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Dimensional analysis in school physics

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DIMENSION ANALYSIS IN SCHOOL PHYSICS

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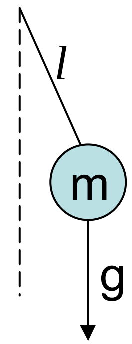
Principal topics

1. Dimensions: are they consequences of the fundamental physical laws?
2. If Galileo Galilei knew dimensions…
3. Gulliver and Lilliputs
4. Dimensions and art
5. Π – theorem
6. Enlarging of the set of dimensions
7. The simplest examples:

- Height of Vesuvio,
- Tsunami 2011
- A useful explosion
- Oscillations of a drop, nucleus and of the Universe

8. Is it possible to discover new fundamental laws?

If G.Galilei knew Dimension Analysis, he could conclude at once:
a pendulum period $$T \sim \sqrt{\frac{l}{g}} \rightarrow \sqrt{\frac{\mathrm{m}}{\mathrm{m/s^2}}} = \mathrm{s}$$



(Principal conclusions:

- It does not depend on the pendulum mass
- It's proportional to the square root of the pendulum length l)

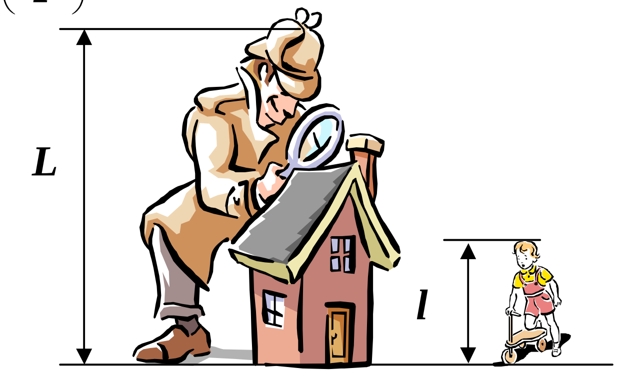
But it was only the beginning of the Numerical Science in his time

Lilliput's scientists has proposed to

provide Gulliver with as many

bread –and–butters as $\left(\frac{L}{l}\right)^3 = 12^3 = 1728$

(L and l–their heights)



They did feel Dimension Analysis!

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Π-Theorem

A correlation (which does not depend on the choice of the dimension quantities system) between n+1 dimensions a, $a_1,\ldots,a_n$ (k of them being independent) may be presented in the form of relation between $n+1-k$ values Π, $\Pi_1,\ldots,\Pi_{n-k}$, which are dimensionless combinations of the n+1 dimension quantities

The best case: $n = k$, then the only one dimensionless parameter does exit!

It’s of use to know

Dimension Analysis

to everybody, for example, to painters

(see Nicolay Rerich picture

“The guests from the far”)

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“The guests from the far” N. Rerich

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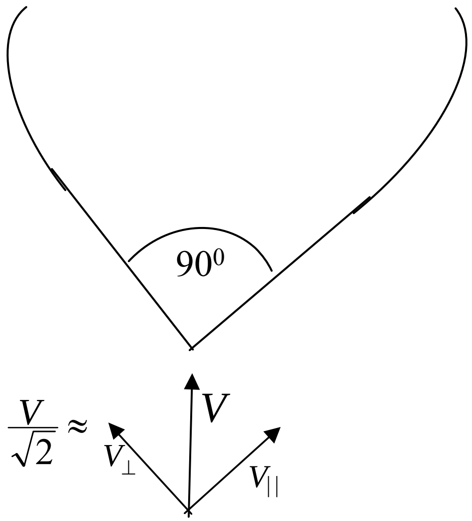
Surface velocity

Ship velocity



$$v_\lambda \sim \sqrt{\lambda\, g} \approx \sqrt{0.5\,\mathrm{m}\cdot 10\frac{\mathrm{m}}{\mathrm{s}^2}} \sim 2\frac{\mathrm{m}}{\mathrm{s}}$$

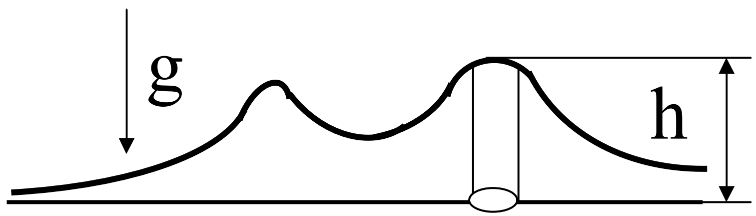
$$V \sim \sqrt{g\,h} = \sqrt{4\cdot 10\cdot(0.5\div 0.3)} \approx (3\div 4)\frac{\mathrm{m}}{\mathrm{s}}$$

$$\frac{V^2}{2\cdot 2} = gh, \qquad v_\lambda < V$$

So, the picture is not realistic…but nice

Why Vesuvio can not be as high as 100 km?

$$h \cong \frac{[\sigma]}{\rho g} \sim \frac{10^8}{3 \cdot 10^3 \cdot 10} \approx 3\,\mathrm{km} \qquad \rho g h \cong [\sigma] \sim 10^8 \,\frac{\mathrm{N}}{\mathrm{m}^2}$$



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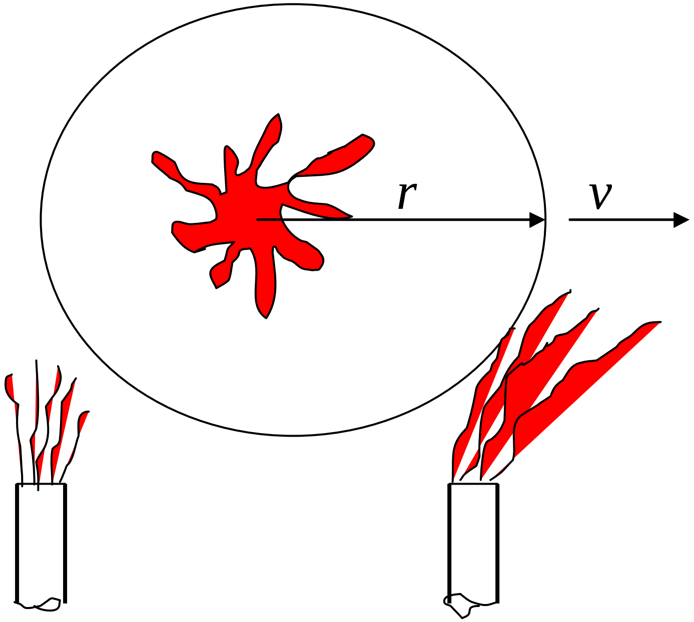
How to blow down a fire with the help of explosion



r	t	v	ρ	E
m	s	$\frac{\mathrm{m}}{\mathrm{s}}$	$\frac{\mathrm{kg}}{\mathrm{m}^3}$	$\frac{\mathrm{kg}\cdot\mathrm{m}^2}{\mathrm{s}^2} = \mathrm{J}$

$$r \sim \left(\frac{E}{\rho}\right)^{1/5} t^{2/5}$$

$$v \sim \left(\frac{E}{\rho}\right)^{1/2} r^{-3/2}$$

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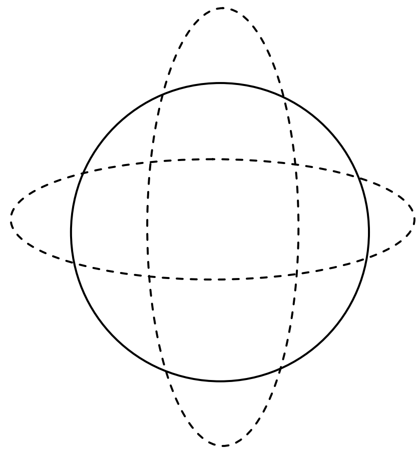
Period of a drop oscillations

$\alpha, \frac{N}{\mathrm{m}} -$ surface tension coefficient (“rigidity”)

$$m = \frac{4}{3}\pi\rho r^3,\ \mathrm{kg} - mass\,(inertia)$$

$$T \sim \sqrt{\frac{m}{\alpha}},\ \mathrm{s}$$



Atomic nuclei oscillates also !

The Globe oscillations

The Earth “surface tension”

α_E~ gravitational potential energy/surface ~

$$\sim \frac{GM_E^2}{R_E}\frac{1}{4\pi R_E^2} = G\rho_E M_E$$

Period of oscillations $$T_E \sim \sqrt{\frac{M_E}{\alpha_E}} \sim \frac{1}{\sqrt{G\rho_E}}$$

Oscillation of the Universe?

$$T_u \sim \frac{1}{\sqrt{G < \rho_u >}} \sim \frac{1}{\sqrt{6 \cdot 10^{-11} \cdot 10 \cdot 1.67 \cdot 10^{-27}}} \approx 10^{18}\,\mathrm{s} \approx 3 \cdot 10^{10}\ \mathrm{year}$$

$<\rho_u> \sim 10 m_p$ – average Universe density

$m_p = 1.67 \cdot 10^{-27}$ kg – proton mass

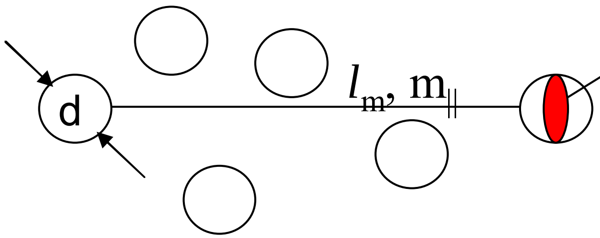
Enlargement of the dimensions set – a useful trick

$$\mathrm{m} \to \mathrm{m}_{\parallel}, \quad \mathrm{m} \to \mathrm{m}_{\perp}$$

Molecule concentration

$$n, \frac{1}{\mathrm{m}^3} \to \frac{1}{\mathrm{m}_{\parallel}\, m_{\perp}^2}$$

Molecule free path



$$\frac{\pi d_m^2}{4} \sim \mathrm{m}_{\perp}^2, \qquad l_m \sim \frac{1}{n\frac{\pi d_m^2}{4}}, \frac{1}{\frac{\mathrm{m}_{\perp}^2}{\mathrm{m}_{\parallel}\mathrm{m}_{\perp}^2}} = \mathrm{m}_{\parallel}$$

The length of visibility in a forest: d_t–a tree diameter,

$n_t, \frac{1}{\mathrm{m}^2} = \frac{1}{\mathrm{m}_{\parallel}\mathrm{m}_{\perp}}$ – surface density of trees

$$l_{vis} \sim \frac{1}{n_t\, d_t}, \frac{1}{\frac{\mathrm{m}_{\perp}}{\mathrm{m}_{\parallel}\mathrm{m}_{\perp}}} = \mathrm{m}_{\parallel}$$

Radiation of a heated body

$$q = f\left(k_B T, h_p, c\right) \rightarrow \frac{\left(k_B T\right)^4}{h_p^3 c^2} \sim \sigma_{SB} T^4$$

J, J·s, $\frac{\text{m}}{\text{s}}$

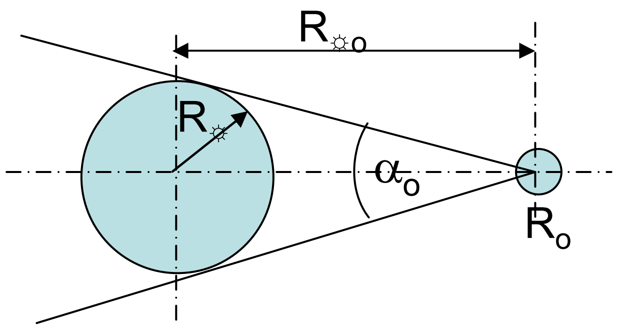
$k_B = 1.4 \cdot 10^{-23}\ \frac{\text{J}}{\text{K}}$ – Boltzmann constant

$h_p = 6.6 \cdot 10^{-34}\ \text{J} \cdot \text{s}$ – Plank constant

$c = 3 \cdot 10^8\ \frac{\text{m}}{\text{s}}$ – speed of light

$\sigma_{SB} = 5.67 \cdot 10^{-8}\ \frac{\text{J}}{\text{m}^2 \cdot \text{K}^4 \cdot \text{s}}$ – Stephan – Boltzmann

Sun temperature $4\pi R_o^2 \sigma T_o^4 \frac{\pi R_\oplus^2}{4\pi R_{o\oplus}^2} = 4\pi R_\oplus^2 \sigma T_\oplus^4$



$$T_o = T_\oplus \frac{2}{\sqrt{\alpha_o}} = 20 \cdot T_\oplus = 6000\,\text{K}$$

o – Sun; ⊕ – Earth

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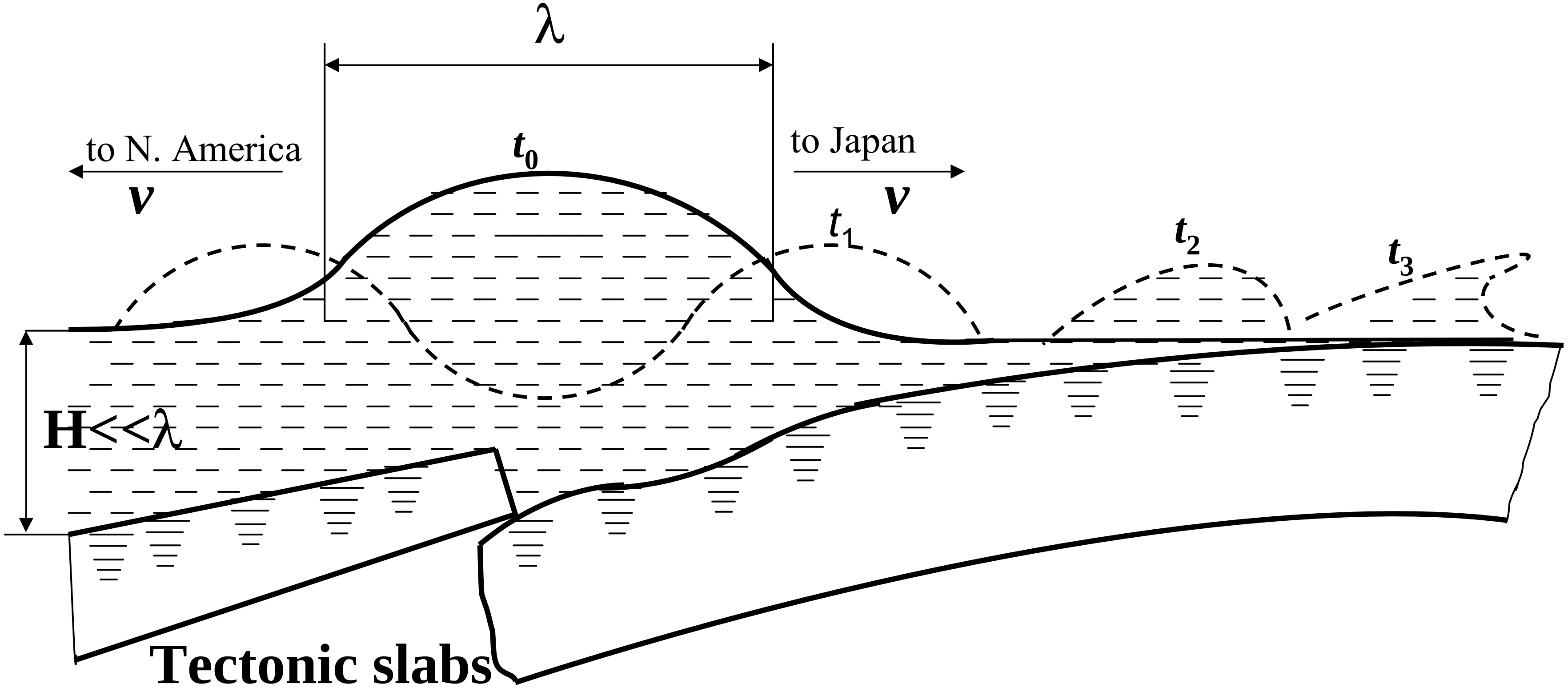
Tsunami 2011



$$v = \sqrt{gH} \sim \sqrt{10\frac{\mathrm{m}}{\mathrm{s}^2}\cdot 4\cdot 10^3\,\mathrm{m}} = 200\frac{\mathrm{m}}{\mathrm{s}}$$

(Theory of small depth – exact formula!)

Is it possible to discover new fundamental Laws of Nature with the help of Dimension Analysis?

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Relevant papers in Quantum

(The Student Magazine of Mathematics and Science)

1. Albert Stasenko. It’s beautiful – but is it science? (There’s something fihy about those waves). Quantum, January 1990, p.8
2. Yuly Bruk and Albert Stasenko. The power of dimensional thinking (priblem solving method). Quantum, May/June 1992, p. 34
3. Yuly Bruk and Albert Stasenko. Hard-core heavenly bodies (ionic cristal, Young’s modulus and planetary mass). Quantum, July/August 1993, p. 34
4. Yuly Bruk and Albert Stasenko. Drops for the crops (limits on the size of droplets). Quantum, March/April 1994, p10
5. Yuly Bruk, Maxim Zelnikov, and Albert Stasenko. Wobbling nuclear drops (macro laws in microworlds). Quantum, January/ February 1997, p.12

And also

Yk. M. Bruk, a.l. Stasenko. On correlation of the chemical compound and thermal-physical characteristics of substance with the parameters of planets. Siberian J. of Physics. 1993, No. 1. p.26