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A primer on gyrokinetic theory and simulation

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Some introductory remarks

My lecture series will cover three interrelated topics:

- A primer on gyrokinetic theory and simulation
- Features of linear and nonlinear gyrokinetics
- Recent progress towards a numerical tokamak

I will attempt to present the material in an accessible way

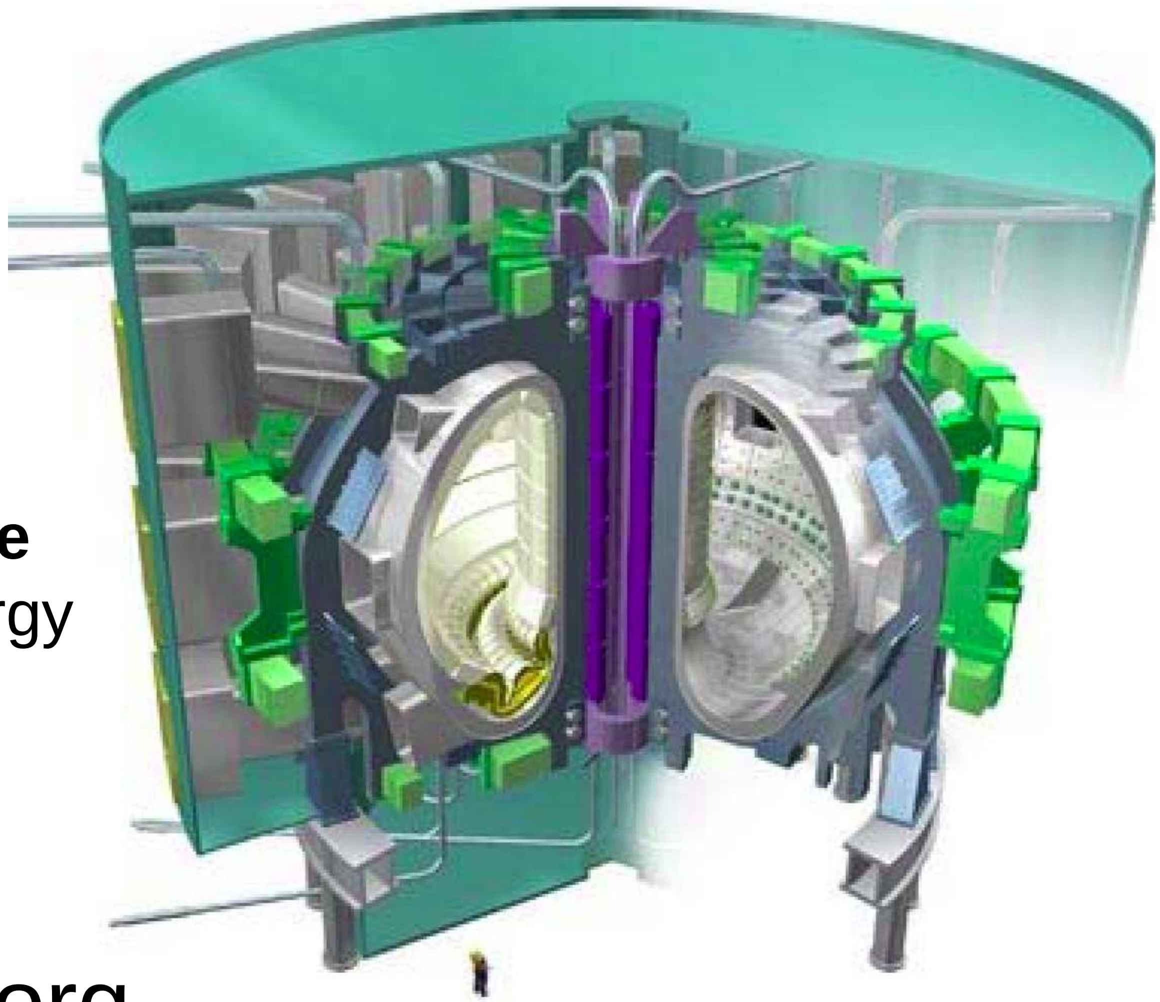
Please feel free to interrupt me if you have a question

Why invent gyrokinetics?

ITER and plasma turbulence

ITER is one of the most challenging scientific projects

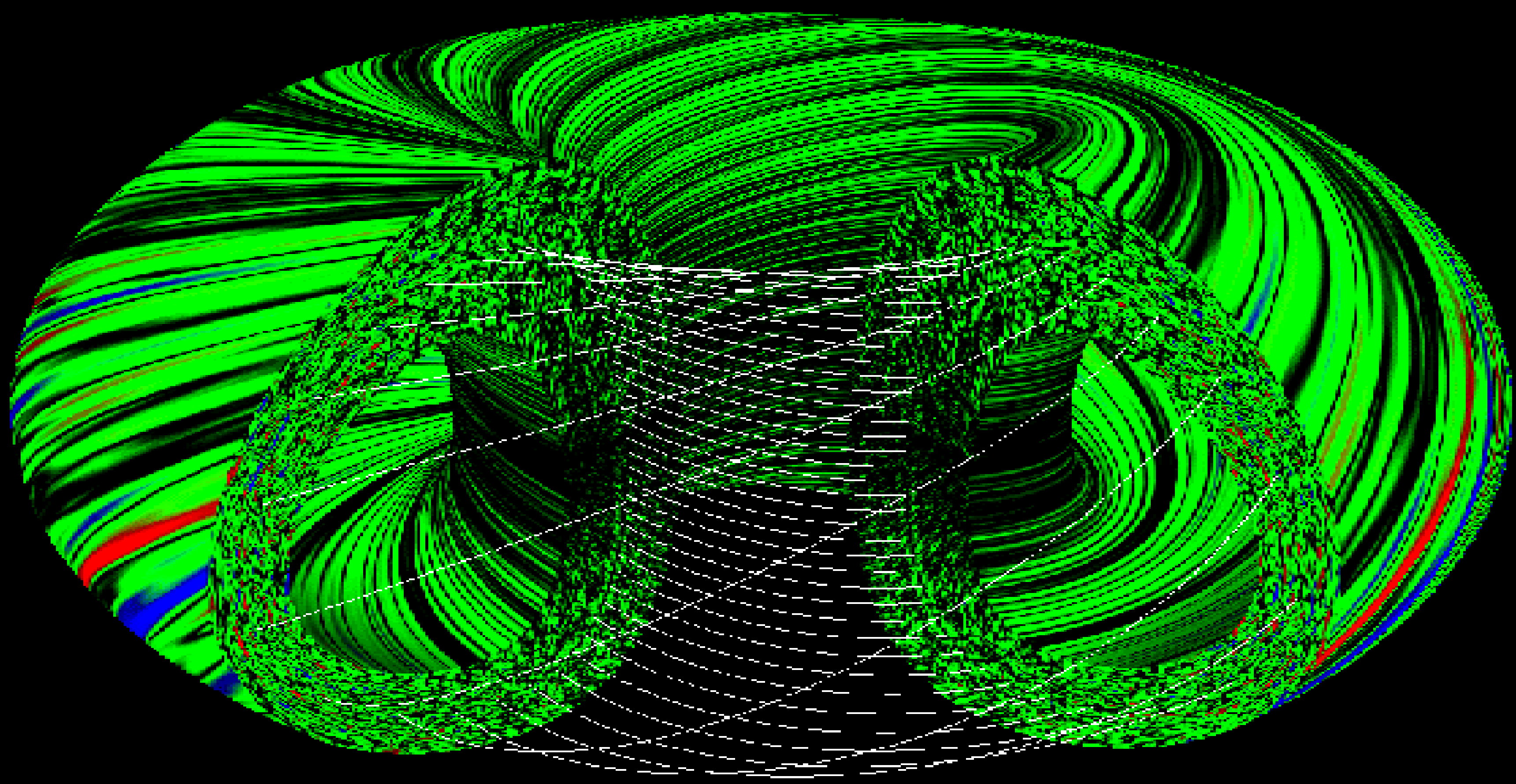
Plasma turbulence determines its energy confinement time



www.iter.org

Turbulent fluctuations are quasi-2D

Reason: Strong background magnetic field



Use field-aligned coordinates and minimize the simulation volume

Turbulent mixing in a tokamak

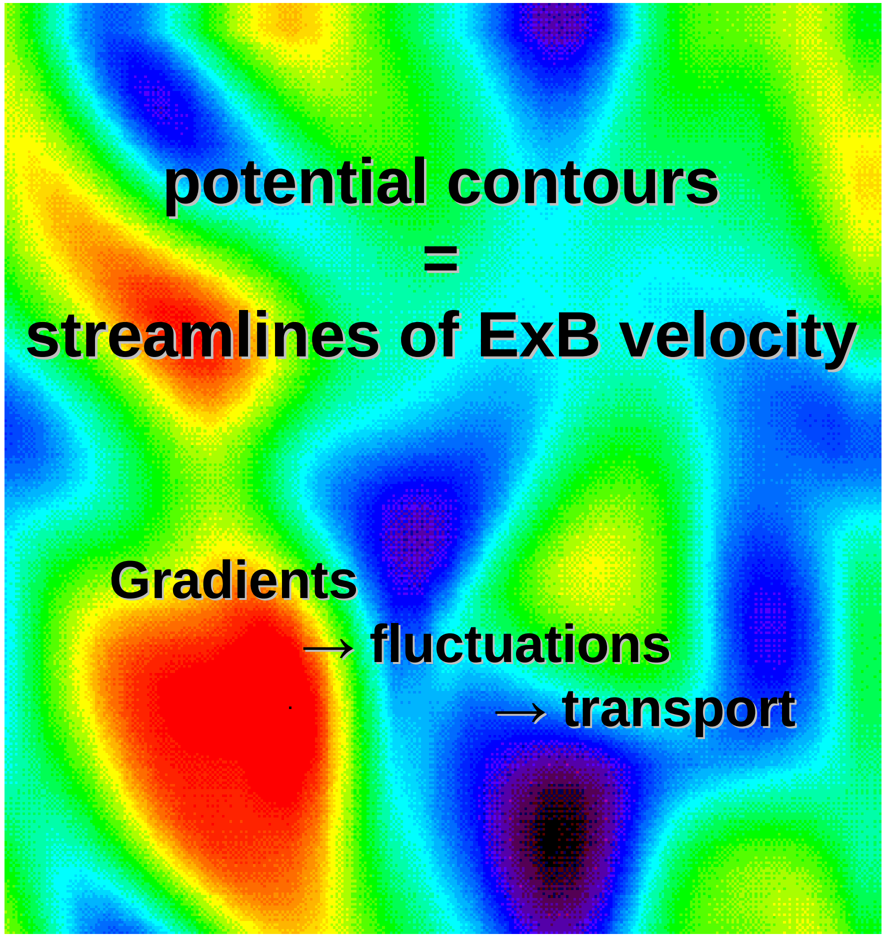
ExB drift velocity

$$\tilde{\mathbf{v}}_E = \frac{c}{B^2}\mathbf{B}\times\nabla\tilde{\phi}$$

$$\mathbf{Q} \equiv \frac{3}{2}\langle\tilde{p}\,\tilde{\mathbf{v}}_E\rangle = -n\chi\nabla T$$

$$\chi \sim \frac{(\delta x)^2}{\delta t} \sim \frac{\rho^2 v_t}{L_T}$$

(random walk/mixing length estimates)



Typical heat and particle diffusivities are of the order of 1 m^2/s.

Gyrokinetic theory:
A brief guided tour

What is gyrokinetic theory?

Dilute and/or hot plasmas are almost collisionless.

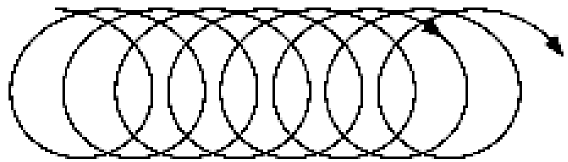
Thus, if kinetic effects (finite Larmor radius, Landau damping, magnetic trapping etc.) play a role, MHD is not applicable, and one has to use a kinetic description!

Vlasov-Maxwell equations

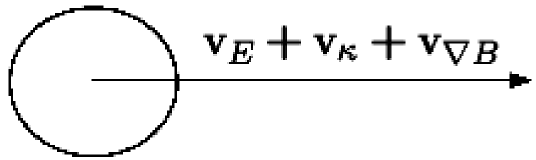
$$\left[\frac{\partial}{\partial t} + \mathbf{v}\cdot\frac{\partial}{\partial \mathbf{x}} + \frac{q}{m}\left(\mathbf{E} + \frac{\mathbf{v}}{c}\times\mathbf{B}\right)\cdot\frac{\partial}{\partial \mathbf{v}}\right] f(\mathbf{x},\mathbf{v},t) = 0$$

Removing the fast gyromotion
leads to a dramatic speed-up

$$\omega \ll \Omega$$



Charged rings as quasiparticles;
gyrocenter coordinates; keep kinetic effects



Details may be found in: Brizard & Hahm, Rev. Mod. Phys. **79**, 421 (2007)

The gyrokinetic ordering

- The gyrokinetic model is a Vlasov-Maxwell on which the GK ordering is imposed:

$\Rightarrow$ Slow time variation as compared to the gyro-motion time scale:

$$\omega/\Omega_i \sim \epsilon_g \ll 1$$

$\Rightarrow$ Spatial equilibrium scale much larger than the Larmor radius:

$$\rho/L_n \sim \rho/L_T \equiv \epsilon_g \ll 1$$

$\Rightarrow$ Strong anisotropy, i.e. only perpendicular gradients of the fluctuating quantities can be large ($k_\perp \rho \sim 1$, $k_\| \rho \sim \epsilon_g$):

$$k_\|/k_\perp \sim \epsilon_g \ll 1$$

$\Rightarrow$ Small amplitude perturbations, i.e. energy of perturbation much smaller than the thermal energy:

$$e\phi/T_e \sim \epsilon_g \ll 1$$

A brief historical review

A. Bottino

- The word "Gyrokinetic" appeared in the literature in the late sixties.
  Rutherford and Frieman, Taylor and Hastie [1968].

  Goal: Provide a adequate formalism for the linear study of kinetic drift-waves in general magnetic configurations, including finite Larmor radius effects.

- First nonlinear set of equations for the perturbed distribution function δF.
  Frieman and Liu Chen [1982].
  $\rightarrow$ Gyrokinetic ordering.

- Littlejohn [1979], Dubin [1983], Hahm[1988], Brizard [1989], ...

  Firm and more transparent theoretical foundation for GK:

  GK equations based on Hamiltonian or Lagrangian variation methods.

Lagrangian
$\Downarrow$
remove gyro-angle dependency in Lagrangian
(change of coordinate system)
$\Downarrow$
equation of motion

A Lagrangian approach

If the Lagrangian of a dynamical system is known...

Example: charged particle motion, in non canonical coordinates $(\vec{x}, \vec{v})$:

$$\begin{aligned} L &= \left(\frac{e}{c}\vec{A}(\vec{x},t) + m\vec{v}\right) \cdot \dot{\vec{x}} - H(\vec{x},\vec{v}) \\ H &= \frac{m}{2}v^2 + e\phi(\vec{x},t) \end{aligned}$$

with $\vec{B} = \nabla \times \vec{A}$ and $\vec{E} = -\nabla\phi - \partial_t \vec{A}/c$.

...the equation of motion are given by the Lagrange equations:

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial L}{\partial \dot{q}_i} - \frac{\partial L}{\partial q_i} = 0 \quad \text{with } i = 1, \ldots, 6$$

Lagrange equation of motion for a charged particle:

$$\begin{aligned} \vec{v} \Rightarrow \quad & -\tfrac{\partial L}{\partial \vec{v}} = 0 && \Rightarrow \dot{\vec{x}} = \vec{v} \\ \vec{x} \Rightarrow \quad & \tfrac{\mathrm{d}}{\mathrm{d}t}\tfrac{\partial L}{\partial \dot{\vec{v}}} - \tfrac{\partial L}{\partial \vec{v}} = 0 && \Rightarrow \dot{\vec{v}} = \frac{e}{m}\left(\vec{E} + \frac{\vec{v}}{c} \times \vec{B}\right) \end{aligned}$$

Guiding center coordinates

GK ordering, but $k_\perp \rho \simeq 1 \Rightarrow k_\perp \rho \ll 1$
$\Rightarrow \vec{B}(\vec{x})$, static magnetic field.

- Single particle Lagrangian:

$$L = \left(\frac{e}{c}\vec{A}(\vec{x}) + m\vec{v}\right) \cdot \dot{\vec{x}} - \frac{m}{2}v^2 + e\phi(\vec{x}, t)$$

- Change of coordinates:

particle coordinates $(\vec{x}, \vec{v}) \Rightarrow$ guiding center coordinates $(\vec{R}, v_\parallel, \mu, \varphi)$

$$\begin{aligned} \vec{x} &= \vec{R} + \vec{\rho} \equiv \vec{R} + \frac{v_\perp}{\Omega}\hat{a}(\vec{R}, \varphi) \\ \mu &= v_\perp^2 / 2B(\vec{R}) \\ v_\parallel &= \vec{v} \cdot \vec{b} \\ \varphi &= \tan^{-1}\left(\frac{\vec{v} \cdot \vec{e}_1}{\vec{v} \cdot \vec{e}_2}\right) \end{aligned}$$

$\vec{R}$ guiding center position; $\Omega \equiv eB/mc$ gyrofrequency.
$\hat{a} \equiv \cos(\varphi)\ \vec{e}_1 + \sin(\varphi)\ \vec{e}_2$
$\vec{e}_1(\vec{R}, \varphi)$, $\vec{e}_2(\vec{R}, \varphi)$ orthogonal unity vectors in the plane perpendicular to $\vec{b} \equiv \vec{B}/B$.

Guiding center coordinates (cont'd)

$$L_{DK} = \left(mv_{\|}\vec{b} + \frac{e}{c}\vec{A}(\vec{R})\right)\cdot\dot{\vec{R}} + \frac{\mu B}{\Omega}\dot{\varphi} - H_{DK}$$

$$H_{DK} = \frac{m}{2}v_{\|}^2 + \mu B + q\phi(\vec{R})$$

- Lagrange equations:

$$\dot{\vec{R}} = v_{\|}\vec{b} + \frac{B}{B_{\|}^*}\left(\vec{v}_{E\times B} + \vec{v}_{\nabla B} + \vec{v}_C\right)$$

$$\dot{v}_{\|} = \left(-\mu\nabla B + e\vec{E}\right)\cdot\frac{\dot{\vec{R}}}{mv_{\|}} \quad ; \quad \dot{\mu} = 0 \quad ; \quad \dot{\varphi} = \Omega$$

$\vec{v}_{E\times B} \equiv$	$\frac{c}{B^2}\,\vec{E}\times\vec{B}$	$E\times B$ drift
$\vec{v}_{\nabla B} \equiv$	$\frac{\mu}{m\Omega}\,\vec{b}\times\nabla B$	∇B drift
$\vec{v}_C \equiv$	$\frac{v_{\|}^2}{\Omega}\,\vec{b}\times(\vec{b}\cdot\nabla)\vec{b}$	Curvature drift

with $\vec{B}^* \equiv \vec{B} + (mc/e)v_{\|}\nabla\times\vec{b} = B(1+\mathcal{O}(\rho_{\|}/L_B))$.

Including fluctuating fields

Particle world, $(\vec{x}, \vec{v})$

$\Downarrow$

static fields

$\Downarrow$

Guiding center, $(\vec{R}, v_{\|}, v_{\perp}, \varphi)$
(remove gyro-angle dependence)

$\Downarrow$

→ Fluctuating fields

$\Downarrow$

Gyrocenter coordinates $(\bar{\vec{R}}, \bar{v}_{\|}, \bar{v}_{\perp}, \bar{\varphi})$
(remove gyro-angle dependence)

$\Downarrow$

Equations of motion
Vlasov equation
Self consistent Poisson equation
(Self consistent Ampere's law)

Resulting Lagrangian 1-form

Eliminate explicit gyrophase dependence via near-identity (Lie) transforms to gyrocenter coordinates:

$$\Gamma \;=\; \left(m v_{||}\mathbf{b}_0 + \frac{e}{c}\,\bar{A}_{1||}\,\mathbf{b}_0 + \frac{e}{c}\,\mathbf{A}_0\right)\cdot d\mathbf{X} + \frac{mc}{e}\,\mu\, d\theta \,- \\ -\left(\frac{m}{2}v_{||}^2 + \mu B_0 + \mu \bar{B}_{1||} + e\,\bar{\phi}_1\right)\, dt$$

$$\bar{\phi}_1 \equiv I_0(\lambda)\,\phi_1\,, \quad \bar{A}_{1||} \equiv I_0(\lambda)\,A_{1||}\,, \quad \bar{B}_{1||} \equiv I_1(\lambda)\,B_{1||}$$

Euler-Lagrange equations

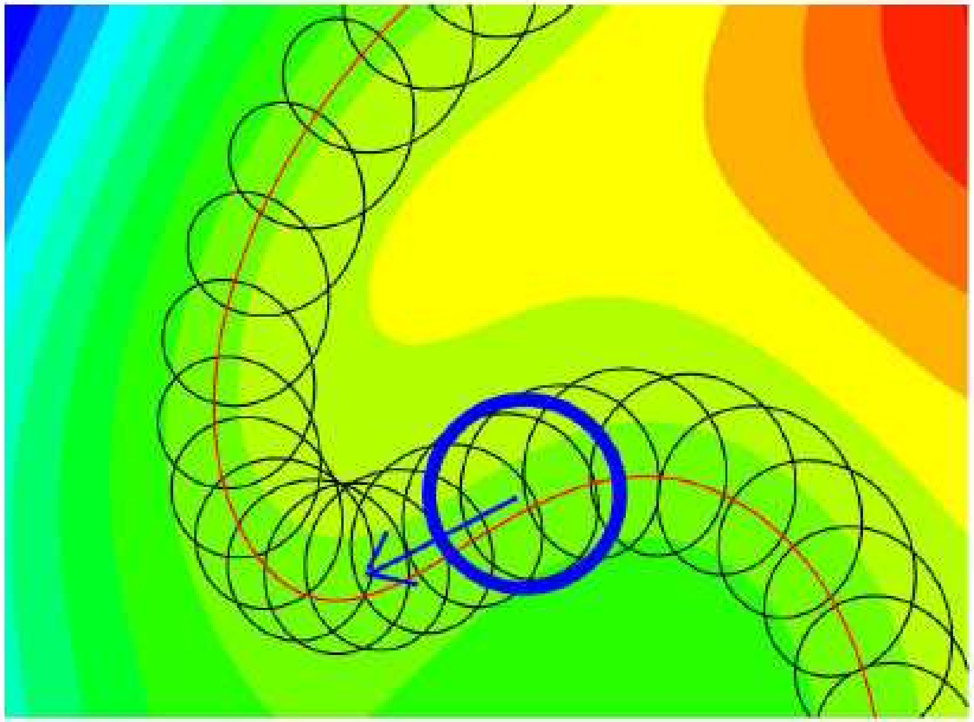
$$\dot{\mathbf{X}} = v_{||}\,\mathbf{b} + \frac{B}{B_{||}^*}\left(\frac{v_{||}}{B}\,\bar{\mathbf{B}}_{1\perp} + \frac{c}{B^2}\,\bar{\mathbf{E}}_1 \times \mathbf{B} + \frac{\mu}{m\Omega}\,\mathbf{b} \times \nabla(B + \bar{B}_{1||}) + \frac{v_{||}^2}{\Omega}\,(\nabla \times \mathbf{b})_\perp\right)$$

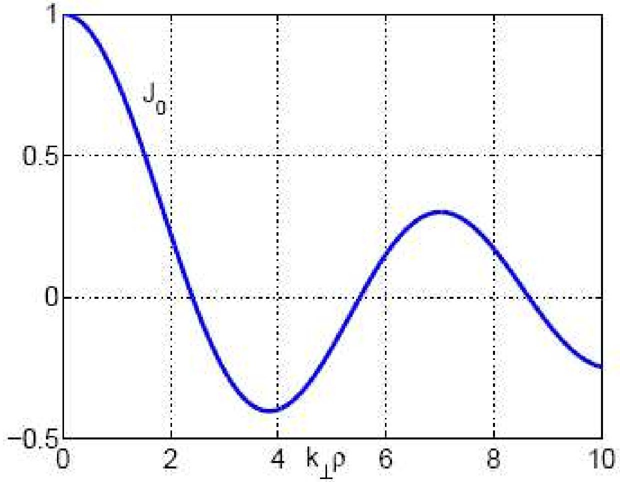
$$\dot{v}_{||} = \frac{\dot{\mathbf{X}}}{m v_{||}} \cdot \left(e\bar{\mathbf{E}}_1 - \mu\nabla(B + \bar{B}_{1||})\right) \qquad \dot{\mu} = 0$$

$$f = f(\mathbf{X}, v_{||}, \mu; t)$$

$$\frac{\partial f}{\partial t} + \dot{\mathbf{X}} \cdot \frac{\partial f}{\partial \mathbf{X}} + \dot{v}_{||}\,\frac{\partial f}{\partial v_{||}} = 0$$

Gyroaveraged potentials





- Full Lorentz dynamics
- Gyrokinetic approx.: $\phi^{\text{eff}}(\vec{x},\rho) = \frac{1}{2\pi}\int_0^{2\pi} d\varphi \Phi(\vec{x}+\vec{\rho})$
  $= \frac{1}{(2\pi)^2}\int_{-\infty}^{\infty} d\vec{k}\, e^{i\vec{k}\vec{x}}\phi(\vec{k})J_0(|\vec{k}|\rho)$

Appropriate field equations

Reformulate Maxwell's equations in gyrocenter coordinates:

$$\nabla_\perp^2 \phi_1 = -4\pi \sum e n_1 \, , \quad \frac{n_1}{n_0} = \frac{\bar{n}_1}{n_0} - \left(1 - \|I_0^2\|\right) \frac{e\phi_1}{T} + \|x I_0 I_1\| \frac{B_{1\|}}{B} \, ,$$

$$\nabla_\perp^2 A_{1\|} = -\frac{4\pi}{c} \sum \bar{J}_{1\|} \, ,$$

$$\frac{B_{1\|}}{B} = -\sum \epsilon_\beta \left(\frac{\bar{p}_{1\perp}}{n_0 T} + \|x I_1 I_0\| \frac{e\phi_1}{T} + \|x^2 I_1^2\| \frac{B_{1\|}}{B} \right) \, ,$$

Nonlinear integro-differential equations in **5 dimensions**...

Major theoretical speedups

G.Hammett

relative to original Vlasov/pre-Maxwell system on a naïve grid, for ITER $1/\rho_* = a/\rho \sim 1000$

- Nonlinear gyrokinetic equations
  - eliminate plasma frequency: $\omega_{pe}/\Omega_i \sim m_i/m_e$ — x10^3
  - eliminate Debye length scale: $(\rho_i/\lambda_{De})^3 \sim (m_i/m_e)^{3/2}$ — x10^5
  - average over fast ion gyration: $\Omega_i/\omega \sim 1/\rho_*$ — x10^3

- Field-aligned coordinates
  - adapt to elongated structure of turbulent eddies: $\Delta_{||}/\Delta_{\perp} \sim 1/\rho_*$ — x10^3

- Reduced simulation volume
  - reduce toroidal mode numbers (i.e., 1/15 of toroidal direction) — x15
  - $L_r \sim a/6 \sim 160\ \rho \sim 10$ correlation lengths — x6

- **Total speedup** — x10^{16}

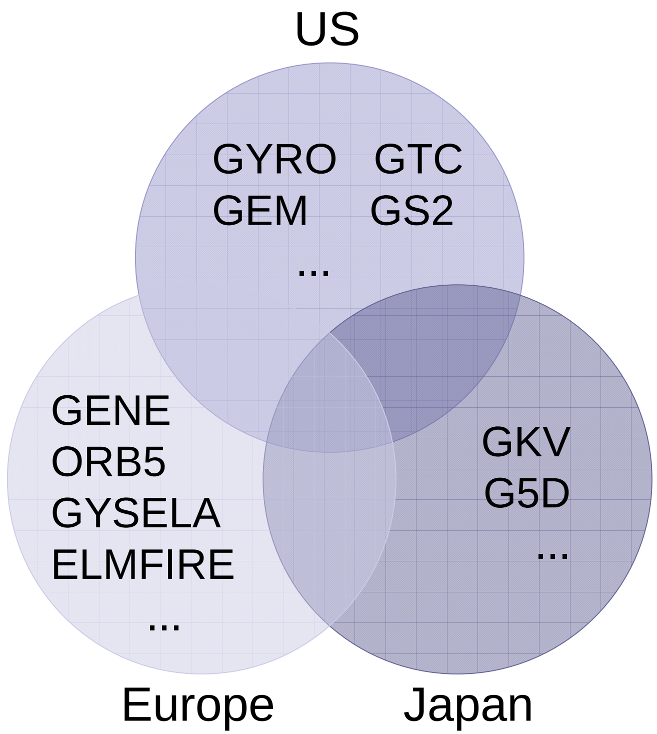
- For comparison: Massively parallel computers (1984-2009) — x10^7

Status quo in gyrokinetic simulation

- over the last decade or so, GK has emerged as the standard approach to plasma turbulence

- a variety of nonlinear GK codes is being used and (further) developed

- these codes differ with respect to their numerics, physics, parallel scalability, and public availability



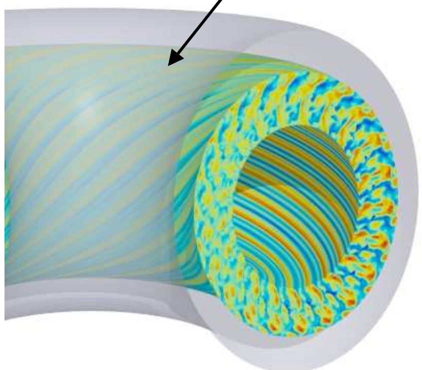
Extreme computing in support of ITER: The GENE code

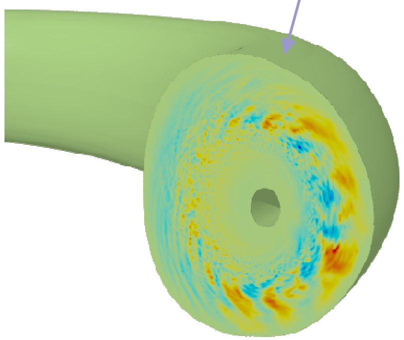
The gyrokinetic Vlasov code GENE

http://gene.rzg.mpg.de

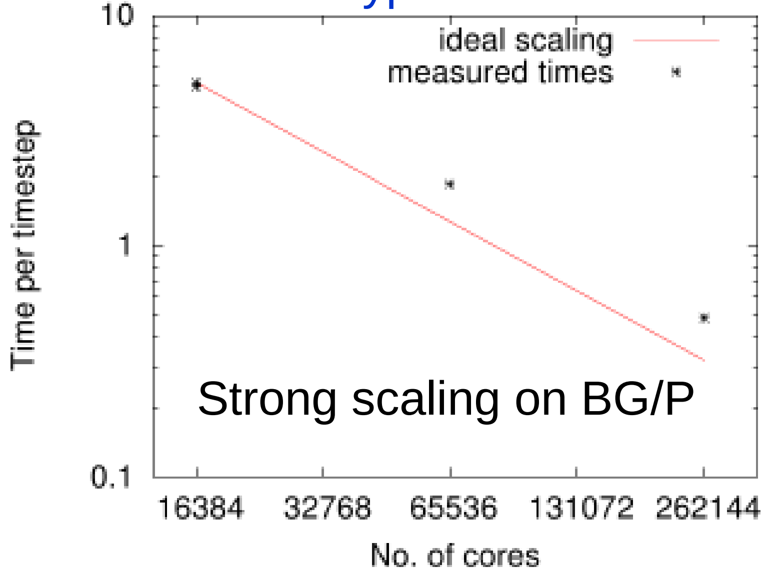
GENE is a physically comprehensive Vlasov code:

- allows for kinetic electrons & electromagnetic fluctuations, collisions, and external ExB shear flows
- is coupled to various MHD equilibrium codes and two transport codes
- can be used as **initial value** or **eigenvalue** solver
- supports **local** (flux-tube) and **global** (full-torus), gradient-driven and flux-driven simulations





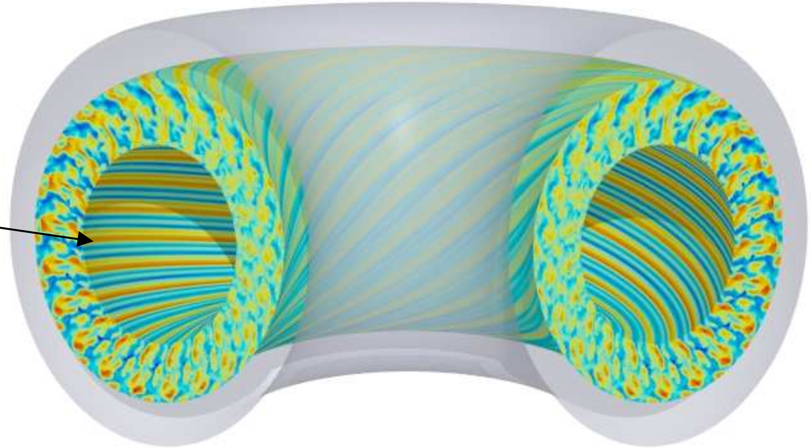
GENE is well benchmarked and hyperscalable



Concepts for efficiency & flexibility

Field-aligned (Clebsch-type) coordinates
to exploit the high anisotropy of plasma turbulence

parallel (z) derivatives taken to be small compared to perpendicular (x,y) ones



δf-splitting:
Apply same approach as in the derivation of the GKE and split the distribution function

$$f = F_0 + \delta f$$

where
F_0: stationary background,
here: local Maxwellian
δf: fluctuating part with $\delta f/F_0 << 1$

Lowest-order nonlinearity kept

$$\sim \frac{\partial\bar{\phi}_1}{\partial x}\frac{\partial f_{1\sigma}}{\partial y} - \frac{\partial\bar{\phi}_1}{\partial y}\frac{\partial f_{1\sigma}}{\partial x}$$

next order

$$-\left\{\frac{q_\sigma}{m_\sigma}\nabla_\parallel\bar{\phi}_1 + c\frac{B_0}{B^*_{0\parallel}}v_\parallel\frac{1}{B_0^2}\left(\mathbf{B}_0\times\frac{\nabla B_0}{B_0}\right)\cdot\nabla\bar{\phi}_1\right\}\frac{\partial f_{1\sigma}}{\partial v_\parallel}$$

can be switched on for testing

Numerical methods – Time stepping scheme

Method of Lines:

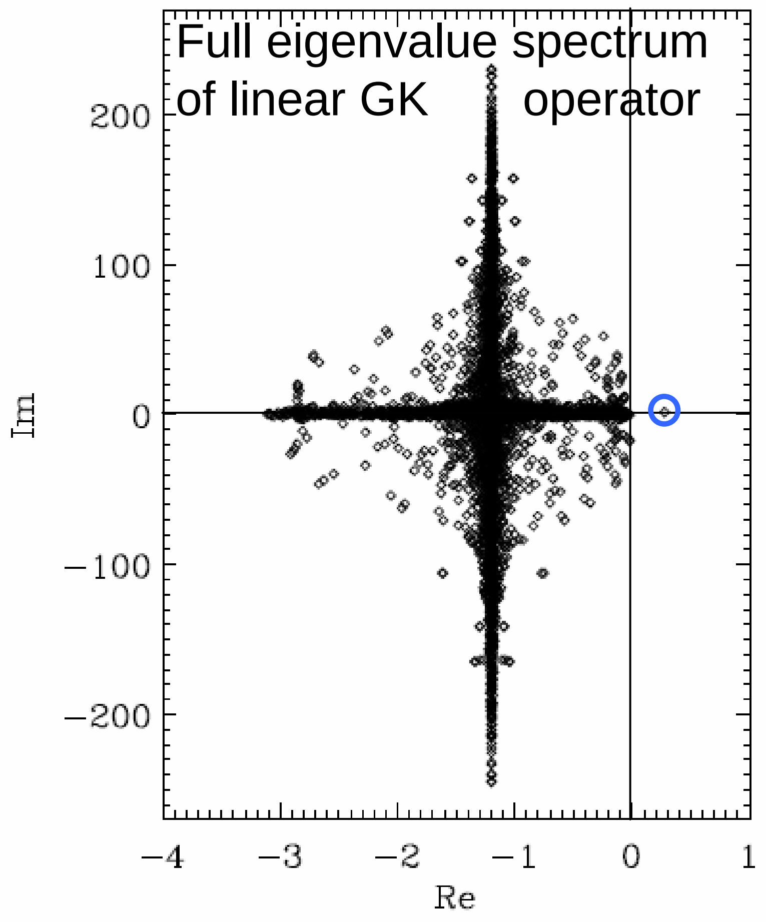
- turn PDEs into ODEs by discretizing the spatial derivative first
- solve for the continuous time coordinate

Time Solver:

Linear system: $$\frac{\partial g}{\partial t} = \mathcal{L}[g]$$

- *Iterative eigenvalue solver*
  based on PETSc/SLEPc/Scalapack lib's
  → solve for largest abs/re/im eigenvalues
  → gain insight in linear stability/physics

- *Explicit Runge-Kutta* (ERK) schemes



Numerical methods – Time stepping scheme (cont'd)

Time Solver:

(Full) **Nonlinear system:**

$$\frac{\partial g}{\partial t} = \mathcal{V}[g] = Z + \mathcal{L}[g] + \mathcal{N}[g]$$

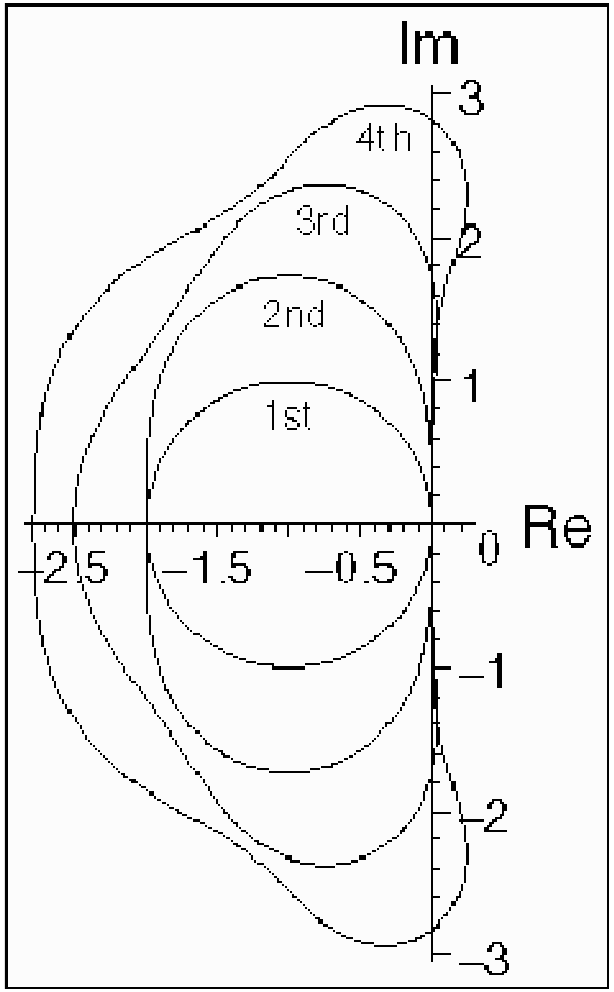
- Several *ERK* methods, e.g. 4th order

$$g_{n+1} = g_n + \frac{\Delta t}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$$k_1 = \mathcal{V}(t_n, g_n),$$
$$k_2 = \mathcal{V}(t_n + \Delta t/2, g_n + k_1 \Delta t/2),$$
$$k_3 = \mathcal{V}(t_n + \Delta t/2, g_n + k_2 \Delta t/2),$$
$$k_4 = \mathcal{V}(t_n + \Delta t, g_n + k_3 \Delta t).$$

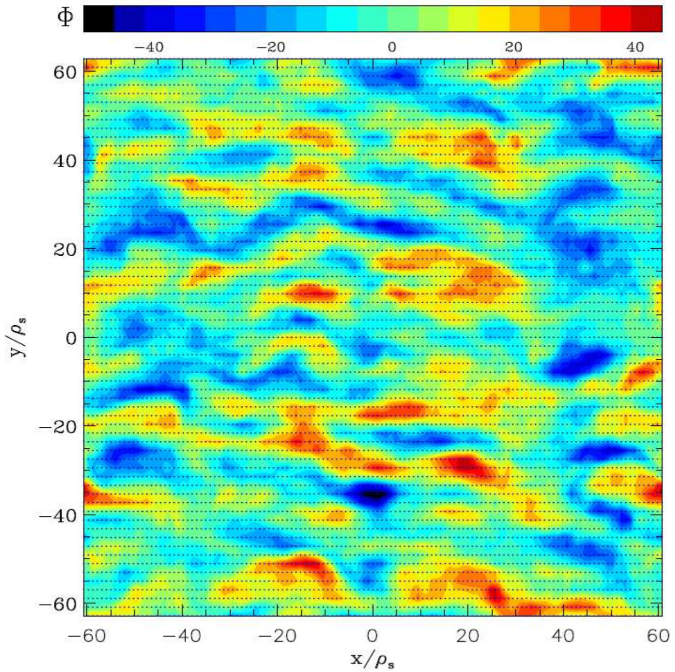
- Optimum linear time step can be precomputed using iterative EV solver
- Adaptive CFL time step adaption for nonlinearity

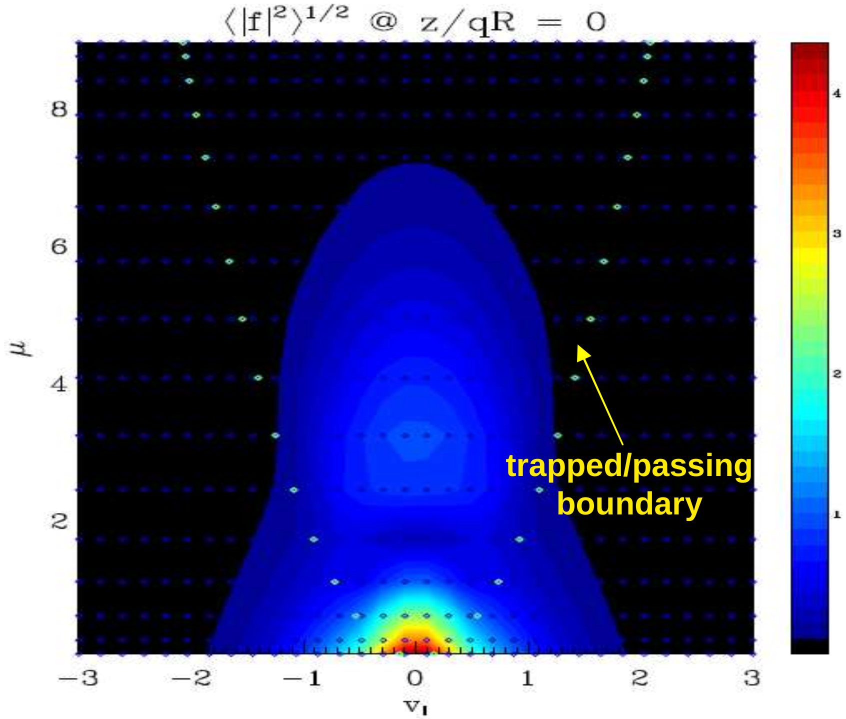
Linear stability regions for low-order ERK schemes



Phase space discretization

- GENE is a *Eulerian* code, representing the 5D distribution function on a fixed grid in (**x**, $v_\parallel$, μ)





radial direction x:	equidistant grid (either configuration or Fourier space)
toroidal direction y:	equidistant grid in Fourier space
parallel direction z:	equidistant grid points
$v_\parallel$-velocity space:	equidistant grid
μ-velocity space:	Gauss-Legendre or Gauss-Laguerre knots

Phase space discretization (cont'd)

$$\frac{\partial f_\sigma}{\partial t} + \dot{\mathbf{X}} \cdot \nabla f_\sigma + \dot{v}_\parallel \frac{\partial f_\sigma}{\partial v_\parallel} = 0$$

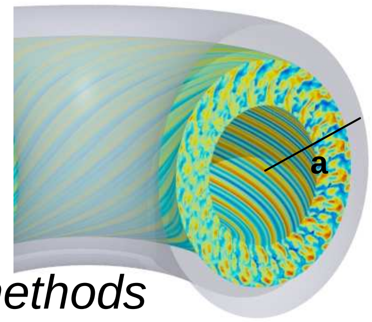
$$\frac{\partial f}{\partial z} \rightarrow \frac{f(z_{(k-2)}) - 8f(z_{(k-1)}) + 8f(z_{(k+1)}) - f(z_{(k+2)})}{12\Delta z}$$

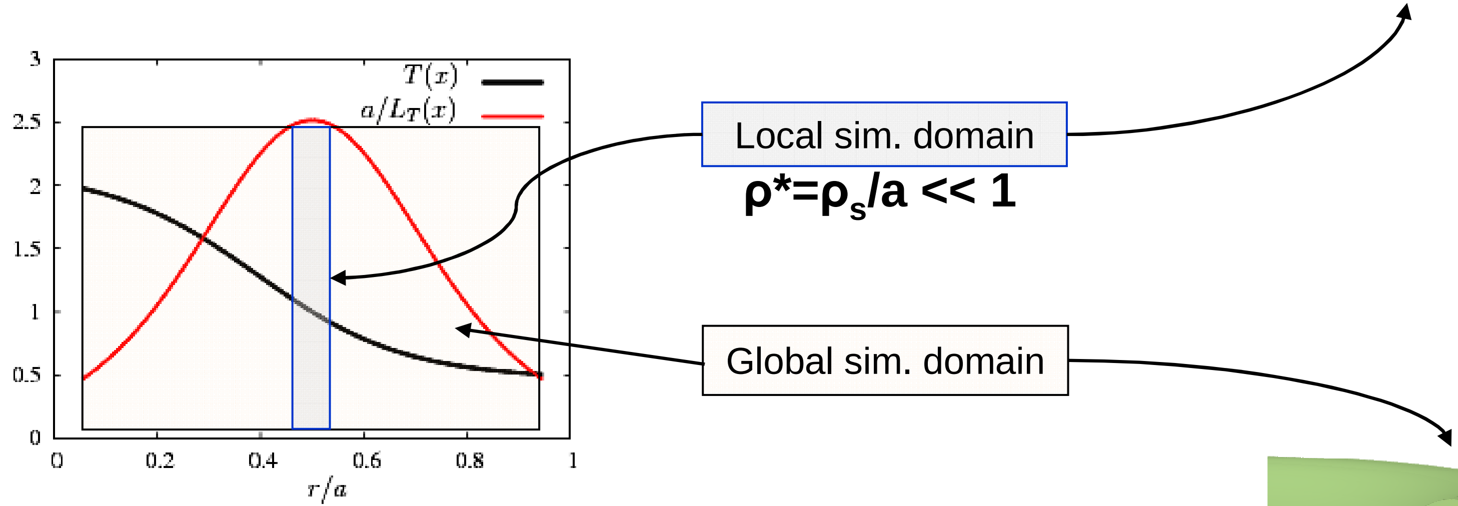
- 4th order finite differences are basic choice
- however, the more elaborate Arakawa-scheme is employed, if possible [A. Arakawa, JCP 135, 103 (1997), reprint]

Exception:
spectral methods are employed in the
- y direction - always
- x direction - depends on the type of operation (local/global)

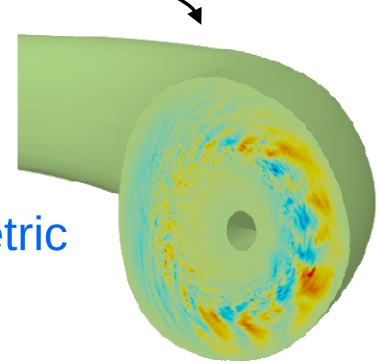
Local vs. global GENE

- **Local:** in the radial direction
  - Simulation domain small compared to machine size; thus, *constant* temperatures/densities and *fixed* gradients
  - Periodic boundary conditions; allows application of *spectral methods*





- **Global:** adding nonlocal features in the *radial* direction
  - Consider full temperature & density profiles; radially varying metric
  - Dirichlet or v. Neumann boundary conditions
  - Heat sources & sinks



Local vs. global GENE: Numerics

Local approach:

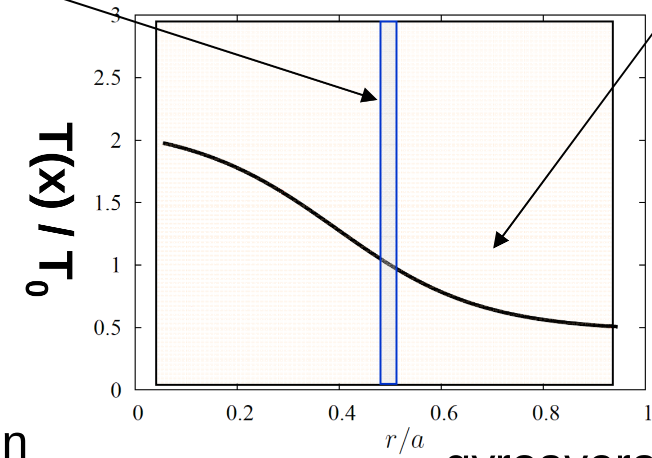
Spectral methods

- derivatives:

$$\frac{\partial f}{\partial x} \rightarrow \mathrm{i} k_x f(k_x)$$

- Gyroaverage & field operators can be given analytically:

$$\langle \phi(\mathbf{x}+\mathbf{r}) \rangle = \sum_{\mathbf{k}_\perp} J_0(k_\perp \rho)\phi(\mathbf{k}_\perp, z)\mathrm{e}^{\mathrm{i}\mathbf{k}_\perp \mathbf{x}}$$



Global approach:

- derivatives: finite difference scheme, typically 4th order

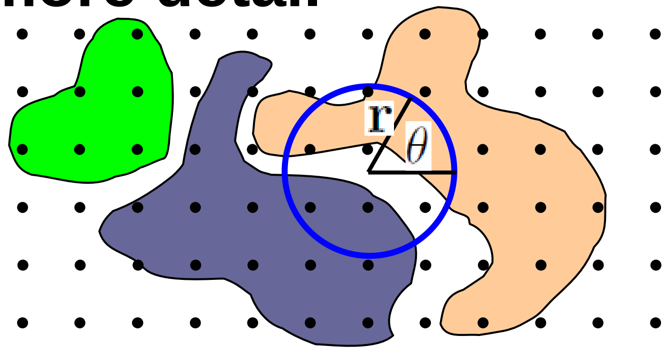
- Use interpolation schemes for gyroaverage & field operators

$$\langle \phi_1(\mathbf{X}+\mathbf{r}) \rangle = \sum_{k_y} \mathrm{e}^{\mathrm{i}k_y Y}\, \mathcal{G}(X, k_y, z, \mu) \cdot \phi_1(X, k_y, z)$$

with gyromatrix $\mathcal{G}(X, k_y, z, \mu)$

Gyro-averaging procedure in more detail

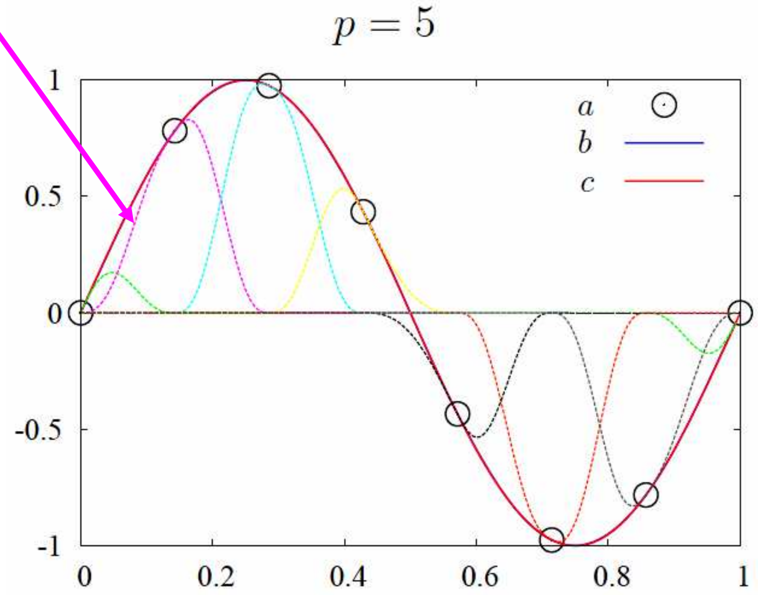
$$\bar{f}(\mathbf{x}) \equiv \frac{1}{2\pi}\oint \mathrm{d}\theta\, f(\mathbf{x}+\mathbf{r}(\theta))$$



- discretize gyro-angle integration
- coordinate transform
- interpolation between grid-cells required
  - here: 1-dimensional problem (y remains in Fourier space)
  - use "finite-elements" which allows easy extraction of gyro-averaged quantities at original grid points *(~ Hermite polynomial interpol.)*

$$f(x) = \mathbf{\Lambda}(x)\cdot\mathbf{f} = \sum_{m=0}^{(p-1)/2} \mathbf{P}_m(x)\mathcal{D}^m\mathbf{f}$$

$$\frac{\partial^u}{\partial x^u}P_{n,m}(x)\bigg|_{x=x_{(i)}} = \delta_{in}\delta_{um}$$



Gyromatrix is constructed at initialization only:

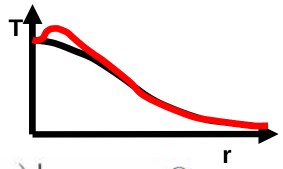
$$\mathcal{G}_{in}(k_y,z,\mu) = \frac{1}{2\pi}\int_0^{2\pi}\mathrm{d}\theta\Lambda_n(x_{(i)}+r^x)\,\mathrm{e}^{\mathrm{i}k_y r^y}$$

$$\bar{\mathbf{f}}(k_y,z,\mu) = \mathcal{G}(k_y,z,\mu)\cdot\mathbf{f}(k_y,z)$$

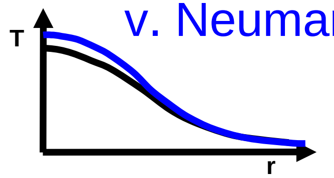
Boundary conditions - overview

radial direction x: local: periodic

global: Dirichlet $f(x,y,z)|_{x\in\mathcal{B}} = 0$



v. Neumann $\partial_x f(x,k_y=0,z)|_{x\in\mathcal{B}} = 0$

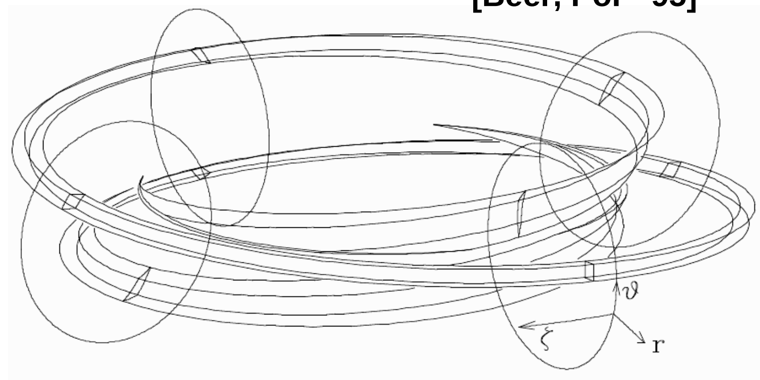


toroidal direction y: periodic

parallel direction z: quasi-periodic

$$f(x,k_y,z+L_z) = f(x,k_y,z)\exp\left(-2\pi i n_0 q(x) j\right)$$

[Beer, PoP '95]



magnetic shear tilts the simulation box → phase factor

$v_\parallel$-velocity space: Dirichlet b.c.

μ-velocity space: Not required (if collisions are neglected)

Source and sink models in GENE

Full domain:

- Radially dependent **heat source/sink** in *whole* domain (density and parallel momentum are unaffected)

$$\frac{df_1}{dt} = -\gamma_h \left[\langle f_1(\vec{X}, |v_{\|}|, \mu) \rangle - \langle f_0(\vec{X}, |v_{\|}|, \mu) \rangle \frac{\langle \int d\vec{v} \langle f_1(\vec{X}, |v_{\|}|, \mu) \rangle \rangle}{\langle \int d\vec{v} \langle f_0(\vec{X}, |v_{\|}|, \mu) \rangle \rangle} \right]$$

Localized sources:

- Krook operator, $\frac{\mathrm{d}f_1}{\mathrm{d}t} = -\nu_{\mathrm{K}}(x) f_1$, to damp fluctuations close to the boundaries
- **Heat source** model (cmp. Grandgirard *et al*., PPCF '07*)*

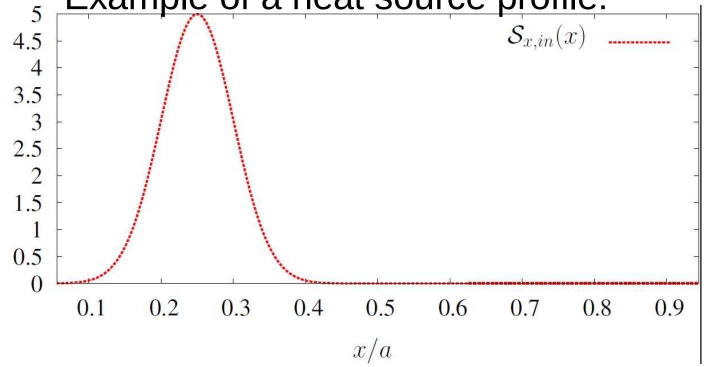
$$\frac{\bar{\mathrm{d}}f_1}{\mathrm{d}t} = \mathcal{S}_H = \mathcal{S}_0 \mathcal{S}_x \mathcal{S}_E$$

$$\mathcal{S}_E = \frac{2}{3} \frac{1}{p_{0\sigma}(x)} \left(\frac{v_{\|}^2 + \mu B_0}{T_{0\sigma}(x)/T_{0\sigma}(x_0)} - \frac{3}{2} \right) f_{0\sigma}$$

$$\mathcal{S}_x = \mathcal{S}_{x,in}(x) \Big/ \int \mathrm{d}^3x \, \mathcal{S}_{x,in}(x) J(x,z)$$

and floating boundary conditions

Example of a heat source profile:



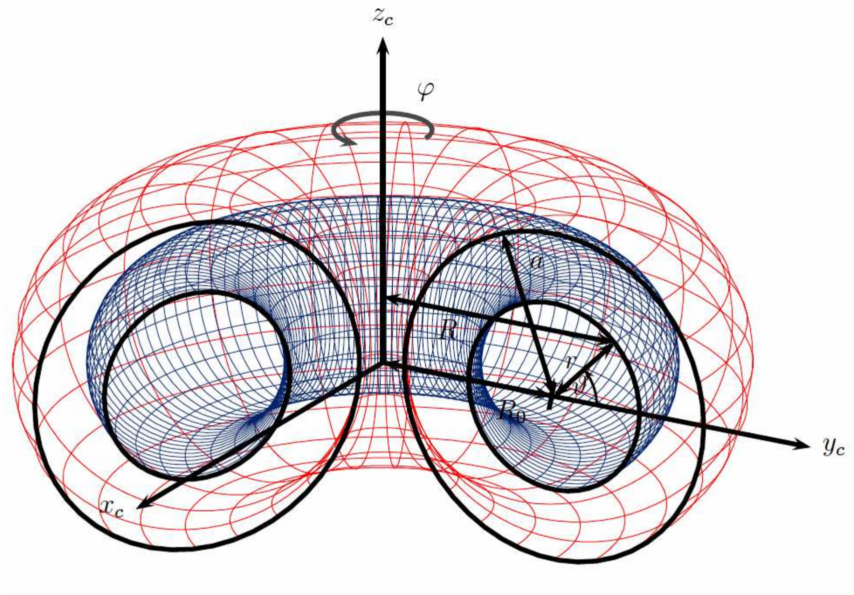
Magnetic geometry

- arbitrary flux-surface shapes can be considered as long as the metric $g = (g^{ij}) = (\nabla u^i \cdot \nabla u^j)$ is provided to compute expressions like

$$\frac{1}{B_0^2}\left(\mathbf{B}_0 \times \nabla\zeta\right)\cdot\nabla = \frac{1}{\mathcal{C}}\frac{g^{1i}g^{2j} - g^{2i}g^{1j}}{\gamma_1}\partial_i\zeta\partial_j$$

- simple choice: circular concentric flux surfaces

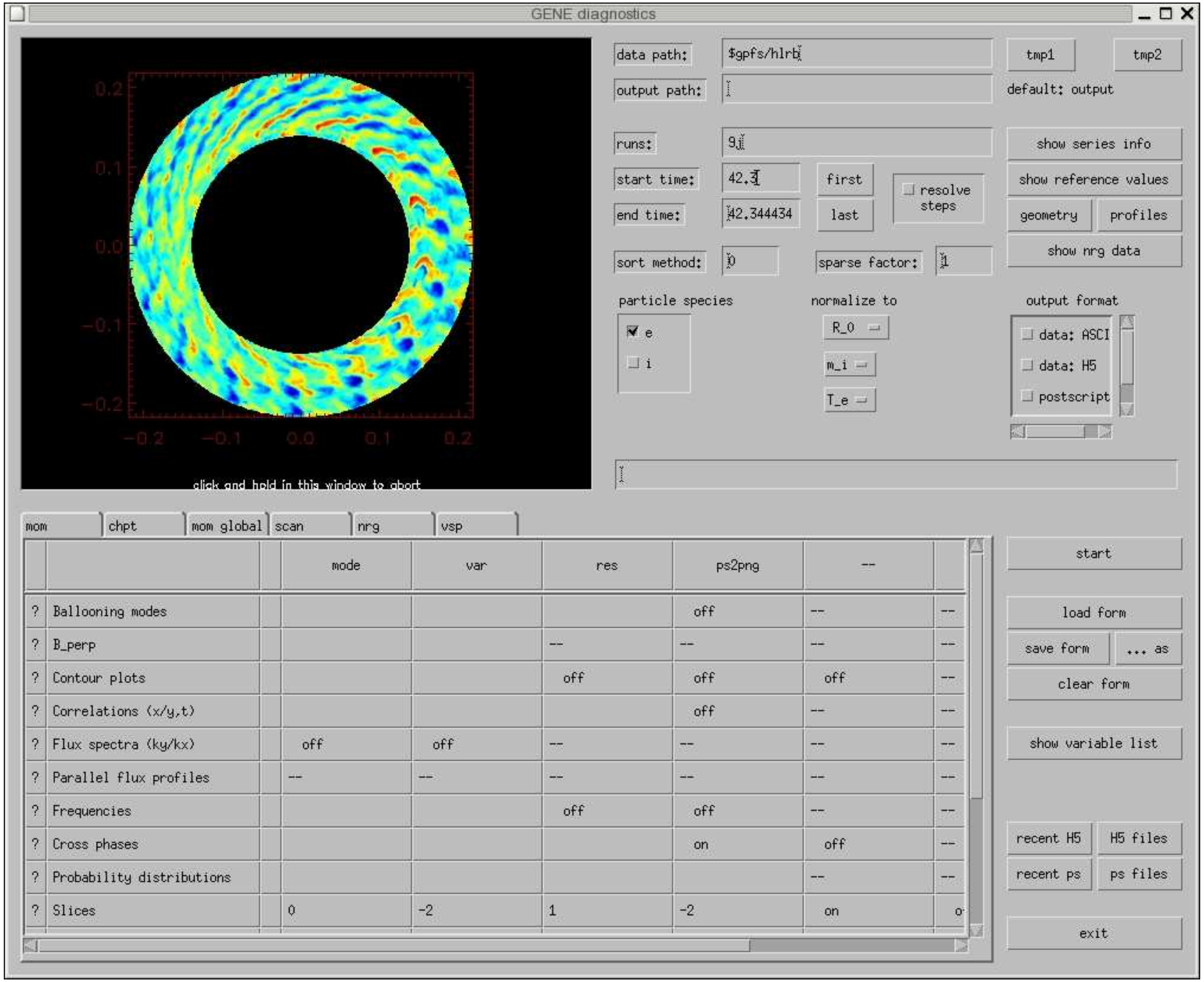
$$\begin{aligned}
g^{xx} &= 1,\\
g^{xy} &= g^{yx} = \hat{s}z - \frac{q}{q_0}\varepsilon_0 \sin z,\\
g^{xz} &= g^{zx} = -\varepsilon\frac{\sin z}{r},\\
g^{yy} &= (\hat{s}z)^2 - 2\frac{q}{q_0}\varepsilon_0\hat{s}z\sin z + \frac{q^2}{q_0^2}\frac{r_0^2}{r^2}\left(1-2\varepsilon\cos\vartheta\right),\\
g^{yz} &= g^{zy} = \frac{1}{r_0}\left[-\hat{s}z\varepsilon_0\sin z + \frac{q}{q_0}\frac{r_0^2}{r^2}\left(1-2\varepsilon\cos\vartheta\right)\right],\\
g^{zz} &= \frac{1}{r^2}\left(1-2\varepsilon\cos\vartheta\right),
\end{aligned}$$



- others can be read in via interfaces to TRACER/GIST (field line tracer) and the equilibrium code CHEASE

Input: GENE Launcher

Output: GENE Diagnostics Tool



Global Gyrokinetic Simulation of Turbulence in

ASDEX Upgrade

GENE

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Summary and outlook

Some outroductory remarks

Goal of this first lecture:

Introduction to gyrokinetic theory and simulation

More info:

Review by Brizard & Hahm, 2007

http://gene.rzg.mpg.de

Topic of next lecture:

Linear and nonlinear gyrokinetics “in action”