Introduction to Isentropic Coordinates:

a new view of mean meridional & eddy circulations

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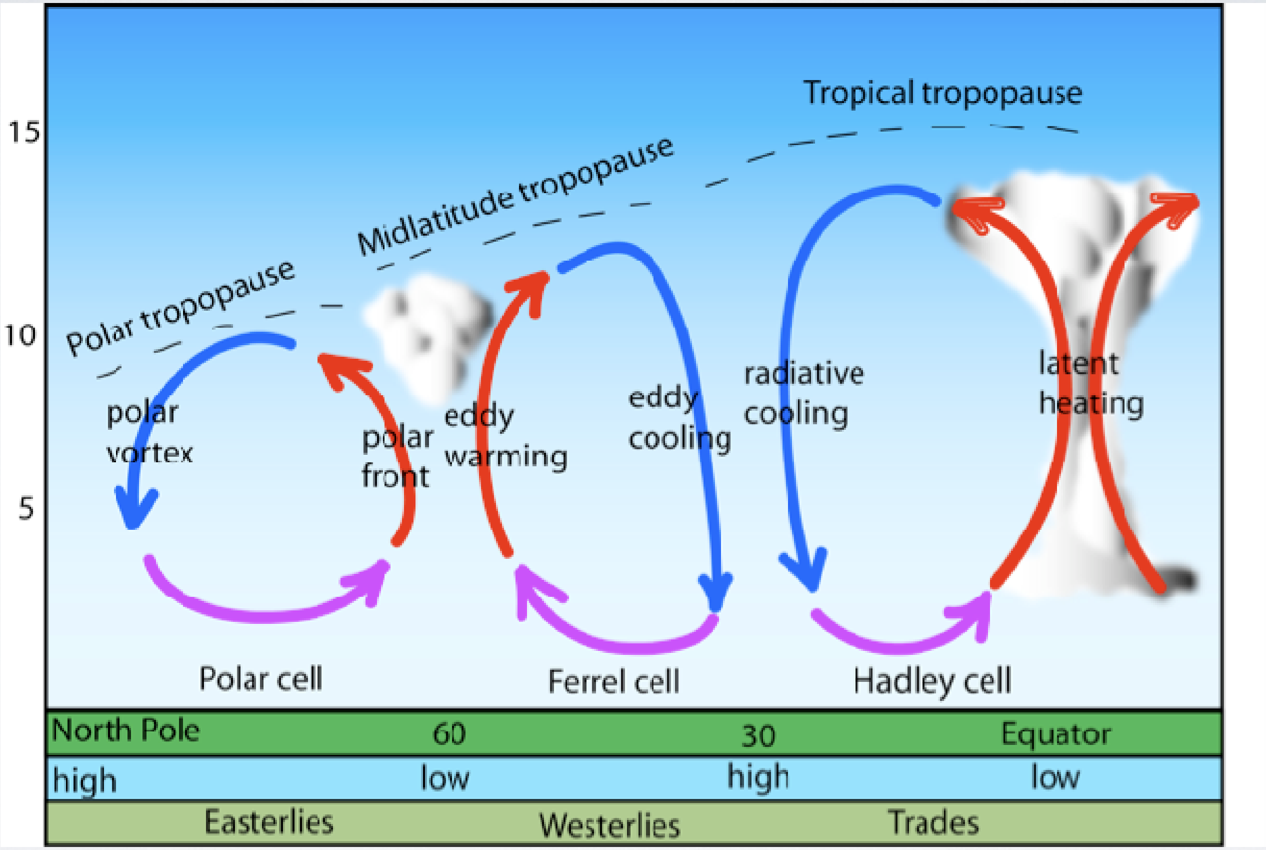
Mean Meridional Circulation

Mechanism for the meridional transports of momentum, energy, and moisture in the atmosphere



What mathematical model do we use to represent these observed features?

- The heating processes at work in the atmosphere are very complicated, and motion-dependent. The heating is very closely related to moist processes, including cloud formation and precipitation.
- The response of the atmospheric circulation to the heating is complicated because of the existence of eddies and their interaction with the mean flow. These eddies are neither purely random, nor purely regular.

Diabatic heating

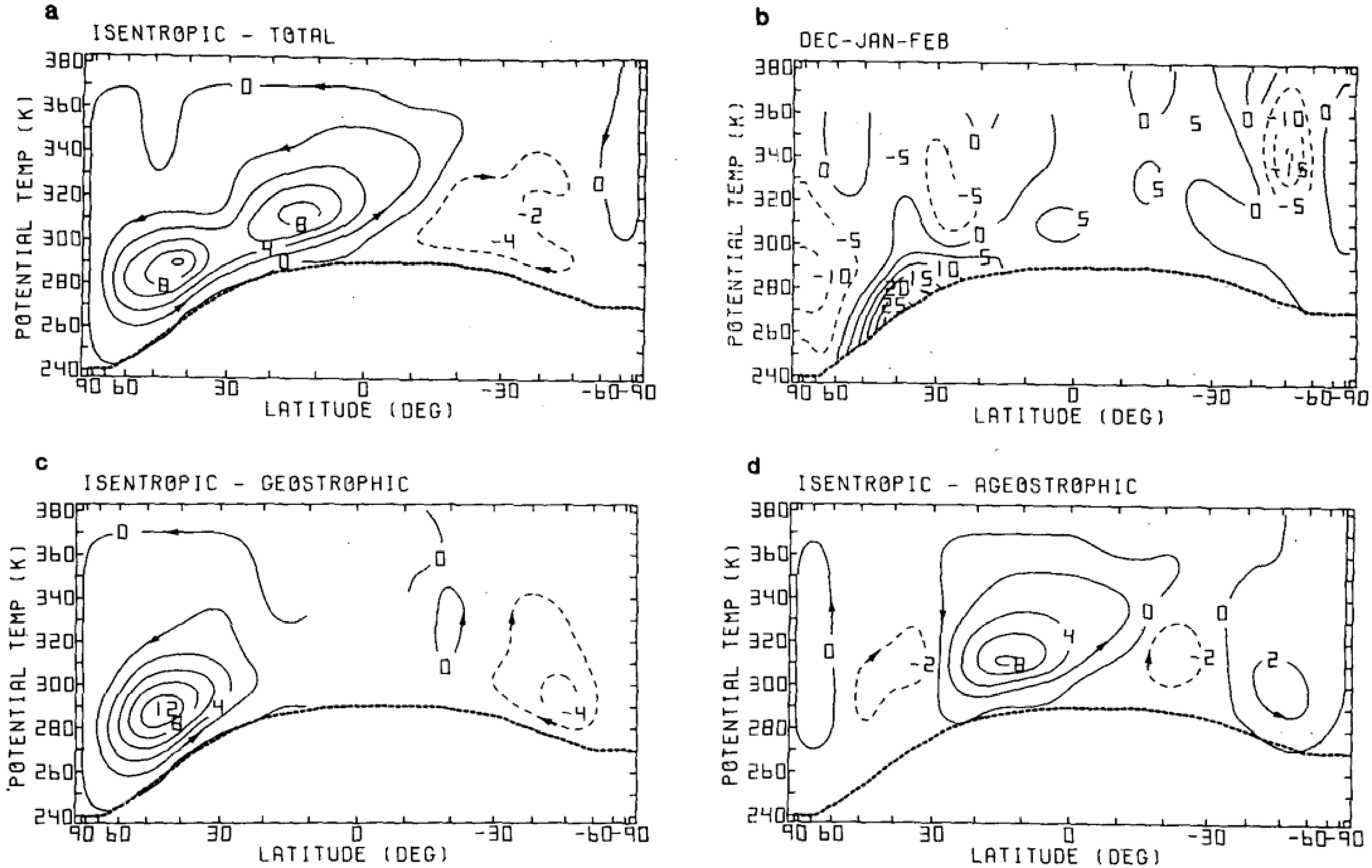


FIG. 7. Mass streamfunction for the (a) isentropic total, (c) geostrophic and (d) ageostrophic mean meridional circulations (10^{10} kg s^{-1}) and (b) meridional cross section of mass-weighted time- and zonally averaged diabatic heating (10^{-1} K day^{-1}) for December 1978–February 1979. Arrows indicate the direction of the circulation.

Townsend and Johnson, 1985

$$F_\phi = -a\cos\phi[(\sigma v)^* u^*]$$ **barotropic wave**

$$F_\theta = \frac{1}{g}\left[p^* \frac{\partial M^*}{\partial \lambda}\right] - a\cos\phi[(\sigma\dot{\theta})^* u^*]$$

baroclinic wave

$$\frac{1}{g}\left[p^* \frac{\partial M^*}{\partial \lambda}\right]$$ **pressure torque exerted by the fluid above an isentrope on that below.**

Introduction

The meridional distribution of the atmosphere can be represented using various coordinates such as:

- geometric latitude, y-coordinate: planar distance in the meridional direction
- latitude, ϕ-coordinate: geographic latitude
- angular momentum, $M = a\cos\phi(u + \Omega a\cos\phi)$, where a is the Earth's radius and Ω the angular speed of the Earth's rotation
- potential vorticity, $q = \frac{1}{\rho}\vec{\zeta_a} \cdot \nabla\theta$, where ρ is thefluid density, $\vec{\zeta_a}$ the absolute vorticity and θ the potential temperature

Review of Potential Vorticity Properties

🕮 Physical Interpretation

$$q = \frac{1}{\rho}\vec{\zeta_a} \cdot \nabla\theta = \frac{1}{\rho}(2\vec{\Omega} + \nabla \times \vec{u}) \cdot \nabla\theta$$

➓ **Isentropic coordinates:**

$$q = \frac{1}{g}(2\Omega \sin\phi + \zeta_\theta)\frac{\partial\theta}{\partial p}$$

with $\zeta_\theta = \frac{1}{a\cos\phi}\left(\frac{\partial u}{\partial \lambda}\right)_\theta - \frac{1}{a}\left(\frac{\partial(v\cos\phi)}{\partial\phi}\right)_\theta$

Potential Vorticity is the absolute circulation of an air parcel that is enclosed between two isentropic surfaces.

Materially conserved for adiabatic flow

$$\frac{Du}{Dt} - \left(2\Omega\sin\phi + \frac{u\tan\phi}{a}\right)v + \left(\frac{\partial M}{a\cos\phi\partial\lambda}\right)_\theta = 0$$

$$\frac{Dv}{Dt} + \left(2\Omega\sin\phi + \frac{u\tan\phi}{a}\right)u + \left(\frac{\partial M}{a\partial\phi}\right)_\theta = 0$$

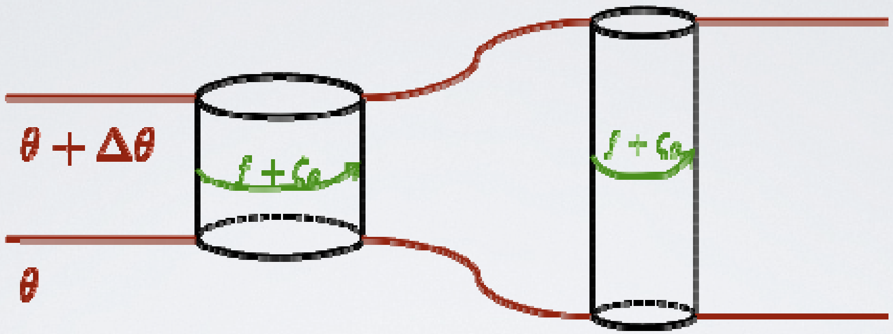
$$\frac{D\sigma}{Dt} + \sigma\left\{\left(\frac{\partial u}{a\cos\phi\partial\lambda}\right)_\theta + \left(\frac{\partial(v\cos\phi)}{a\cos\phi\partial\phi}\right)_\theta + \frac{\partial\dot{\theta}}{\partial\theta}\right\} = 0$$

$$\frac{Dq}{Dt} = \frac{1}{\sigma}\left\{\frac{\partial}{\partial\theta}(\sigma q\dot{\theta}) - \nabla\cdot\mathbf{J}_\theta\right\}$$

$$\boxed{\frac{Dq}{Dt} = 0}$$

$$\mathbf{J}_\theta = \left(\dot{\theta}\frac{\partial(v\cos\phi)}{\partial\theta}, -\frac{\partial(u\cos\phi)}{\partial\theta}, 0\right)$$

- A parcel will conserve its potential vorticity if it moves along an adiabatic surface.



- Difference in potential temperature between the top and bottom is the same for the two cylinders.
- PV conserved, and the cylinder is stretched then static stability is decreasing and absolute vorticity must increase
- PV conserved, and the cylinder is squashed then the static stability is increasing and absolute vorticity must decrease

Invertibility Principle*

- It allows to obtain the geopotential, wind, temperature, and static stability when potential vorticity, boundary conditions and temperature at the surface are known and under certain balance conditions.

*Hoskins et al., 1985

Primitive equations in potential-vorticity, potential temperature coordinates

Zonal Momentum $U = u\cos\phi$

$$\left\{\frac{\partial}{\partial t}(mU)\right\}_{q,\theta} - \frac{1}{a}\left\{\frac{\partial}{\partial\lambda}\left(\frac{mUU}{1-\mu^2}\right)\right\}_{q,\theta} + \left\{\frac{\partial}{\partial q}(m\dot{q}U)\right\}_{\lambda,\theta} + \left\{\frac{\partial}{\partial\theta}(m\dot{\theta}U)\right\}_{\lambda,q}$$

$$-2\Omega\mu V + \frac{1}{a}\left\{m\left(\frac{\partial M}{\partial\lambda}\right)_{q,\theta} - \frac{m}{h}\left(\frac{\partial M}{\partial q}\right)_{\lambda,q}\left(\frac{\partial\mu}{\partial\lambda}\right)_{q,\theta}\right\} = 0$$

where the "pseudo-density", m, is given by

$$m = \sigma h = -\frac{h}{g}\left\{\left(\frac{\partial p}{\partial\theta}\right)_{\lambda,q} - \frac{1}{h}\left(\frac{\partial p}{\partial q}\right)_{\lambda,\theta}\left(\frac{\partial\mu}{\partial\theta}\right)_{\lambda,q}\right\}$$

and the "potential-vorticity thickness", h, is defined as

$$h = \left(\frac{\partial\mu}{\partial q}\right)_{\lambda,\theta}$$

Meridional Momentum $V = v\cos\phi$

$$\left\{\frac{\partial}{\partial t}(mV)\right\}_{q,\theta} + \frac{1}{a}\left\{\frac{\partial}{\partial\lambda}\left(\frac{mUV}{1-\mu^2}\right)\right\}_{q,\theta} + \left\{\frac{\partial}{\partial q}(m\dot{q}V)\right\}_{\lambda,\theta} + \left\{\frac{\partial}{\partial\theta}(m\dot{\theta}V)\right\}_{\lambda,q}$$

$$+2m\mu\left(\Omega U + \frac{K}{a}\right) + \frac{\left(1-\mu^2\right)}{a}\frac{m}{h}\left(\frac{\partial M}{\partial q}\right)_{\lambda,\theta} = 0$$

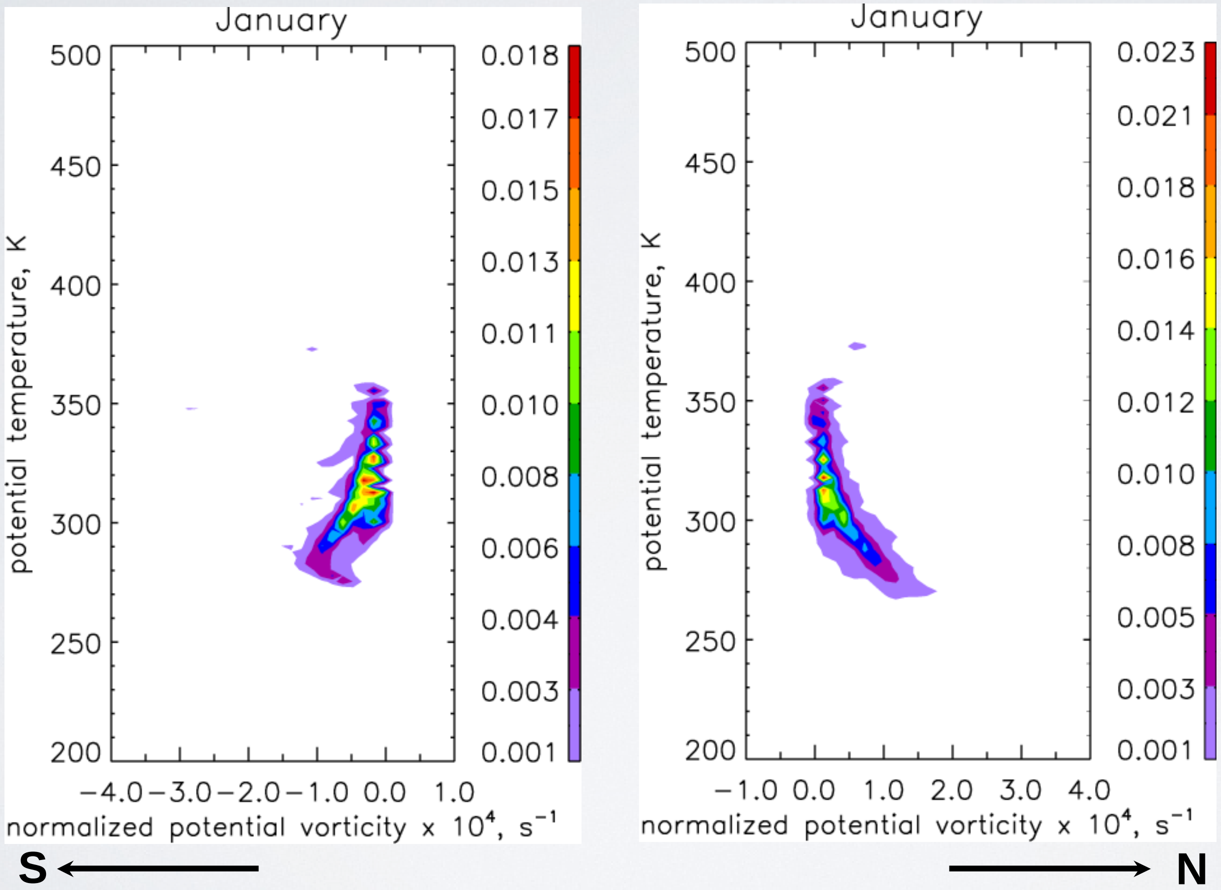
where,

$$K = \frac{1}{2}\left(\frac{U^2+V^2}{1-\mu^2}\right)$$

represents the horizontal kinetic energy per unit mass.

Continuity Equation

$$\left(\frac{\partial m}{\partial t}\right)_{q,\theta} + \frac{1}{a}\left\{\frac{\partial}{\partial \lambda}\left(\frac{mU}{1-\mu^2}\right)\right\}_{q,\theta} + \left\{\frac{\partial}{\partial q}(m\dot{q})\right\}_{\lambda,q} + \left\{\frac{\partial}{\partial \theta}(m\dot{\theta})\right\}_{\lambda,q} = 0$$



Hydrostatic Equation

$$\theta\left(\frac{\partial\Pi}{\partial\theta}\right)_{\lambda,q}+g\left(\frac{\partial z}{\partial\theta}\right)_{\lambda,q}-\frac{\theta}{h}\left(\frac{\partial\Pi}{\partial q}\right)_{\lambda,q}\left(\frac{\partial\mu}{\partial\theta}\right)_{\lambda,q}-\frac{g}{h}\left(\frac{\partial z}{\partial q}\right)_{\lambda,\theta}\left(\frac{\partial\mu}{\partial\theta}\right)_{\lambda,q}=0$$

Geographical latitude of a PV contour

$$\left(\frac{\partial\mu}{\partial t}\right)_{q,\theta}+\frac{U}{a(1-\mu^2)}\left(\frac{\partial\mu}{\partial\lambda}\right)_{q,\theta}+\dot{q}\left(\frac{\partial\mu}{\partial q}\right)_{\lambda,\theta}+\dot{\theta}\left(\frac{\partial\mu}{\partial\theta}\right)_{\lambda,q}=\frac{V}{a}$$

V is the velocity of fluid parcel moving along a surface of constant PV, or the velocity of a fluid parcel that conserves its PV moving on an isentropic surface.

The Zonal Mean Equations in PV, PT Coordinates

Zonal Mean and mass-weighted zonal mean

$$[A] = \frac{1}{2\pi}\int_0^{2\pi} A(\lambda)\,d\lambda \qquad \hat{A} \equiv \frac{[mA]}{[m]}$$

Departure from the Zonal Mean or "eddy component"

$$A^* = A - [A]$$

Statistics of interest

$$[A^*] = 0$$

$$\frac{1}{2\pi}\int_0^{2\pi}\left(\frac{\partial A}{\partial \lambda}\right)d\lambda = 0$$

$$\underbrace{[vA]}_{\text{total flux}} = \underbrace{[A][v]}_{\text{symmetric circulation}} + \underbrace{[A^*v^*]}_{\text{flux due to eddies}}$$

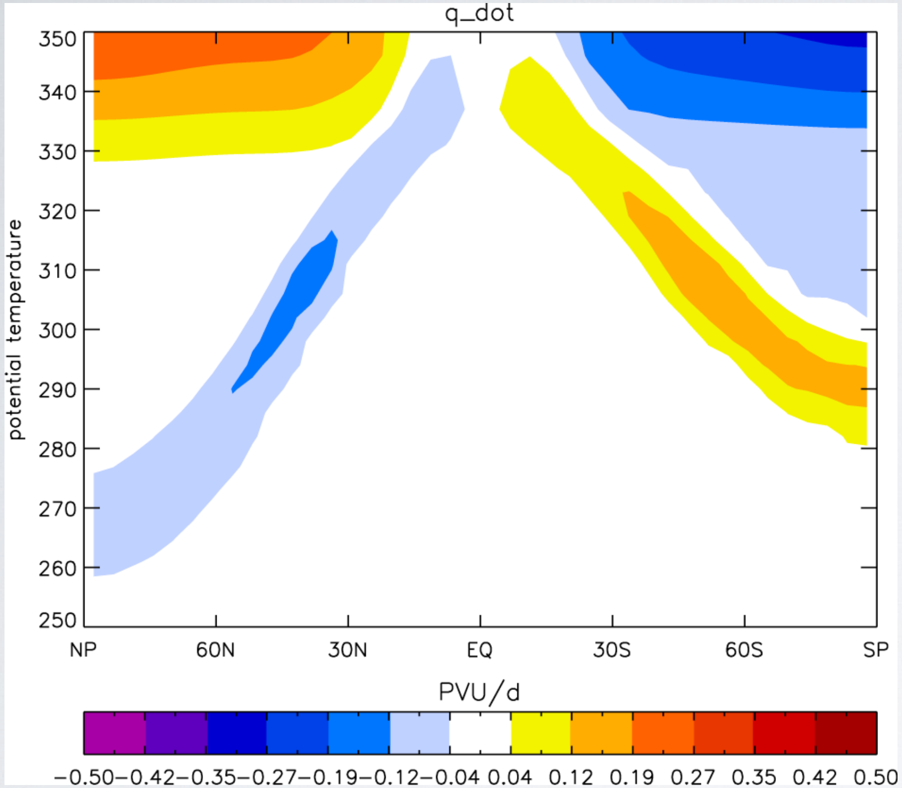
Mean Meridional Circulation

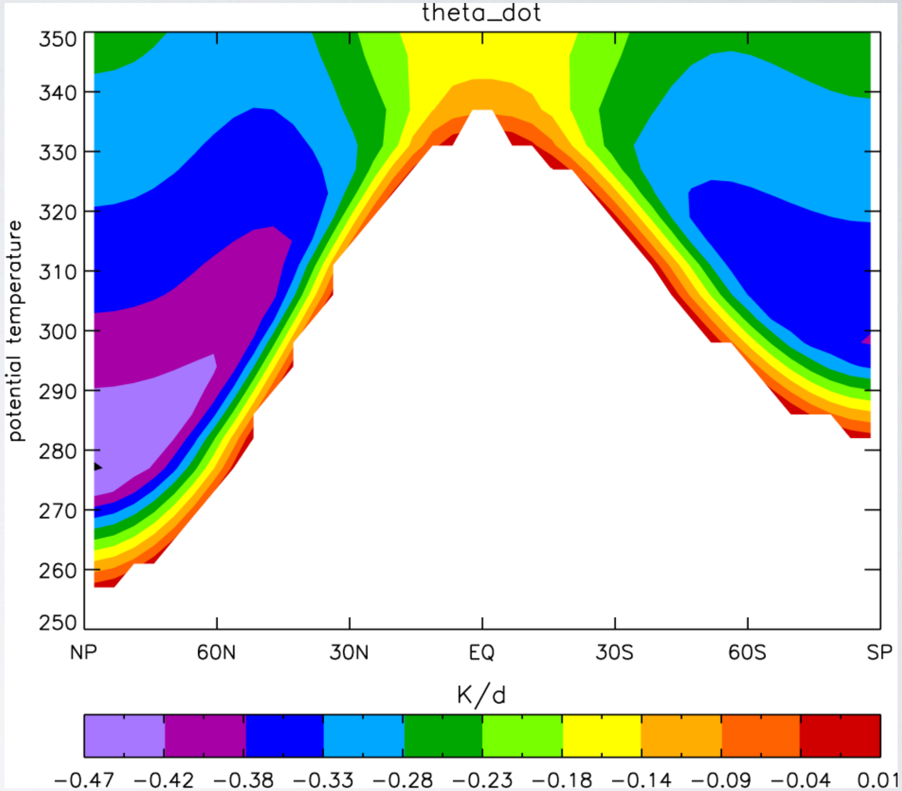
$$\left(\frac{\partial m}{\partial t}\right)_{q,\theta} + \frac{1}{a}\left\{\frac{\partial}{\partial \lambda}\left(\frac{mU}{1-\mu^2}\right)\right\}_{q,\theta} + \left\{\frac{\partial}{\partial q}(m\dot{q})\right\}_{\lambda,q} + \left\{\frac{\partial}{\partial \theta}(m\dot{\theta})\right\}_{\lambda,q} = 0$$

$$\frac{\partial([m]\hat{\dot{q}})}{\partial q} + \frac{\partial([m]\hat{\dot{\theta}})}{\partial \theta} = 0$$

- **heating induces a meridional circulation across the PV surfaces**
- **mass convergence (divergence) across PV surfaces requires an increase (decrease) of the upward mass flux across isentropic surfaces in regions of cooling.**

$\hat{\dot{q}}$ $\hat{\dot{\theta}}$





- **an equatorial flow at lower levels of 0.2 PVU/day and poleward at upper levels of 0.35 PVU/day dominates in each hemisphere.**
- **cool air is transported equatorward at lower levels and warm air is transported poleward aloft.**

Relationship between blocking and large-scale circulation regime over the Pacific

- **blocking***

- intrusion around 60N of a “blob” of low potential vorticity from the lower latitudes yielding to a quasi-stationary state characterized by meridional flow

- influences on baroclinic wave activity and surface weather

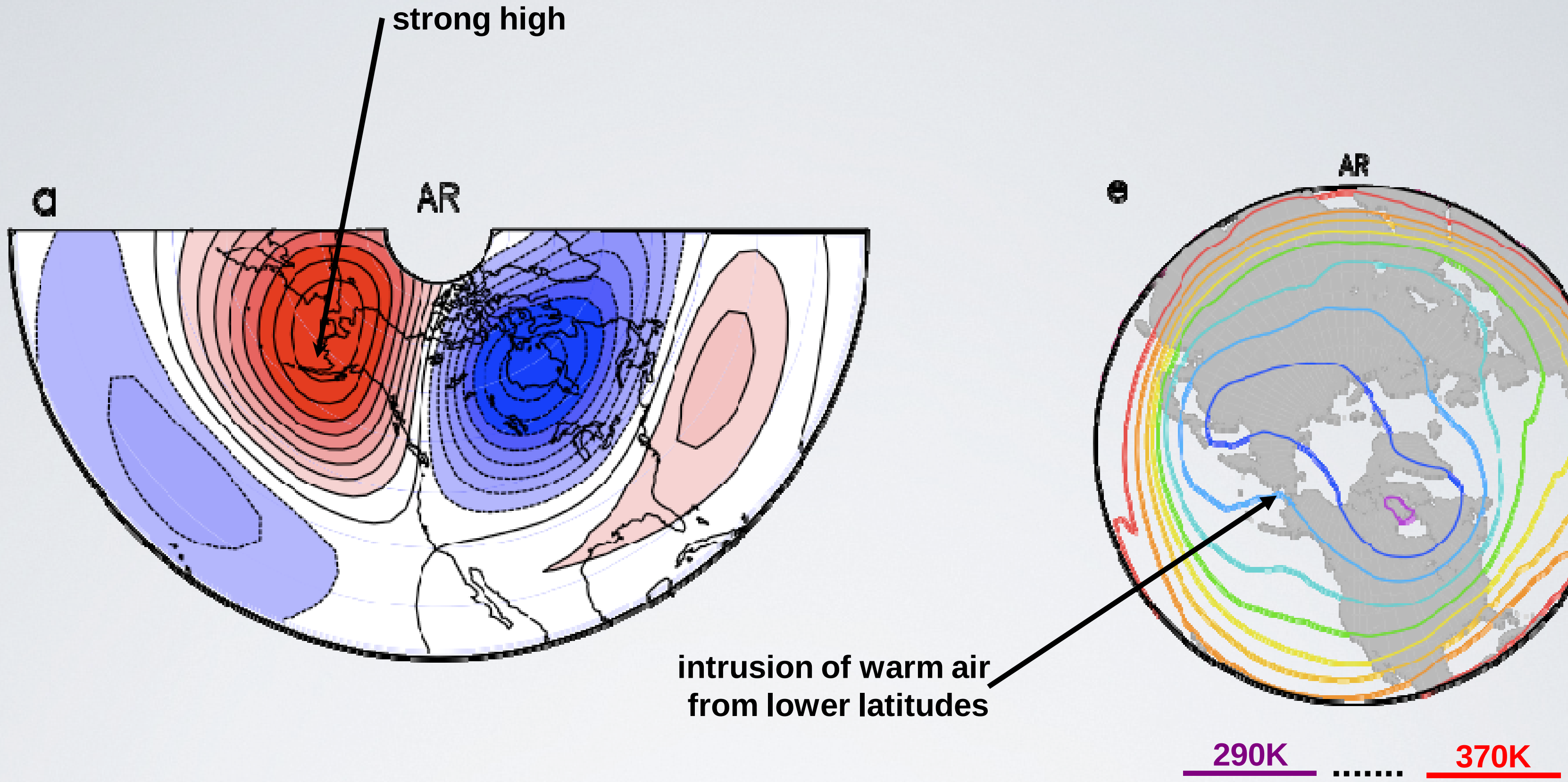
- accompanied by an increased number of extreme events

- **large-scale circulation regimes**

- preferred states associated with planetary wave patterns

- associated with extreme events

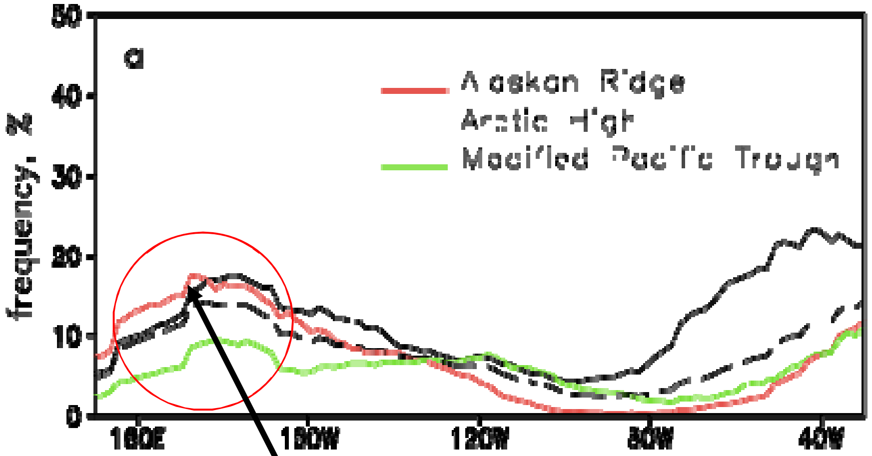
***Nakamura and Walllace 1990**

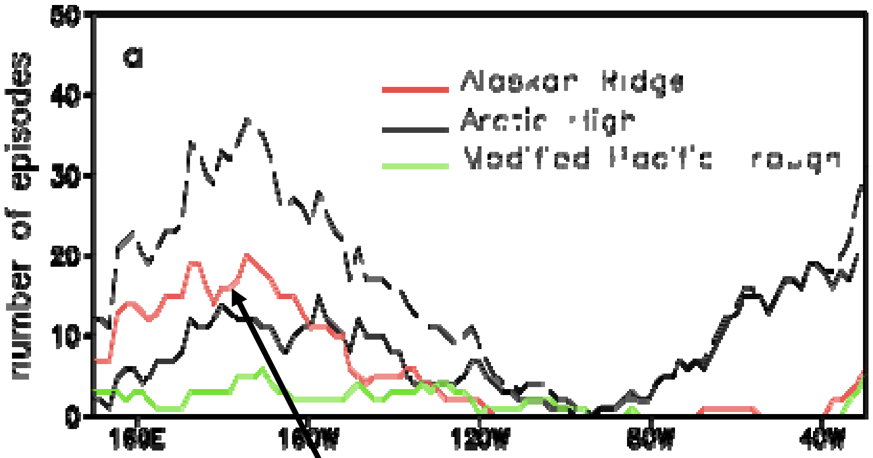


- Does the occurrence of the AR regimes implies that blocking is present?
- Do all the Pacific blocking events occur during the AR regime?

Blocking Frequency

Count of Blocking Episodes





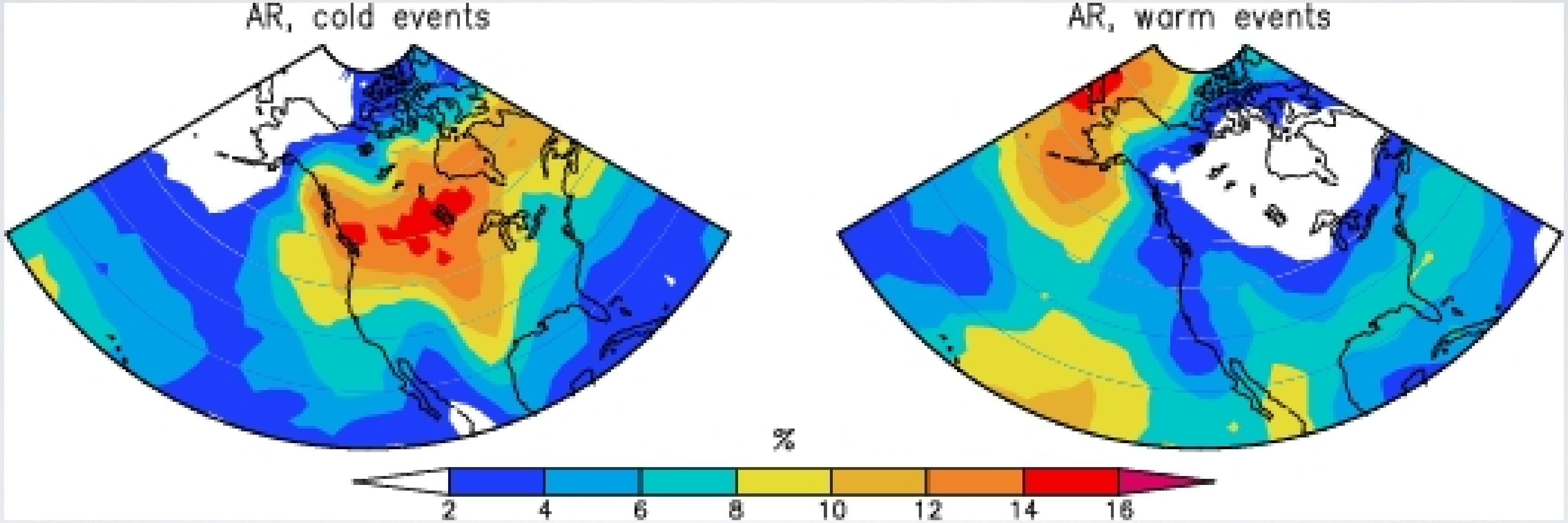
AR regime

AR regime

AR regime frequency: 21%

Extreme Events

Frequency



Extreme warm (cold) events: days when the daily mean 1000-hPa temperature resides in the highest (lowest) 5% of the ranked time series.