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Advanced School on Scaling Laws in Geophysics: Mechanical and Thermal Processes in Geodynamics

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DEEP CONVECTION III
(Mantle Convection Planform and Convective Wavelengths)

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Mantle convection planform and convective wavelengths

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ICTP Advanced School on Scaling Laws in Geophysics
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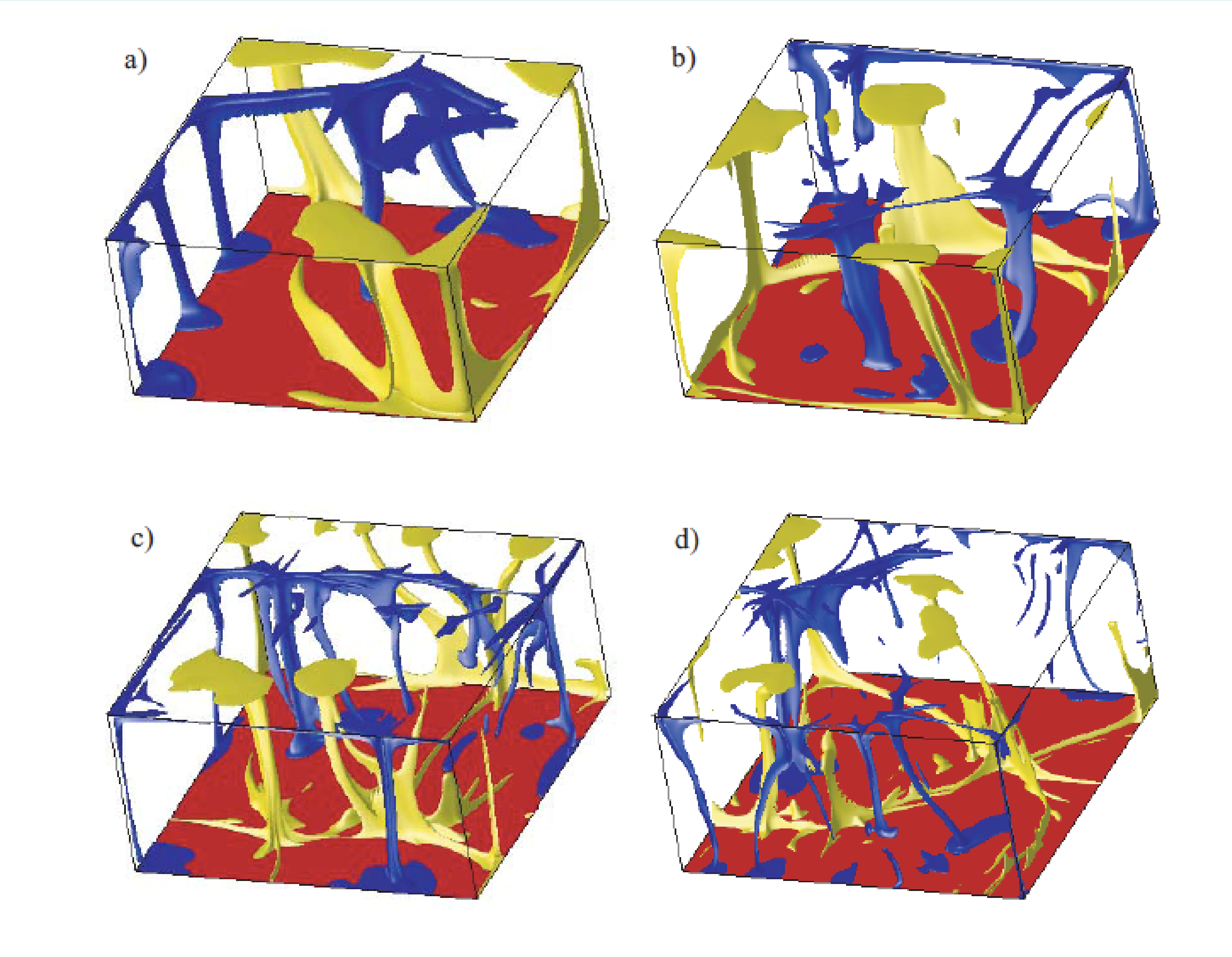
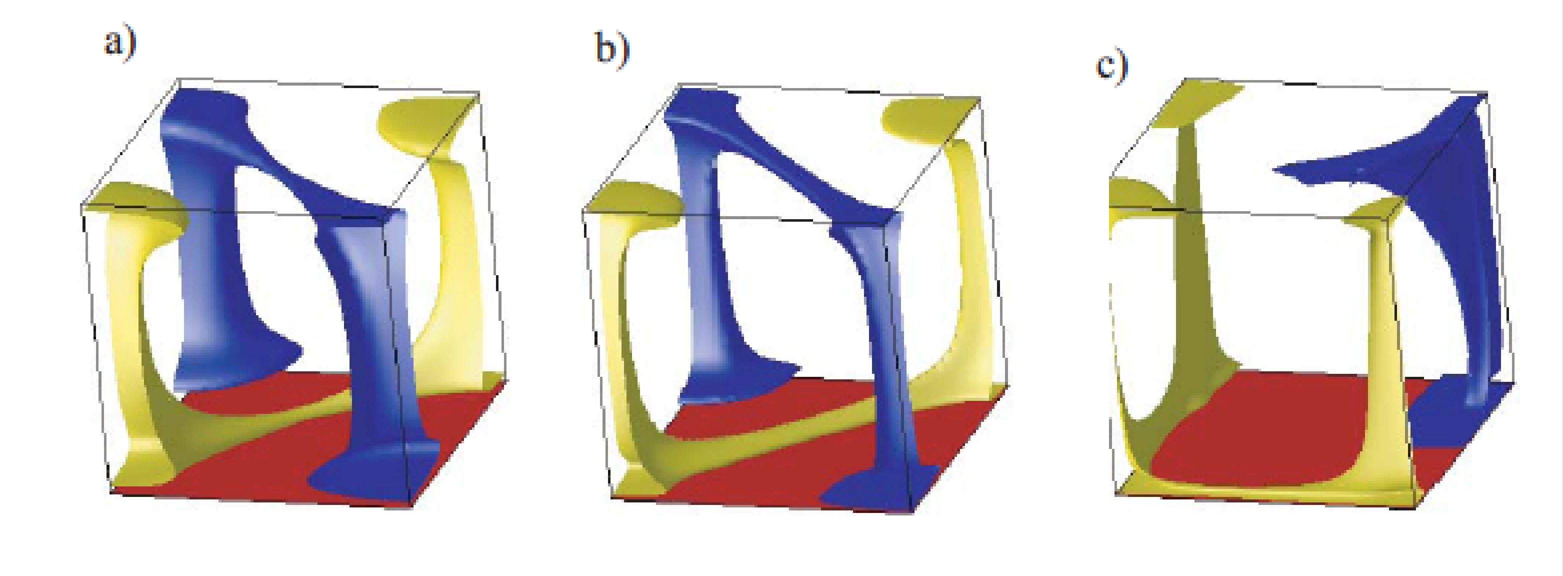
Constant viscosity convection at different Ra (3e6, 1e7, 3e7 and 1e8)



Constant viscosity convection at different Ra (3e4, 3e5, and 3e6)



A scaling analysis for plume populations (Zhong, 2005)

Suppose that for thermal convection in 3-D, the number of plumes including downwelling and upwelling $N_p \sim Ra^n$, then the average spacing between downwelling and upwelling plumes $\lambda \sim D(1/N_p)^{1/2} \sim DRa^{-n/2}$ (notice that for basal heating and isoviscous convection the average spacing between upwelling plumes is 2λ and that the number of upwelling plumes is $\sim N_p/2$). Let us consider the bottom TBL that thickens with time starting from below a downwelling. The critical TBL thickness $\delta \sim (\kappa t)^{1/2}$ where $t=\lambda/u_b$ where u_b is an averaged horizontal velocity at the bottom boundary. Suppose that $u_b \sim (\kappa/D)Ra^{v}$, then we have $\delta \sim (\kappa\lambda/u_b)^{1/2} \sim DRa^{-n/4-v/2}$. Also because $Nu \sim Ra^{\beta}$, $\delta \sim DNu^{-1} \sim DRa^{-\beta}$. Therefore, we have the following relationship between the scaling exponents n, β and v:

$$n = 4\beta - 2v. \quad (6)$$

For thermal convection in 2-D, a similar analysis leads to $n=2\beta-v$.

Scaling for plume vertical velocity and radius

force balance for a plume:

$$\pi R_{up}^2 \rho g \alpha \Delta T_{up} = 2\pi R_{up} \eta (dV/dr)_{r=R_{up}},$$

$$V_{up} \sim \lambda R_{up} \rho g \alpha \Delta T / \eta,$$

$$V_{up} \sim \frac{\kappa}{D^3} \lambda R_{up} Ra \sim \frac{\kappa}{D^2} R_{up} Ra^{1-n/2},$$

The energy balance leads to

$$N_p \rho C \Delta T_{up} V_{up} \pi R_{up}^2 \sim k \frac{\Delta T}{D} Nu \sim k \frac{\Delta T}{D} Ra^{\beta}.$$

$$R_{up} \sim D Ra^{(\beta-1-n/2)/3}.$$

$$V_{up} \sim \frac{\kappa}{D} Ra^{2(1-n+\beta/2)/3}.$$

For β=1/3 and n=0, the exponents for plume radius and velocity are -2/9 and 7/9, respectively.

Table 1. Model parameters and Nu and u_b.

Case	Ra	Box size	Mesh size[a]	Nu	u_b
1	10^4	$1 \times 1 \times 1$	$48 \times 48 \times 48$	4.72	31.77
2	3×10^4	$1 \times 1 \times 1$	$48 \times 48 \times 48$	7.32	72.5
3	10^5	$1 \times 1 \times 1$	$48 \times 48 \times 48$	11.24	165.4
4	3×10^5	$1 \times 1 \times 1$	$48 \times 48 \times 48$	16.10	335.7
5	10^6	$1 \times 1 \times 1$	$64 \times 64 \times 64$	23.45	710.5
6	3×10^6	$1 \times 1 \times 1$	$64 \times 64 \times 64$	29.28(1.51)[b]	1472(110)
7	10^4	$3 \times 3 \times 1$	$96 \times 96 \times 48$	5.12[4.54][c]	49.8[63.7]
8	3×10^4	$3 \times 3 \times 1$	$96 \times 96 \times 48$	7.44[6.21]	104.3[123.1]
9	10^5	$3 \times 3 \times 1$	$96 \times 96 \times 48$	10.8[8.82]	223.3[244]
10	3×10^5	$3 \times 3 \times 1$	$96 \times 96 \times 48$	14.3[–]	397.9[–]
11	3×10^6	$2 \times 2 \times 1$	$128 \times 128 \times 64$	27.1(1.0)	1642(153)
12	10^7	$2 \times 2 \times 1$	$192 \times 192 \times 96$	41.2(1.9)	3085(348)
13	3×10^7	$2 \times 2 \times 1$	$256 \times 256 \times 96$	57.3(1.9)	5162(507)
14	10^8	$2 \times 2 \times 1$	$320 \times 320 \times 96$	82.1(2.4)	11012(1018)

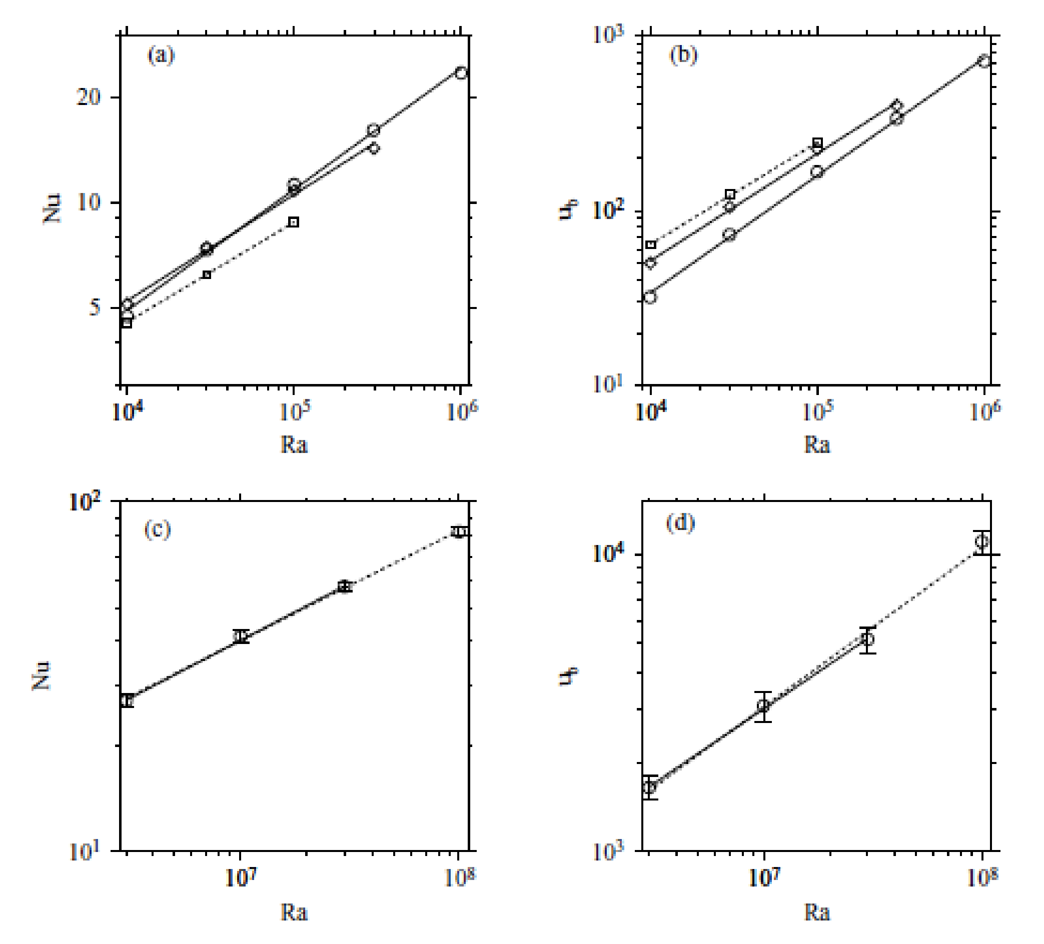


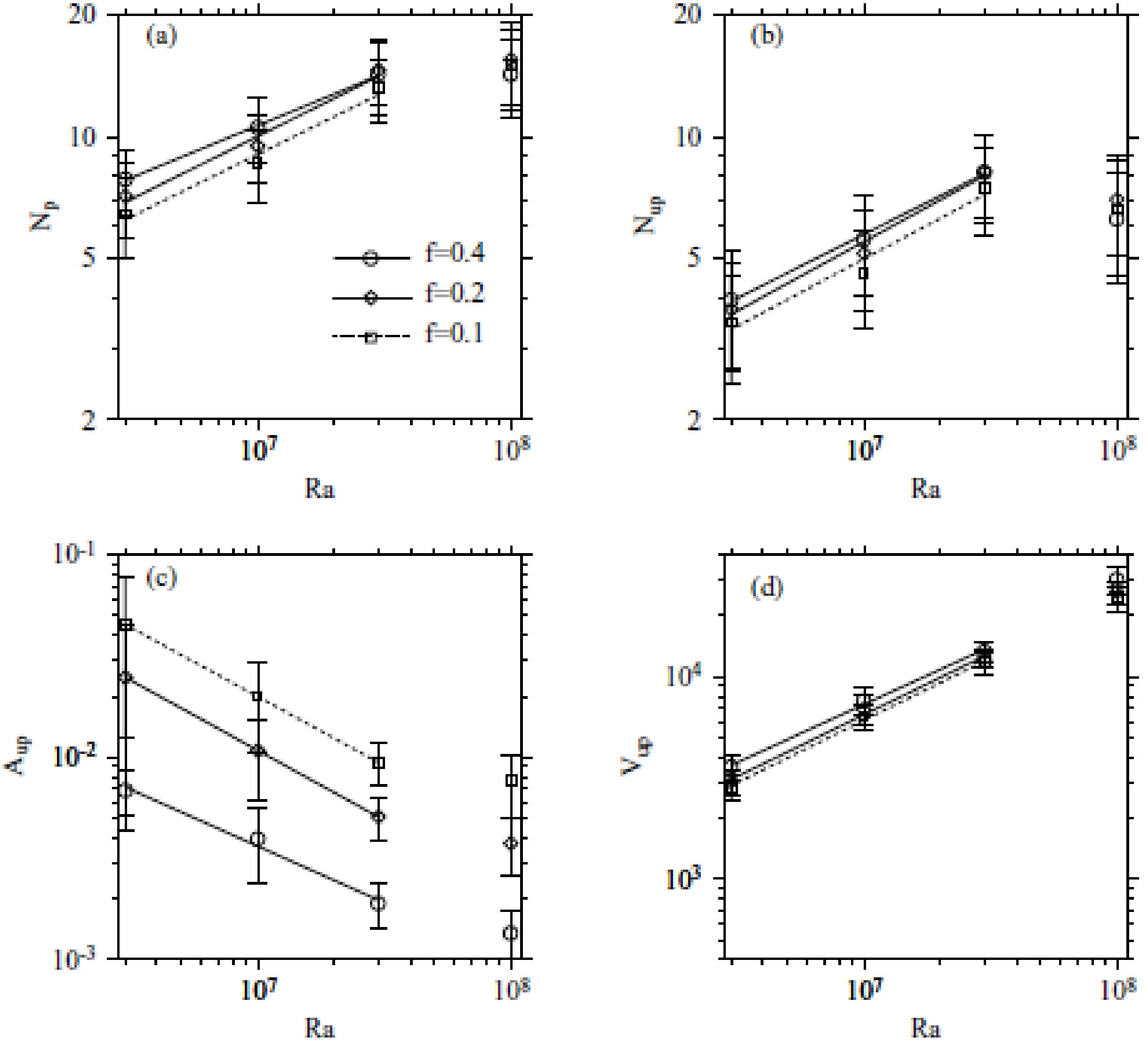
Table 2. Coefficient p and exponent q for fitting a power-law function pRa^q.

	Cases 1–5	Cases 7–10[a]	Cases 7–9[a]	Cases 11–14	Cases 11–13
Nu[b]	(0.201, 0.347)	(0.322, 0.303)	(0.326, 0.286)	(0.250, 0.315)	(0.210, 0.326)
u_b	(0.0683, 0.673)	(0.182, 0.613)	(0.298, 0.583)	(0.542, 0.536)	(0.985, 0.498)
n[c]	0.04	−0.01	−0.02	0.188	0.308

[a]Cases 7–10 are for the quasi-steady-state solutions, while cases 7–9 are for the final steady state.

[b]p and q are given as (p, q).

[c] n is the predicted exponent for plume number and is equal to $4(q_{Nu} - q_u/2)$, where q_{Nu} and q_u are exponents for Nu and u_b, respectively (they are also given as β and υ).



(a)
(b)
(c)
(d)
20
10
5
2
N_p
N_{up}
A_{up}
V_{up}
10^{-1}
10^{-2}
10^{-3}
10^4
10^3
10^7
10^8
Ra
f=0.4
f=0.2
f=0.1

Table 3. The number, area, and vertical velocity of plumes for cases 11–14.

	f	P_f	Case 11[a]	Case 12	Case 13	Case 14	(p, q)[b]
$\langle N_{\mathrm{up}}\rangle$	0.4	0.05	3.95(1.27)	5.59(1.53)	8.16(2.00)	6.26(1.90)	(0.0358, 0.315)
	0.2	0.05	3.74(1.11)	5.15(1.42)	8.23(1.90)	7.00(1.95)	(0.0222, 0.342)
	0.1	0.05	3.48(1.04)	4.61(1.24)	7.51(1.83)	6.63(2.12)	(0.0237, 0.332)
$\langle N_{\mathrm{p}}\rangle$	0.4	0.05	7.85(1.44)	10.5(1.93)	14.3(3.03)	14.3(3.09)	(0.164, 0.259)
	0.2	0.05	7.13(1.49)	9.49(1.80)	14.6(2.61)	15.5(3.55)	(0.0678, 0.310)
	0.1	0.05	6.44(1.41)	8.60(1.75)	13.2(2.25)	15.1(3.36)	(0.0622, 0.309)
$\langle N_{\mathrm{up}}\rangle$	0.4	0.10	3.32(1.55)	4.76(1.52)	6.71(1.65)	4.96(1.84)	(0.0345, 0.306)
	0.2	0.10	2.90(0.99)	4.34(1.41)	6.85(1.74)	5.35(1.84)	(0.0109, 0.373)
	0.1	0.10	2.82(0.95)	3.80(1.19)	6.08(2.01)	4.99(1.88)	(0.0191, 0.333)
$\langle N_p\rangle$	0.4	0.10	6.89(1.50)	9.21(1.93)	11.8(2.53)	10.8(2.74)	(0.217, 0.232)
	0.2	0.10	5.97(1.42)	8.06(1.90)	12.3(2.51)	11.6(3.39)	(0.0565, 0.311)
	0.1	0.10	5.52(1.37)	7.08(1.76)	11.0(2.34)	11.1(3.22)	(0.0637, 0.297)
$\langle A_{\mathrm{up}}\rangle$ $(\times 10^{-3})$	0.4	0.05	6.87(1.71)	3.96(1.61)	1.89(0.48)	1.35(0.38)	(29.5, −0.558)
	0.2	0.05	24.8(20.5)	10.8(4.59)	5.08(1.23)	3.76(1.18)	(728, −0.690)
	0.1	0.05	45.2(32.8)	20.1(9.39)	9.44(2.25)	7.65(2.62)	(1150, −0.680)
$\langle V_{\mathrm{up}}\rangle$ $(\times 10^{3})$	0.4	0.05	3.59(0.52)	7.58(1.13)	13.3(1.51)	30.2(4.38)	(0.737, 0.571)
	0.2	0.05	3.04(0.42)	6.88(1.19)	12.3(1.27)	26.2(3.44)	(0.368, 0.606)
	0.1	0.05	2.85(0.41)	6.35(0.99)	11.7(1.40)	24.5(3.41)	(0.314, 0.613)

[a] The numbers in the round brackets under each case column are the standard deviation.
[b] p and q are the coefficient and exponent of a power-law function pRa^q for cases 11–13.

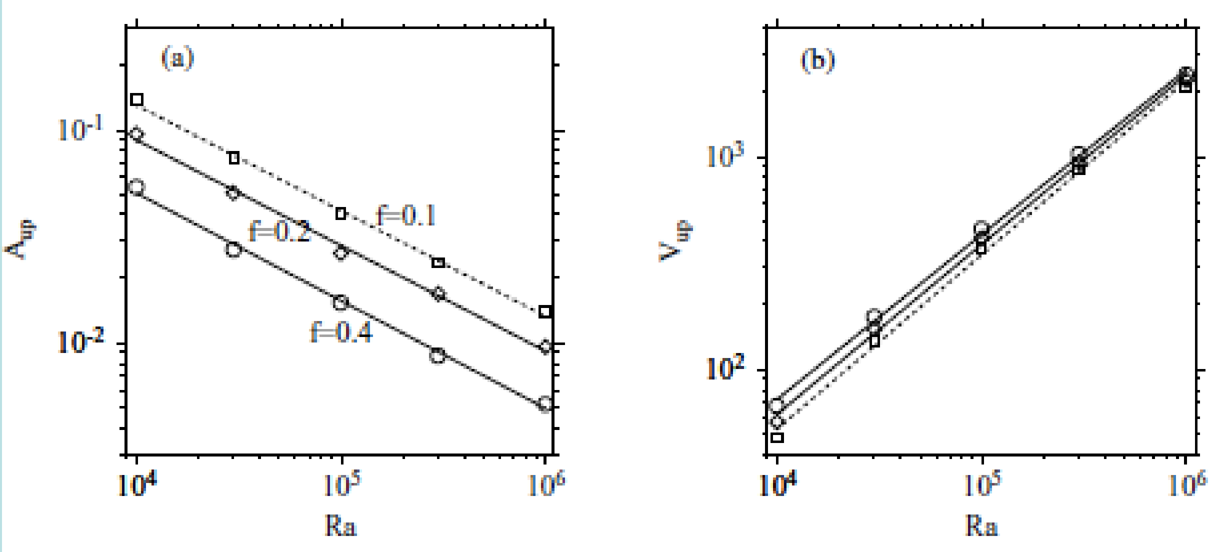
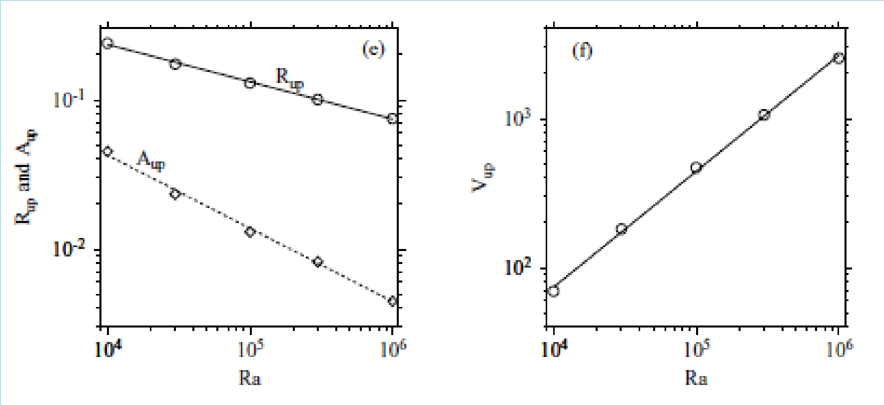
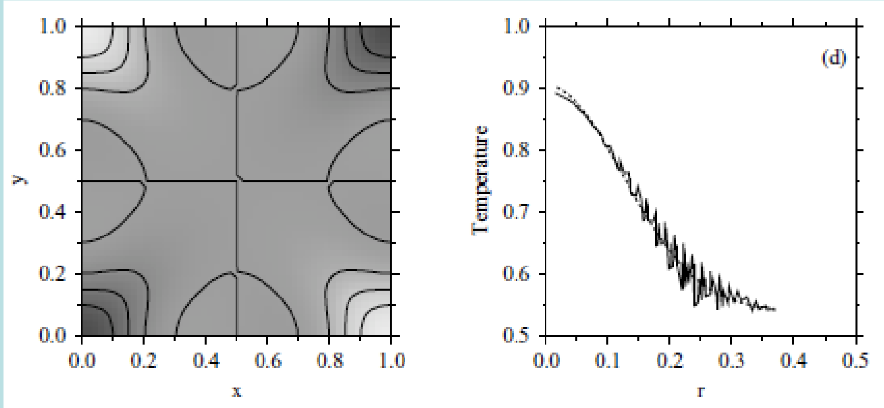


Table 4. The area and vertical velocity of upwelling plumes for cases 1–5.

	f	Case 1	Case 2	Case 3	Case 4	Case 5	$(p, q)^a$
A_{up} ($\times 10^{-2}$)	0.4	5.40	2.73	1.55	0.868	0.513	(5.47, –0.508)
	0.2	9.61	5.07	2.64	1.69	0.952	(8.77, –0.497)
	0.1	13.9	7.39	4.06	2.38	1.40	(12.8, –0.497)
V_{up}	0.4	67.4	176.9	458.2	1030.6	2452	(0.0561, 0.777)
	0.2	56.5	154.5	416.6	942.6	2286	(0.0388, 0.800)
	0.1	47.6	135.8	370.4	870.4	2120	(0.0272, 0.821)

[a] p and q are the coefficient and exponent of a power-law function pRa^q for cases 1–5.
[b] R_{up} is from fitting temperature to an exponential function in eq. (5). A_{up} and V_{up} below the line for this radius are the area and average vertical velocity of plumes defined by this radius. Also notice that the plumes are not perfectly cylindrical.





For β=1/3 and n=0, the exponents for plume radius and velocity are predicted to be -2/9 (~-0.222) and 7/9 (~0.777), respectively.

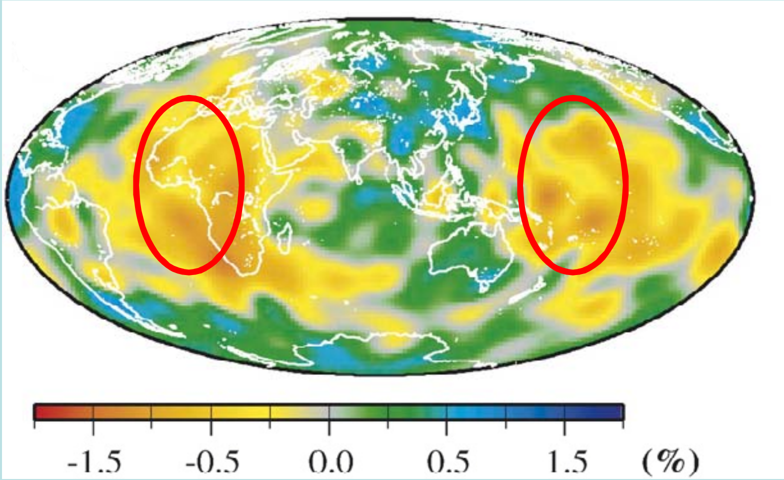
Table 4. The area and vertical velocity of upwelling plumes for cases 1–5.

	f	Case 1	Case 2	Case 3	Case 4	Case 5	$(p, q)^{a}$
A_{up} $(\times 10^{-2})$	0.4	5.40	2.73	1.55	0.868	0.513	(5.47, −0.508)
	0.2	9.61	5.07	2.64	1.69	0.952	(8.77, −0.497)
	0.1	13.9	7.39	4.06	2.38	1.40	(12.8, −0.497)
V_{up}	0.4	67.4	176.9	458.2	1030.6	2452	(0.0561, 0.777)
	0.2	56.5	154.5	416.6	942.6	2286	(0.0388, 0.800)
	0.1	47.6	135.8	370.4	870.4	2120	(0.0272, 0.821)
R_{up}^{b}	—	0.240	0.175	0.130	0.101	0.0749	(2.35, −0.250)
A_{up} $(\times 10^{-2})$	—	4.51	2.34	1.30	0.825	0.449	(3.93, −0.491)
V_{up}	—	70.0	181.8	470.2	1059	2544	(0.0579, 0.777)

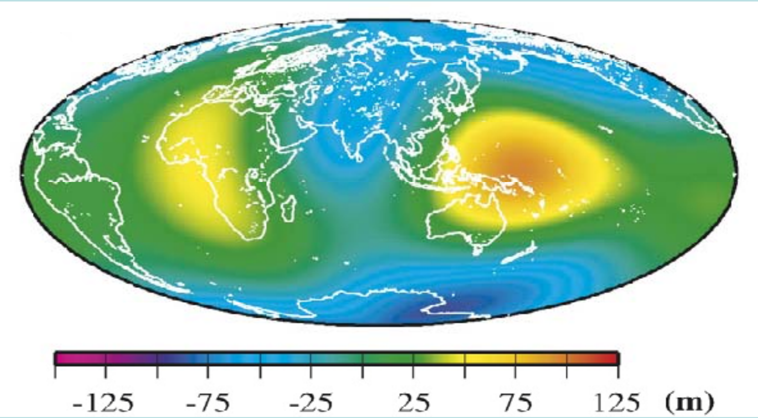
[a] p and q are the coefficient and exponent of a power-law function pRa^{q} for cases 1–5.
[b] R_{up} is from fitting temperature to an exponential function in eq. (5). A_{up} and V_{up} below the line for this radius are the area and average vertical velocity of plumes defined by this radius. Also notice that the plumes are not perfectly cylindrical.

African and Pacific Superplumes
-- Spherical harmonic degree-2 Structure

Shear-wave anomalies at 2300 km depth from S20RTS [Ritsema et al., 1999]

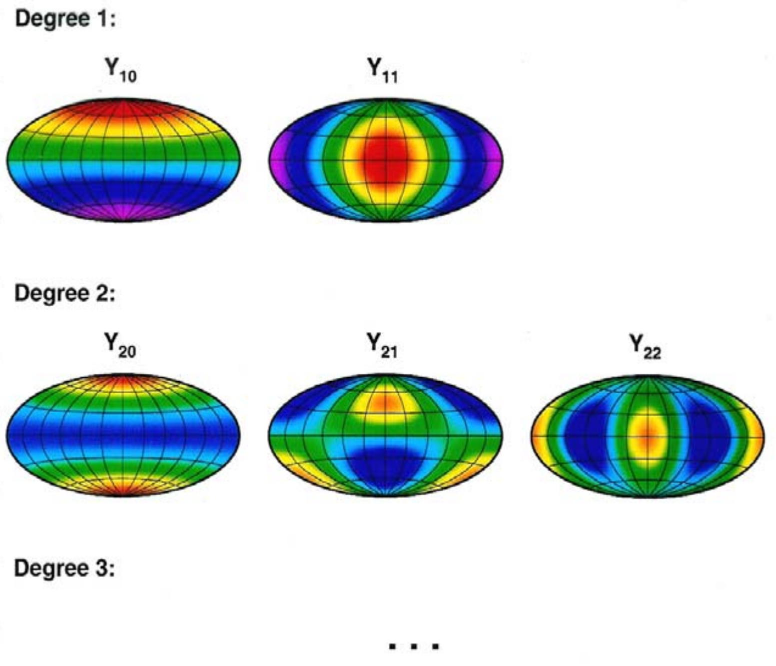


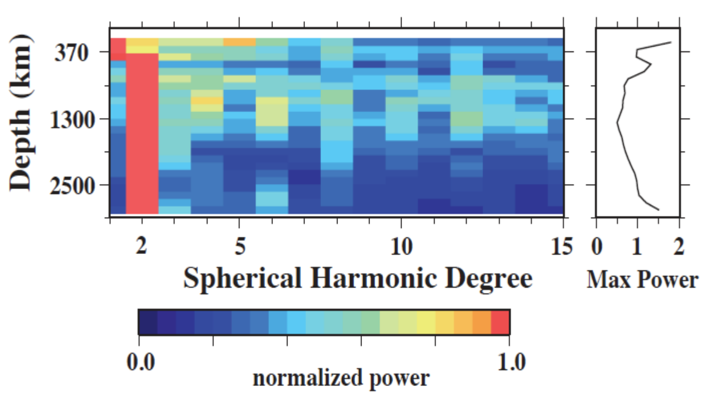
Degree-2 structure:
Dziewonski et al. [1984], van der Hilst et al. [1997], Masters et al. [1996, 2000], Romanowicz and Gung [2002], and Grand [2002].



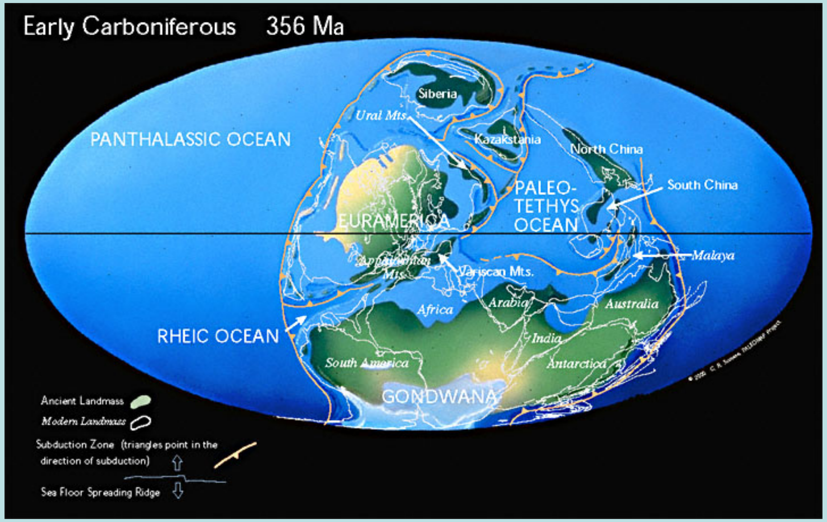
Long-wavelength geoid (degrees 2-3)

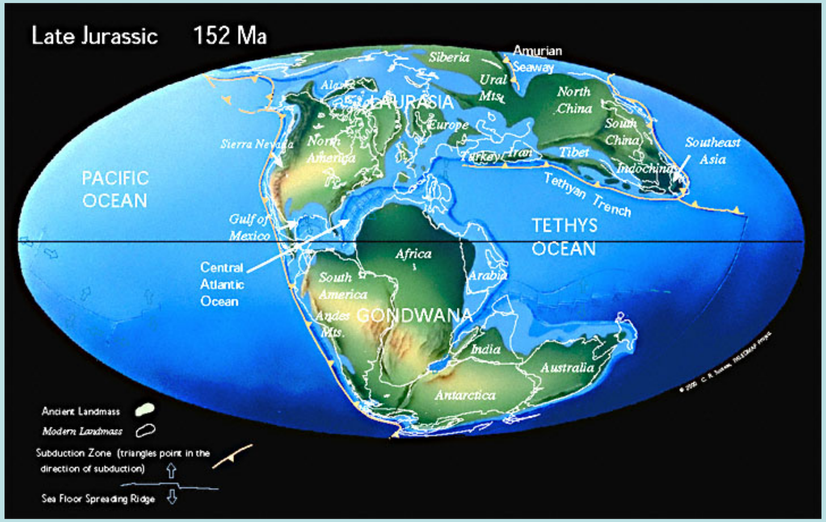
Spherical harmonic functions $Y_{lm}(\theta,\phi)$

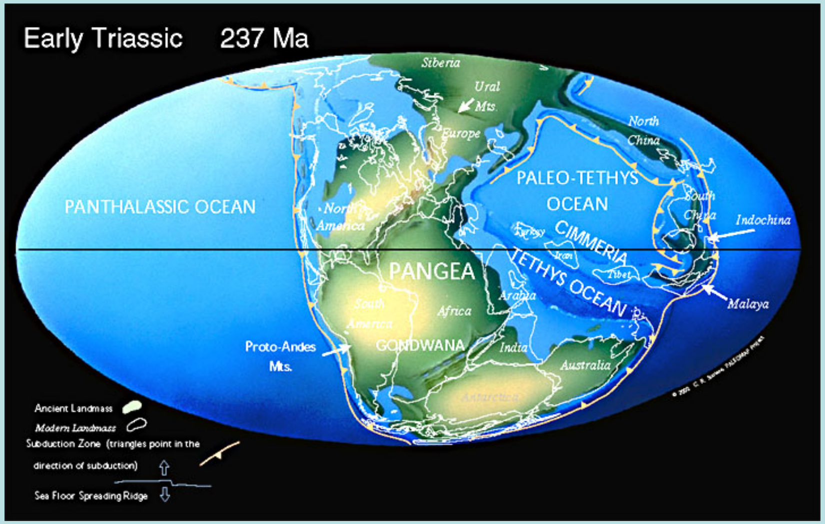


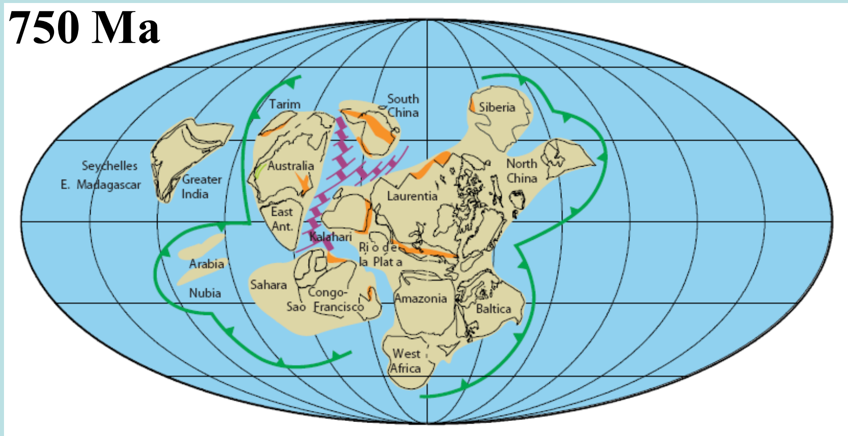


Supercontinent Pangea (330 -- 180 Ma)
and Supercontinent Rodinia (900 -- 750 Ma)









[Smith et al., 1982, and Scotese, 1997]

[Li et al., 2008; Hoffman, 1991; Dalziel, 1991; Torsvik, 2003].

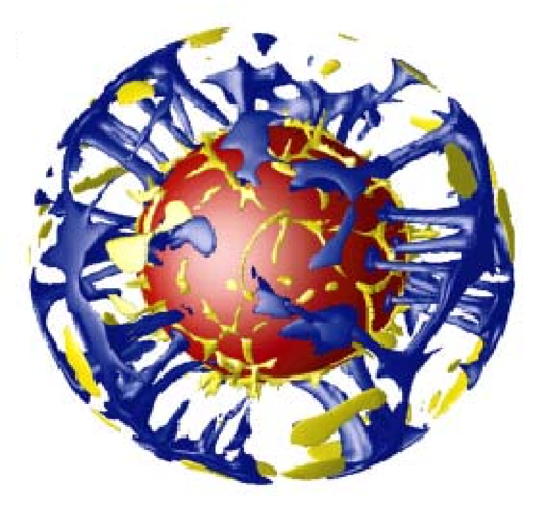
A couple of first-order questions

1. Why should a supercontinent form? Why are supercontinent events cyclic?
2. How do we understand the present-day seismic structure (e.g., two antipodal African and Pacific slow anomalies) and supercontinent events in a general framework?

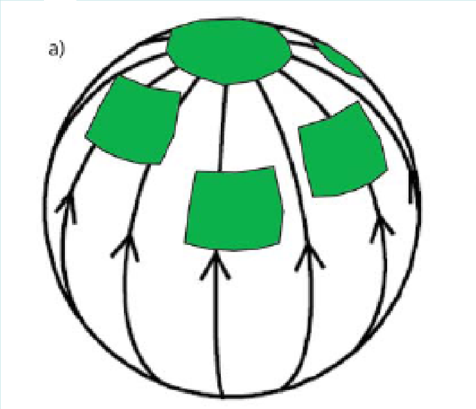
Thermal convection in the mantle is the key to all these questions.

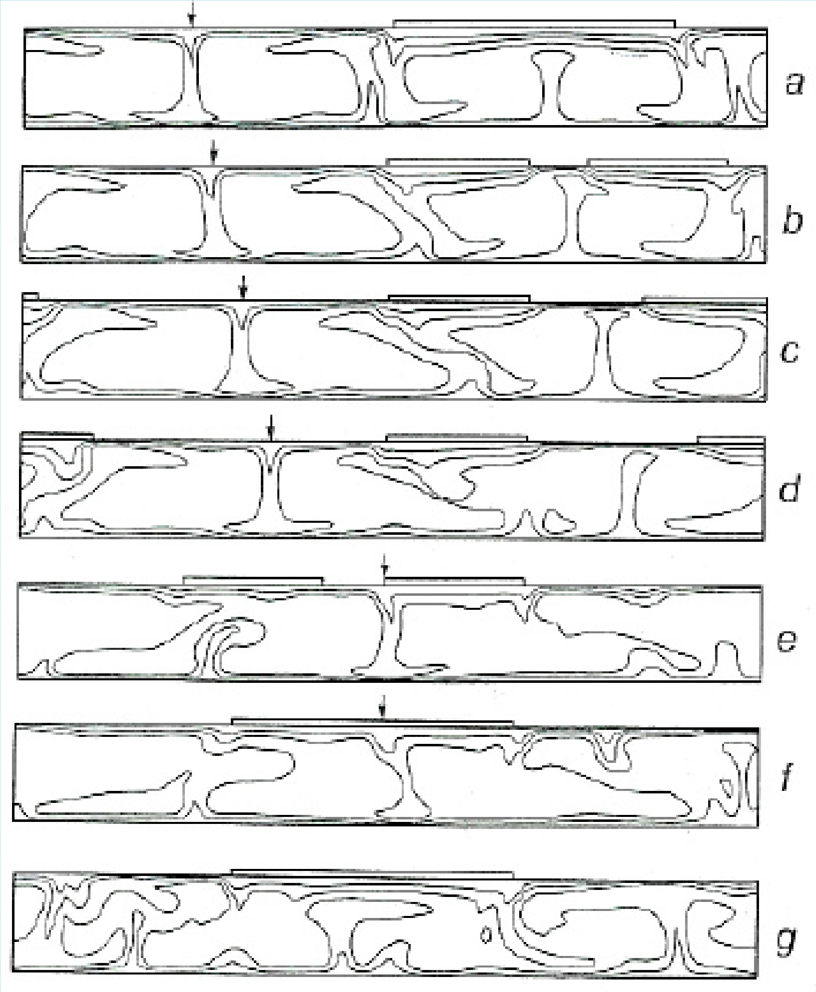
Thermal convection in the mantle

But …



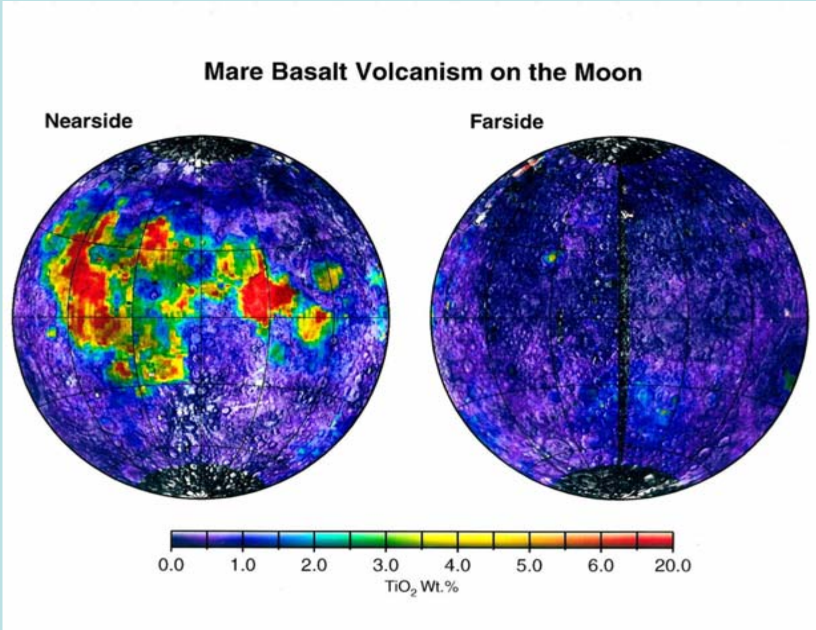
Degree-1 flow?



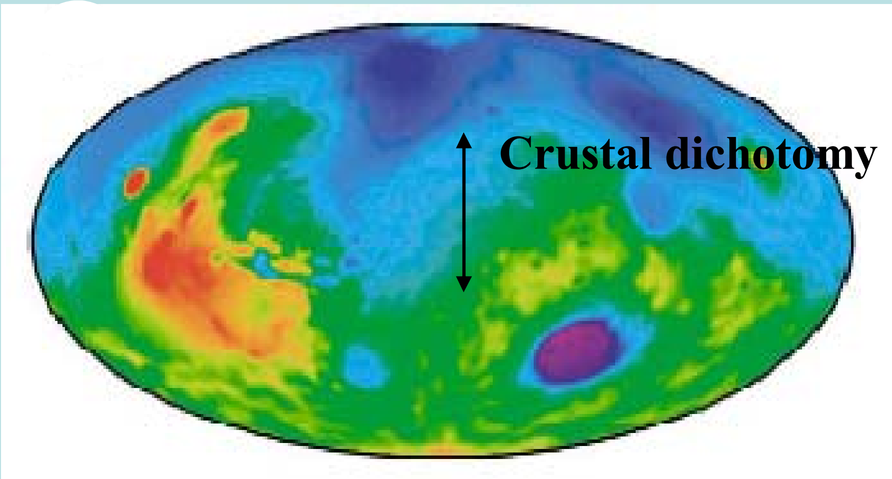


Gurnis [1988]

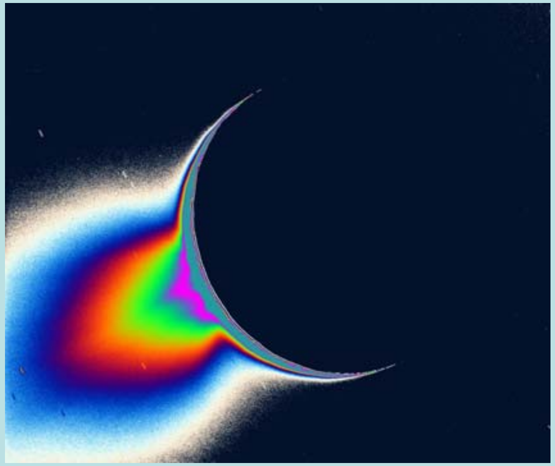
Degree-1 or hemispherically asymmetric structures for the other planetary bodies?



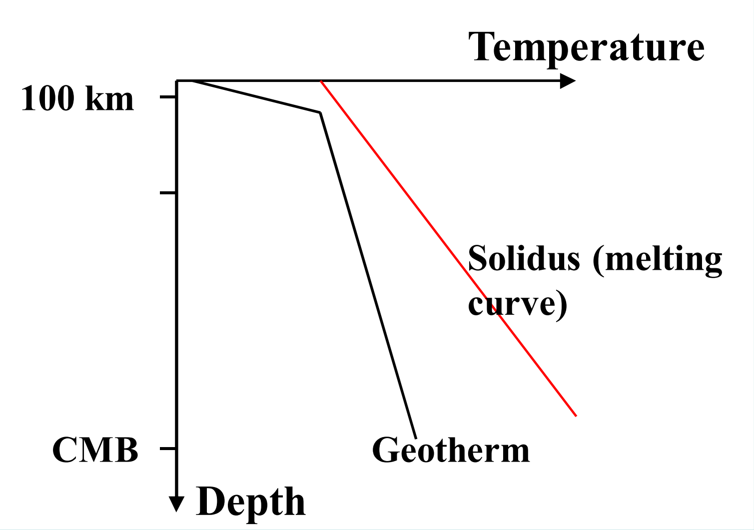
Surface topography on Mars

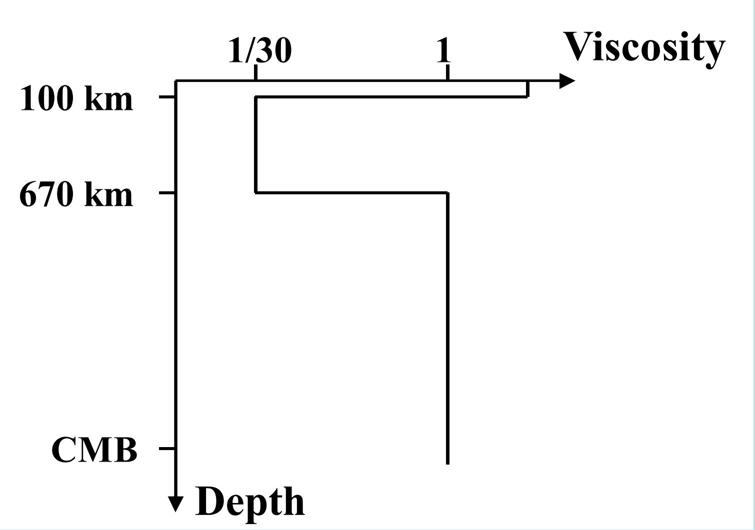


Icy satellite Enceladus



How to generate degree-1 mantle convection?
-- the effect of a weak upper mantle





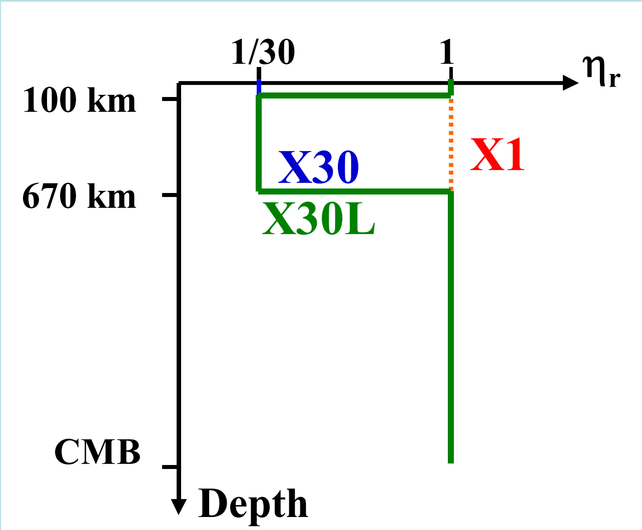
Constrained by postglacial rebound and gravity observations [Hager, 1991; Mitrovica et al., 2007]

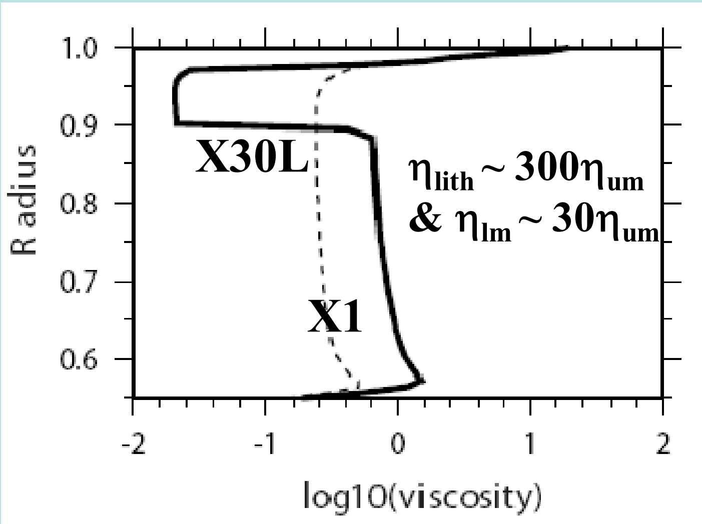
Mantle viscosity depends on temperature (T), stress (σ) and pressure (P).

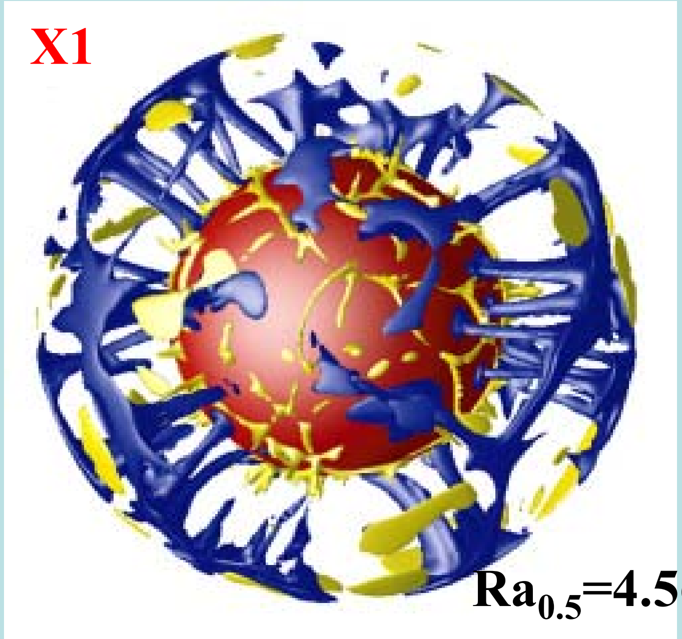
$$\eta = A\sigma^{n-1}\exp[(E+PV)/RT].$$

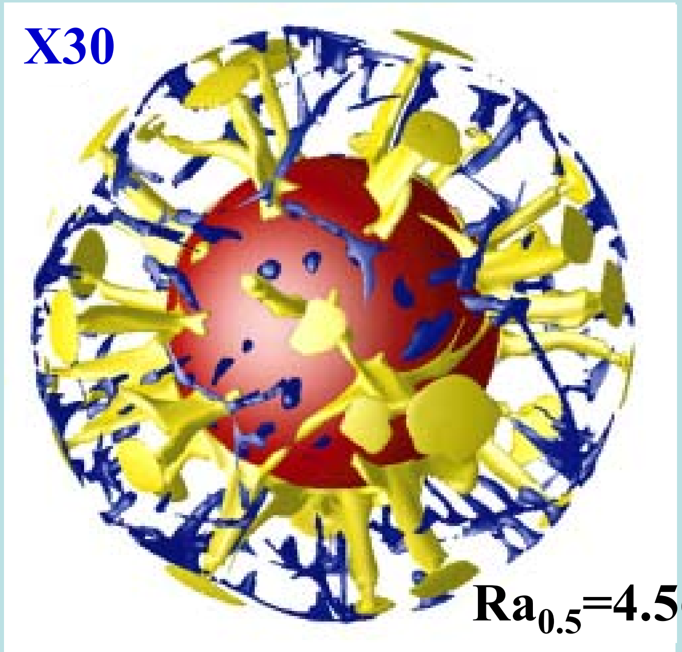
Degree-1 mobile-lid convection with realistic mantle viscosity

$\eta = \eta_r \exp[E(0.5 - T)]$

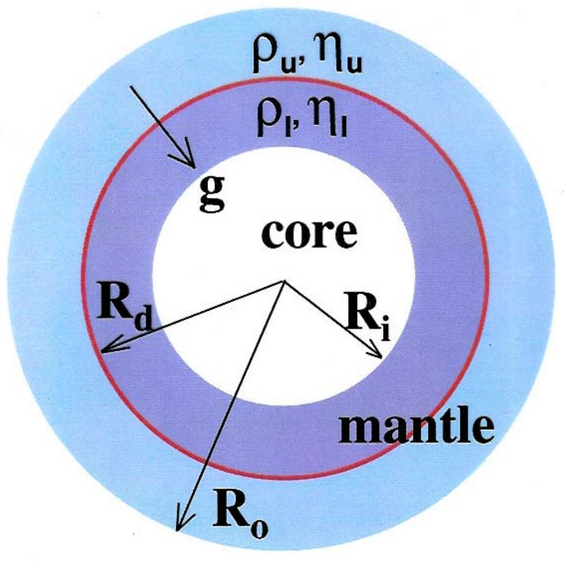






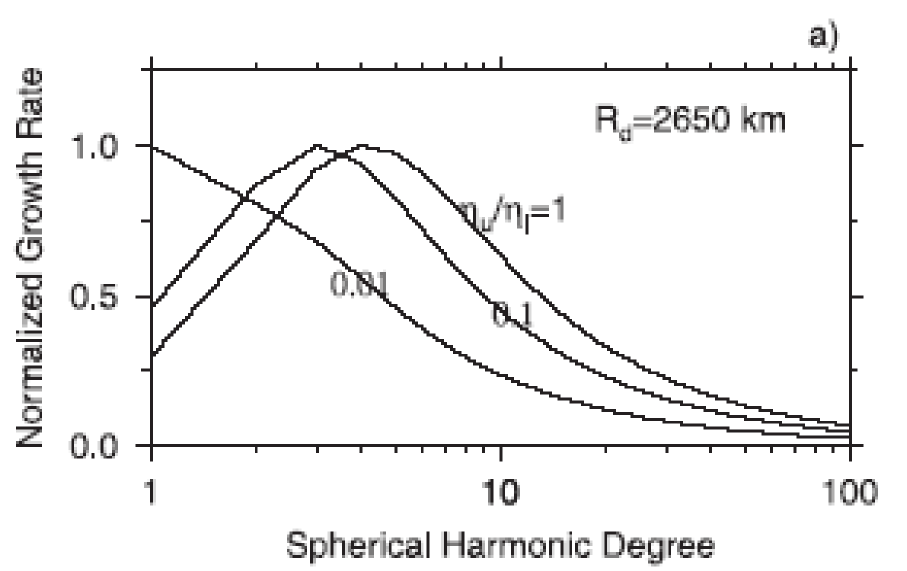


A Rayleigh-Taylor analysis



unstable, if $\rho_l < \rho_u$.

Zhong and Zuber, 2001



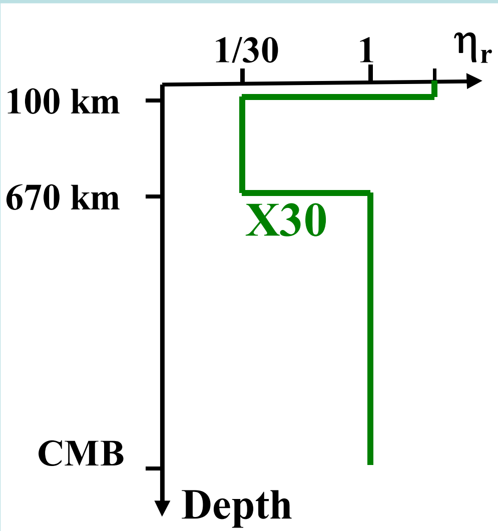
Movie 1: Evolving to degree-1 convective structure

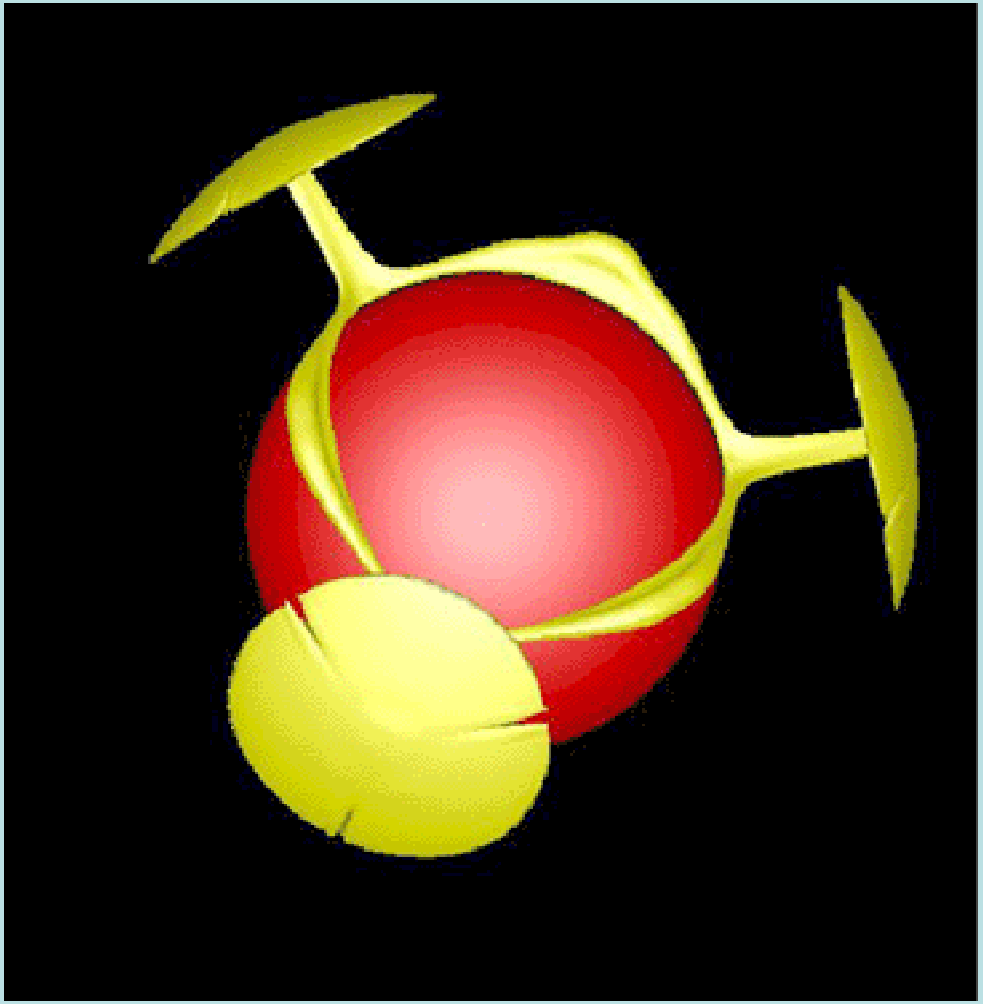
[Zhong et al., 2007]

Viscosity:

η(T, depth).

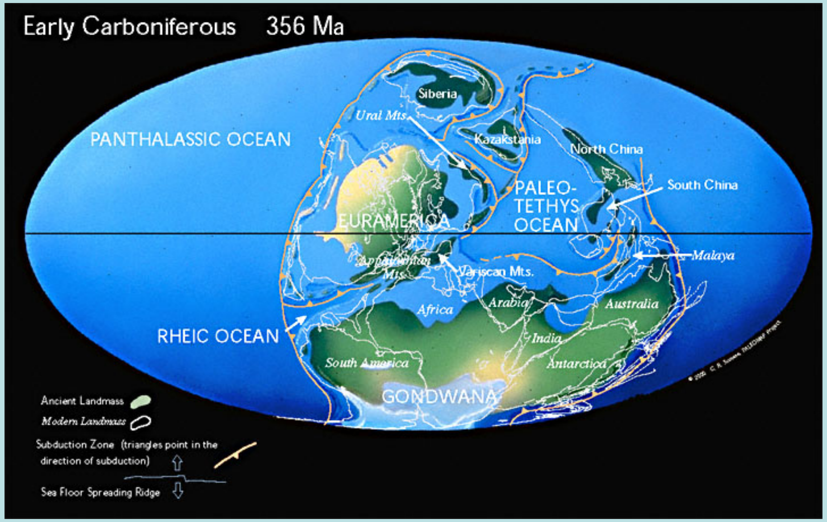
η_{lith}>~200η_{um}
& η_{lm}~30η_{um}

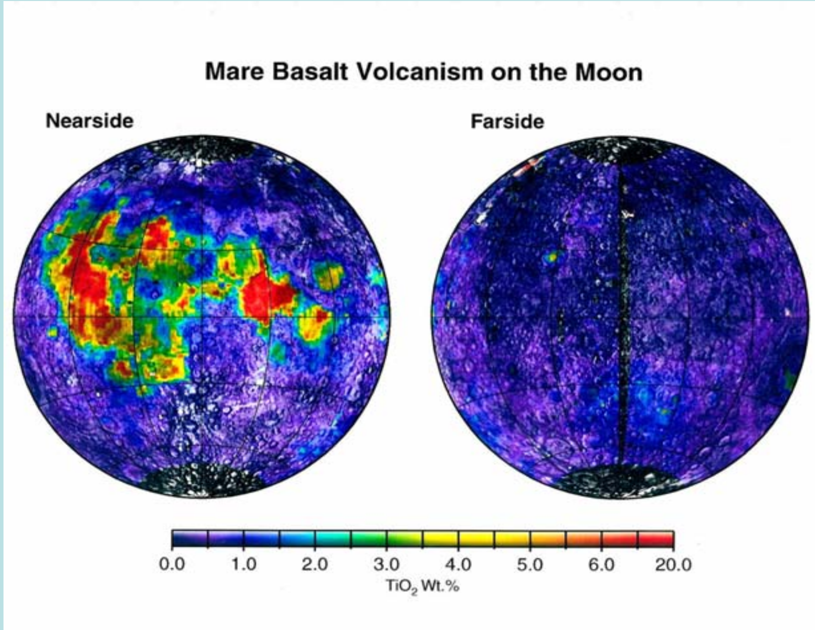




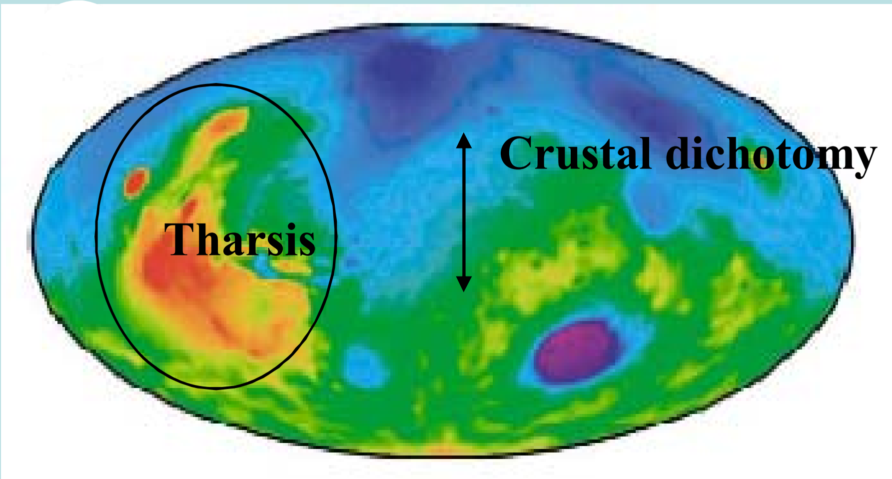
Independent of Ra, heating mode, & initial conditions.
Cause supercontinent formation over the downwelling?

Degree-1 or hemispherically asymmetric structures for planetary bodies?

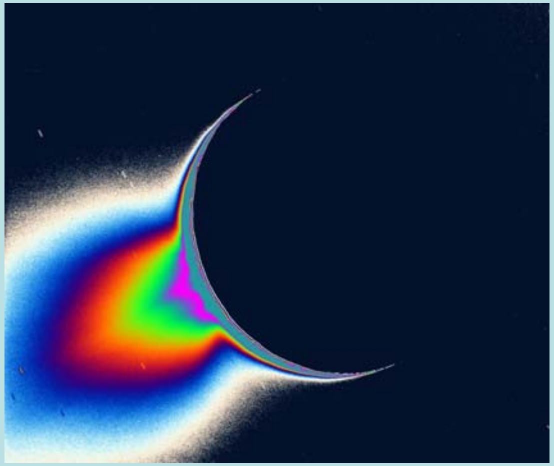




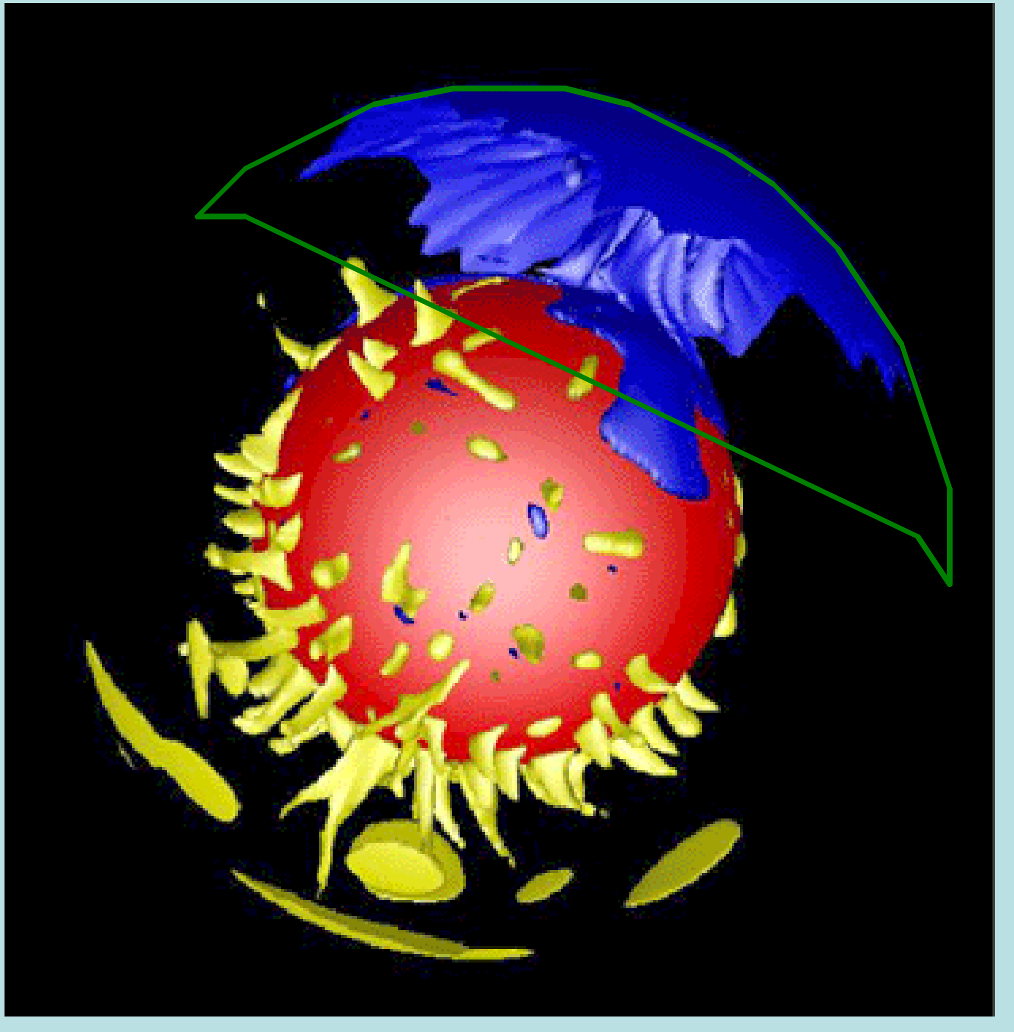
Surface topography on Mars



Icy satellite Enceladus



Movie 2: A supercontinent turns initially degree-1 to degree-2 structures



Conclusion

- For isoviscous convection, convective wavelength (plume population) is mostly constrained by thickness of the layer, and is independent to Ra.
- Convective wavelength and planform are significantly affected by radial viscosity structure. This important problem is not well understood.