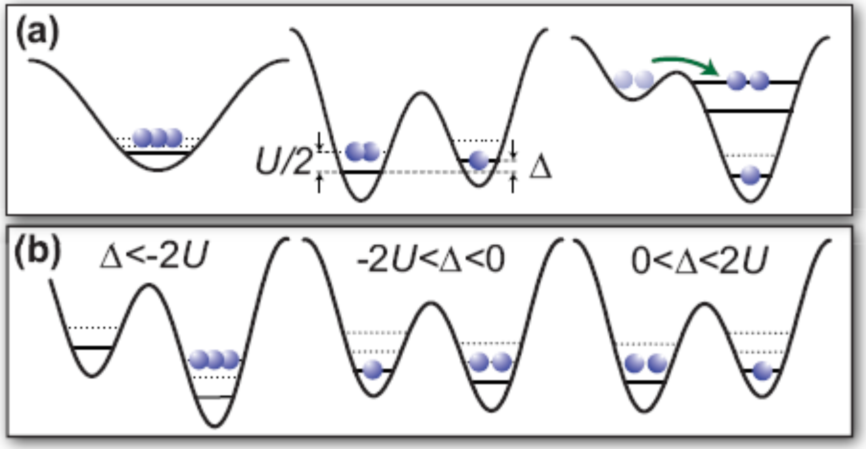


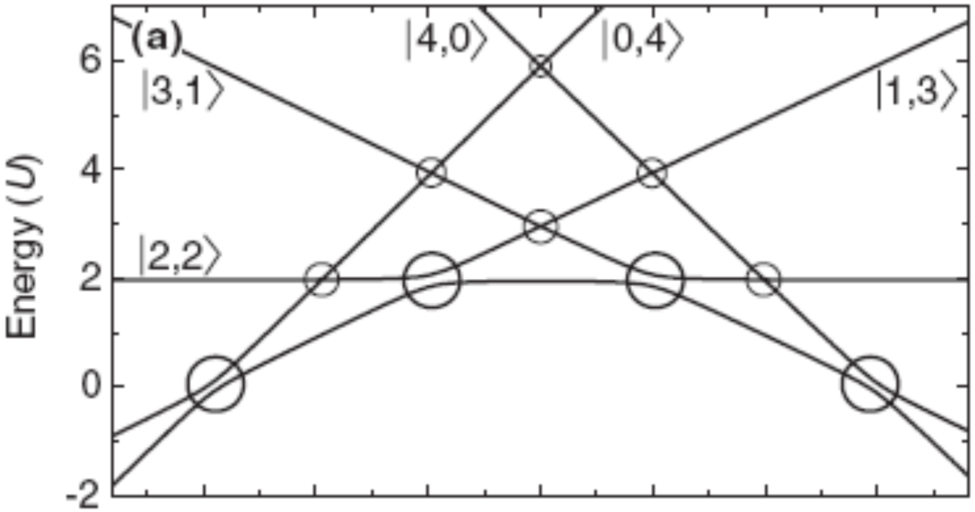
J. Kotthaus et al, Nature Nanotechnology 2008

Optical Lattices

$$H = -J(\hat{a}_L^\dagger \hat{a}_R + a_R^\dagger a_L) - \frac{\Delta}{2}(\hat{n}_L - \hat{n}_R)$$

$$H_U = \frac{U}{2}[\hat{n}_L(\hat{n}_L - 1) + \hat{n}_R(\hat{n}_R - 1)]$$



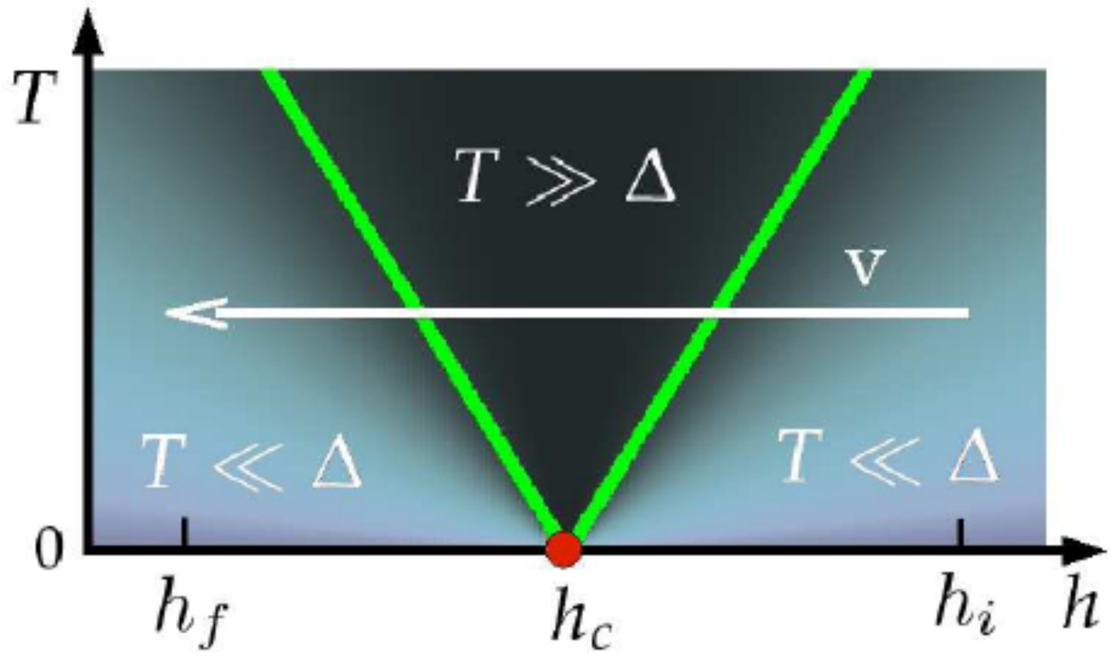


Bloch et at, 2008

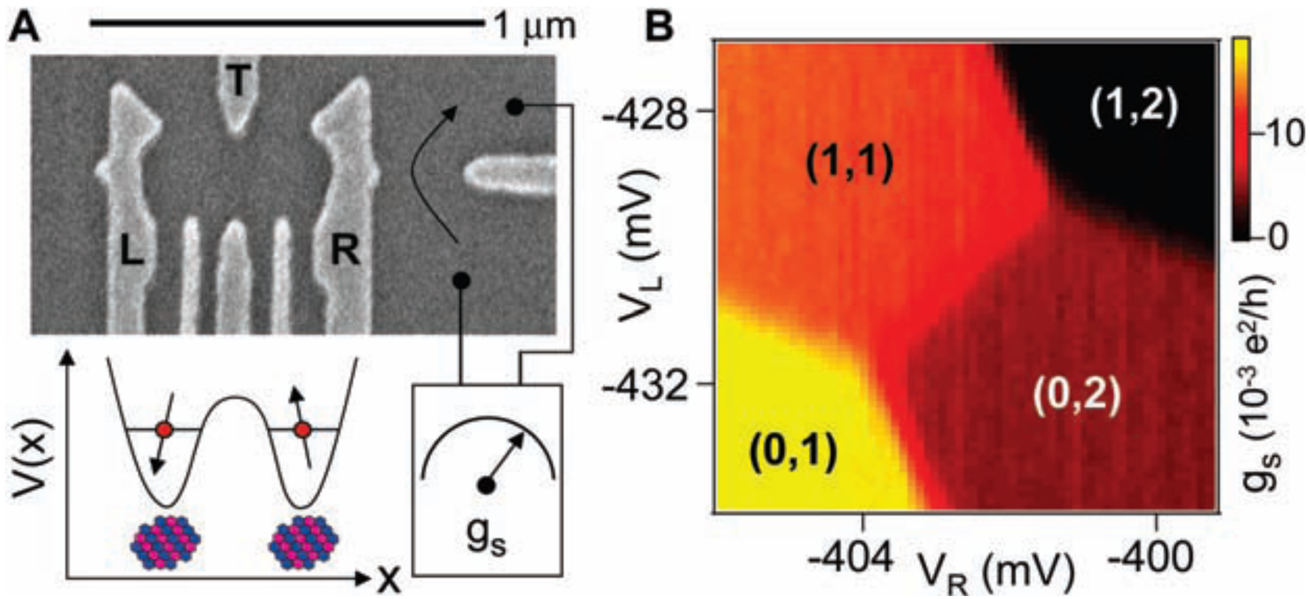
Quantum quenches

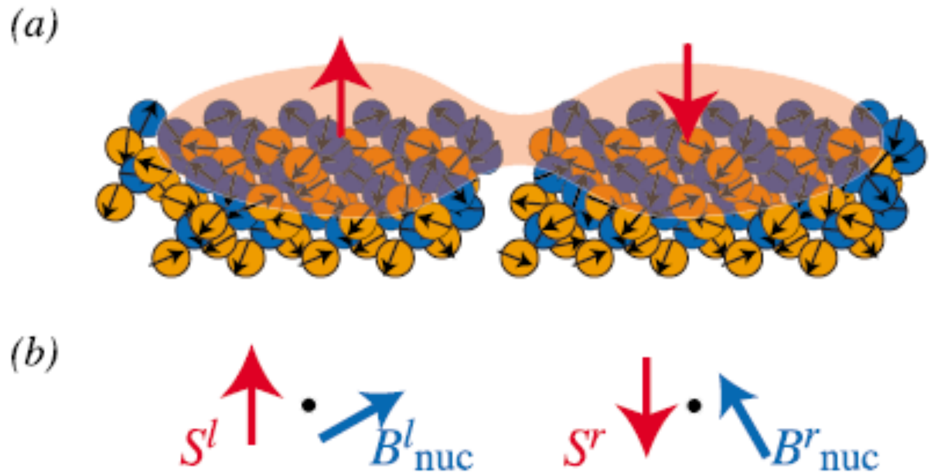
$$H = -h(t) \sum_i \sigma_i^x - \sum_i \sigma_i^z \sigma_{i+1}^z$$

Quantum Ising model

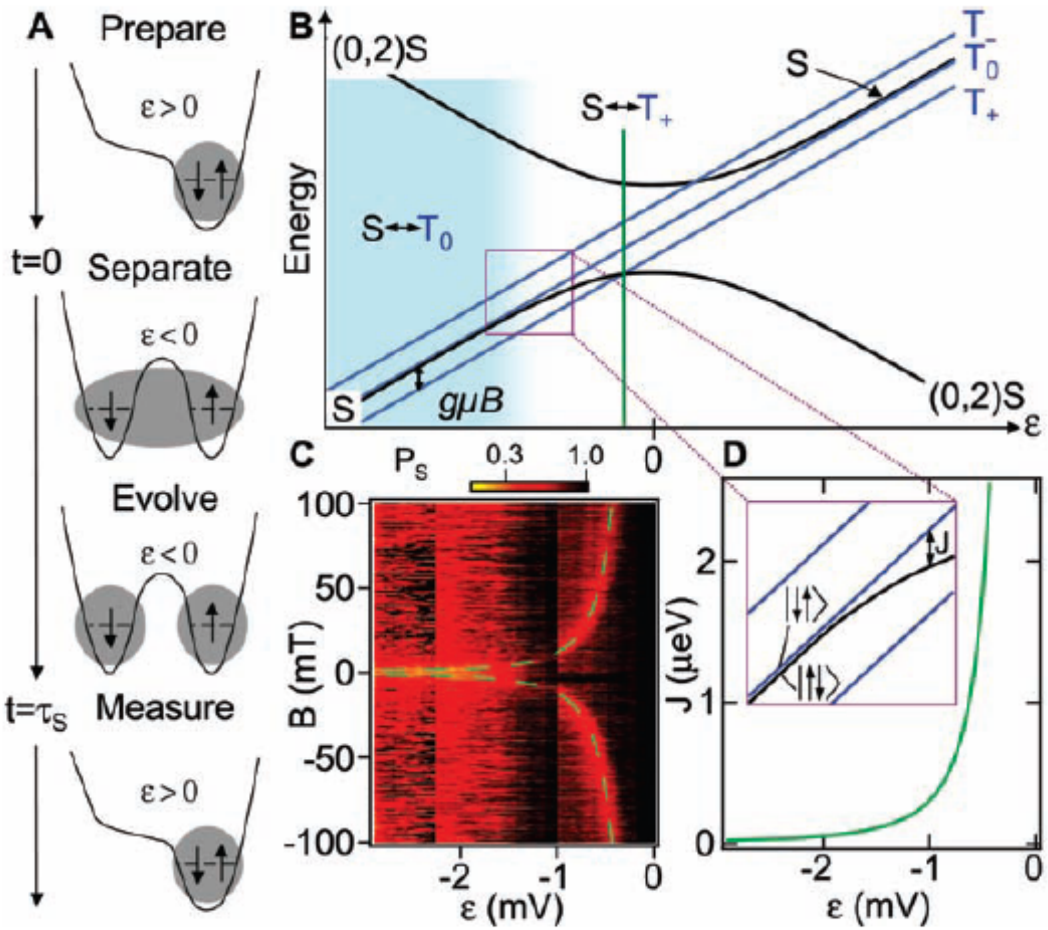


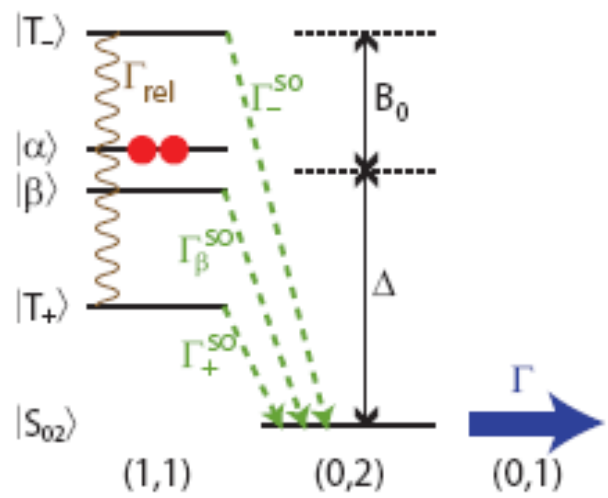
LZ transition: spin blockade in DQD devices (I)





LZ transition: spin blockade in DQD devices (III)





$$H_e = -\Delta |S_{02}\rangle\langle S_{02}|$$

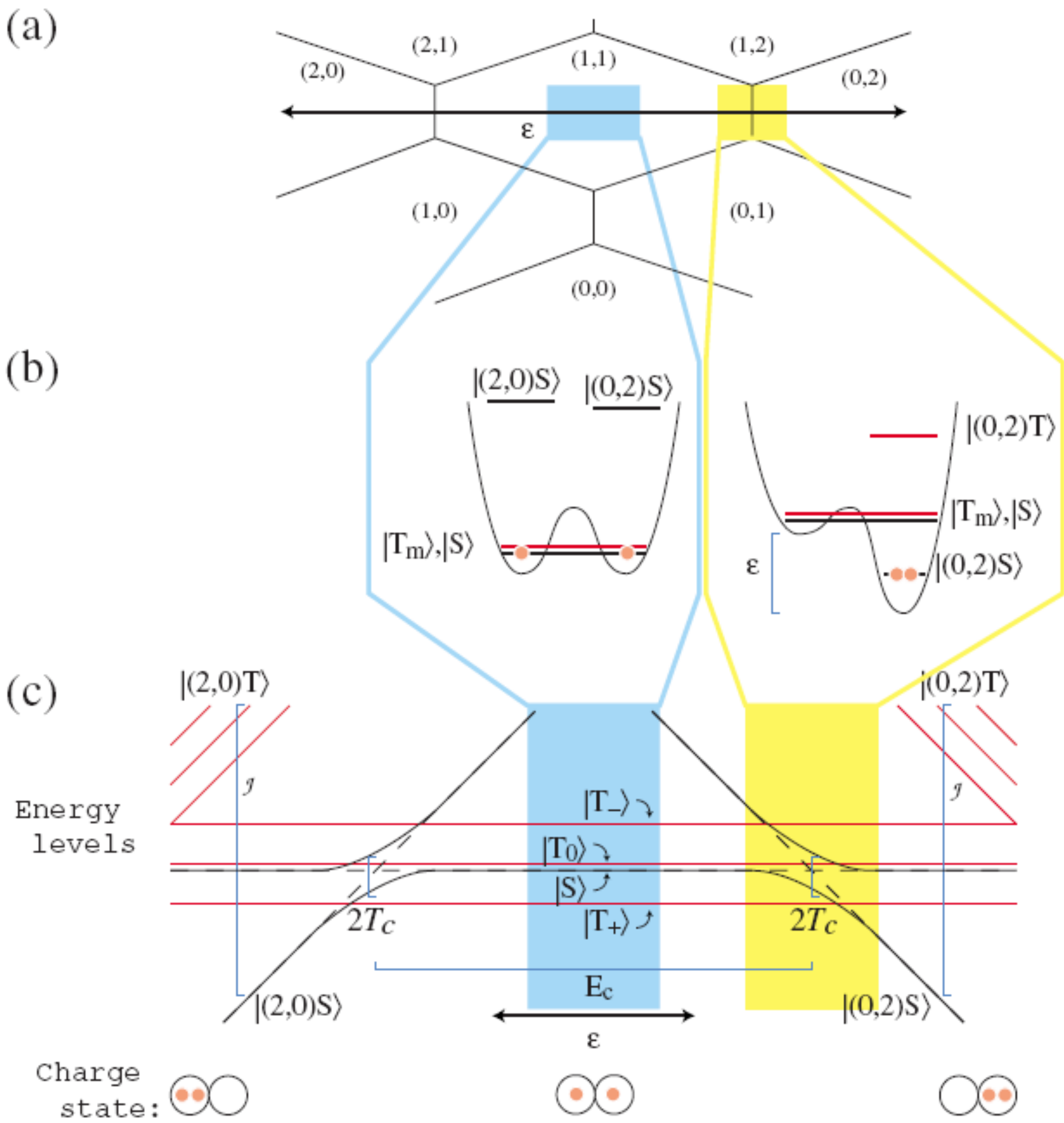
$$H_t = t_0 |S_{02}\rangle\langle S_{11}| + h.c.$$

$$H_B = B_0 (S^z_L + S^z_R)$$

$$H_K = \vec{K}_L \cdot \vec{S}_L + \vec{K}_R \cdot \vec{S}_R$$

Experiment: Foletti et al, 2008
Theory (no LZ): Nazarov et al, 2008, 2009

LZ transition: spin blockade in DQD devices (II)



Classical noise in LZ theory

Q: How to solve the LZ problem with noise?

A: Use density matrix equation

$$H = \frac{vt}{2}\sigma^z + f_\alpha(t)\sigma^\alpha$$

Noise-induced LZ transition

$$i\frac{d\widehat{\rho}}{dt} = [H\widehat{\rho}]$$

Bloch Equation

$$\dot{\vec{g}} = -\vec{b} \times \vec{g}$$

$$\vec{g} = \begin{pmatrix} 2\mathrm{Re}\rho_{12} \\ 2\mathrm{Im}\rho_{12} \\ \rho_{11} - \rho_{22} \end{pmatrix}$$

$$\mathrm{Tr}\widehat{\rho}^2 = 1$$

$$\Updownarrow$$

$$(\vec{g})^2 = 1$$

$$\vec{b} = \begin{pmatrix} f_x(t) \\ f_y(t) \\ \frac{vt}{2} \end{pmatrix}$$

Noise induced LZ transition: Bloch equation

$$\frac{d}{dt}g_z(t) = -4\int_{-\infty}^{t} dt_1 \cos\left(\frac{v}{2}[t^2 - t_1^2]\right) f_+(t) f_-(t_1) g_z(t_1)$$

$$f_\pm(t) = f_x(t) \pm i f_y(t)$$

Initial condition $$g_z(t = -\infty) = 1$$

$$F_\alpha(\tau) = \langle f_\alpha(t) f_\alpha(t+\tau)\rangle = J^2 \exp(-\gamma|\tau|)$$

Q: How to perform a statistical average?

A: It depends whether the noise is fast or slow.

Noise induced LZ transition: x-Fast Noise $\tau_{LZ} \gg 1/\gamma$

When noise is fast, write a master equation

$$\frac{d}{dt} g_z(t) = -4 \int_{-\infty}^{t} dt_1 \cos\left(\frac{v}{2}[t^2 - t_1^2]\right) f_x(t) f_x(t_1) g_z(t_1)$$

⬇

$$\frac{d}{dt}\langle g_z(t)\rangle = -4 \int_{-\infty}^{t} dt_1 \cos\left(\frac{v}{2}[t^2 - t_1^2]\right) \langle f_x(t) f_x(t_1)\rangle \langle g_z(t_1)\rangle$$

⬇

$$F(\tau) = \langle f_x(t) f_x(t+\tau)\rangle = J^2 \exp(-\gamma|\tau|)$$

⬇

$$P_{\uparrow\to\downarrow} = \frac{1}{2}\left(1 - \exp\left(-\frac{4\pi F(0)}{v}\right)\right) = \frac{1}{2}\left(1 - \exp\left(-\frac{4\pi J^2}{v}\right)\right)$$

White noise $F(\tau) \to \xi\delta(\tau)$ ➡ $P_{\uparrow\to\downarrow} = \frac{1}{2}$

Pokrovsky, Sinitsyn 2003, 2004

Fast-noise induced LZ transition

Message 1

$$P_{\uparrow\to\downarrow} = \frac{1}{2}\left(1 - \exp\left(-\frac{4\pi\langle f_x(t) f_x(t)\rangle}{v}\right)\right)$$

Averaging the argument of exponent !

Message 2

Q: How to sum up noises in x and y directions?

A: Just do it in the exponent !

$$P_{\uparrow\to\downarrow} = \frac{1}{2}\left(1 - \exp\left(-\frac{4\pi[\langle f_x(t) f_x(t)\rangle + \langle f_y(t) f_y(t)\rangle]}{v}\right)\right)$$

Message 3

Transition probability depends non-analytically on v in the adiabatic limit $v \to 0$

Pokrovsky, Sinitsyn 2003, 2004

Noise induced LZ transition: Slow Noise $\tau_{LZ} \ll 1/\gamma$

Solve the Bloch equation in given realization, then average!

$$\frac{d}{dt} g_z(t) = -4 \int_{-\infty}^{t} dt_1 \cos\left(\frac{v}{2}[t^2 - t_1^2]\right) f_+(t) f_-(t_1) g_z(t_1)$$

⬇

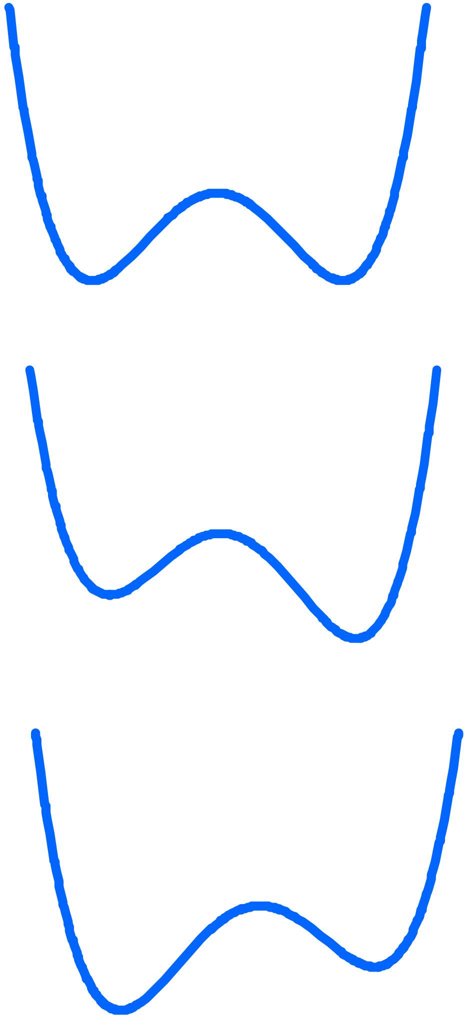
$$P_{\uparrow\to\downarrow} = \langle P_{\uparrow\to\downarrow}^{[\text{given realization}]} \rangle_{[\text{all realizations}]}$$

Q: How to average over all realizations?

Random 2 x 2 matrices

$$\begin{pmatrix} \varepsilon & \Delta^* \\ \Delta & -\varepsilon \end{pmatrix}$$

Keldysh Model: I

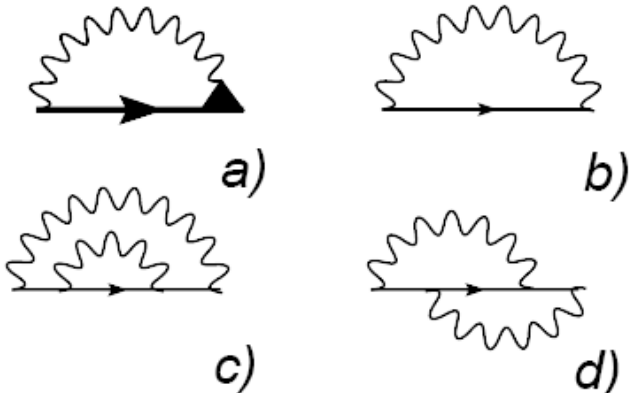


$$H = (h_0^z + h^z(t))\sigma^z + \Delta_0 \sigma^x$$

$$\langle h^z(t) h^z(t+\tau) \rangle = 2\pi\zeta^2 \exp(-\gamma|\tau|)$$

Slow fluctuations of a longitudinal magnetic field

$$G^R_{j,s}(\varepsilon) = g_j(\varepsilon)\left[1 + \sum_{n=1}^{\infty} A_n \zeta^{2n} g_j^{2n}(\varepsilon)\right]$$



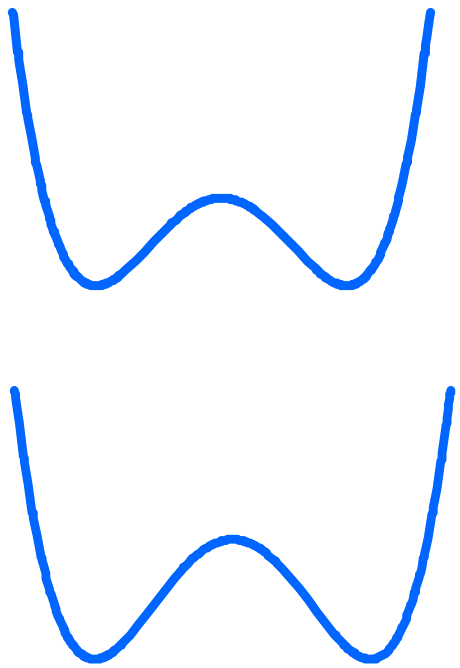
Combinatorics

$$A_n = (2n-1)!!$$

scalar model

$$G^R_{l,s}(\varepsilon) = \frac{1}{\zeta\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-z^2/2\zeta^2} \frac{dz}{\varepsilon - z + i\delta}$$

Keldysh Model: II



$$H = h_0^z \sigma^z + \Delta^x(t)\sigma^x + \Delta^y(t)\sigma^y$$

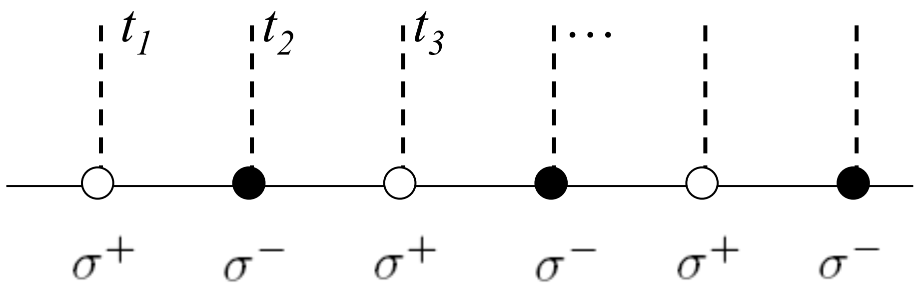
$$\langle \Delta^+(t)\Delta^-(t+\tau)\rangle = 4\pi\xi^2 \exp(-\gamma|\tau|)$$

$$\Delta^\pm(t) = \Delta^x(t) \pm i\Delta^y(t)$$

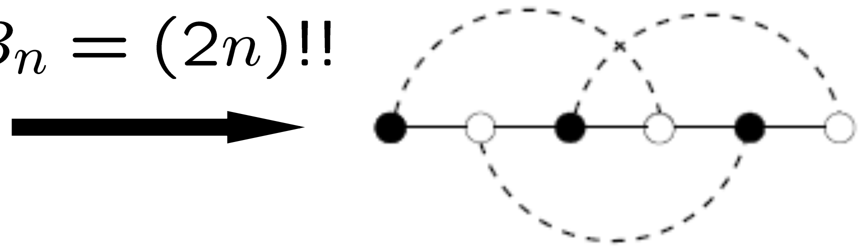
Slow fluctuations of the barrier

Planar model

$$G^R_{j,p}(\varepsilon) = g_j(\varepsilon) + \sum_{n=1}^{\infty} B_n \xi^{2n} g_j^{2n+1}(\varepsilon)$$

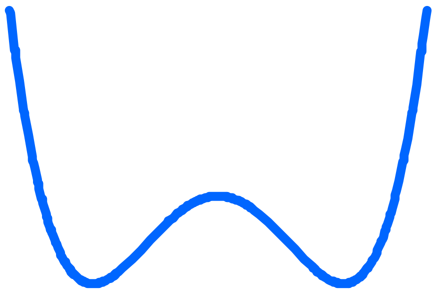


$$B_n = (2n)!!$$



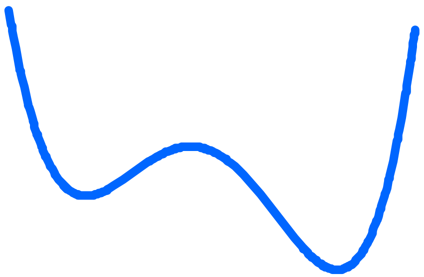
$$G^R_{j,v}(\epsilon) = \frac{1}{2}\int_{-\infty}^{+\infty} \frac{dx e^{-x^2/2\xi^2}}{\xi\sqrt{2\pi}} \int_{-\infty}^{+\infty} \frac{dy e^{-y^2/2\xi^2}}{\xi\sqrt{2\pi}} \left[\frac{1}{\epsilon - \sqrt{x^2+y^2} + i\delta} + \frac{1}{\epsilon + \sqrt{x^2+y^2} + i\delta}\right]$$

Keldysh Model: III



$$H = h^z(t)\sigma^z + \Delta^x(t)\sigma^x + \Delta^y(t)\sigma^y$$

$$\langle h^z(t)h^z(t+\tau)\rangle = 2\pi\zeta^2 \exp(-\gamma|\tau|)$$



$$\langle \Delta^+(t)\Delta^-(t+\tau)\rangle = 4\pi\xi^2 \exp(-\gamma|\tau|)$$

Slow fluctuations of magnetic field

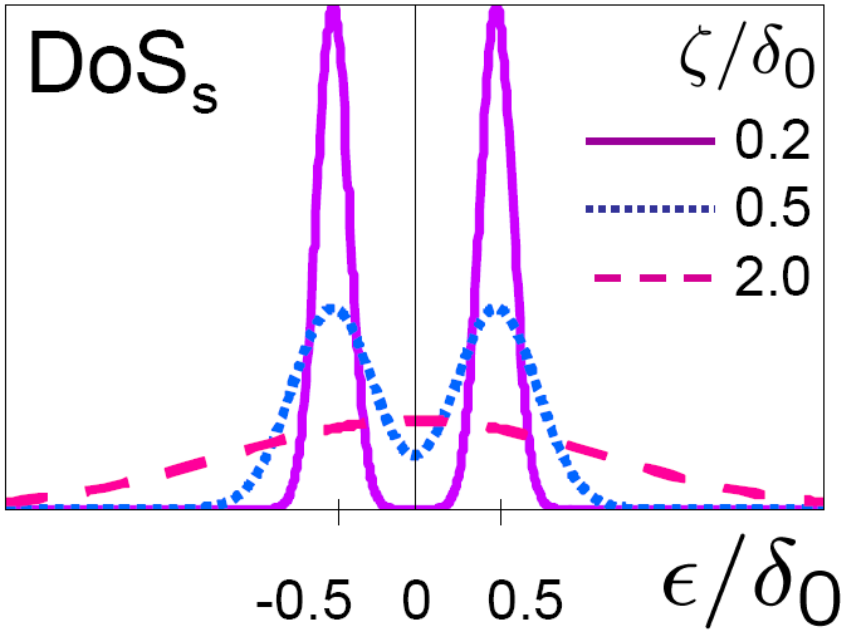
vector model

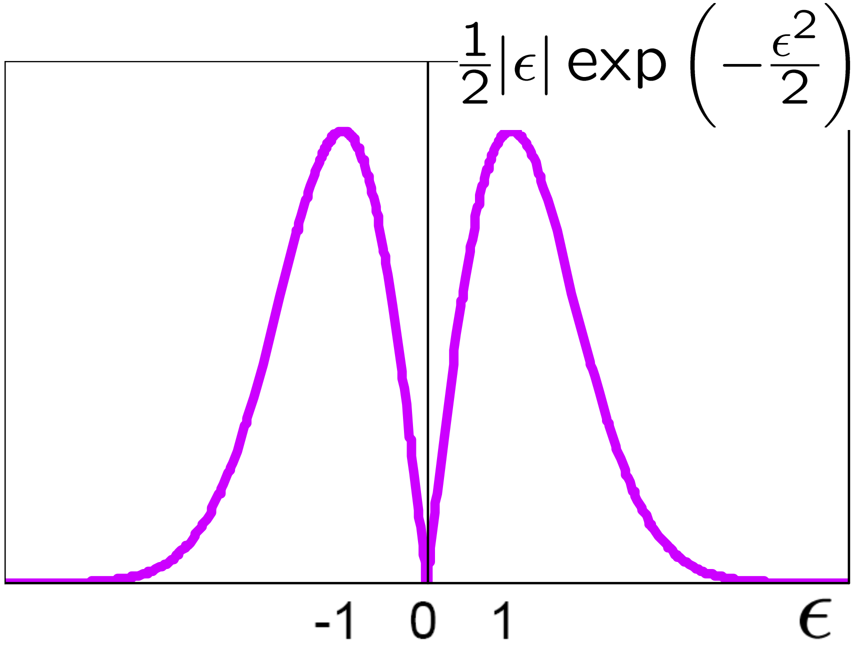
Isotropic Keldysh model $\xi = \zeta = \lambda$

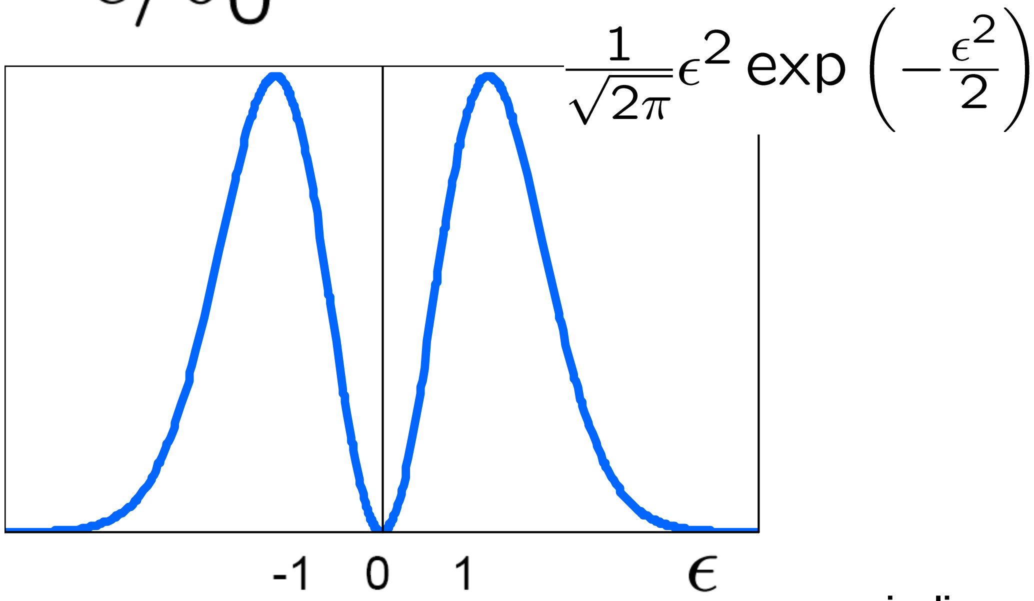
$$G^R_{j,v}(\varepsilon) = g_j(\varepsilon) + \sum_{n=1}^{\infty} C_n \lambda^{2n} g_j^{2n+1}(\varepsilon) \quad \Longrightarrow \quad C_n = (2n+1)!!$$

$$G^R_{j,v}(\epsilon) = \frac{1}{\lambda^3\sqrt{2\pi}} \int_0^{\infty} \rho^2 d\rho \left(\frac{1}{\epsilon - \rho + i\eta} + \frac{1}{\epsilon + \rho + i\eta} \right) e^{-\rho^2/2\lambda^2}$$

Density of States $\mathrm{DoS}(\epsilon) = -\frac{1}{\pi}\mathrm{Im}G^R(\epsilon)$







in dimensionless variables

Keldysh Model: Summary

scalar $$G^R_{l,s}(\varepsilon) = \frac{1}{\zeta\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{-z^2/2\zeta^2}\frac{dz}{\varepsilon - z + i\delta}$$

planar $$G^R_{j,p}(\epsilon) = \int_0^\infty \frac{udu}{2\xi^2}\left(\frac{1}{\epsilon - u + i\eta} + \frac{1}{\epsilon + u + i\eta}\right)e^{-u^2/2\xi^2}$$

vector $$G^R_v(\epsilon) = \frac{1}{2\xi}\int_0^\infty d\rho\rho\exp\left(-\frac{\rho^2}{2\xi^2}\right)\frac{\mathrm{erf}\left(\rho\sqrt{\frac{\xi^2-\zeta^2}{2\xi^2\zeta^2}}\right)}{\sqrt{\xi^2-\zeta^2}}\left(\frac{1}{\epsilon-\rho+i\eta} + \frac{1}{\epsilon+\rho+i\eta}\right).$$

Message 1 Averaging over slow noise is nothing but averaging of the correlator calculated in given realization with the Gaussian distribution.

Message 2 Only the modulus of the classical field fluctuates, while the phases are uniformly distributed

Message 3 Combinatorics of scalar and planar Keldysh Model can be used for accounting X and XY noises in LZ theory

Averaging over slow noise

$$P_{\uparrow\rightarrow\downarrow} = \langle P_{\uparrow\rightarrow\downarrow}^{[\text{given realization}]} \rangle_{[\text{all realizations}]}$$

where for any function G

$$\langle G \rangle = \frac{1}{J\sqrt{2\pi}} \int_{-\infty}^{\infty} dX \, \exp\left(-\frac{X^2}{2J^2}\right) G(X)$$

Averaging the LZ transition probability !

Noise-induced LZ transition: Slow Noise

Message 1 Slow "x-direction" noise induced transition

Kayanuma, 1985

$$P_{\uparrow\to\downarrow} = 1 - \frac{1}{\sqrt{1+\frac{4\pi J^2}{v}}}$$

Slow noise makes transition probability non-exponential in the adiabatic limit $v \to 0$

Message 2

Q: How to sum up noises in x and y directions ?

$$\textbf{A:}\quad P_{\uparrow\to\downarrow} = 1 - \left(\frac{1}{\sqrt{1+\frac{4\pi J^2}{v}}}\right)_x \times \left(\frac{1}{\sqrt{1+\frac{4\pi J^2}{v}}}\right)_y = \frac{4\pi J^2}{v+4\pi J^2}$$

Slow noise makes transition probability analytic in the adiabatic limit

Noise-assisted LZ transition: Slow Noise

Message 3

$$H = \frac{vt}{2}\sigma^z + [J_0 + f_x(t)]\sigma^x$$

$$\langle f_x(t) f_x(t+\tau)\rangle = J^2 e^{-\gamma|\tau|}$$

$$P_{\uparrow\to\uparrow} = \frac{1}{\sqrt{1+4\pi J^2/v}} \exp\left(-\frac{2\pi J_0^2/v}{1+4\pi J^2/v}\right)$$

adiabatic transition

$v \to 0$

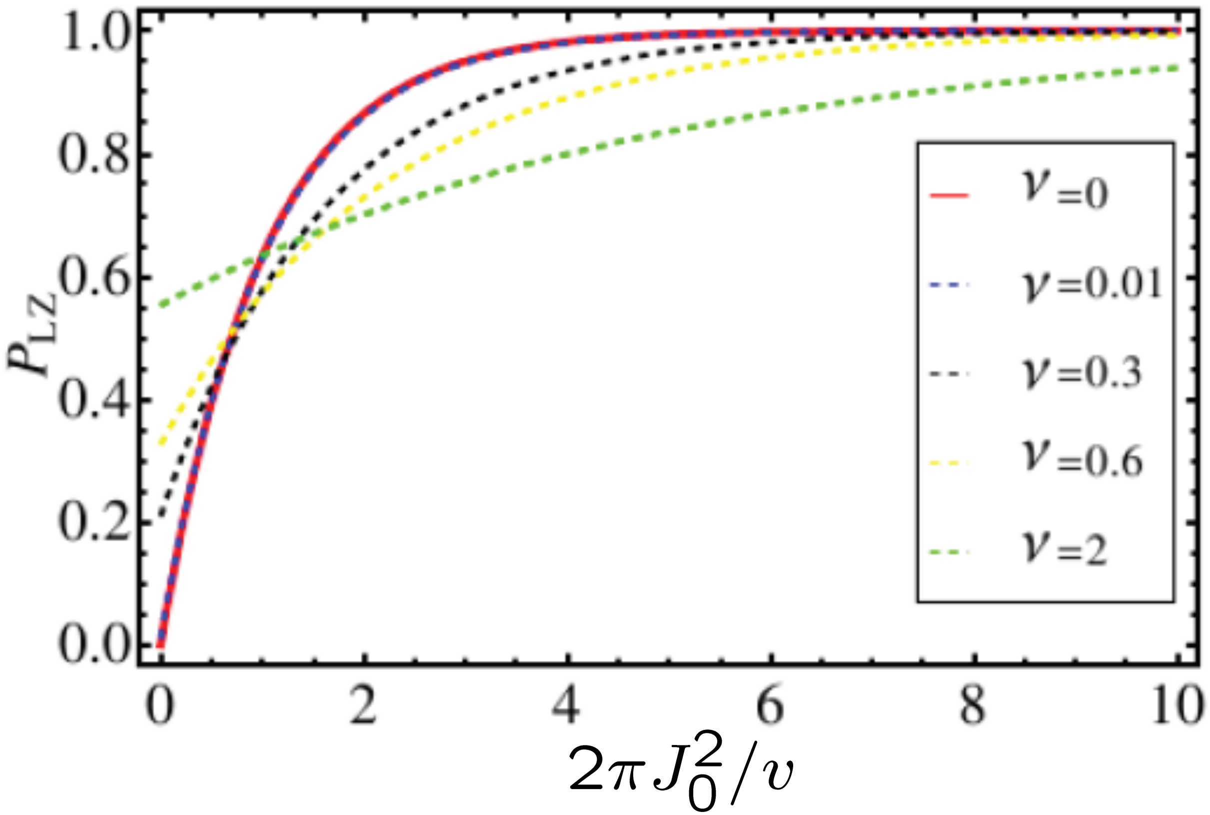
does not depend on velocity

$$P_{\uparrow\to\uparrow} = \sqrt{\frac{v}{4\pi J^2}} \exp\left(-\frac{J_0^2}{2J^2}\right)$$

noise determines pre- exponent

Noise-assisted LZ transition

$$P_{\uparrow\to\downarrow} = 1 - \frac{1}{\sqrt{1+4\pi J^2/v}} \exp\left(-\frac{2\pi J_0^2/v}{1+4\pi J^2/v}\right)$$



$$\nu = \frac{2\pi J^2}{v}$$

Noise-assisted LZ transition: Slow Noise

Message 4

Fluctuation Dissipation Theorem: $\langle f_x^2(t)\rangle = A \cdot T$

noise is classical **coupling constant**

$$P_{\uparrow\to\uparrow} = \sqrt{\frac{v}{4\pi AT}}\exp\left(-\frac{E}{T}\right) \qquad E = J_0^2/(2A)$$

$$\gamma\sqrt{A \cdot T} \ll \gamma J_0 \ll v \ll A \cdot T \ll J_0^2$$

Message 5 **Q: What happens if noise is slow in one direction and fast in another one?**

A: Fast noise contributes to the argument of LZ exponent while slow noise both determines pre – exponent and renormalizes the coupling.

Slow Noise induced LZ transition: finite time probabilities

Sudden transition: perturbative in $2\pi J^2/v \ll 1$
solution of the Bloch equation

$$P_{\uparrow\to\downarrow}(t) \approx \frac{2\pi J^2}{v} F(t)$$

$$F(t) = \frac{1}{2}\left[\left(\frac{1}{2} + C\left(\sqrt{\frac{v}{\pi}}t\right)\right)^2 + \left(\frac{1}{2} + S\left(\sqrt{\frac{v}{\pi}}t\right)\right)^2\right]$$

$$F(t \to +\infty) = 1$$

cos - Fresnel integral - sin

Iteration solution of the Bloch equation results in:

$$P_{\uparrow\to\downarrow}(t) = 1 - \frac{1}{\sqrt{1+\frac{4\pi J^2}{v}[F(t)+\delta F(t)]}}$$

$$\delta F(\pm\infty) = 0$$

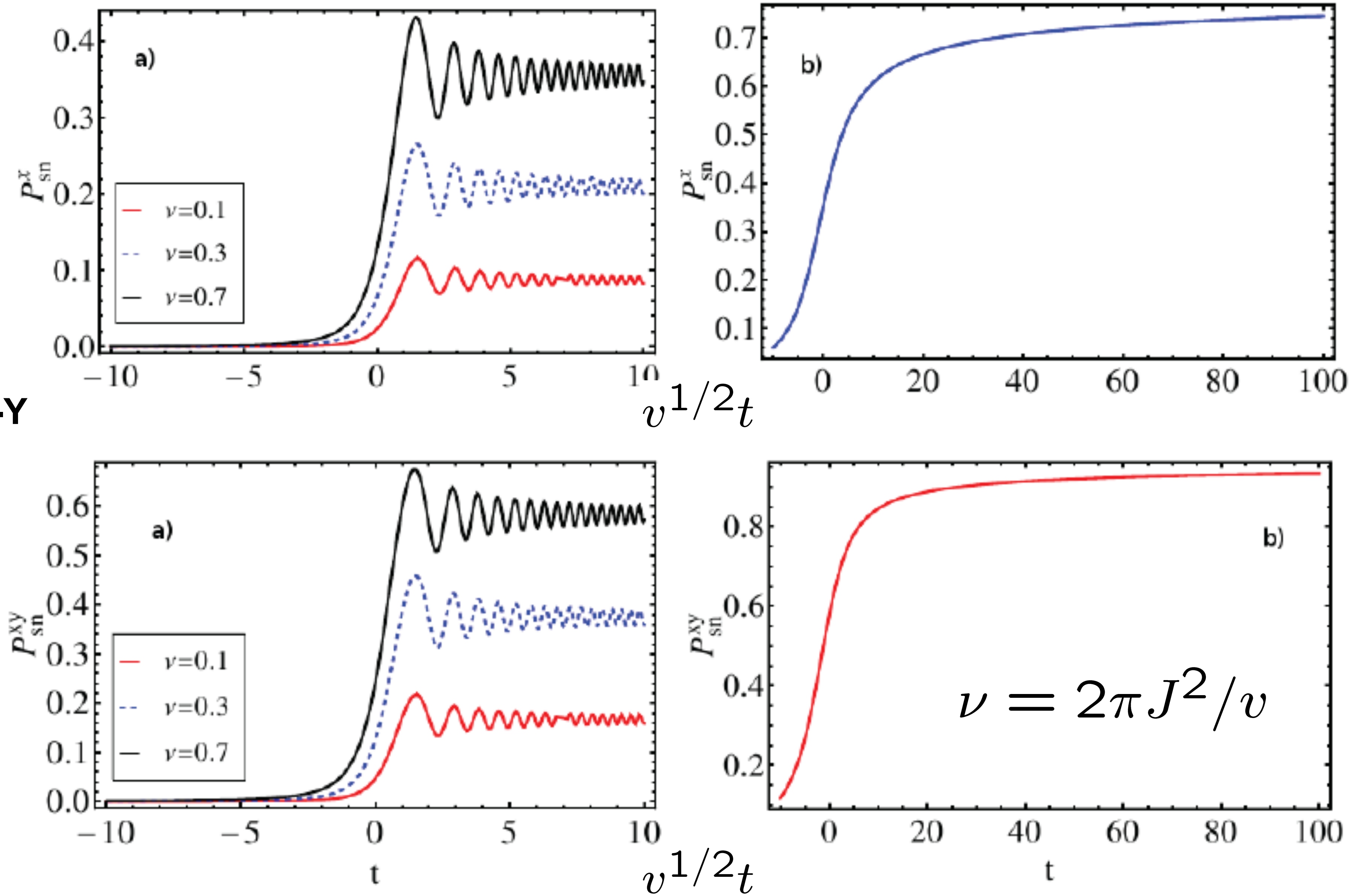
Levitov et al, 2008

LZ transition: Slow Noise

sudden **adiabatic**

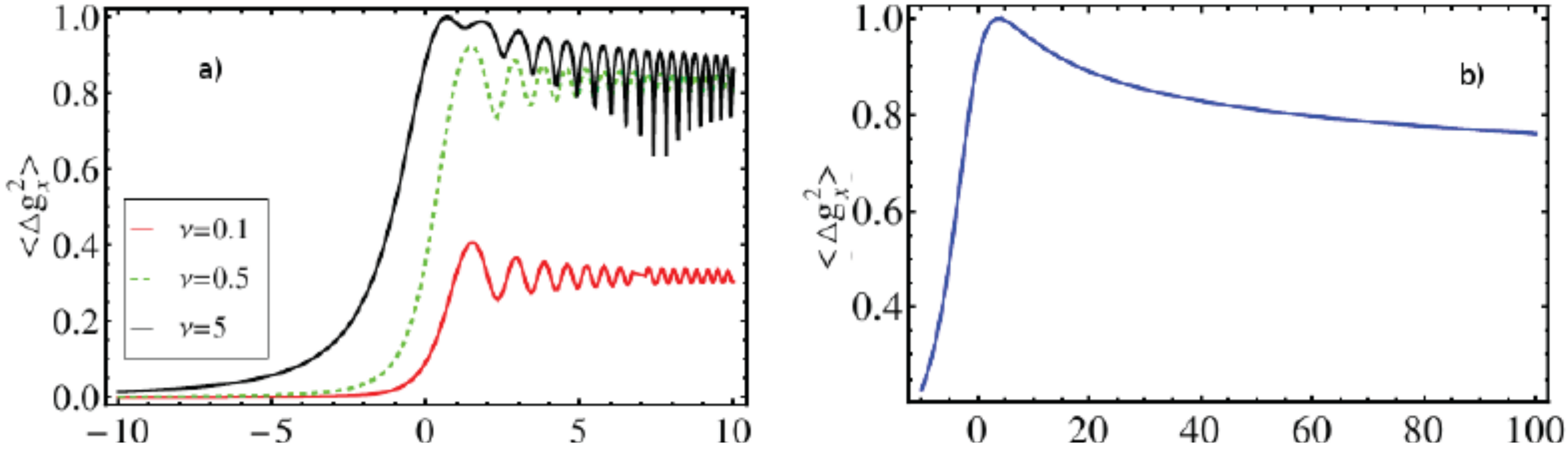
X

X-Y

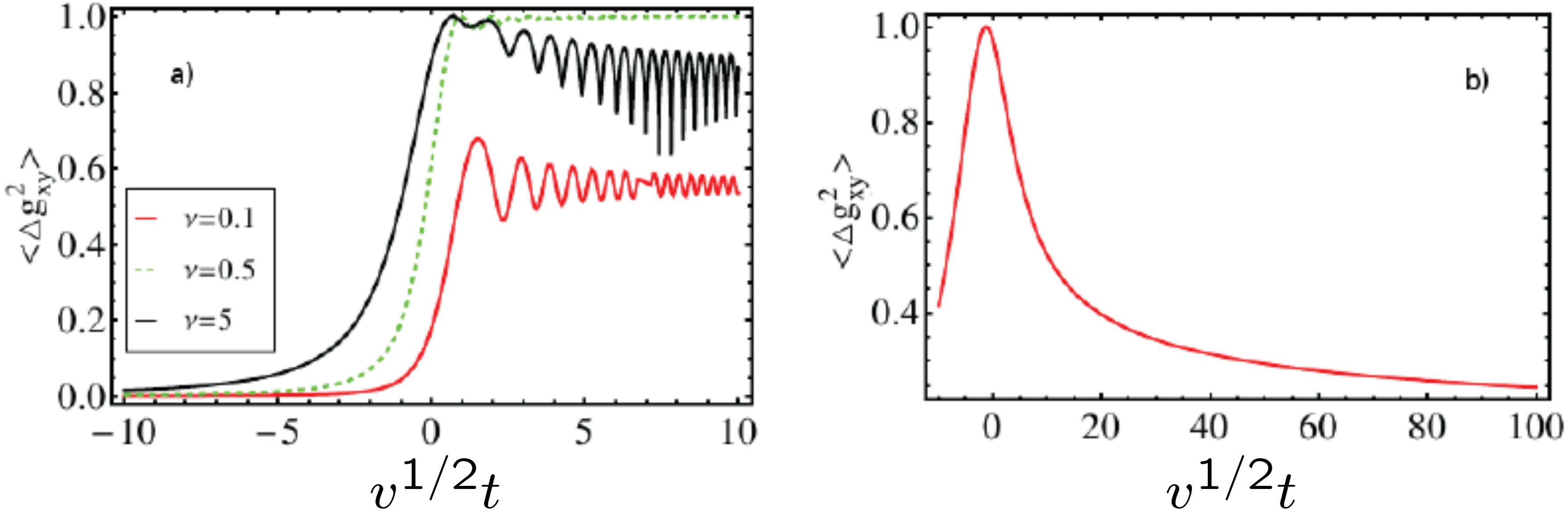


LZ transition: Bloch vector's fluctuations $\langle(\Delta\vec{g})^2\rangle = 1 - \langle g_z\rangle^2$

X



XY



Perspectives

- Periodic drive + noise
- Interaction + noise
- Dissipation + noise
- Multi-level LZ + noise
- Many-particle LZ + noise
- etc

Phien Ho (now in Maryland)

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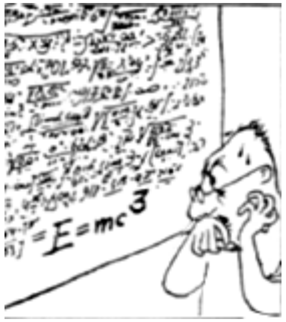

Conclusions

• Fast noise only contributes to the argument of LZ exponent while slow noise both determines pre – exponent and renormalizes the coupling.

• Slow noise makes transition probability analytic (v) in the adiabatic limit, while fast noise does not.

• Classical Noise = temperature. Landau-Zener probability is described by activation exponent.

Thanks!