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On the physics description of fusion plasmas 2

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Foundations of kinetic theory

We have already discussed kinetic theory as something intuitively clear. However, it is important to also have its foundation clear. Kinetic theory stems from the most general representation of particles in phase space, $(\mathbf{r},\mathbf{v})$.

In order to realize which approximations that are made in the descriptions of plasmas that we generally use, it is instructive to start from the most general description which includes all individual particles and their correlations in the six dimensional phase space $(\mathbf{r},\mathbf{v})$.

General particle description, Liouville and Klimontovich equations.

In the absence of particle sources or sinks we must have a contimuity equation for the delta function density *N*:

$$N(X,t) = \sum_{i=1}^{N} (X - X_i(t)) \tag{2.1}$$

$$\frac{\partial}{\partial t} N + \sum_i \frac{\partial}{\partial r_i} (N \frac{\partial r_i}{\partial t}) + \sum_i \frac{\partial}{\partial \mathrm{v}_i} (N \frac{\partial \mathrm{v}_i}{\partial t}) = 0, \tag{2.2}$$

Since we have included all particles, this system conserves energy if we ignore radiation. Thus there must be a Hamiltonian for the system and we use the Hamiltonian equations:

Integration along characteristics

$$\frac{\partial r_i}{\partial t} = \frac{\partial H}{\partial \mathrm{v_i}} \qquad \frac{\partial \mathrm{v}_i}{\partial t} = -\frac{\partial H}{\partial \mathrm{r_i}} \Rightarrow$$

$$\frac{\partial}{\partial t} N + \sum_i \frac{\partial r_i}{\partial t}\frac{\partial N}{\partial r} + \sum_i \frac{\partial \mathrm{v}_i}{\partial t}\frac{\partial N}{\partial \mathrm{v_i}}) = 0, \qquad (2.3)$$

Using acceleration due to the Lorenz force we then get:

$$\left\{ \frac{\partial}{\partial t} + \vec{\mathrm{v}} \cdot \frac{\partial}{\partial \vec{r}} + \frac{e}{m}\vec{E} \cdot \frac{\partial}{\partial \vec{\mathrm{v}}} + \Omega_c (\vec{\mathrm{v}} \times \widehat{\mathrm{e}}_{\parallel}\ ) \cdot \frac{\partial}{\partial \vec{\mathrm{v}}} \right\} N(X,t) = 0 \qquad (2.4)$$

Where we introduced e**B**/m = Ω_c e$_{\parallel}$.

The BBGKY hierarchy

Eq (2.4) is written in the way $\frac{DN}{Dt} = 0$

where $\frac{D}{Dt}$

- is the total operator in (2.4). Equation (2.4) is the Liouville or Klimontovich equation. Since $N(X,t)$, as given by (2.1), contains the simultaneous location of all particles in phase space, it can be considered as a probability density in phase space. It gives the probability of finding a particle in the location ($\mathbf{r}$,$\mathbf{v}$) given the simultaneous locations ($\mathbf{r}_\mathbf{i}$,$\mathbf{v}_\mathbf{i}$) of all the other particles. This is an enormous amount of information which is usually not needed. This information can be reduced by integrating over the positions of several other particles giving an hierarchy of distribution functions (the BBGKY hierarchy) where the evolution of each distribution

Expansion in the plasma parameter

- function, giving the probability of the simultaneous distribution of n particles, depends on that of n+1 particles. Thus we need to close this hierarchy in some way. This is usually done by expanding in the plasma parameter:

$$g = \frac{1}{n\lambda_d^{\,3}}; \qquad \lambda_d = \sqrt{\frac{T}{4\pi e n}}$$

Which is the inverse number of particles in a Debyesphere. When the plasmaparameter tends to zero only collective interactions remain between the particles which form a continous charge distribution in phase space. When we study the equation of the one particle distribution function and include effects of the two particle distribution function (describing pair collisions) as expanded in g we get the equation:

The Vlasov and Fokker-Planck equations

$$\left\{\frac{\partial}{\partial t}+\vec{\mathrm{v}}\cdot\frac{\partial}{\partial \vec{r}}+\frac{e}{m}\vec{E}\cdot\frac{\partial}{\partial \vec{\mathrm{v}}}+\Omega(\vec{\mathrm{v}}\times\hat{\mathrm{e}}_{\|})\cdot\frac{\partial}{\partial \vec{\mathrm{v}}}\right\}f(\mathbf{r},\mathbf{v},\mathrm{t})=\left(\frac{\partial f}{\partial t}\right)_{coll} \qquad (2.5)$$

where f is the one particle distribution function and the right hand side approximates close collisions (first order in g). Here various approximations like Boltzmanns or the Fokker-Planck collision terms are used. If we can ignore close collisions completely we have the Vlasov equation:

$$\left\{\frac{\partial}{\partial t}+\vec{\mathrm{v}}\cdot\frac{\partial}{\partial \vec{r}}+\frac{e}{m}\vec{E}\cdot\frac{\partial}{\partial \vec{\mathrm{v}}}+\Omega_c(\vec{\mathrm{v}}\times\hat{\mathrm{e}}_{\|})\cdot\frac{\partial}{\partial \vec{\mathrm{v}}}\right\}f(\mathbf{r},\mathbf{v},\mathrm{t})=0 \qquad (2.6)$$

Limitations of Kinetic theory

- It is, however, not always certain that the BBKY expansion works. Due to the reversible form of the Liouville equation it could, in principle, try to return back to its original state introducing higher order correlations. Such questions have been discussed by Klimontovich (Yu.L.Klimontovich. Statistical theory of open systems. Kluwer (1995), Theoretical and Mathematical Physics Volume 92, Number 2, 909-921).

Vlasov and Fokker-Planck equations

- The kinetic equations (2.5) and (2.6) are the equations usually used by plasma physicists. Equation (2.6) is reversible like (2.4). This means that processes can go back and forth. Equation (2.6) describes only collective motions. An example of this is wave propagation. It is also able to describe temporary damping (in the linearized case) of waves, so called Landau damping, due to resonances between particles and waves. Since the plasma parameter g in typical laboratory plasmas is of the order 10^{-8}, collective phenomena usually dominate over phenomena related to close collisions. We mentioned above the Fokker-Planck collision term for close collisions. However, as we already noted in (34) in a random phase situation also turbulent collisions can be described by a Fokker-Planck equation. We will now consider the Vlasov and gyrokinetic equations which are those most frequently used by fusion physicists

Orbit integration

To solve the Vlasov equation we usually divide the distribution Function into an unperturbed and a perturbed part taking the perturbation to be small.

$$f \equiv f_0 + f^1; f_0 >> f^1$$

$$\frac{\partial f^1}{\partial t} + \mathbf{v} \cdot \nabla f^1 + \mathbf{a}_0 \cdot \frac{\partial f^1}{\partial \mathbf{v}} = -\mathbf{a}^1 \cdot \frac{\partial f_0}{\partial \mathbf{v}}$$

$$\frac{D_l f^1}{Dt} = -\mathbf{a}^1 \cdot \frac{\partial f_0}{\partial \mathbf{v}} \qquad \frac{D_l}{Dt} \equiv \frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla + \mathbf{a}_0 \cdot \frac{\partial}{\partial \mathbf{v}} \qquad \mathbf{a}^1 \equiv \frac{q}{m}(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

$$f^1 = -\int_{orb} \mathbf{a}^1 \cdot \frac{\partial f_0}{\partial \mathbf{v}} dt$$

Integration along unperturbed orbit

- This can be generalized by integrating along the perturbed orbit or adding a nonlinear integrand.
- We can also divide the integration into gyroperiods thus deriving a gyrokinetic equation

The Vlasov and Gyrokinetic equations

The Vlasov equation is usually valid for collective modes in the bulk of fusion plasmas. As we have already noted, the MHD and microturbulence modes in a fusion plasma fulfil the condition $\omega \ll \Omega_{ci}$. This thus concerns stability and transport. Heating is thus excluded from this discussion. It is obvious that we can save a lot of time in simulations by first averaging the Vlasov equation over the fast gyro timescale. Thus we get the *Gyrokinetic equation* which we already used. It can in its linear electromagnetic form be written:

$$(\omega - \omega_D(\mathrm{v}_\parallel{}^2, \mathrm{v}_\perp{}^2) - k_\parallel \mathrm{v}_\parallel)(f^{(1)}{}_{k,\omega} + \frac{q\phi_{k,\omega}}{T} f_0)e^{-iL_k} = \left[(\omega - \omega_*)\frac{q}{T}(\phi_{k,\omega} - \mathrm{v}_\parallel \mathrm{A}_\parallel)J_0(\xi_k) - i\frac{\mathrm{v}_\perp}{\mathrm{k}_\perp}(\hat{e}_\parallel \times \mathbf{k})\cdot \mathbf{A_k} J_0'\right] f_0$$

(2.7)

Fluid and gyrofluid equations

- While the Vlasov equation describes the dynamics of a smeared out particle density in phase space, the gyrokinetic equation describes the dynamics of guiding centres. Because of this we get the ordinary fluid equations when taking moments of the Vlasov equation and gyrofluid equations when taking moments of the gyrokinetic equation. Of course the fluid equations are more general than the gyrofluid equations. However, the low frequency expansion of the fluid drifts should give us the same macroscopic description as the gyrofluid equations. An important difference is that while the fluid drifts contain the diamagnetic drifts, the gyrofluid drifts are guiding centre drifts. However, as they should, they give the same density perturbation, i.e.

Fluid and gyrofluid equations

$$\nabla \cdot (n\mathbf{v}_{*j}) = \nabla \cdot (n\mathbf{v}_{\mathbf{D_j}}) = \frac{1}{T}\mathbf{v}_{\mathbf{D_j}} \cdot \nabla P_j$$

- More generally the difference between fluid and particle current is the magnetization current $\mathbf{j_m}$ which fulfills **div $\mathbf{j_m}$ =0.** One interesting comparison is for the ion motion along the magnetic field. Here the gyrofluid equations give directly for electrostatic perturbations:

$$\frac{\partial \delta u_{\|}}{\partial t} + 2\mathbf{v}_{\mathbf{D}} \cdot \nabla \delta u_{\|} = -\hat{e}_{\|} \cdot \nabla(\delta p + en\phi) \qquad (2.8)$$

Where **u** is the guiding centre drift. However it has to be equal to the fluid drift in the parallel direction

Fluid and Gyrofluid equations

- It is here significant that we obtain a convective magnetic drift in (2.8). Of course, the magnetic drift is not a fluid drift. Nevertheless Eq (2.8) is recovered also by using fluid equations, this time through the stress tensor. Since the gyrofluid equations do not have the pressure or stress tensor gradients they are simpler to deal with although obtaining the convective magnetic drift term certainly is more complicated than obtaining convective terms in the derivation of fluid equations. Using gyrofluid equations is sometimes referred to as a new development which is supposed to be more advanced than the ordinary fluid equations. However,
- *gyrofluid equations are approximations of fluid equations*!

Turbulent spectrum- correlation length

The calculation of transport requires a knowledge of the scale length of the turbulence. That depends on sources and sinks.

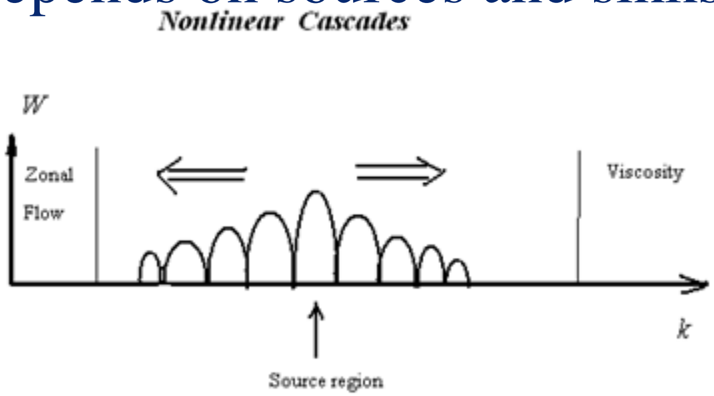

Fig 2.1 Sources and sinks for tokamak turbulence

- We have found in turbulence simulations that with absorbing boundaries in k-space we can use the inverse of the modenumber of the fastest growing mode as correlation length. We can then calculate transort as if this is the only modenumber! This has recently also been found to reproduce stiffness with rotation (J. Weiland and P. Mantica , EPS 2011)

Transport

- The main purpose of studying low frequency perturbations in toroidal magnetized plasmas is to predict transport. We will here give a brief overview. What we are primarily interested in is fluxes:

$$\Gamma_n = < v_{Er}\delta n > \qquad (2.9)$$

$$\Gamma_T = < v_{Er}\delta T > \qquad (2.10)$$

We are here considering only ExB transport. Clearly steady state fluxes are produced by the nonlinear beating of velocity and respective perturbations. Thus

Transport can never be linear!

Quasilinear kinetic models

- The question of whether the transport is quasilinear or fully nonlinear depends on if nonlinear frequency shifts are included in the relation between density or temperature perturbations and potential. In Quasilinear theory we use only the linear eigenfrequency.
- Recently a frequency "width" has been added to a quasilinear model (QualiKiz). This width has then been fitted to fully nonlinear simulations with the nonlinear gyrokinetic code Gyro. This is probably a good way of obtaining an efficient code with better particle pinches than a quasilinear. However, it should not be called quasilinear. This can lead to confusion regarding the physics of quasilinear models and regarding the first quasilinear kinetic models with particle trapping that were presented in 1989 which had very weak particle pinches.

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Saturation level

- Another aspect of nonlinear effects is that we need to know the turbulence level. This is usually obtained by balancing the linear growthrate with the main (ExB) nonlinearity. This leads to:

$$\frac{e\phi}{T_e} = \frac{\gamma}{k_\theta c_s k_r \rho_s} = \frac{\gamma}{\omega_*}\frac{1}{k_r L_n} \qquad (2.11)$$

This typically gives a level of a few percent which is characteristic of experiment. Thus the growthrate is fully nonlinear although it is the linear growthrate that appears in (2.11). This is due to a renormalization by Dupree. It is well known that turbulence is damped by turbulent diffusion at the rate k^2D. Then the instantaneous growthrate is:

$$\gamma(t) = \gamma_{linear} - k_r^{\,2} D \qquad (2.12)$$

Dupree renormalization

- Now, D increases with the turbulence level. Thus we get saturation where $\gamma(t)=0$ and thus

$$D = \gamma_{linear}/\mathrm{k_r}^2 \qquad (2.13)$$

Eq (2.13) is a Markovian form of (1.1). The saturation level (2.11) is actually consistent with (1.1). The non-Markovian feature of (1.1) is obviously the dependence on the real eigenfrequency. The non-Markovian physics involved in (1.1) is that turbulent eddies are rocking at the mode frequency and this reduces the step length.

Momentum transport

- Recently there has been a strong interest in momentum transport. The main reason for this is the need to understand transport barriers, both internat (ITB) and at the edge (ETB) associated with the H- mode barrier. Both ITB's and ETB's are associated with plasma flows, particularly in the poloidal direction.
- Theoretically the leading theories are suggesting the generation of poloidal flows by the Reynolds stress, i.e. the nonlinear convective part of the inertia.

Poloidal spinup due to Reynolds stress

The radial flux of poloidal momentum

$$\frac{\partial U_\theta}{\partial t} + \frac{\partial}{\partial r}\Gamma_p = S_v \qquad (2.14a)$$

$$\Gamma_p = < \mathrm{v}_{\mathrm{Er}} \mathrm{v}_\theta > = -D_B{}^2 k_r k_\theta \frac{1}{2}\hat{\phi}^* \left[\hat{\phi} + \frac{1}{\tau}\hat{P}_i\right] + c.c \qquad (2.14b)$$

We note that we included also the diamagnetic drift in the poloidal velocity but not in the radial. This is because the radial velocity here has the character of a convecting velocity.

The diamagnetic drift is sensitive to the fluid resonance through the temperature perturbation.

We have obtained both ITB’s and ETB’s through the spinup of poloidal velocity.

Toroidal momentum transport

We may approximate the toroidal momenum with the parallel momentum. It is described by the equation:

$$m_i n_i \left(\frac{\partial}{\partial t} + 2\vec{U}_{Di} \cdot \vec{\nabla} \right) \delta u_{\|} = -m_i n_i \vec{u}_E \cdot \nabla U_{\|0} - \left[\hat{e}_{\|} \cdot \nabla + U_{\|0} \frac{m_i \vec{U}_{D_i}}{T_i} \cdot \nabla \right] \left(\delta p_i + e n_i \phi - \frac{\omega + \omega_{*e}(1+\eta_e)/\tau}{k_{\|} c} A_{\|} \right) \tag{2.15}$$

We recognize the convective magnetic drift from (2.8). However, in (2.15) we have added also a background parallel flow $U_{\|0}$. This leads to the new magnetic drift term in the right hand side. Eq (2.15) was obtained from a fluid derivation, including the stress tensor. (J. Weiland et. al. Nnuclear Fusion 2009) However, the same term can also be obtained from gyrofluid equations including the Coriolis acceleration. (Hahm et. Al PoP 2008, Peeters et. al. PRL 2008).

Simulations

- Momentum is, in principle, consereved in the absence of sources. Nevertheless a poloidal spinup can be obtained in transport barriers

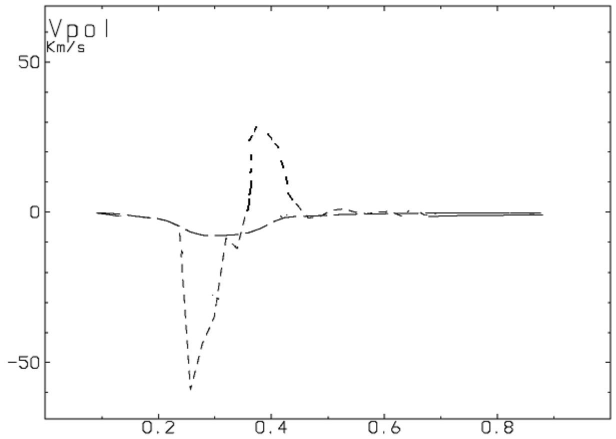


Fig 2.2 Qualitative and semiquantitative poloidal spinup with fixed temperatures

- Momentum is, in principle, consereved in the absence of sources. Nevertheless a poloidal spinup can be obtained in transport barriers by producing a balansing rotation in another region (See Fig 2.2)

Poloidal spinup

- In a self consistent simulation with varying temperatures, poloidal momentum is not conserved.

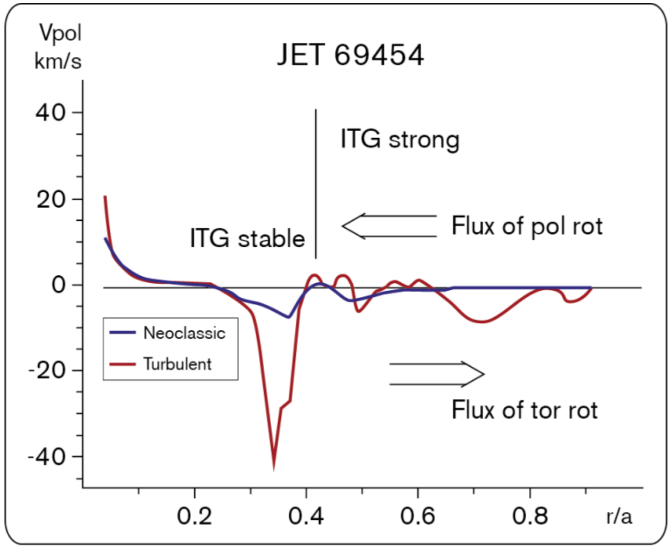


Fig 2.3 Qualitative and semiquantitative poloidal spinup with varying temperatures

General features of model

The model includes the following features:

Our usual electromagnetic fluid model for ITG and TE modes
with transport of energy and momenta (includes pressure gradient drive)
Current gradient (kink) drive
Collisions on both trapped and free electrons

This gives the following modes:
ITG (both toroidal and slab), *TE* modes, collisionless (driven by electron or density gradients) and collision dominated
MHD and kinetic *Ballooning* modes
Peeling modes
Resistive ballooning modes

Edge barrier

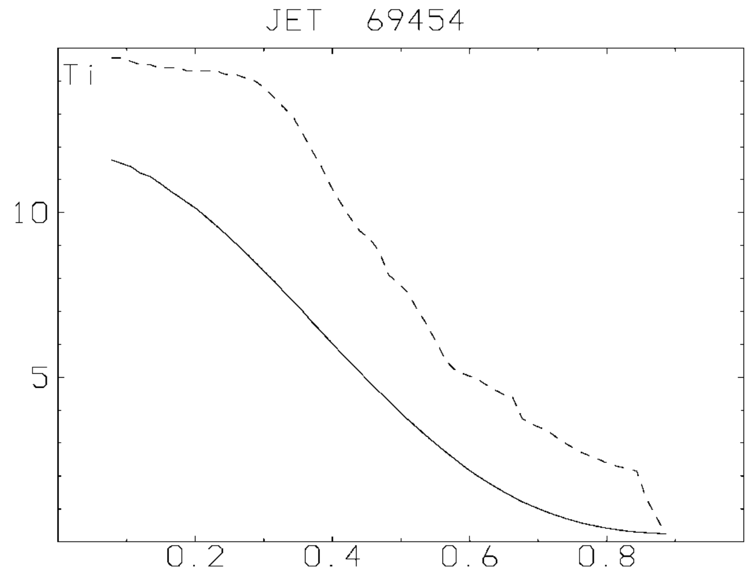


Fig 2.4

Start profile

Simulation

Experimental T_i at r/a = 0.9 was around 1.5 KeV. B_p =0.2T

Similarities between Transport barriers in Core and Edge

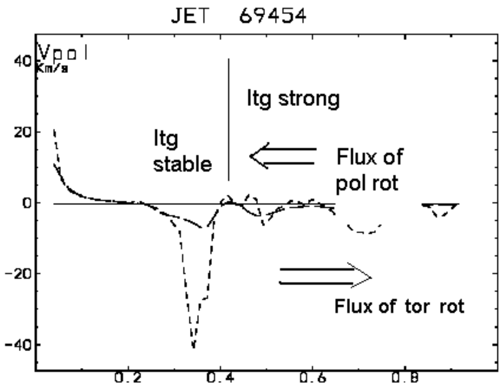
Electromagnetic – Nonlocal simulations

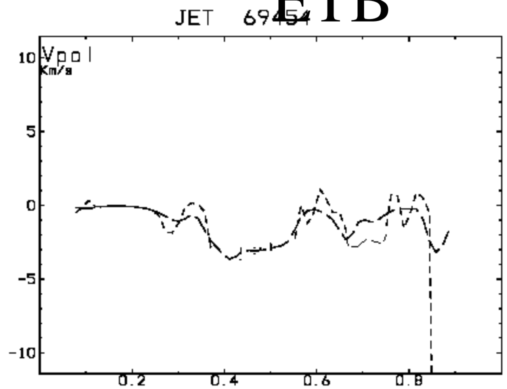
Fig 2.6

- ITB

Fig 2.5

ETB





J. Weiland et. al EPS Dublin 2010

J. Weiland TTG Cordoba 2010

Strong poloidal spinup both in internal barrier (ITB) and in edge barrier (ETB). Both electromagnetic and nonlocal effects needed for the internal barrier. For the edge barrier we also need nonlocal effects but electromagnetic effects reduce the barrier.

Flowshear

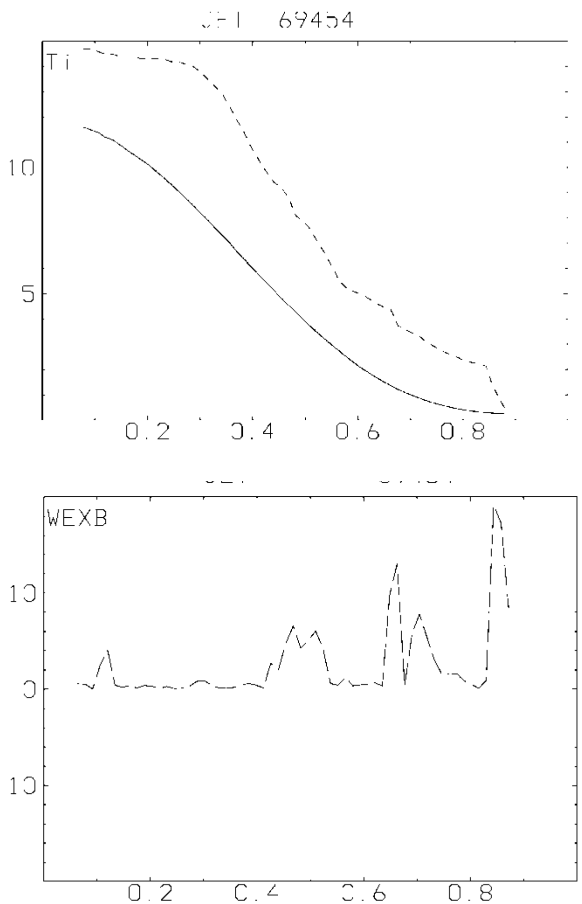


Fig 2.7a,b

Ion temperature and Flowshear profiles showing why we get stabilization at the edge. Note that this was obtained self-consistently in a global simulation The flowshear is driven primarily by the poloidal nonlinear spinup of rotation. Careful study of simulation data shows that a mode propagating in the electron drift direction is unstable at the edge point and at the first point inside the edge.

Peeling

Preliminary simulations have also been made with the inclusion of a kink term (peeling)

$$\frac{\partial n_{ef}}{\partial t} + \nabla \cdot \left[n_{ef} \left(\mathbf{v}_{\mathbf{E}} + \mathbf{v}_{*\mathbf{e}} + \mathrm{v}_{\parallel} \frac{\delta \mathbf{B}_{\perp}}{\mathrm{B}} + \mathrm{v}_{\parallel} \hat{\mathbf{e}}_{\parallel} \right) \right] = 0$$

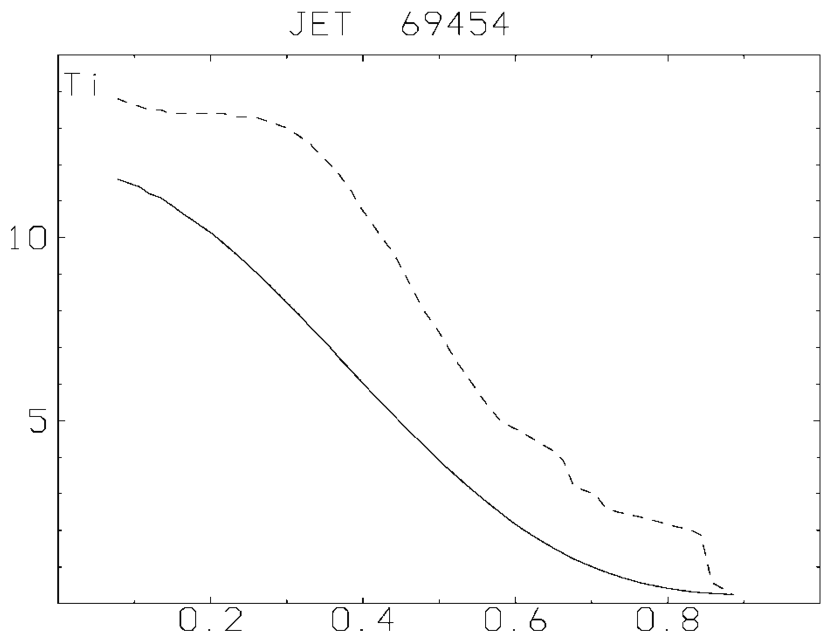


Fig 2.8. This case corresponds to Fig 2.4, As seen also without peeling, a mode rotating in the electron drift direction gets unstable at the outer end of the barrier. This trend gets stronger when peeling is included.

Peeling

Peeling tends to create a shelf with smaller slope at the outer edge of the barrier while the remaining barrier gets steeper

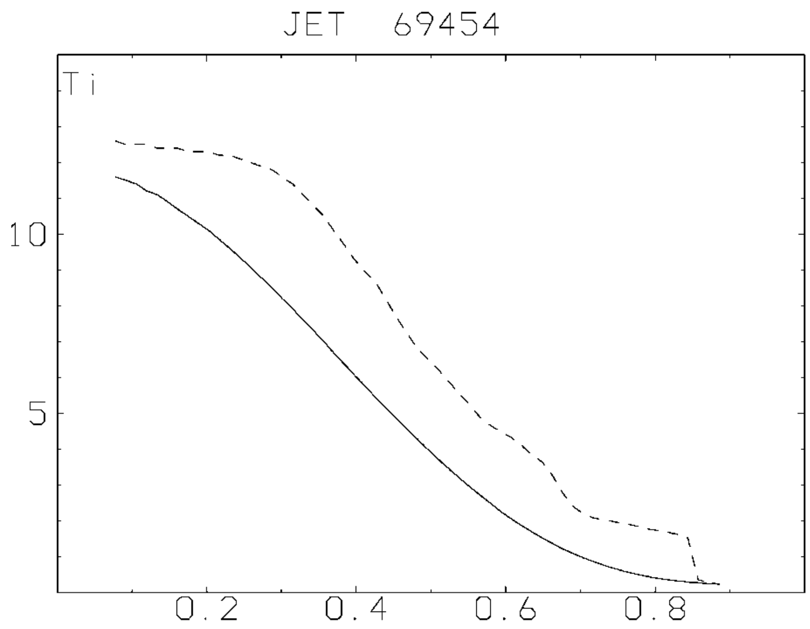


Fig 1.4. This case has 50% increase in B_p and experimental edge density

Discussion

We have here applied a transport code for both ITB's and ETB's. The principle justifying this is the same as for core transport, i.e. in a phase mixed situation we can use the correlation length corresponding to the inverse mode number of the fastest growing mode. This means that in a phase mixed situation with a broad spectrum, the sidebands studied in low dimensional nonlinear systems will be part of the broadband turbulence giving the correlation length as the inverse modenumber of the fastest growing mode. As it turns out, nonlocal and electromagnetic effects are important for both ITB and ETB just as in turbulence simulations.

In the broadband, phase mixed situation we can use the model of Hinton and Staebler (Phys. Fluids B**5**, 1281 (1993)) modified to dominating poloidal flow, to describe the bifurcation.

Summary

Previous results on the formation of an internal transport barrier have been extended to include also the edge barrier.

Electromagnetic and **nonlocal** effects play dominant roles in both cases.

The **turbulent spinup** of poloidal rotation is instrumental for both transitions.

Our parameter dependent correlation length gives a realistic description of turbulence also in the edge barrier.

The peeling mechanism leads to further excitation of an electron mode close to the outer boundary.

Overall summary

The field of confinement in toroidal fusion devices is quite complicated but most aspects are now under control. However, in particular the questions of the edge pedestals are still partly unresolved. The fluid model described here gives fusion Q close to 9 for a pedestal height of 4 Kev and with the density used in the ITER design. However this is partly due to help from a toroidal momentum pinch. If we, on the other hand use the particle transport in the model a particle pinch leads to substantially higher Q.