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Supersymmetry at LHC - I

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Supersymmetry and the LHC

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Plan for the Lectures

Lecture I: Supersymmetry Introduction

1. Why supersymmetry?
2. Basics of Supersymmetry
3. R Symmetries (a theme in these lectures)
4. SUSY soft breakings
5. MSSM: counting of parameters
6. MSSM: features

Lecture II: Microscopic supersymmetry: supersymmetry breaking

1. Nelson-Seiberg Theorem (R symmetries)
2. O'Raifeartaigh Models
3. The Goldstino
4. Flat directions/pseudomoduli: Coleman-Weinberg vacuum and finding the vacuum.
5. Integrating out pseudomoduli (if time) – non-linear lagrangians.

Lecture III: Dynamical (*Metastable*) Supersymmetry Breaking

1. Non-renormalization theorems
2. SUSY QCD/Gaugino Condensation
3. Generalizing Gaugino condensation
4. A simple – *dumb* – approach to Supersymmetry Breaking: Retrofitting.
5. Other types of metastable breaking: ISS (Intriligator, Shih and Seiberg)
6. Retrofitting – a second look. Why it might be right (cosmological constant!).

Lecture IV: Mediating Supersymmetry Breaking

1. Gravity Mediation
2. Minimal Gauge Mediation (one – really three) parameter description of the MSSM.
3. General Gauge Mediation
4. Assessment.

Supersymmetry

Virtues

1. Hierarchy Problem
2. Unification
3. Dark matter
4. Presence in string theory (often)

Hierarchy: Two Aspects

1. Cancelation of quadratic divergences
2. Non-renormalization theorems (holomorphy of gauge couplings and superpotential): if supersymmetry unbroken classically, unbroken to all orders of perturbation theory, but can be broken beyond: exponentially large hierarchies.

But reasons for skepticism:

1. Little hierarchy
2. Unification: why generic (grand unified models; string theory?)
3. Hierarchy: landscape (light higgs anthropic?)

Reasons for (renewed) optimism:

1. The study of metastable susy breaking (ISS) has opened rich possibilities for model building; no longer the complexity of earlier models for dynamical supersymmetry breaking.
2. Supersymmetry, even in a landscape, can account for hierarchies, as in traditional thinking about naturalness ($e^{-\frac{8\pi^2}{g^2}}$)
3. Supersymmetry, in a landscape, accounts for stability – i.e. the very existence of (metastable) states.

Supersymmetry Review

Basic algebra:

$$\{Q_\alpha, \bar{Q}_{\dot{\beta}}\} = 2\sigma^{\mu}_{\alpha\dot{\beta}} P_\mu.$$

Superspace

It is convenient to introduce an enlargement of space-time, known as *superspace*, to describe supersymmetric systems. One does not have to attach an actual geometric interpretation to this space (though this may be possible) but can view it as a simple way to realize the supersymmetry algebra. The space has four additional, anticommuting (Grassmann) coordinates, $\theta_\alpha, \bar\theta_{\dot\alpha}$. Fields (superfields) will be functions of $\theta, \bar\theta$ and x^μ. Acting on this space of functions, the Q's and $\bar{Q}$'s can be represented as differential operators:

$$Q_\alpha = \frac{\partial}{\partial\theta_\alpha} - i\sigma^\mu_{\alpha\dot\alpha}\bar\theta^{\dot\alpha}\partial_\mu; \qquad \bar{Q}^{\dot\alpha} = \frac{\partial}{\partial\bar\theta_{\dot\alpha}} - i\theta^\alpha\sigma^\mu_{\alpha\dot\beta}\epsilon^{\dot\beta\dot\alpha}\partial_\mu. \tag{1}$$

Infinitesimal supersymmetry transformations are generated by

$$\delta\Phi = \epsilon Q + \bar{\epsilon}\bar{Q}. \tag{2}$$

It is also convenient to introduce a set of *covariant derivative* operators which anticommute with the Q_α's, $\bar{Q}_{\dot{\alpha}}$'s:

$$D_\alpha = \frac{\partial}{\partial\theta_\alpha} + i\sigma^\mu_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}\partial_\mu; \qquad \bar{D}^{\dot{\alpha}} = -\frac{\partial}{\partial\bar{\theta}_{\dot{\alpha}}} - i\theta^\alpha\sigma^\mu_{\alpha\dot{\beta}}\epsilon^{\dot{\beta}\dot{\alpha}}\partial_\mu. \tag{3}$$

Chiral and Vector Superfields

There are two irreducible representations of the algebra which are crucial to understanding field theories with $N = 1$ supersymmetry: chiral fields, Φ, which satisfy $\bar{D}_{\dot{\alpha}}\Phi = 0$, and vector fields, defined by the reality condition $V = V^\dagger$. Both of these conditions are invariant under supersymmetry transformations, the first because $\bar{D}$ anticommutes with all of the Q's. In superspace a chiral superfield may be written as

$$\Phi(x, \theta) = A(x) + \sqrt{2}\theta\psi(x) + \theta^2 F + \dots \quad (4)$$

Here A is a complex scalar, ψ a (Weyl) fermion, and F is an auxiliary field, and the dots denote terms containing derivatives.

More precisely, Φ can be taken to be a function of θ and

$$y^\mu = x^\mu - i\theta\sigma^\mu\bar\theta. \tag{5}$$

Under a supersymmetry transformation with anticommuting parameter ζ, the component fields transform as

$$\delta A = \sqrt{2}\zeta\psi, \tag{6}$$

$$\delta\psi = \sqrt{2}\zeta F + \sqrt{2}i\sigma^\mu\bar\zeta\partial_\mu A, \qquad \delta F = -\sqrt{2}i\partial_\mu\psi\sigma^\mu\bar\zeta. \tag{7}$$

Vector fields can be written, in superspace, as

$$V = i\chi - i\chi^\dagger + \theta\sigma^\mu\bar{\lambda}A_\mu + i\theta^2\bar{\theta}\bar{\lambda} - i\bar{\theta}^2\theta\lambda + \frac{1}{2}\theta^2\bar{\theta}^2 D. \quad (8)$$

Here χ is a chiral field.

In order to write consistent theories of spin one fields, it is necessary to enlarge the usual notion of gauge symmetry to a transformation of V and the chiral fields Φ by superfields. In the case of a $U(1)$ symmetry, one has

$$\Phi_i \to e^{q_i \Lambda} \Phi_i \quad V \to V - \Lambda - \Lambda^\dagger. \tag{9}$$

Here Λ is a chiral field (so the transformed Φ_i is also chiral). Note that this transformation is such as to keep

$$\Phi^{i\dagger} e^{q_i V} \Phi^i \tag{10}$$

invariant. In the non-abelian case, the gauge transformation for Φ_i is as before, where Λ is now a matrix valued field.

For the gauge fields, the physical content is most transparent in a particular gauge (really a class of gauges) know as Wess-Zumino gauge. This gauge is analogous to the Coulomb gauge in QED. In that case, the gauge choice breaks manifest Lorentz invariance (Lorentz transformations musts be accompanied by gauge transformations), but Lorentz invariance is still a property of physical amplitudes. Similarly, the choice of Wess-Zumino gauge breaks supersymmetry, but physical quantities obey the rules implied by the symmetry. In this gauge, the vector superfield may be written as

$$V = -\theta\sigma^\mu \bar{\lambda} A_\mu + i\theta^2 \bar{\theta}\bar{\lambda} - i\bar{\theta}^2 \theta\lambda + \frac{1}{2}\theta^2\bar{\theta}^2 D. \tag{11}$$

The analog of the gauge invariant field strength is a chiral field:

$$W_\alpha = -\frac{1}{4}\bar{D}^2 D_\alpha V \tag{12}$$

or, in terms of component fields:

$$W_\alpha = -i\lambda_\alpha + \theta_\alpha D - \frac{i}{2}(\sigma^\mu \bar{\sigma}^\nu \theta)_\alpha F_{\mu\nu} + \theta^2 \sigma^\mu_{\alpha\dot{\beta}} \partial_\mu \bar{\lambda}^{\dot{\beta}}. \tag{13}$$

In the non-Abelian case, the fields V are matrix valued, and transform under gauge transformations as

$$V \rightarrow e^{-\Lambda^\dagger} V e^{\Lambda} \tag{14}$$

Correspondingly, for a chiral field transforming as

$$\Phi \rightarrow e^{\Lambda}\Phi \tag{15}$$

the quantity

$$\Phi^\dagger e^V \Phi \tag{16}$$

is gauge invariant.

The generalization of W_α of the Abelian case is the matrix-valued field:

$$W_\alpha = -\frac{1}{4}\bar{D}^2 e^{-V} D_\alpha e^V, \tag{17}$$

which transforms, under gauge transformations, as

$$W_\alpha \to e^{-\Lambda} W_\alpha e^{\Lambda}. \tag{18}$$

Supersymmetric Actions

To construct an action with $N = 1$ supersymmetry, it is convenient to consider integrals in superspace. The integration rules are simple:

$$\int d^2\theta\theta^2 = \int d^2\bar{\theta}\bar{\theta}^2 = 1; \quad \int d^4\theta\bar{\theta}^2\theta^2 = 1, \tag{19}$$

all others vanishing. Integrals $\int d^4x d^4\theta F(\theta, \bar{\theta})$ are invariant, for general functions θ, since the action of the supersymmetry generators is either a derivative in θ or a derivative in x. Integrals over half of superspace of *chiral* fields are invariant as well, since, for example,

$$\bar{Q}_{\dot{\alpha}} = \bar{D}_{\dot{\alpha}} + 2i\theta^{\alpha}\sigma^{\mu}_{\alpha\dot{\alpha}}\partial_{\mu} \tag{20}$$

so, acting on a chiral field (or any function of chiral fields, which is necessarily chiral), one obtains a derivative in superspace.

In order to build a supersymmetric lagrangian, one starts with a set of chiral superfields, Φ^i, transforming in various representations of some gauge group $\mathcal{G}$. For each gauge generator, there is a vector superfield, V^a. The most general renormalizable lagrangian, written in superspace, is

$$\mathcal{L} = \sum_i \int d^4\theta \Phi_i^\dagger e^V \Phi_i + \sum_a \frac{1}{4g_a^2} \int d^2\theta W_\alpha^2 \tag{21}$$

$$+c.c. + \int d^2\theta W(\Phi_i) + c.c.$$

Here $W(\Phi)$ is a holomorphic function of chiral superfields known as the superpotential.

Component lagrangians

In terms of the component fields, the lagrangian includes kinetic terms for the various fields (again in Wess-Zumino gauge):

$$\mathcal{L}_{kin} = \sum_i \left(|D\phi_i|^2 + i\psi_i \sigma^\mu D_\mu \psi_i^* \right) - \sum_a \frac{1}{4g_a^2} \left(F^a_{\mu\nu} F^{a\mu\nu} - i\lambda^a \sigma^\mu D_\mu \lambda^{a*} \right) .$$

There are also Yukawa couplings of “matter" fermions (fermions in chiral multiplets) and scalars, as well as Yukawa couplings of matter and gauge fields:

$$\mathcal{L}_{yuk} = i\sqrt{2} \sum_{ia} (g^a \psi^i T^a_{ij} \lambda^a \phi^{*j} + c.c.) + \sum_{ij} \frac{1}{2} \frac{\partial^2 W}{\partial \phi^i \partial \phi^j} \psi^i \psi^j. \quad (23)$$

We should note here that we will often use the same label for a chiral superfield and its scalar component; this is common practice, but we will try to modify the notation when it may be confusing. The scalar potential is:

$$V = |F_i|^2 + \frac{1}{2}(D^a)^2. \tag{24}$$

The auxiliary fields F_i and D_a are obtained by solving their equations of motion:

$$F_i^\dagger = -\frac{\partial W}{\partial \phi_i} \qquad D^a = g^a \sum_i \phi_i^* T^a_{ij} \phi_j. \tag{25}$$

A Simple Free Theory

To illustrate this discussion, consider first a theory of a single chiral field, with superpotential

$$W = \frac{1}{2} m \phi^2. \tag{26}$$

Then the component Lagrangian is just

$$\mathcal{L} = |\partial \phi|^2 + i \psi \sigma^\mu \partial_\mu \psi + \frac{1}{2} m \psi \psi + \mathrm{c.c.} + m^2 |\phi|^2. \tag{27}$$

So this is a theory of a free massive complex boson and a free massive Weyl fermion, each with mass m^2. (I have treated here m^2 as real; in general, one can replace m^2 by $|m|^2$).

An Interacting Theory – Supersymmetry Cancelations

Now take

$$W = \frac{1}{3}\lambda\phi^3. \tag{28}$$

The interaction terms in $\mathcal{L}$ are:

$$\mathcal{L}_I = \lambda\phi\psi\psi + \lambda^2|\phi|^4. \tag{29}$$

The model has an R symmetry under which

$$\phi \to e^{2i\alpha/3}\phi; \quad \psi \to e^{-2i\alpha/3}\psi; \quad W \to e^{2i\alpha W}. \tag{30}$$

Aside: R Symmetries

Such symmetries will be important in our subsequent discussions. They correspond to the transformation of chiral fields:

$$\Phi_i \rightarrow e^{ir_i\alpha}\Phi_i; \quad \theta \rightarrow e^{i\alpha\theta} \tag{31}$$

Then

$$Q \rightarrow e^{-i\alpha}Q; W \rightarrow e^{2i\alpha W} \tag{32}$$

and

$$\phi_i \rightarrow e^{ir_i\alpha}\phi_i; \quad \psi_i \rightarrow e^{(r_i-1)\alpha}\psi_i \quad F_i \rightarrow e^{i(r_i-2)\alpha}F_i. \tag{33}$$

The gauginos also transform:

$$\lambda \rightarrow e^{i\alpha}\lambda. \tag{34}$$

Supersymmetry Cancelation and Soft Breaking (continued)

The symmetry means that there can be no correction to the fermion mass, or to the superpotential. Let's check, at one loop, that there is no correction to the scalar mass. Two contributions:

1. Boson loop:

$$\delta m_\phi^2 = 4\lambda^2 \frac{d^4k}{(2\pi)^4} \frac{1}{k^2} \tag{35}$$

2. Fermion loop:

$$\delta m_\phi^2 = -2\lambda^2 \frac{d^4k}{(2\pi)^4} \frac{\mathrm{Tr}(\sigma^\mu k_\mu \bar{\sigma}^\nu k_\nu)}{k^4}. \tag{36}$$

In the first expression, the 4 is a combinatoric factor; in the second, the minus sign arises from the fermion loop; the 2 is a combinatoric factor.

These two terms, each separately quadratically divergent, cancel.

Now add to the lagrangian a “soft", non-supersymmetric term,

$$\delta\mathcal{L} = -m^2|\phi|^2. \tag{37}$$

This changes the scalar propagator above, so

$$\delta m_\phi^2 = 4\lambda^2 \int \frac{d^4k}{(2\pi)^4}\left(\frac{1}{k^2+m^2} - \frac{1}{k^2}\right) \tag{38}$$

$$= \int \frac{d^4k}{(2\pi)^4}\frac{-m^2}{k^2(k^2+m^2)}$$

$$\approx \frac{\lambda^2 m^2}{16\pi^2}\log(\Lambda^2/m^2).$$

Here Λ is an ultraviolet cutoff. Note that these corrections vanish as $m^2 \to 0$.

More generally, possible soft terms are:

1. Scalar masses
2. Gaugino masses
3. Cubic scalar couplings.

All have dimension less than four.

MSSM:A supersymmetric generalization of the SM.

1. Gauge group $SU(3) \times SU(2) \times U(1)$; corresponding (12) vector multiplets.
2. Chiral field for each fermion of the SM: $Q_f, \bar{u}_f, \bar{d}_f, L_f, \bar{e}_f$.
3. Two Higgs doublets, H_U, H_D.
4. Superpotential contains a generalization of the Standard Model Yukawa couplings:

$$W_y = y_U H_U Q \bar{U} + y_D H_D Q \bar{D} + y_L H_D \bar{E}. \tag{39}$$

y_U and y_D are 3×3 matrices in the space of flavors.

Soft Breaking Parameters

Need also breaking of supersymmetry, potential for quarks and leptons. Introduce *explicit soft breakings*:

1. Soft mass terms for squarks, sleptons, and Higgs fields:

$$\mathcal{L}_{scalars} = Q^* m_Q^2 Q + \bar{U}^* m_U^2 \bar{U} + \bar{D}^* m_D^2 \bar{D} \qquad (40)$$
$$+ L^* m_L^2 L + \bar{E}^* m_E \bar{E}$$
$$+ m_{H_U}^2 |H_U|^2 + m_{H_U}^2 |H_U|^2 + B_\mu H_U H_D + \text{c.c.}$$

   m_Q^2, m_U^2, etc., are matrices in the space of flavors.

2. Cubic couplings of the scalars:

$$\mathcal{L}_{\mathcal{A}} = H_U Q\, A_U\, \bar{U} + H_D Q\, A_D\, \bar{D} \qquad (41)$$
$$+ H_D L\, A_E \bar{E} + \text{c.c.}$$

   The matrices A_U, A_D, A_E are complex matrices

3. Mass terms for the U(1) (b), $SU(2)$ (w), and $SU(3)$ (λ) gauginos:

$$m_1 bb + m_2 ww + m_3 \lambda\lambda \qquad (42)$$

Counting the Soft Breaking Parameters

1. $\phi\phi^*$ mass matrices are 3×3 Hermitian (45 parameters)
2. Cubic terms are described by 3 complex matrices (54 parameters
3. The soft Higgs mass terms add an additional 4 parameters.
4. The μ term adds two.
5. The gaugino masses add 6.

There appear to be 111 new parameters.

But Higgs sector of SM has two parameters.
In addition, the supersymmetric part of the MSSM lagrangian has symmetries which are broken by the general soft breaking terms (including μ among the soft breakings):

1. Two of three separate lepton numbers
2. A “Peccei-Quinn" symmetry, under which H_U and H_D rotate by the same phase, and the quarks and leptons transform suitably.
3. A continuous "R" symmetry, which we will explain in more detail below.

Redefining fields using these four transformations reduces the number of parameters to 105.
If supersymmetry is discovered, determining these parameters, and hopefully understanding them more microscopically, will be the main business of particle physics for some time. The phenomenology of these parameters has been the subject of extensive study; we will focus on a limited set of issues.

Constraints

Direct searches (LEP, Fermilab) severely constrain the spectrum. E.g. squark, gluino masses $>$ 100's of GeV, charginos of order 100 GeV. Spectrum must have special features to explain

1. Absence of Flavor Changing Neutral Currents (suppression of $K \leftrightarrow \bar{K}$, $D \leftrightarrow \bar{D}$ mixing; $B \rightarrow s + \gamma$, $\mu \rightarrow e + \gamma, \ldots$)
2. Suppression of CP violation (d_n; phases in $K\bar{K}$ mixing).

Might be accounted for if spectrum highly degenerate, CP violation in soft breaking suppressed.

The little hierarchy: perhaps the greatest challenge for Supersymmetry

Higgs mass and little hierarchy:
Biggest contribution to the Higgs mass from top quark loops. Two graphs; cancel if supersymmetry is unbroken. Result of simple computation is

$$\delta m_{H_U}^2 = -6 \frac{y_t^2}{16\pi^2} \tilde{m}_t^2 \ln(\Lambda^2/\tilde{m}_t^2) \quad (43)$$

Even for modest values of the coupling, given the limits on squark masses, this can be substantial.

But another problem: $m_H > 114$ GeV. At tree level $m_H \leq m_Z$. Loop corrections involving top quark: can substantially correct Higgs quartic, and increase mass. But current limits typically require $\tilde{m}_t > 800$ GeV. Exacerbates tuning. Typically worse than 1 %.

$$\delta\lambda \sim 3\frac{y_t^4}{16\pi^2}\log(\tilde{m}_t^2/m_t^2). \tag{44}$$

Possible solution: additional physics, Higgs coupling corrected by dimension five term in superpotential or dimension six in Kahler potential.

$$\delta W = \frac{1}{M}H_U H_D H_U H_D \quad \delta K = Z^\dagger Z H_U^\dagger H_U H_U^\dagger H_U. \tag{45}$$

Aside on Two Component Spinors

We have been using two component spinors up to now, but these may be unfamiliar to some of you. So the following few pages demonstrate how four component spinors are equivalent to two component spinors, and how *everything* can be described in terms such two component spinors.

Writing a Relativistic Equation for Massless Fermions

If we were living in 1930, and wanted to write a relativistic wave equation for massless fermions, we might proceed as follows. Write:

$$\sigma^\mu \partial_\mu \chi = 0. \tag{46}$$

We want χ to satisfy the Klein-Gordan equation. This will be the case if we can find a set of matrices, $\bar{\sigma}^\mu$, which satisfy

$$\bar{\sigma}^\mu \sigma^\nu + \bar{\sigma}^\nu \sigma^\mu = 2g^{\mu\nu}. \tag{47}$$

Unlike the massive case, we can satisfy this requirement with 2×2 matrices:

$$\sigma^\mu = (1, \vec{\sigma}); \quad \bar{\sigma}^\mu = (1, -\vec{\sigma}). \tag{48}$$

In momentum space, this equation is remarkably simple:

$$(E - \vec{p} \cdot \vec{\sigma})\chi = 0. \tag{49}$$

For positive energies, this says that the spin is aligned along the momentum. For negative energy spinors, the spin is aligned opposite to the momentum.

Exercise: Write the mode expansion for $\chi(x)$, and identify suitable creation and destruction operators.

Connecting to Four Component Spinors

Adopt the following basis for the γ matrices:

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix} \tag{50}$$

In this basis,

$$\gamma_5 = i\gamma^o\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \tag{51}$$

so the projectors

$$P_\pm = \frac{1}{2}(1 \pm \gamma_5) \tag{52}$$

are given by:

$$P_+ = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \quad P_- = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}. \tag{53}$$

We will adopt some notation, following the text by Wess and Bagger:

$$\psi = \begin{pmatrix} \chi_\alpha \\ \phi^{*\dot{\alpha}} \end{pmatrix}. \tag{54}$$

Correspondingly, we label the indices on the matrices σ^μ and $\bar{\sigma}^\mu$ as

$$\sigma^\mu = \sigma^\mu_{\alpha\dot{\alpha}} \qquad \bar{\sigma}^\mu = \bar{\sigma}^{\mu\beta\dot{\beta}}. \tag{55}$$

This allows us to match upstairs and downstairs indices, and will prove quite useful. We define complex conjugation to change dotted to undotted indices. So, for example,

$$\phi^{*\dot{\alpha}} = (\phi^\alpha)^*. \tag{56}$$

Then we define the anti-symmetric matrices $\epsilon_{\alpha\beta}$ and $\epsilon^{\alpha\beta}$ by:

$$\epsilon^{12} = 1 = -\epsilon^{21} \qquad \epsilon_{\alpha\beta} = -\epsilon^{\alpha\beta}. \tag{57}$$

The matrices with dotted indices are defined identically. Note that, with upstairs indices, $\epsilon = i\sigma_2$, $\epsilon_{\alpha\beta}\epsilon^{\beta\gamma} = \delta_\alpha^\gamma$. We can use these matrices to raise and lower indices on spinors. Define $\phi_\alpha = \epsilon_{\alpha\beta}\phi^\beta$, and similarly for dotted indices. So

$$\phi_\alpha = \epsilon_{\alpha\beta}(\phi^{*\dot\beta})^*. \tag{58}$$

Finally, we will define complex conjugation of a product of spinors to invert the order of factors, so, for example, $(\chi_\alpha \phi_\beta)^* = \phi^*_{\dot\beta} \chi^*_{\dot\alpha}$.
With this in hand, the reader should check that the action for our original four component spinor is:

$$S = \int d^4x \mathcal{L} = \int d^4x \left(i\chi_{\dot\alpha} \bar\sigma^{\mu\dot\alpha\alpha} \partial_\mu \chi_\alpha + i\phi^\alpha \sigma^\mu_{\alpha\dot\alpha} \partial_\mu \phi^{*\dot\alpha} \right) \quad (59)$$

$$= \int d^4x \mathcal{L} = \int d^4x \left(i\chi^\alpha \sigma^\mu_{\alpha\dot\alpha} \partial_\mu \chi^{*\dot\alpha} + i\phi^\alpha \sigma^\mu_{\alpha\dot\alpha} \partial_\mu \phi^{*\dot\alpha} \right).$$

At the level of Lorentz-invariant lagrangians or equations of motion, there is *only one* irreducible representation of the Lorentz algebra for massless fermions.
It is instructive to describe quantum electrodynamics with a massive electron in two-component language. Write

$$\psi = \begin{pmatrix} e \\ \bar{e}^* \end{pmatrix}. \qquad (60)$$

In the lagrangian, we need to replace ∂_μ with the covariant derivative, D_μ. e contains annihilation operators for the left-handed electron, and creation operators for the corresponding anti-particle. $\bar{e}$ contains annihilation operators for a particle with the opposite helicity and charge of e, and $\bar{e}^*$, and creation operators for the corresponding antiparticle.

The mass term, $m\bar{\psi}\psi$, becomes:

$$m\bar{\psi}\psi = me^{\alpha}\bar{e}_{\alpha} + me^{*}_{\dot{\alpha}}\bar{e}^{*\dot{\alpha}}. \tag{61}$$

Again, note that both terms preserve electric charge. Note also that the equations of motion now couple e and $\bar{e}$.

It is helpful to introduce one last piece of notation. Call

$$\psi\chi = \psi^\alpha\chi_\alpha = -\psi_\alpha\chi^\alpha = \chi^\alpha\psi_\alpha = \chi\psi. \tag{62}$$

Similarly,

$$\psi^*\chi^* = \psi^*_{\dot\alpha}\chi^{*\dot\alpha} = -\psi^{*\dot\alpha}\chi^*_{\dot\alpha}\chi^*_{\dot\alpha}\psi^{*\dot\alpha} = \chi^*\psi^*. \tag{63}$$

Finally, note that with these definitions,

$$(\chi\psi)^* = \chi^*\psi^*. \tag{64}$$

Exercise: Starting with the action for the four component electron, *with a mass term*, work verify the lagrangian in two component notation for the massive electron. Make sure to work out the covariant derivatives.