Theories and Models of the General Circulation of the Ocean

A Theory of the Meridional Overturning Circulation

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The sciences do not try to explain... they mainly make models.
John von Neumann

Theories, Models and Understanding

Is a theory the same as a model?

(A good debate to have at the bar.)

Theories and Models

Almost anything can be a model (analog, digital, toy, complicated, a virtual reality, a GCM).

A theory is a special type of model that has:

1. Generality (to explain more than the phenomenon at hand).
2. Simplicity (else it is just a description of the phenomenon at hand).
3. Predictive power, or testability.

A theory need not have equations, but ideally has some quantitative aspects.

Understanding

Understanding is needed for two purposes:

1. Satisfaction. An end in itself.
2. To improve our models and so make better predictions.

The problem, and why we care

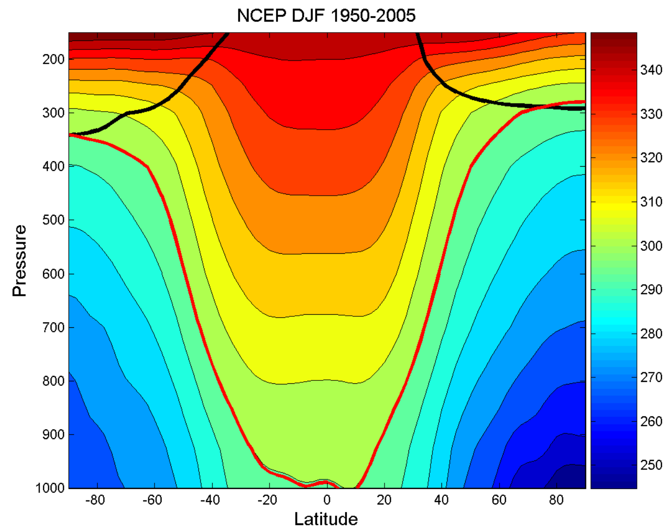
What?

- What processes determine the stratification and deep circulation of the ocean?
  - Mixing, advection, winds, surface buoyancy gradients. . .
- Previous theories. Primarily mixing driven (e.g., Stommel-Arons), but observational and numerical evidence (e.g., Toggweiler-Samuels) that wind over the Southern Ocean plays a key role.

Why?

- It is simply fundamental.
- Deep ocean (the meridional overturning circulation or MOC, aka the ‘thermohaline circulation’ or THC) carries a large fraction of the ocean’s meridional heat transport.
  - Its variability would give rise to large variations in climate (inc. global warming and paleoclimate).
- Many results from GCMs, especially regarding climate variability, lack interpretation.
  - **Motto:** *Be wary of the results from a GCM unless there is some interpretation to go along with it.*

Atmospheric Stratification



The atmosphere is heated from below and cooled from above. So the forcing itself has a tendency towards being statically unstable.
Stratification might be maintained by:

1. Vertical convection, moist or dry.
   - Moist convection is almost certainly the dominant process maintaining stratification in the tropics. (Having a given vertical structure simplifies remaining theory; leads to 'quasi-equilibrium theory' etc.)
2. Baroclinic instability. Moves energy upwards and sideways.

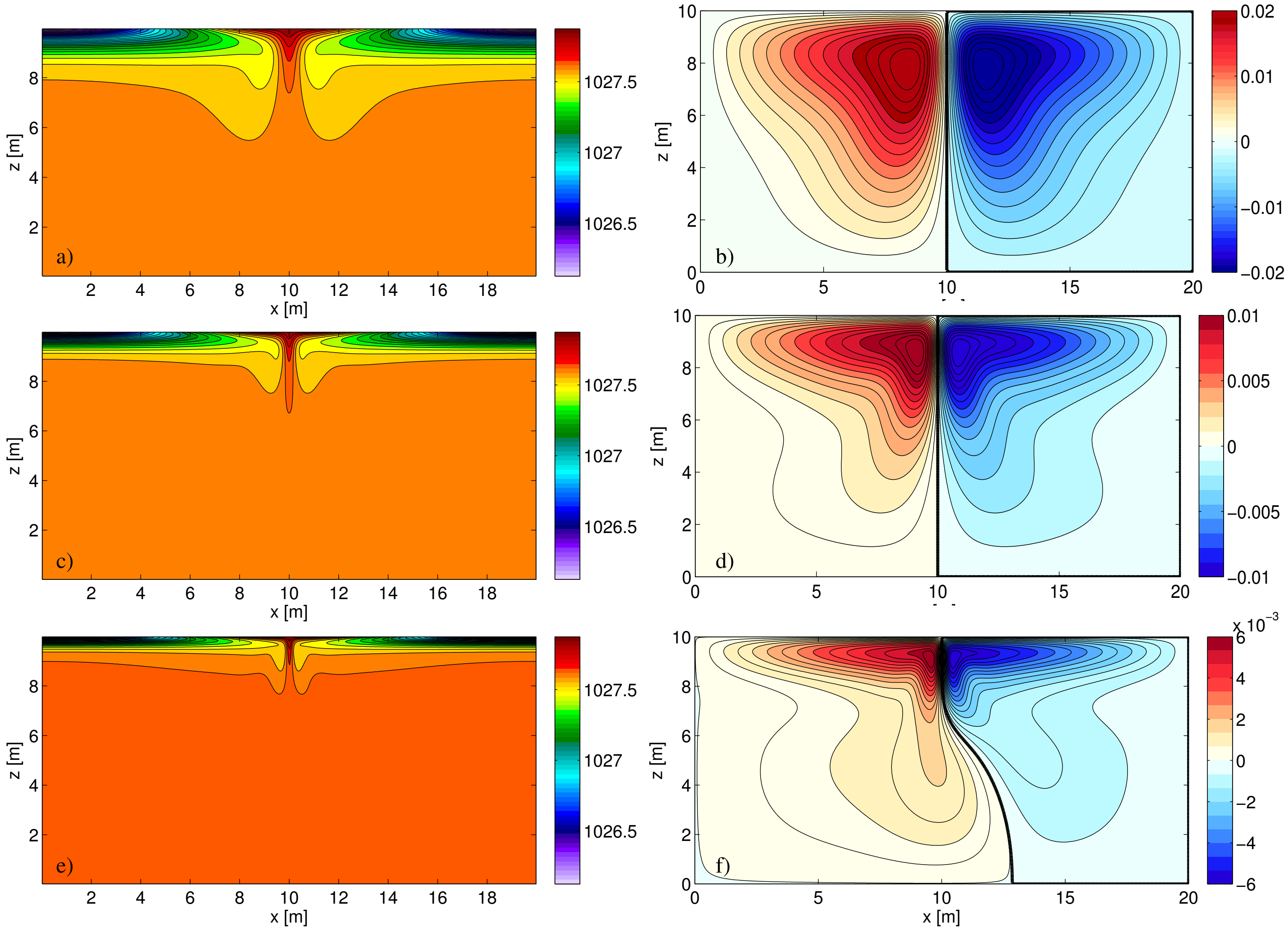
There is no shortage of ideas for maintaining stratification in the atmosphere!

The Ocean

But the ocean is heated *and* cooled from above. In the absence of winds and mixing there will be no deep stratification.

Horizontal Convection with Mehmet Ilicak

A fluid heated and cooled from above will generate no deep flow and no deep stratification at high Rayleigh number.



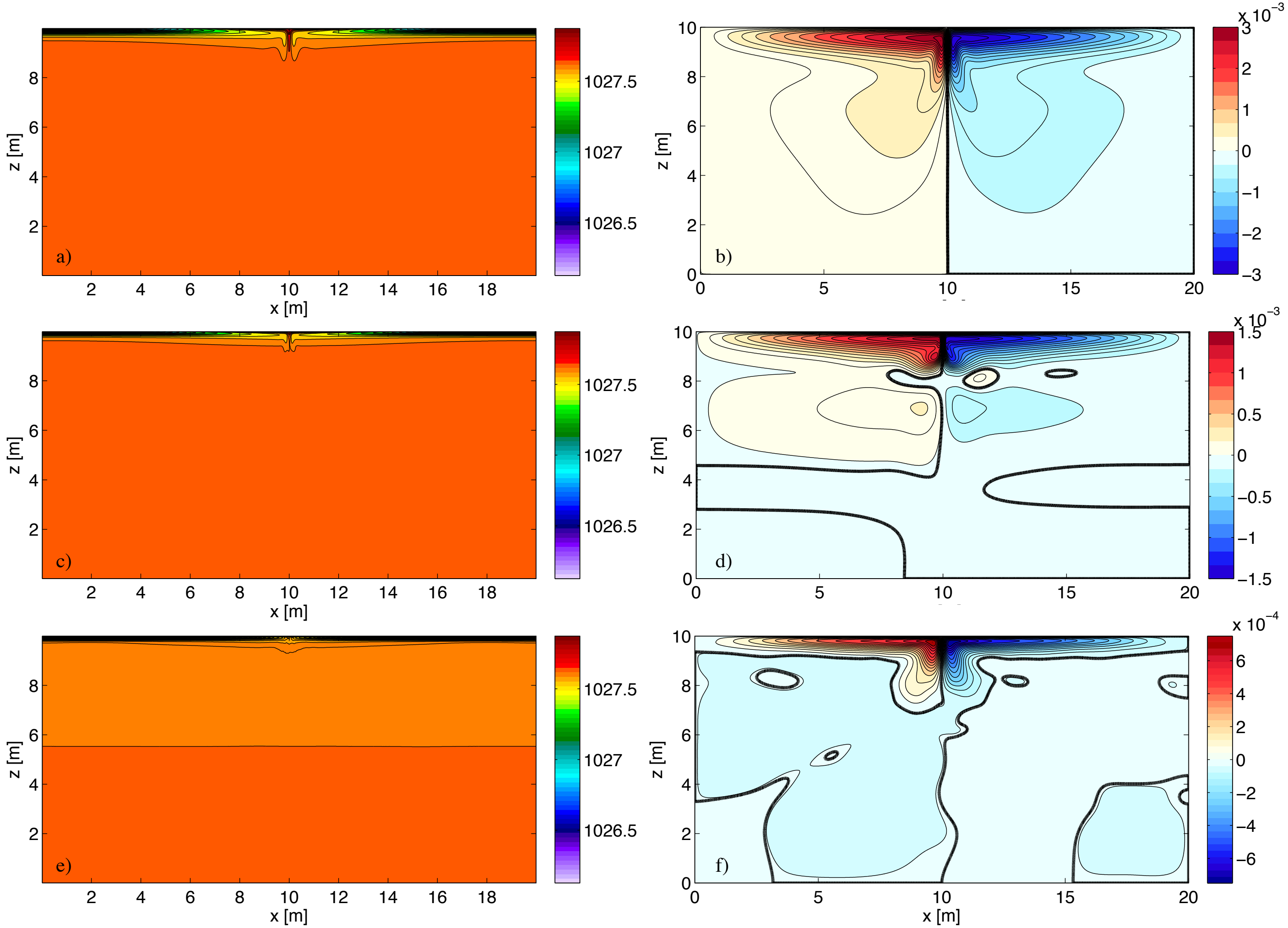
$Ra = 10^6$

$Ra = 10^7$

$Ra = 10^8$

Horizontal Convection continued

Still higher Rayleigh number.



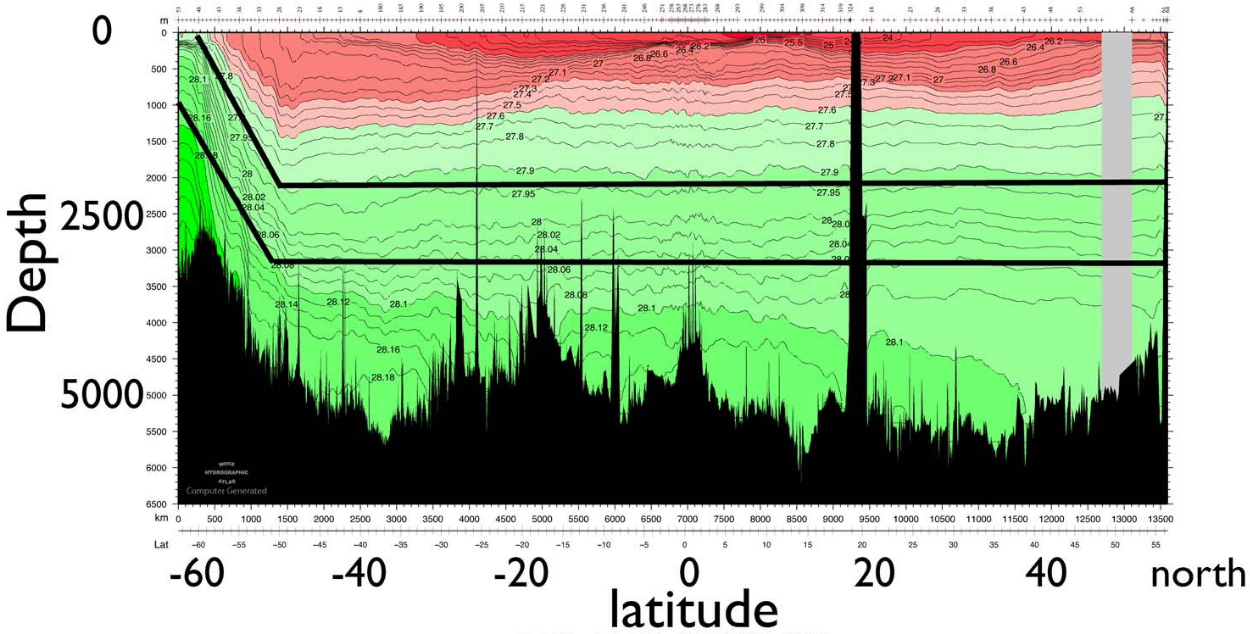
$Ra = 10^9$

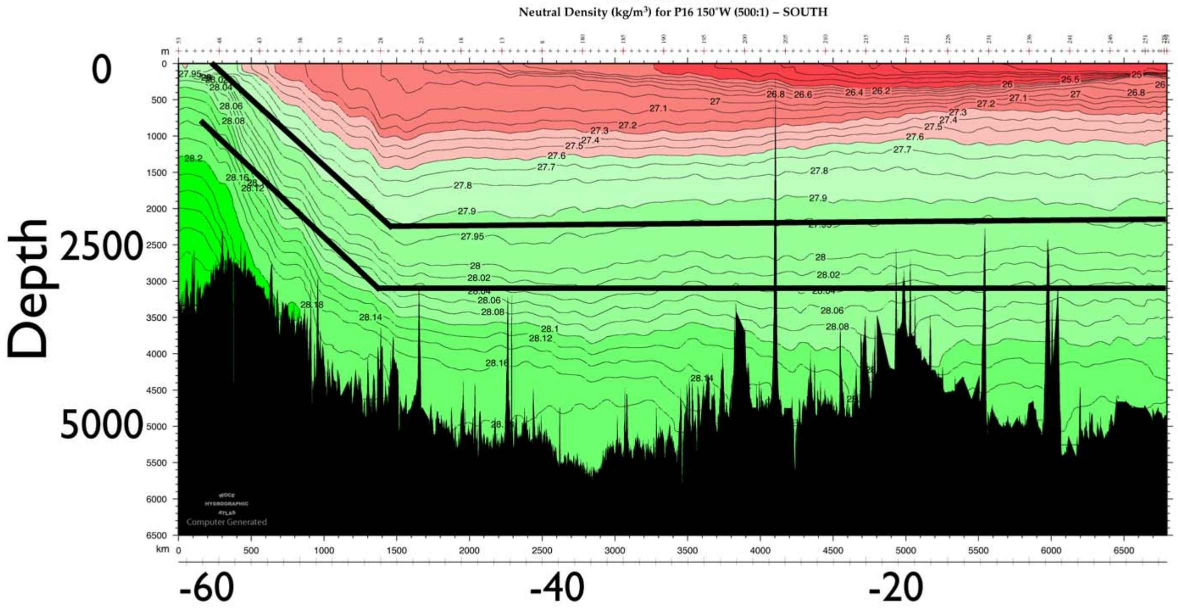
$Ra = 10^{10}$

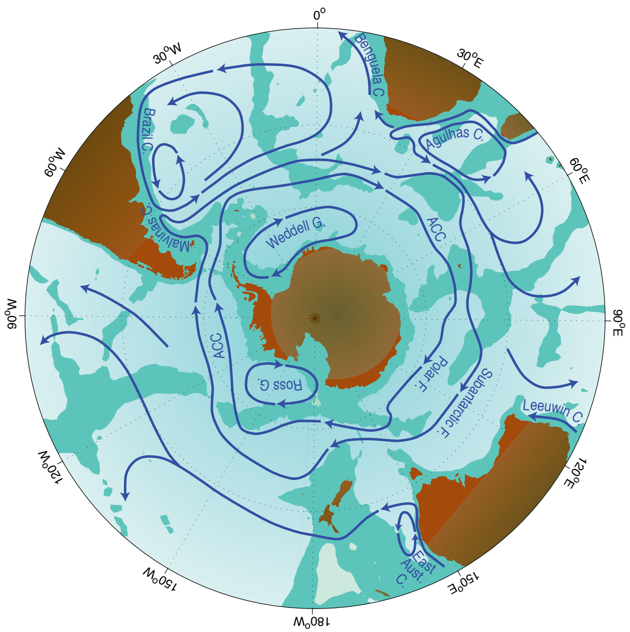
$Ra = 10^{11}$

For real ocean: $Ra \approx 10^{30}$.

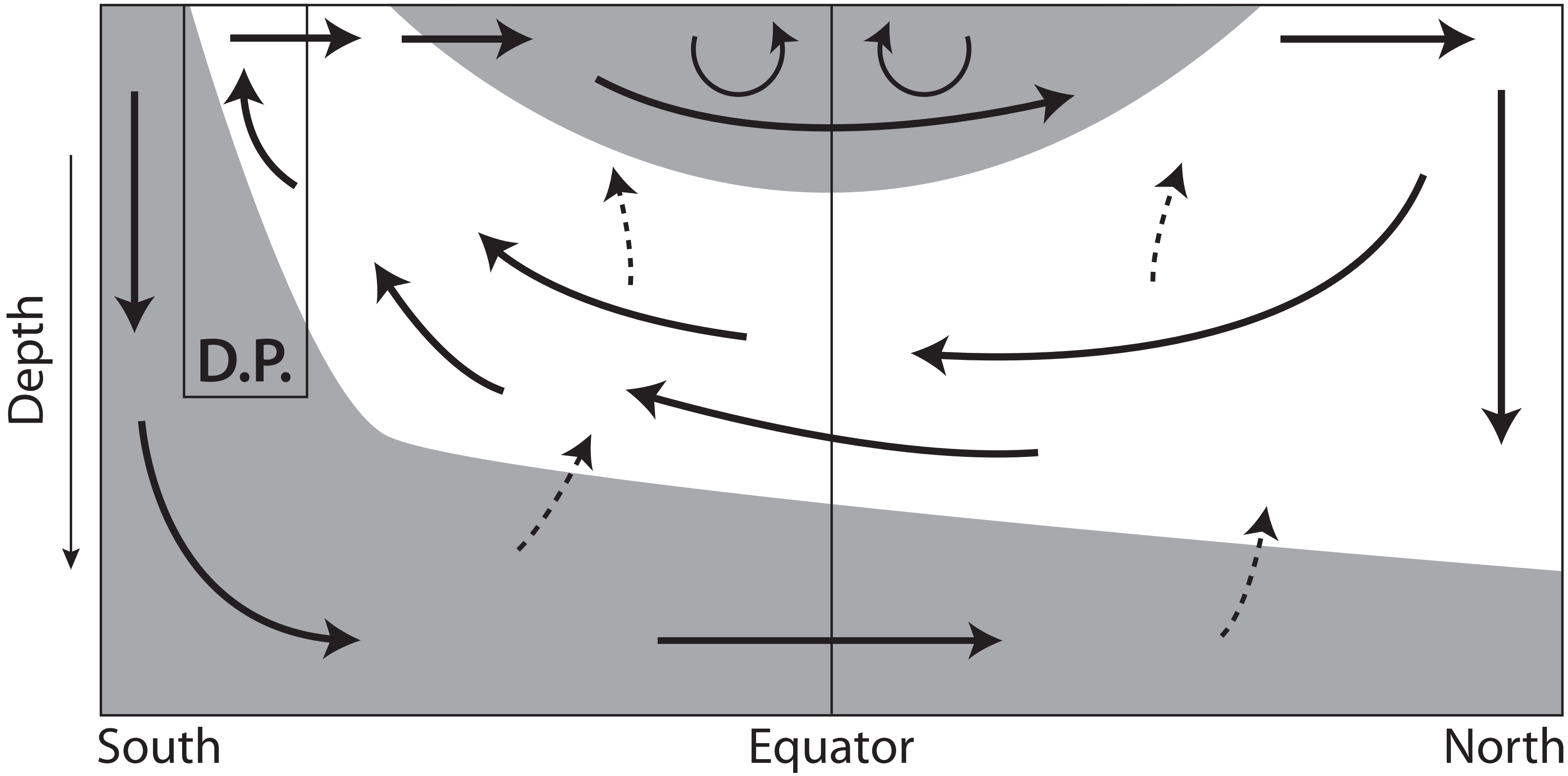
Ocean Stratification. Pacific. WOCE. 150°W





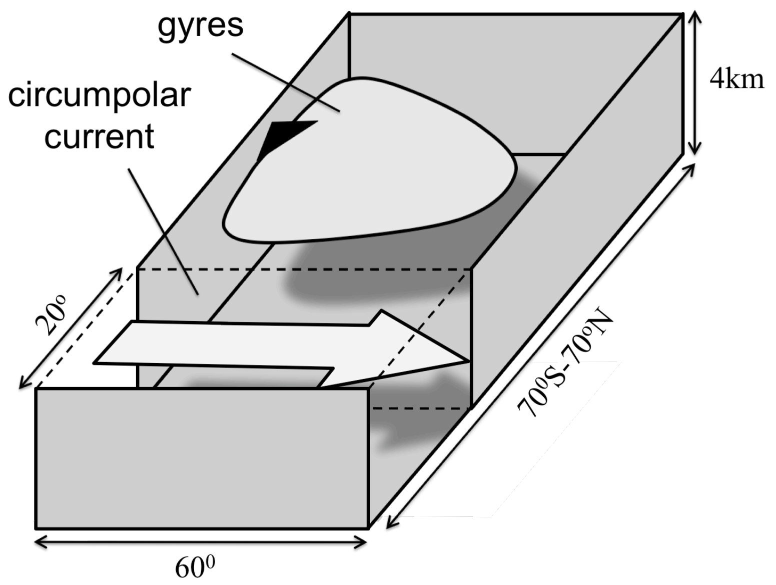


(Unequal contours)



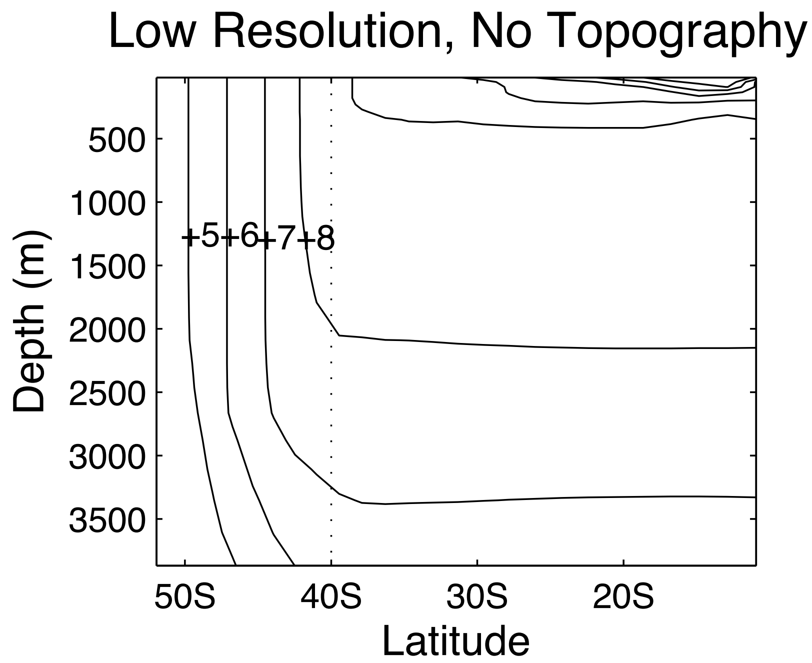
Schematic from an undergraduate book

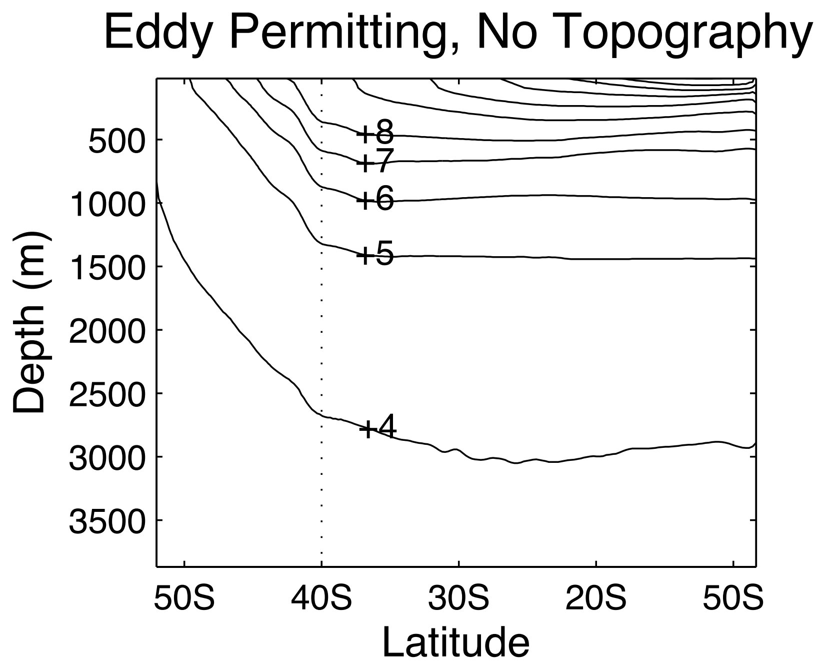
Wind and Eddies in the ACC



Use a 3D primitive equation model in simplified geometry to build intuition and phenomenology.

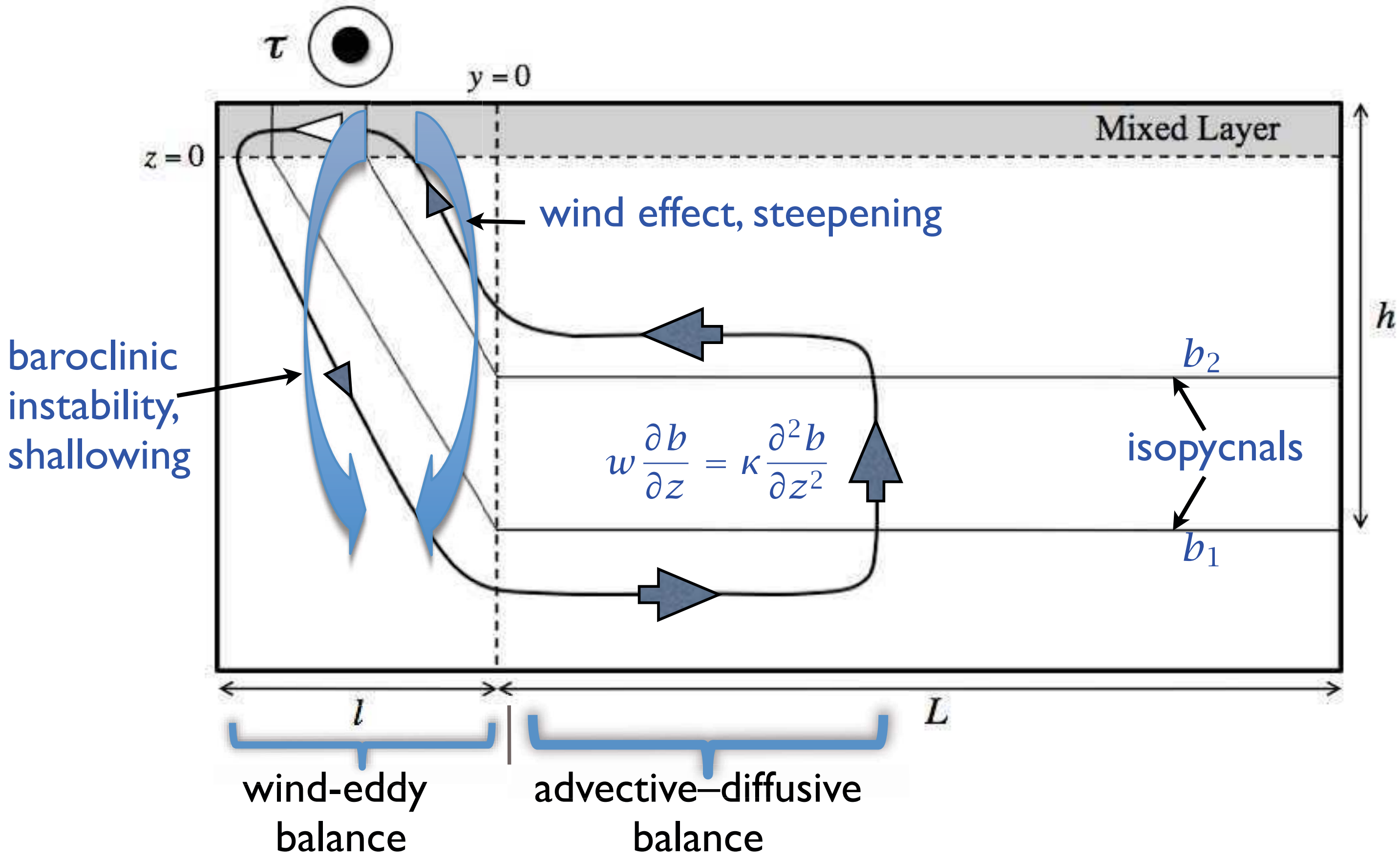
- Stratification in the ACC maintained by a balance between wind (steepens) and mesoscale eddies (slumping).
- Stratification then extends through the rest of the ocean.





A Theory for Deep Stratification

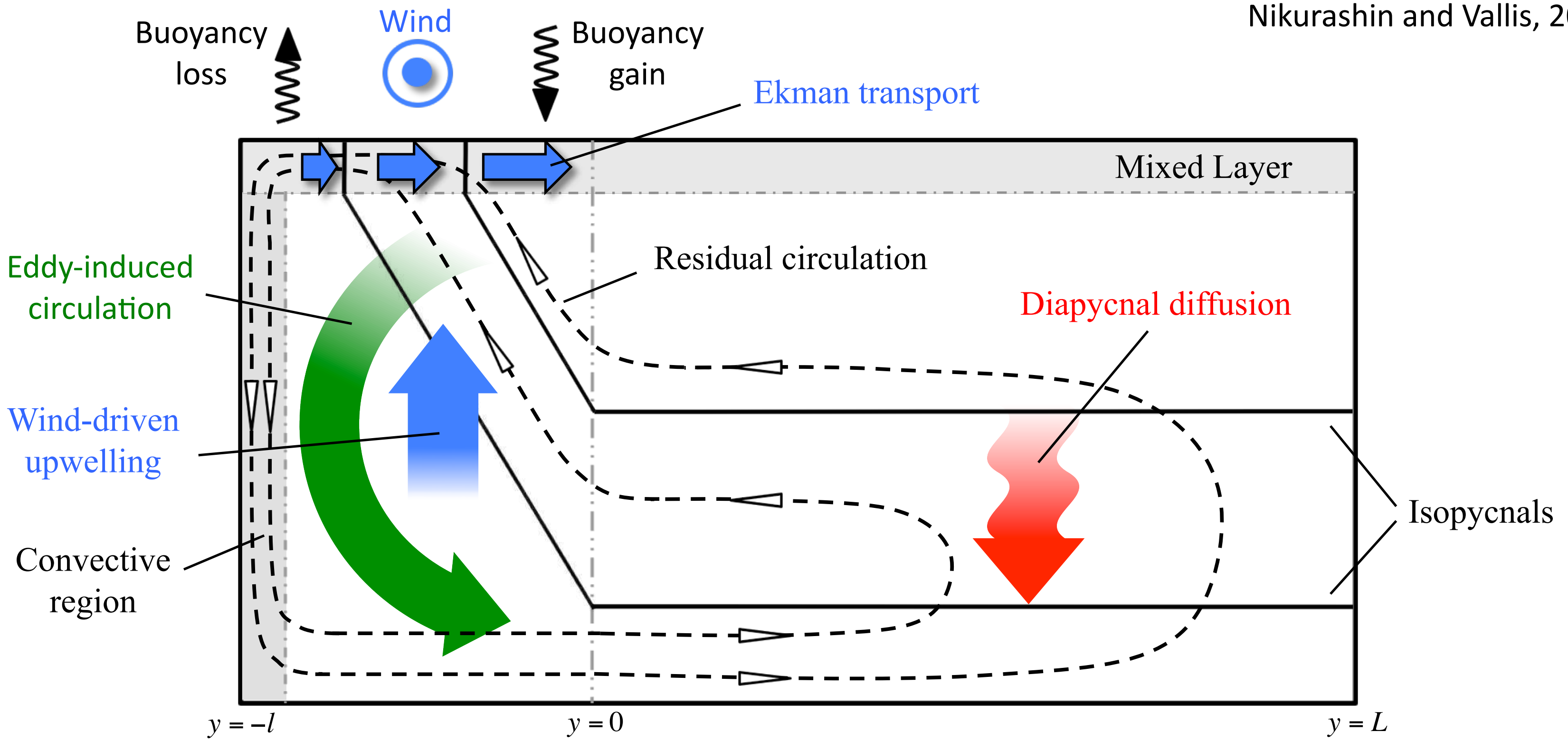
That is, a conceptual model that makes testable predictions



- In channel, the wind induced circulation causes generates a clockwise 'Deacon Cell' that rotates tries to make the isopycnals vertical.
- The baroclinic eddies try to make the isopycnals horizontal.
- The circulation in the channel must connect smoothly with that in the basin.

Theory

Nikurashin and Vallis, 2011



In the channel

$$-\psi_z b_y + \psi_y b_z = 0$$

and

$$\psi = -\frac{\tau}{f} - K\frac{b_y}{b_z}$$

(e.g. Marshall and Radko, 2003)

In the basin

$$\psi_y b_z = \kappa b_{zz}$$

or

$$\psi\big|_{y=0} = -\kappa L\frac{b_{zz}}{b_z}$$

(e.g. Munk and Wunsch, 1998)

τ - wind stress K - isopycnal eddy diffusivity κ - diapycnal diffusivity $(v,w) = (-\psi_z, \psi_y)$ - velocity

Equations

In the channel

Momentum equation: wind, Coriolis and eddies in TEM form

$$-fv^{*} = \frac{\partial \tau}{\partial z} + f\frac{\partial}{\partial z}\left(\frac{\overline{v'b'}}{\partial_z \overline{b}}\right)$$

becomes

$$\psi = -\frac{\tau}{f} + K_e S_b$$

where $S_b = -\partial_y b/\partial_z b$ is the slope of isopycnals and K_e is a baroclinic eddy diffusivity.

Buoyancy equation:

$$J(\psi, \overline{b}) = \kappa_v \frac{\partial^2 \overline{b}}{\partial z^2}$$

Boundary condition at edge of channel:

$$\psi|_{y=0} = -\kappa_v L \frac{\partial_{zz}\overline{b}}{\partial_z \overline{b}}.$$

Nondimensionalization and Scaling

Let $y = l\hat{x}$, $z = h\hat{z}$, $\psi = \hat{\psi}(\tau_0/f_0)$, etc.

Buoyancy evolution: $$\partial_y \hat{\psi} + \hat{s}_\rho \partial_z \hat{\psi} = -\epsilon \left(\frac{l}{L}\right) \frac{\partial_{zz}\hat{b}}{\partial_z \hat{b}},$$

Momentum balance: $$\hat{\psi} = -\frac{\hat{\tau}}{\hat{f}} + \Lambda \hat{s}_\rho,$$

Boundary condition: $$\hat{\psi}|_{y=0} = -\epsilon \frac{\partial_{zz}\hat{b}}{\partial_z \hat{b}},$$

where

$$\Lambda = \frac{\text{Eddies}}{\text{Wind}} = \frac{K_e}{\tau_0/f_0}\frac{h}{l} \sim 1 \qquad \text{and} \qquad \epsilon = \frac{\text{Mixing}}{\text{Wind}} = \frac{\kappa_v}{\tau_0/f_0}\frac{L}{h} \sim 0.1 - 1,$$

The parameter h is a characteristic depth of the stratification, and will be a part of the solution.

Scaling and Solutions

We proceed in four ways:

I Scaling: gives basic parameter dependencies on wind, diffusivity, Coriolis parameter etc., in limits of weak and strong diffusion.

II Analytic solutions: Obtainable, in limit of weak diffusion only, by regular asymptotics and method of characteristics.

III Numerical solution of equations of the theory.

IV Full solution of primitive equations using a comprehensive ocean GCM.

Scaling

Weak diffusiveness

$$\epsilon \ll 1, \qquad \Lambda = 1, \qquad \text{and} \qquad \frac{l}{L} \ll 1$$

$$\boxed{h = \frac{\tau_0/f_0}{K_e} l, \qquad \Psi = \kappa_v \frac{K_e}{\tau_0/f_0} \frac{L}{l}}.$$

Depth of stratification is determined by wind and eddies only. Circulation is weak, and goes to zero with the diffusivity.

Strong diffusiveness

$$\epsilon \gg 1, \qquad \Lambda = \epsilon, \qquad \text{and} \qquad \frac{l}{L} \ll 1$$

$$\boxed{h = \sqrt{\frac{\kappa_v}{K_e} L l}, \qquad \Psi = \sqrt{\kappa_v K_e \frac{L}{l}}}$$

Depth of stratification is determined by diffusion and eddies. Circulation is stronger, goes as half power of diffusivity.

Analytic Solutions

If diffusion is weak then the surface conditions are propagated into the interior along characteristics (helpfully found by my Russian collaborator). Requires particular forcing at the surface. Choosing

$$b_0(y) = \Delta b \left(1 + \frac{y}{l}\right)^2,$$

we obtain

$$\psi^{(0)} = 0$$

and

$$\psi^{(1)}(y, z) = \kappa_v \frac{K_e}{\tau/f} \frac{L}{l} \left(1 + \frac{y}{l} - \frac{K_e}{\tau/f} \frac{z}{l}\right)^{-1}$$

Arguably, solutions are of no especial interest in themselves, except that we can compare them against numerical solutions to check the theory.

Numerical Solutions of Equations of the Theory

Provide a bridge between analytics and full GCM. Step forward the zonally-averaged TEM buoyancy equation:

$$\frac{\partial \overline{b}}{\partial t} + J(\psi, \overline{b}) = \kappa_v \frac{\partial^2 \overline{b}}{\partial z^2}$$

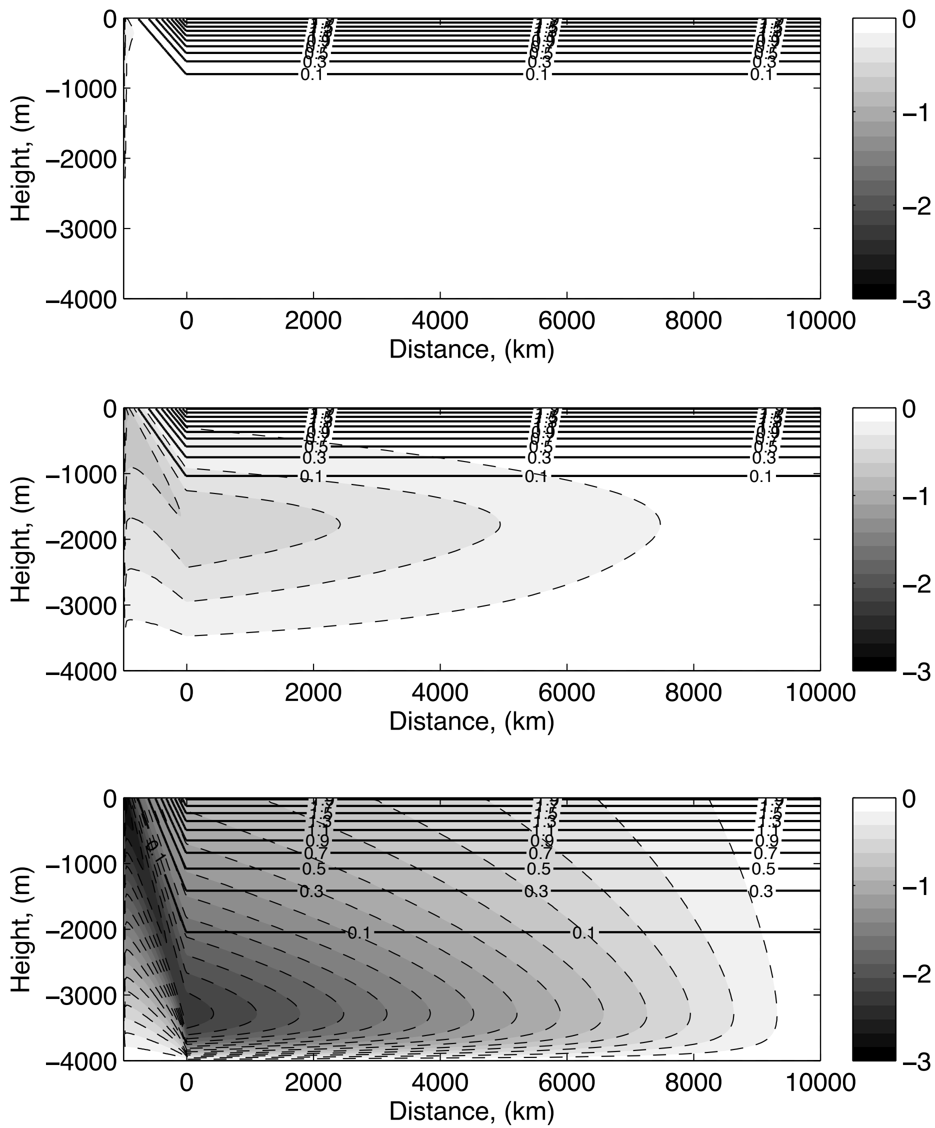
with

$$\psi = -\frac{\tau}{f} + K_e S_b.$$

and boundary conditions:

At top: $\overline{b}|_{z=0} = b_0(y).$

At channel edge: $\overline{\psi}|_{y=0} = -\kappa_v \frac{\partial^2 \overline{b}/\partial z^2}{\partial \overline{b}/\partial z}.$

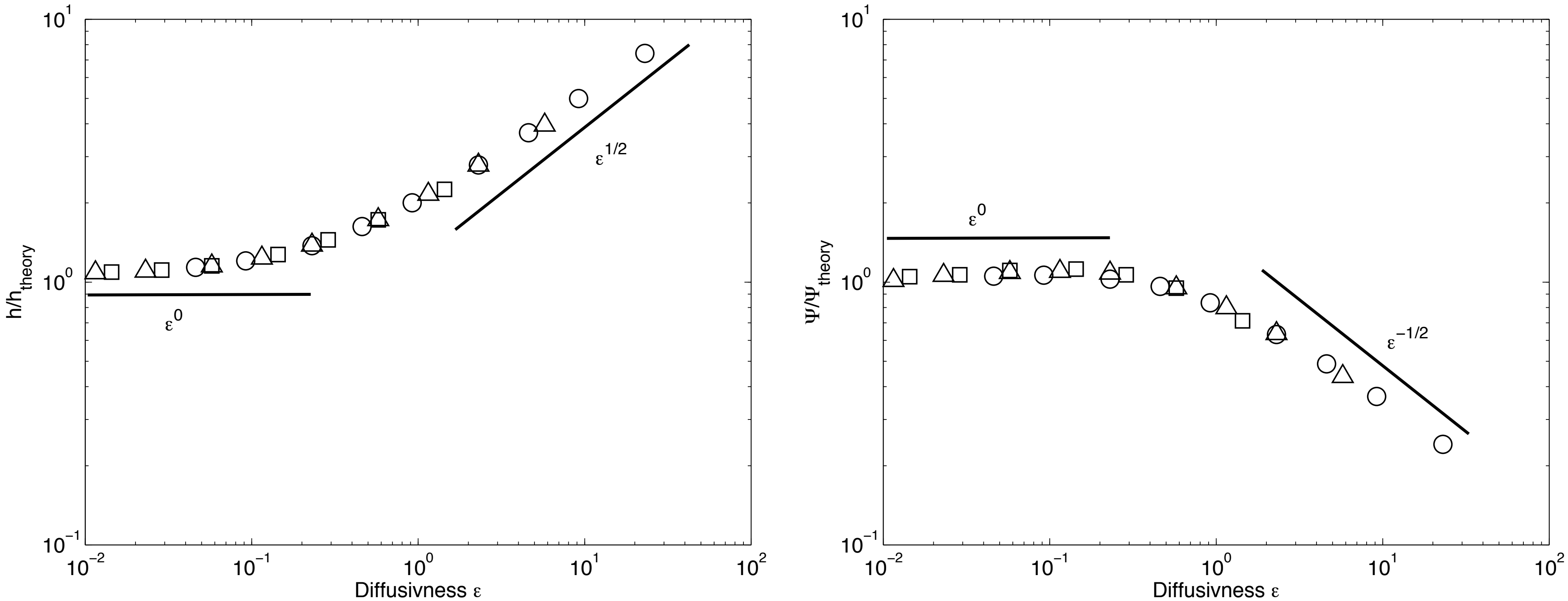


Figures (from top):
Buoyancy distribution and circulation with low, medium and high diffusion.

Tests of Scaling

Against numerical solution of theoretical equations

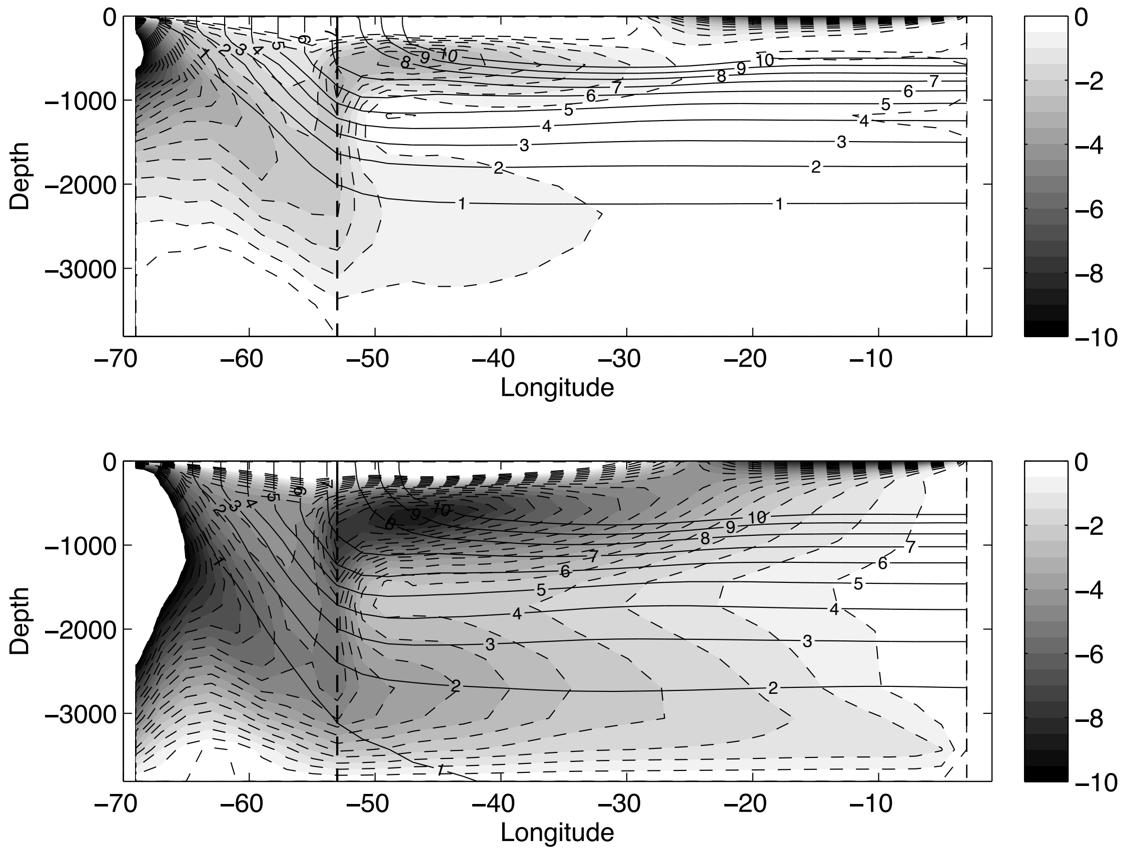
Obtain many solutions by independently varying wind-stress and diffusivity. Plot in non-dimensional form against diffusiveness, $\epsilon = L\kappa_v/(h\tau/f)$



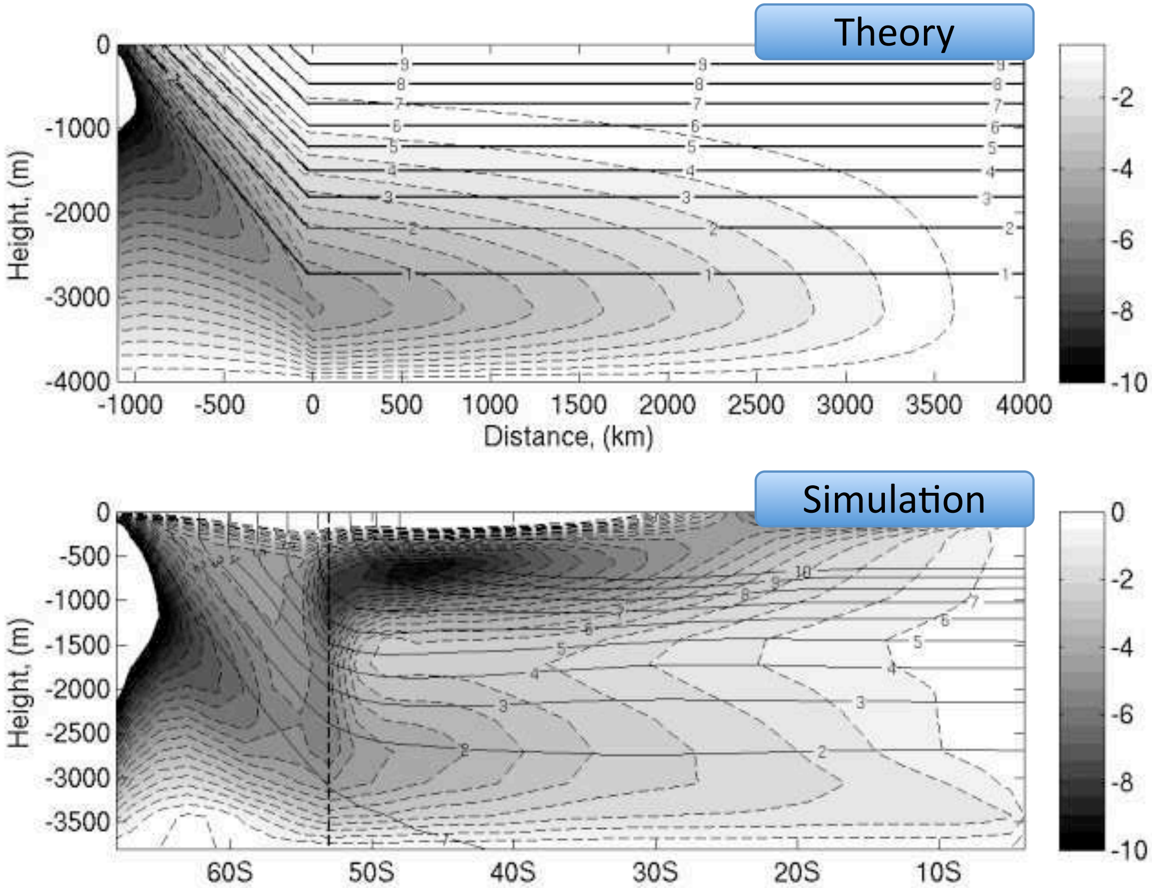
Solutions collapse on to the scalings predicted in low and high diffusiveness limits.

Ocean GCM Simulations

Integrate a full OCGM (MOM) in idealized geometry. Figure shows two different values of diapycnal mixing: $10^{-5}\,m^2\,s^{-1}$ (upper panel) and $10^{-4}\,m^2\,s^{-1}$ (lower panel).



Theory and GCM Simulations



Theory and simulations are consistent in deep ocean.

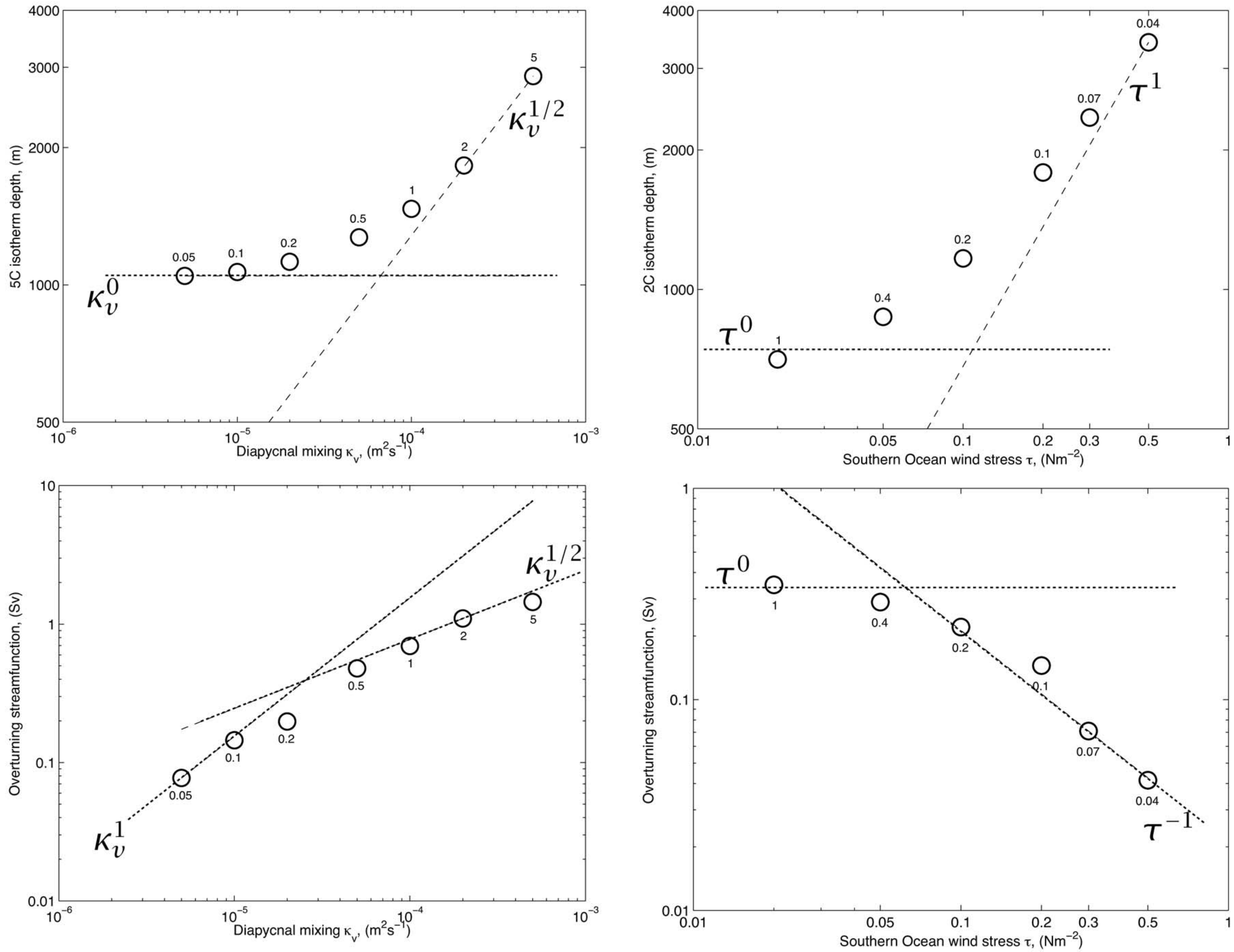
Test of Scaling with General Circulation Model

$$h = \begin{cases} \dfrac{\tau/f_0}{K_e}\, l \\[2ex] \sqrt{\dfrac{\kappa_v}{K_e} L l} \end{cases}$$

h

$$\Psi = \begin{cases} \kappa_v \dfrac{K_e}{\tau/f_0} \dfrac{L}{l} \\[2ex] \sqrt{\kappa_v K_e \dfrac{L}{l}} \end{cases}$$

Ψ



Diffusivity

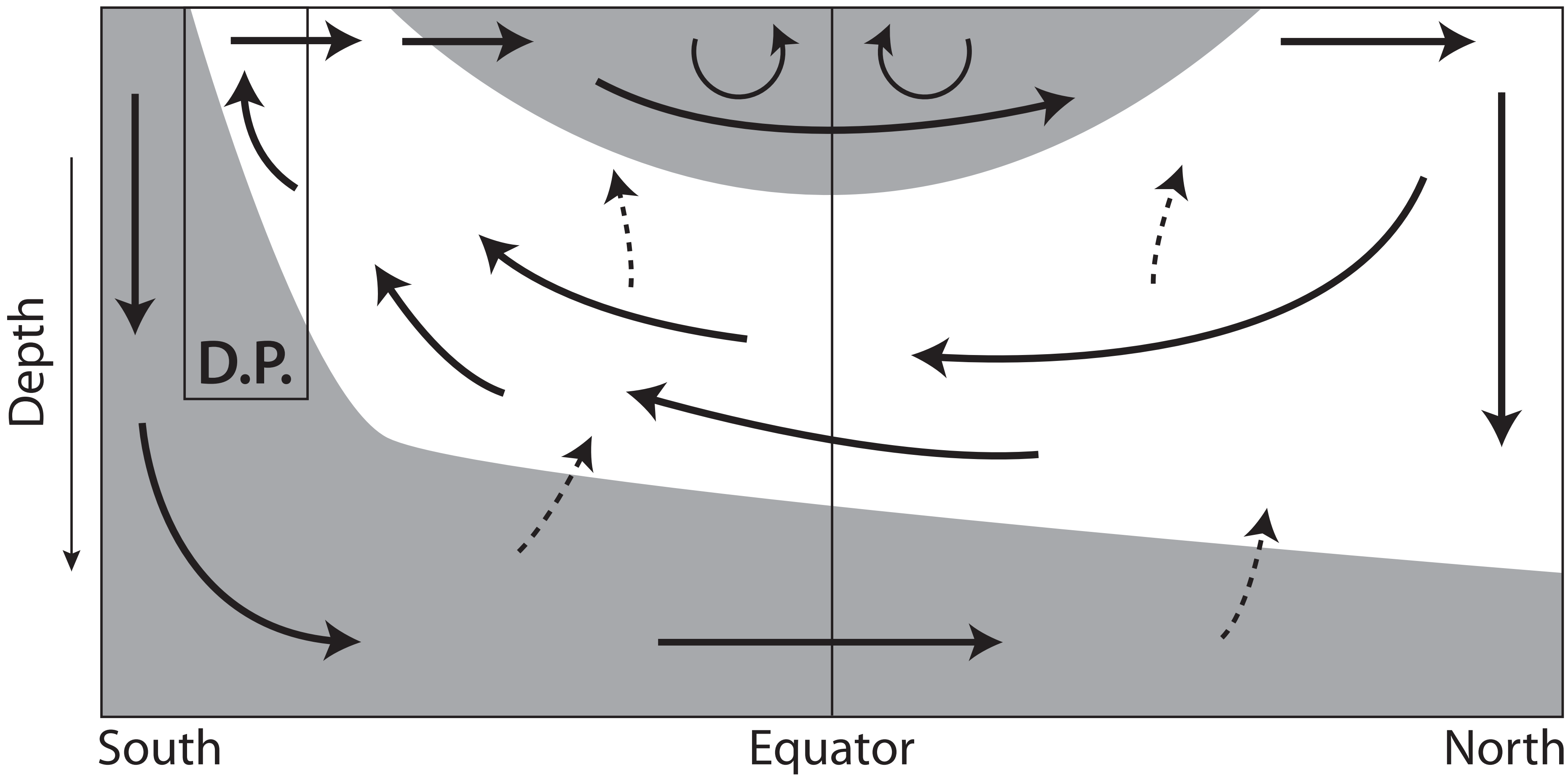
Wind stress

Conclusions

- Theory provides solutions for zonally-averaged stratification and overturning streamfunction. Interplay between mesoscale eddies, wind and diffusivity.
- In limit of weak mixing:
  - Stratification is set by wind and eddies in Southern Ocean.
  - Overturning is driven by diapycnal mixing in ocean basin.
- In limit of strong mixing
  - Diapycnal mixing drives overturning *and* sets the stratification.
- Interhemispheric circulation not included. But wait!

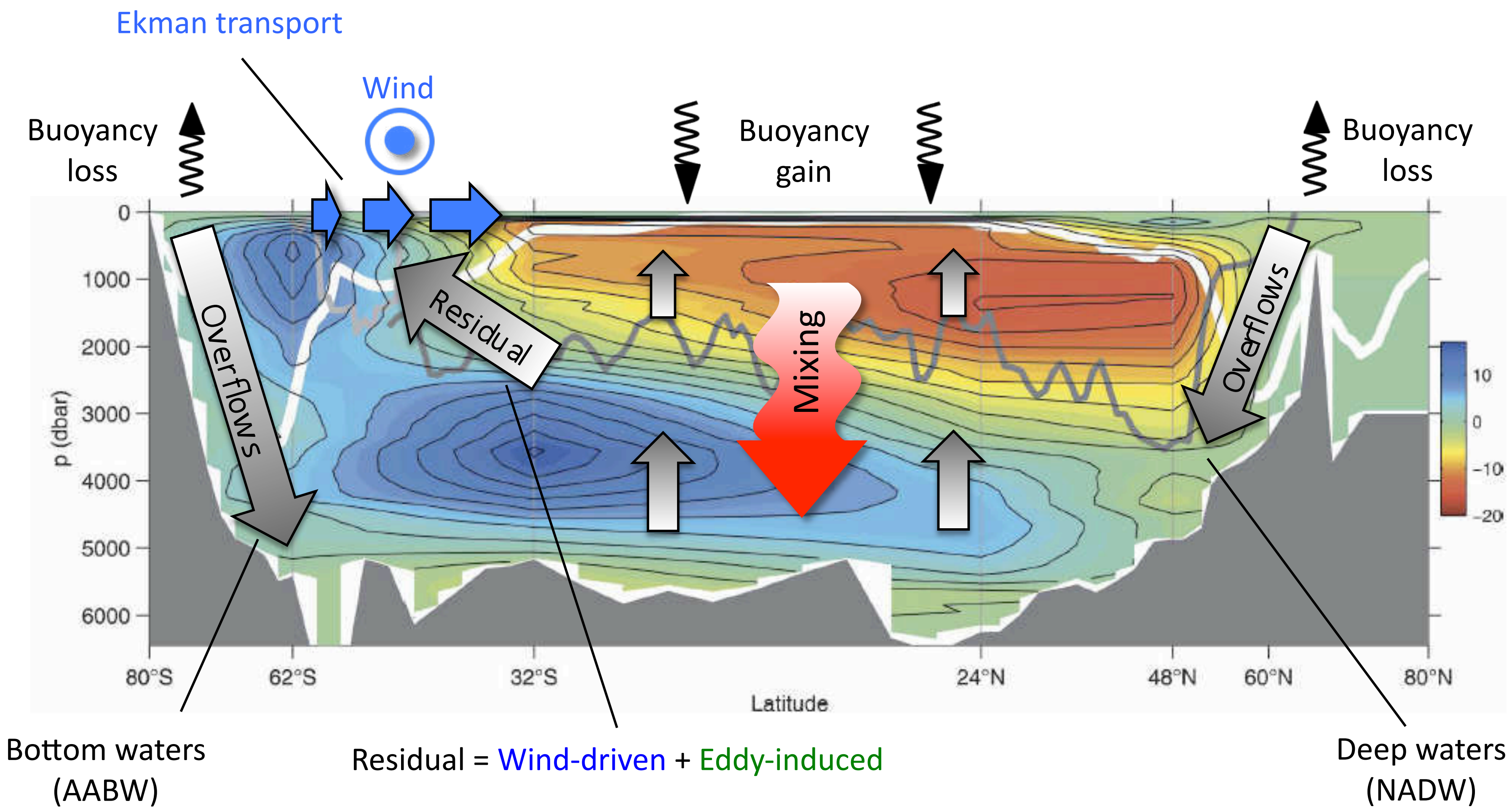
Interhemispheric Circulation

Simple plot



Interhemispheric Circulation

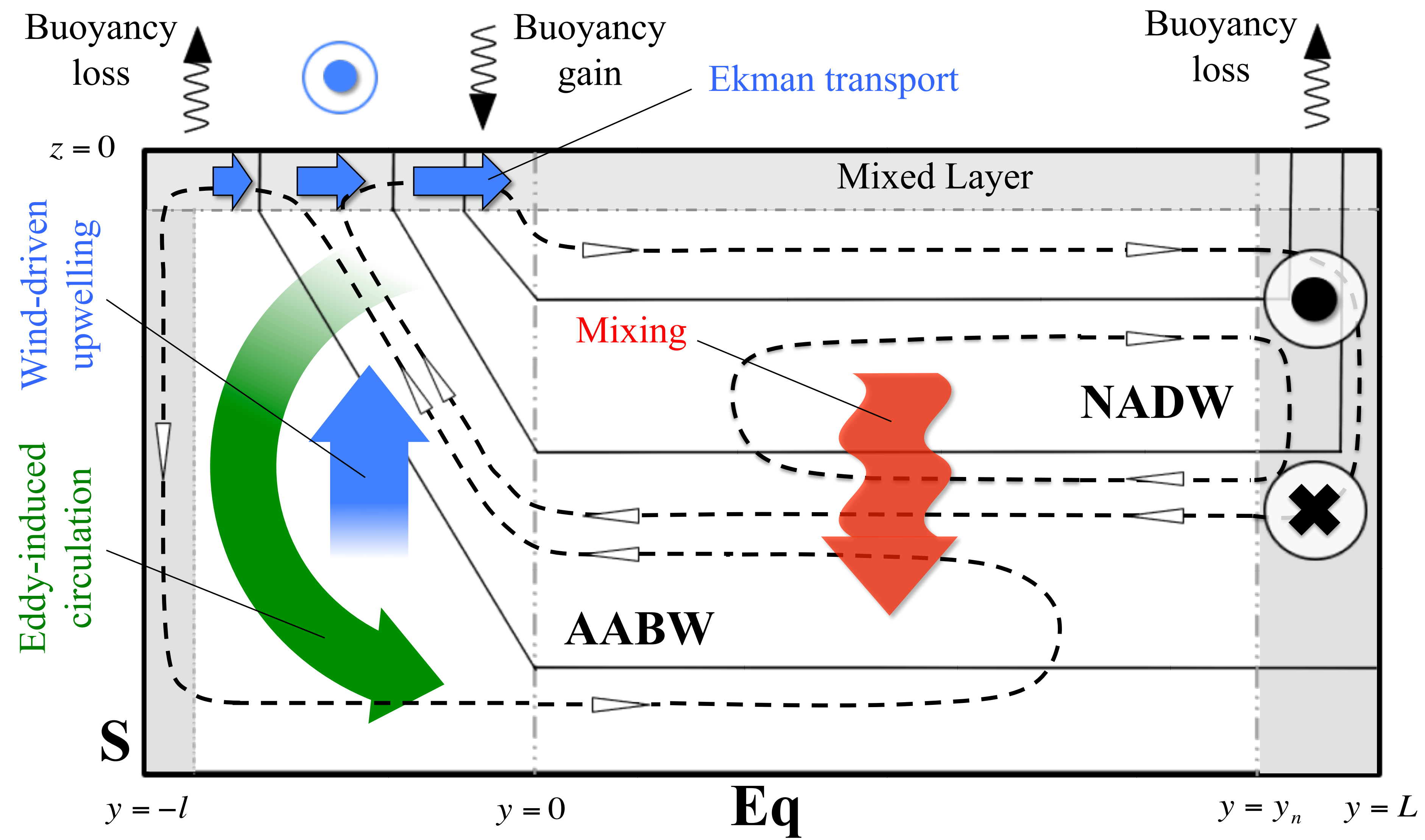
More complicated plot



To maintain a deep stratification and circulation we need:

- Gradients of buoyancy fluxes (to get 'water masses')
- Mixing and/or winds.

Schematic for Interhemispheric Flow



The Theory is an Algorithm

- SH Circumpolar Channel
  - (i) Buoyancy: Nearly adiabatic buoyancy advection.
  - (ii) Momentum: Advection by residual flow. Balance between wind driving and eddy (GM) effects
  - (iii) Boundary conditions on buoyancy:
    (i) Surface fluxes, (ii) matching to basin region.
- The Ocean Basin
  - (i) Isopycnals are flat
  - (ii) Advective-diffusive balance: $w\partial_z b = \partial_z(\kappa_v \partial_z b)$.
  - (iii) Boundary conditions on buoyancy (surface restoring).
- NH convective region
  - (i) Buoyancy distribution from the basin interior is matched convectively to the surface buoyancy profile.
  - (ii) Meridional buoyancy gradient drives eastward flow in the upper ocean and the return westward flow in the deep ocean (thermal wind).
  - (iii) Zonal flows are connected to the meridional circulation at the western boundary.

The Theory is Equations

1. SH Circumpolar Channel

Buoyancy advection: $$J(\psi_1, b_1) = \kappa_v \frac{\partial^2 b_1}{\partial z^2}$$

Momentum: $$\psi_1 = -\frac{\tau(y)}{f} - K_{GM} s$$

Surface boundary: $$-\kappa \frac{\partial b_1}{\partial z} \,|_{z=0} = \lambda(b^* - b_1)$$

Match to interior: $$b_1(z)|_{y=y_c} = b_2(z)$$

2. The Ocean Basin

Advective diffusive: $$\frac{(\psi_3 - \psi_1)}{L} \frac{\partial b_2}{\partial z} = \kappa_v \frac{\partial^2 b_1}{\partial z^2}$$

Surface boundary: $$-\kappa \frac{\partial b_2}{\partial z} \,|_{z=0} = \lambda(b^* - b_2)$$

3. NH convective region

Convective matching: $$b_2(z) \Rightarrow b_3(y, z)$$

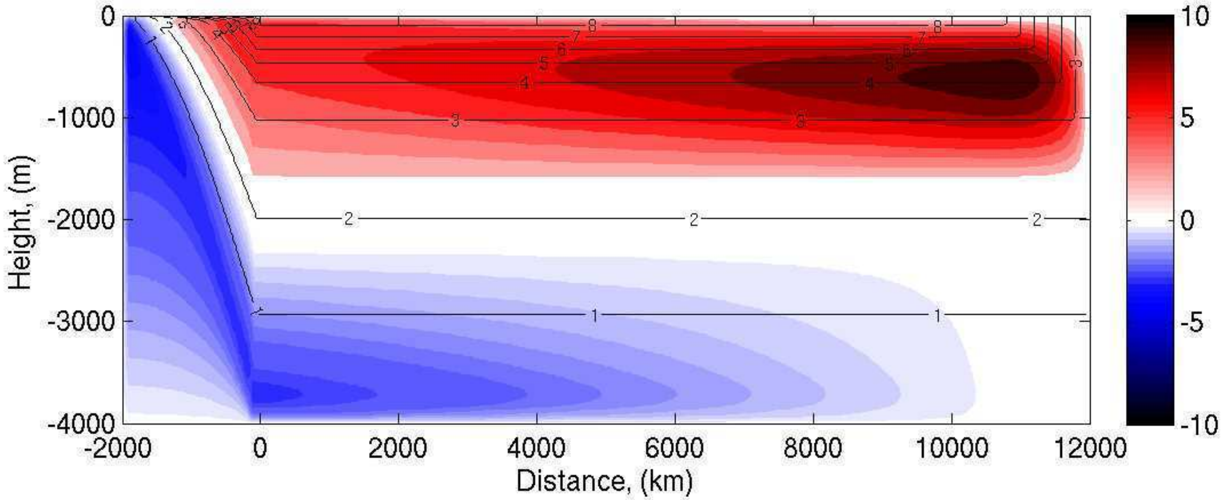
Thermal wind: $$f \frac{\partial u_3}{\partial z} = \frac{\partial b_3}{\partial y}$$

Mass Continuity: $$\psi_3(z) = \iint u_3 \, \mathrm{d}y \, \mathrm{d}z$$

Results

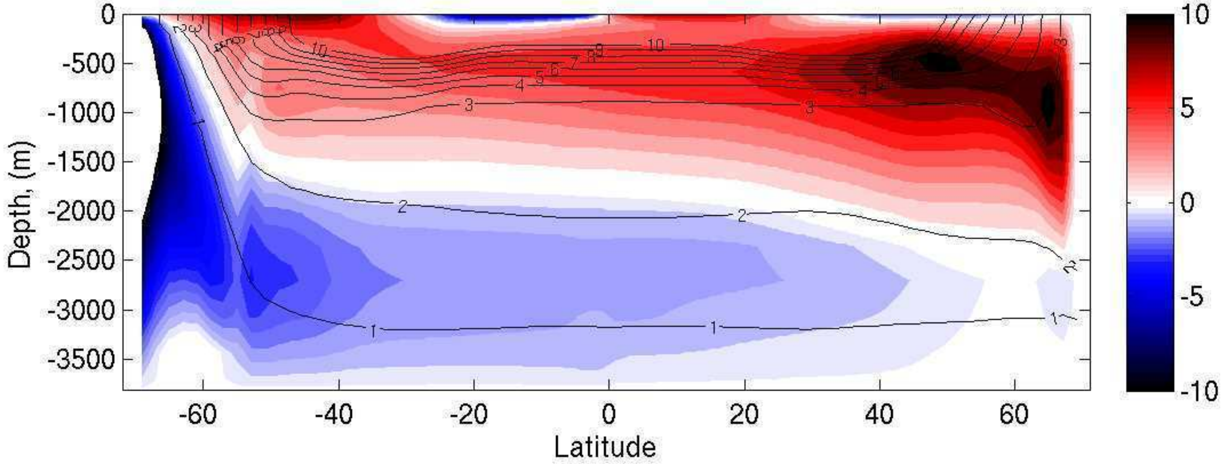
Theory:

Temperature (lines) and overturning circulation (Sv, colour).



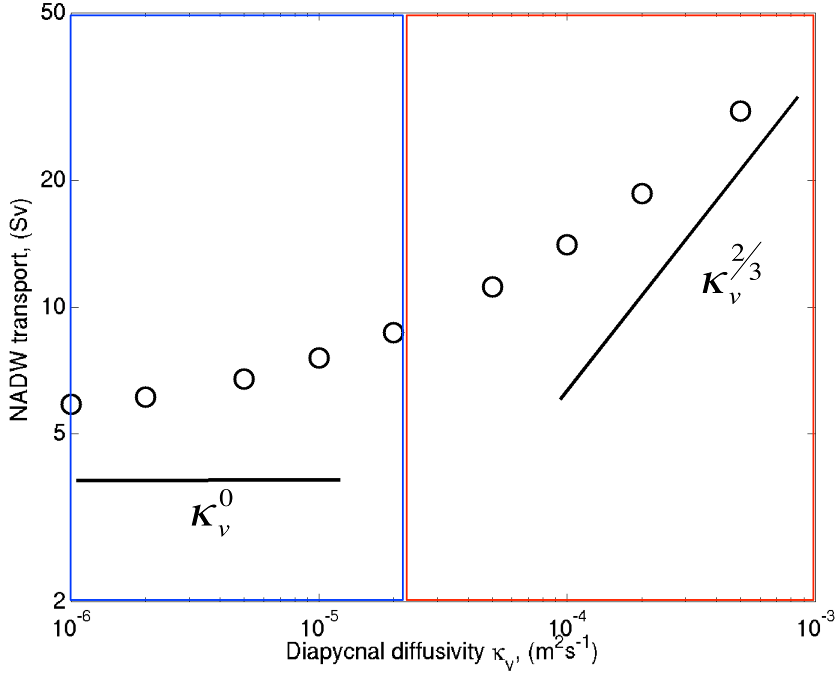
GCM simulation:

(MOM)

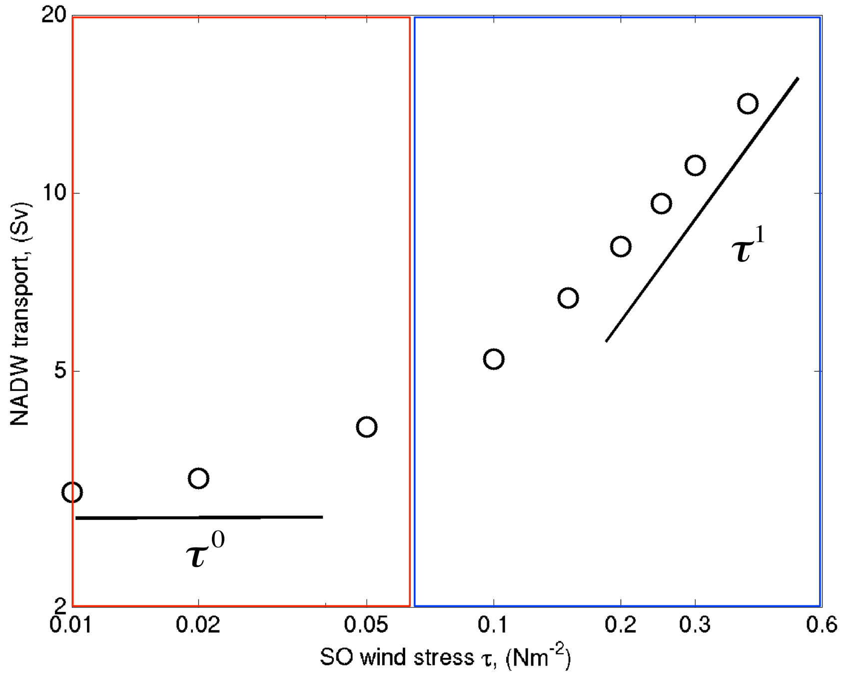


Scaling

NADW strength



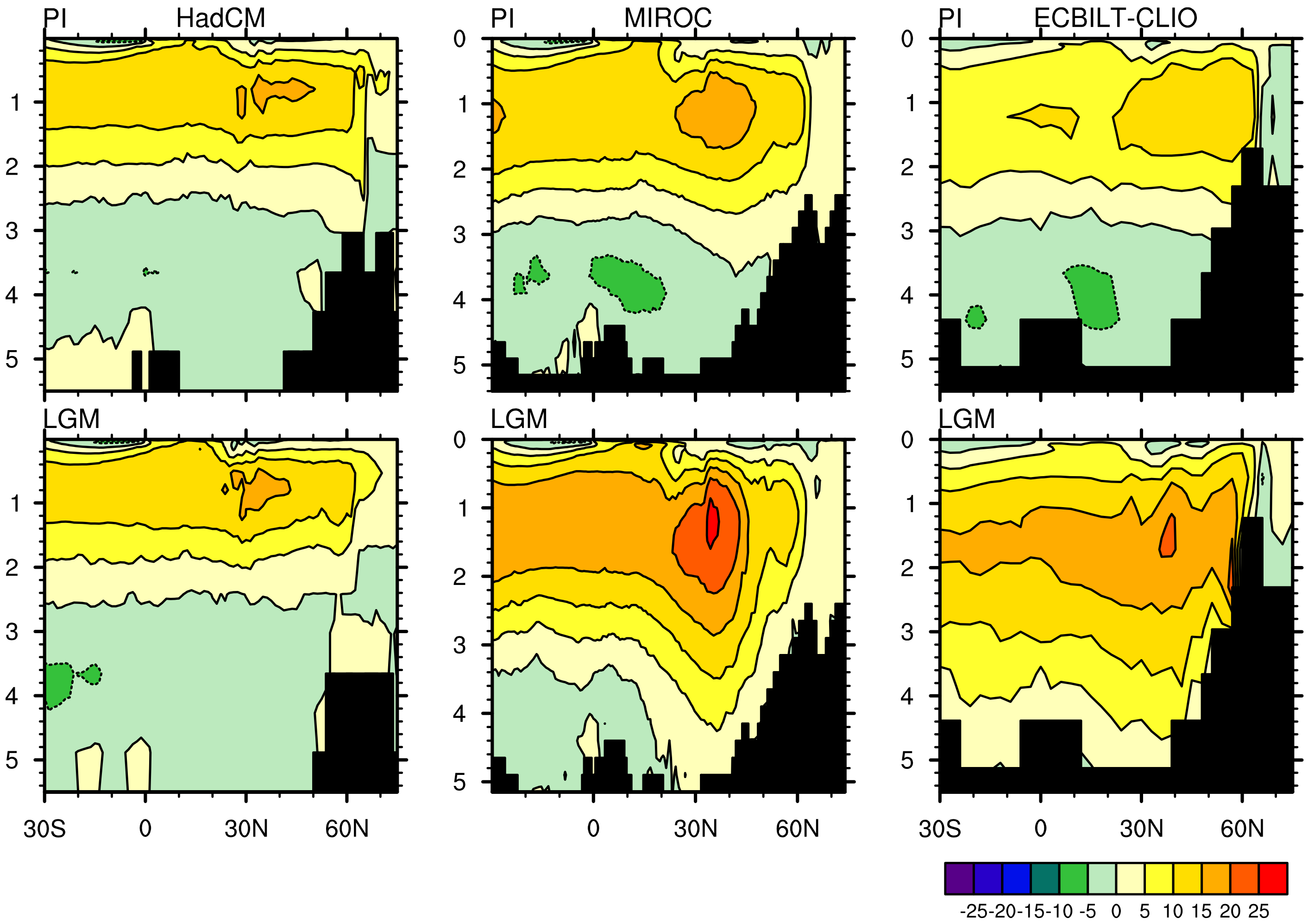
Diapycnal diffusivity



SO wind stress

Paleoclimate Implications

GCM results



Present Day

LGM

Otto-Bliesner et al 2007

- Relative sizes of top and bottom cells (NADW and AABW) was different in Last Glacial Maximum (LGM). (Curry and Oppo, 2005, others).
- GCM results are all over the place (Otto-Bliesner et al, 2007)
- Large implications for carbon cycle (Sigman et al 2010).

Conclusions

- Developed a theoretical model for deep stratification and overturning circulation. Differs in fundamental ways from Stommel Arons.
- Stratification depends on mesoscale eddies and wind in ACC.
- Overturning circulation depends on mixing and Northern Hemisphere temperature.
- Stay tuned for implications for carbon cycle and paleoclimate.