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2265-12

Advanced School on Understanding and Prediction of Earthquakes and other Extreme Events in Complex Systems

26 September - 8 October, 2011

**Boolean Delay Equations (BDEs)on Networks:
Applications to Economic and Seismic Damage Propagation**

Michael Ghil

Ecole Normale Supeerieure, Paris
France
&
University of California, Los Angeles
USA

Adv. School: Understand & Predict EQs & Other Extreme Events in Complex Systems

ICTP, Trieste, 28 Sept. 2010

Boolean Delay Equations (BDEs) on Networks:
Applications to Economic and Seismic Damage Propagation

Michael Ghil
Ecole Normale Supérieure, Paris, and
University of California, Los Angeles;
with B. Coluzzi and G. Weisbuch (ENS), S. Hallegatte (CIRED),
V. I Keilis-Borok (UCLA) and I. Zaliapin (UNR)

Please visit these sites for more info.

http://www.atmos.ucla.edu/tcd/
http://www.environnement.ens.fr/

Outline

- ***What for BDEs?***

  - life is sometimes too complex for ODEs and PDEs

- ***What are BDEs?***

  - formal models of complex feedback webs

  - classification and major results

- ***Applications to climate modeling***

  - ENSO – interannual variability in the Tropics

- ***"Partial" BDEs – spatio-temporal models***

  - theoretical results + connections to Boolean networks & CAs

- ***Seismic applications***

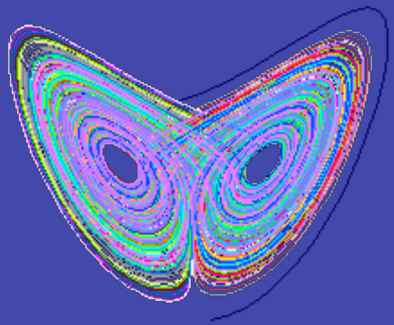
  - modeling, theory and predictions

- ***Socio-economic applications***

  - economic damage propagation on a network

- ***Concluding remarks***

  - bibliography & future work



Lorenz Lecture – AGU'05; SynCline – Bad Honnef, 27 May 2010

MPI für PKS, Dresden'06; Committee on Mathematical Geophysics (CMG'06)

Boolean Delay Equations: A Simple Way of Looking at Complex Systems

Michael Ghil

Ecole Normale Supérieure, Paris, &
University of California, Los Angeles

Work with *B. Coluzzi* (ENS, Paris), *D. P. Dee* (ECMWF, Reading), *S. Hallegatte* (CIRED, Paris), *V. Keilis-Borok* (IGPP, UCLA, & MITPAN, Moscow), *A. P. Mullhaupt* (Wall Street & SUNY; Erdös # = 2), *P. Pestiaux* (UCL & Total, France), *A. Saunders* (UCLA & LAUSD), *G. Weisbuch* (ENS) & *I. Zaliapin* (U. of Nevada, Reno); see http://www.environnement.ens.fr/ & http://www.atmos.ucla.edu/tcd/

Motivation

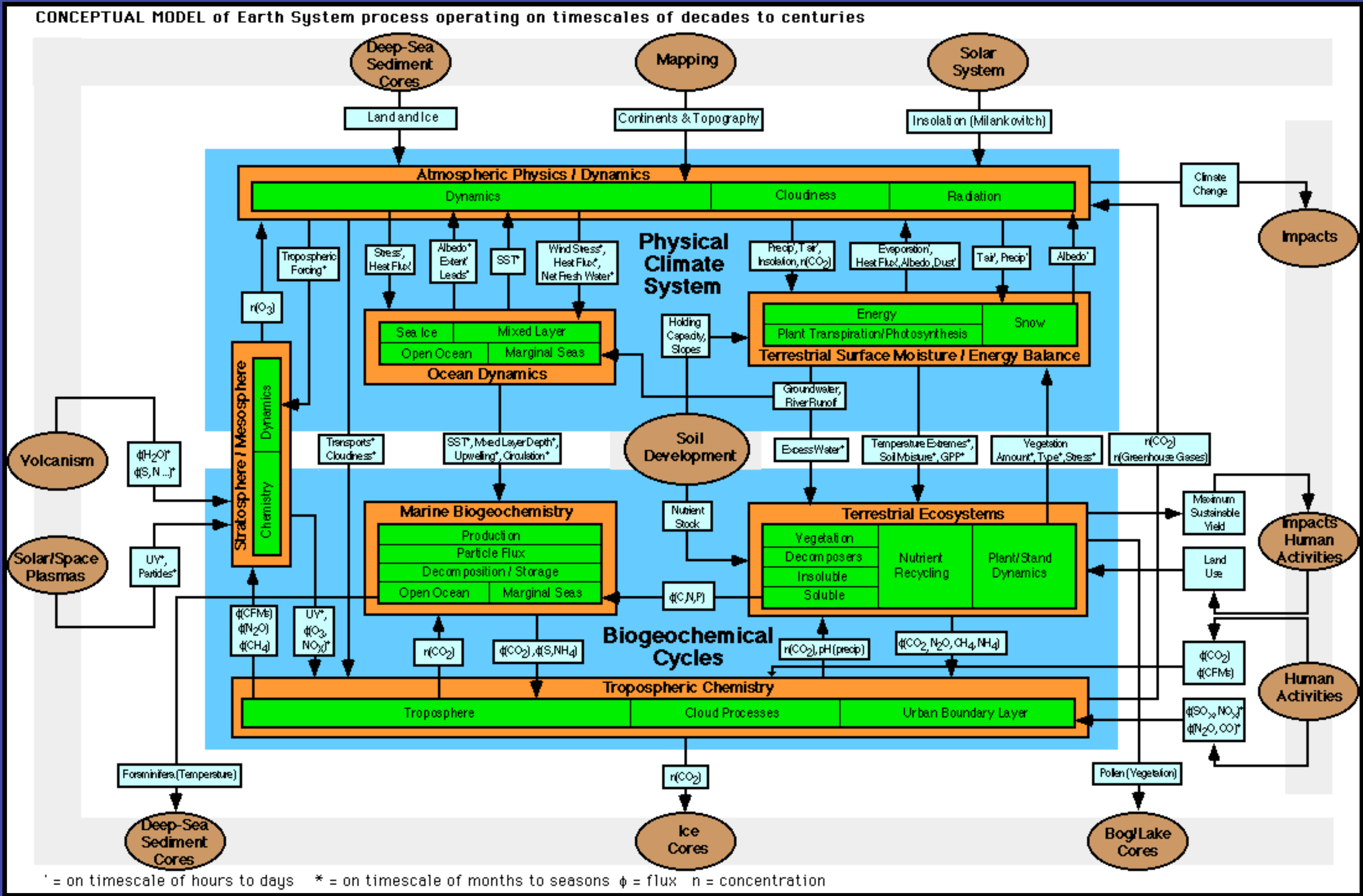
1. ***Components***
    - solid earth (crust, mantle)
    - fluid envelopes (atmosphere, ocean, snow & ice)
    - living beings on and in them (fauna, flora, people)

2. ***Complex feedbacks***
    - positive and negative
    - nonlinear - small pushes, big effects?

3. ***Approaches***
    - reductionist
    - holistic

4. ***What to do?*** - Let's see!

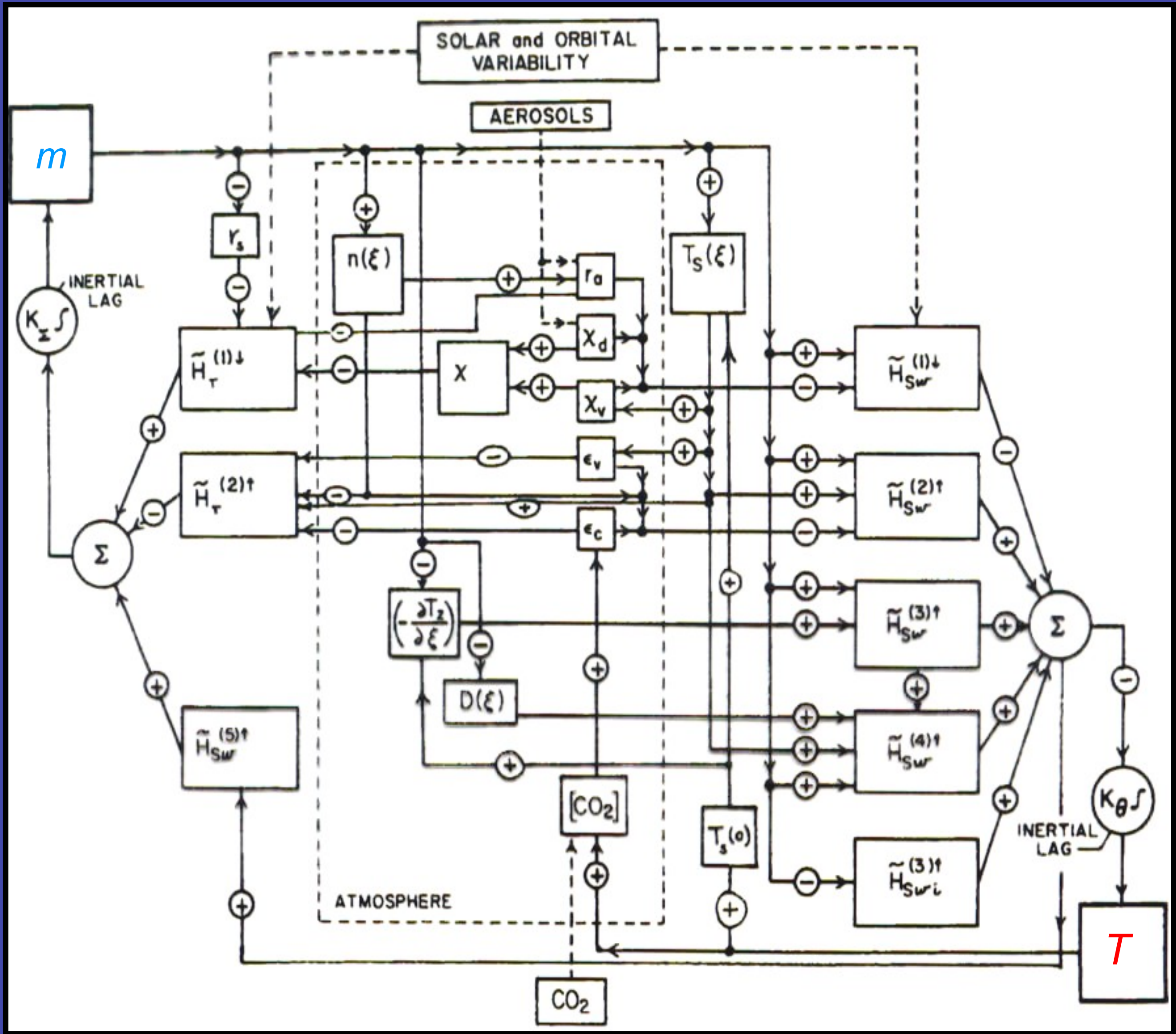
F. Bretherton's "horrendogram" of Earth System Science



Earth System Science Overview, *NASA Advisory Council*, 1986

The climate system on long time scales

“Ambitious” diagram



Flow diagram showing feedback loops contained in the dynamical system for ice-mass *m* and ocean temperature variations *T*.

Constants for ODE & PDE models are poorly known. Mechanisms and effective delays are easier to ascertain.

B. Saltzman, Climatic system analysis, *Adv. Geophys.*, **25**, 1983

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  - *- theoretical results + connections to Boolean networks & CAs*
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Introduction

Binary systems Examples: Yes/No, True/False (ancient Greeks)

Classical logic (*Tertium not datur*)

Boolean algebra (19^{th} cent.)

Propositional calculus (20^{th} cent.)

(syllogisms as trivial examples)

Genes*: on/off*

Descriptive – Jacob and Monod (1961)

Mathematical *genetics – L. Glass, S. Kauffman, M. Sugita (1960s)*

Symbolic dynamics of differentiable dynamical systems (DDS): S. Smale (1967)

Switches*:* on/off, 1/0

Modern *computation* (EE & CS)

- cellular automata (CAs) J. von Neumann (1940s, 1966), S. Ulam, Conway (the game of life), S. Wolfram (1970s, '80s)
- spatial increase of complexity – infinite number of channels
- conservative logic Fredkin & Toffoli (1982)
- kinetic logic: importance of distinct delays to achieve temporal increase in complexity (synchronization, operating systems & parallel computation), R. Thomas (1973, 1979,…)

Introduction (cont'd)

M.G.'s immediate motivation:

Climate dynamics – complex interactions
(reduce to binary), C. Nicolis (1982)

Joint work on developing and applying BDEs to climate dynamics
with D. Dee, A. Mullhaupt & P. Pestiaux (1980s)
& with A. Saunders (late 1990s)
Work of L. Mysak and associates (early 1990s)

Recent applications to solid-earth geophysics
(*earthquake modeling and prediction*)
with V. Keilis-Borok and I. Zaliapin

Recent applications to the biosciences
(*genetics and micro-arrays*)
Oktem, Pearson & Egiazarian (2003) *Chaos*
Gagneur & Casari (2005) *FEBS Letters*

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What are BDEs?

Short answer:

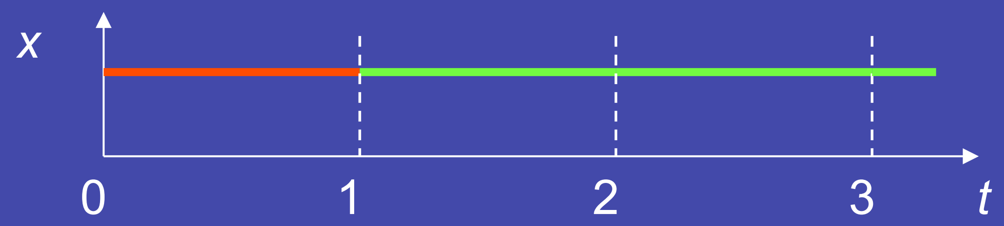
Maximum simplification of nonlinear dynamics
(non-differentiable time-continuous dynamical system)

Longer answer:

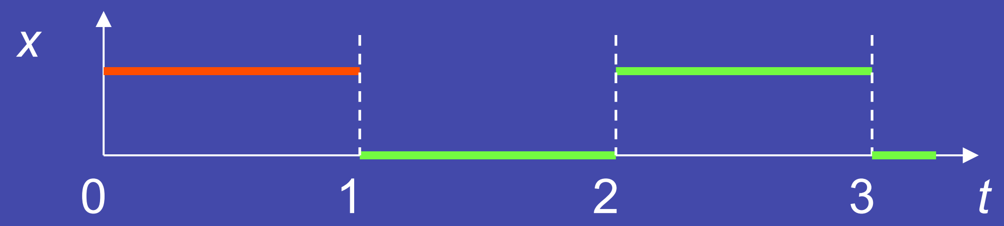
1) $x \in B = \{0,1\}$

$x(t) = x(t-1)$

(simplest EBM: $x = T$)



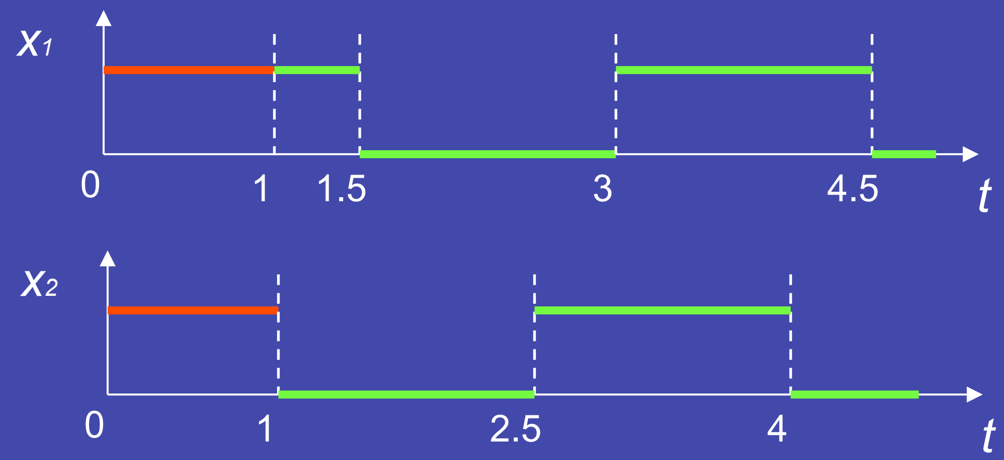
2) $x(t) = \overline{x}(t-1)$



3) $x_1, x_2 \in B = \{0,1\}; 0 < \theta \leq 1$

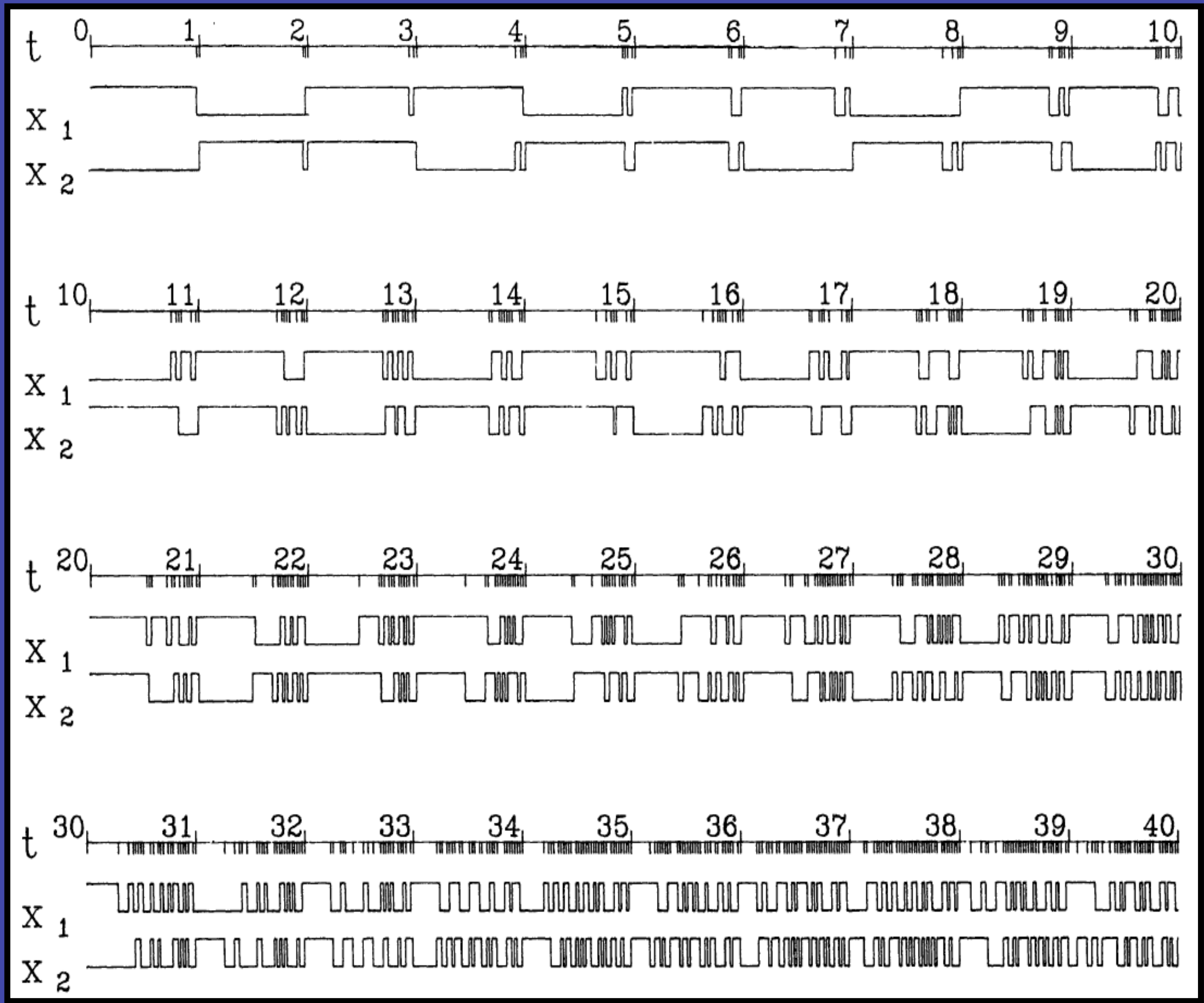
$$\begin{cases} x_1(t) = x_2(t-\theta), \theta = 1/2 \\ x_2(t) = \overline{x}_1(t-1) \end{cases}$$

Eventually periodic with
a period $= 2(1+\theta)$
(simplest OCM: $x_1 = m$, $x_2 = T$)



$$\begin{cases} x_1(t) = x_2(t-\theta) \\ x_2(t) = x_2(t-1)\nabla x_1(t-\theta) \end{cases}$$

θ is irrational



Increase in complexity!

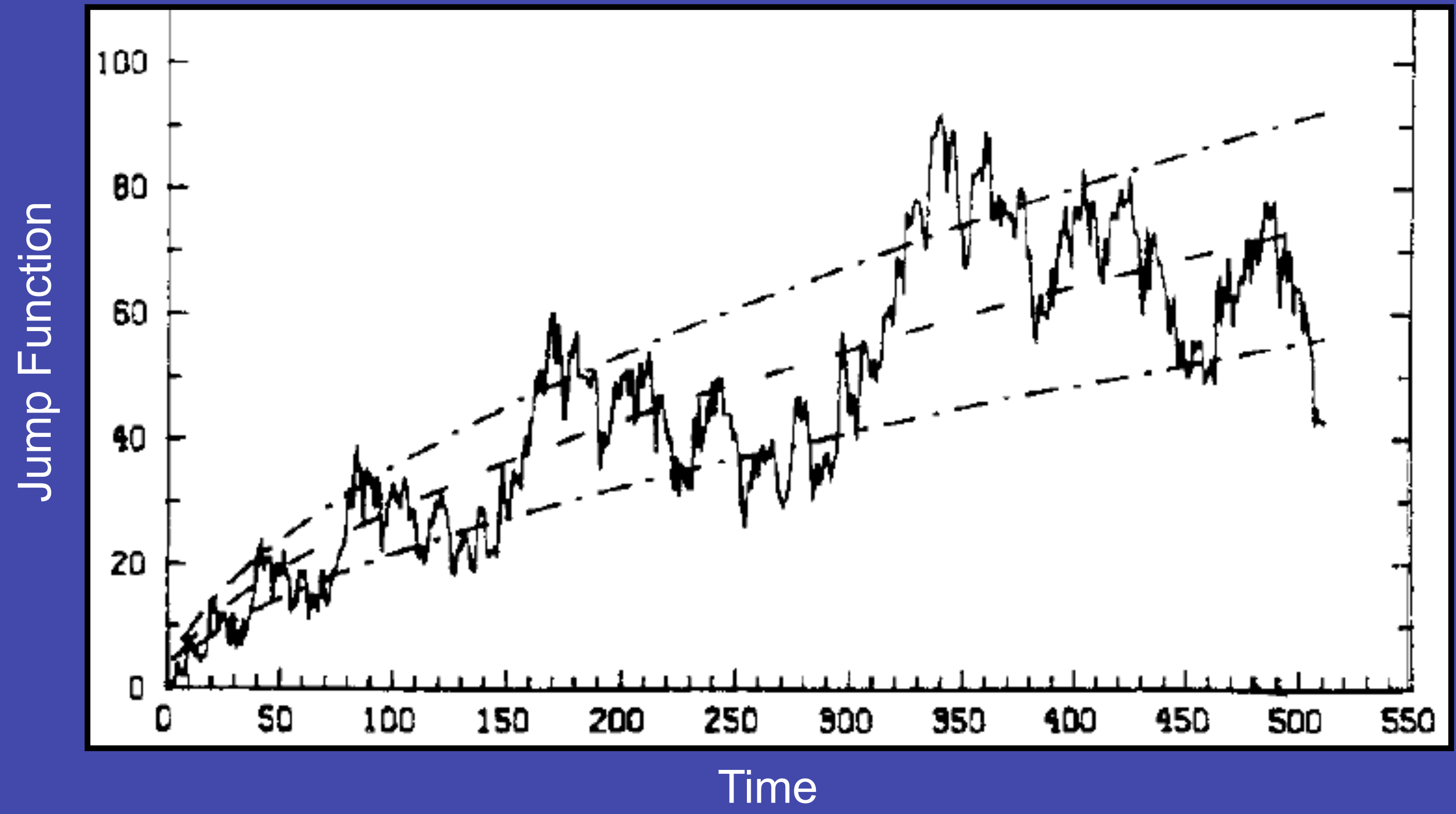
Evolution: biological, cosmogonic, historical

But how much?

Dee & Ghil, *SIAM J. Appl. Math*. (1984), **44**, 111-126

Aperiodic solutions with *increasing complexity*

$$x(t) = x(t-1)\nabla x(t-\theta), \qquad \theta = \frac{\sqrt{5}-1}{2} = \text{"golden ratio"}$$

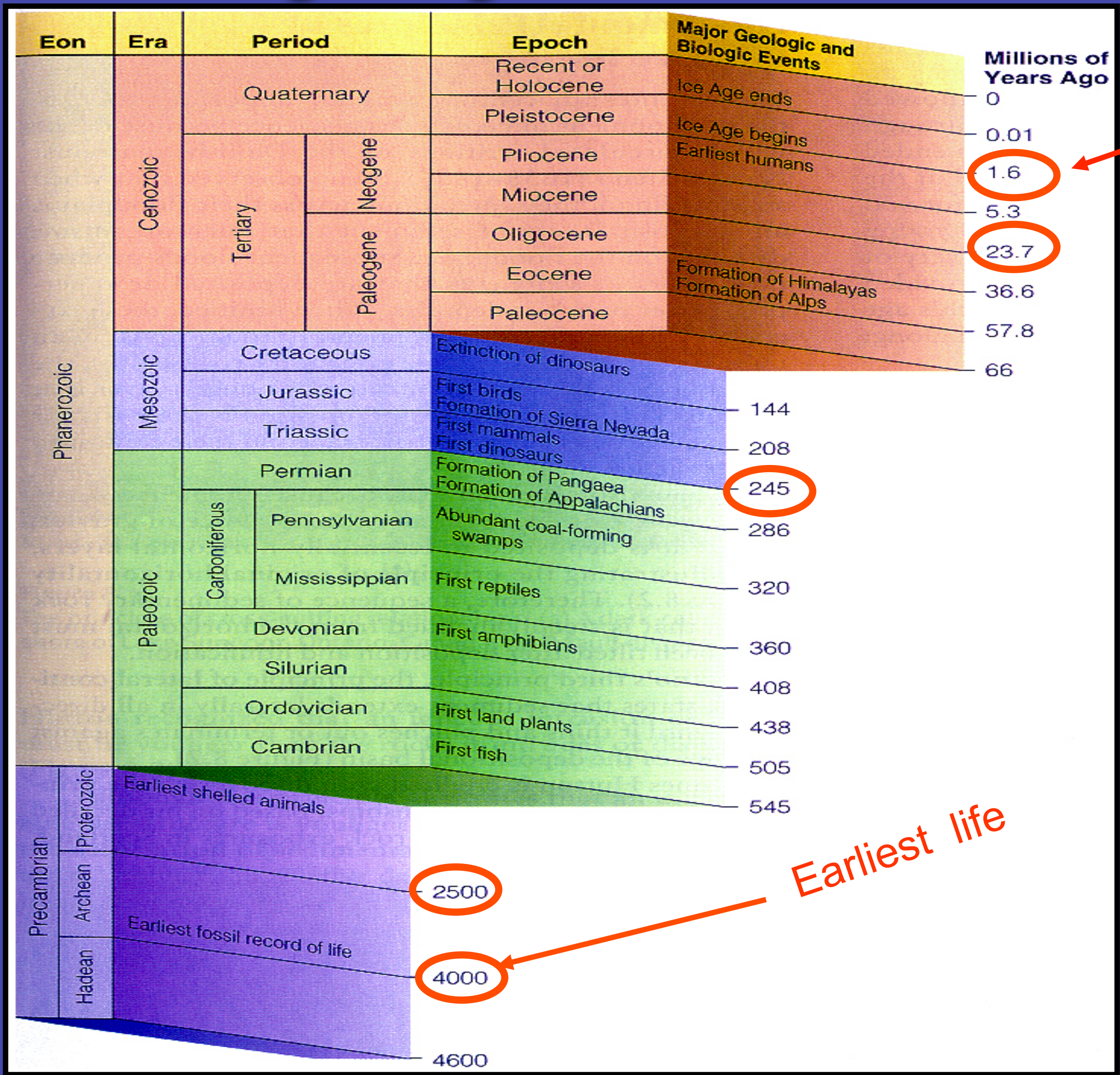


Theorem:

Conservative BDEs with irrational delays have aperiodic solutions with a *power-law increase in complexity.*

N.B. Log-periodic behavior!

The geological time scale

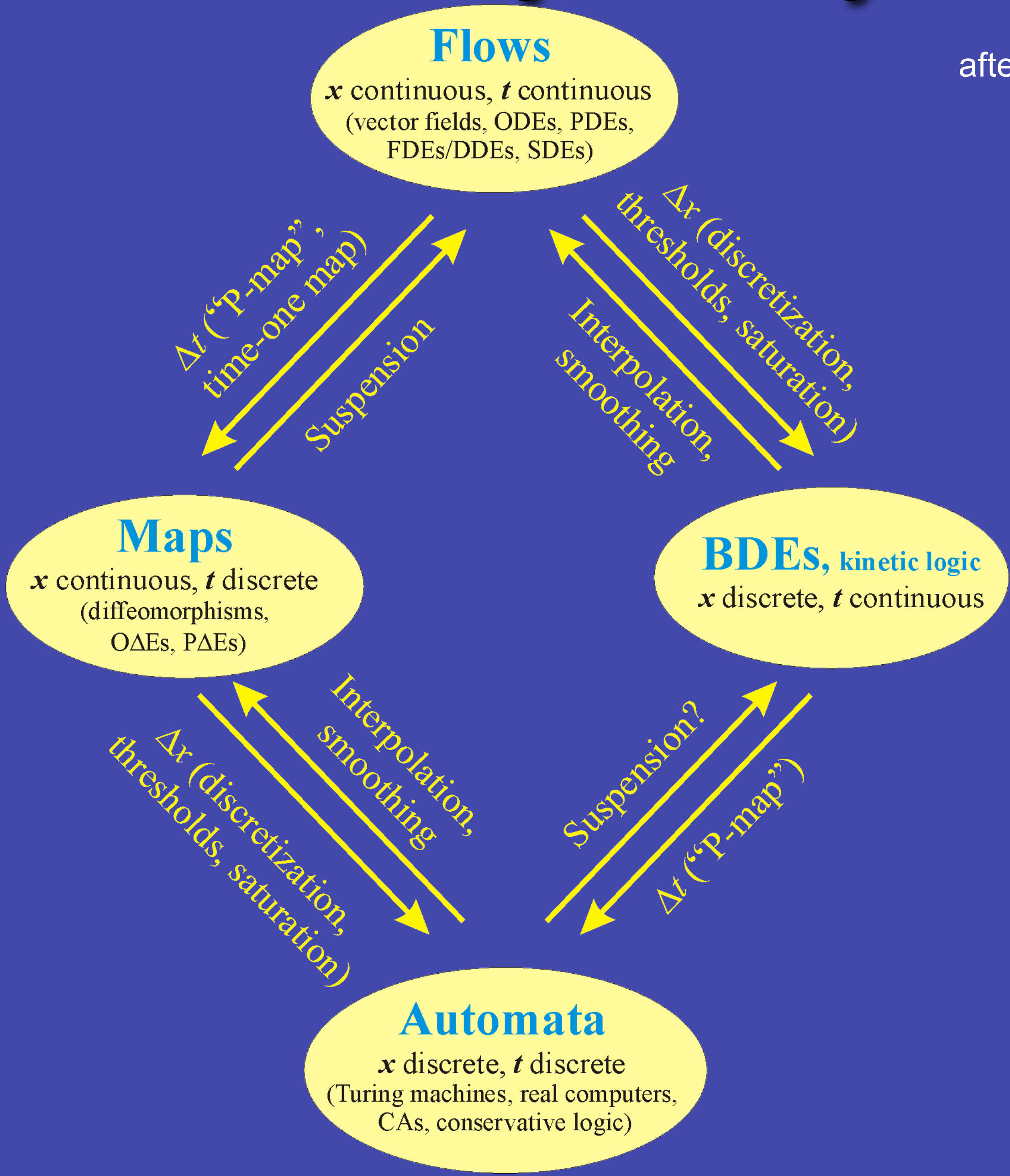


Density of events $\cong \log(t)$

http://www.yorku.ca/esse/veo/earth/image/1-2-2.JPG

The place of BDEs in dynamical system theory

after *A. Mullhaupt* (1984)



Classification of BDEs

Definition: A BDE is *conservative* if its solutions are immediately periodic, i.e. no transients; otherwise it is *dissipative*.

Remark: Rational vs. irrational delays.

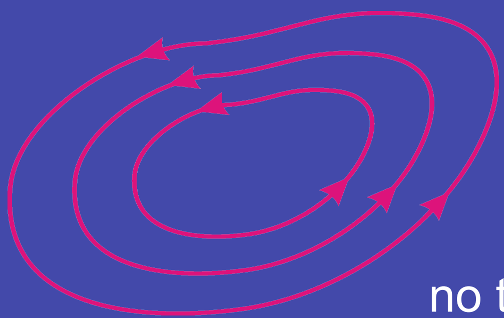
Example:

1) Conservative $$x(t) = \overline{x}(t-1)$$

2) Dissipative $$x(t) = x(t-1) \wedge x(t-\theta)$$

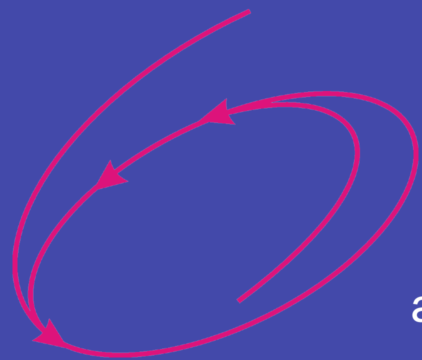
Analogy with ODEs

Conservative – Hamiltonian



no transients

Dissipative – limit cycle



attractor

M. Ghil & A. Mullhaupt, *J. Stat. Phys*., **41**, 125-173, 1985

Examples. Convenient shorthand for scalar 2nd order BDEs

$$x = y \circ z \Leftrightarrow x(t) = x(t-1) \circ x(t-\theta)$$

1. Conservative

$$x = y \nabla z = y \oplus z = y + z \pmod{2}$$

$$x = y \Delta z = 1 \oplus y \oplus z$$

Remarks: i) Conservative $\Leftarrow$ linear (mod 2)
ii) $\exists$ few conservative connectives (~ ODEs)

2. Dissipative

$$x = y \wedge z \tilde{\Rightarrow} x \to 0$$

$$x = y \vee z \tilde{\Rightarrow} x \to 1$$

Theorem

Conservative $\Leftarrow$ reversible
$\Leftarrow$ invertible

A. Mullhaupt, Ph.D. Thesis, May 1984, CIMS/NYU
M. Ghil & A. Mullhaupt, *J. Stat. Phys.*, **41**, 125-173, 1985

Classification of BDEs

Structural stability & bifurcations

Theorem

BDEs with periodic solutions only are structurally stable,and conversely

Remark. They are dissipative.

Meta-theorems, by example.

The asymptotic behavior of

$$x(t) = x(t-\theta) \wedge \overline{x}(t-\tau)$$

is given by

$$x(t) = x(t-\theta)$$

Hence, if $\tau < \theta = 1$ then solutions are asymptotically periodic;

if however $\theta < \tau = 1$ then solutions tend asymptotically to 0.

Therefore, as θ passes through τ, one has Hopf bifurcation.

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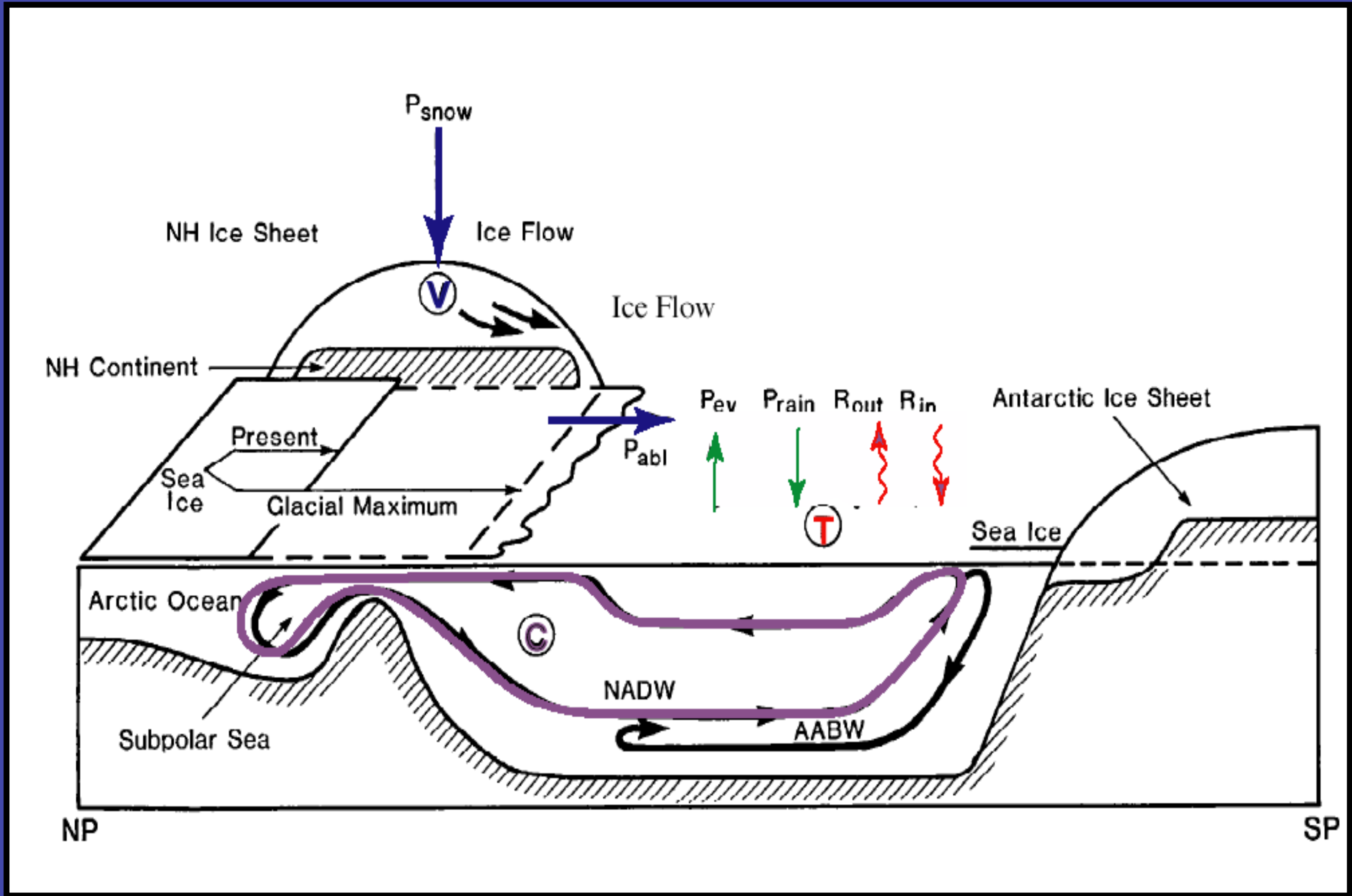
Paleoclimate application

Thermohaline circulation and glaciations



Logical variables

T - global surface temperature;

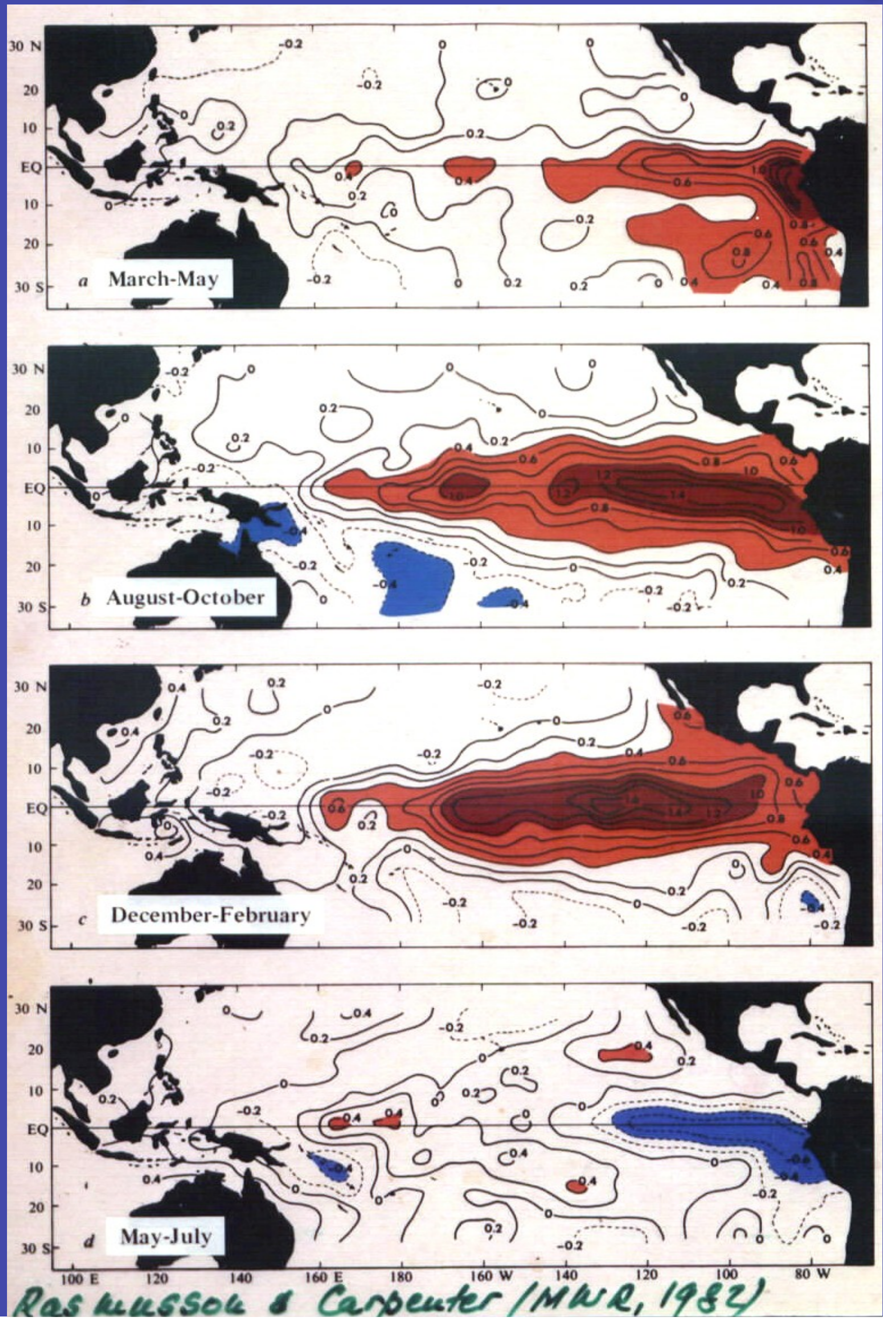
V_N - NH ice volume, V_N = V;

V_S - SH ice volume, V_S = 1;

C - deep-water circulation index

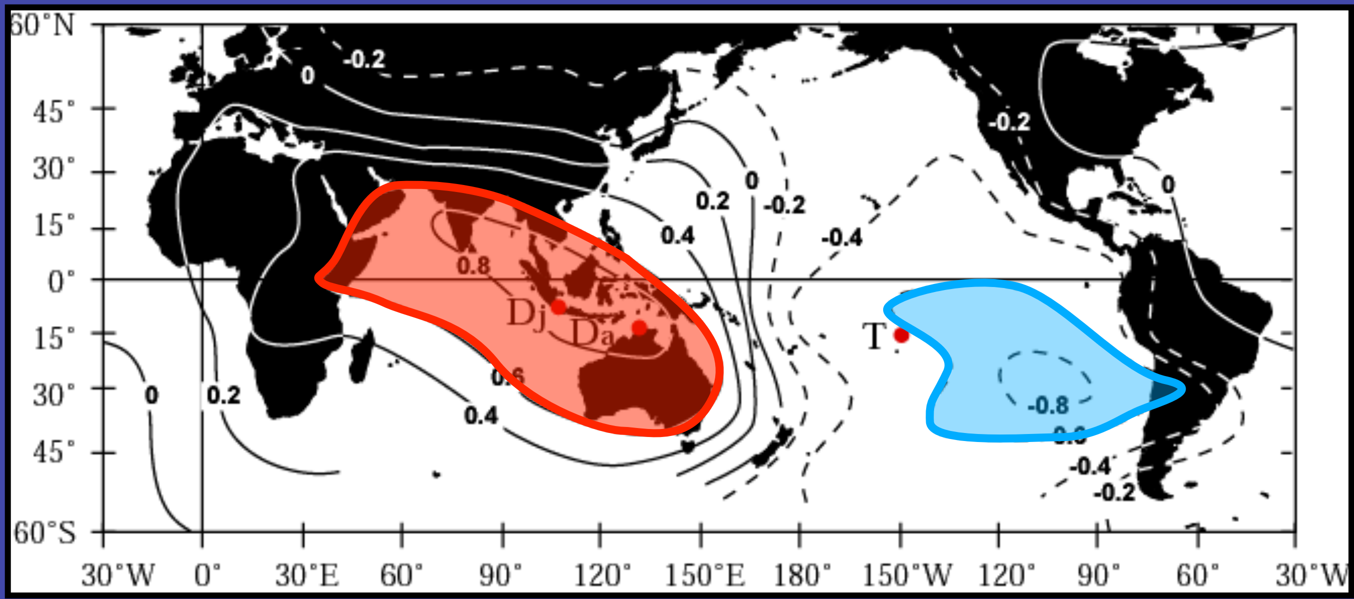
M. Ghil, A. Mullhaupt, & P. Pestiaux, *Climate Dyn.*, **2**, 1-10, 1987.

Spatio-temporal evolution of ENSO episode



Scalar time series that capture ENSO variability

The large-scale Southern Oscillation (SO) pattern associated with El Niño (EN), as originally seen in surface pressures



Neelin (2006) *Climate Modeling and Climate Change*, after Berlage (1957)

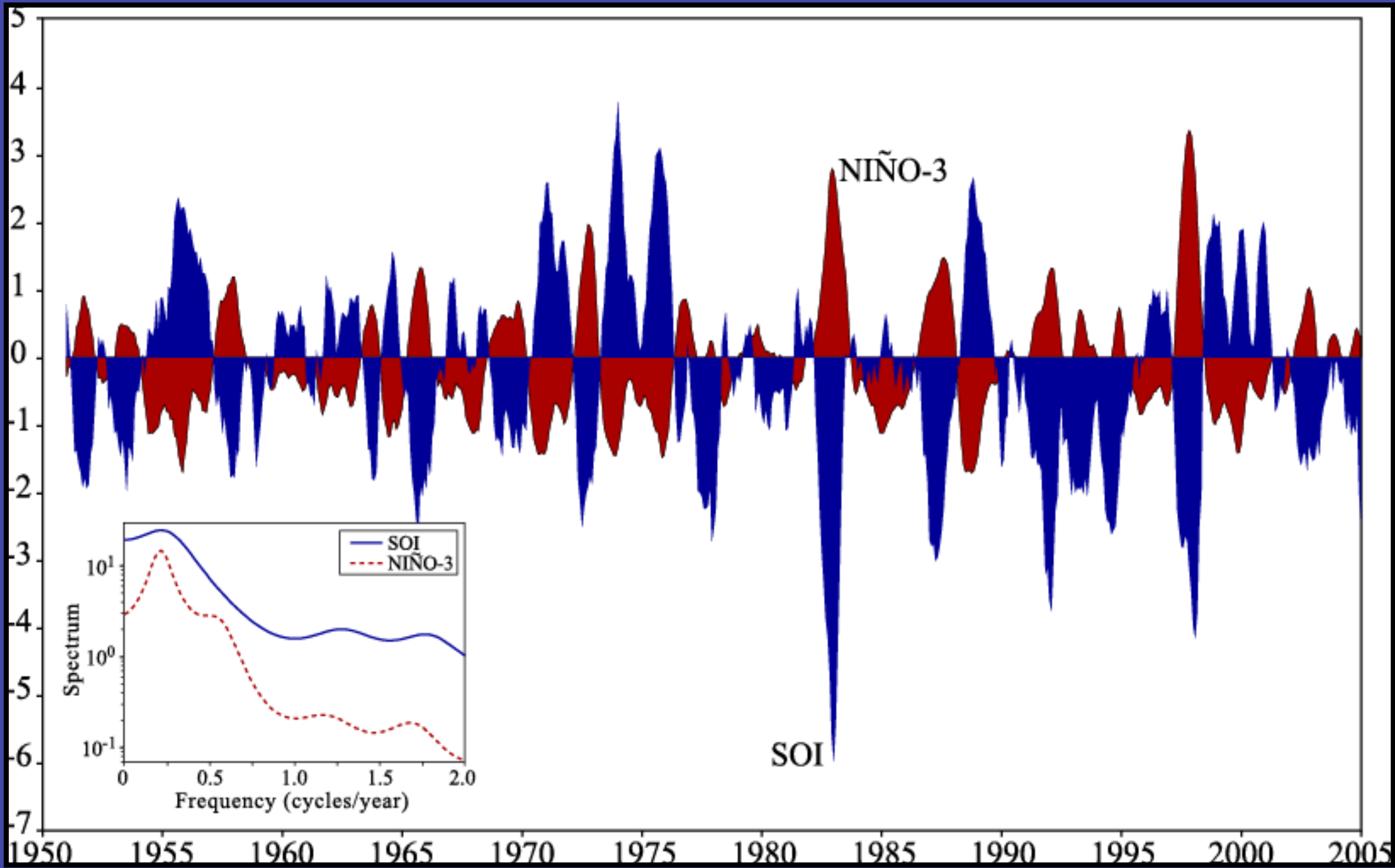
Southern Oscillation:

The seesaw of sea-level pressures p_s between the two branches of the Walker circulation

Southern Oscillation Index (SOI) = normalized difference between p_s at Tahiti (T) and p_s at Darwin (Da)

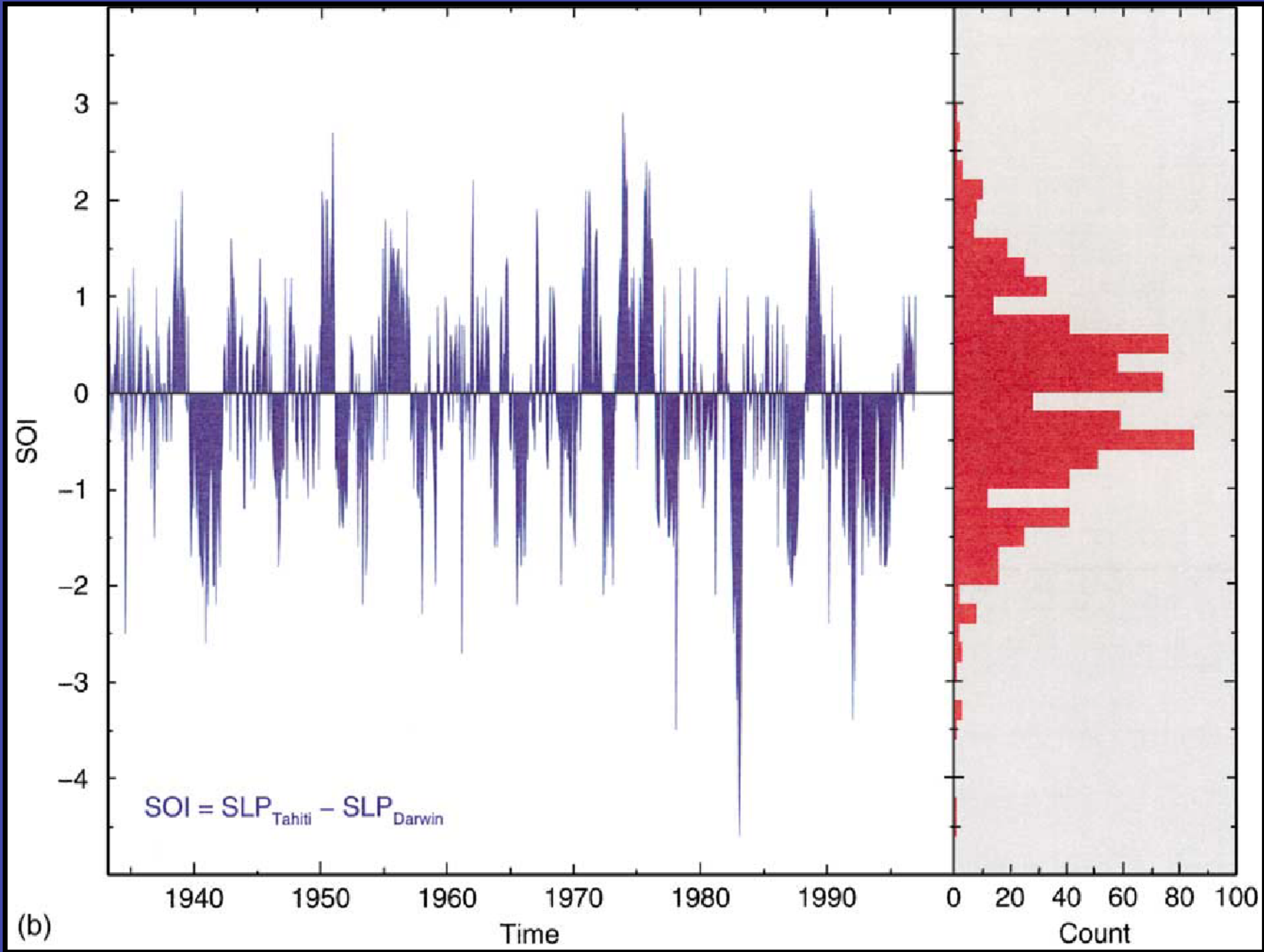
Scalar time series that capture ENSO variability

Time series of ***atmospheric pressure***
and ***sea surface temperature*** (SST) indices



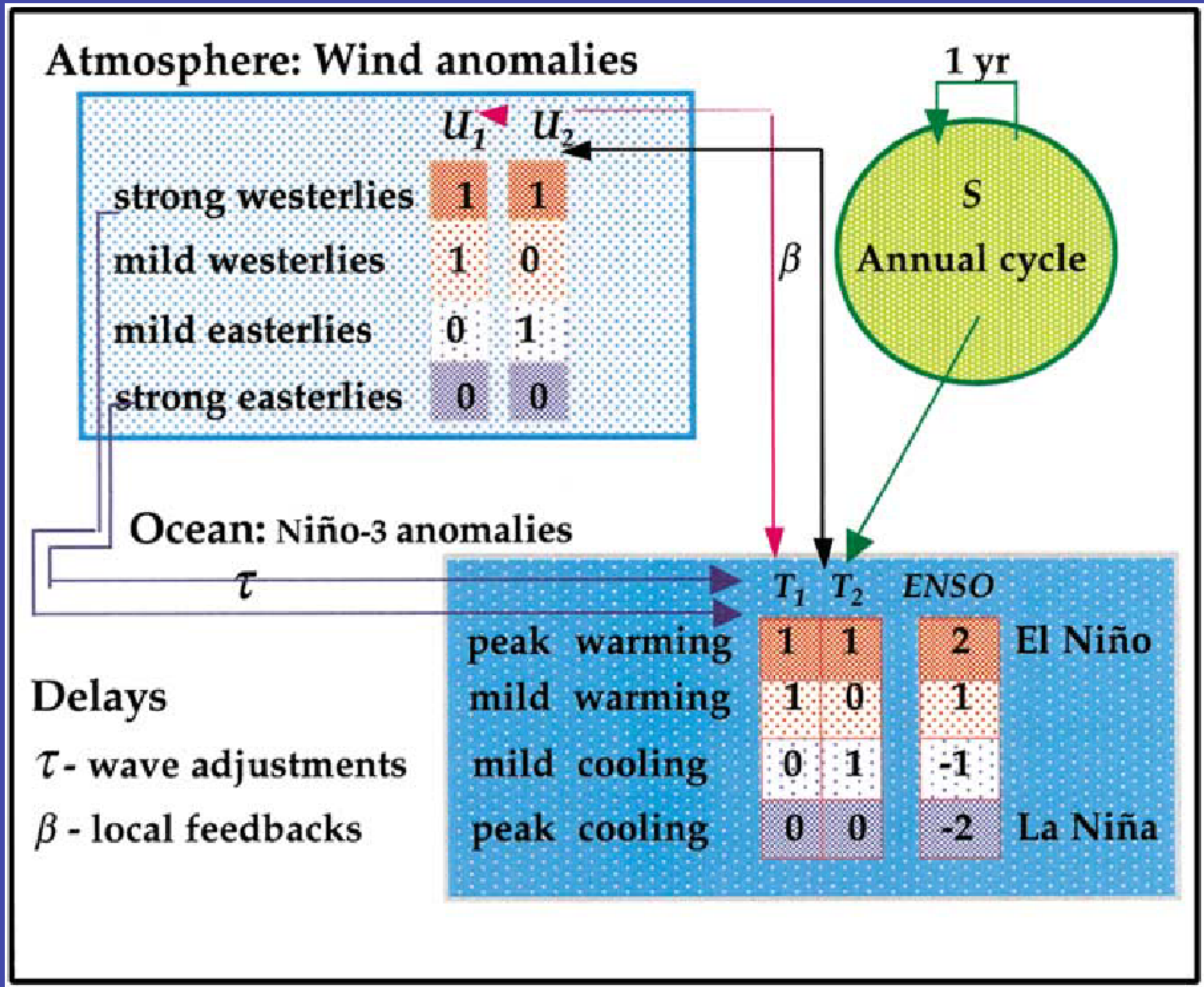
Data courtesy of NCEP's Climate Prediction Center
Neelin (2006) *Climate Modeling and Climate Change*

Histogram of size distribution for ENSO events



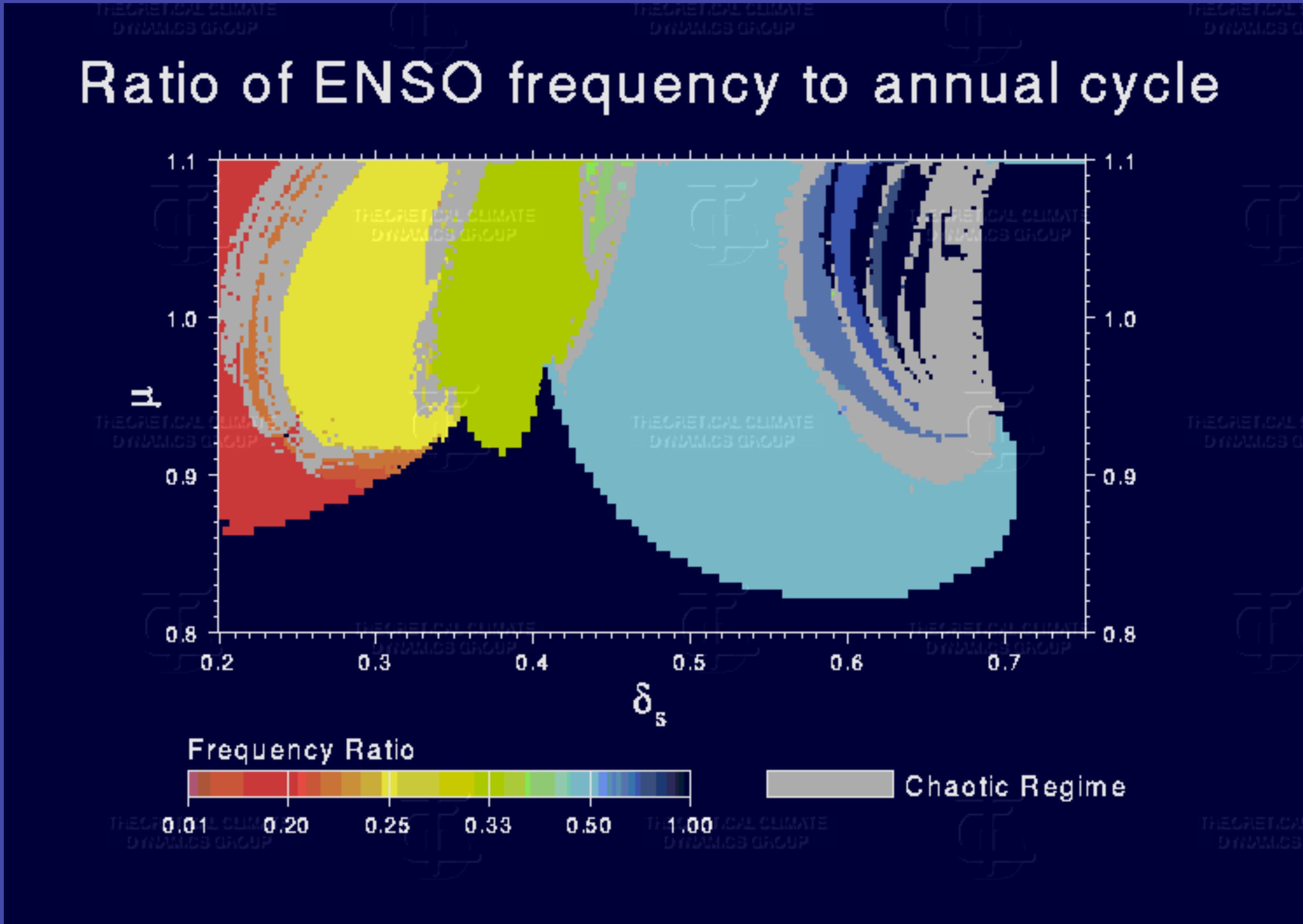
A. Saunders & M. Ghil, *Physica D,* **160**, 54–78, 2001
(courtesy of Pascal Yiou)

BDE Model for ENSO: Formulation



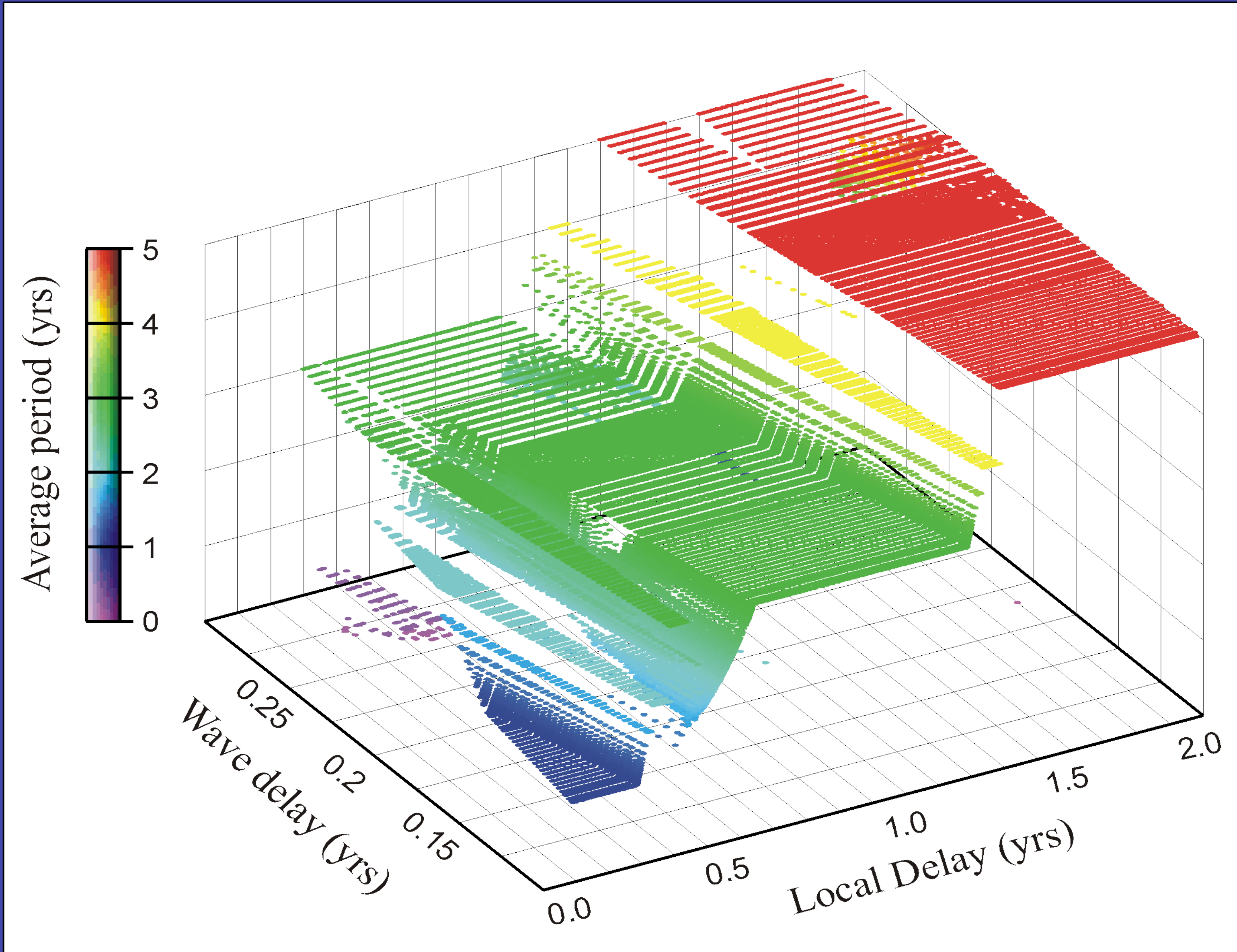
A. Saunders & M. Ghil, *Physica D*, **160**, 54–78, 2001

Devil's Bleachers in a 1-D ENSO Model



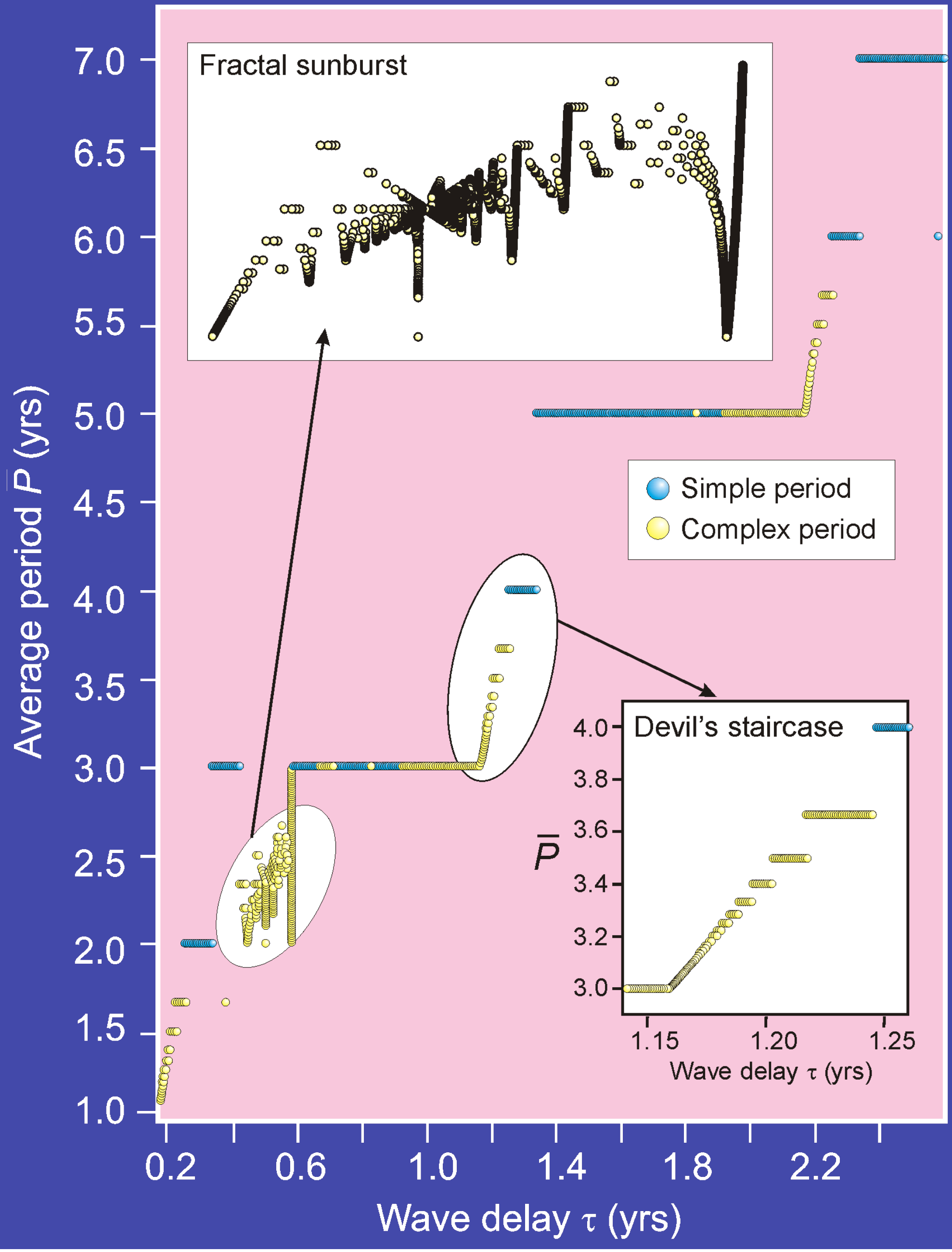
F.-F. Jin, J.D. Neelin & M. Ghil, *Physica D*, **98**, 442-465, 1996

Devil's Bleachers in the BDE Model of ENSO

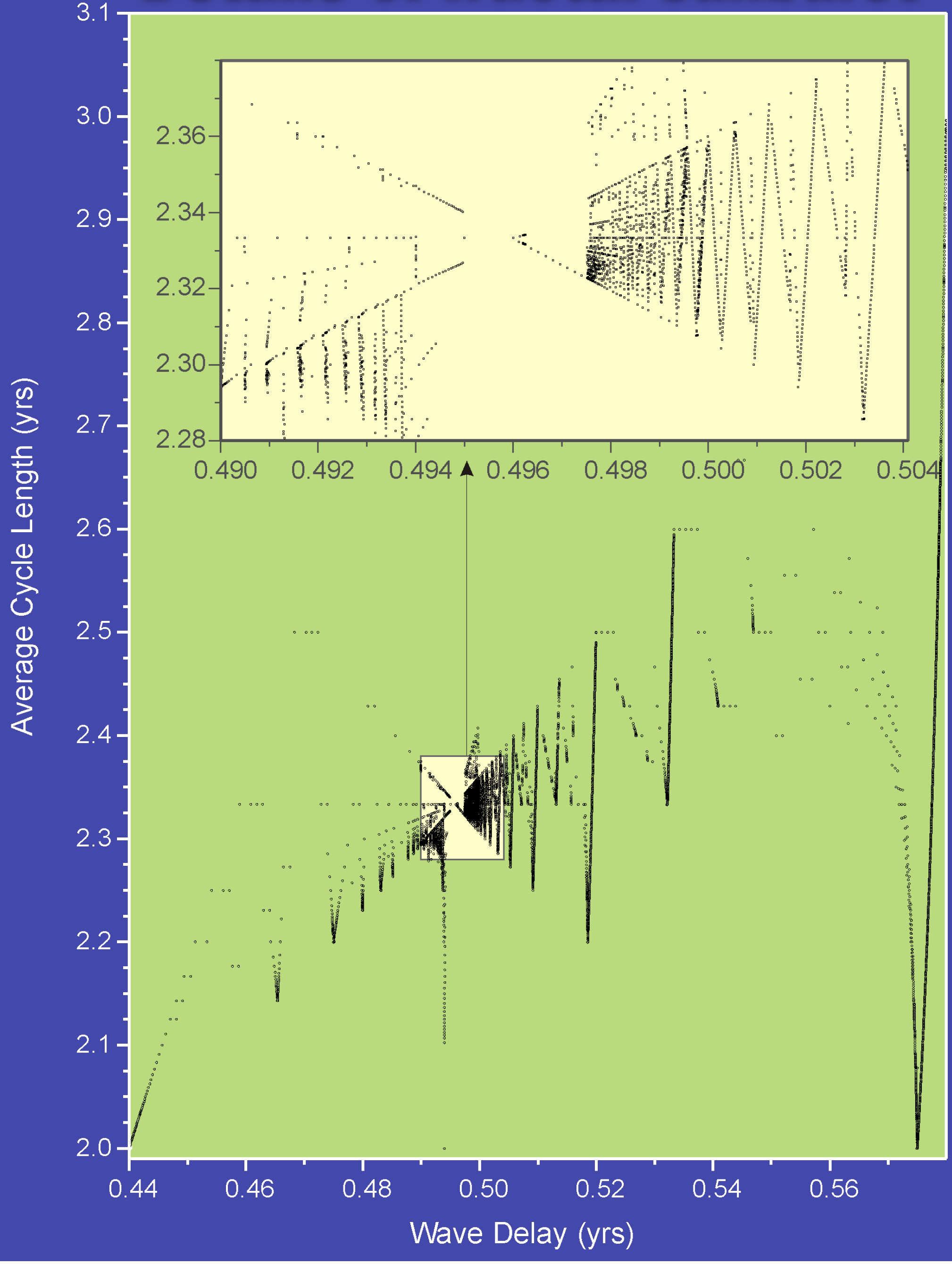


A. Saunders & M. Ghil, *Physica D*, **160**, 54–78, 2001

Devil's staircase and fractal sunburst



Details of fractal sunburst



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"Partial" BDEs – An Introduction, I

Consider on-off sites $u_i(t)$ on a line and

$u_i(t) = u_{i-1}(t\text{-}\vartheta_t)\ \Delta\ u_i(t\text{-}\vartheta_t)\ \Delta\ u_{i+1}(t\text{-}\vartheta_t)$,

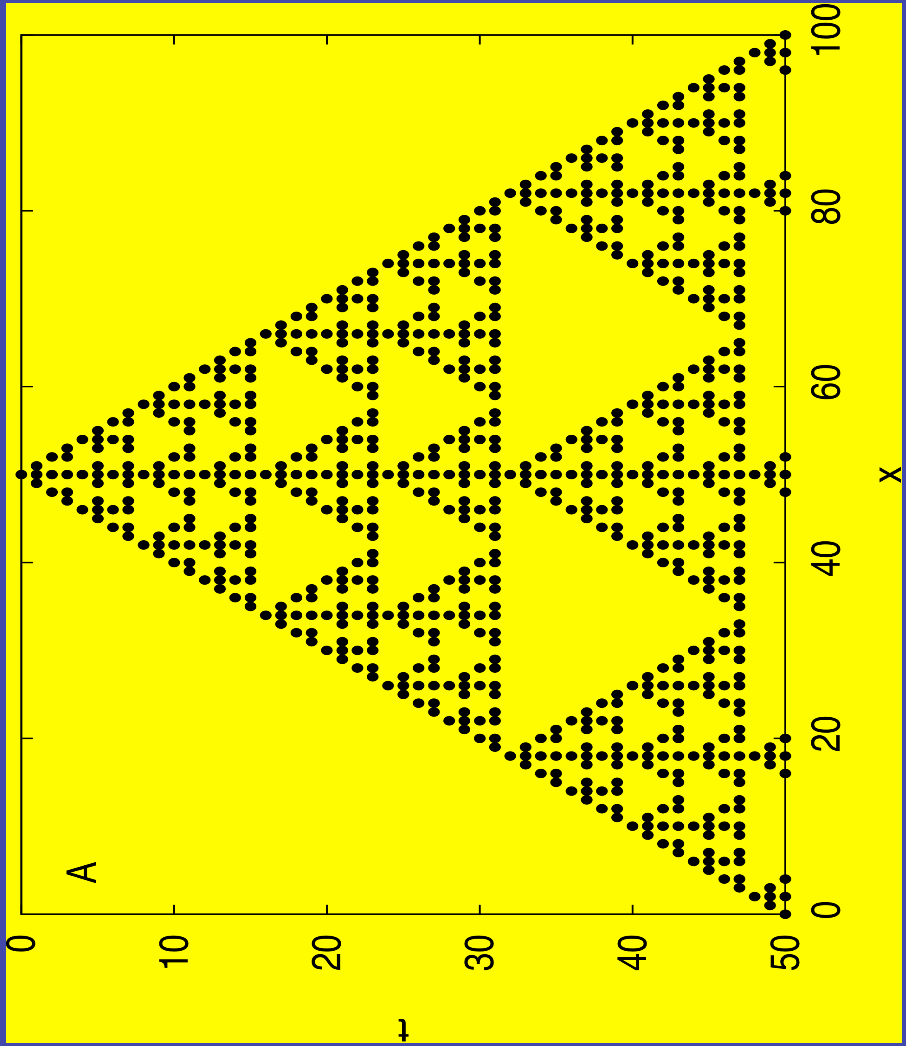
where Δ is the XOR operator, and
$\vartheta_t = const.$ for now is the time delay.

We use periodic boundary conditions,

$u_i(t) = u_{i+N}(t)$,

and thus have $n = 2N$ "ordinary" BDEs.

The initial state is $u_0(0) = 1$,
with all other $u_i(0) = 0$.



The evolution of the solution is the "Pascal's triangle" in the figure.
For $\vartheta_t = const.$ it is equivalent to an elementary CA (ECA).

Ghil *et al.* (*Physica D*, 2008)

"Partial" BDEs – An Introduction, II

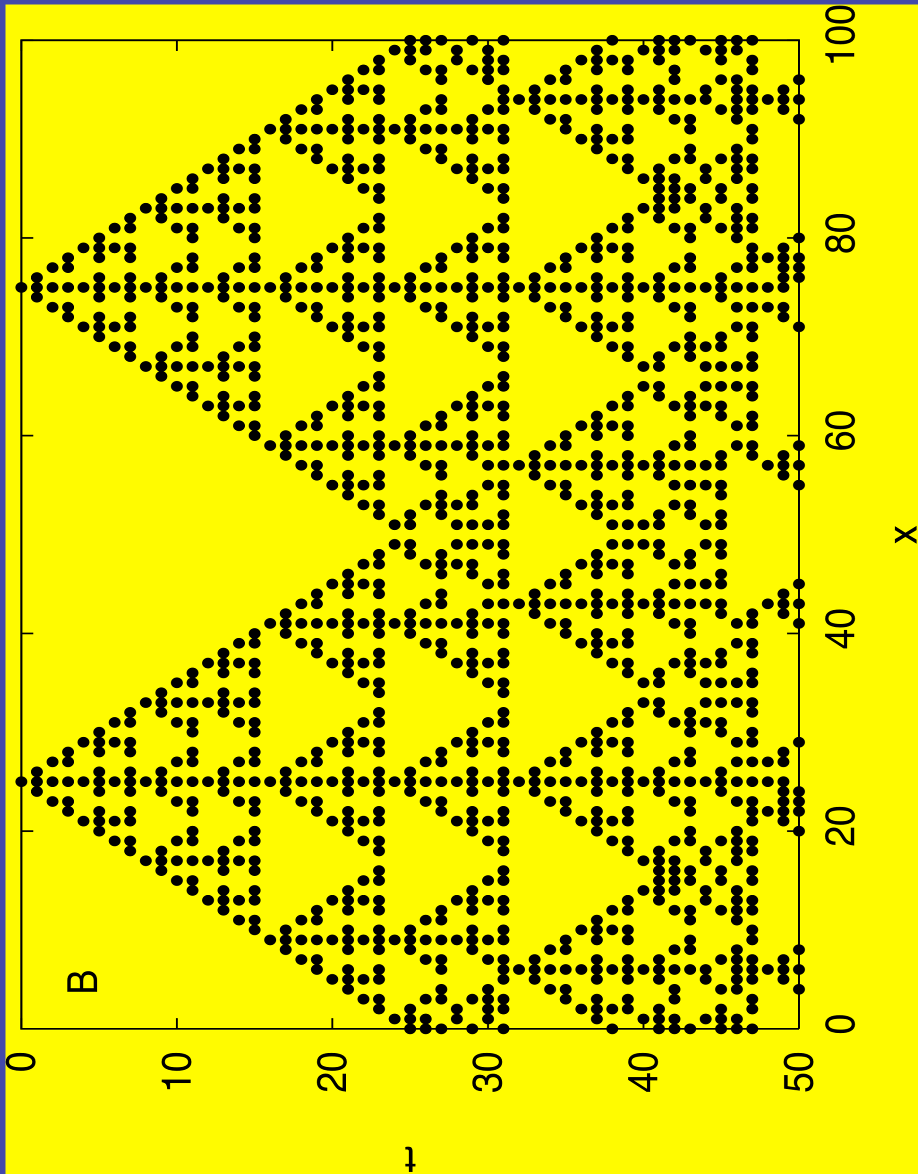
The figure now shows the "collision" of two waves, each started from an "on" site, while all other sites are "off."

Thus the solution in the previous slide is a "Green's function" of the partial BDE (PBDE) before.

This behavior is still equivalent to that of an ECA, as long as $\vartheta_t = const.$

But more interesting things will happen when that is no longer the case.

Empty sites, $u_i(t) = 0$ in white, while occupied sites, $u_i(t) = 0$ are in black.



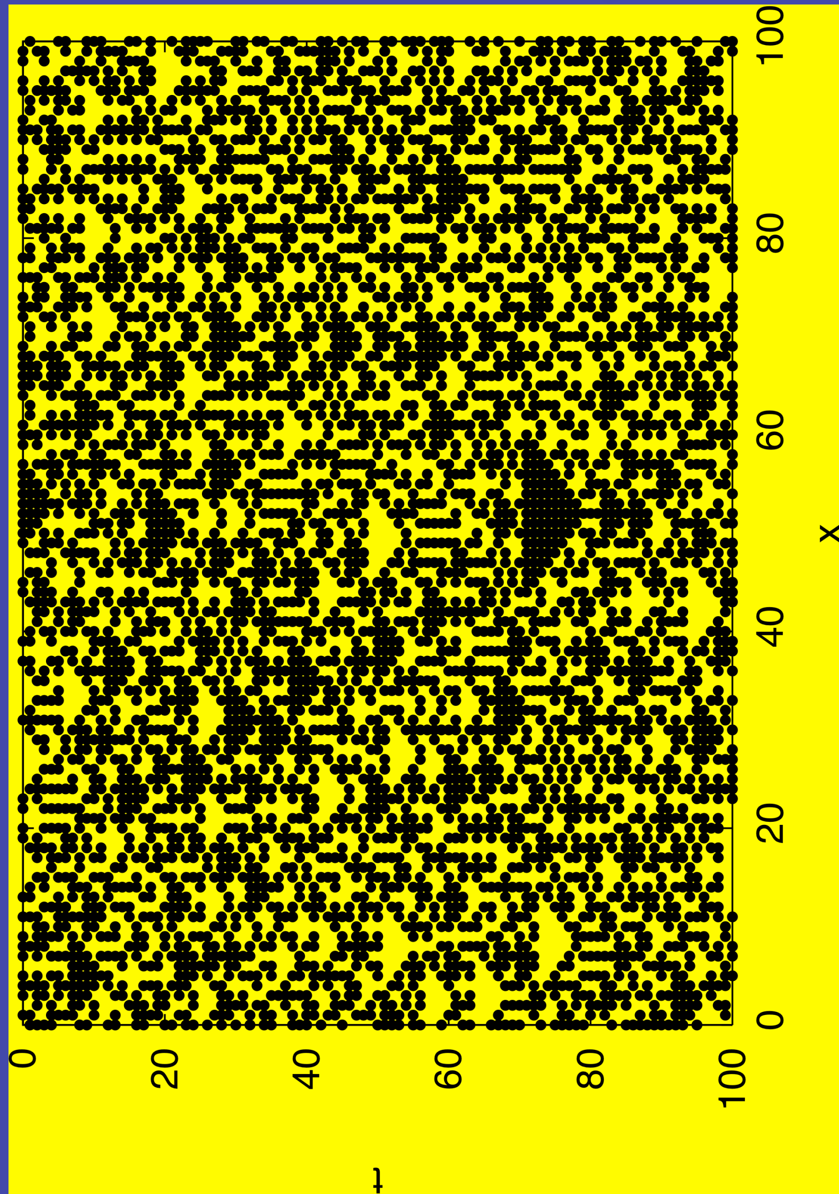
Ghil *et al.* (*Physica D,* 2008)

"Partial" BDEs – An Introduction, III

The figure now shows the solution of the same PBDE, when the initial state is a random distribution of "on" and "off" sites.

The qualitative behavior is characterized by "triangles" of empty (white) or occupied (black) sites, without any recurrent pattern.

This behavior does not depend on the particular random initial state.



Ghil *et al.* (*Physica D,* 2008)

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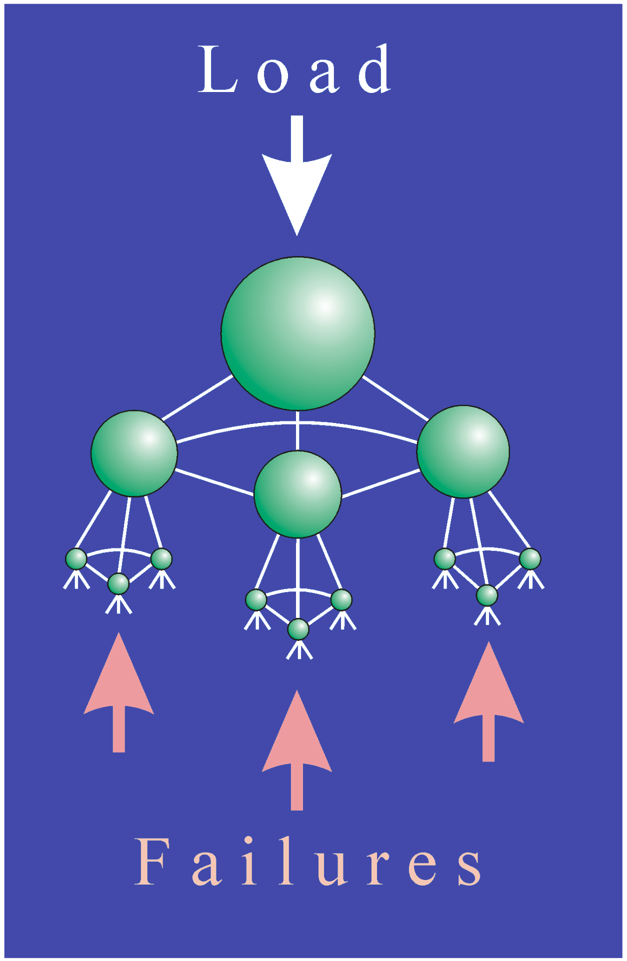
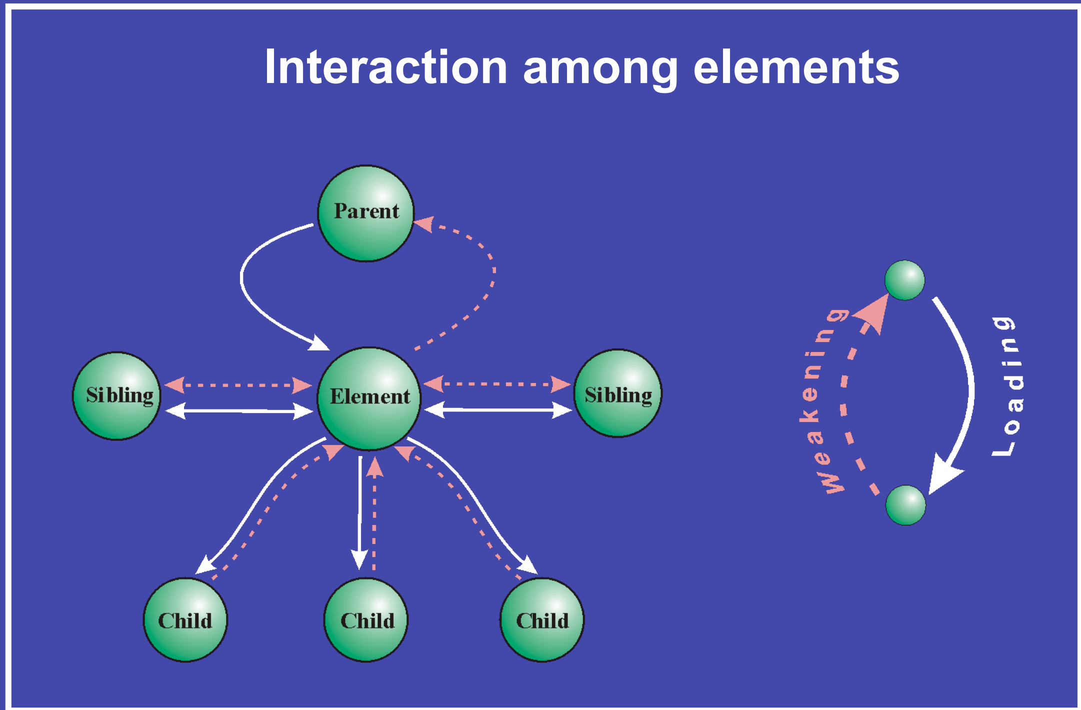
Colliding-Cascade Model



1. **Hierarchical structure**

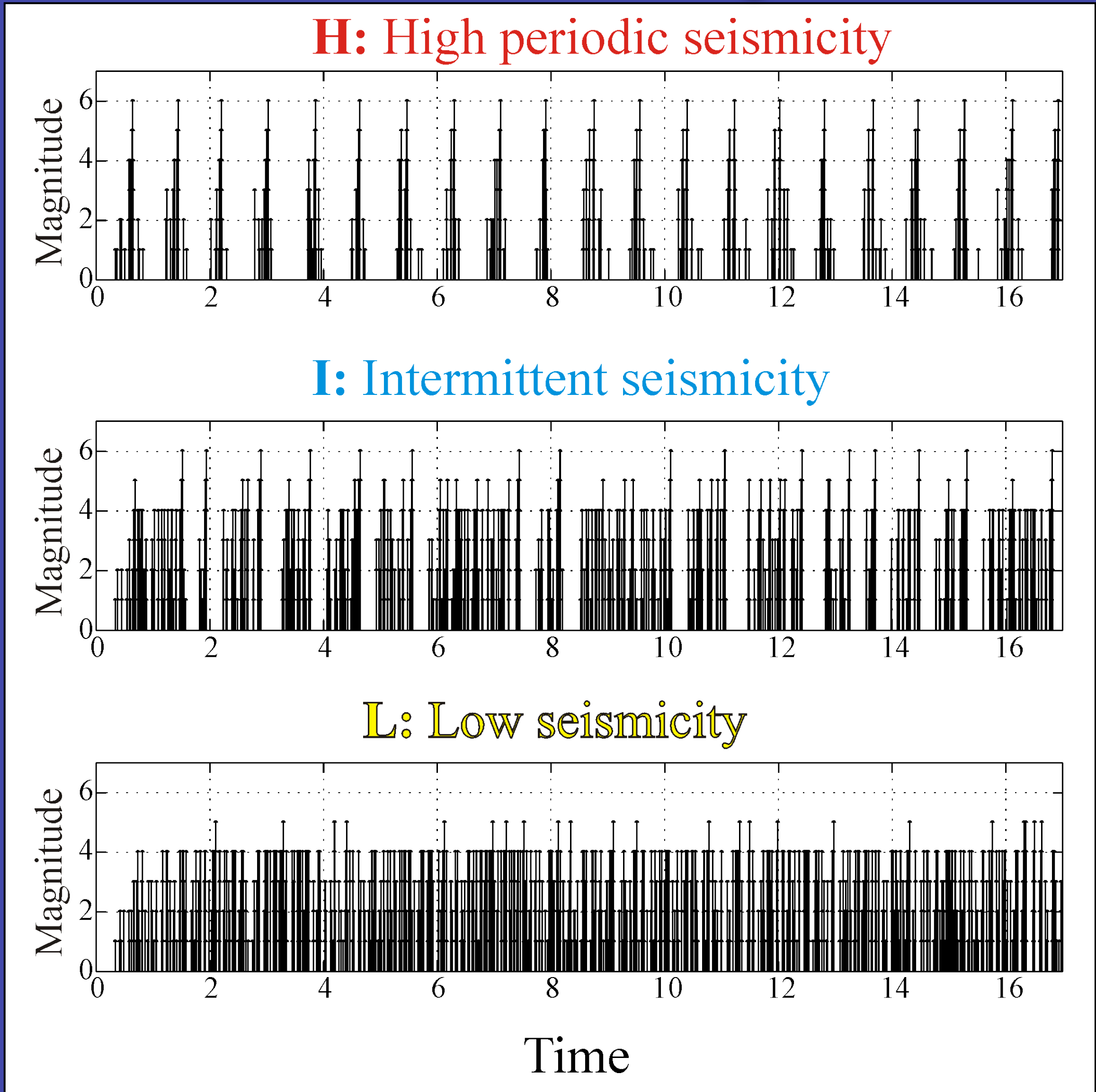
2. **Loading by external forces**

3. **Elements' ability to fail & heal**



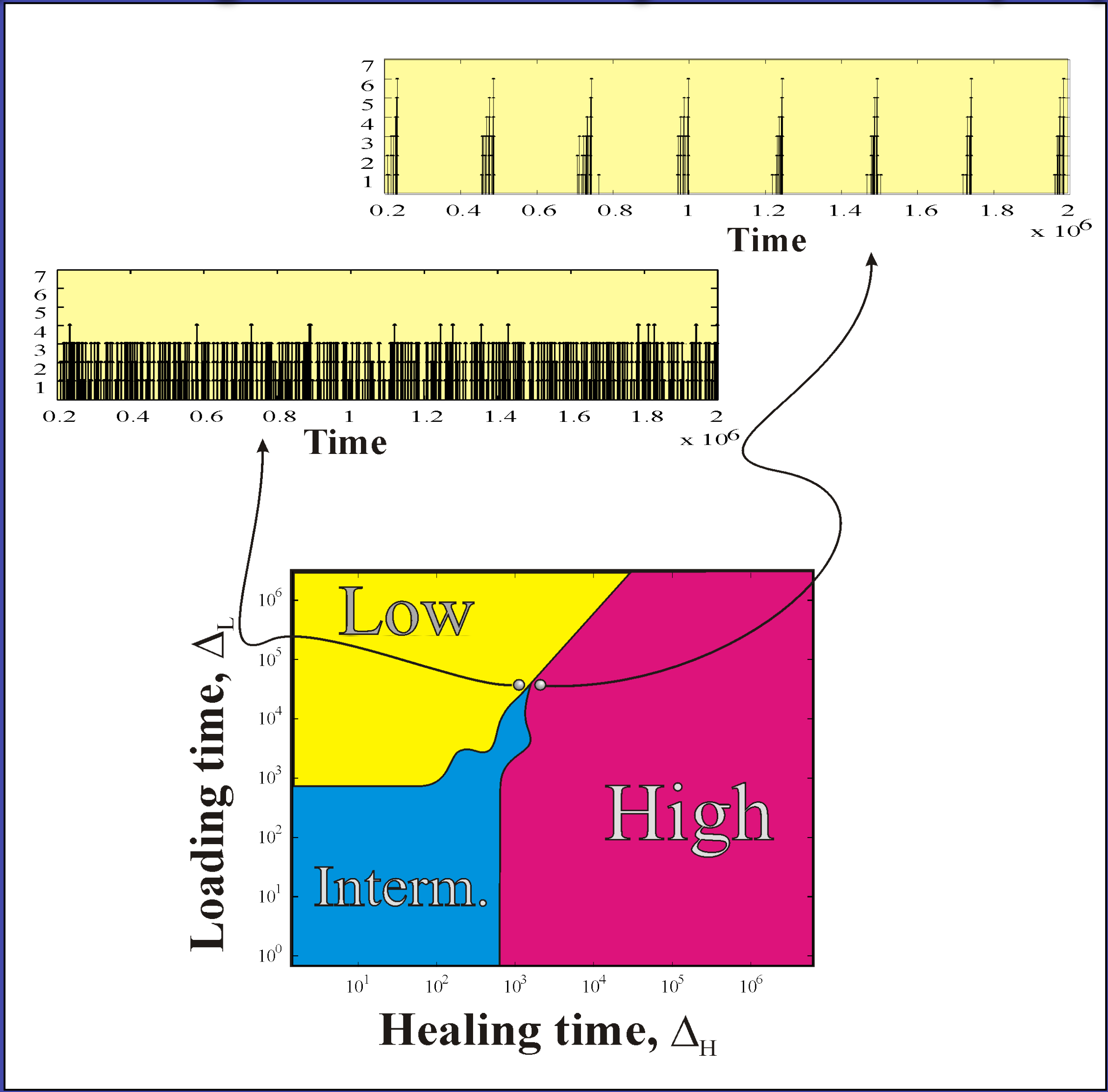
A. Gabrielov, V. Keilis-Borok, W. Newman, & I. Zaliapin (2000a, b, *Phys. Rev. E*; *Geophys. J. Int.*)

BDE model of colliding cascades: Three seismic regimes



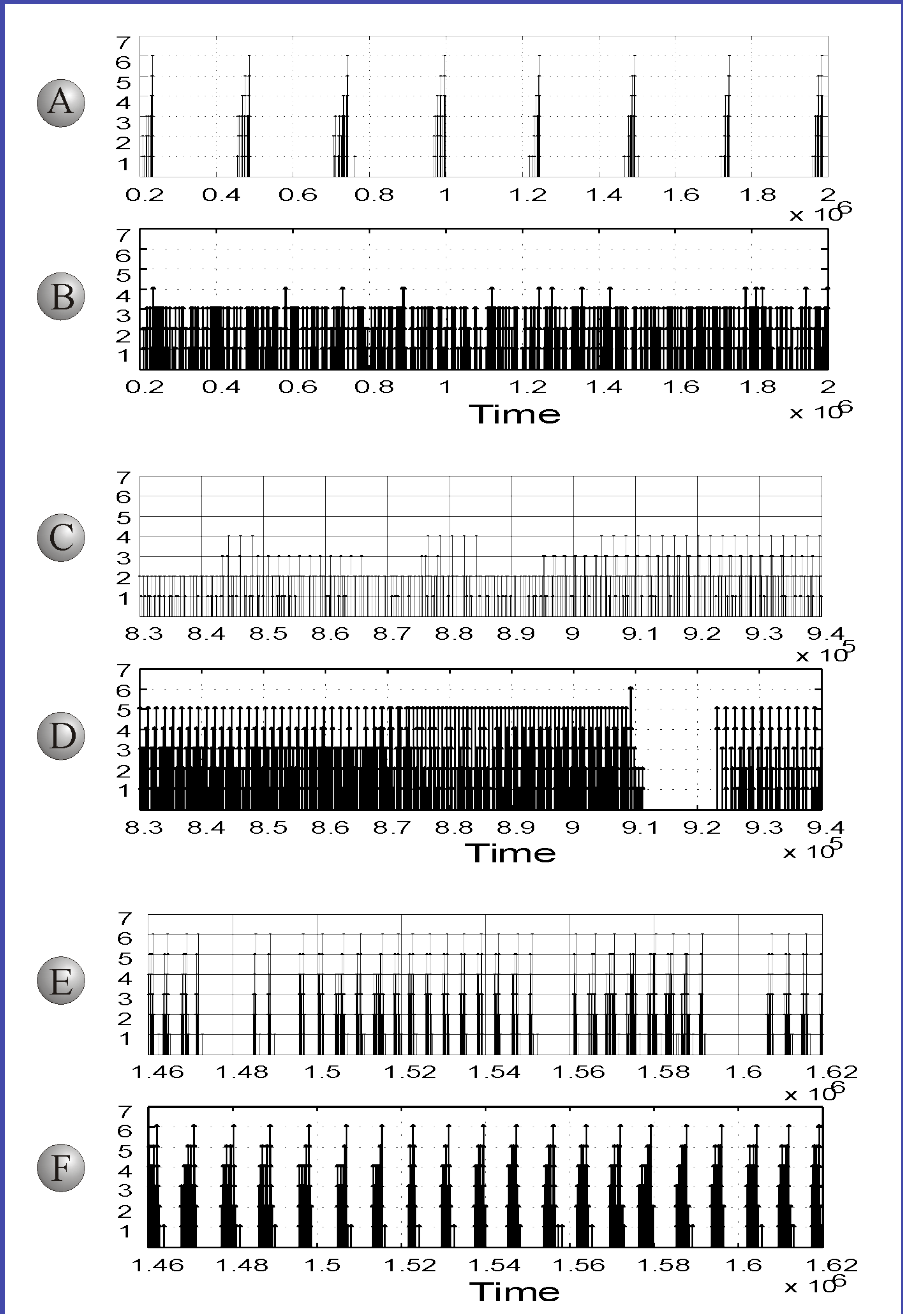
I. Zaliapin, V. Keilis-Borok & M. Ghil (2003a, *J. Stat. Phys.*)

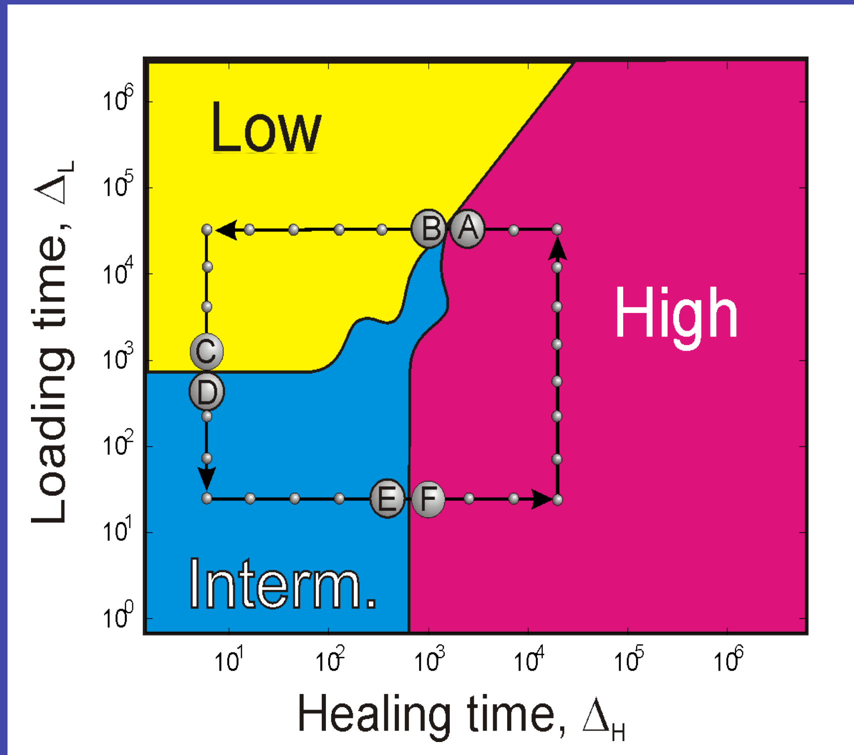
BDE model of colliding cascades
Regime diagram: Instability near the triple point



I. Zaliapin, V. Keilis-Borok & M. Ghil (2003a, *J. Stat. Phys.*)

BDE model of colliding cascades
Regime diagram:
Transition between regimes





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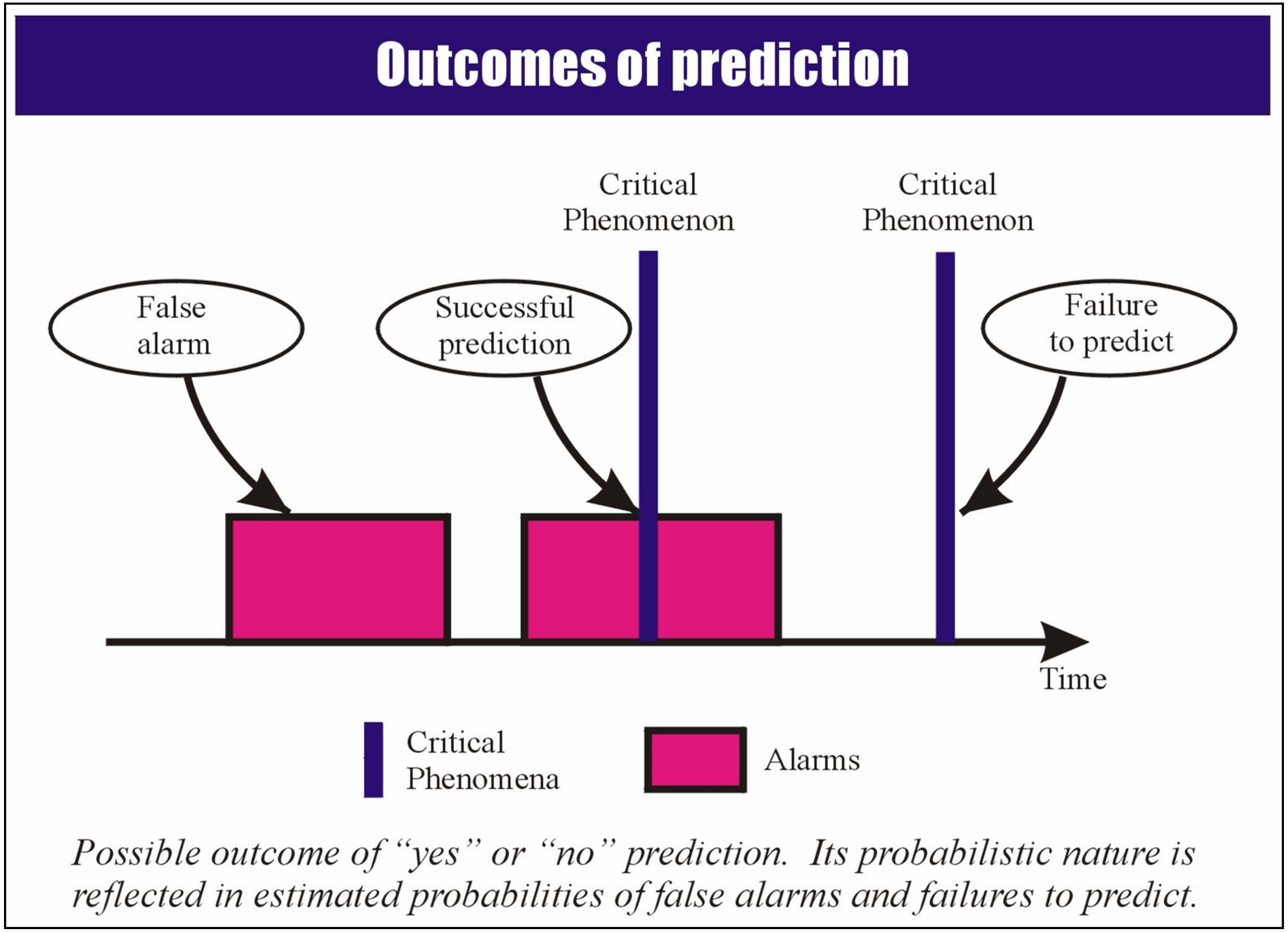


Forecasting algorithm for natural & social systems: can we beat statistics-based approaches?

Ghil & Robertson, 2002, *PNAS*; Keilis-Borok. 2002, *Annu. Rev. EPS*.

Minimax prediction strategy

P – set of parameters for precursor Π
(e.g. magnitude threshold, time window, *etc.*)

$\Pi_t(P)$ – Boolean alarm process

$\tau(P)$ – fractional time covered by alarms

$n(P)$ – fractional number of unpredicted target events

$f(P)$ – fractional number of false alarms

Minimax prediction strategy 1:

$$P = \arg\min\left[f(P)\right]$$

$$A_{\text{collective}} = \Pi_1 \vee \Pi_2 \vee \ldots \vee \Pi_n$$

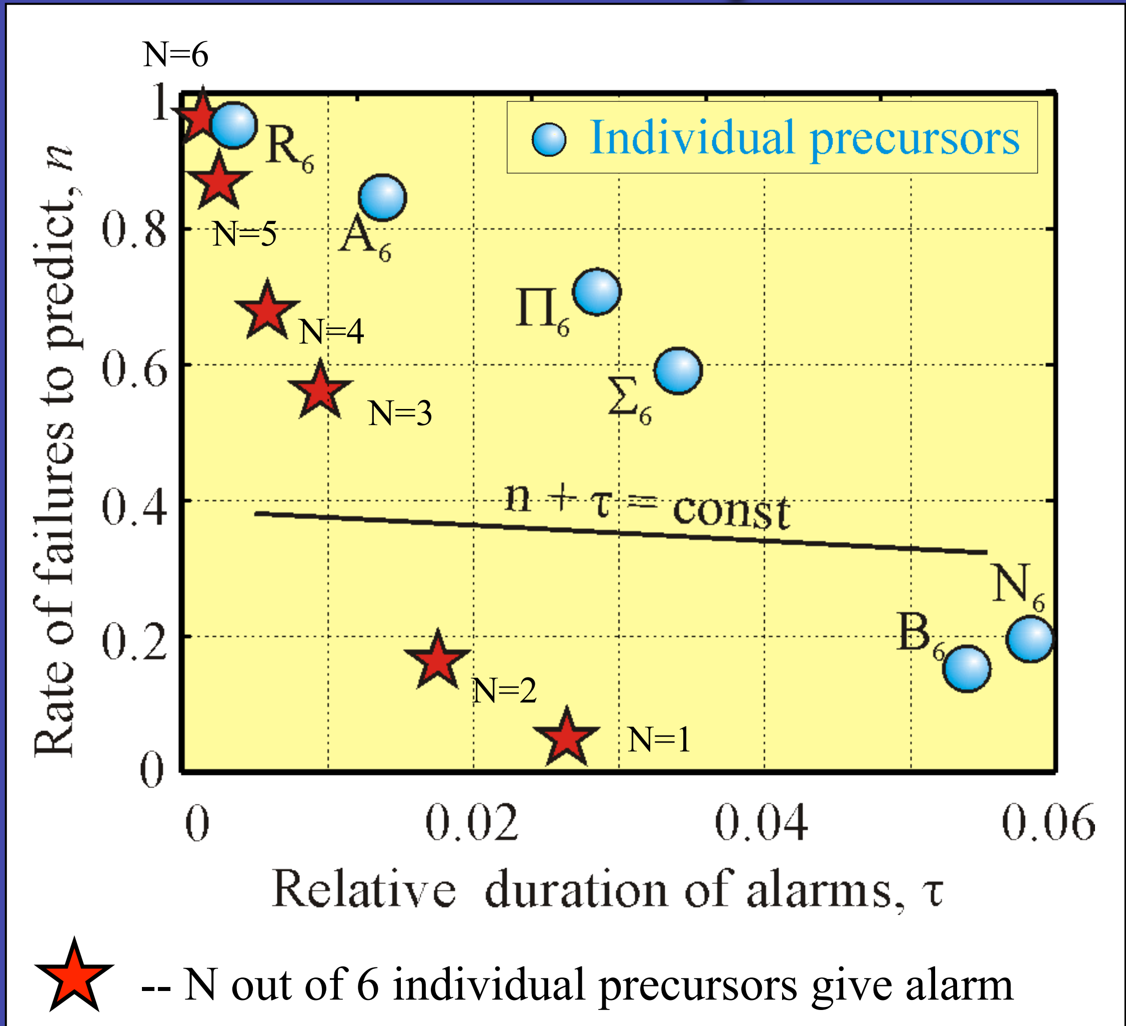
Minimax prediction strategy 2:

$$P = \arg\min\left[n(P)\right]$$

$$A_{\text{collective}} = \Pi_1 \wedge \Pi_2 \wedge \ldots \wedge \Pi_n$$

BDE model

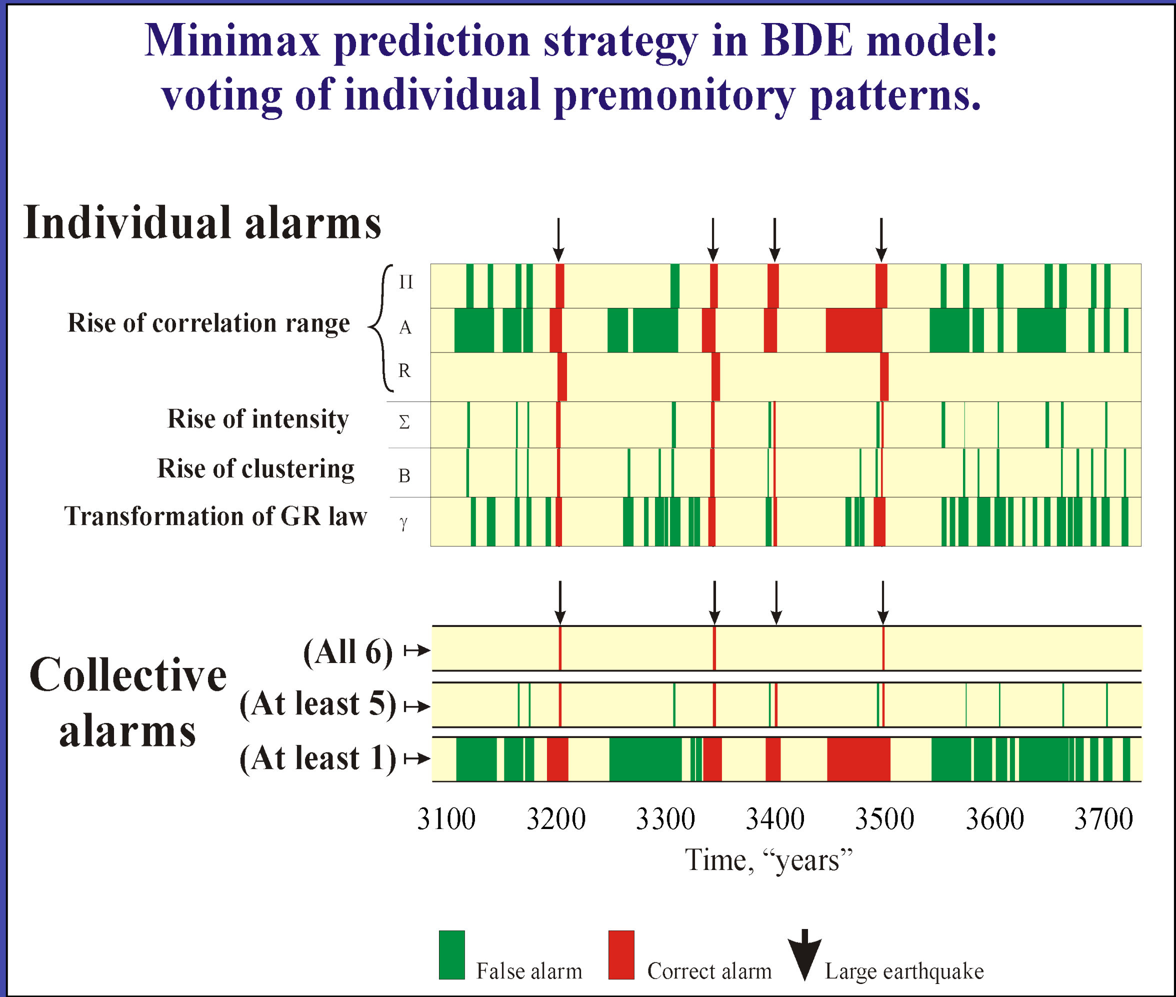
Minimax prediction strategy 1



Individual patterns are tuned to eliminate false alarms
at the cost of having more failures to predict.
Collectively, errors of both kinds are drastically reduced.

After Zaliapin, Keilis-Borok, & Ghil (2003b, *J. Stat. Phys.*)

BDE model   Minimax prediction strategy 2



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Outline

- ***What for BDEs?***
  - *life is sometimes too complex for ODEs and PDEs*
- ***What are BDEs?***
  - *formal models of complex feedback webs*
  - *classification and major results*
- ***Applications to climate modeling***
  - *ENSO – interannual variability in the Tropics*
- ***"Partial" BDEs – spatio-temporal models***
  - *theoretical results + connections to Boolean networks & CAs*
- ***Seismic applications***
  - *modeling, theory and predictions*
- ***Socio-economic applications***
  - *economic damage propagation on a network*
- ***Concluding remarks***
  - *bibliography & future work*

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NG42A. *Complex Networks in Geosciences*

San Francisco,
16 December 2010

Boolean Delay Equations (BDEs) on Networks: An Application to Economic Damage Propagation

Michael Ghil
Ecole Normale Supérieure, Paris, and
University of California, Los Angeles;
with B. Coluzzi and G. Weisbuch (ENS), and
S. Hallegatte (CIRED and Météo-France)

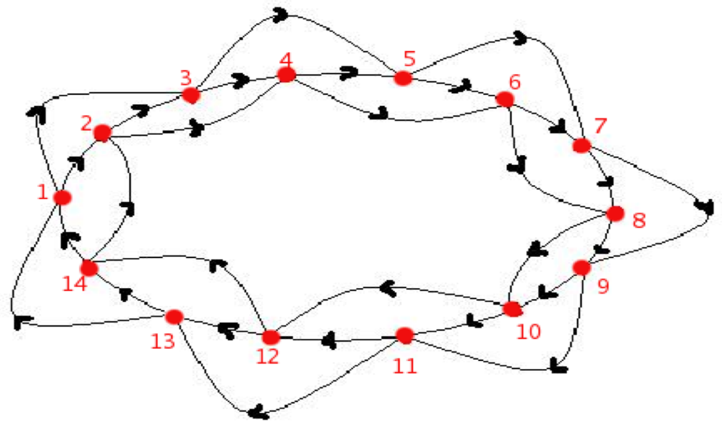
Please visit these sites for more info.
http://www.atmos.ucla.edu/tcd/
http://www.environnement.ens.fr/

Motivation

- Aggregate modeling of the economy — long-term growth models (Solow, 1956) or general equilibrium models (Arrow and Debreu, 1954) — cannot account for firm-to-firm damage propagation
- Input-output models (Leontieff, 1986) can account for multiplicity of firms and paths, but not for distinct lags
- Heterogeneous production-and-exchange networks have been shown to strongly affect propagation of damage & shortages
- We use here a Boolean delay equation (BDE) model with distinct delays ➔ quick look at the effects of network topology
  – open *vs*. closed system (free *vs*. forced)
  – fixed *vs*. random geometry
  – deterministic *vs*. random delays
- Conclusions
  – in a closed system, damage asymptotes to a fixed fraction $\rho \leq 1.0$
  – in an open system, "waves" of damage can occur

Network topologies

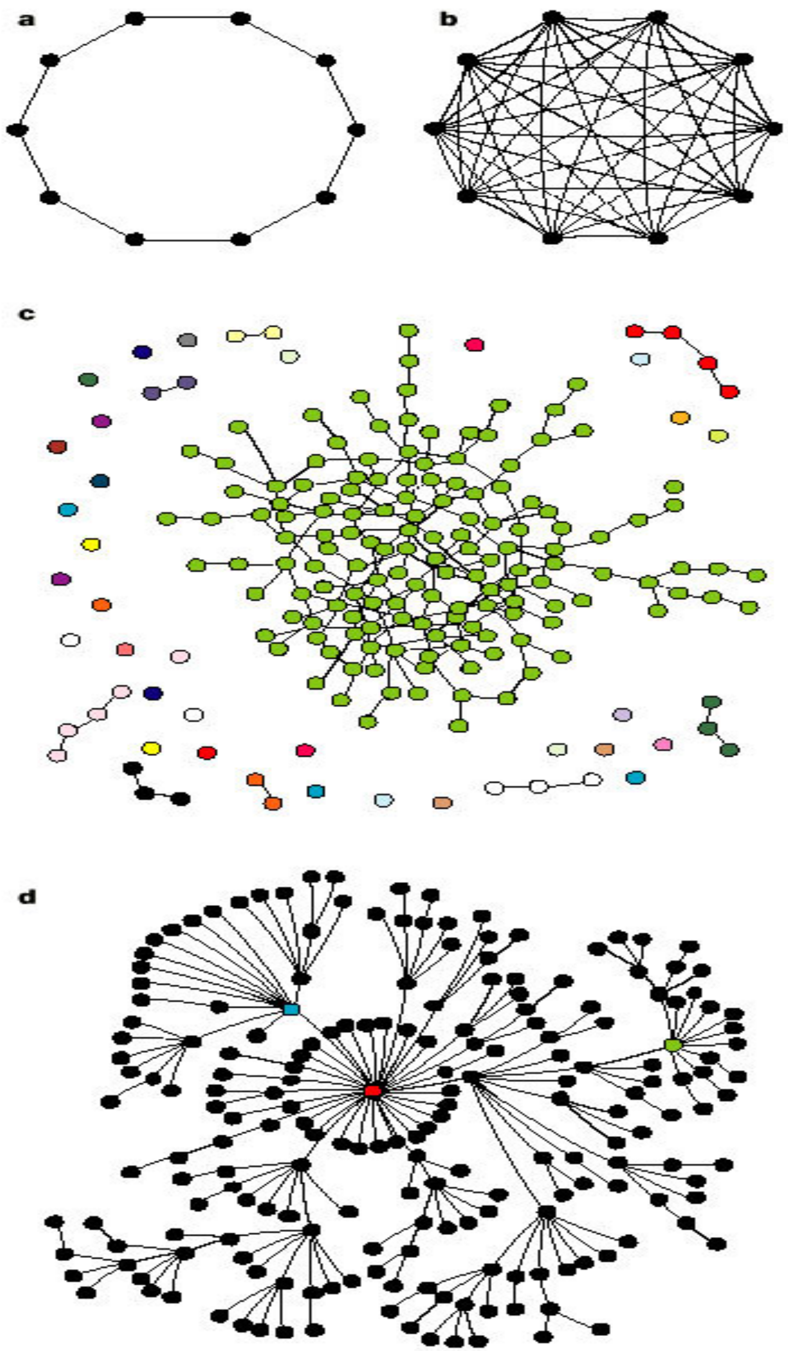
Braid structure, input-output degree $k = 2$



Four types of networks:

a) periodic, $k = 1$;
b) fully connected, $k = N - 1$;
c) random, $z = \langle k \rangle > N/2$;
d) scale-free, $\mathrm{Prob}(k) \sim k^{-\alpha}, \alpha > 0\,.$

Strogatz (*Nature*, 2001)



Model formulation, I – Network

We consider a network of *N* firms on the *vertices* of a *directed graph* or *digraph.*
The case of a *directed random graph (DRG)* or random Erdös-Rényi (1961) graph will be of particular interest.

The *edges* of the digraph are described by the *connectivity matrix A,* with

$$A_{ij} = 1, \text{ if firm } i \text{ needs the output of firm } j,$$
$$\text{and } A_{ij} = 0, \text{ otherwise.}$$

We analyze, w.l.o.g., the impact of damage $x_1(t : 0 \leq t \leq \tau_c)$ to a single firm on the production $\{x_i : 1 \leq i \leq N\}$ of all the firms. We let

$$x_i = 0 \text{ if firm } i \text{ is damaged,}$$
$$\text{and } x_i = 1 \text{ if it is is not.}$$

We study the effect on damage propagation of
(i) the network topology, i. e. of the matrix *A*, and
(ii) the distribution of delays, $\tau_{\min} \leq \tau_{ij} \leq \tau_{\max}$.

Mean density:

(i) *deterministic case,* $\rho(t) \equiv \frac{1}{N}\sum_{i=1}^{N} x_i(t)$;

(ii) *random case* $\langle\rho(t)\rangle \equiv \int \prod_{i,j} d\tau_{i,j}\mathcal{P}(\tau_{i,j}) \int \prod_{h,k} dA_{hk}\mathcal{P}(A_{hk})\rho(t)$

Model formulation, II – BDE model

The availability (of the stock) S_{ji} of a good produced by firm j for firm i obeys:

$$S_{ji}(t) = x_j(t - \tau_{i,j}) \text{ for free (autonomous) models, and}$$
$$S_{ji}(t) = \overline{x}_i(t - \tau_{i,j}) \vee x_j(t - \tau_{i,j}) \text{ for forced models (with re-supply).}$$

x_i	x_j	S_{ji}	Free models
0	0	0	j and i inactive, the stock can not be reconstituted
0	1	1	j active and i inactive, the good is stocked
1	0	0	j inactive and i active, the stock is finished
1	1	1	j active and i active, the stock is updated
x_i	x_j	S_{ji}	Forced models
0	0	1	j and i inactive, the stock is supplied from outside
0	1	1	j active and i inactive, the good is stocked
1	0	0	j inactive and i active, the stock is finished
1	1	1	j active and i active, the stock is updated

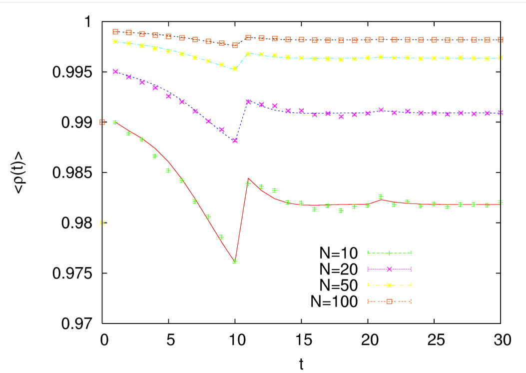
Table 1: The input-output table of the stock S_{ji} of a given product as a Boolean function of the activities (x_i, x_j) of the customer firm i and supplier firm j, in the free models described by Eq. (1) and in the forced models described by Eq. (2), respectively.

Selected results – I, $\langle\rho(t)\rangle$

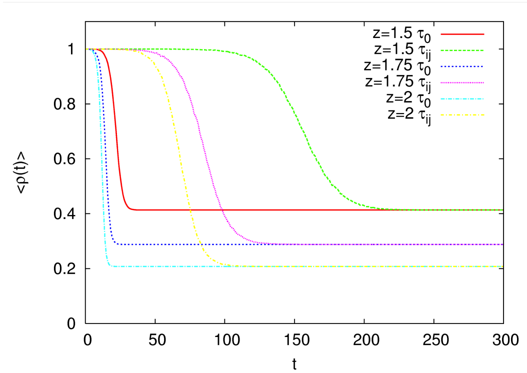
Braid structure, $k = 1$, random delays

$\tau_c = \tau_{\min}\ , \quad \tau_{\max} = 10\ \text{days}$

Random digraph (DRG) with average connectivity $z > 1$

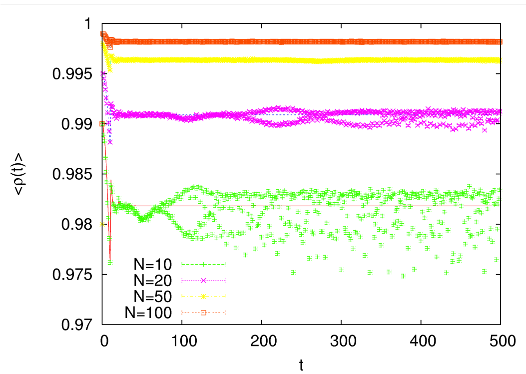


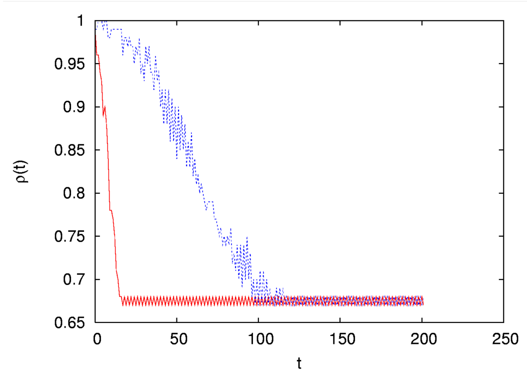
Free network



Short times, $0 \leq t \leq 30$ days

Fixed delays, $\tau_0 = 1$ day, *vs.* random ones, $\tau_{\min} \leq \tau_{ij} \leq \tau_{\min}$.





Long times, $0 \leq t \leq 500$ days

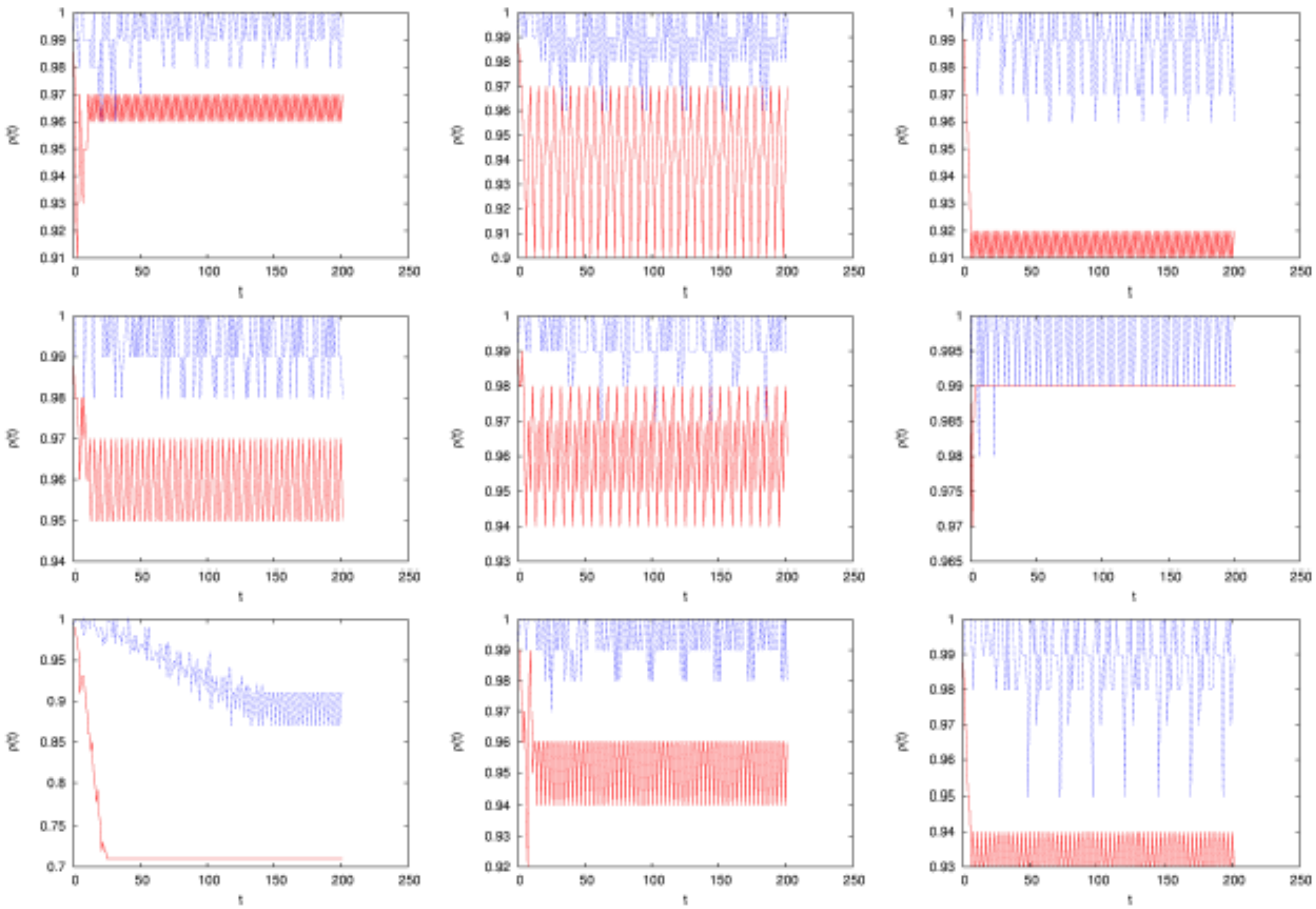
$\langle\rho(t)\rangle \sim\!\rightarrow 1 - \frac{0.182}{N}$ as $t \rightarrow \infty$

Fixed delays *vs.* random, $z = 0.525$, $N = 100$

Selected results – II, $\langle \rho(t) \rangle$

Free network

Periodic solutions in a DRG with average connectivity $z = 0.525 < 1$, $N = 100$



Fixed delays, $\tau_0 = \tau_c = 1$ day (red curves), *vs.* random ones, $\tau_{\min} \leq \tau_{ij} \leq \tau_{\max}$ (blue curves).

Concluding remarks, I – General

Summary

- BDEs on networks provide great flexibility in modeling complex systems
- The behavior of these systems is rich and varied.
- Just starting to be explored for economic applications.

Main conclusions (repeated)

- In a closed system, damage asymptotes to a fixed density,

$$\langle \rho(t) \rangle \to \rho_\infty,\ \rho_\infty < 1 \text{ or } \rho_\infty = 1.$$

- In an open system, (cyclostationary) “waves” of damage may occur, $\langle \rho(t) \rangle \to \rho_\infty(t),\ \rho_\infty(t+T) = \rho_\infty(t).$

Concluding remarks, II – Specific

Free models

Mean damage is nonzero and possibly total if:

- mean input connectivity is larger than 1;
- τ_C is larger than the shortest propagation time between nodes.

Damage spreading velocity

depends on network topology: the number of affected nodes increases

- – linearly in time for the braid chain, and
- – exponentially for random digraphs (DRGs)

Forced models

External supplies limit damage and damage waves move across the structure

- The transient up to asymptotic behavior diverges exponentially with *N*;
- a shorter transient to effectively constant mean damage equals the passage time of the first wave through the system's connected component; and
- This behavior is obtained even for shorter initial damage.

Outline

- ***What for BDEs?***

  - life is sometimes too complex for ODEs and PDEs
- ***What are BDEs?***

  - formal models of complex feedback webs

  - classification of major results
- ***Applications to climate modeling***

  - paleoclimate – Quaternary glaciations;

  - interdecadal – climate variability in the Arctic

  - ENSO – interannual variability in the Tropics
- ***"Partial" BDEs – spatio-temporal models***

  - theoretical results
- ***Socio-economic applications***

  - economic damage propagation on a network
- ***Concluding remarks***

  - bibliography & future work

Concluding remarks

1. BDEs have rich behavior:
   periodic, quasi-periodic, aperiodic, increasing complexity

2. BDEs are relatively easy to study

3. BDEs are natural in a digital world

4. Two types of applications

   - strictly discrete (genes, computers)
   - saturated, threshold behavior (nonlinear oscillations, climate dynamics, population biology, earthquakes)

5. Can provide insight on a very qualitative level
   (~ symbolic dynamics)

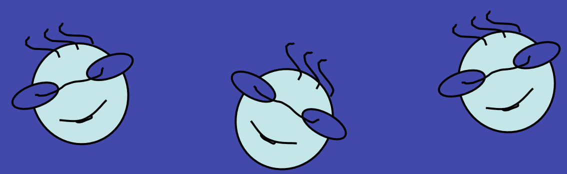
6. Generalizations possible
   (spatial dependence – “partial” BDEs;
   stochastic delays &/or connectives)

Conclusions

Hmmm, this is interesting!

But what does it all mean?

Needs more work!!!



Short BDE bibliography

Theory

Dee & Ghil (1984, *SIAM J. Appl. Math.*)
Ghil & Mullhaupt (1985, *J. Stat. Phys.*)

Applications to climate

Ghil *et al*. (1987, *Climate Dyn.*)
Mysak *et al*. (1990, *Climate Dyn.*),
Darby & Mysak (1993, *Climate Dyn.*),
Saunders & Ghil (2001, *Physica D*)

Applications to solid-earth problems

Zaliapin, Keilis-Borok & Ghil (2003a, b, *J. Stat. Phys.*)

Applications to genetics

Oktem, Pearson & Egiazarian (2003, *Chaos*)
Gagneur & Casari (2005, *FEBS Letters*)

Socio-economic applications

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Review paper

Ghil, Zaliapin & Coluzzi (2008) Boolean delay equations: A simple way of looking at complex systems, *Physica D*, **237**, 2967–2986, doi: 10.1016/j.physd.2008.07.006.

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Zaliapin, I., V. Keilis-Borok, and M. Ghil, 2003: A Boolean delay equation model of colliding cascades. I: Multiple seismic regimes. *J. Stat. Phys*., **111**, 815–837, 2003.

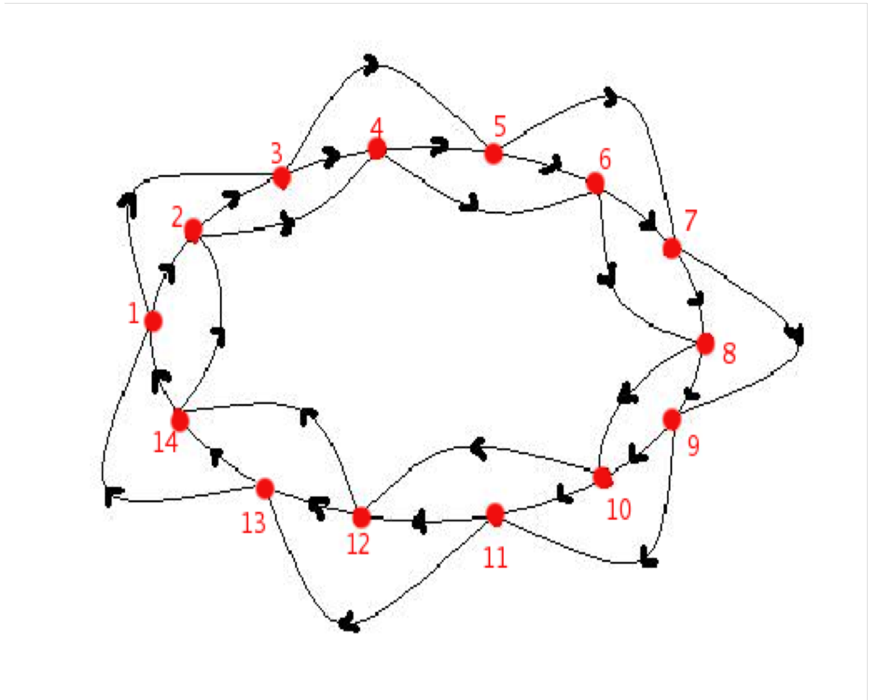
RESERVE SLIDES

Edward Norton Lorenz
born May 23, 1917

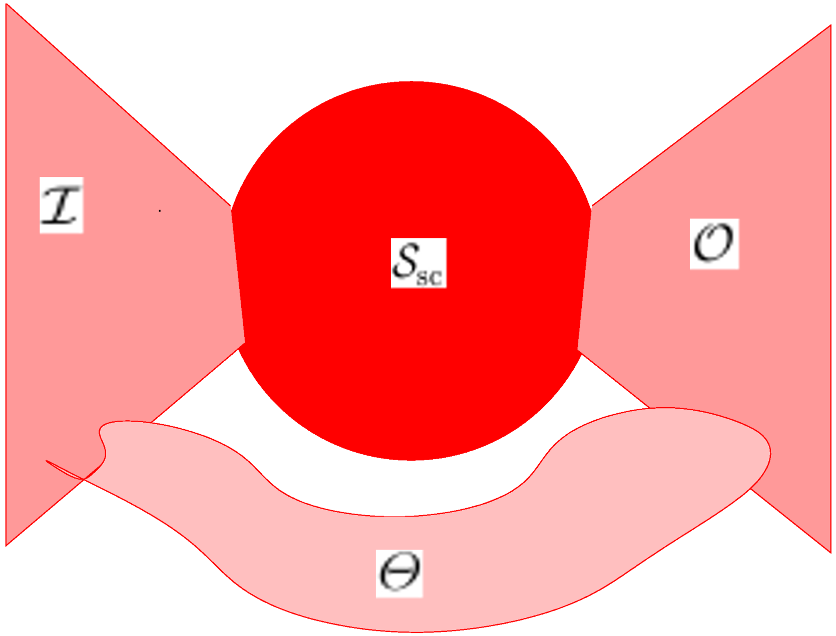
Jule Gregory Charney
January 1, 1917 – June 16, 1981

Network topologies

Braid structure, input-output degree $n = 2$



Bow-tie structure, e. g. the Web



The two bows correspond to the giant components $\mathcal{I}\backslash\mathcal{S}_{sc}$ and $\mathcal{O}\backslash\mathcal{S}_{sc}$ respectively, whereas the tie represents the giant strongly connected component $\mathcal{S}_{sc} = \mathcal{I}\cap\mathcal{O}$. We consider here the most general case, in which $\mathcal{W} = \mathcal{I}\cup\mathcal{O}\cup\mathcal{T}$.

Here W is the weakly connected component.

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What for BDEs?

- life is sometimes too complex for ODEs and PDEs

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- colliding-cascades model of seismic activity
- intermediate-term prediction

"Partial" BDEs – spatio-temporal models

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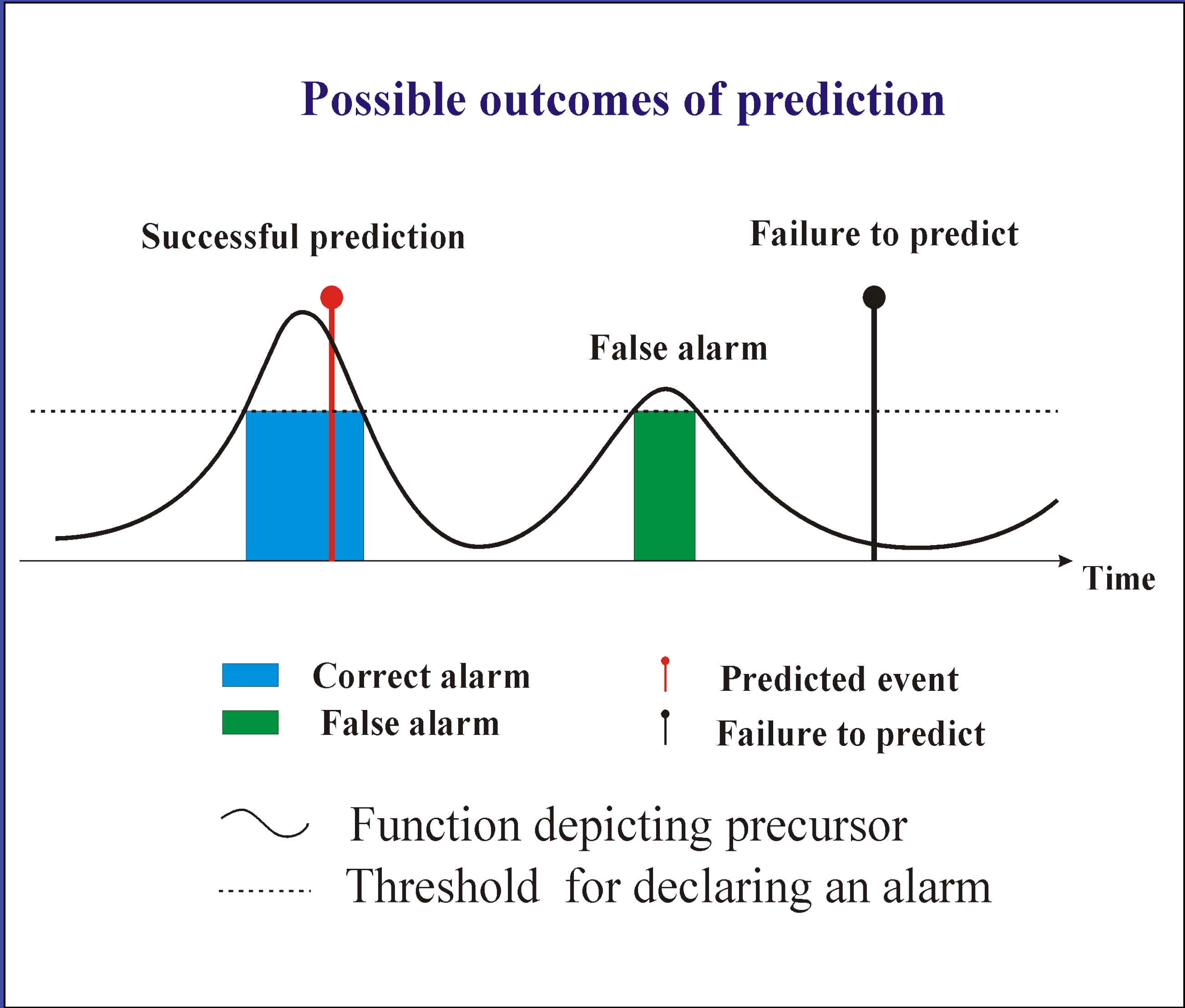
- bibliography & future work

Forecasting algorithms for natural and social systems: Can we beat statistics-based approach?



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Keilis-Borok (2002, *Annu. Rev. Earth Planet. Sci.*)

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