



**The Abdus Salam  
International Centre for Theoretical Physics**



**2240-4**

**Advanced School on Scaling Laws in Geophysics: Mechanical and  
Thermal Processes in Geodynamics**

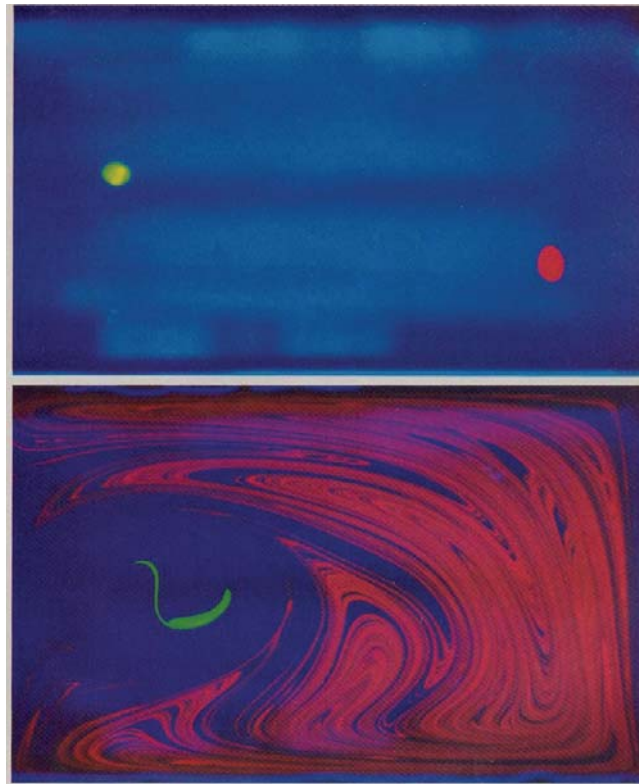
*23 May - 3 June, 2011*

**Mixing and Volcanos**

M.Manga  
*Univ. of California  
Berkeley  
USA*

# Mixing

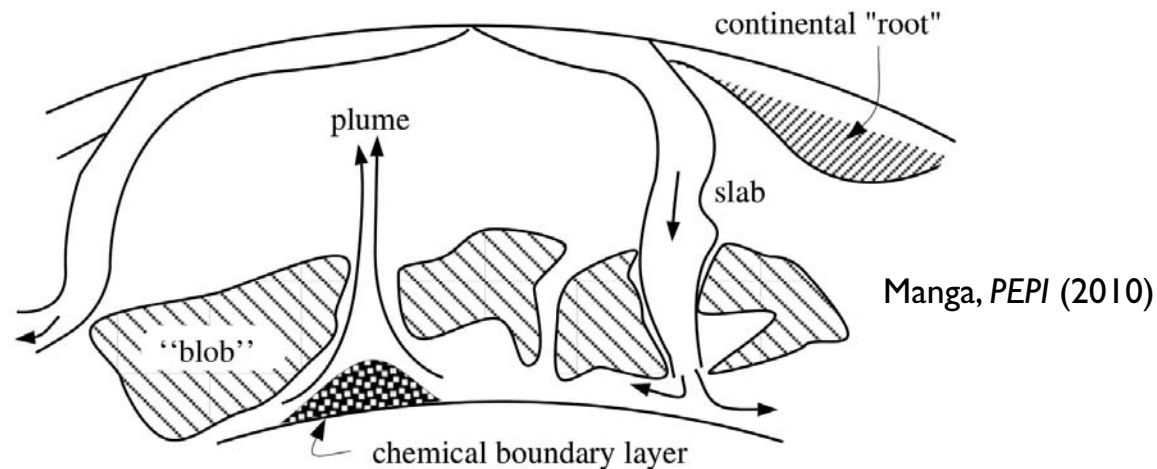
Michael Manga  
University of California, Berkeley



“Virtually everyone agrees that mixing is complicated”  
Ottino, Ann Rev Fluid Mech (1990)

# Why study mixing?

To provide a quantitative framework to interpret geochemical and isotopic variations in magmas, or structures we image with seismology (present structure and evolution of structures, rates of mass and energy exchange, evolution of mantle composition); magmatic processes within the crust



# Outline

- A bit of terminology
- Physics of mixing
- Characterization of mixing
- Mixing in the mantle

# Not covered

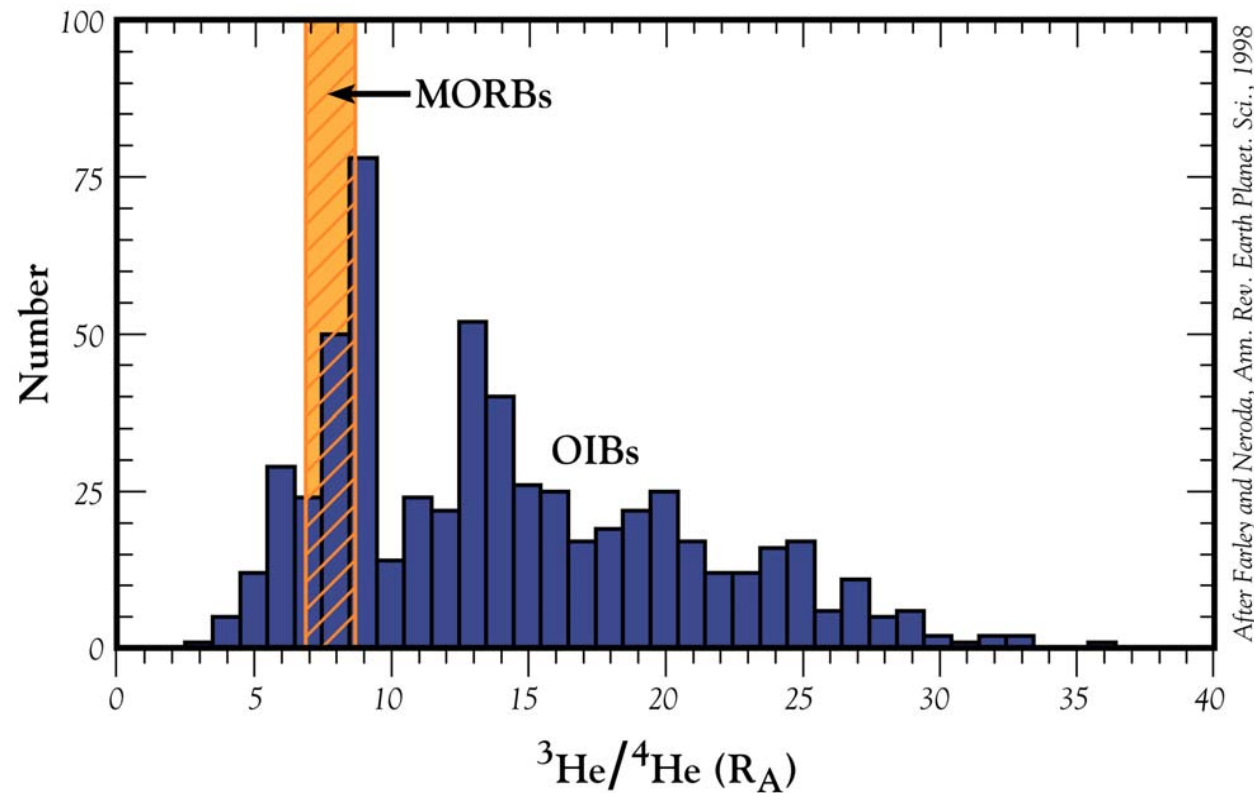
- How convection works (see other lectures)
- The geochemistry we want to interpret
- Numerical and computational challenges  
(see van Keken et al *JGR* 1997 for a discussion)
- Turbulent mixing (only low Reynolds number, laminar mixing)

# Main points

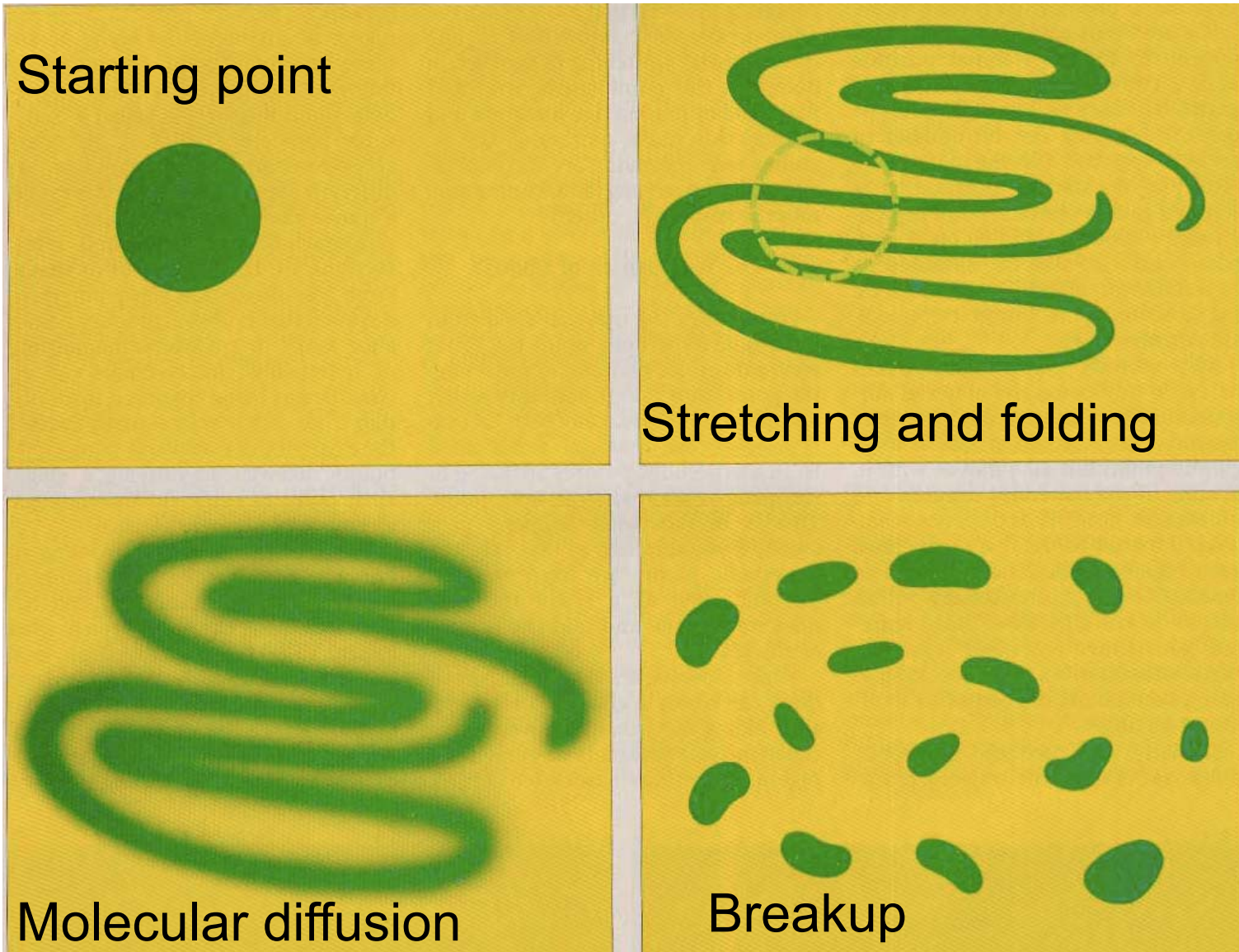
- Flow type matters
- Time dependence matters
- Properties of heterogeneity matter
- Convection both creates and destroys heterogeneity

# Some observations that we can interpret in the context of mixing

Global scale: mantle contains well-mixed regions and heterogeneity



# How does mixing occur?





# Definitions

**Stirring:** *stretching and folding* of material surfaces to reduce length scales

**Mixing:** homogenization by stirring and diffusion

**Passive tracer:** is convected with the flow  $\mathbf{u}(\mathbf{x},t)$  and does not influence the flow

**Active heterogeneities:** owing to differences in density and/or rheology, modify the flow

# Stretching: flow type matters

The deformation of a material filament from  $d\mathbf{X}$  to  $d\mathbf{x}$  is given by

$$d\mathbf{x} = \mathbf{F} \cdot d\mathbf{X}$$

where  $\mathbf{F}$  is the deformation tensor (which can be related to the velocity  $\mathbf{u}$ ).

The magnitude of stretching is

$$\lambda = \lim_{|d\mathbf{X}| \rightarrow 0} \frac{|d\mathbf{x}|}{|d\mathbf{X}|}$$

and the rate of stretching is

$$\frac{D(\ln \lambda)}{Dt} = \mathbf{E} : \mathbf{m}\mathbf{m} \quad \text{with} \quad \mathbf{m} = d\mathbf{x}/|d\mathbf{x}|$$

with  $\mathbf{E} = \frac{1}{2}[\nabla\mathbf{u} + (\nabla\mathbf{u})^T]$  is the stretching tensor.

# Stretching: flow type matters

Lets consider linear 2D flows in the  $x$ - $y$  plane

$$v_x = Gy \quad \text{and} \quad v_y = KGx$$

For long times, if  $K = 0$  (simple shear)

$$\lambda \sim Gt$$

and if  $K = 1$  (pure shear, hyperbolic flow)

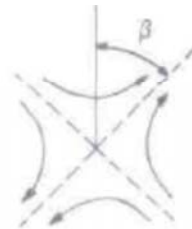
$$\lambda \sim e^{Gt}$$



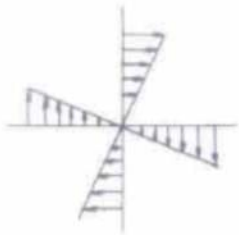
$K=-1$



$K=0$



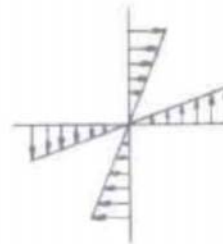
$K=1$



(a)



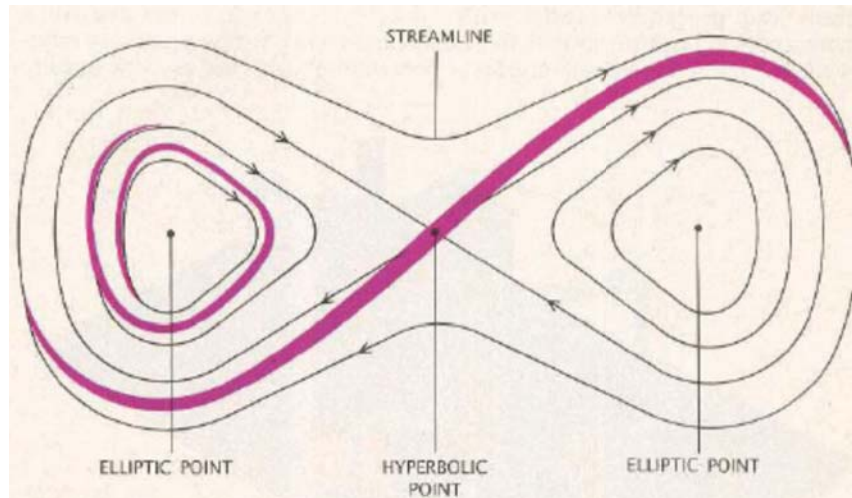
(b)



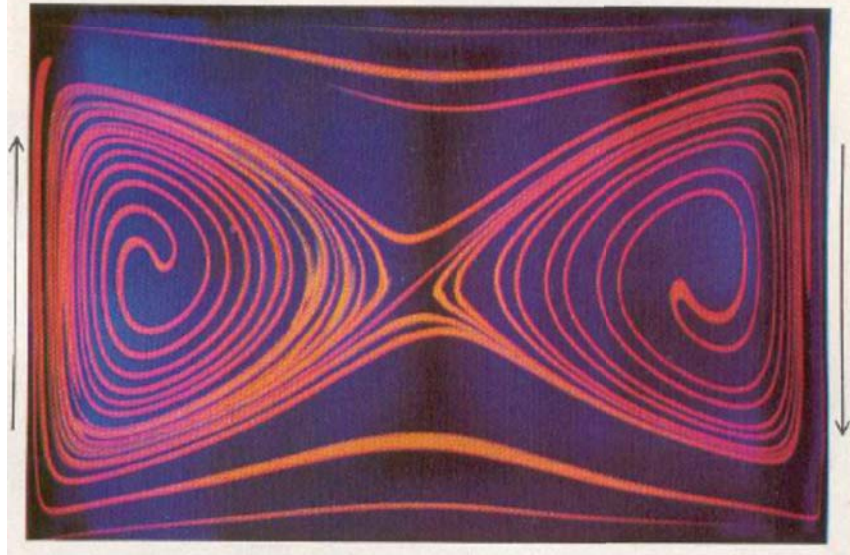
(c)

In a more complex flows, regions with pure shear (hyperbolic streamlines) will cause most of the *stretching*

# Two types of building blocks for flows: Elliptic and hyperbolic points

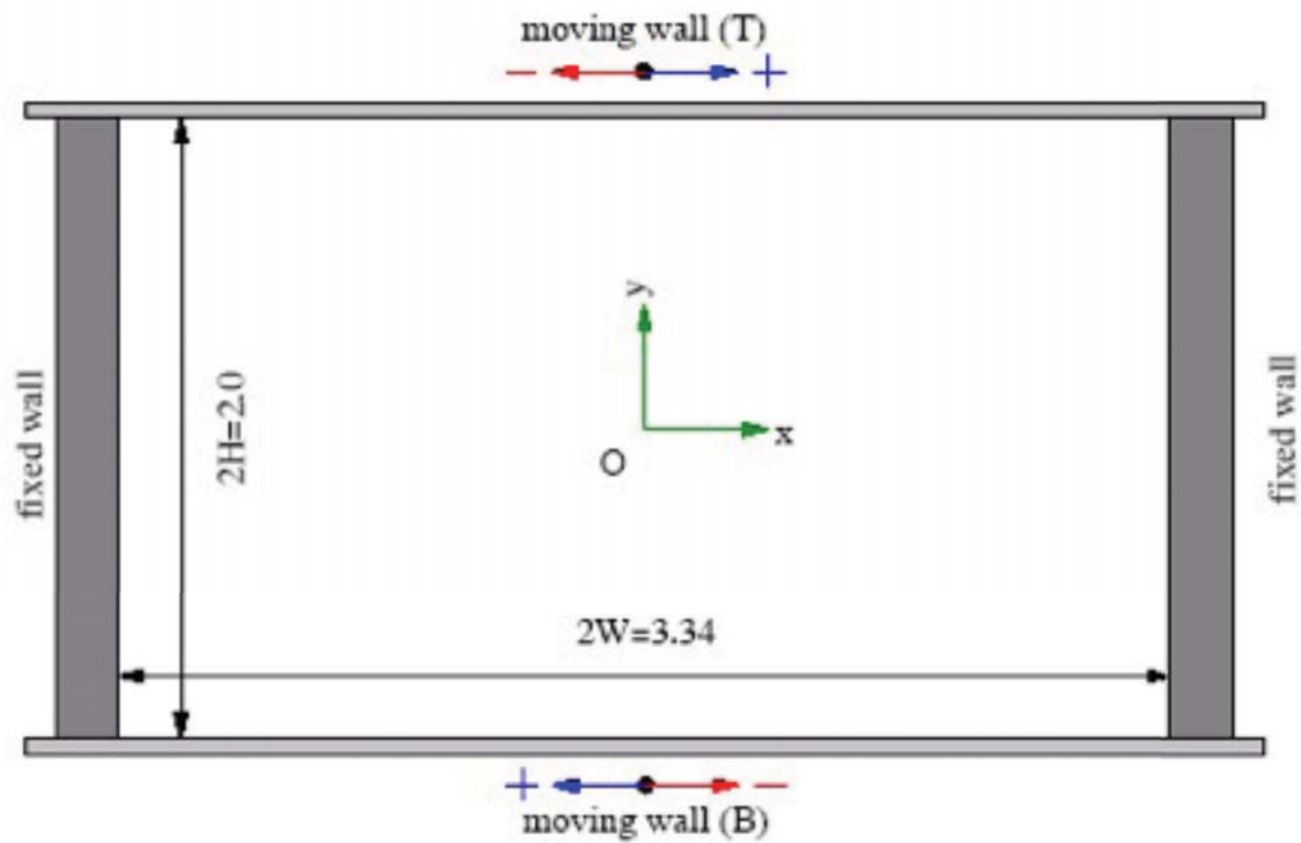


Ottino, *Scientific American* 1989



Steady two-dimensional flows are cannot mix well (no way to cross streamlines)

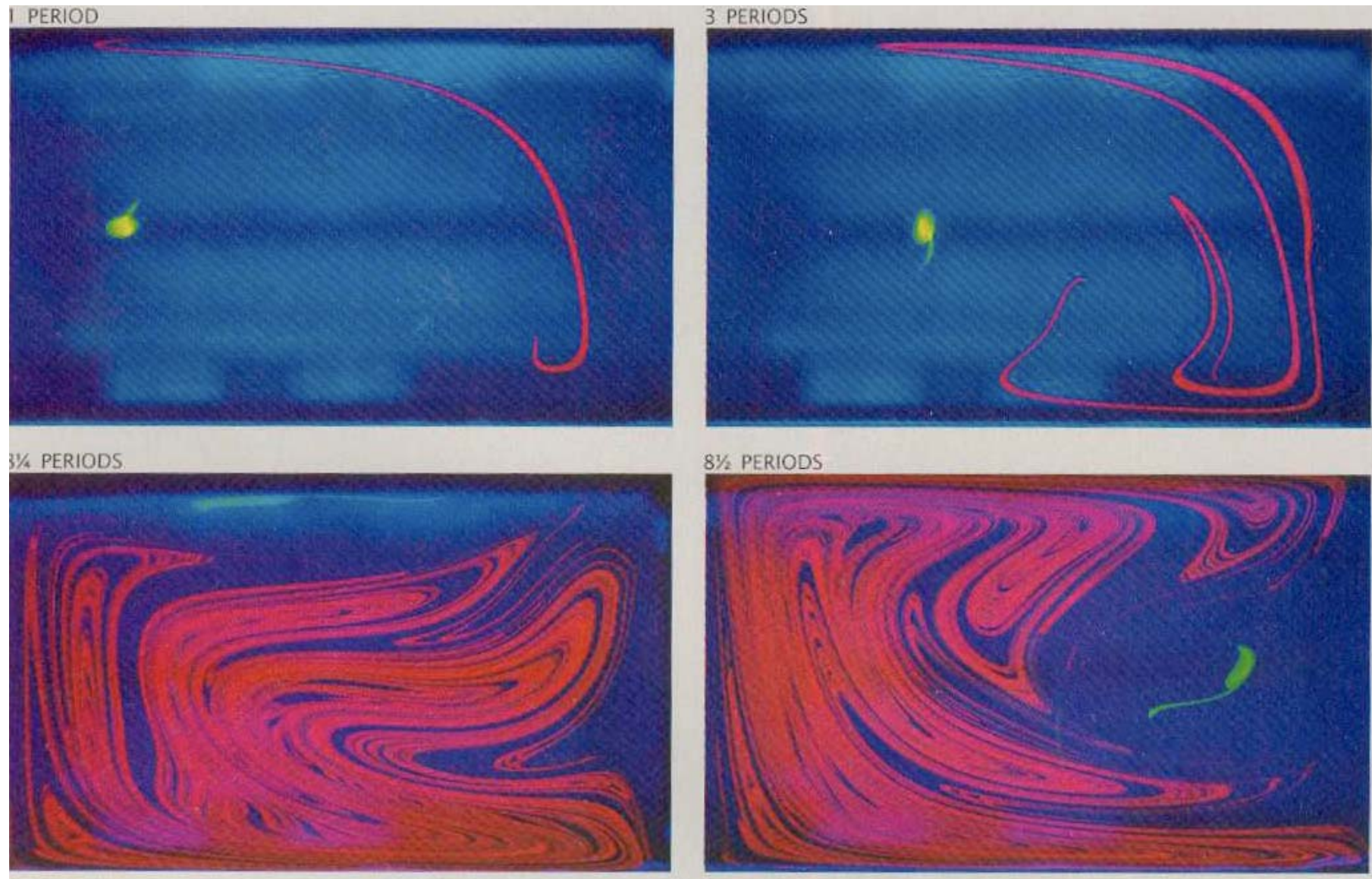
but, Aref (J Fluid Mech 1984) 2D time-periodic flows can mix effectively



Dimensionless displacement:  $D = \frac{VT}{2W}$ ,  $T$  is the duration of a period

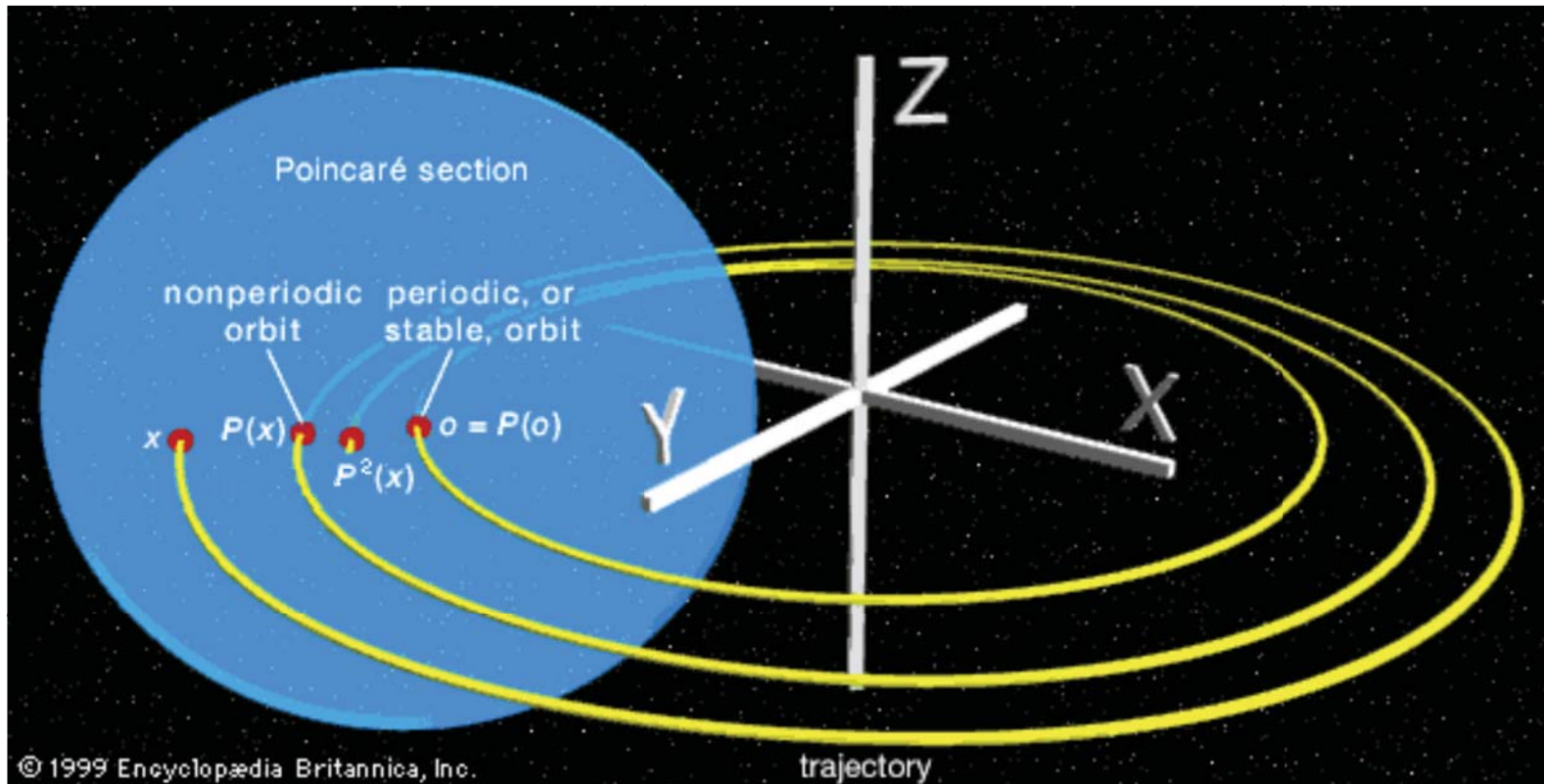


Add time-dependence (periodic motion of boundaries)  
well-mixed and not-well-mixed regions coexist



# Poincare sections

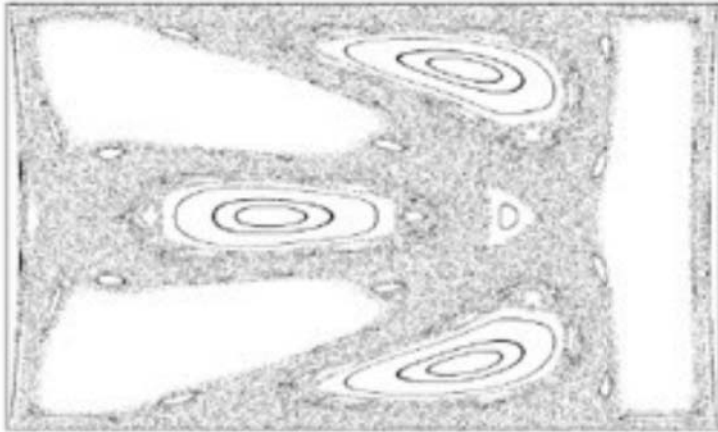
(reduces dimensionality by converting flow into a map; convenient way to show the character of solutions for all possible initial conditions)



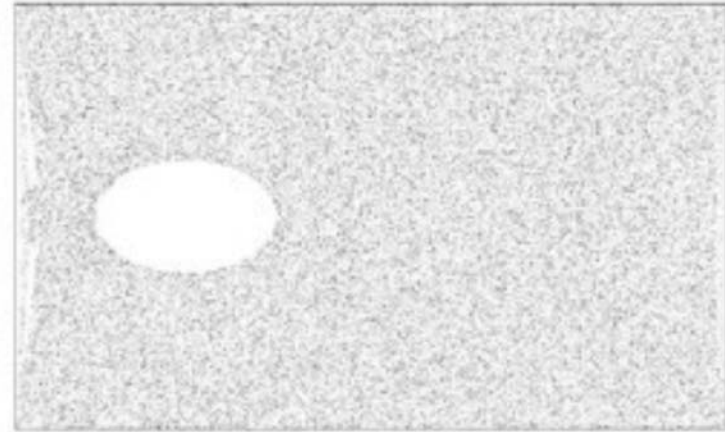


# Poincare sections

(reduces dimensionality by converting flow into a map; convenient way to show the character of solutions for all possible initial conditions)



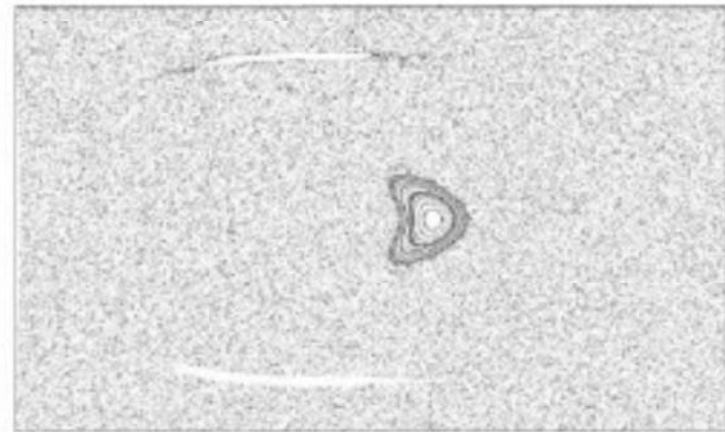
$D = 3$



$D = 6.24$

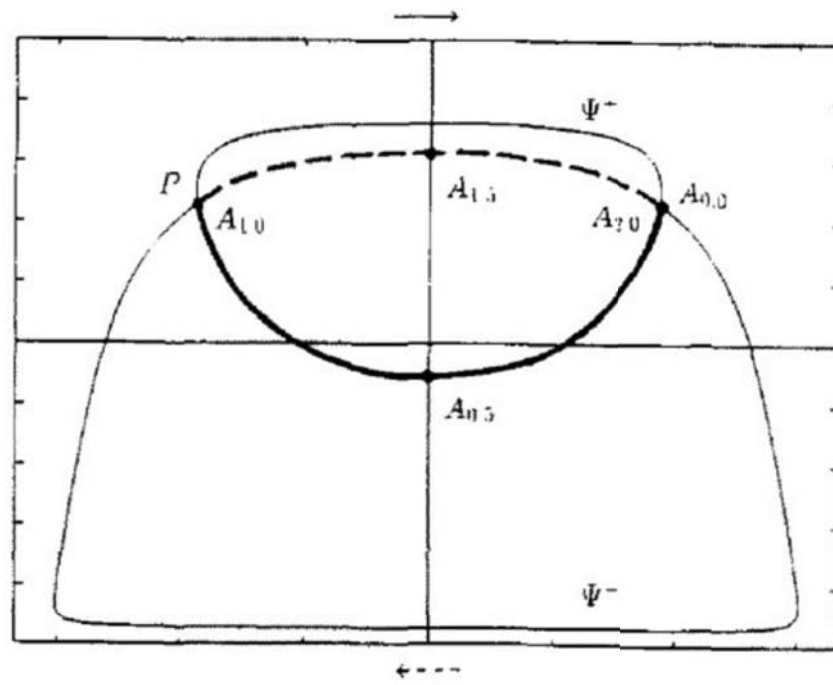


$D = 15$

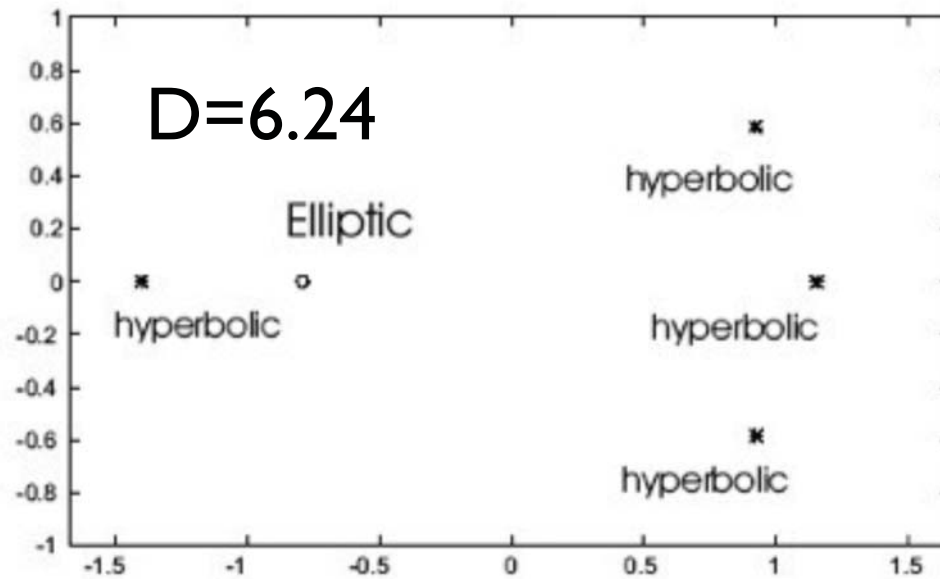
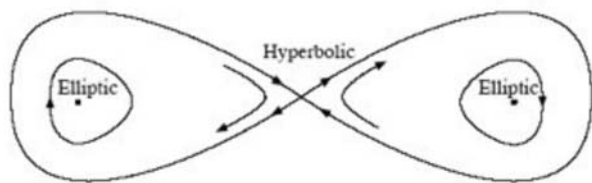


$D = 20$





first-order periodic points



# Stirring

Can produce complex structures AND unmixed islands

Under what circumstances does a deterministic flow widespread and efficient stretching of material surfaces (lines in 2D)?

(Mathematical) definition of chaotic flows

- The flow stretches and folds
- The trajectories of tracers are sensitive to initial conditions
- The flow has homoclinic and/or heteroclinic points
- The flow produces horseshoe maps

# Mathematical characterization of stretching

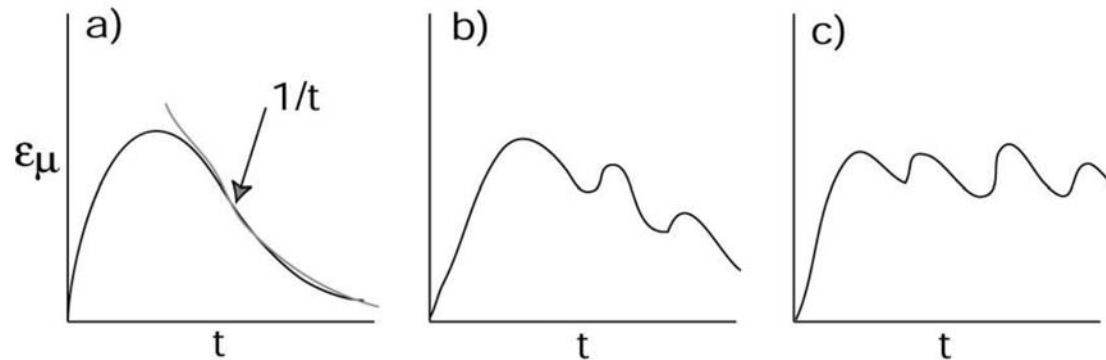
The magnitude of stretching is

$$\lambda = \lim_{|d\mathbf{X}| \rightarrow 0} \frac{|d\mathbf{x}|}{|d\mathbf{X}|}$$

The stretching efficiency is

$$e_\lambda = \frac{D(\ln \lambda)/Dt}{(\mathbf{E} : \mathbf{E})^{1/2}} \leq 1$$

For simple shear,  $e \rightarrow 0$  for large  $t$  (a random sequence of shears has a maximum of  $e = 0.28$ ). For pure shear,  $e \rightarrow 2/3$ , but this requires an unbounded fluid. Hence, for good mixing, we need reorientation.

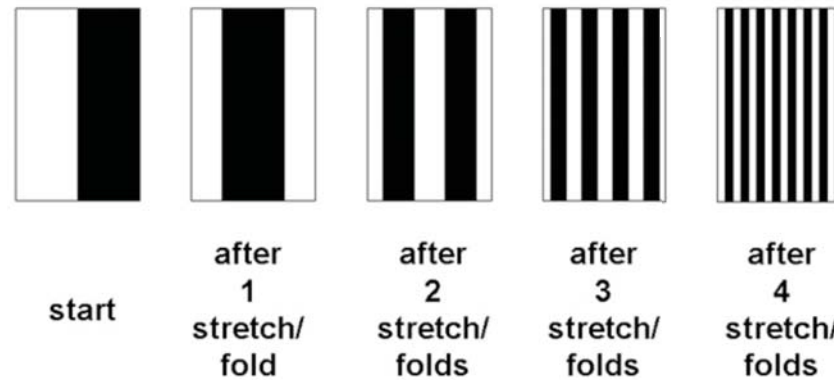
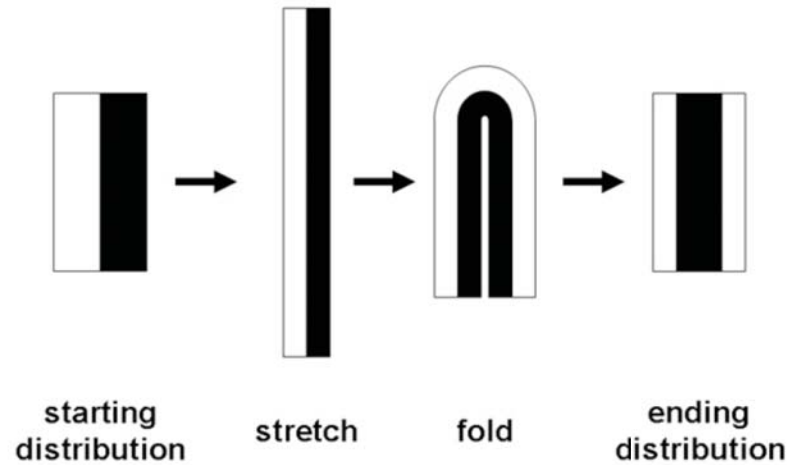


Another way to characterize mixing is with the Lyapunov exponents

$$\sigma = \lim_{|d\mathbf{X}| \rightarrow 0; t \rightarrow \infty} \left[ \frac{1}{t} \ln \lambda \right]$$

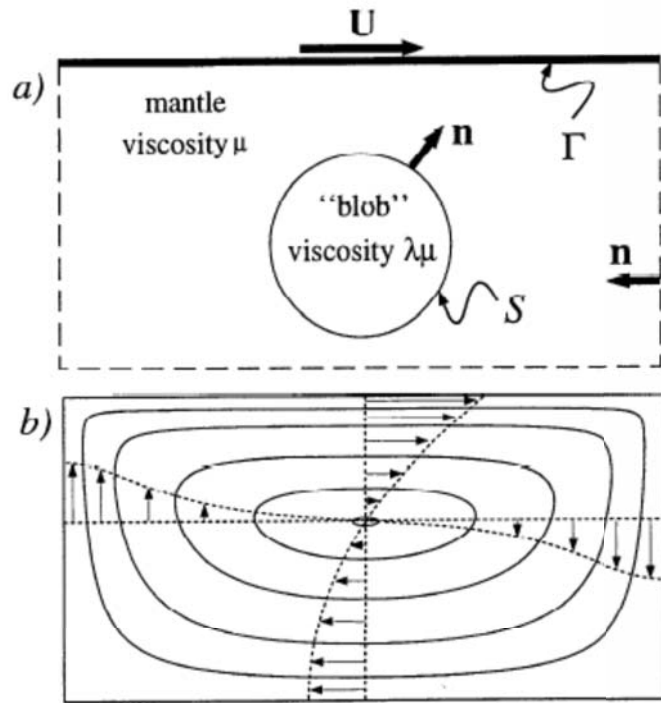
(not the same as  $e$  because  $\mathbf{E} : \mathbf{E}$  varies in space and time). At a given point there is one  $\sigma$  in each direction and the sum is 0. Worry about the largest one.

# Horseshoe maps

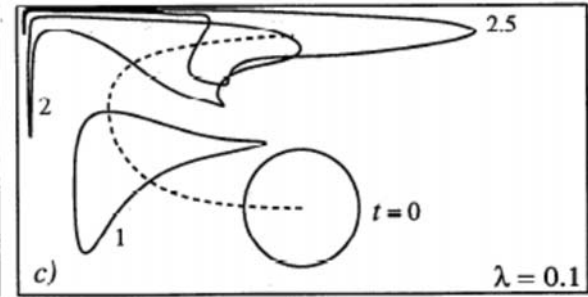
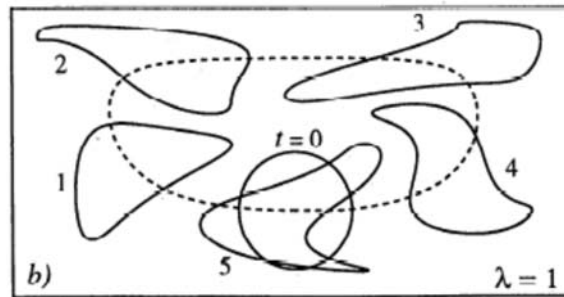
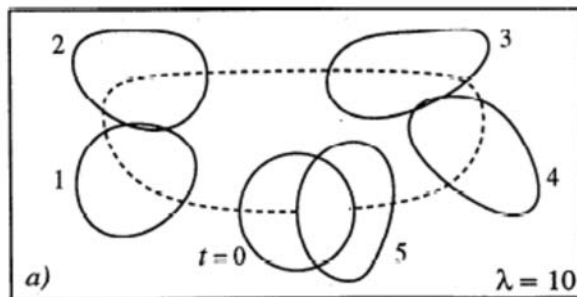


Flow must be capable of stretching and folding and returning it (stretched and folded) to its initial location – called a horseshoe map

# Active heterogeneity: viscosity differences affect stretching

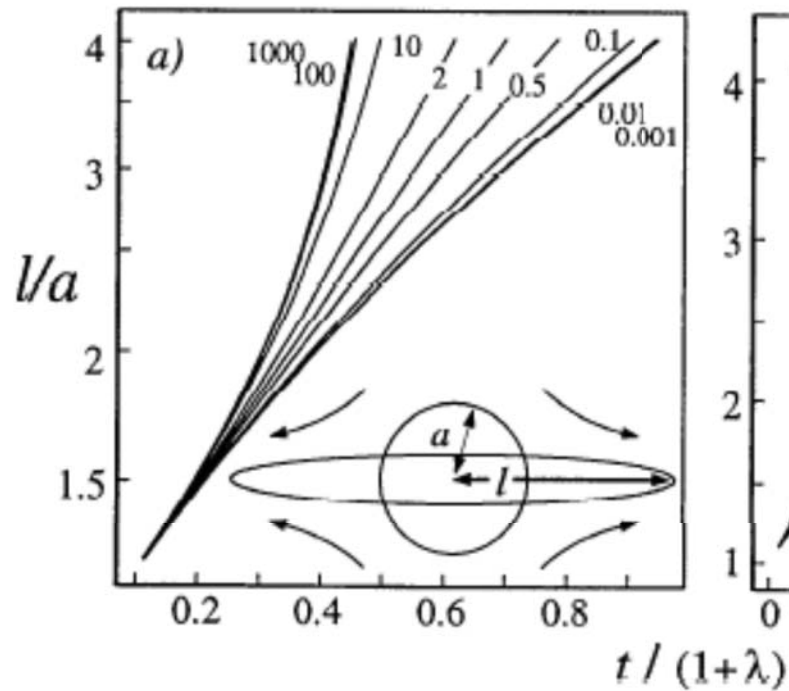


More viscous ← Isoviscous → Less viscous

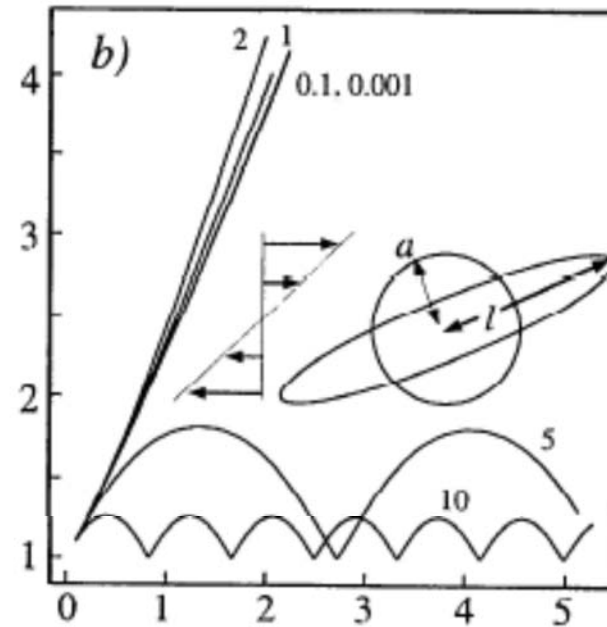


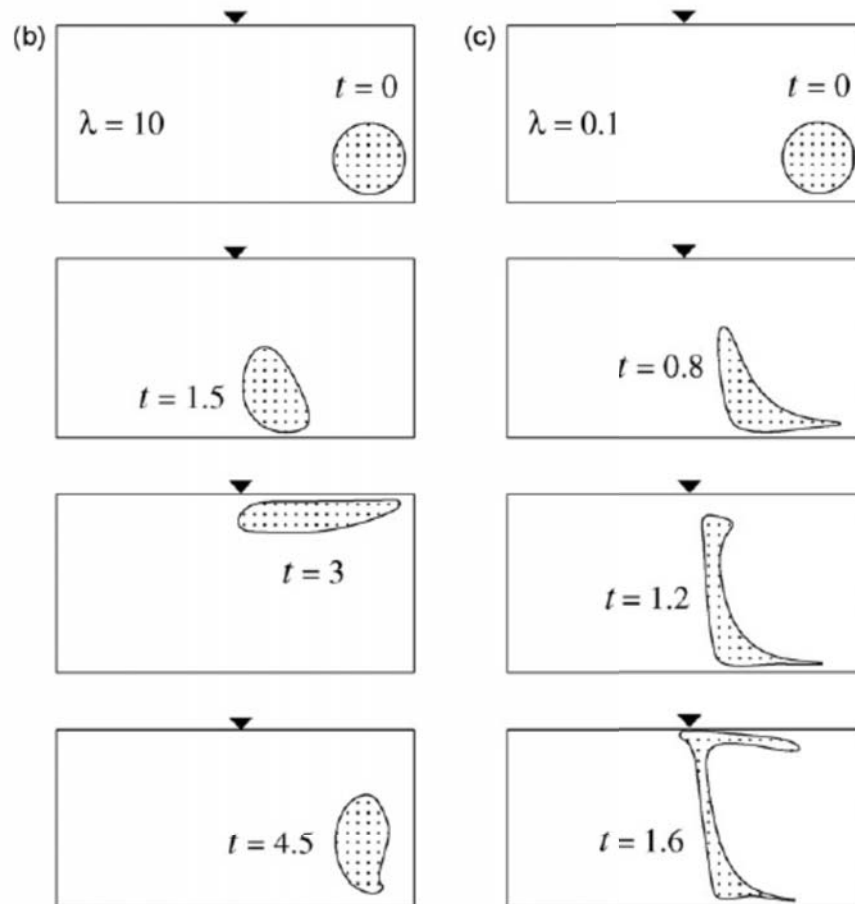
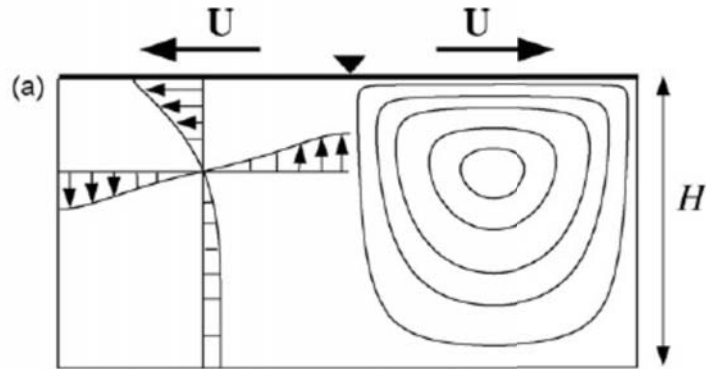
# Active heterogeneity: viscosity differences affect stretching

exponential stretching



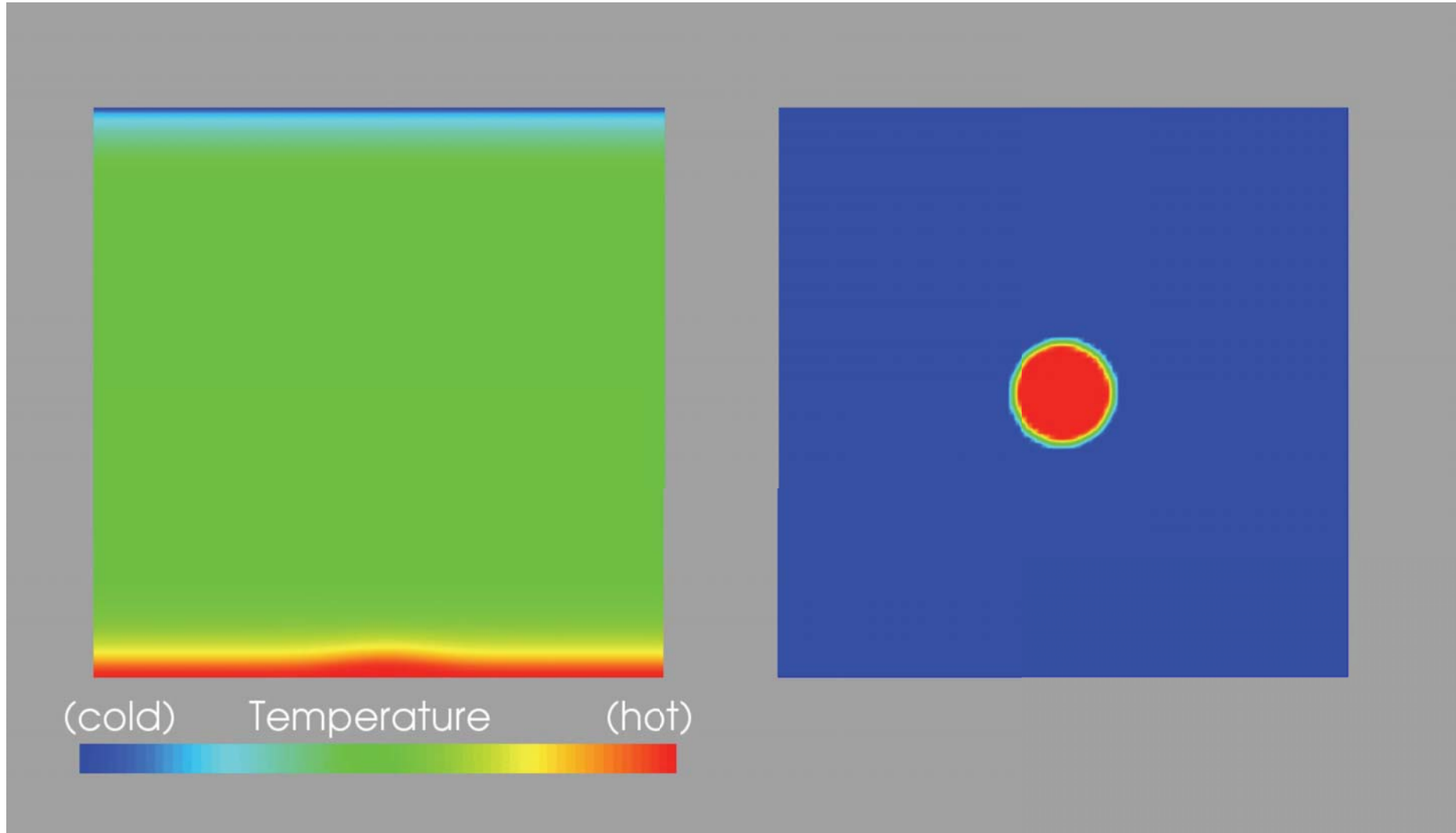
linear stretching





Active heterogeneity:  
viscosity differences  
affect stretching and  
hence flow

# Active heterogeneity: viscosity differences affect stretching and hence flow

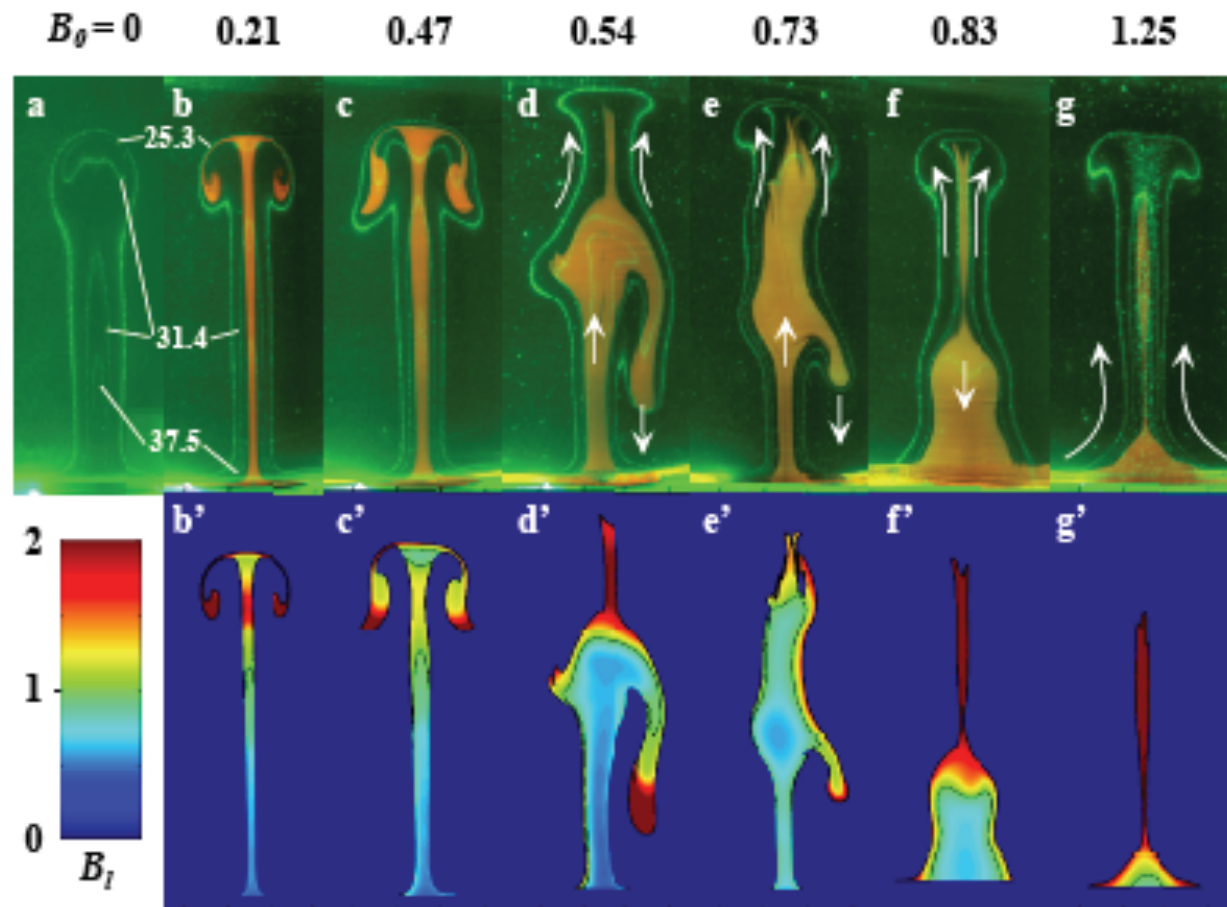


From Henri Samuel



# Active heterogeneity

density differences influences velocity field



$$B_0 = \Delta\rho_{\text{Xeff}} / \rho\alpha\Delta T_{\text{eff}}$$

Kumagai et al., *GRL* 2008

# Mixing in 3D

- Arnold (C R Acad Sci Paris Ser A 1965) showed that 3D steady flows can have chaotic streamlines)
- Steady, isoviscous thermal convection in a spherical shell, however, is not chaotic (Schmalzl et al. JGR 1996)
- Plate motion changes this story . . .

# Mixing associated with plate motion

## Poloidal vs toroidal flow

- Poloidal flow: no vertical (radial) vorticity
- Toroidal flow: rotations in horizontal (confined to spherical shells) plane

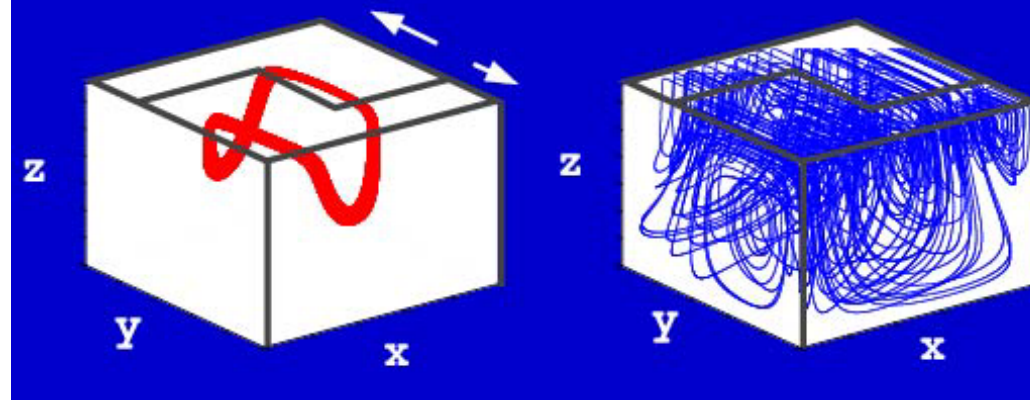
Surface manifestations

Poloidal motion: ridges and trenches

Toroidal motion: transform boundaries

Roughly equal in magnitude

## Examples of trajectories

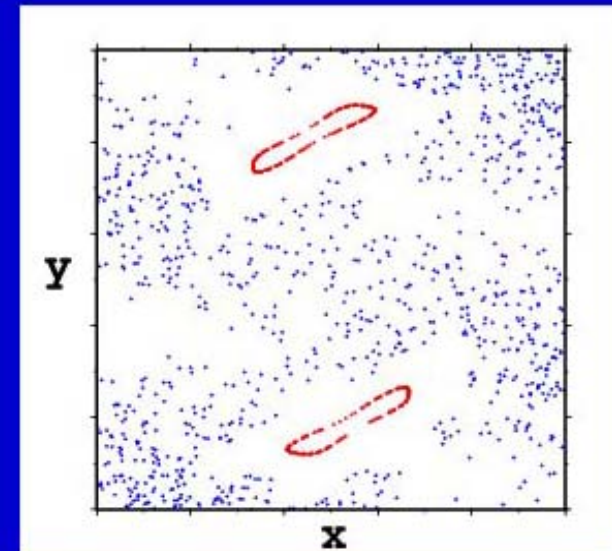


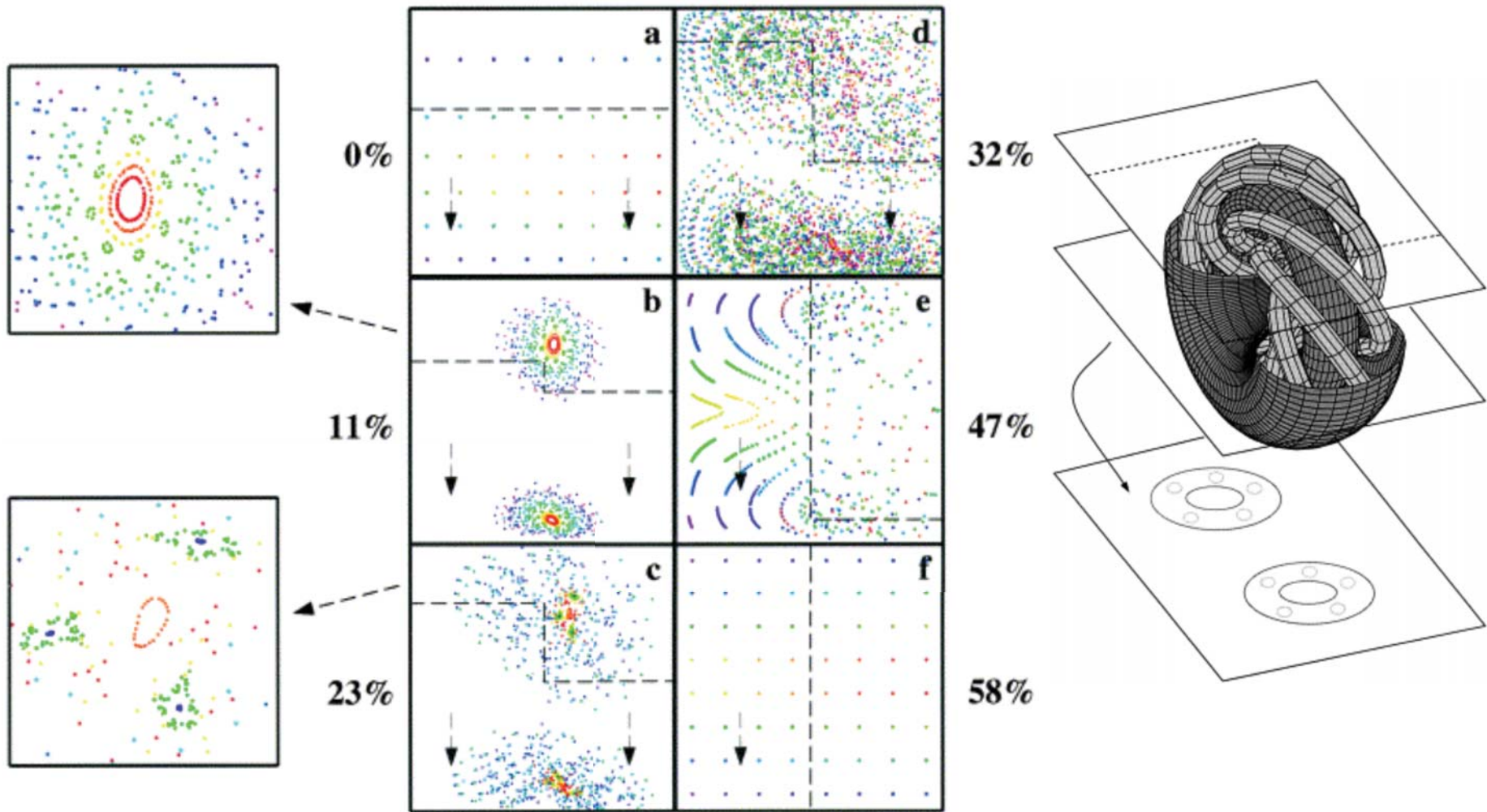
Ferrachat and Ricard, *JGR* 2001

Chaotic trajectories in steady-state plate driven flows

Why? Hyperbolic points do the stretching, toroidal motion does the reorientation

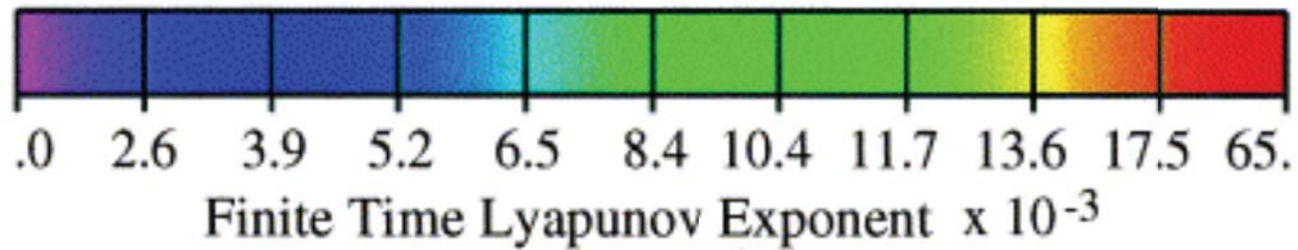
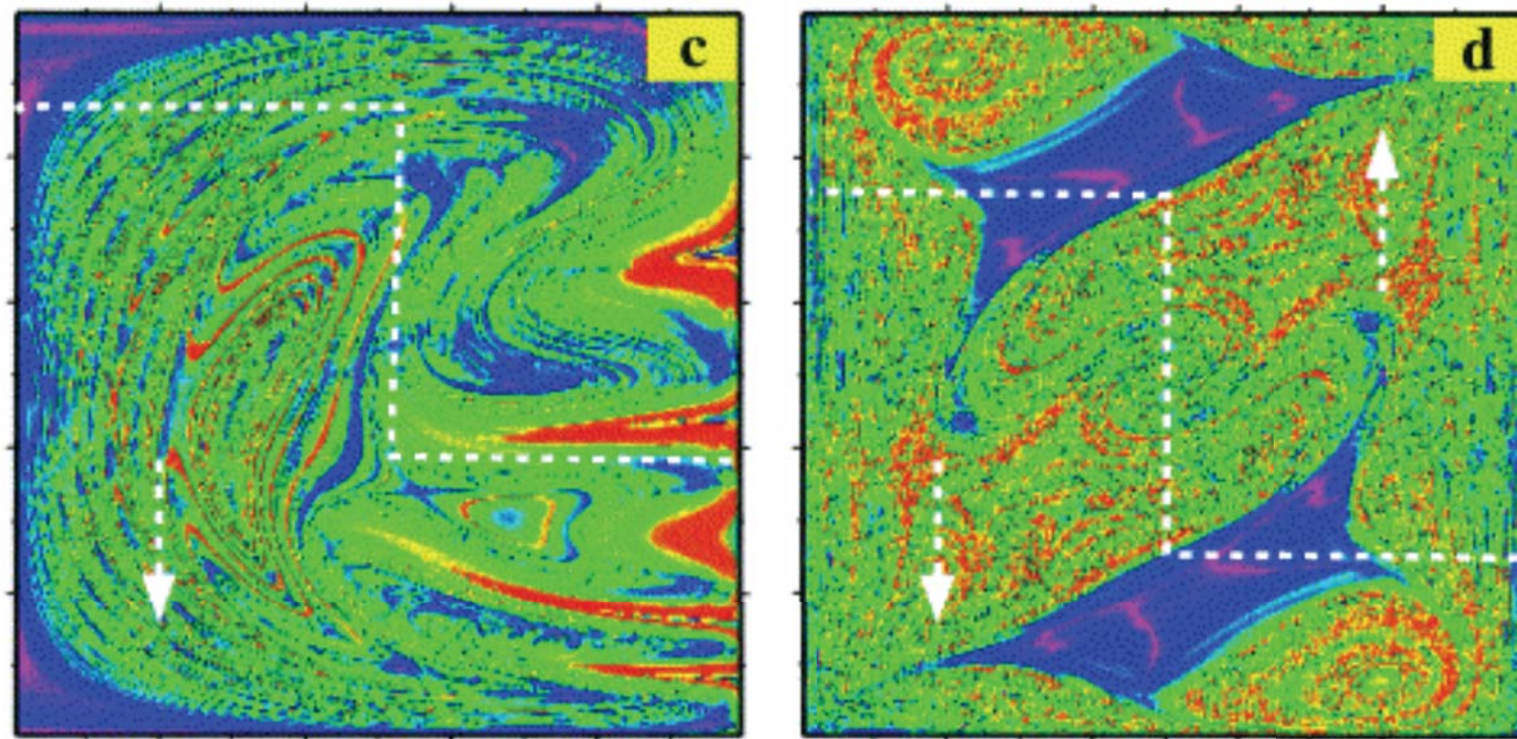
Corresponding Poincare section:





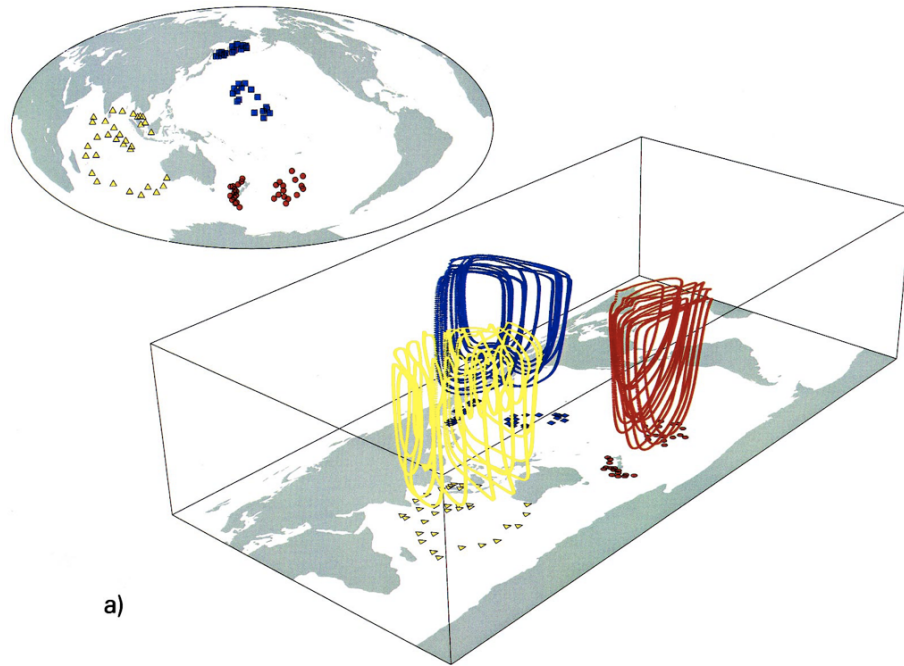
Ferrachat and Ricard, *JGR* 2001





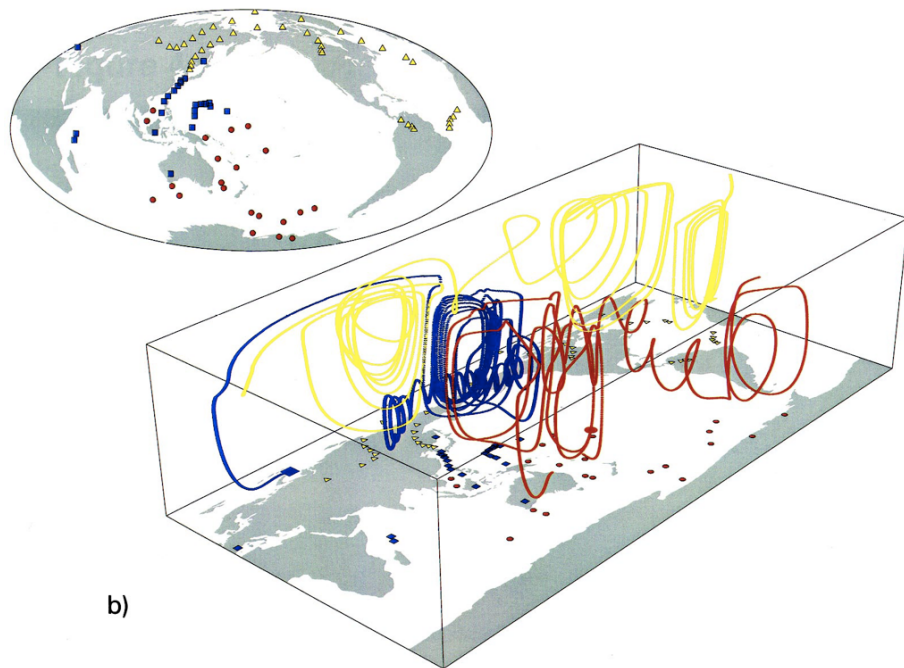
Lyapunov exponents  $\sigma$  estimated by tracking tracers:  
 Both chaotic and laminar mixing are observed

Ferrachat and Ricard, *JGR* 2001



With plate motion, well mixed and poorly mixed regions

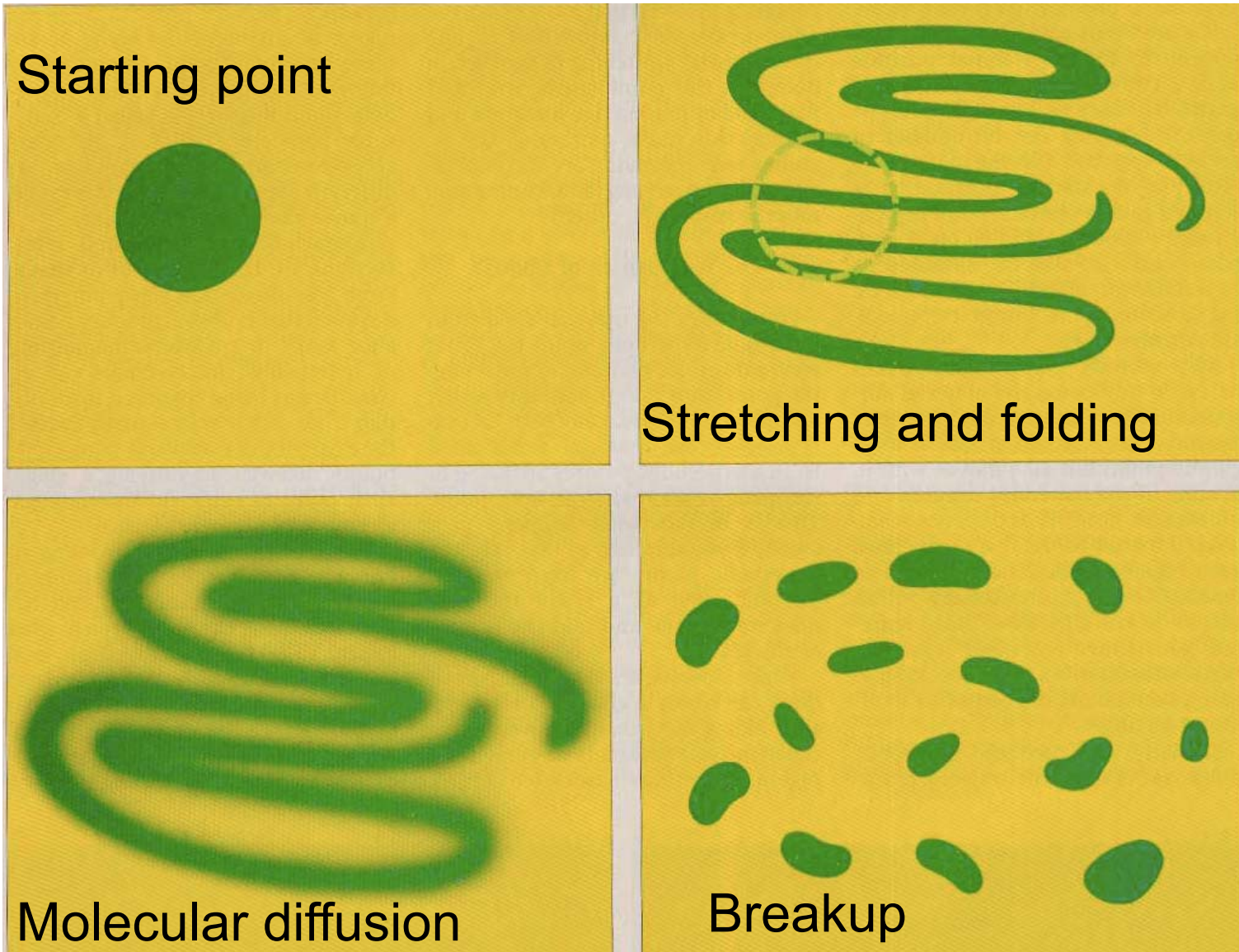
Take steady flow driven present day plate motion and trace particles for 4 Ga



van Keken and Zhong, EPSL 1999



# How does mixing occur?





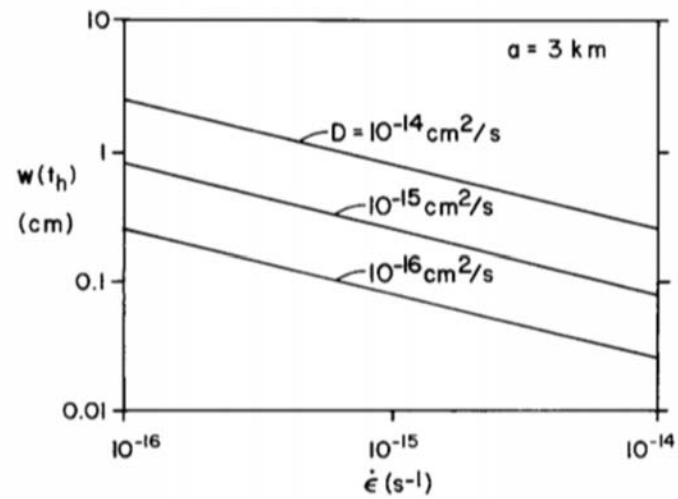
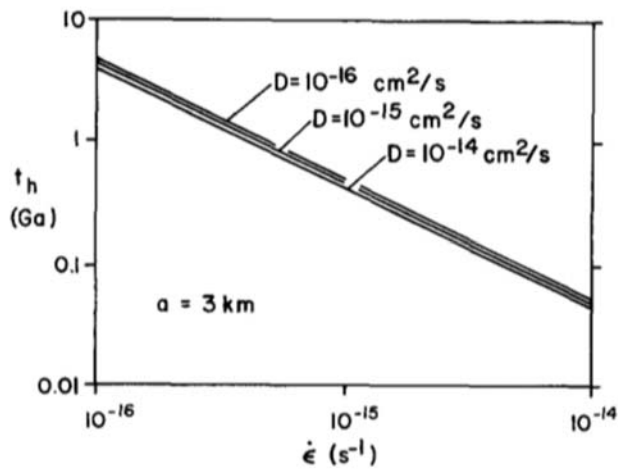
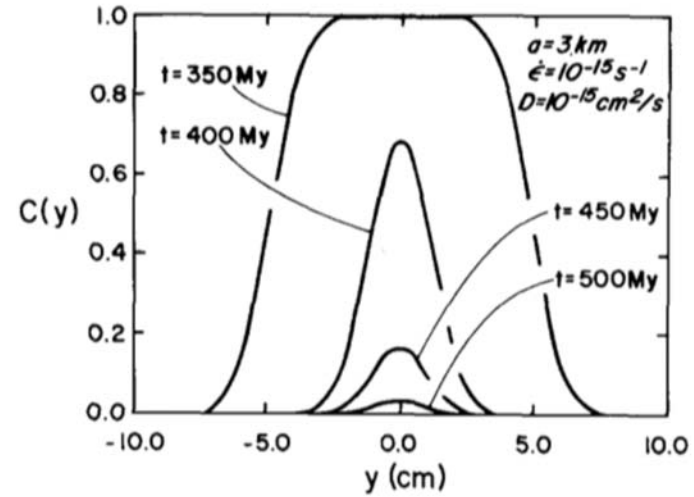
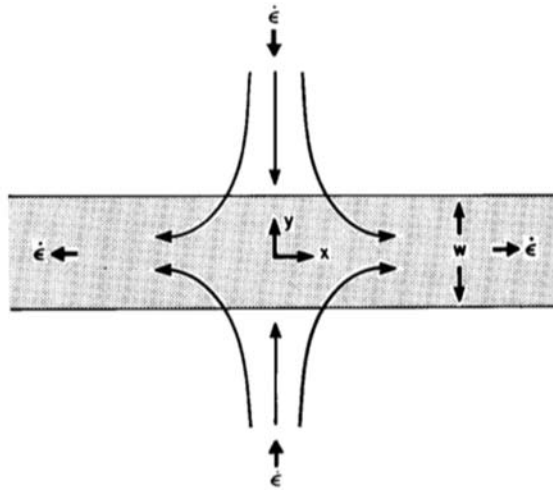
What about chemical diffusion?

$$\begin{array}{ccc} & \frac{DC}{Dt} = \kappa \nabla^2 C & \\ \swarrow & & \searrow \\ O(\Delta C/t) & & O(\kappa \Delta C/\delta^2) \\ & \delta \sim \sqrt{\kappa t} & \end{array}$$

Diffusivities are  $10^{-18} - 10^{-20}$  m<sup>2</sup>/s in the mantle  
 $10^{-11}$  m<sup>2</sup>/s in magmas

In 4 Ga, diffusion over  $< 1$  m in the mantle  
In 30 ka, diffusion over 1 m in magmas

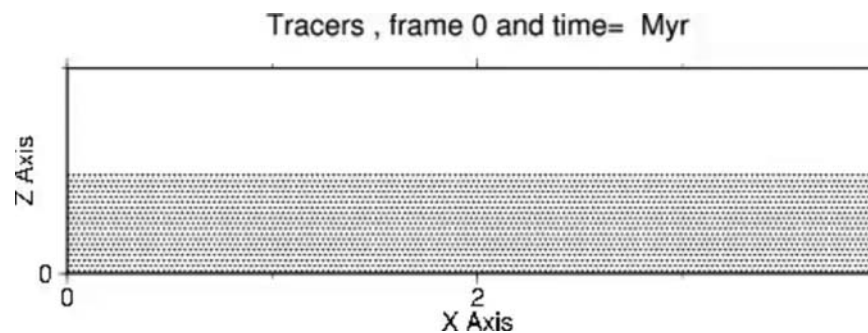
# What about chemical diffusion?



Kellogg and Turcotte, 1987 EPSL

# Some ways to analyze mixing in models of the mantle

- Dispersal of heterogeneities (visually or using statistical methods)
- Compute derived isotopic signatures



# 4 Ga of processing mantle at ridges (Geoff Davies)



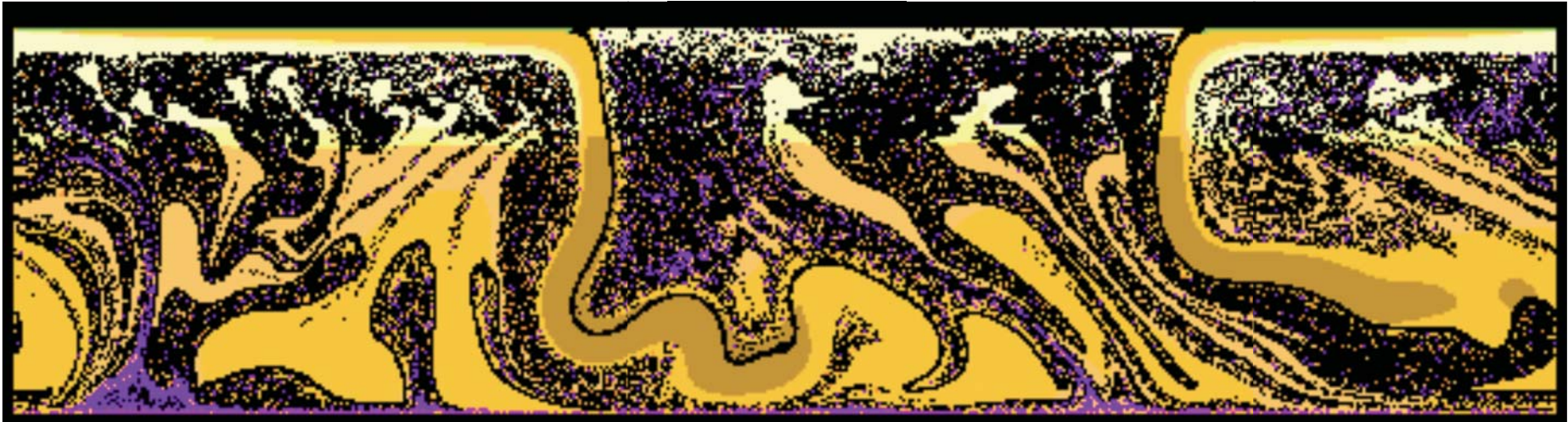
250,000 tracer particles (initially orange)

Crust (stuff melted below ridges) in black

Crust that gets within 20 km of the CMB in purple

Darkness scales with viscosity

# Stirring and segregation (Geoff Davies)



Tracers are more dense than surroundings  
Segregation of depleted mantle from crust

# Mixing – simplest analysis

(time, no spatial dimensions)

## Mid-ocean ridge mass flux

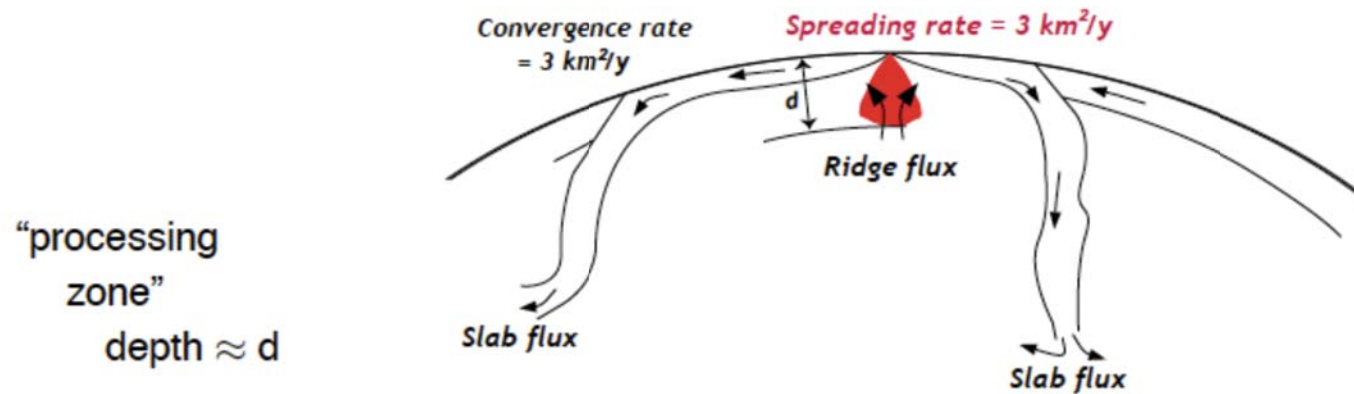


Plate creation rate  $a \approx 3 \text{ km}^2/\text{y}$   
 Mass flux in zone  $\dot{M}_r = ad\rho$   
 Mass of upper mantle  $M_{um} = 1 \cdot 10^{24} \text{ kg}$

“Turnover” time:  $\tau_{um} = M_{um}/\dot{M}_r$

From Rick O’Connell, Harvard

depth	$\tau_{um}$ (upper mantle)	$\tau_{em}$ (entire mantle)
50 km	2 Gy	8 Gy
100 km	1 Gy	4 Gy



## Ridge migration over mantle

Ridges move  
relative to  
one another

Total ridge length  $L \approx 56,000$  km  
Ridge migration rate  $u \geq 2$  cm/y  
Surface area  $A = 5 \cdot 10^{14}$  m<sup>2</sup>

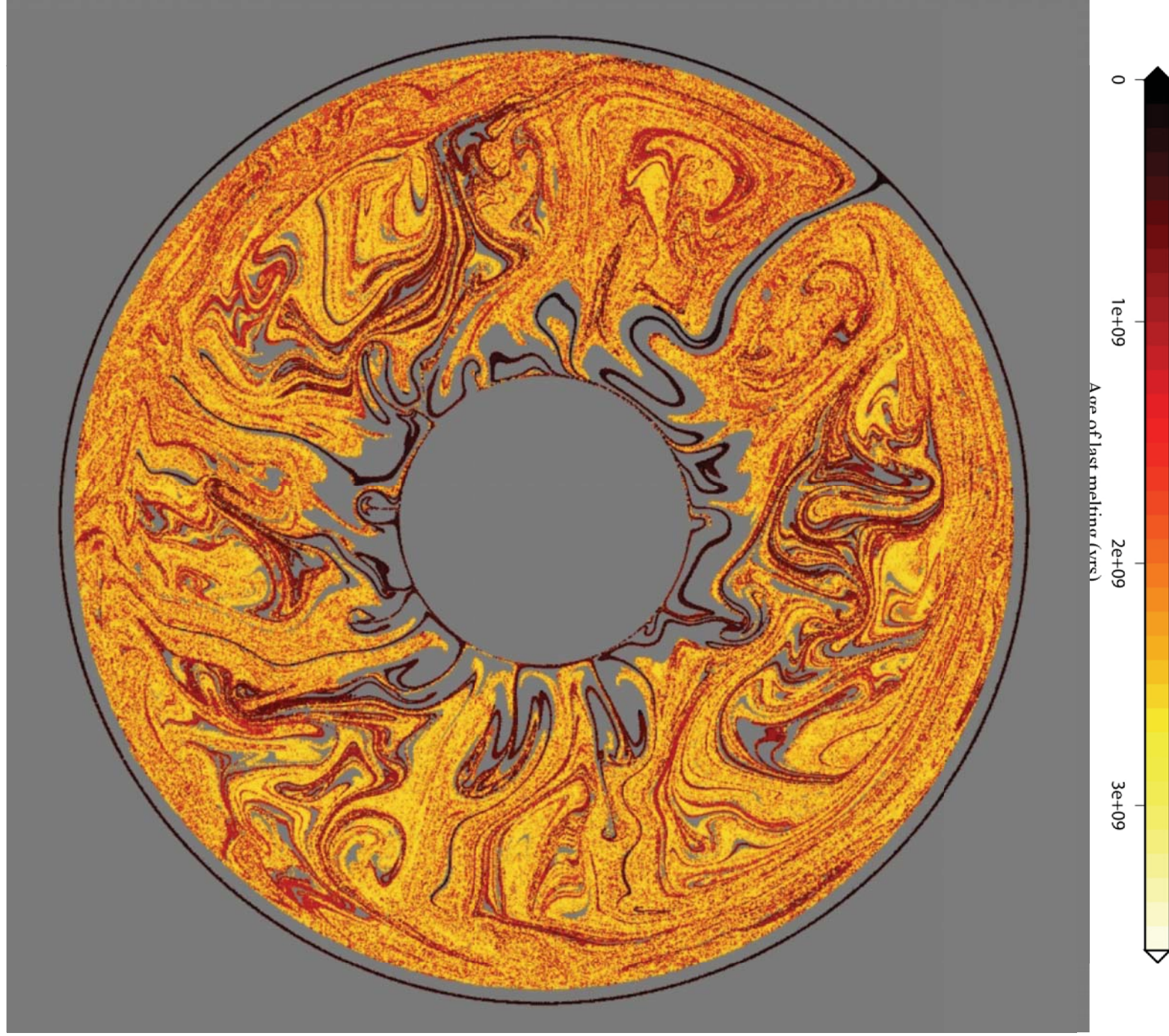
*Ridges sweep over Earth's surface  
with time scale:*



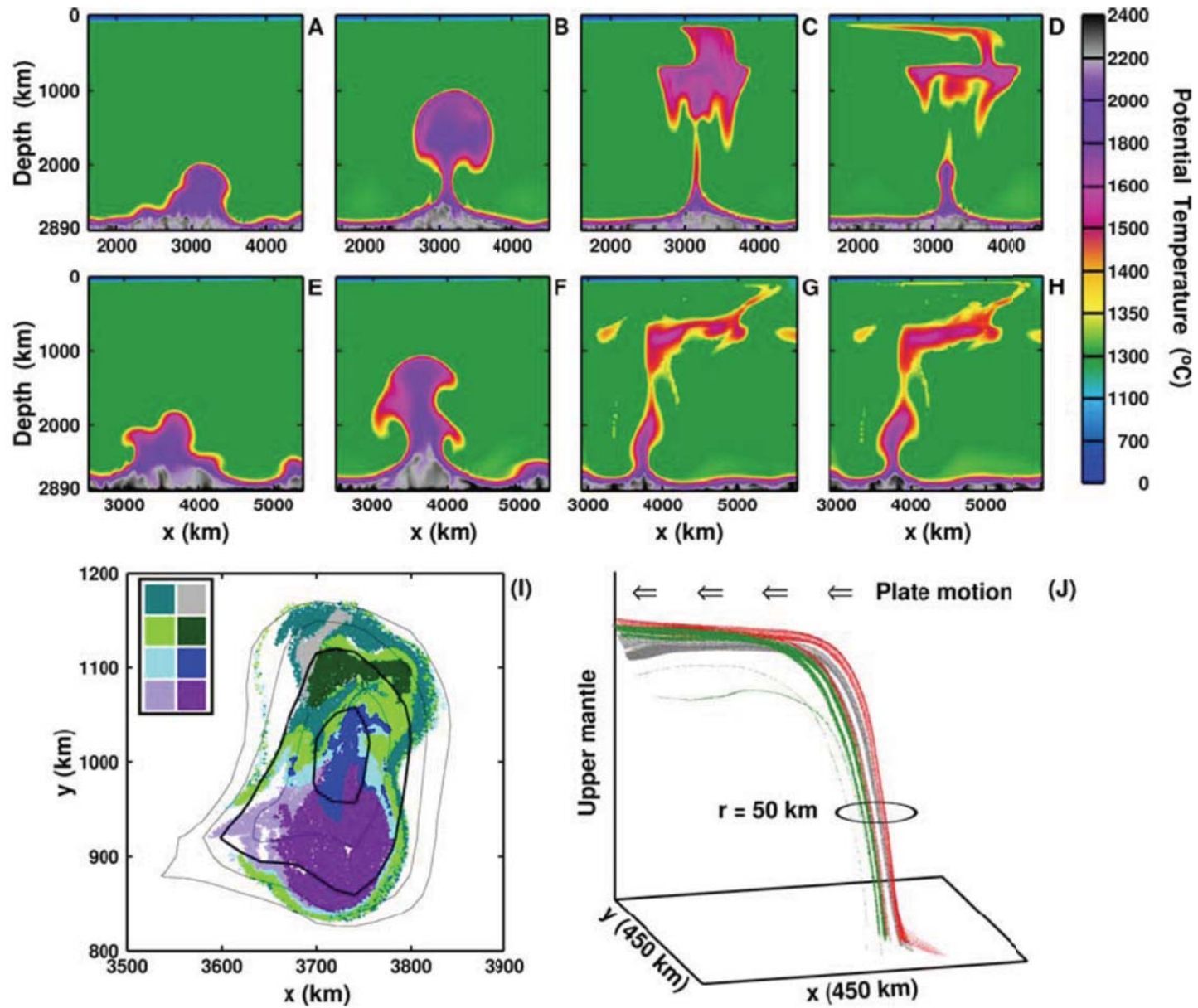
“Passover” time, whole Earth  $\frac{A}{Lu} \leq 500$  My

From Rick O’Connell, Harvard

Brandenburg et al., *EPSL* 2008



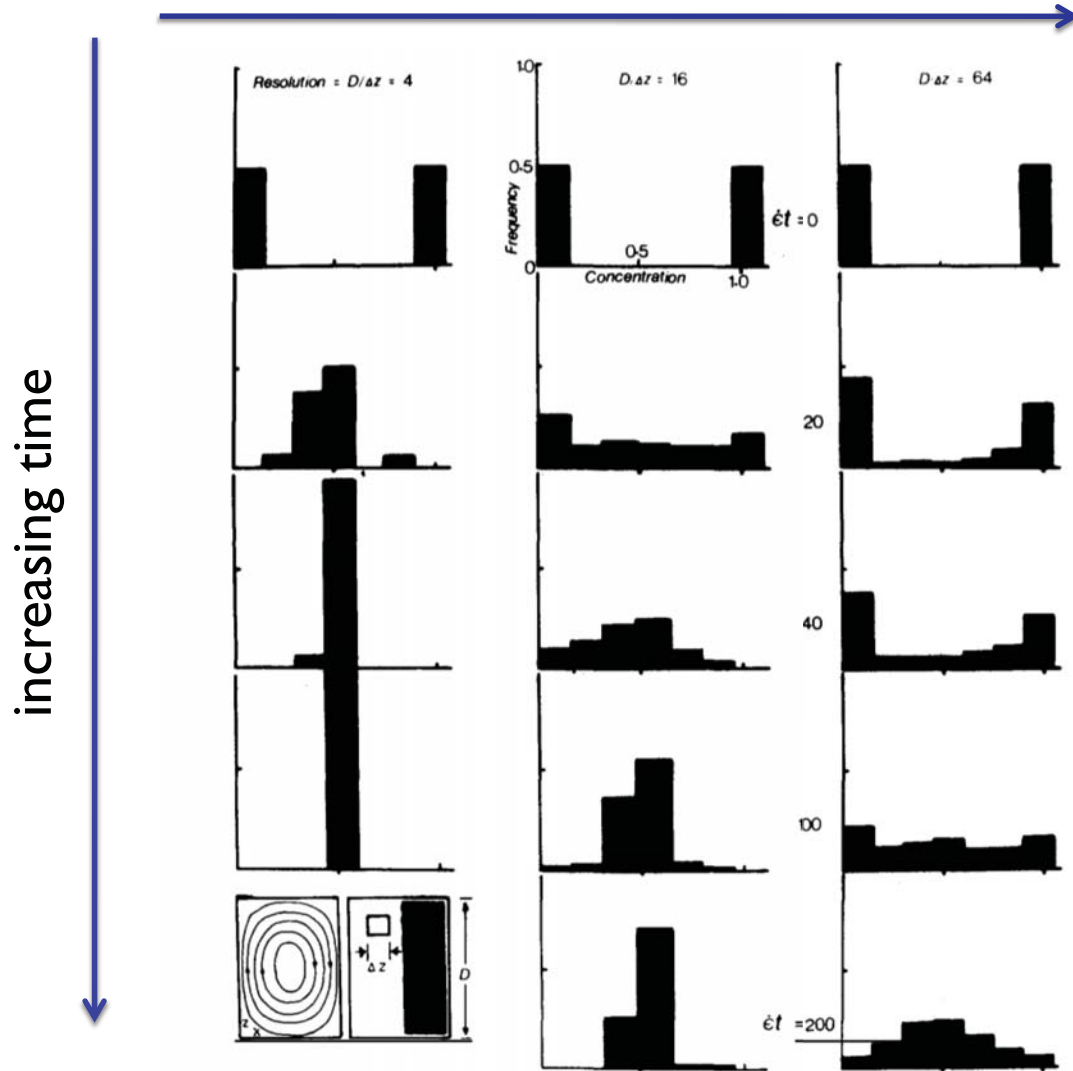




Farnetani and Samuel, *GRL* 2005

# Sampling filter

decreasing sampling volume



Olson et al., *PEPI* 1984

Fig. 7. Histograms of anomalous concentration versus time and sample resolution for a large scale heterogeneity in Bénard convection. The insert shows the initial distribution.

# Characterization of structure

“Important to distinguish between mixing measure and the process producing mixing . . . The measure should be selected according to the application, and the measurements should be related to the fluid mechanics.” Ottino, *Kinematics of mixing* 1989

- $e$  and  $\sigma$  characterize effectiveness of a given *flow* at stirring
- Other measures can be used to characterize observed *structures* (e.g., spectral analysis, fractal analysis)
- Easiest: striation thickness  $s$  (ID)
- Use characterization of structure can be used to distinguish between mixing processes

# Evidence for length scale reduction in the mantle, recorded in an exposed peridotite

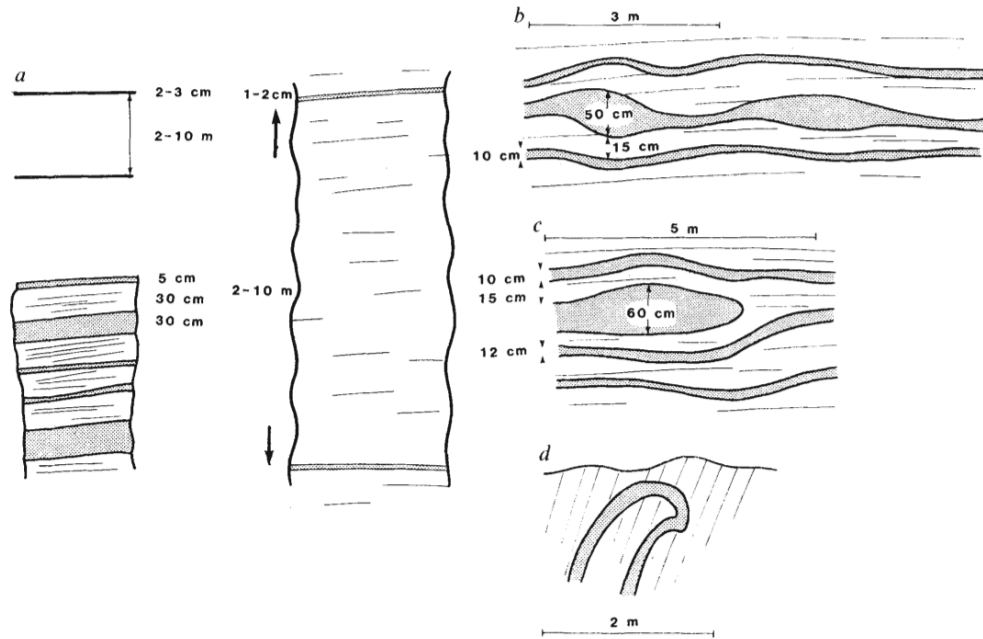


Fig. 2 Occurrences of pyroxenite layers in the Beni Bousera high-temperature peridotite. Grey, pyroxenite; white, lherzolite with foliation. a, Occurrences in an outcrop with no folding; b-d, occurrences with folding and boudinage.

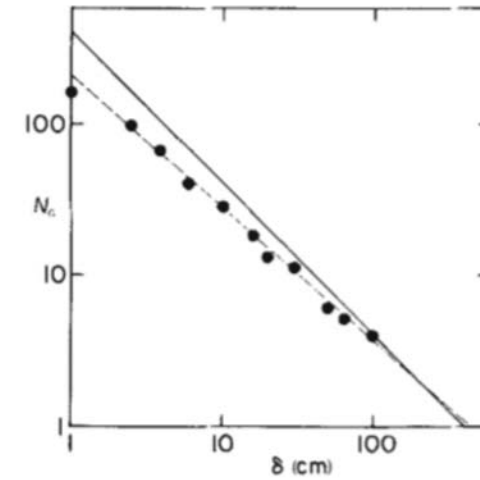


Fig. 3 Number of pyroxenite layers exposed at Beni Bousera with a thickness greater than  $\delta$ ,  $N_c$ , as a function of  $\delta$ . Points, observations; dashed line,  $N_c \propto \delta^{-0.87}$ ; solid line,  $N_c \propto \delta^{-1}$ .

Allègre and Turcotte, *Nature* 1986

The scale of heterogeneity led Allègre and Turcotte (1986) to support their 'marble cake' structure to the mantle



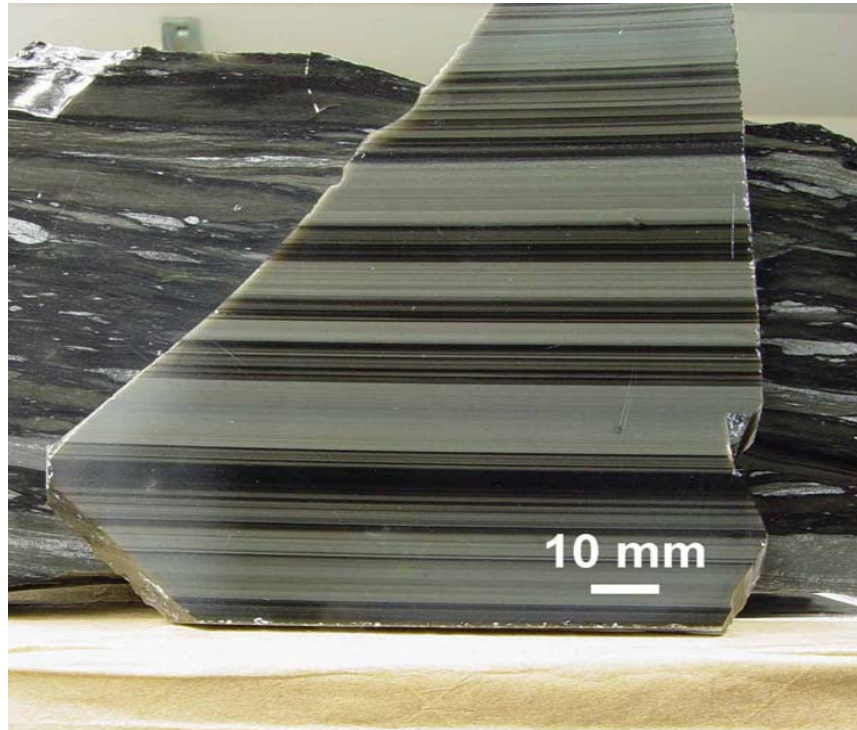
Static.ifood.tv



Easier to see in magmas . .



## Obsidian is banded at all scales



Do these bands (in some cases) record how the obsidian deformed?

# Terminology

*Scale invariance:* Attributes do not change if lengths are changed (no specific scale can be identified - all scales are equally important)

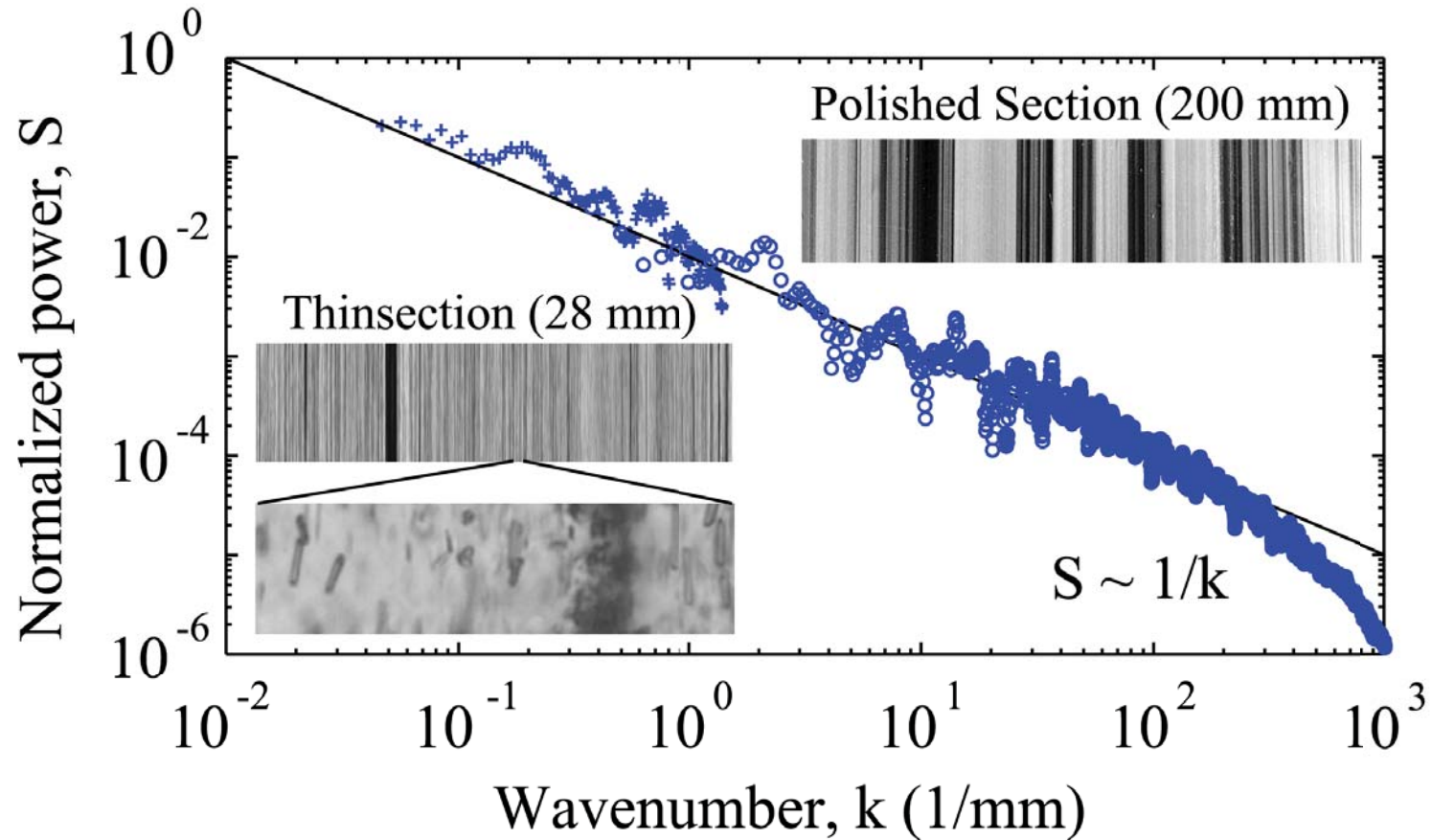
*Fractal:* A fractal is generally "a rough or fragmented geometric shape that can be split into parts, each of which is (at least approximately) a reduced-size copy of the whole," [1] a property called self-similarity. Roots of mathematical interest on fractals can be traced back to the late 19th Century, the term however was coined by Benoît Mandelbrot in 1975 and was derived from the Latin fractus meaning "broken" or "fractured."

*Multifractal:* A single exponent is insufficient, and a continuous spectrum of exponents is needed; around any point, there is a local power law and the "singularity distribution" describes its variation

*Multiplicative:* recursive process that produce interdependencies in different scales, results in multifractal properties

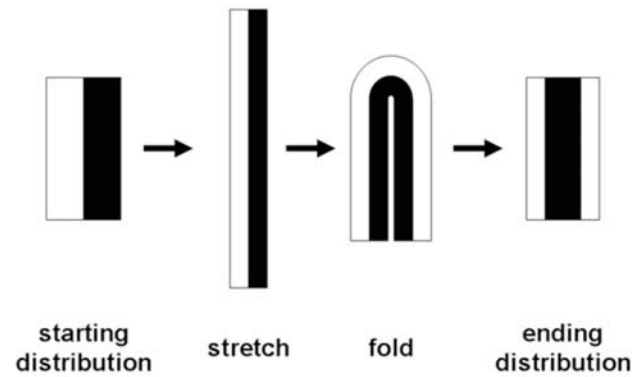
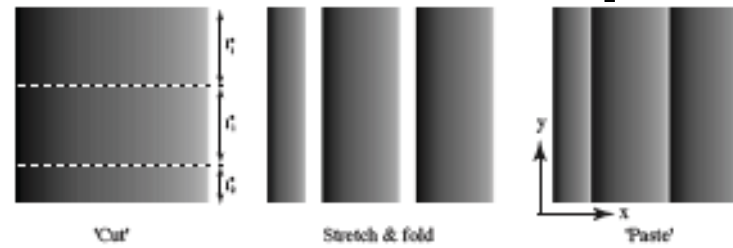


# Power spectrum: Scale invariant banding

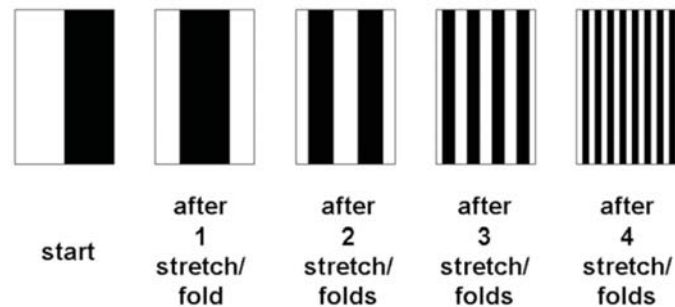


Band widths are scale invariant over 4 orders of magnitude

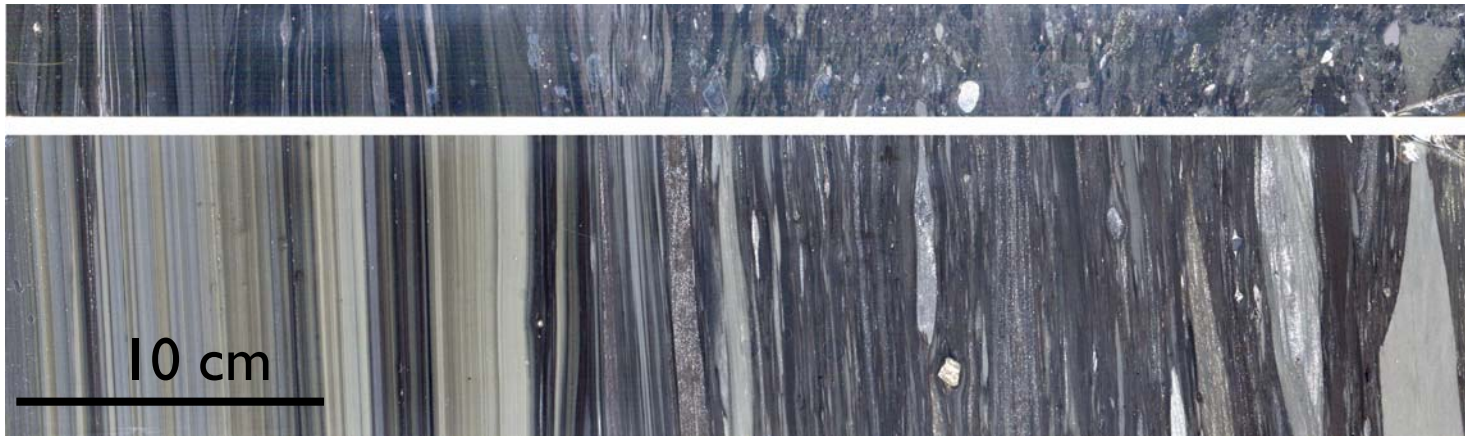
# Baker's map



## Horseshoe maps



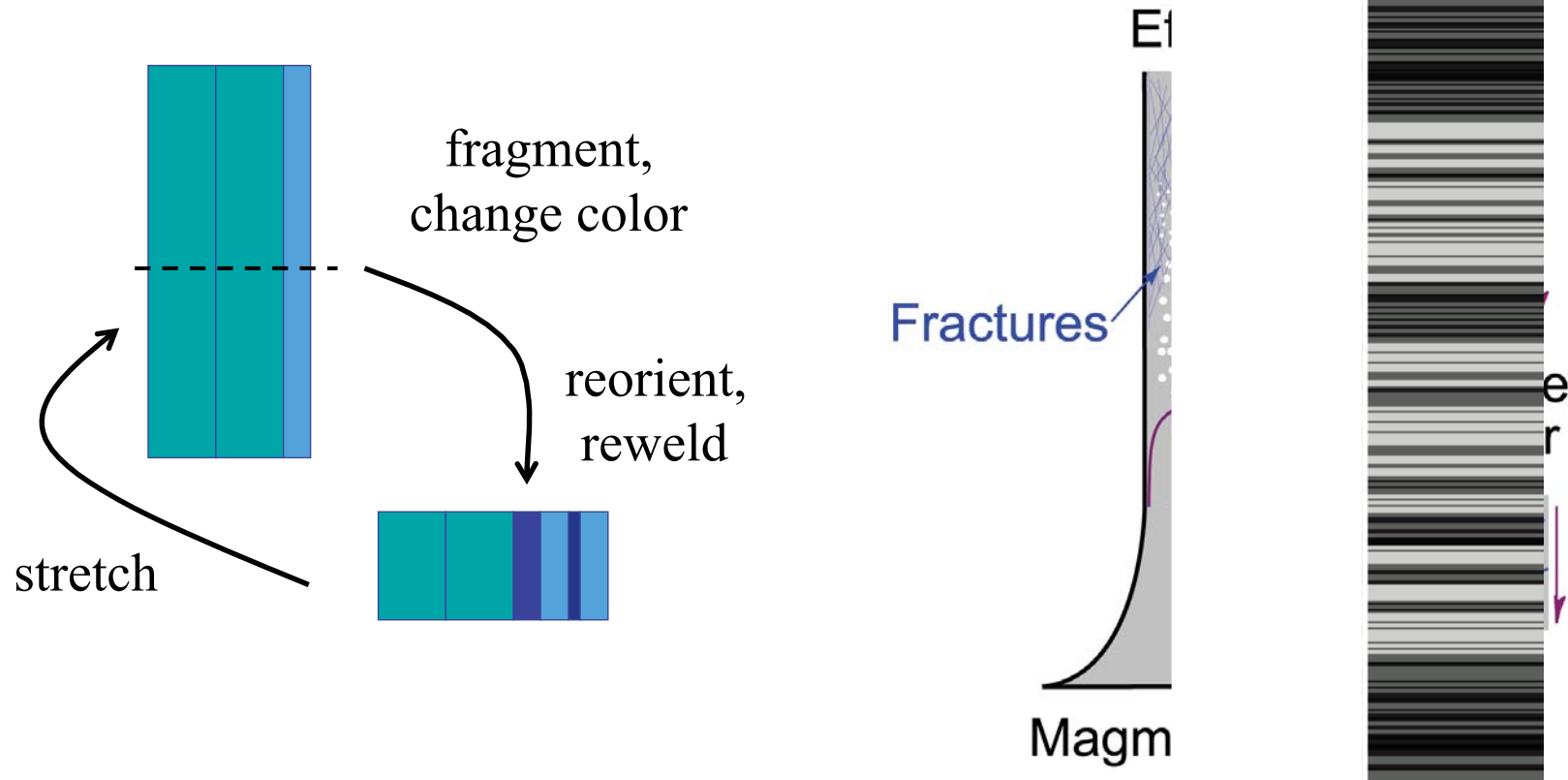
# Brecciation, rewelding and deformation



## Cantor set



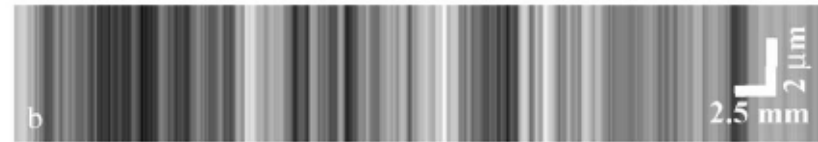
# A representative model Cantor model



Bands consistent with repeated brecciation, reorientation of fragments, welding (stick back together) and stretching (reproduce power law and multifractal characteristic of bands)



BGM (200 mm)



MI (55 mm)



Cantor set



B&W Cantor set



BGM color sorted



Random redistribution



4 step Baker



6 step Baker

Table 1  
Comparison of obsidian samples

Record	MF <sup>a</sup>	$(S \sim k^{-1})^b$	MP <sup>c</sup>	Implications
Big Glass Mountain (BGM)	Y	Y	Y	Concurrent microlite growth and deformation into bands
Mayor Island (MI)	Y	Y	Y	Concurrent formation of variable vesicularity and deformation
Cantor (MC)	Y	Y	Y	Concurrent development of heterogeneity and deformation
Cantor binary	N	N	N	No binomial measure
BGM randomized	N	N	N	Decoupled microlite growth and deformation into bands
Baker's map	Y	N	N	Decoupled microlite growth and deformation into bands

Cantor map with hypothesis tests.

<sup>a</sup> Multifractal.

<sup>b</sup>  $S$  is spectral power and  $k$  is wavenumber.

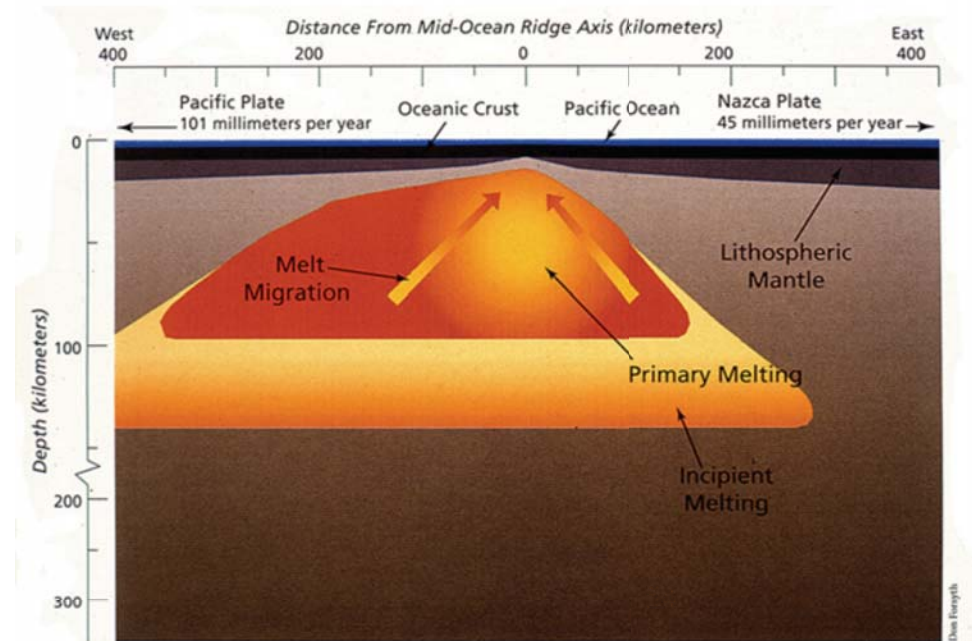
<sup>c</sup> Multiplicative process.

**Baker's map should describe convective stirring**



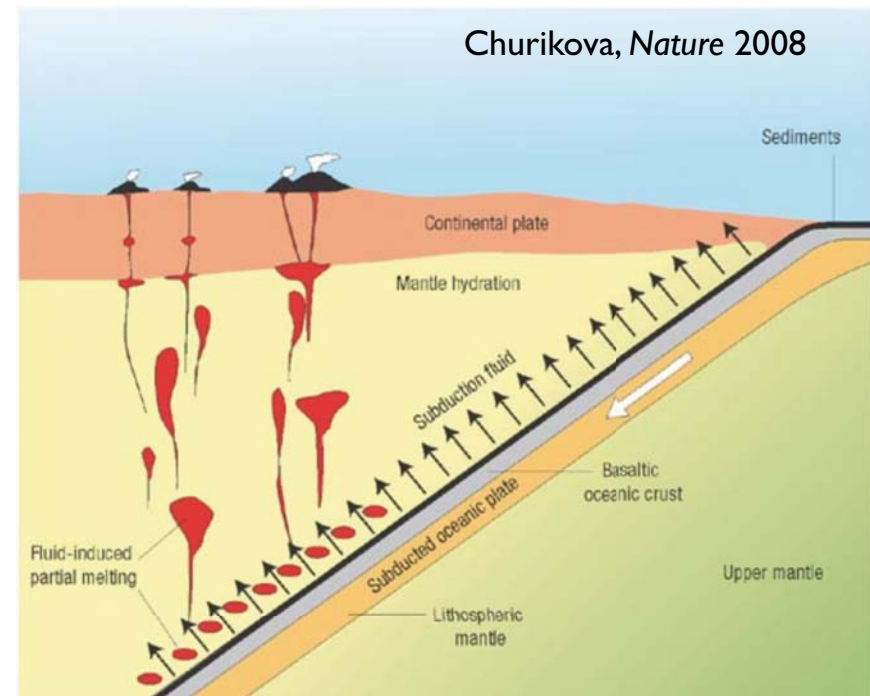
# Convection is a source and sink of heterogeneity

- Melting at ridges
- Fluid migration and melting at subduction zones
- Melting at mid mantle phase transitions?
- Melting at the base of the mantle
- Chemical reactions between the mantle and core



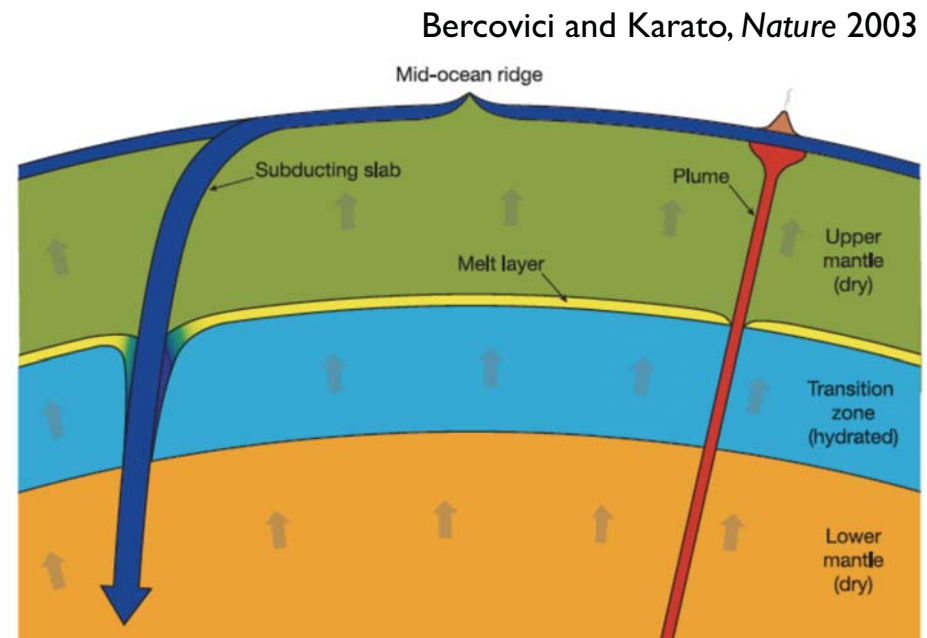
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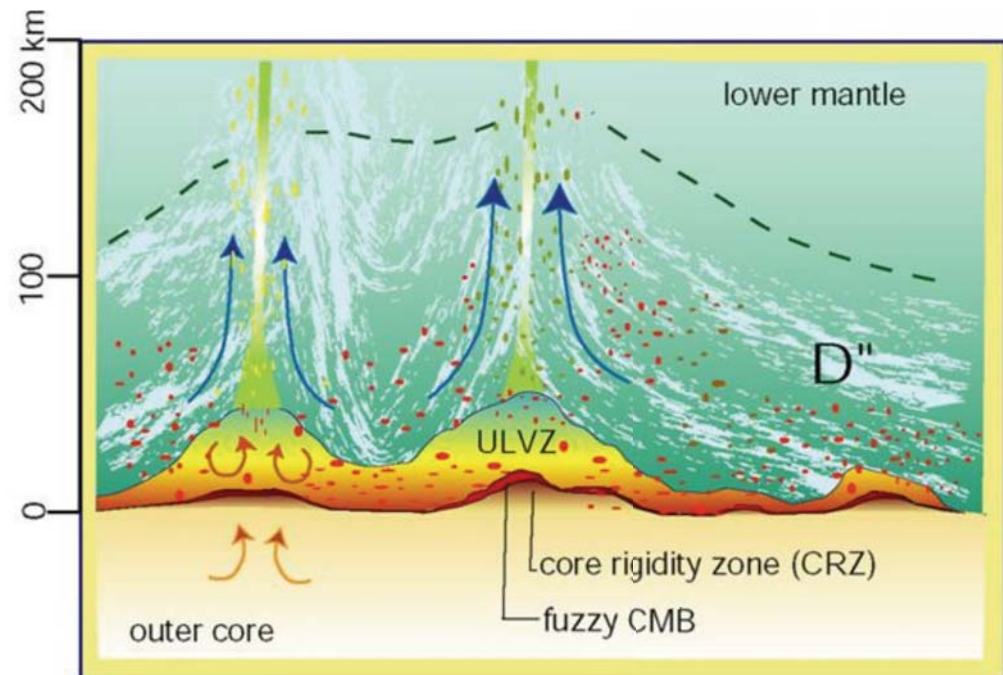
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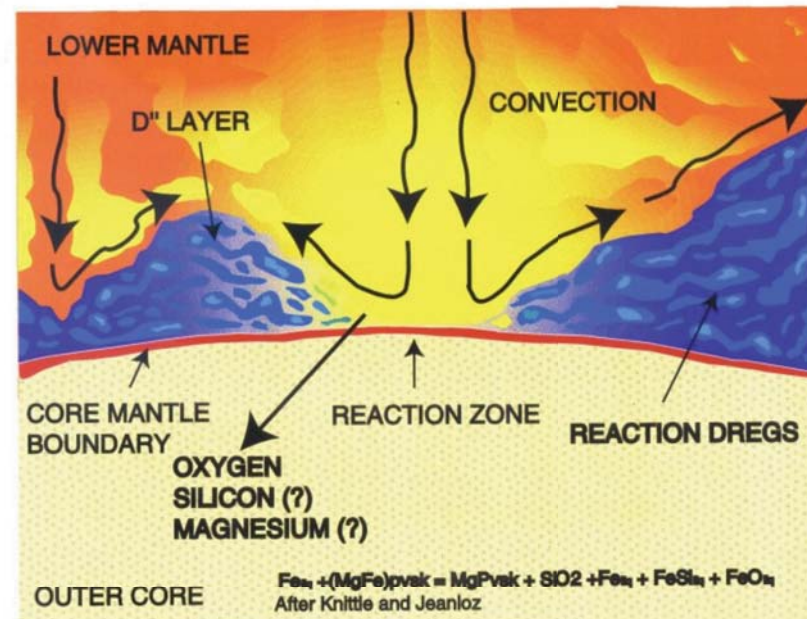
# Convection is a source and sink of heterogeneity

- Melting at ridges
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- Melting at mid mantle phase transitions?
- **Melting at the base of the mantle**
- Chemical reactions between the mantle and core



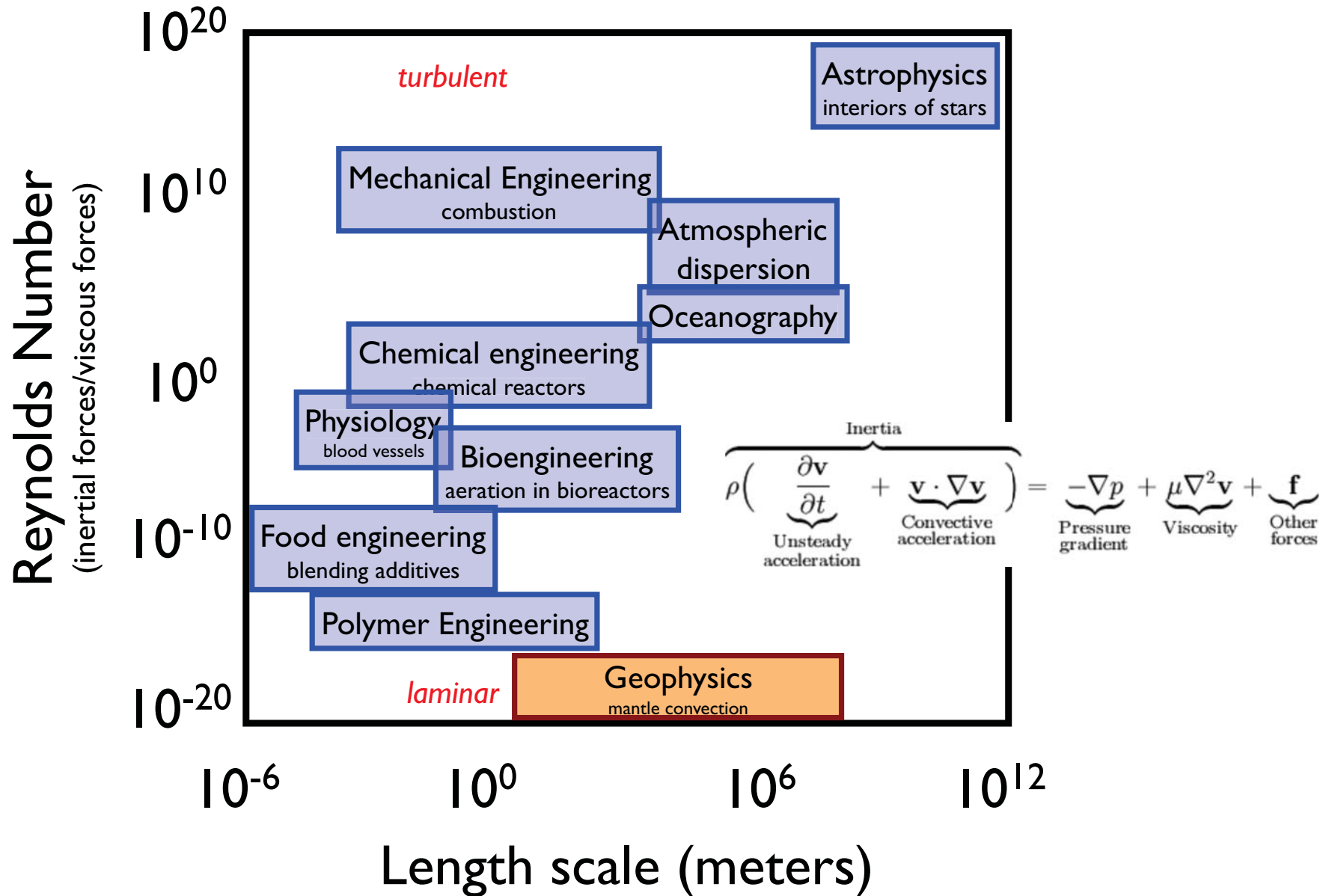
# Convection is a source and sink of heterogeneity

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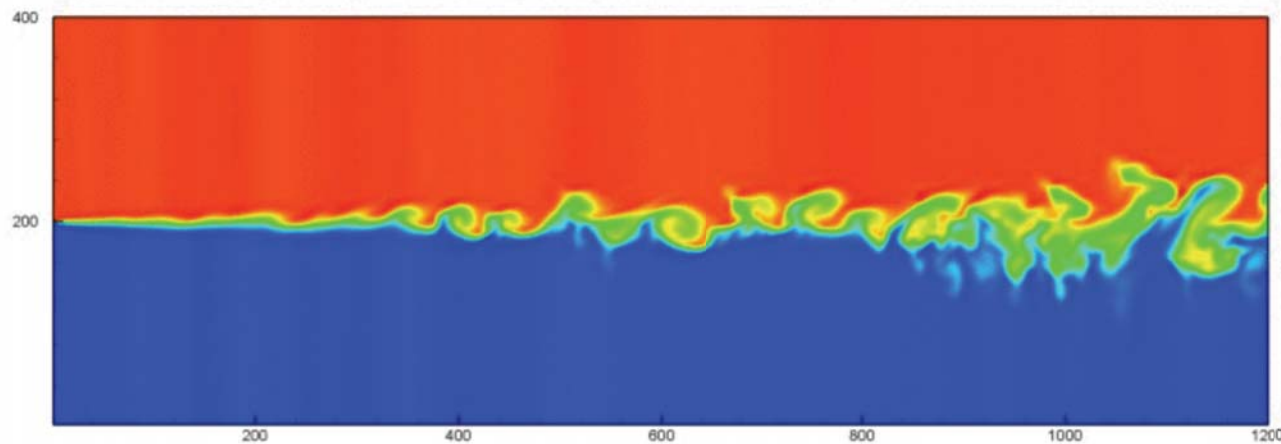


# Some mixing scales

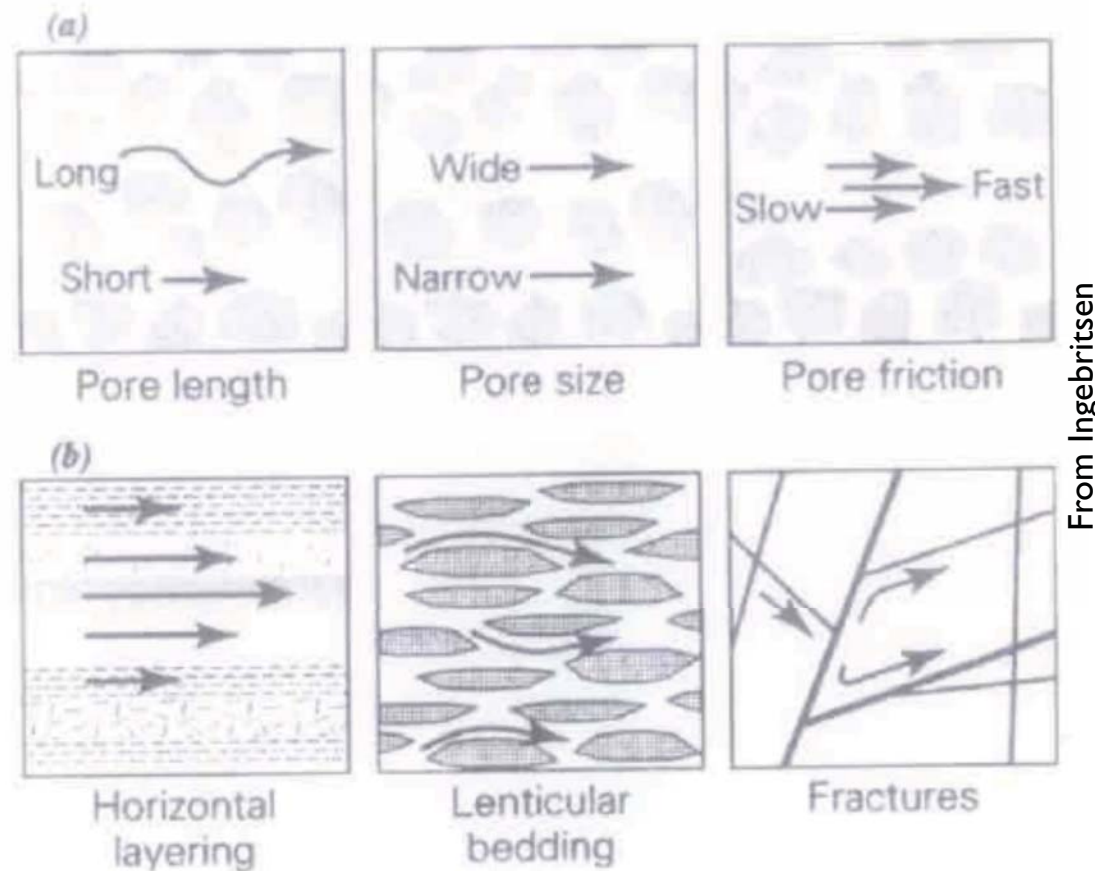


## Turbulent mixing

- Energy transfer from large to small scales
- Intermittency in space and time
- Velocity is a complicated function of time



# Dispersion in porous materials



- Complexity in flow paths, spatial variation in velocity greatly enhance mixing (dispersion)

“Virtually everyone agrees that mixing is complicated”

“However there is no agreement as to the source of the complications . . . What makes mixing complex? Usually realistic mixing problems have been regarded as nearly intractable from a modeling viewpoint owing to the complexity of the flow fields. Also in many problems of interest the fluids themselves are rheologically complex . . . . .

Mixing problems have been attacked traditionally on a case by case basis. However . . . merging of kinematics with dynamical systems and chaos are providing a paradigm for the analysis of mixing from a rather general viewpoint.”

Ottino, *Ann Rev Fluid Mech* 1990

# Main points

- Flow type matters
- Time dependence matters
- Properties of heterogeneity matter (active heterogeneity is different from passive tracers)
- Mixing will depend on history of Earth and properties of interior (all of which have uncertainty), hence a stochastic approach may be useful
- Convection both creates and destroys heterogeneity

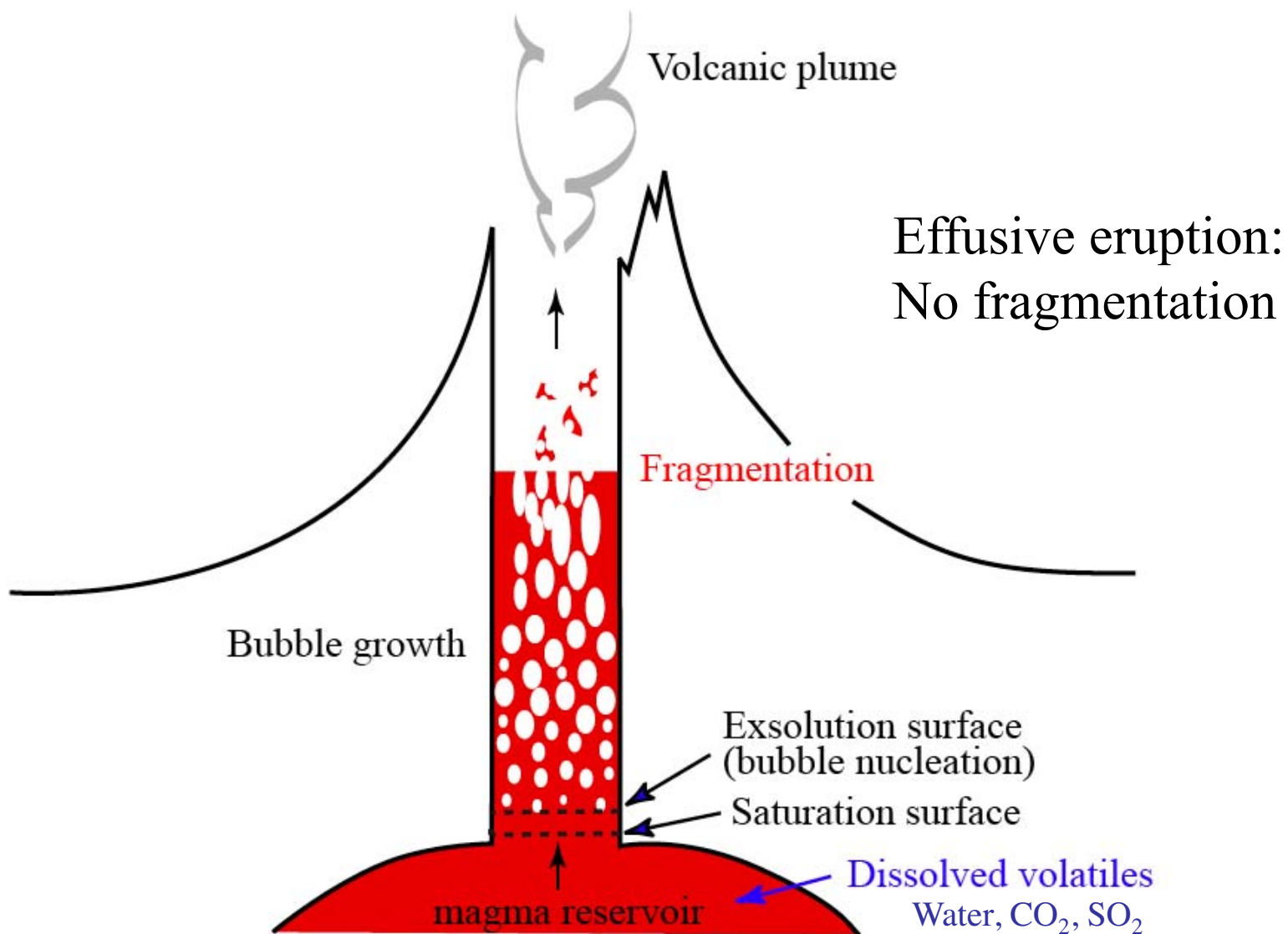


# Why do volcanoes (only sometimes) erupt explosively?

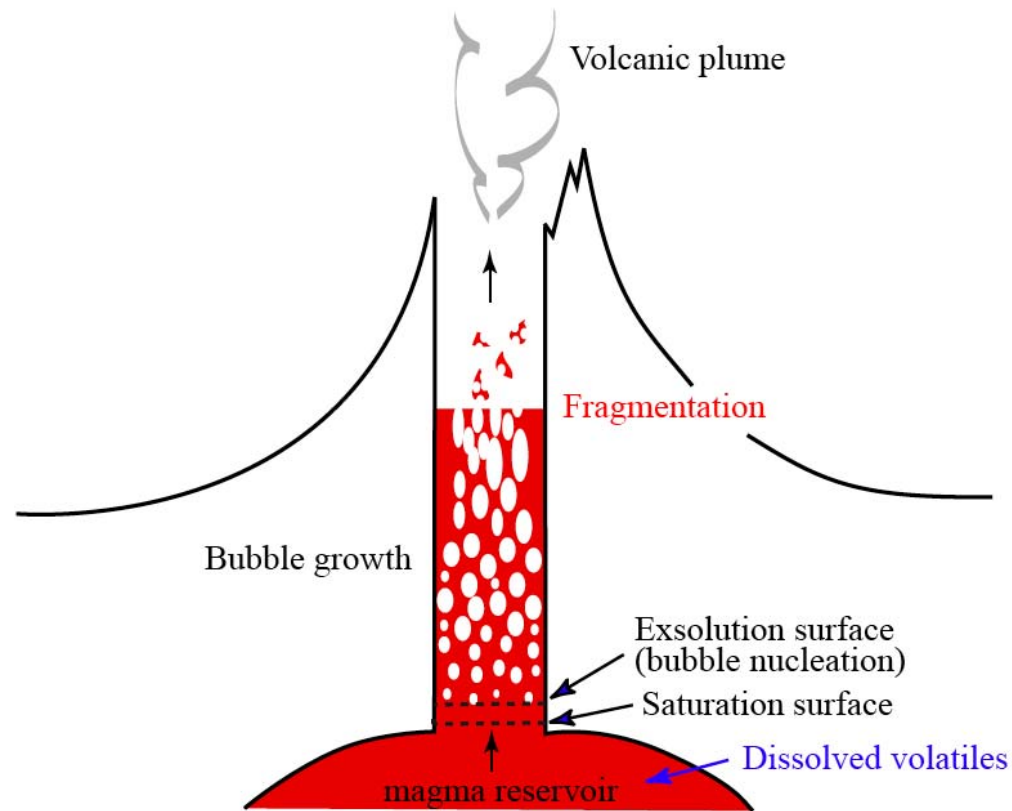


Gonnermann and Manga,  
The fluid mechanics inside a volcano,  
*Annual Reviews of Fluids Mechanics*,  
2007

# Why do volcanoes erupt explosively? (textbook version)



# Why do volcanoes erupt explosively?

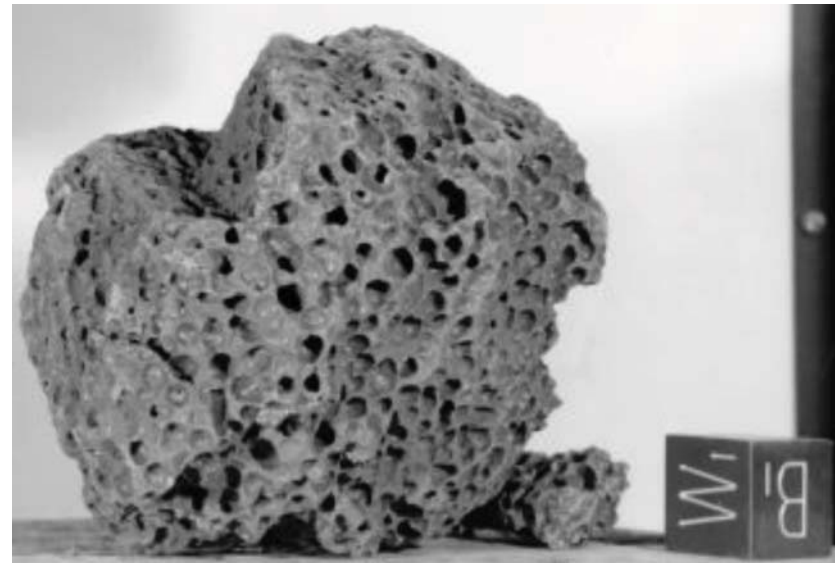
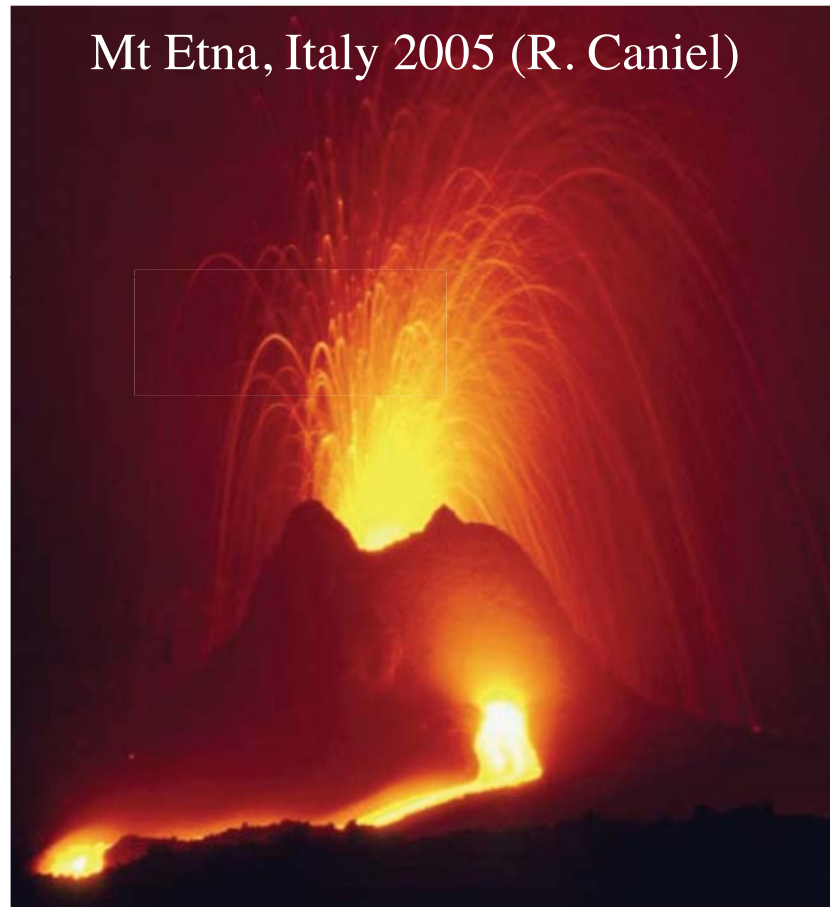


Open questions:

- When, where and how does fragmentation occur?
- Why so much diversity in eruption style?

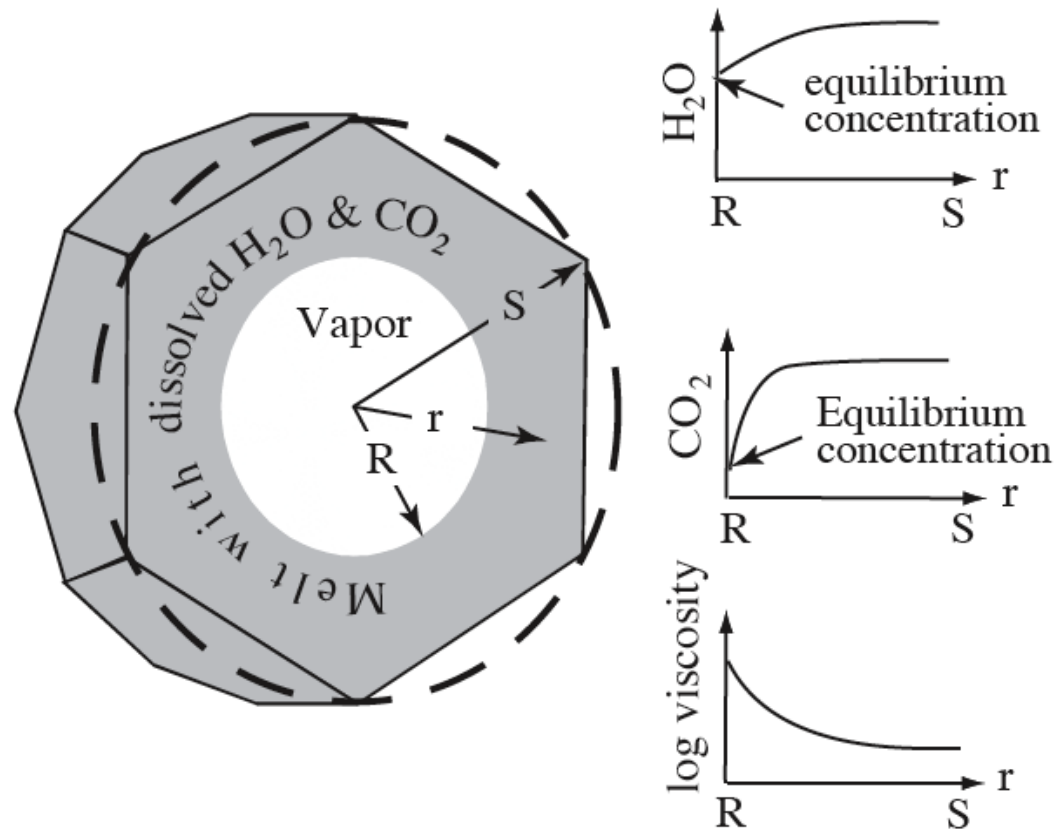
# Three key processes

## 1. Bubble nucleation, exsolution and bubble growth



vesicular basalt (from the moon)

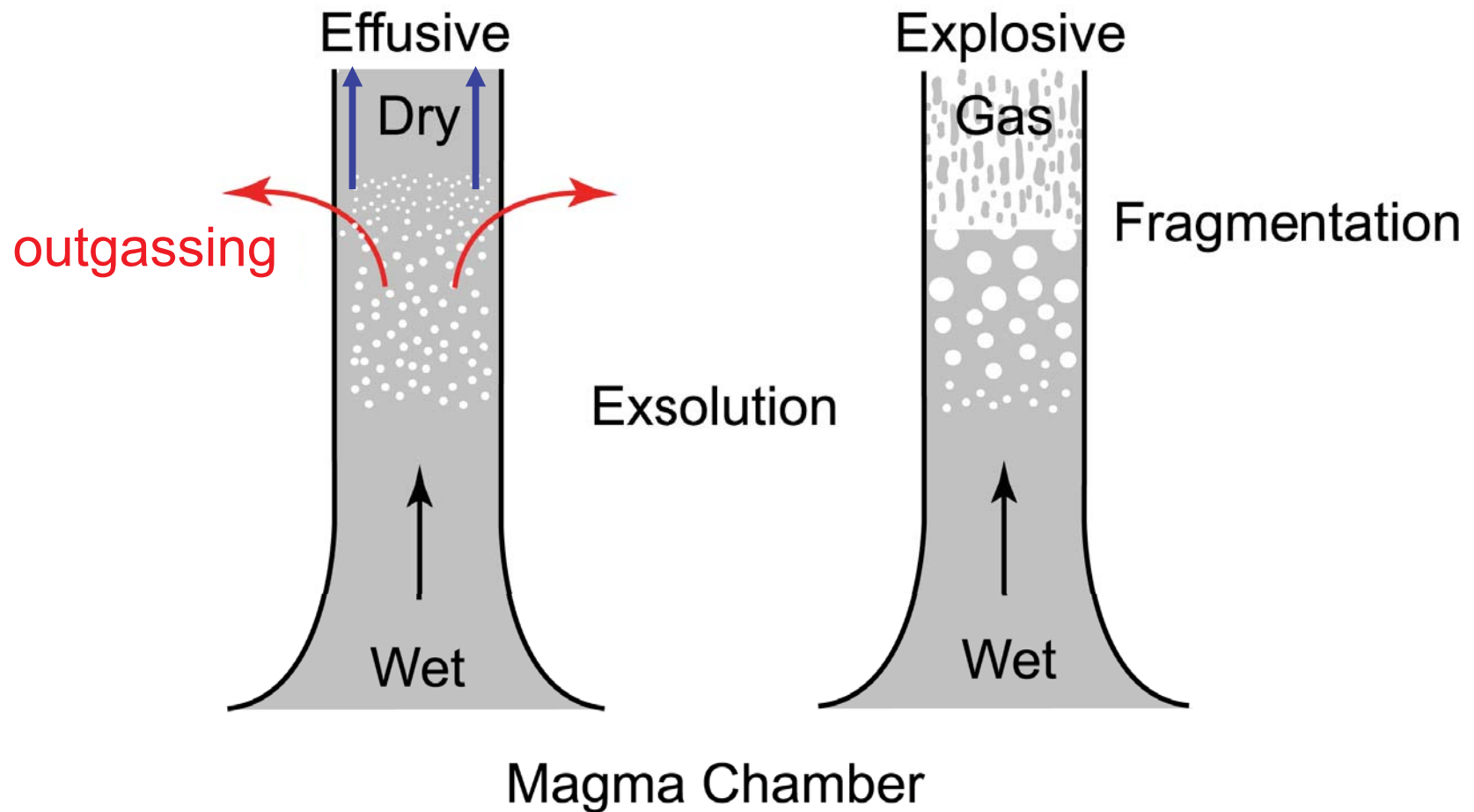
# Volatile exsolution and bubble growth





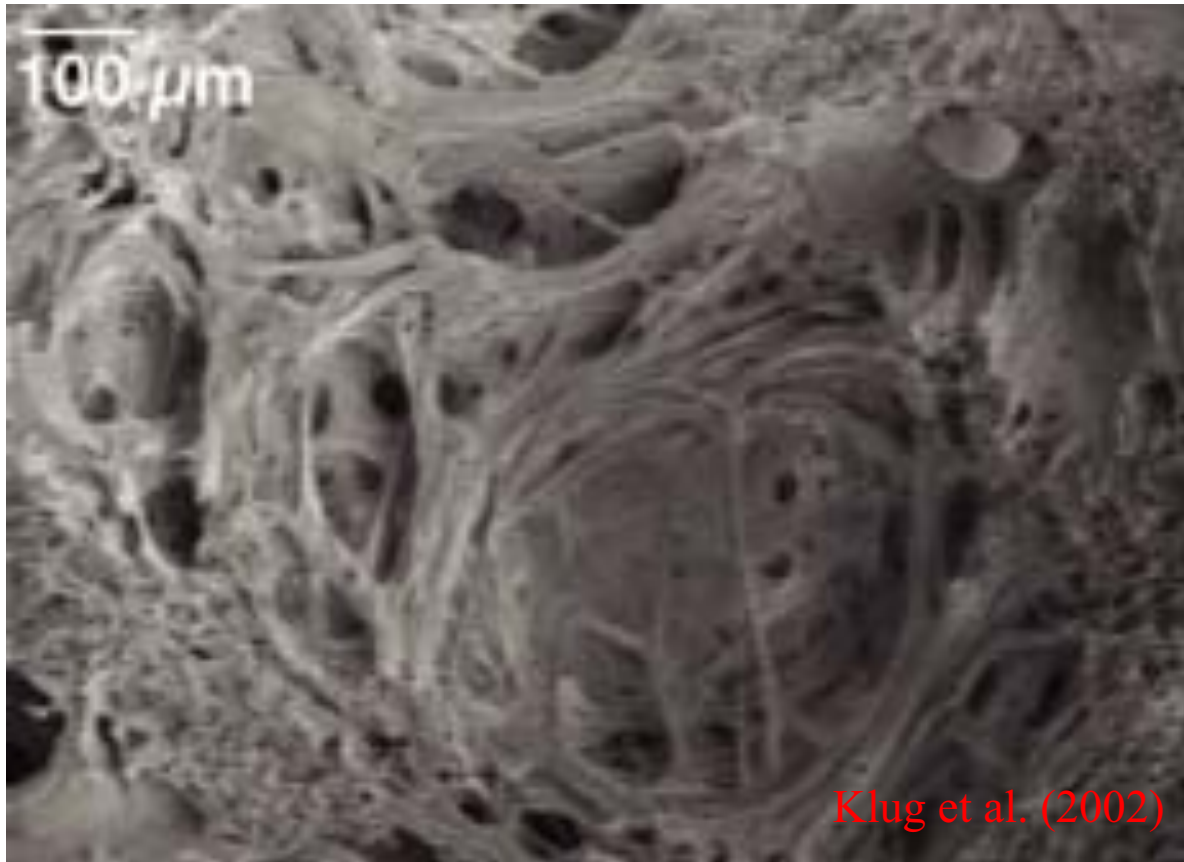
# Three key processes

2. Loss of gases, called **outgassing**,  
supresses eruption





## Vesicular magma is permeable



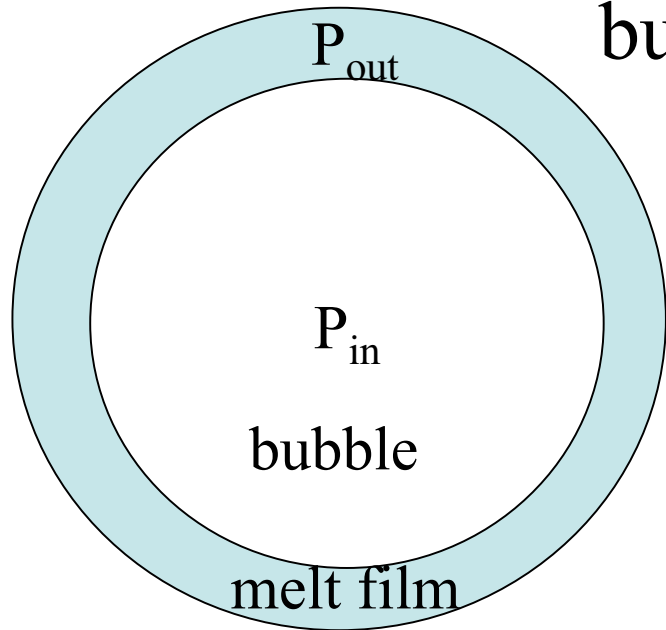
Connections between bubbles allow gases to escape from magma

Permeability depends on vesicularity and bubble size

# Three key processes

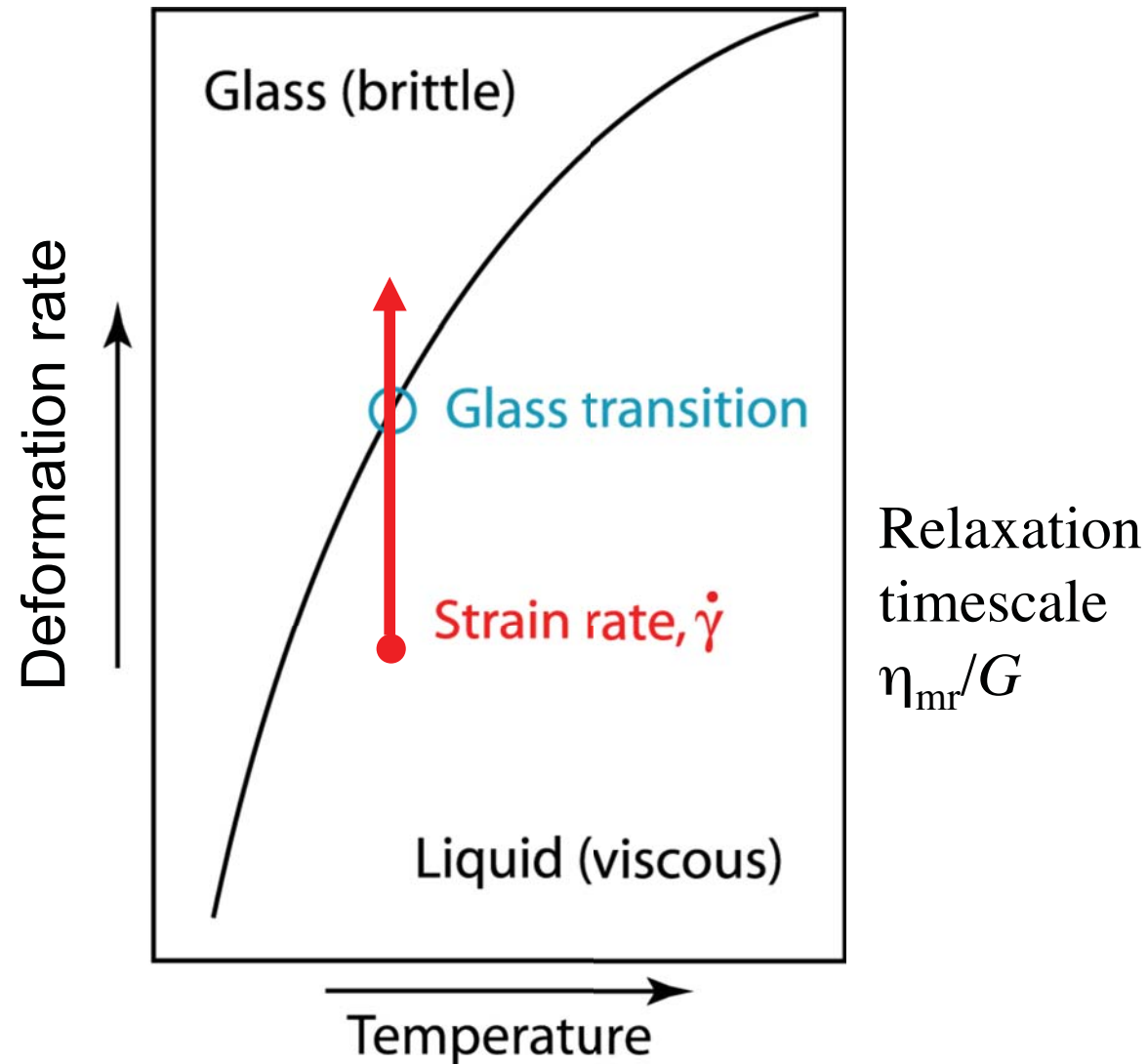
## 3. Fragmentation

If stresses in film surrounding  
bubbles too large



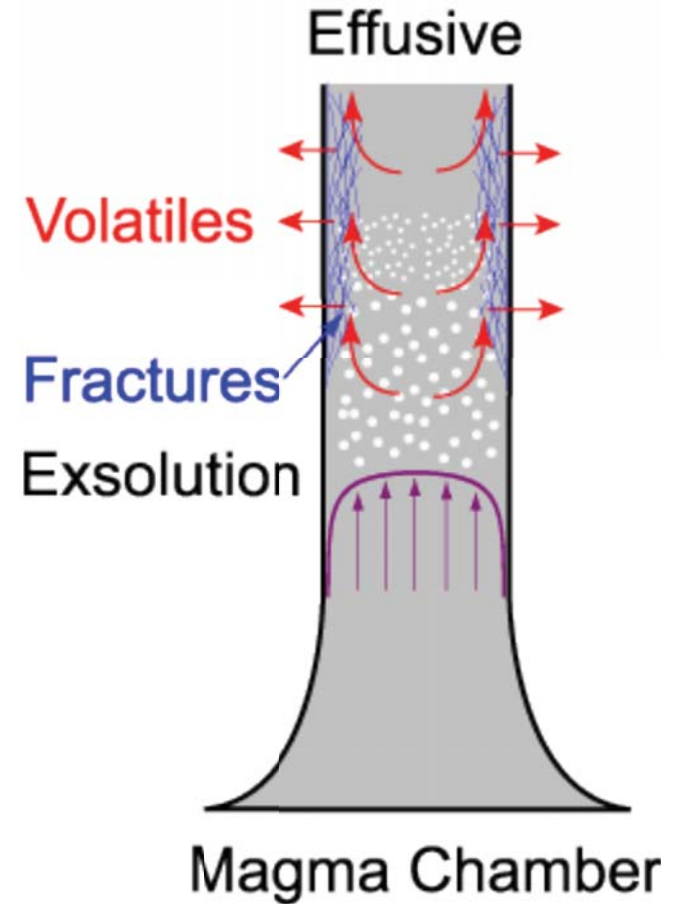
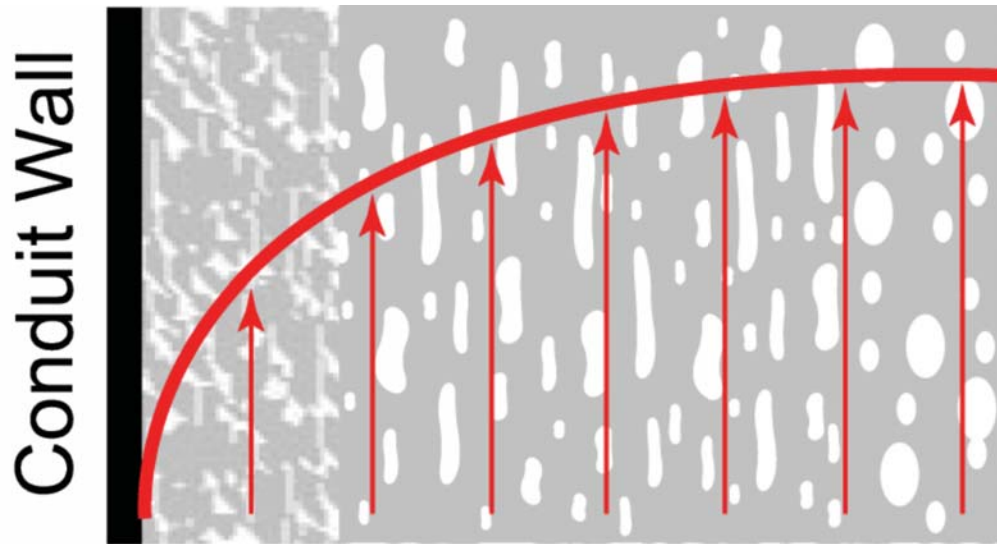
If  $P_{in} - P_{out} >$  critical value  
then film ruptures

## A second way to break magmas . . .



Condition: strain rate  $> CG/\eta_{mr}$  with  $C \sim 0.01$

Are deformation rates high enough to fragment ascending magma?



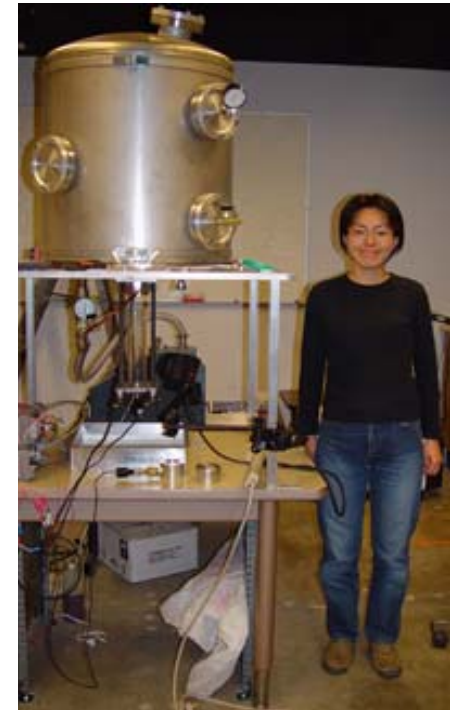
we will refer to this **brecciation**

# Three key processes

- 1) Nucleation (forming new) and growth of bubbles
- 2) Outgassing (loss of gas from the magma)
- 3) Fragmentation and brecciation (breaking magma into pieces)

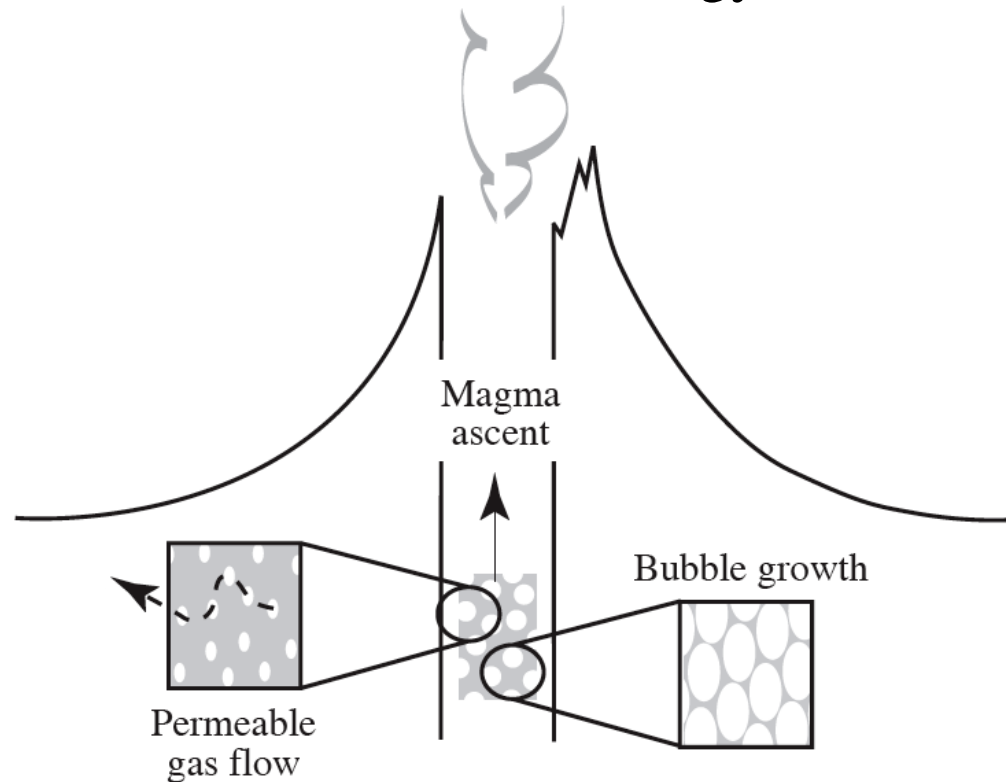
## Approach

1. Lab experiments and theoretical models to study individual processes and properties
2. Computer simulations
3. Test models with measurements made on rocks



# Numerical model

Solve equations for conservation of mass,  
momentum, energy at two scales



1) Conduit flow:  
magma (bubbles+ melt) is  
locally homogeneous

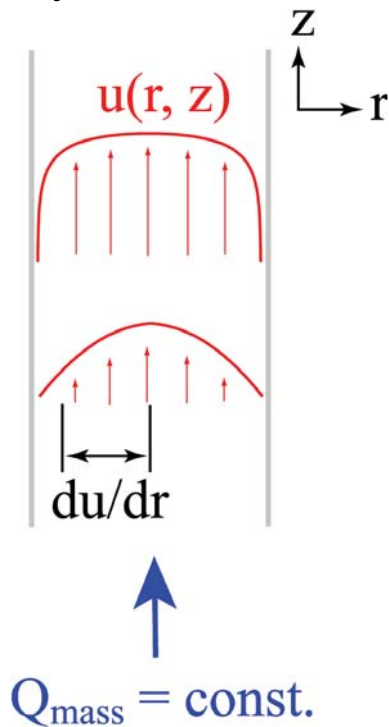
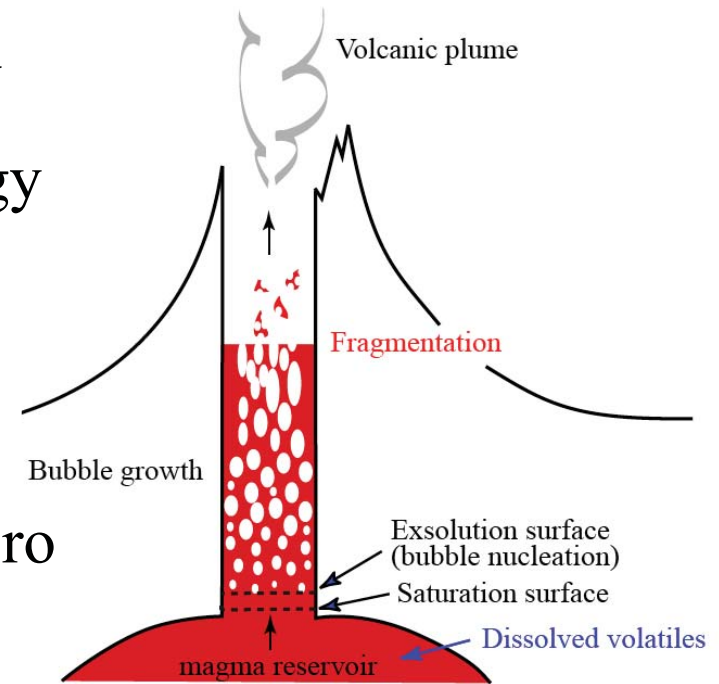
2) Bubble-scale:  
Solve for growth of  
bubbles, determine  
rheology

Feedbacks between scales through temperature, pressure



# Conduit flow

- conservation of mass, momentum, energy (include viscous dissipation; density, rheology from subgrid model)
- non-turbulent, no fragmentation,
- “single” phase magma (melt + bubbles)
- cylindrical conduit, radial velocity is zero
- steady flow



$$-\frac{r}{2} \left( \frac{\partial p_m}{\partial z} + \rho g \right) = -\eta \frac{du_z}{dr}$$

$$\eta = \eta(\dot{\gamma}, T_m, \phi, R, c_w)$$

$$\frac{DT_m}{Dt} = \left[ D_T \left( \frac{\partial^2 T_m}{\partial r^2} + \frac{1}{r} \frac{\partial T_m}{\partial r} \right) - \frac{1}{\rho_m c_{pm}} \left( \sigma_{rz} \frac{\partial u_z}{\partial r} \right) \right]$$

## Conduit flow

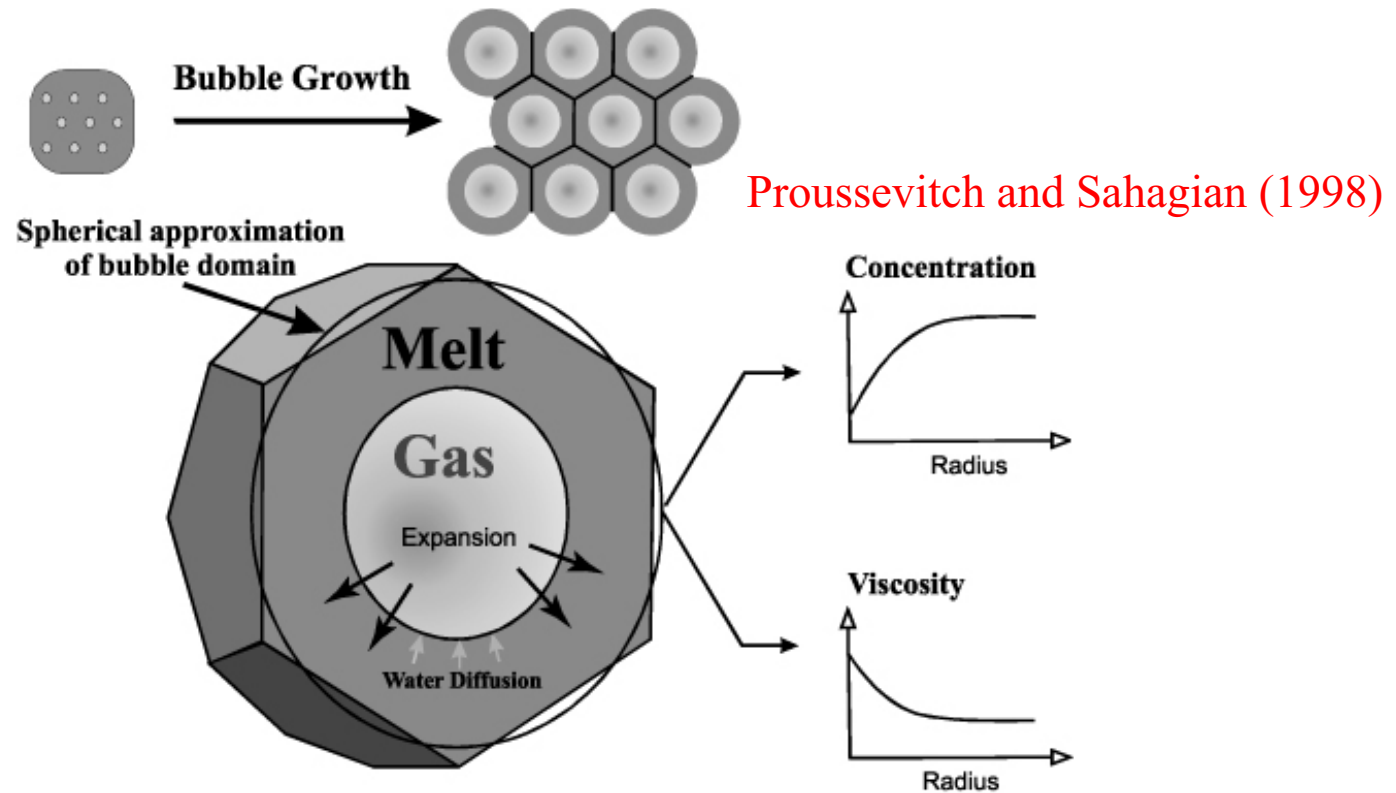
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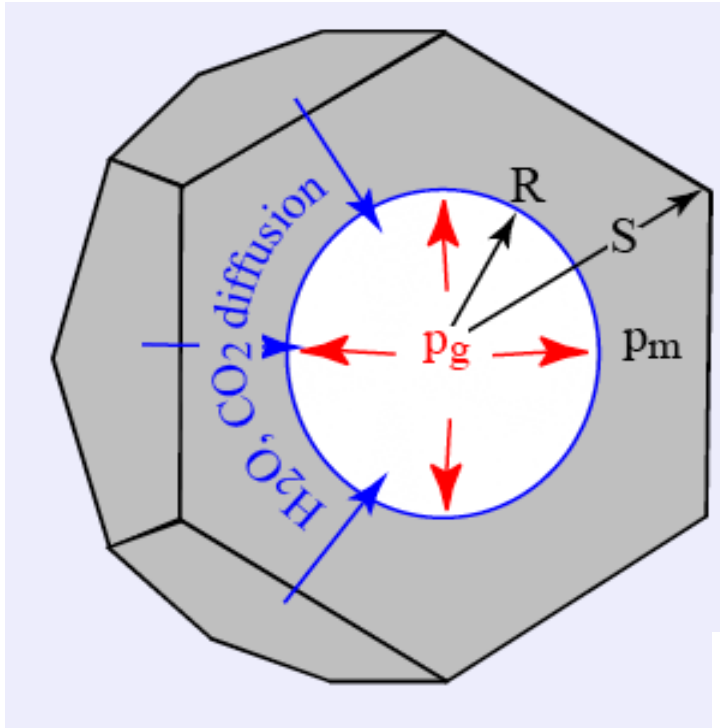
# Subgrid model: Volatile exsolution and bubble growth



Solubility of  $H_2O$ ,  $CO_2$  from Liu et al. (2005)

Diffusivity of  $H_2O$ ,  $CO_2$  from Zhang and Behrens (2000)

# Subgrid model: Volatile exsolution and bubble growth



Conservation of mass, momentum and energy, coupled with solubility model and modified Redlich-Kwong equation of state for water-CO<sub>2</sub> mixtures

$$\frac{d}{dt} (\rho_g R^3) = 3R^2 \rho_m \sum_i D_i \left( \frac{\partial c_i}{\partial r} \right)_{r=R}$$

$$p_g - p_m = \frac{2\gamma}{R} + 12v_R R^2 \int_R^S \frac{\eta_{melt}(r)}{r^4} dr.$$

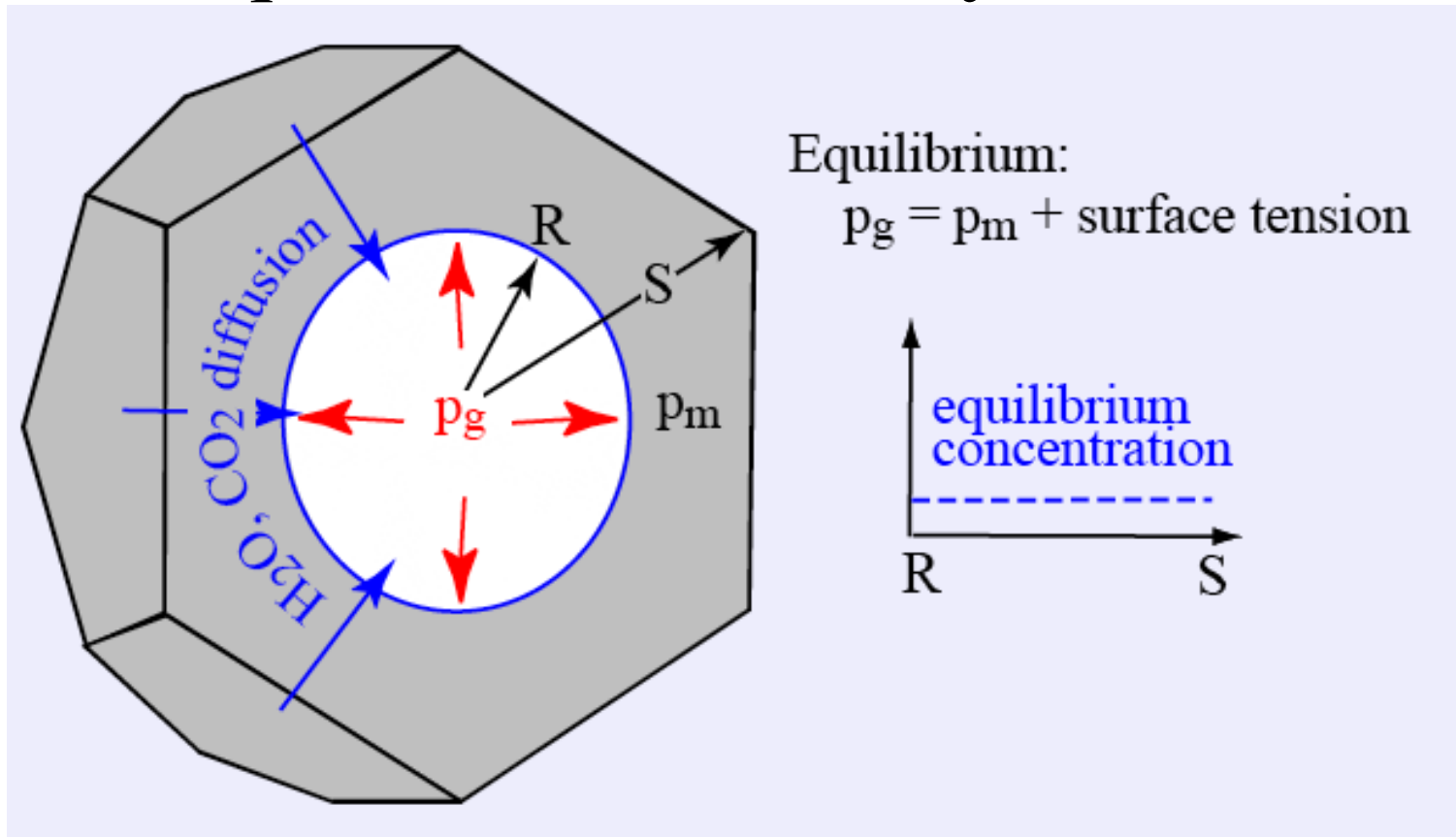
Lensky et al. (2001)

$$\frac{dT_g}{dt} = \Pi \left[ \rho_m c_{pm} D_T \left( \frac{\partial T_m}{\partial r} \right)_{r=R} - \sum_i \Delta H_{ev} D_i \rho_m \left( \frac{\partial c_i}{\partial r} \right)_{r=R} + \frac{R}{3} \frac{dp_g}{dt} \right] \quad \Pi = 4\pi R^2 / (n c_{pg} M_g)$$

$$\frac{\partial T_m}{\partial t} + v_r \frac{\partial T_m}{\partial r} = \frac{1}{r^2} \frac{\partial}{\partial r} \left( D_T r^2 \frac{\partial T_m}{\partial r} \right) + \frac{2\eta}{\rho_m c_{pm}} \left[ \left( \frac{\partial v_r}{\partial r} \right)^2 + 2 \left( \frac{v_r}{r} \right)^2 - \frac{1}{3} \left( \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 v_r) \right)^2 \right]$$

Bird et al. (1960)

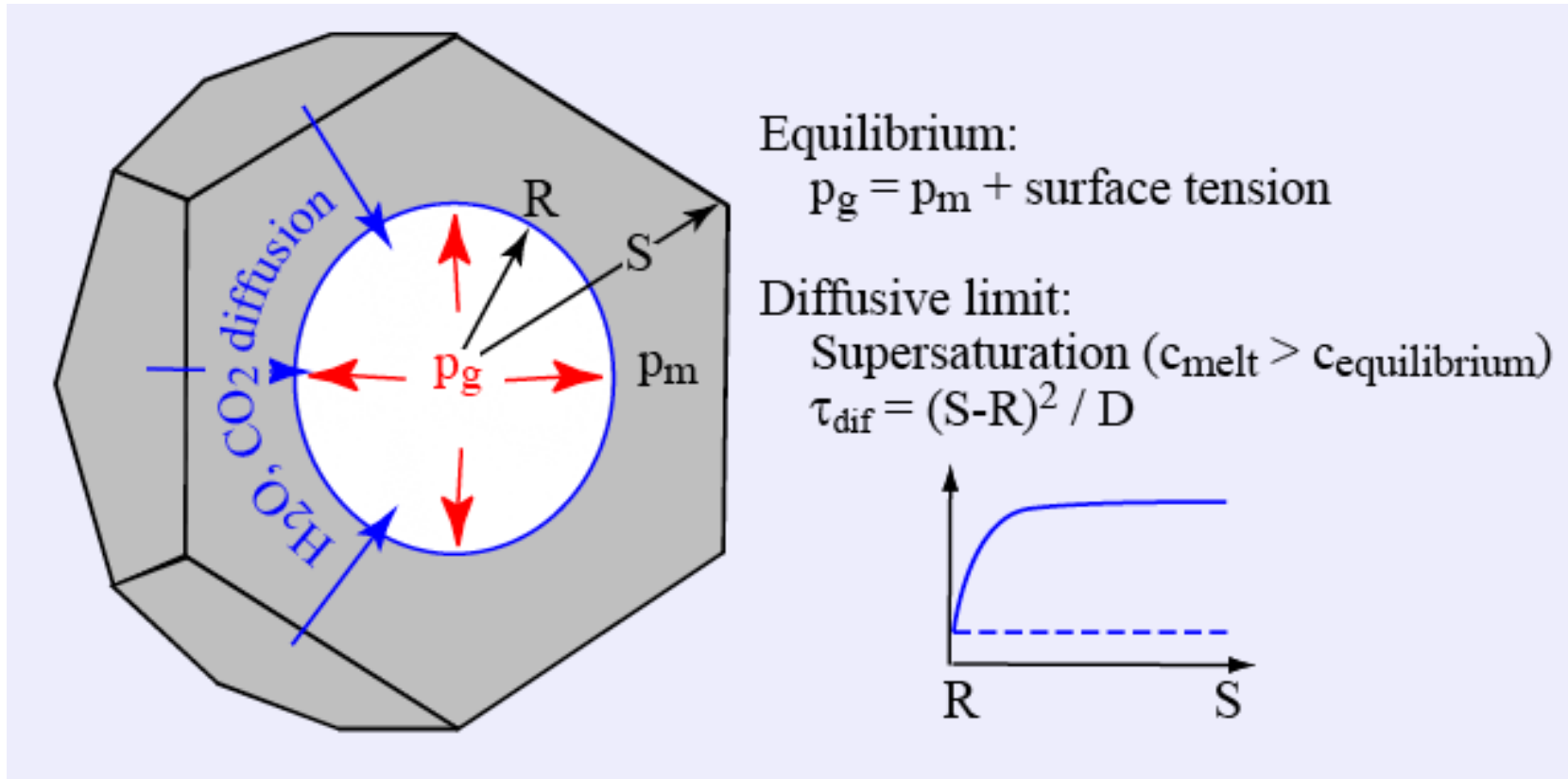
### 3 Regimes of bubble growth: Equilibrium (solubility-limited)



Growth is governed by changes in solubility

Decompression time scale  $\tau_{dec} = p_m / \dot{p}_m$

### 3 Regimes of bubble growth: Diffusion-limited

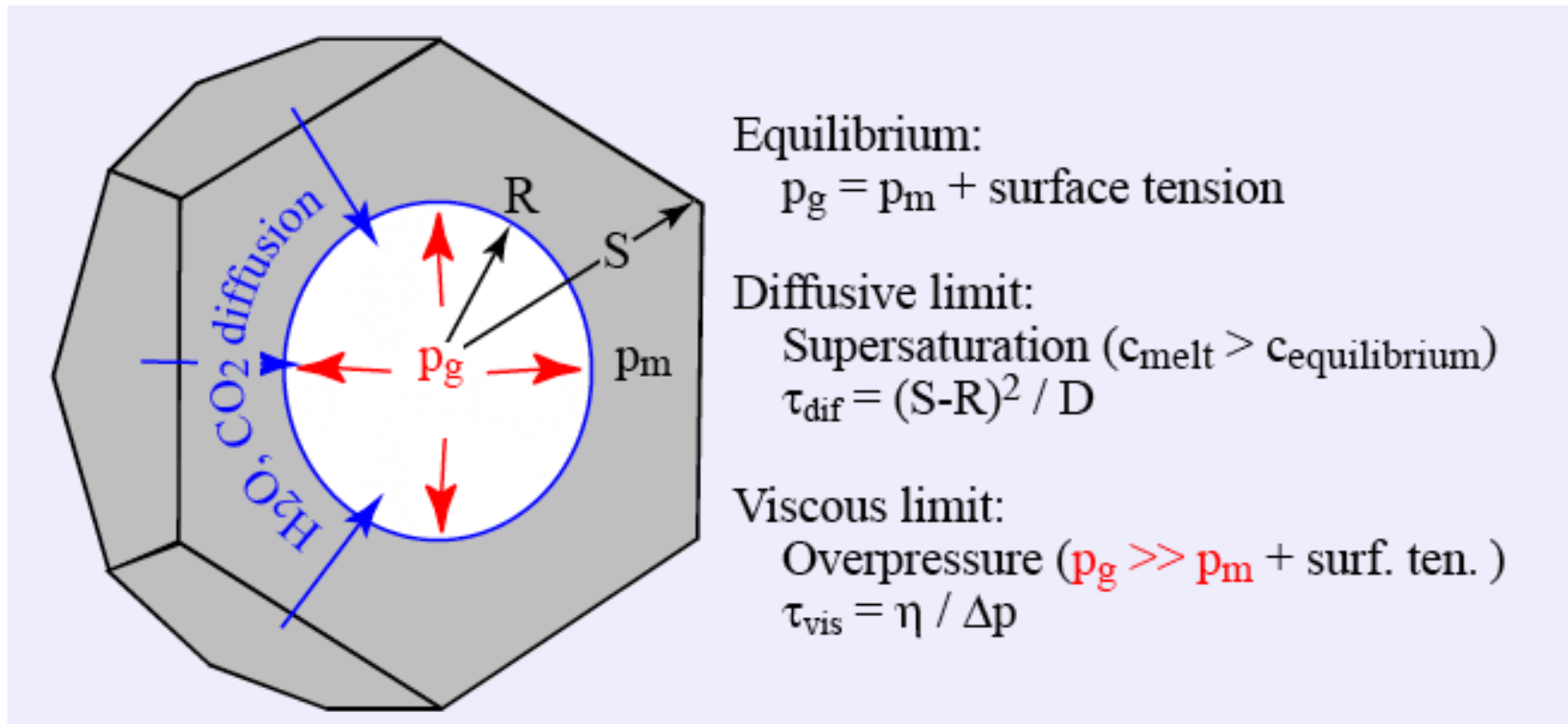


Growth is by diffusion-limited when  $Pe_{dif} = \frac{\tau_{dif}}{\tau_{dec}} \gg 1$

$S-R$  determined by number density of bubbles  $N_d$



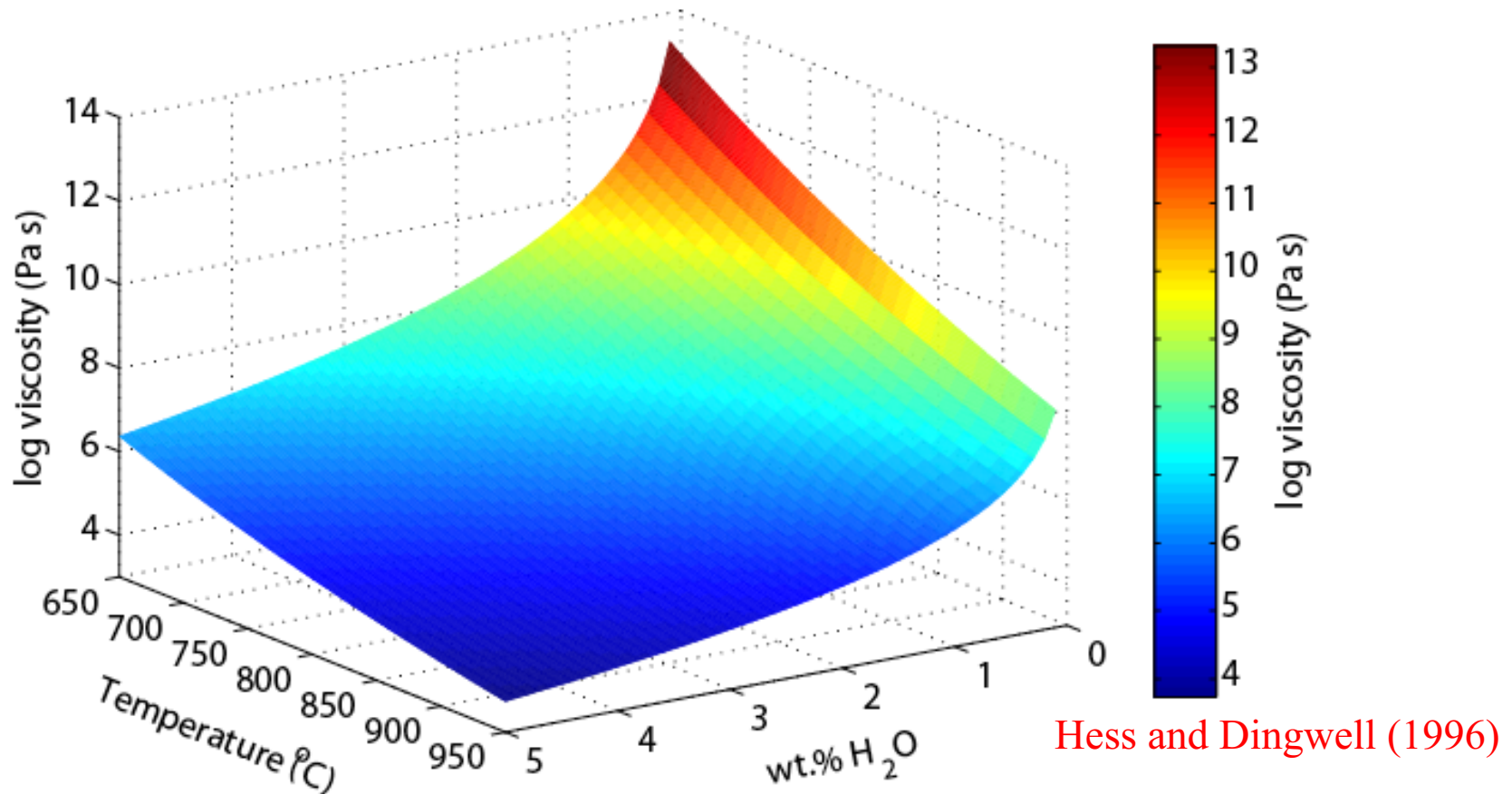
### 3 Regimes of bubble growth: Viscosity-limited



Growth is by viscosity-limited when

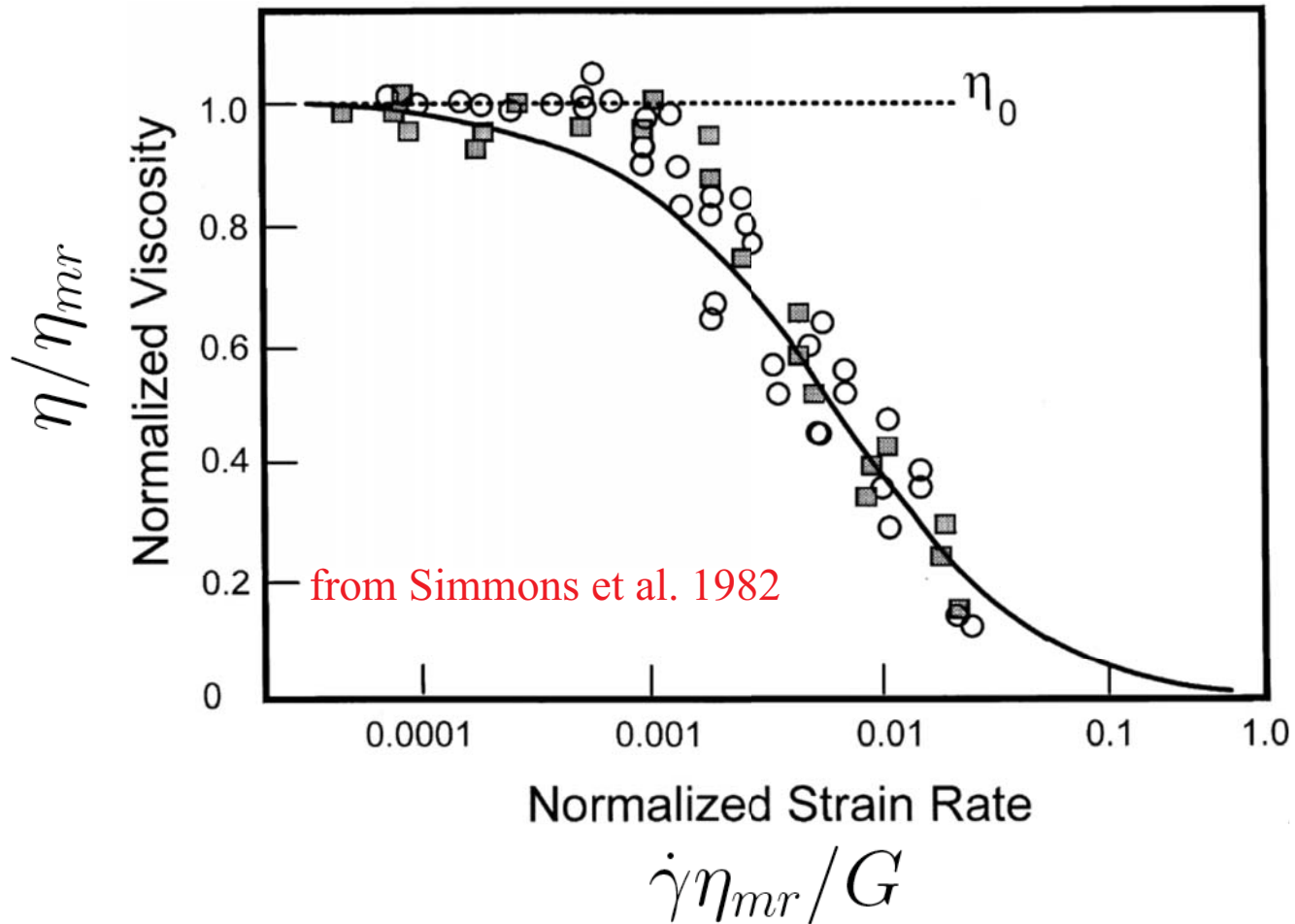
$$Pe_{vis} = \frac{\tau_{vis}}{\tau_{dec}} \gg 1$$

- Melt viscosity depends on amount of dissolved water and temperature (and composition)



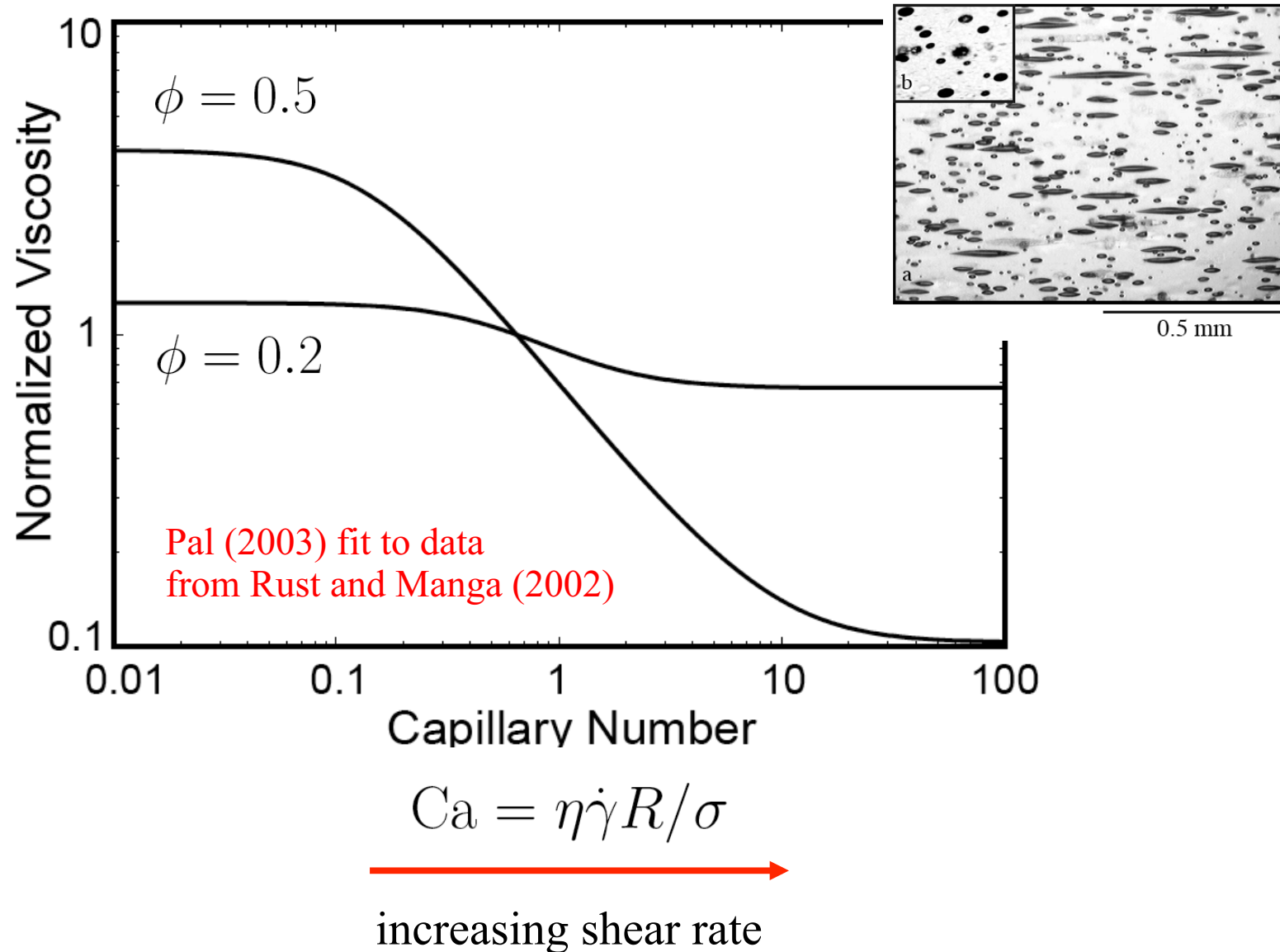
- Melt viscosity depends on deformation rate
- Magma viscosity affected by presence and properties of bubbles and crystals

# Strain-rate dependent viscosity of melt phase

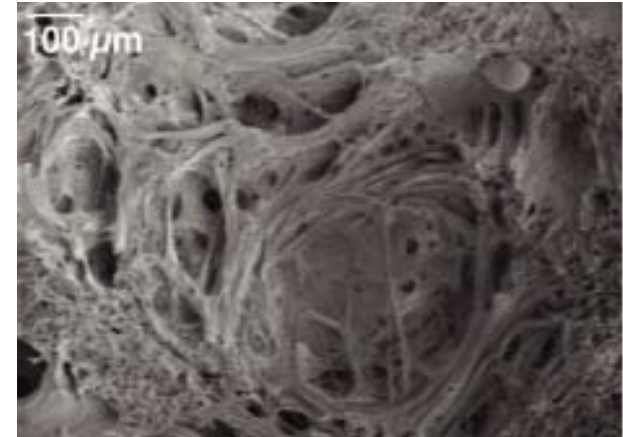
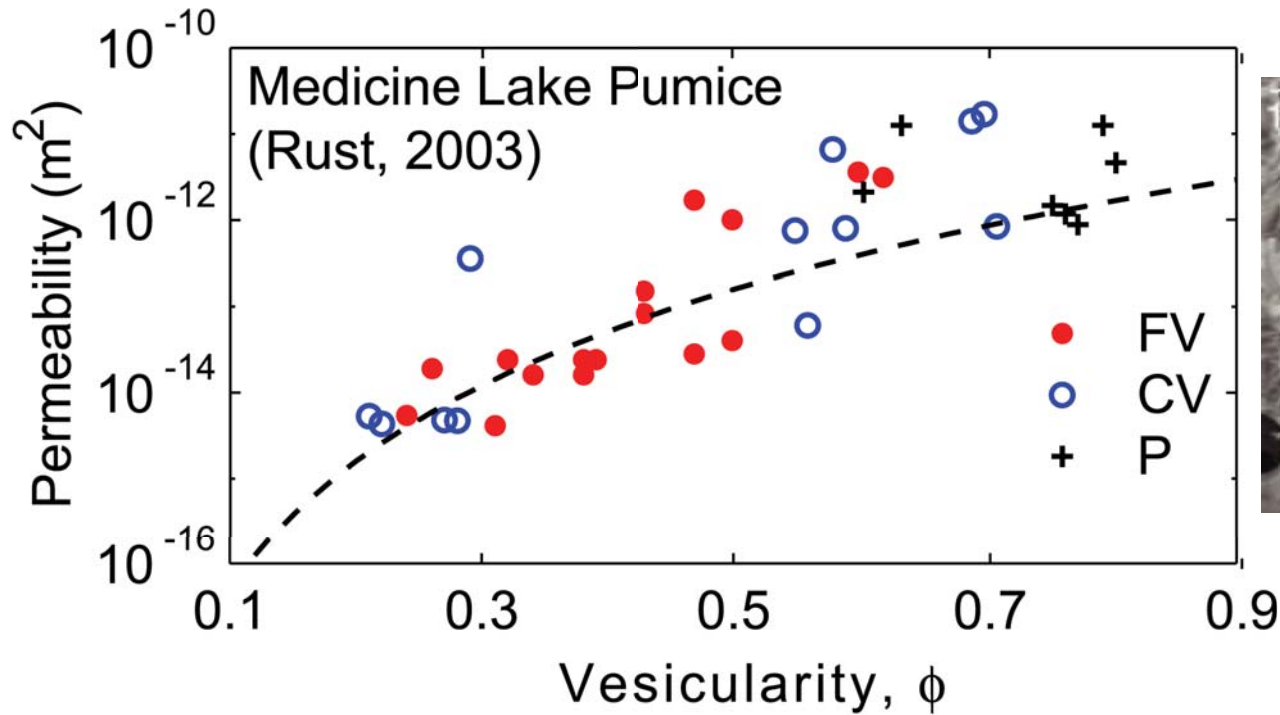


Silicic magmas are similar (Webb and Dingwell)

# Strain-rate dependent viscosity of bubbly suspension



# Vesicular magma is permeable



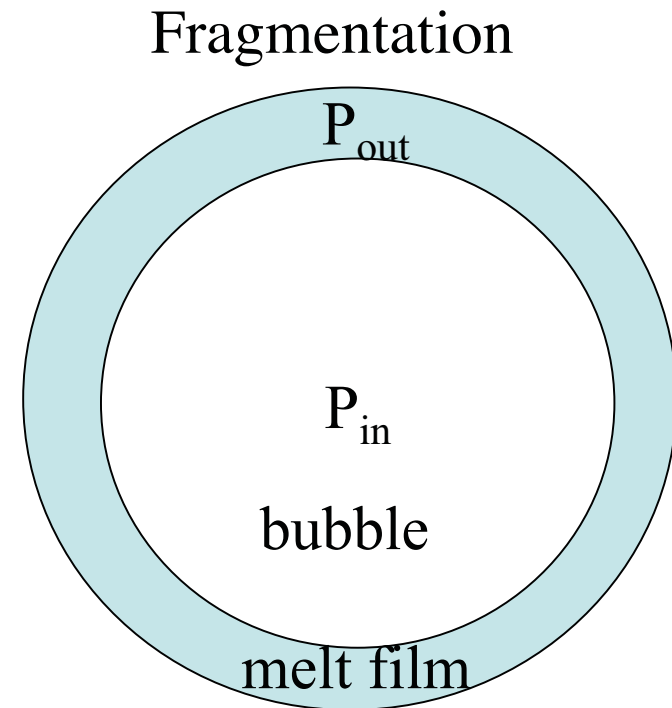
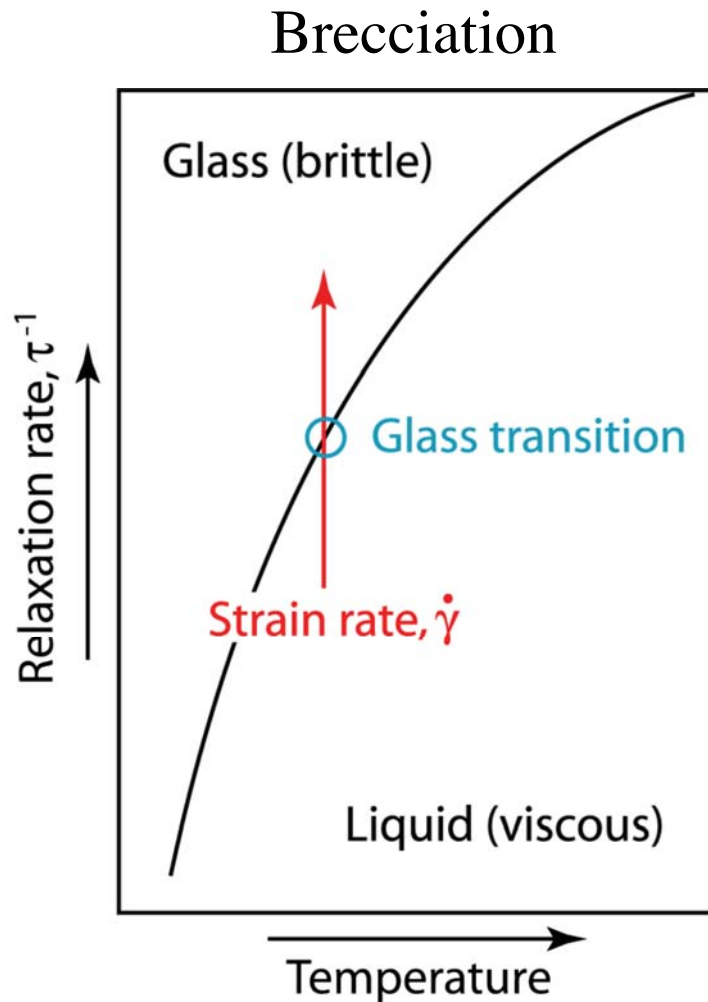
Klug et al. (2002)

Connections between bubbles allow gases to escape from magma

Permeability depends on vesicularity and bubble size  $k \propto \phi^\beta$

Outgassing efficient when  $-\frac{\rho_g k}{\eta_g} \frac{dp_g}{dz}$  exceeds rate of gas exsolution

# Fragmentation criteria: thresholds determined experimentally



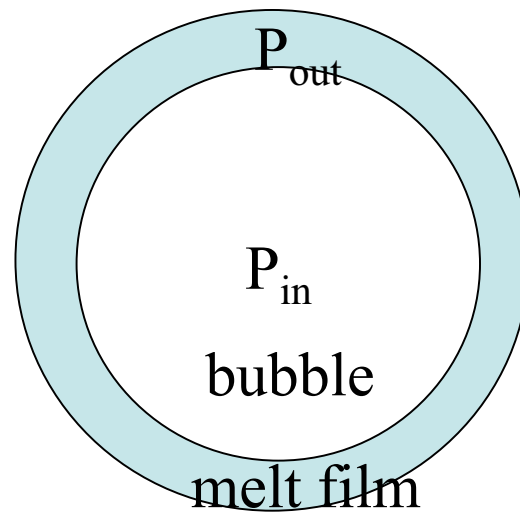
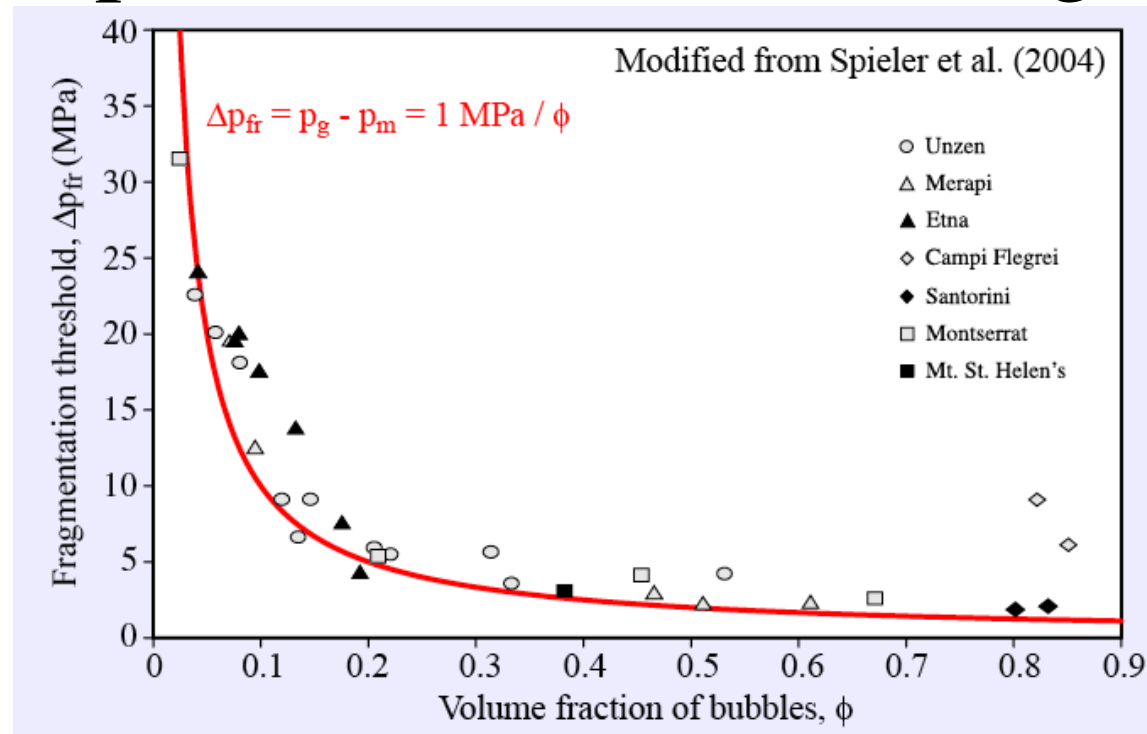
If  $P_{in} - P_{out} >$  critical value  
then film ruptures

Condition: strain rate  $> CG/\eta_{mr}$  with  $C \sim 0.01$

e.g., Webb and Dingwell (1990), Webb (1997), Papale (1998)

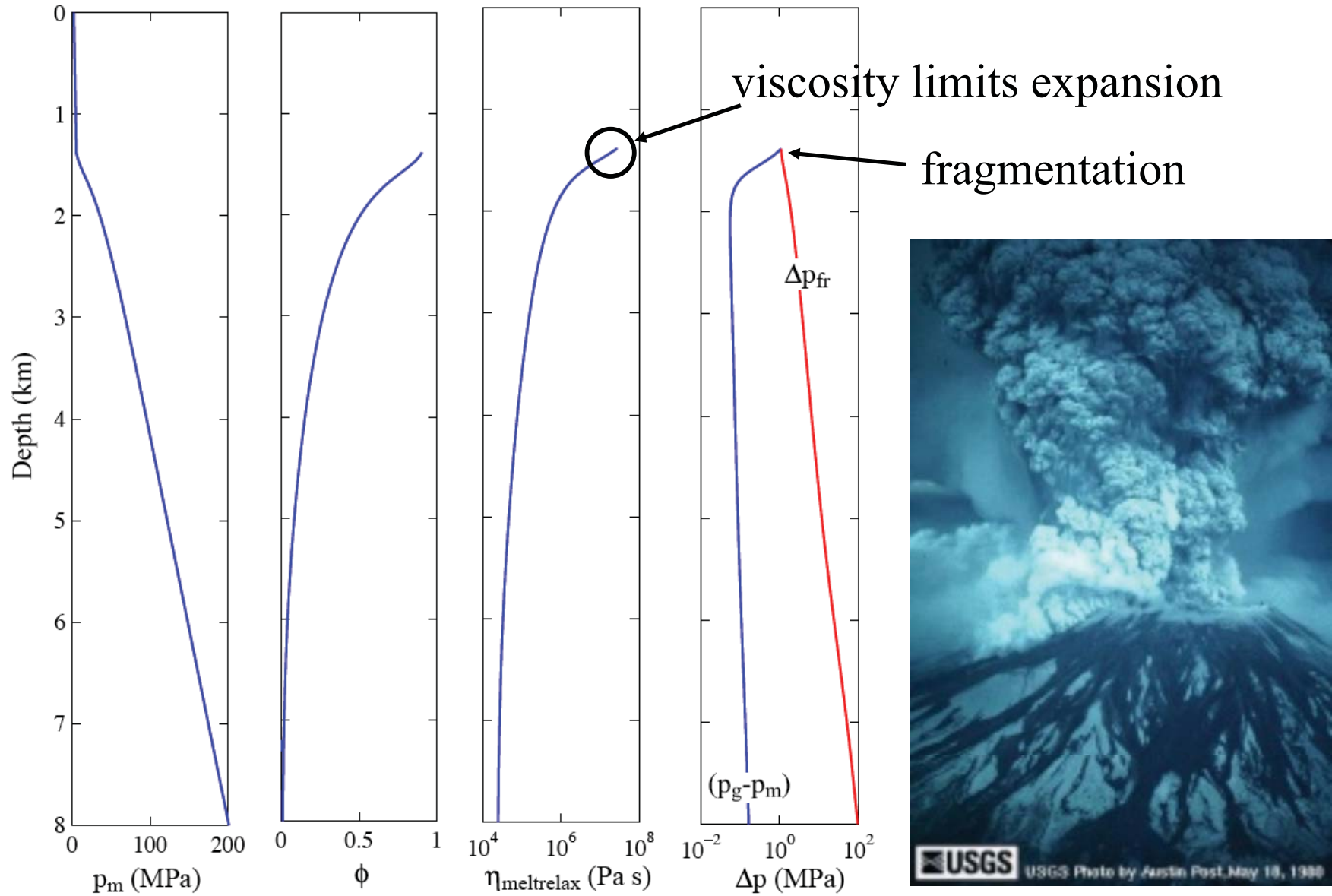


# Experiments with real magma

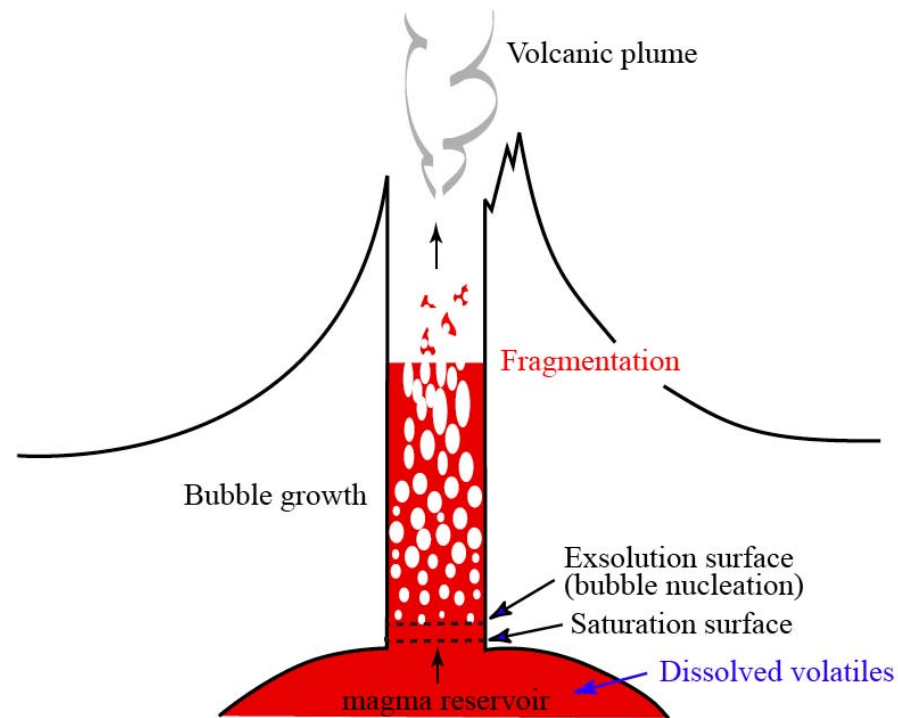


If  $P_{in} - P_{out} > 1 \text{ Mpa}/\phi$   
then film ruptures

# Example: Mount St Helens 1980 conditions



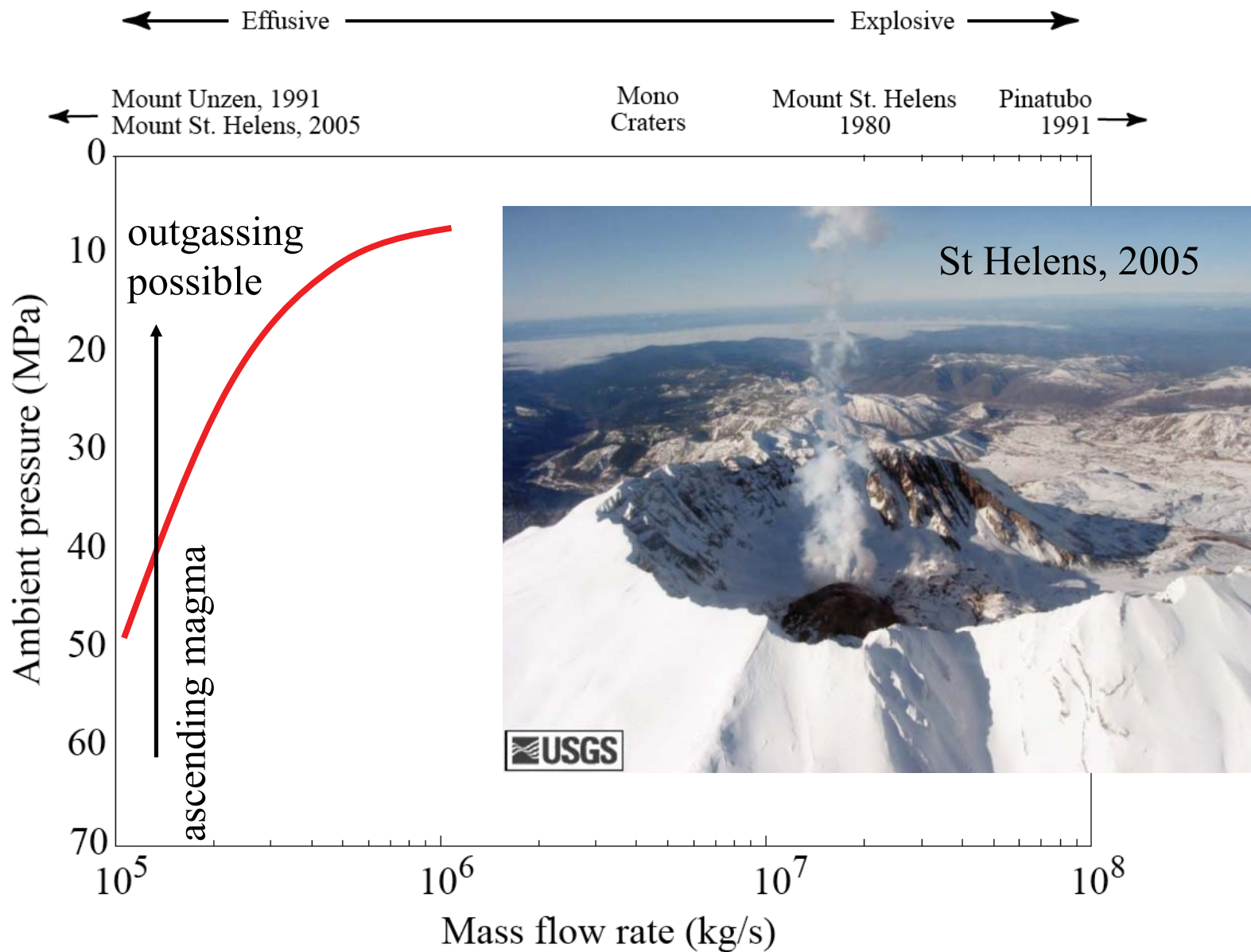
# Why do volcanoes erupt explosively?



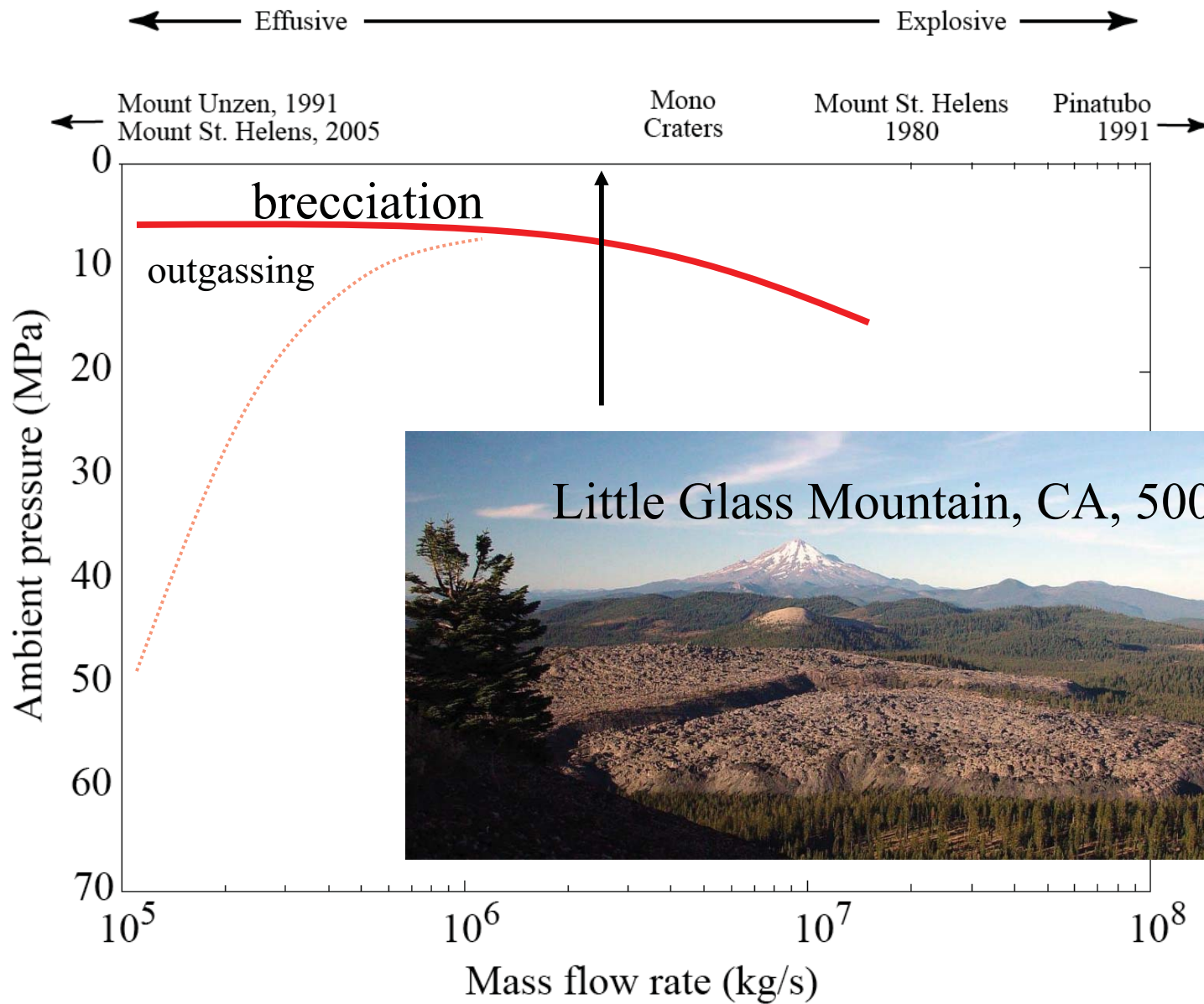
## Open questions:

- When, where and how does fragmentation occur?
- Why so much diversity in eruption style?

# Change in eruption style with changing ascent rate

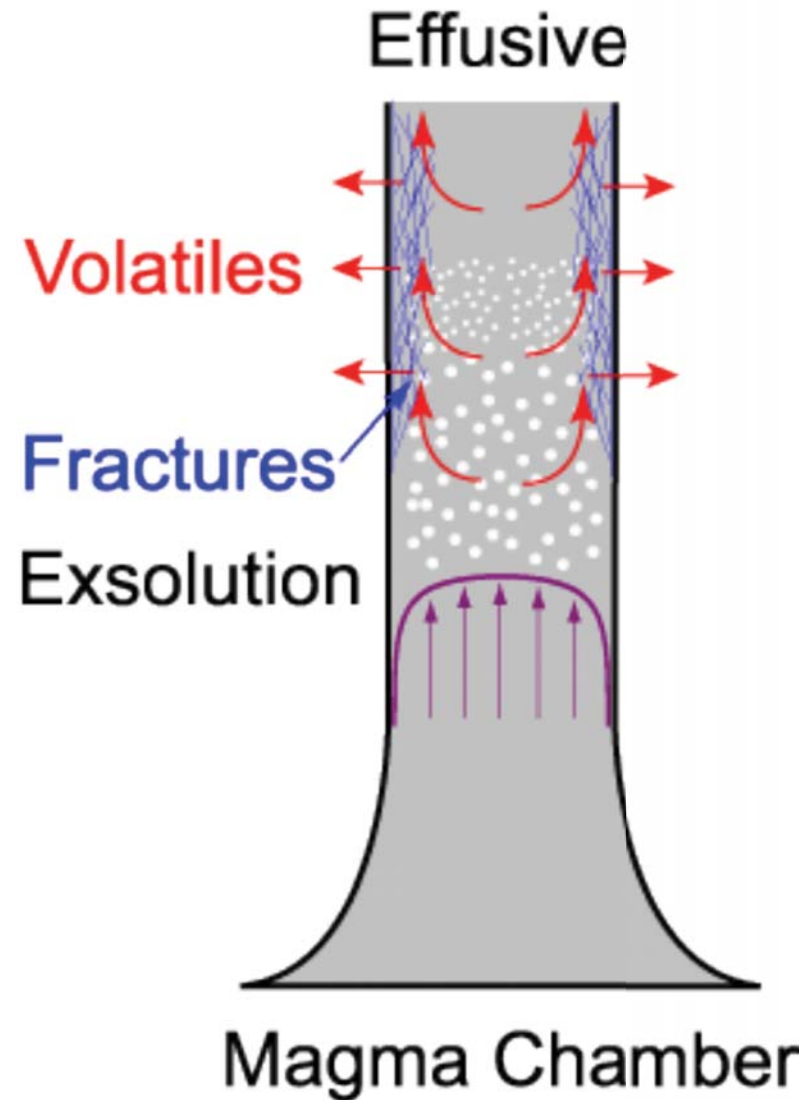


# Change in eruption style with changing ascent rate



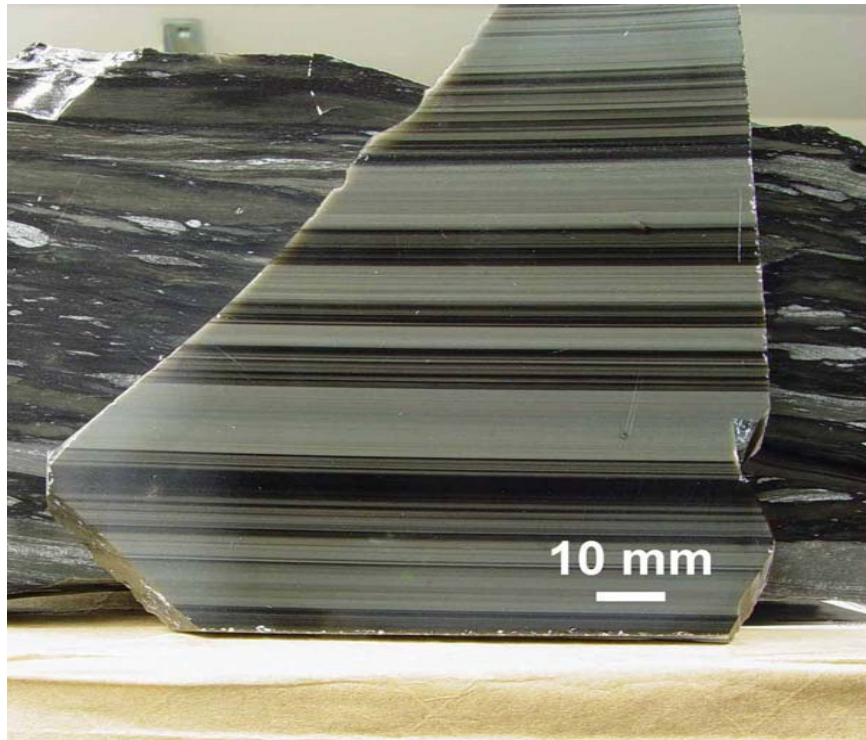
We predict that flow-induced fragmentation (brecciation) occurs at the sides of conduits

Is there any evidence that this occurs?



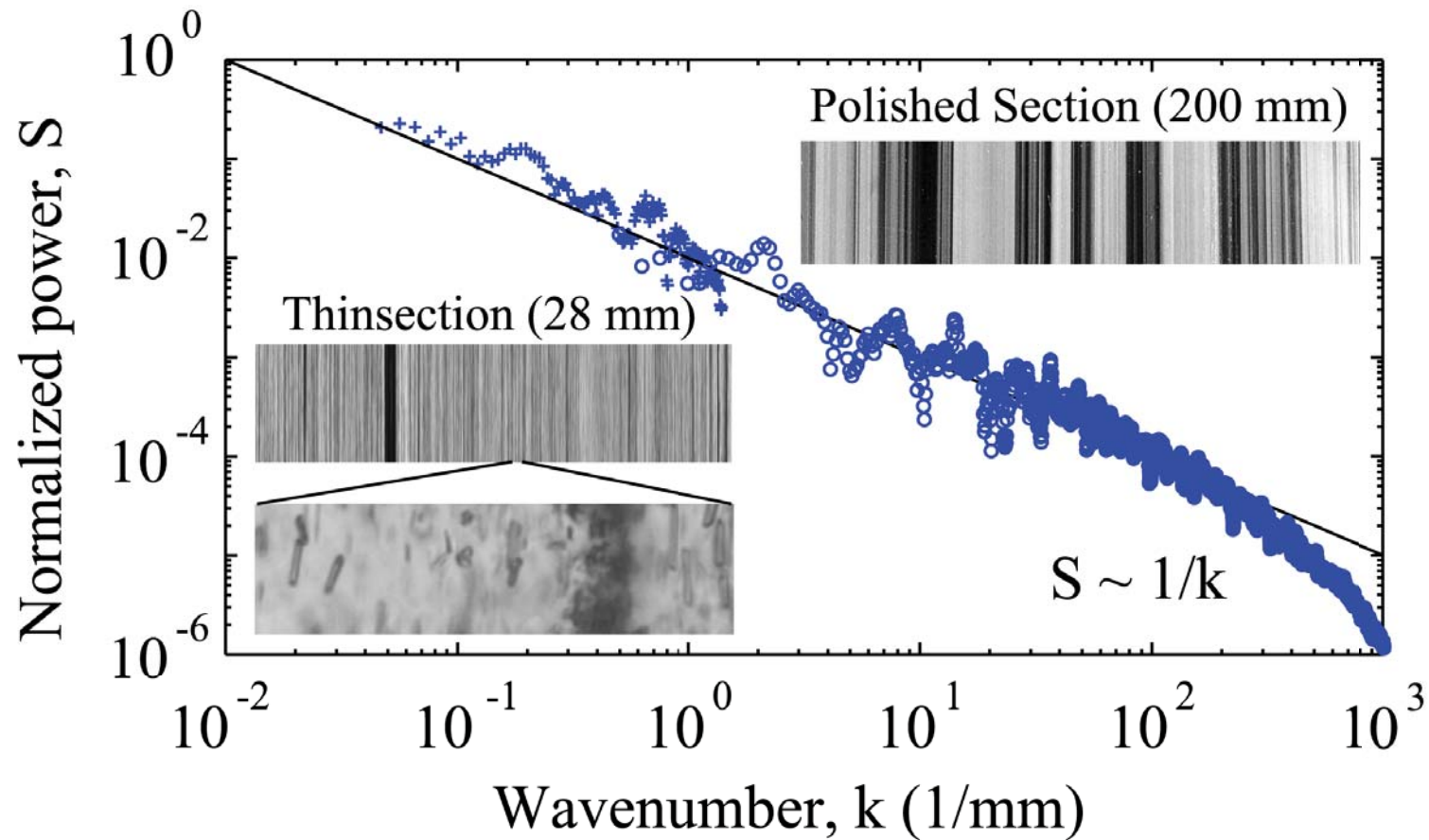


# Obsidian is banded at all scales



Do these bands (in some cases) record fragmentation?

# Power spectrum: Scale invariant banding

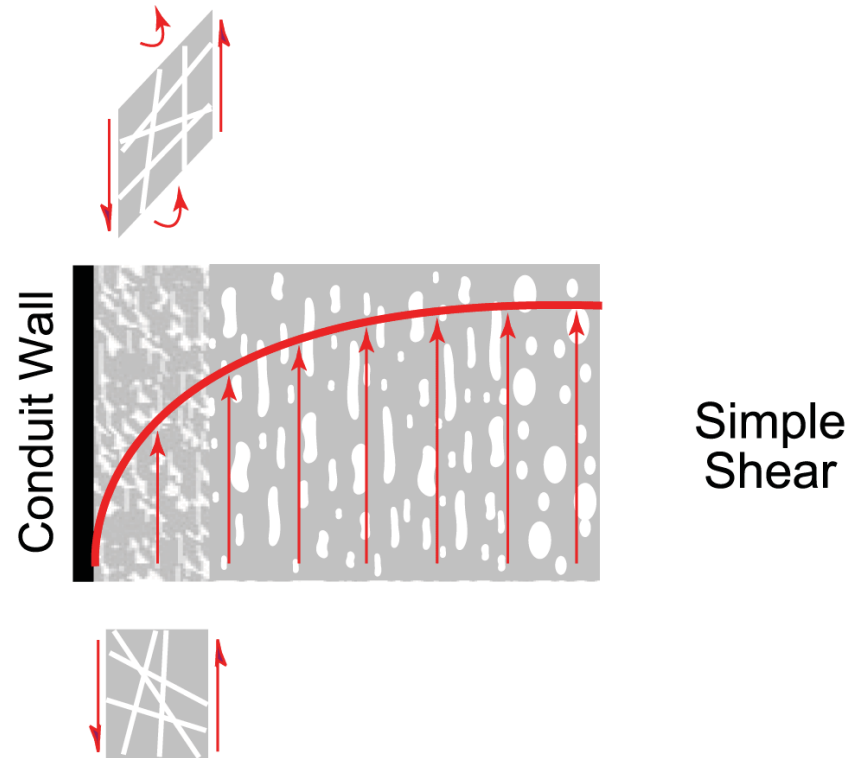


Band widths are scale invariant over 4 orders of magnitude

# Brecciation, rewelding and deformation



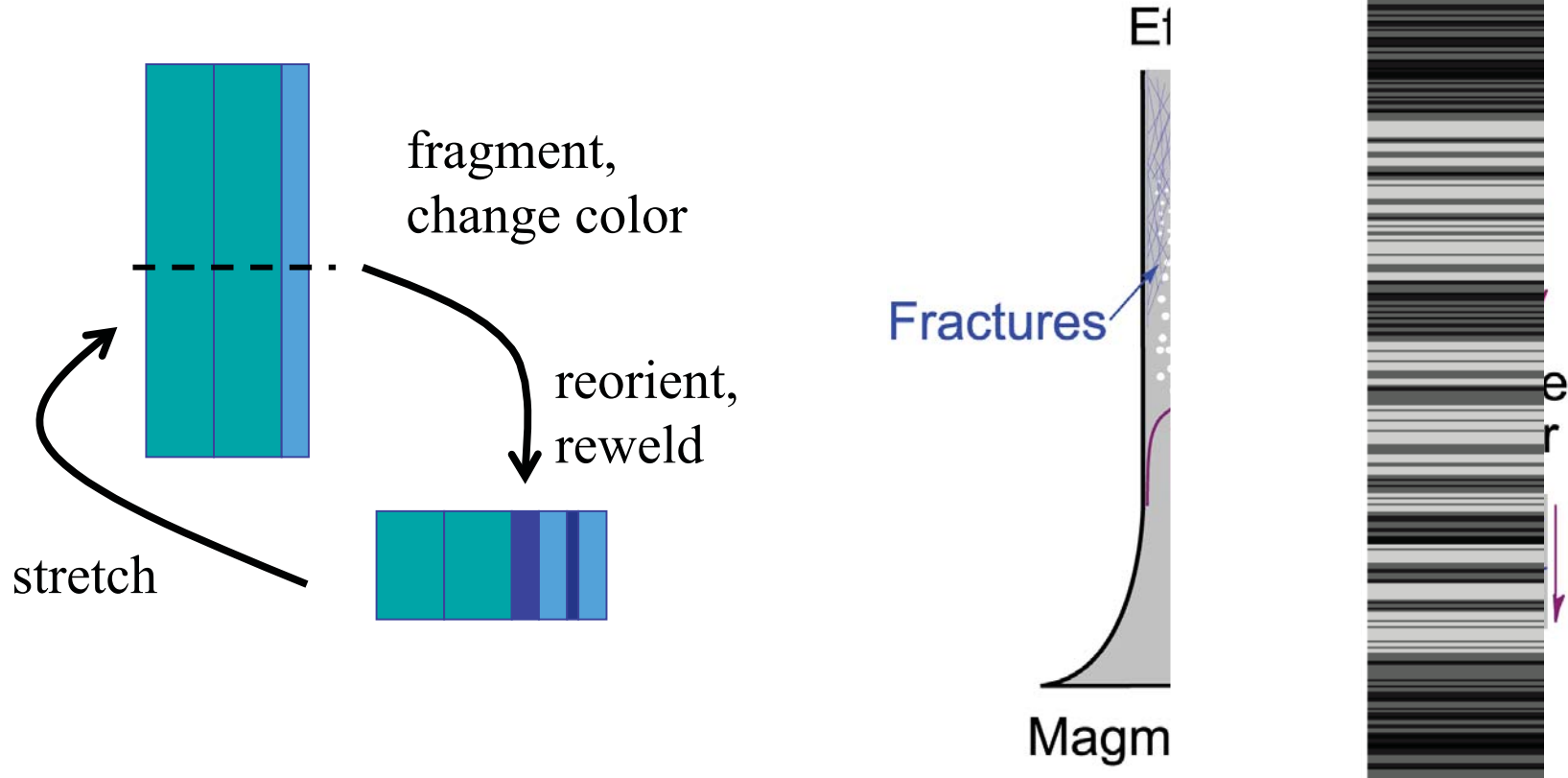
Simple shear . . . .



. . . . rotation and stretching

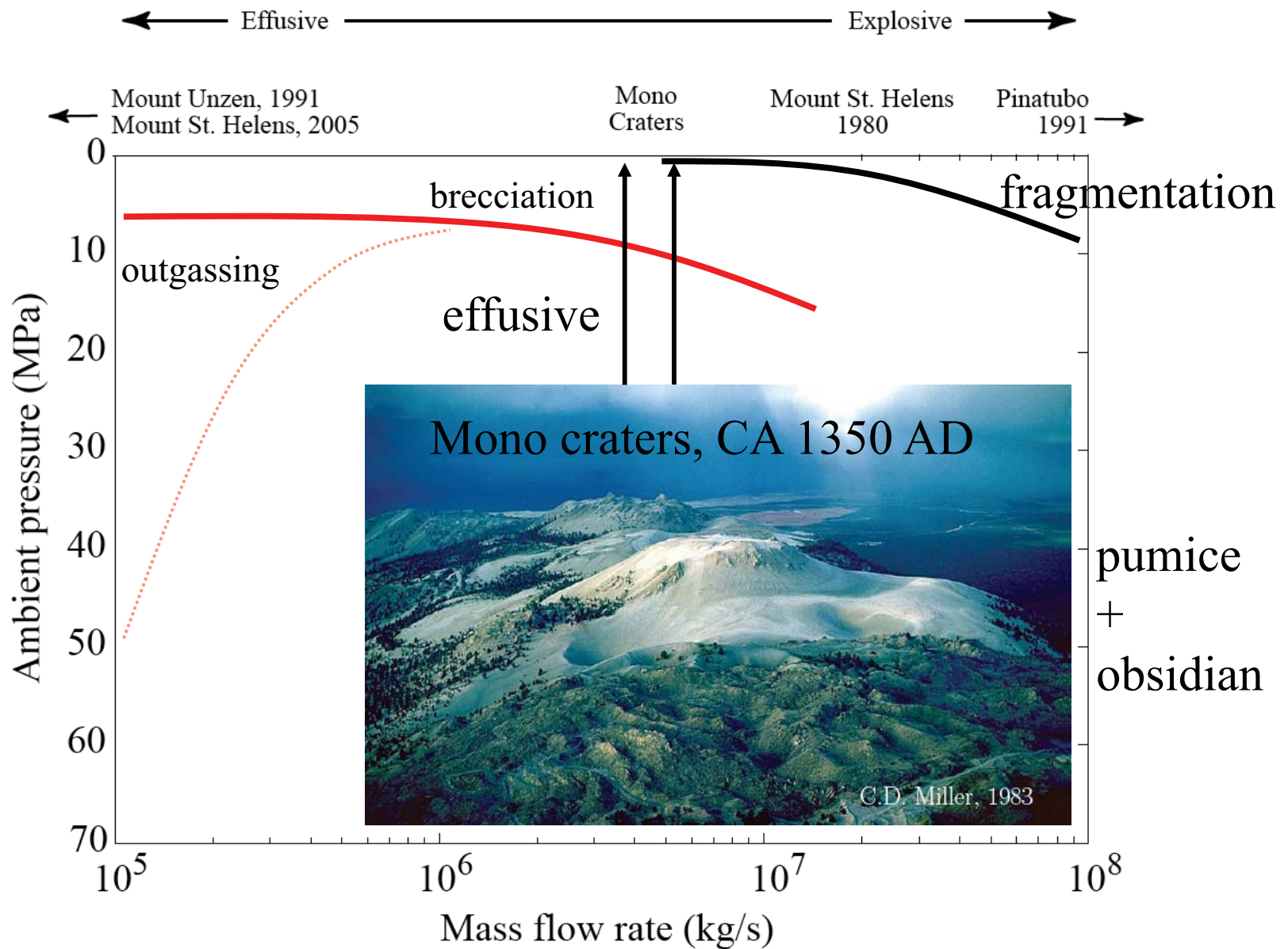


# A representative model Cantor model



Bands consistent with repeated brecciation, reorientation of fragments, welding (stick back together) and stretching (reproduce power law and multifractal characteristic of bands)

# Change in eruption style with changing ascent rate



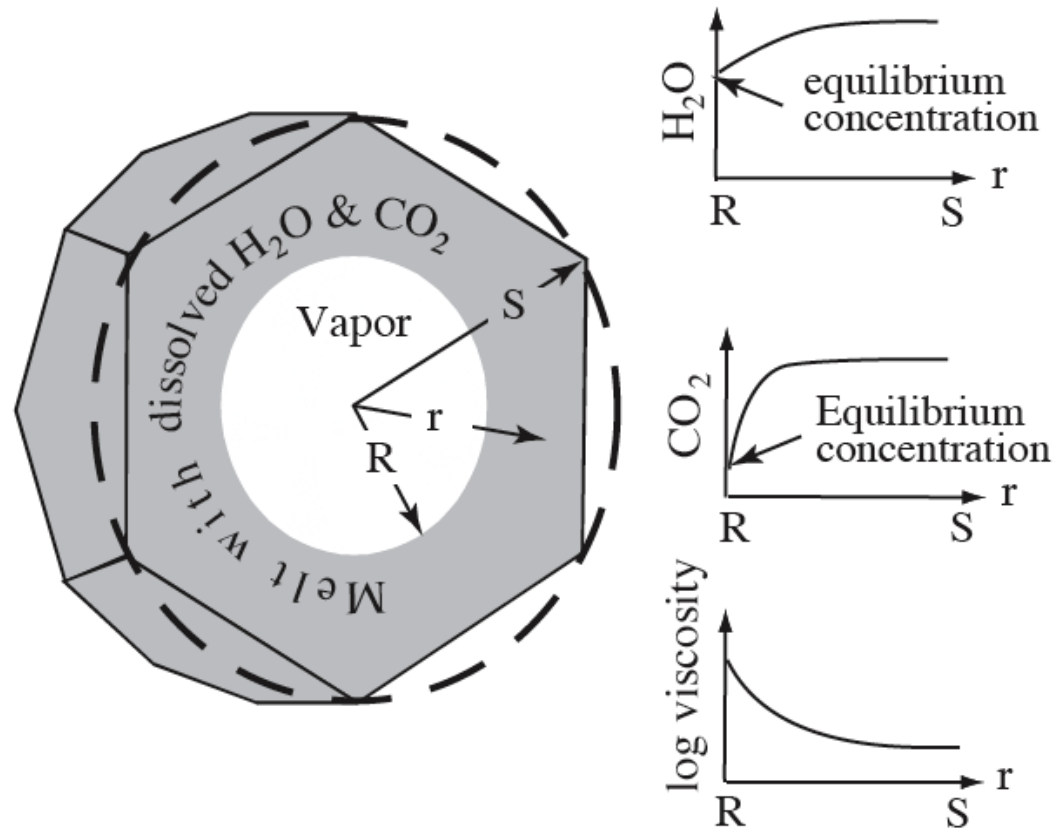


# Mono Crater, CA



Test models using the measured  
concentration of water and  $\text{CO}_2$

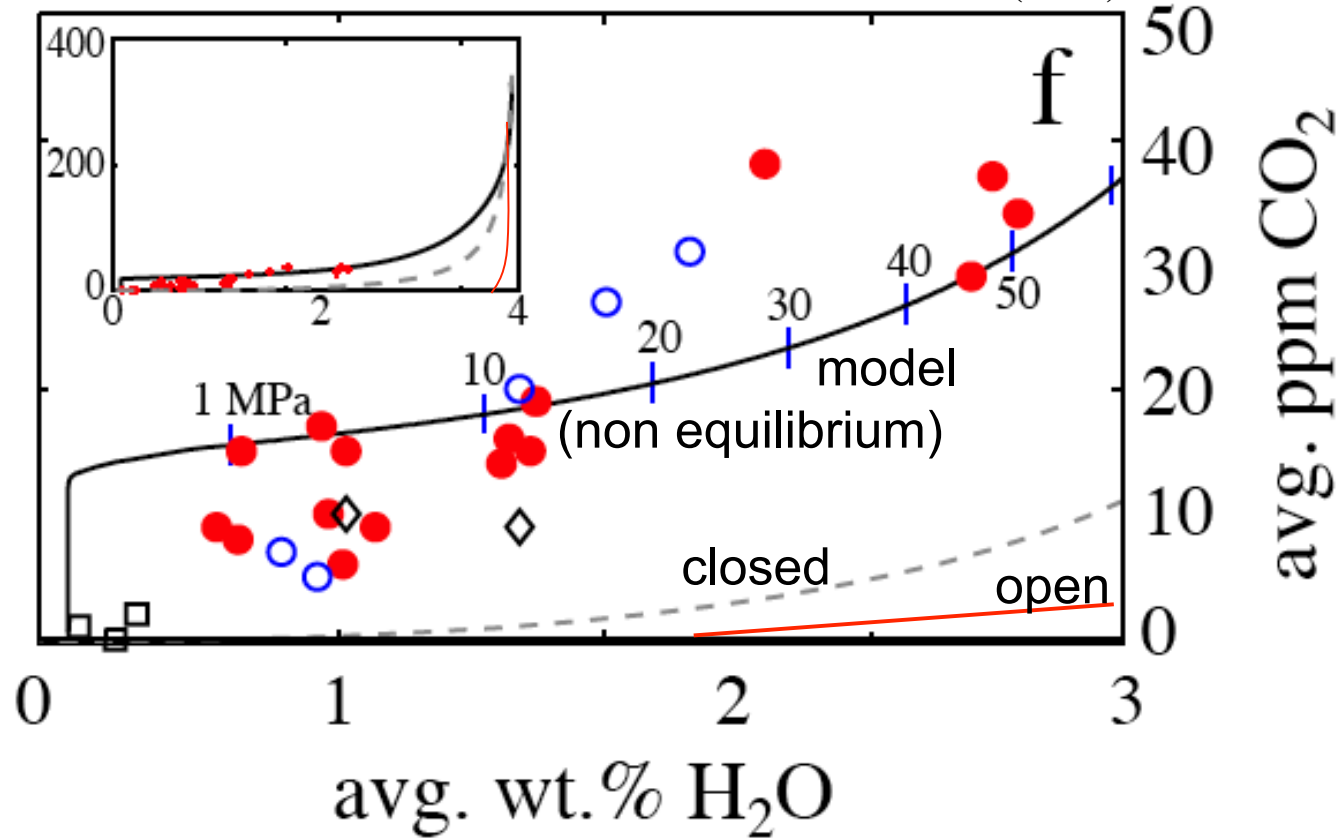
# Water diffuses faster than CO<sub>2</sub>



Concentration of gases in bubbles is not necessarily in equilibrium with that in the melt (diffusion limited growth)

# Water diffuses faster

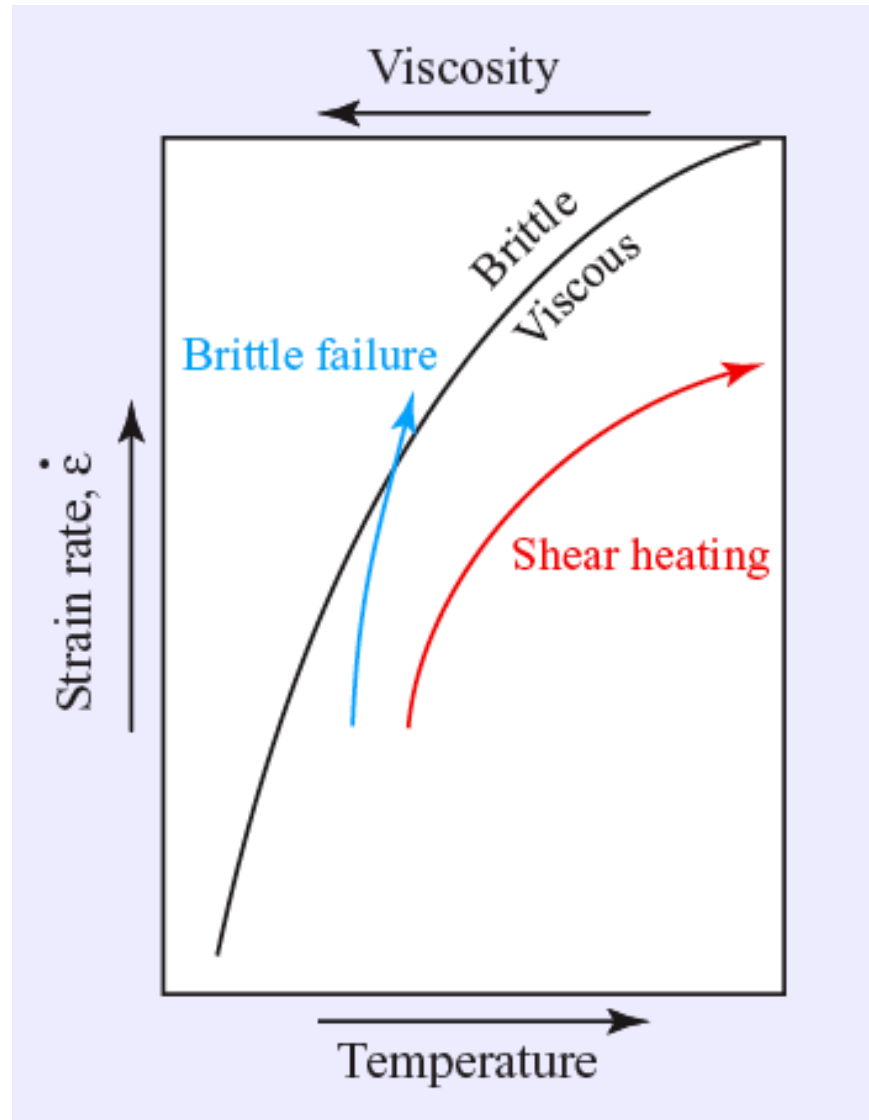
Data from Neuman et al. (1989)



Ascent rate to match data similar to other estimates

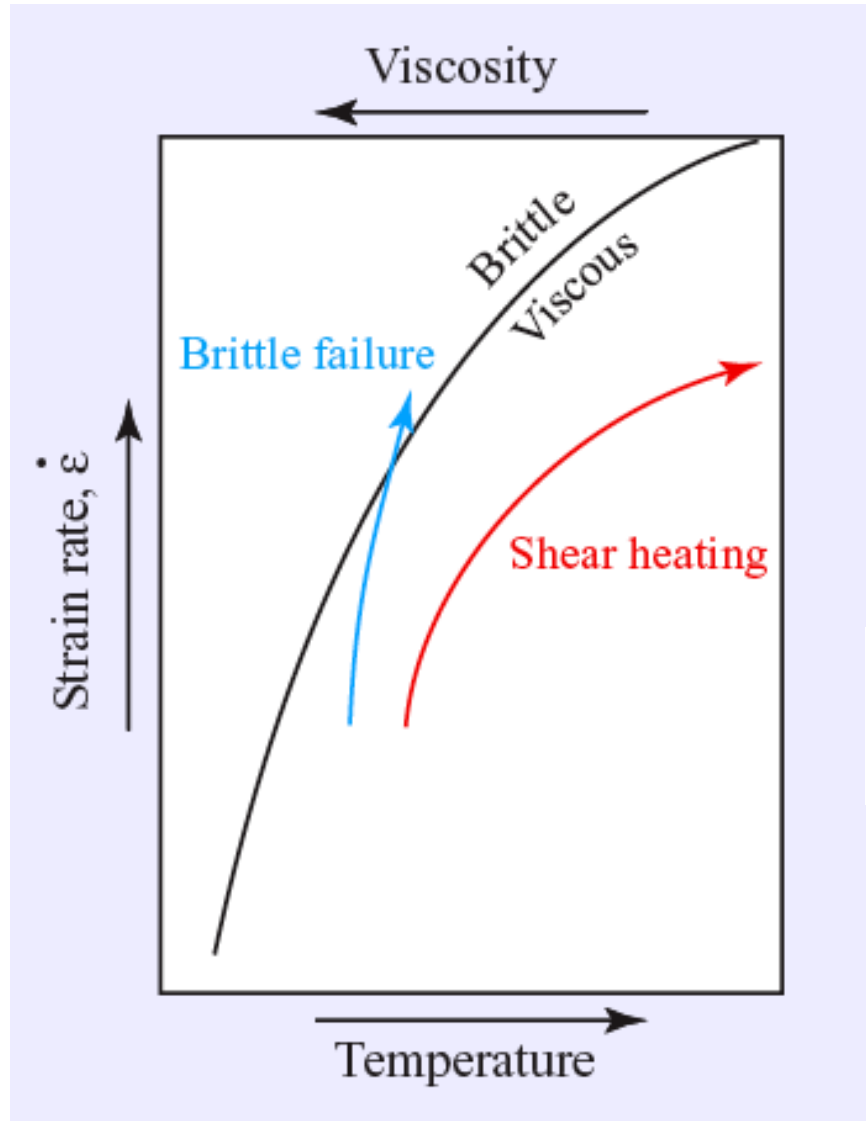
# Does brecciation always happen?

Not if the magma rises fast enough



# Does brecciation always happen?

Not if the magma rises fast enough

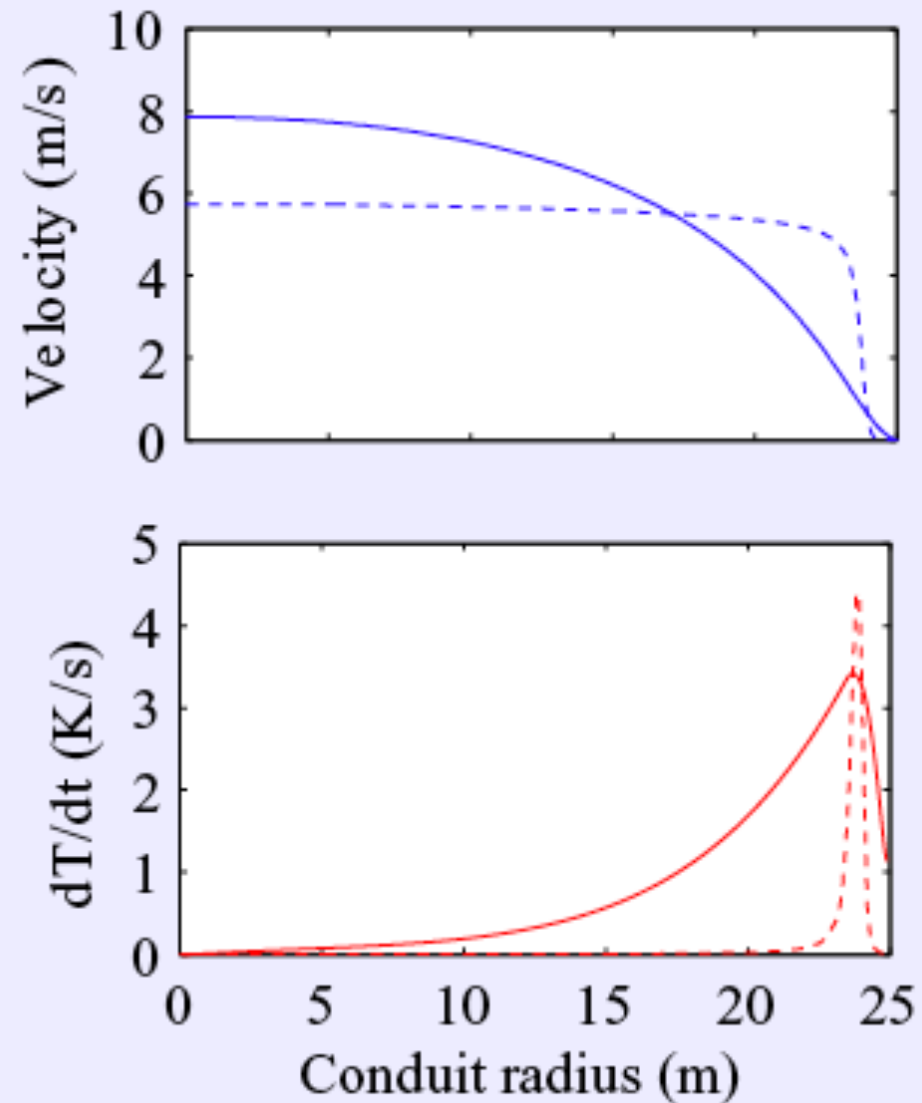
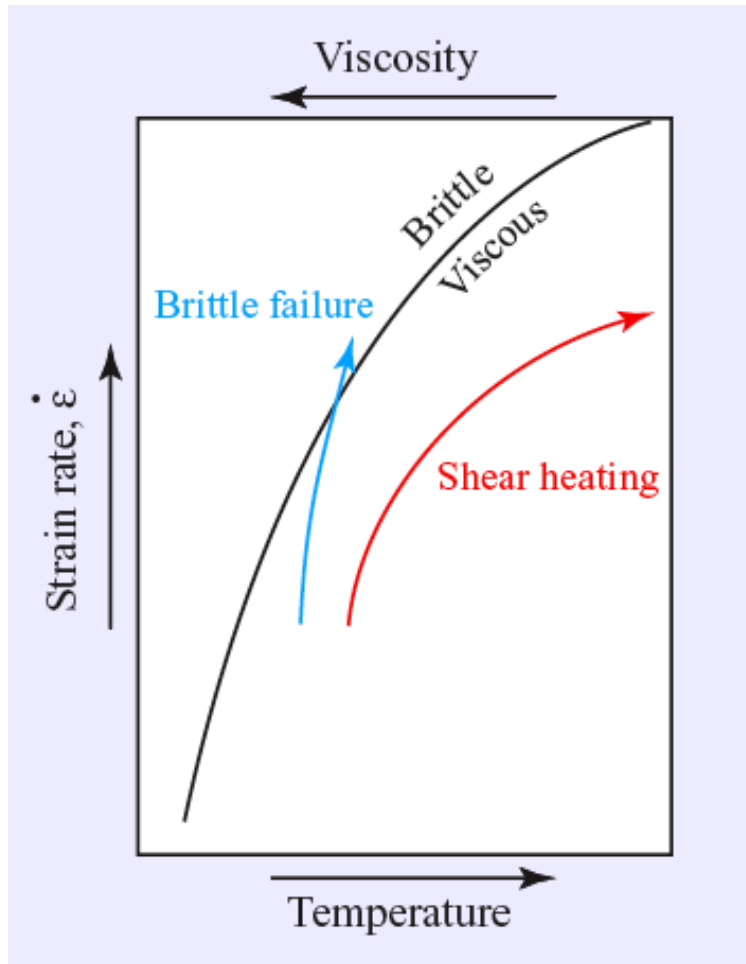


Viscous dissipation important when Brinkman number (viscous dissipation/heat diffusion)

$$\text{Br} = \frac{\eta \dot{Q}_m^2}{c_{pm} \rho_m^3 D_T \Delta T a^4 (1 - \phi)^2}$$

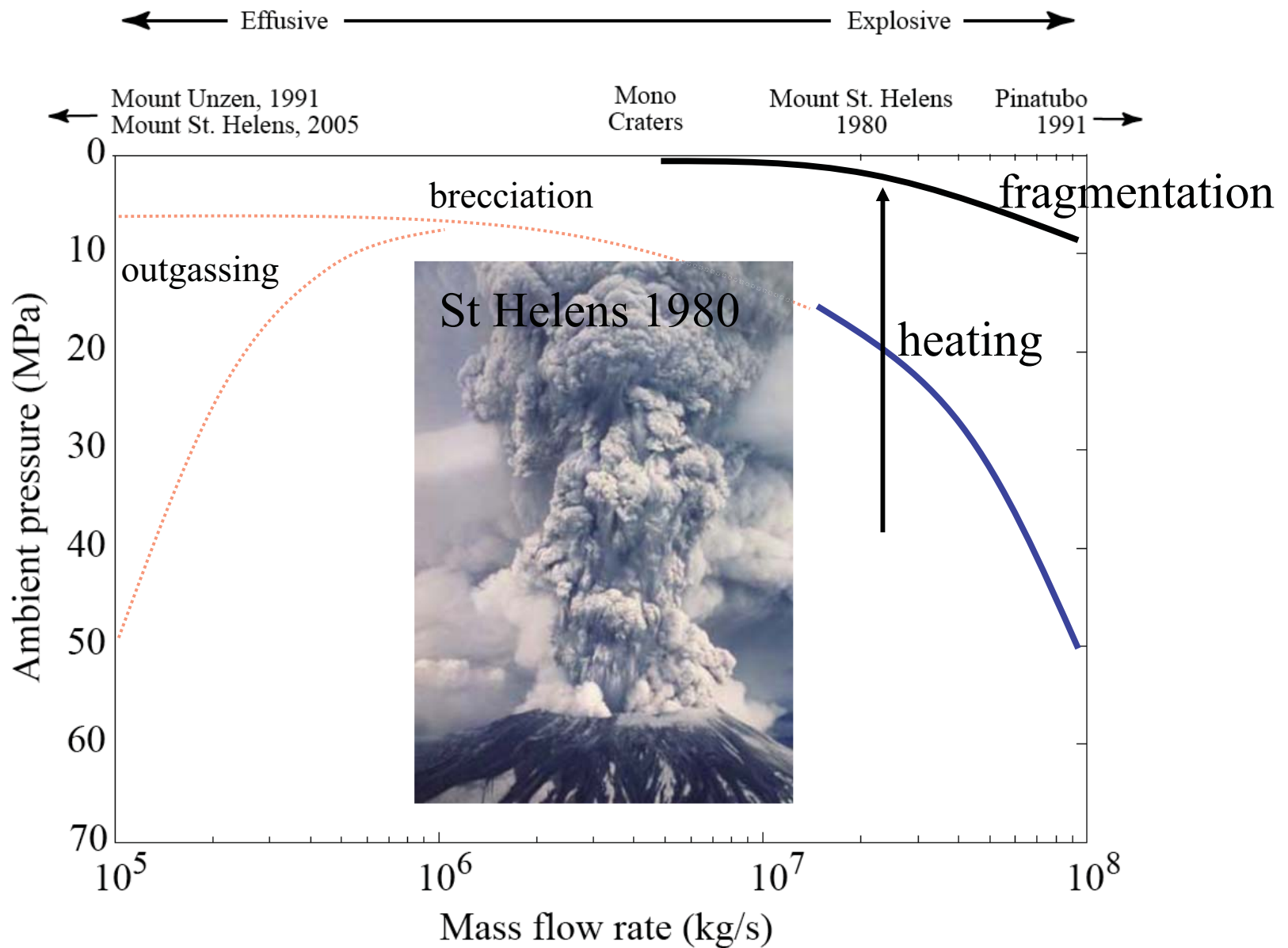
becomes large

Implications:  
no brecciation, “blunt” velocity profiles

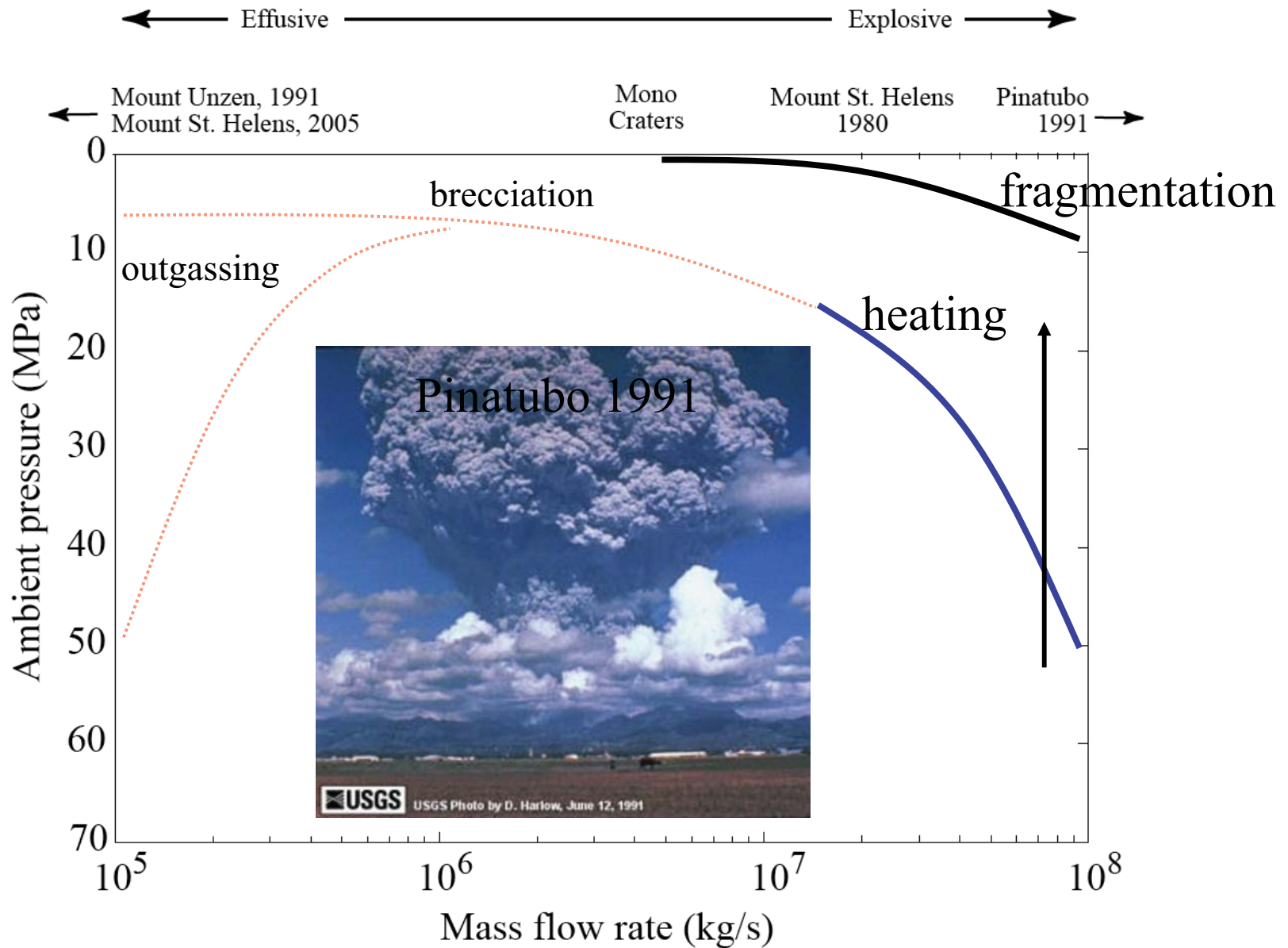




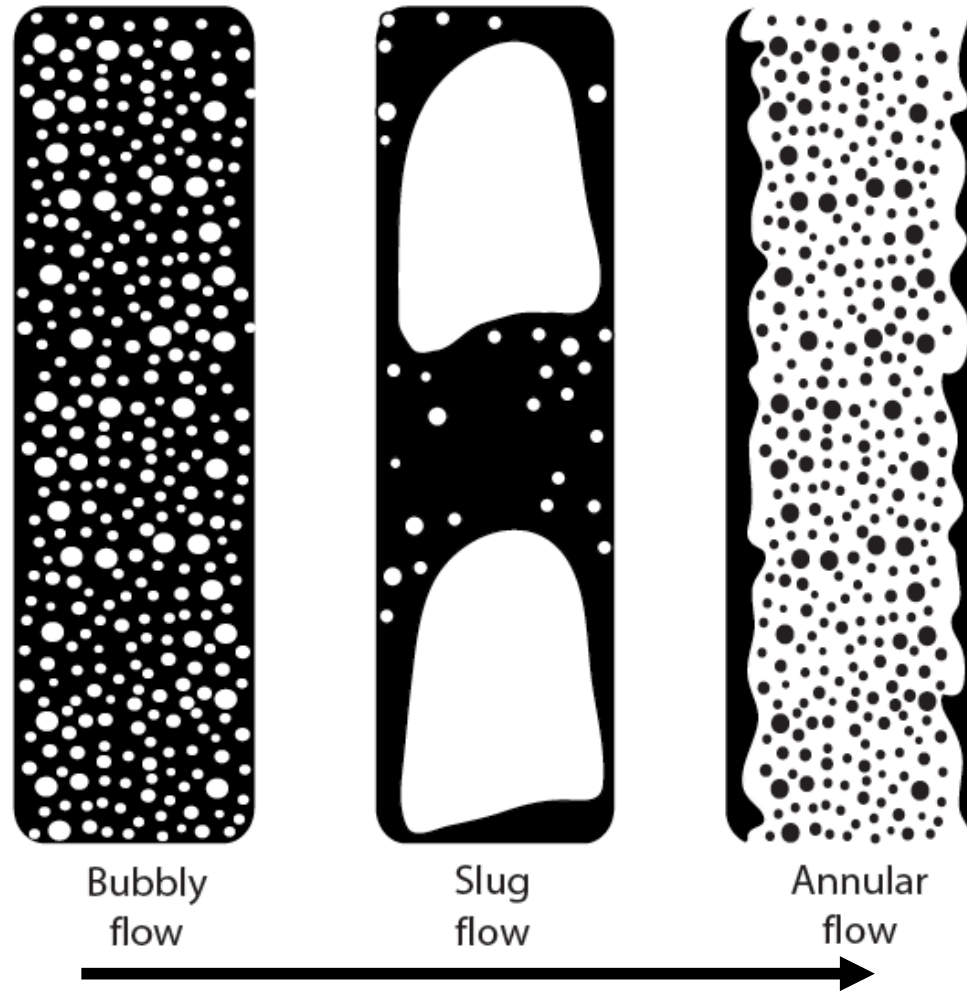
# Change in eruption style with changing ascent rate



# Change in eruption style with changing ascent rate

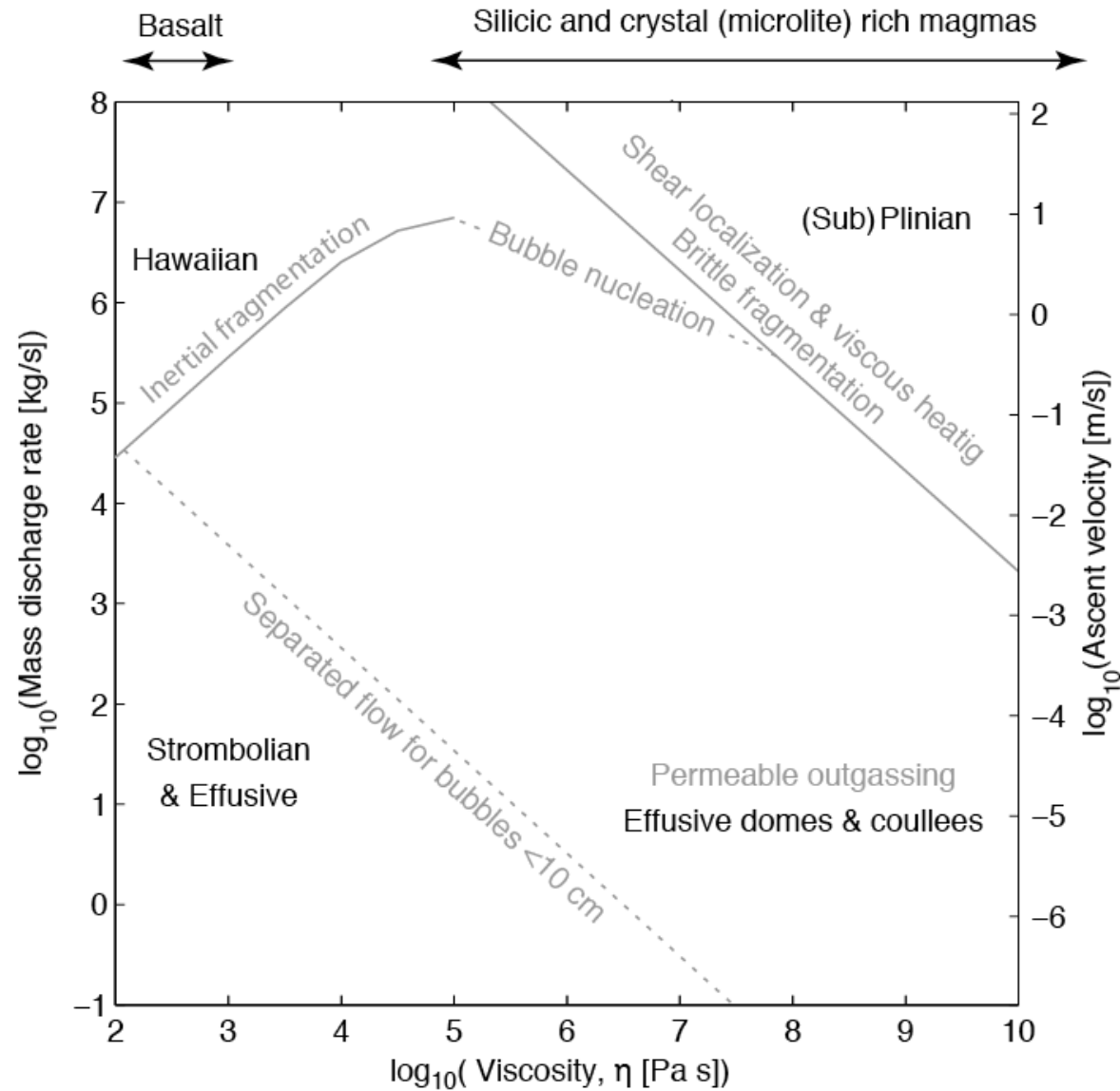


# Basaltic (low viscosity) eruptions

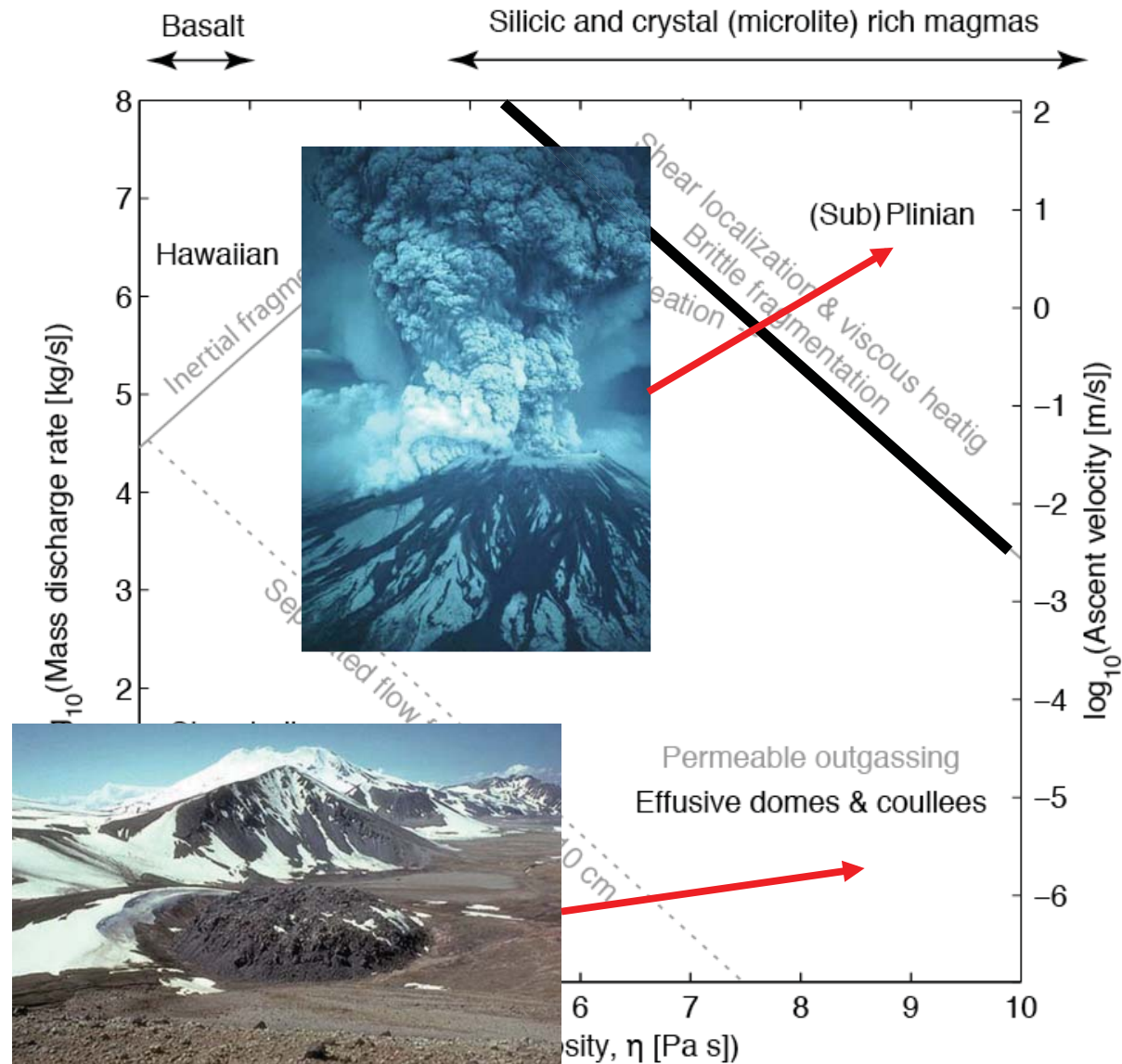


Increasing bubble/melt speed and volume fraction of bubbles

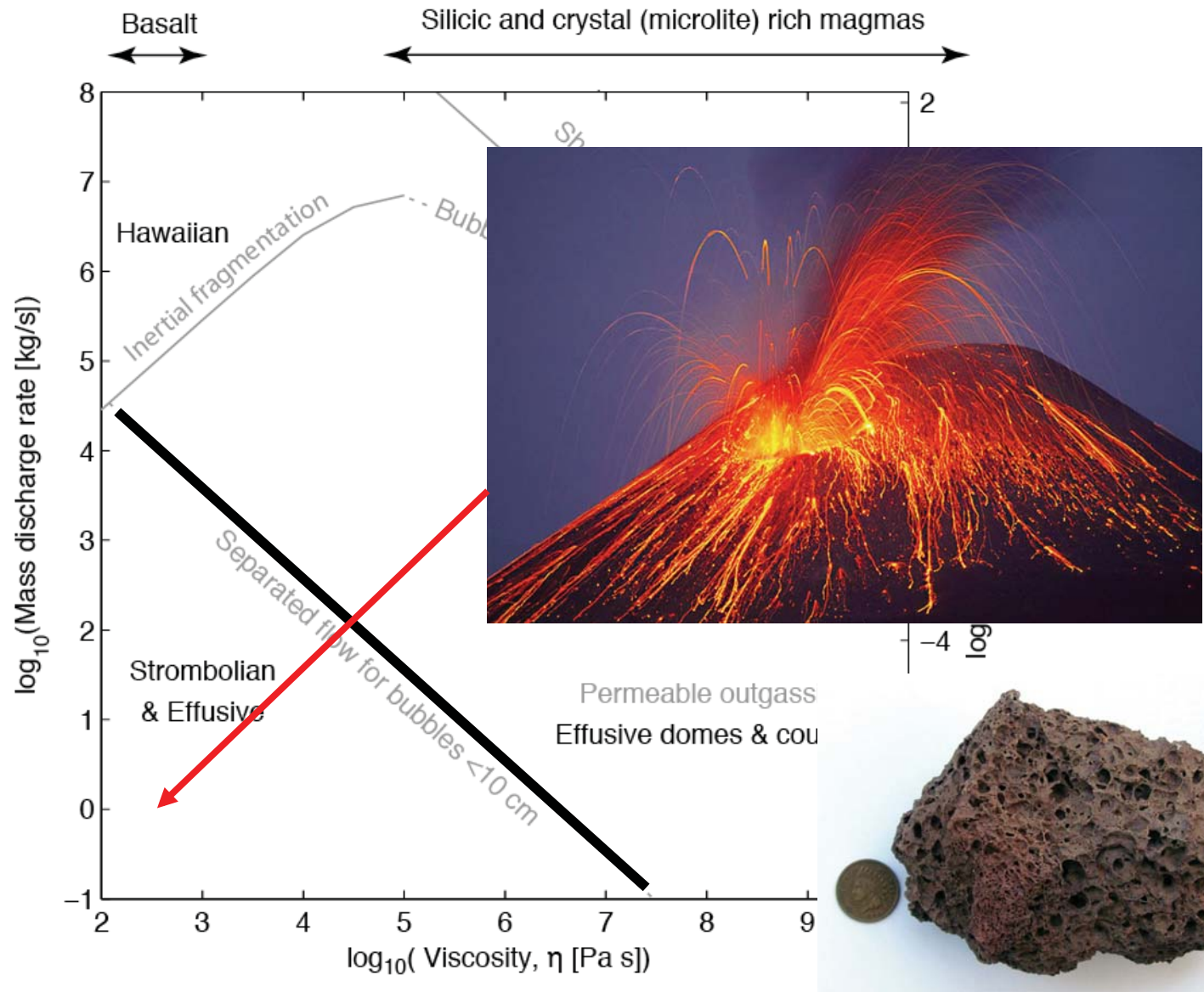
# Basaltic eruption styles



# Basaltic eruption styles

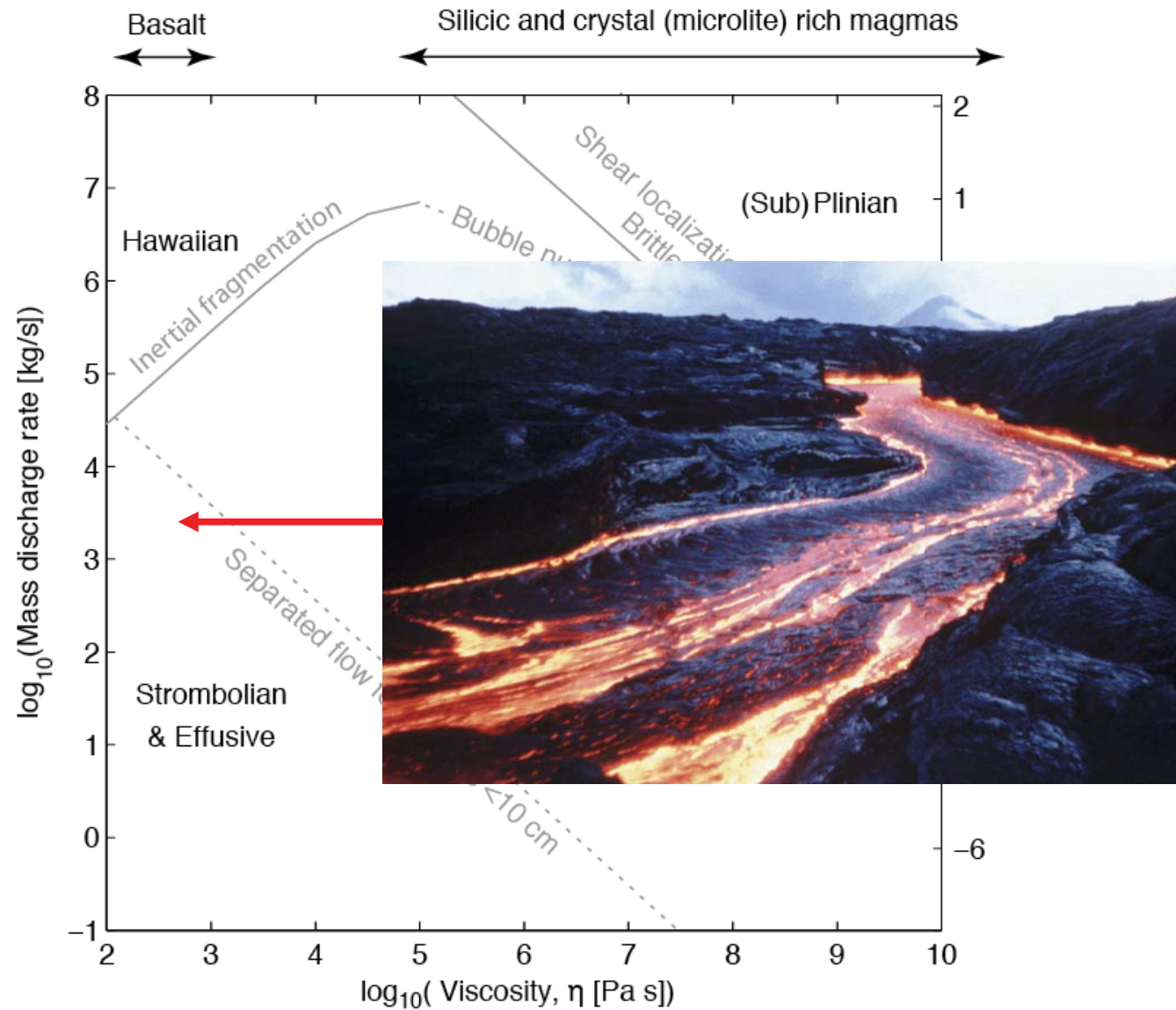


# Basaltic eruption styles

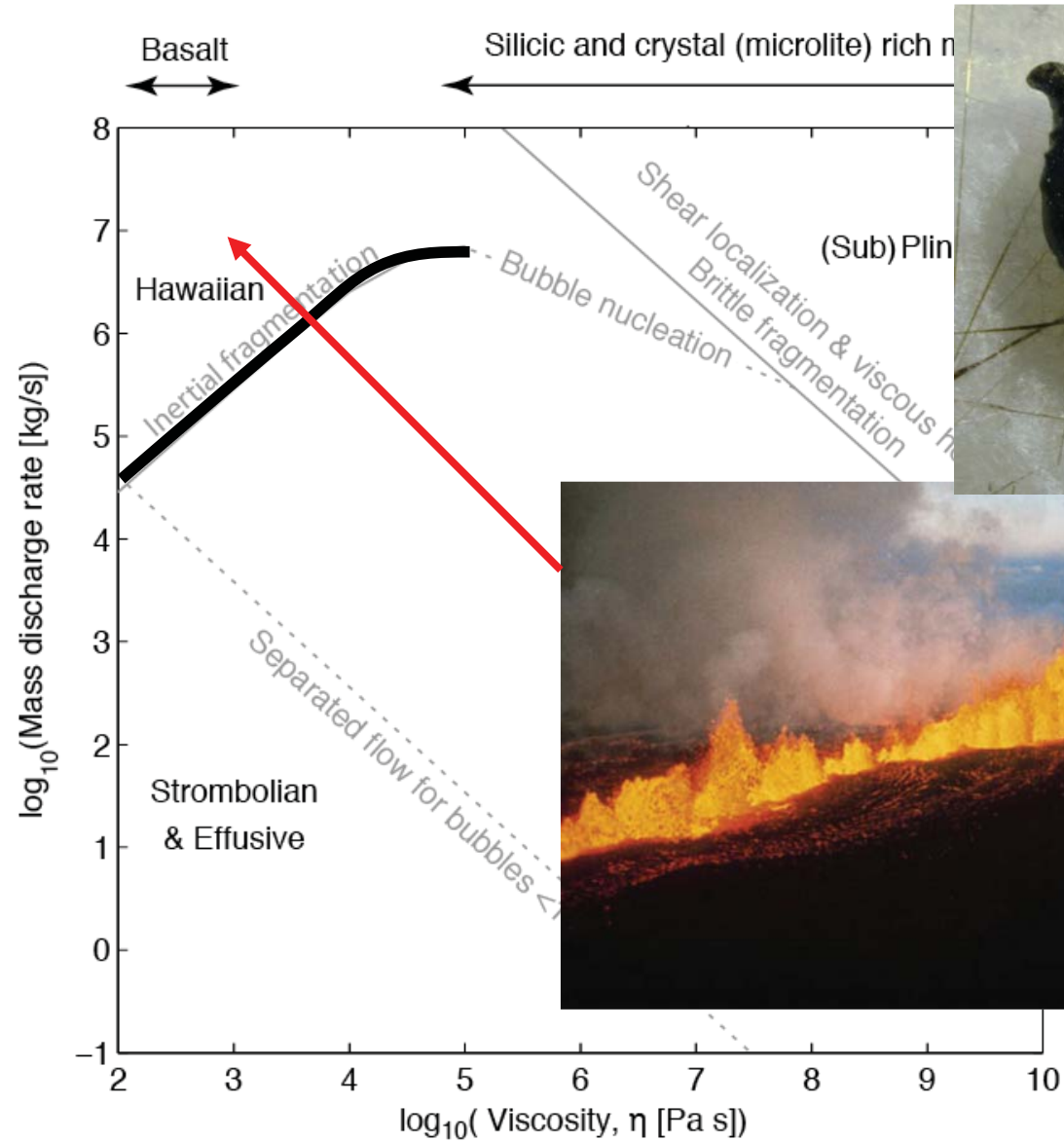




# Basaltic eruption styles



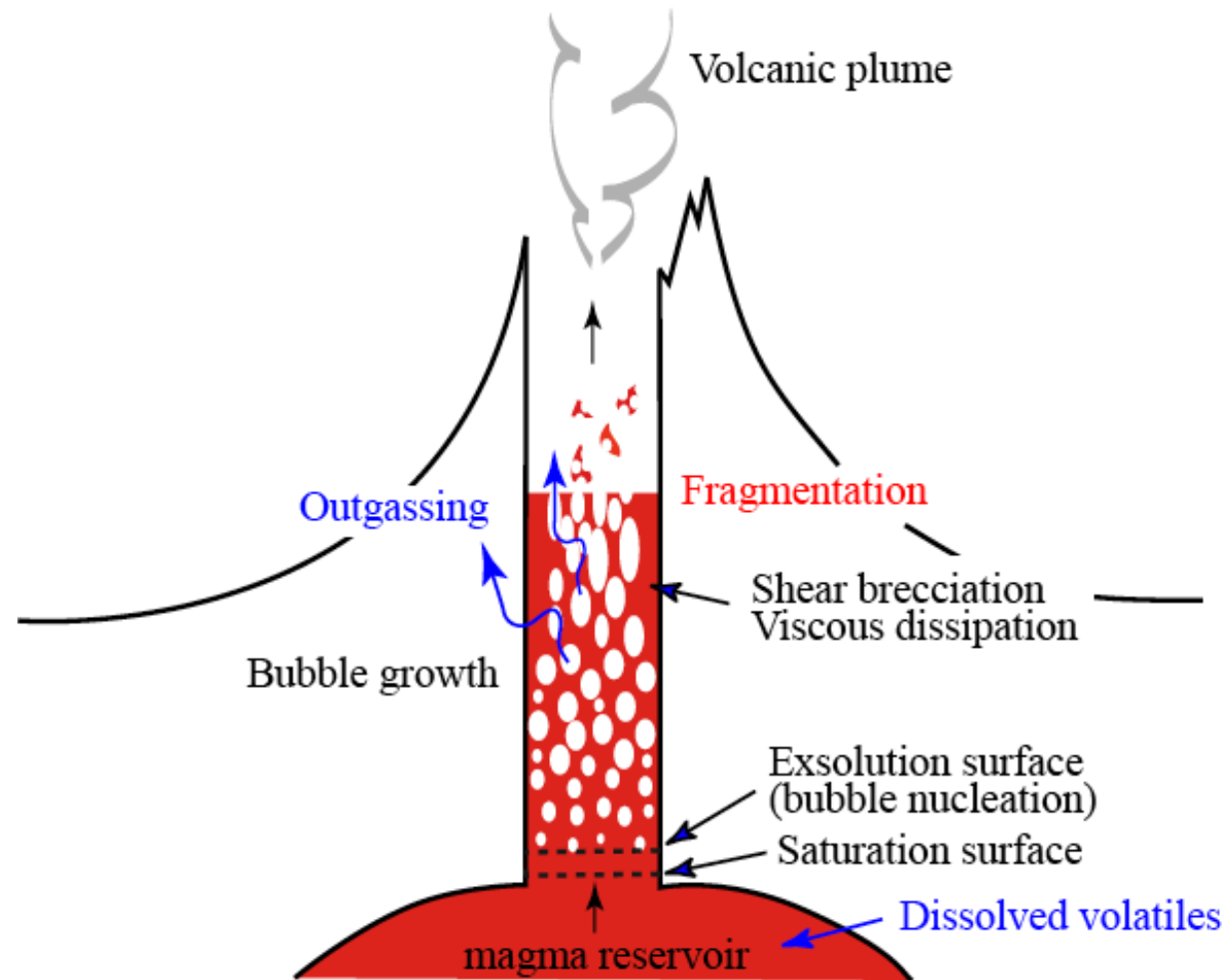
# Basaltic eruption styles



# Governing physical processes: summary

Dimensionless number	Process	Value and effect
Reynolds number (inertia/viscous forces)	Bubble growth Magma ascent	$\ll 1$ $< 10^3$ ; laminar flow prior to fragmentation
Peclet number (diffusion/decompression timescale)	Diffusive growth	$\gg 1$ for low $N_d$ ; supersaturation, nucleation new bubbles
Peclet number (viscous/decompression timescale)	Bubble expansion	$\gg 1$ is viscosity high enough; overpressure, fragmentation
Brinkman number (viscous dissipation/diffusion of heat)	Viscous heating at conduit walls	if large enough, lowers viscous and prevents shear brecciation
Dimensionless shear rates (shear stress/surface tension or shear rate x relaxation time of melt)	Magma ascent	if large enough, shear thinning and blunt velocity profiles; larger still, becciation
Ascent rate bubbles/magma	Bubble separation	

# Why do volcanoes (only sometimes) erupt explosively?



- Interplay between bubble growth, brecciation, outgassing, and fragmentation governs eruption style

# How do bubbly fluids respond to rapid decompression?

## Experimental model

