Stream Function and Mean Meridional circulation Variability

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Objective

- Understanding Basic features of General Circulation

What do we mean by circulation?
Outlines

• Introduction to General Atmospheric Circulation
• Hadley Cell
• Ferrel Cell
Cartoon of proposed general circulation
Meridional circulation \((v, w)\)

Are not useful to see the circulation
Stream Function ($\Psi$)

The mass divergence in the meridional plane is zero (conservation of mass)

$$\nabla \cdot V = 0$$

The stream function is a scalar field whose relationship to $V$ is carefully selected to automatically satisfy the continuity

$$[v] = \frac{g}{2\pi \cos \varphi} \frac{\partial \psi}{\partial p}$$

$$[\omega] = -\frac{g}{2\pi \cos \varphi} \frac{\partial \psi}{\partial \varphi}$$

$$\psi_p(\phi, p) = \frac{2\pi \cos \phi}{g} \int_p^{ps} [v] dp$$
Properties of Stream Function:

1. Stream function always satisfies the continuity equation.

2. Stream function is constant for a stream line.

3. Stream function values vary from one stream line to another.

4. Stream function represents volumetric flux, i.e., volumetric flow rate per unit area. Units of stream function are kg/ms.
Zonal Average Views

Zonal Average of $x = [x]$

$x - [x] = x^* = \text{deviation from the zonal average}$

$u-[u] = u^*$

$v-[v] = v^*$

$t-[t] = t^*$
\[
[\omega] = -\frac{g}{2\pi \cos \varphi} \frac{\partial \psi}{\partial \varphi}
\]
\[
[v] = \frac{g}{2\pi \cos \varphi} \frac{\partial \psi}{\partial p}
\]
Thermally induced cell

\[ \bar{\psi} > 0 \]

Indirect cell

\[ \bar{\psi} < 0 \]
NCEP Mass Stream Function DJF

\[
\frac{\partial \psi}{\partial p} > 0 \Rightarrow \nu > 0
\]

\[
\frac{\partial \psi}{\partial \phi} > 0 \Rightarrow \omega < 0
\]

\[
\frac{\partial \psi}{\partial p} < 0 \Rightarrow \nu < 0
\]

\[
\frac{\partial \psi}{\partial \phi} < 0 \Rightarrow \omega > 0
\]
Hadley cell Standard deviation shows maximum variability however, Ferrel cell standard deviation shows minimum variability. Since geostrophic balance and hydrostatic balance mostly hold all the time in midlatitude.

Oort and Yienger 1996 J. of Climate
Ferrel Cell (indirect cell)

Eddy transport

Departure from mean zonal flow (subtropical jet stream) destroys thermal wind balance, which induces the meridional circulation to maintain thermal wind balance again

\[
\frac{\partial^2 \overline{\chi}}{\partial y^2} + \frac{f_0^2}{N^2} \rho_0 \frac{\partial}{\partial z} \left( \frac{1}{\rho_0} \frac{\partial \overline{\chi}}{\partial z} \right) = \frac{\rho_0}{N^2} \left[ \frac{\partial}{\partial y} \left( \frac{k \overline{J}}{H} - \frac{R}{H} \frac{\partial}{\partial y} \left( \overline{v'T} \right) \right) \right]
\]

\[
-f_0 \left( \frac{\partial^2}{\partial z \partial y} (u'v') - \frac{\partial \overline{X}}{\partial z} \right)
\]

Diagnosis equation for stream function
\[ -\chi \alpha - \frac{\partial}{\partial y} \text{ (diabatic heating)} + \frac{\partial^2}{\partial y^2} \text{ (large scale eddy heat flux)} \]

\[ + \frac{\partial^2}{\partial y \partial z} \text{ (large scale eddy momentum flux)} + \frac{\partial}{\partial z} \text{ (zonal drag force)} \]

\[ \frac{\partial^2 \bar{\chi}}{\partial y^2} + \frac{f_0^2}{N^2} \rho_0 \frac{\partial}{\partial z} \left( \frac{1}{\rho_0} \frac{\partial \bar{\chi}}{\partial z} \right) = \frac{\rho_0}{N^2} \left[ \frac{\partial}{\partial y} \left( \frac{\kappa \bar{J}}{H} - \frac{R}{H} \frac{\partial}{\partial y} (v \bar{T}) \right) \right] \]

\[ - f_0 \left( \frac{\partial^2}{\partial z \partial y} (u' v') - \frac{\partial \bar{X}}{\partial z} \right) \]
Eddy heat flux maximum at 50 N

Eddy momentum flux
References
1-J. Holton, An Introduction to Dynamical Meteorology
2-D. Strauss, presentation
3-Ortt and Yienger, observed interanual in the hadley circulation and its connection to ENSO, J. of Climate, 1996
Thanks for your Attentions