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Introduction to Microturbulence in Tokamaks

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I. Introduction to Microturbulence in Tokamaks

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Outline

Properties of Tokamak Core Turbulence

Implications on Tokamak Confinement Scaling

Self-organized Structures in Torus

Radially Elongated Eddys

Zonal Flows

Emphasis:

Study of underlying Physics Mechanisms leading to Paradigm Shift

Properties of Tokamak Core Turbulence

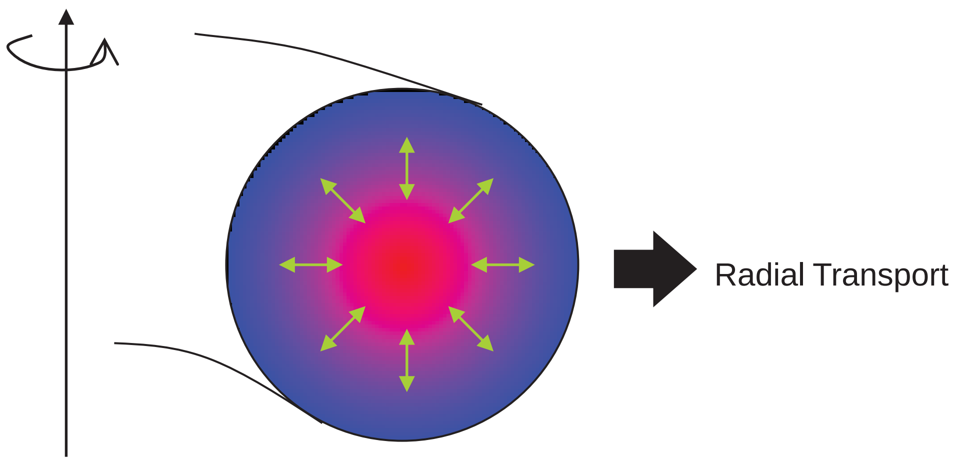
Microinstabilities in Tokamaks

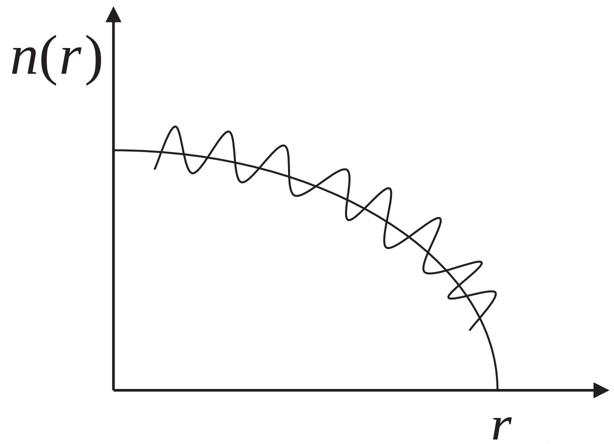
. Tokamak transport is usually anomalous, even in the absence of large-scale magneto-hydro-dynamic (MHD) instabilities.

• Caused by small-scale collective instabilities driven by gradients in temperature, density, …

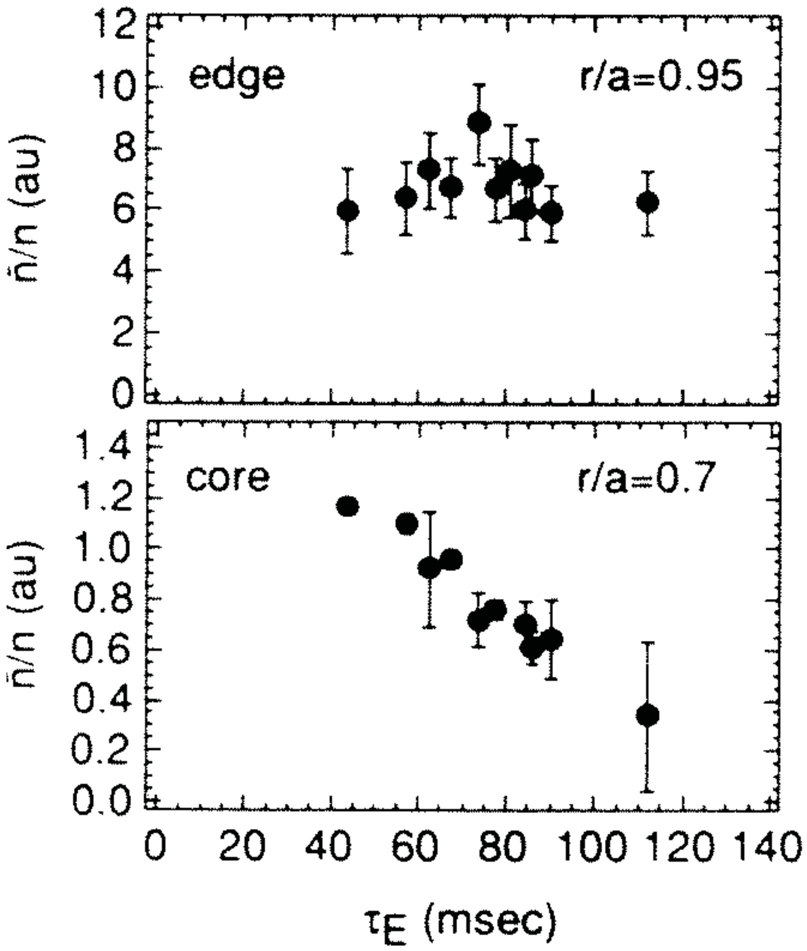
• Instabilities saturate at low amplitude due to nonlinear mechanisms

• Particles **E** x **B** drift radially due to fluctuating electric field





Confinement gets worse with increasing Turbulence Level



L-mode

Global confinement scales with

core **turbulence level**

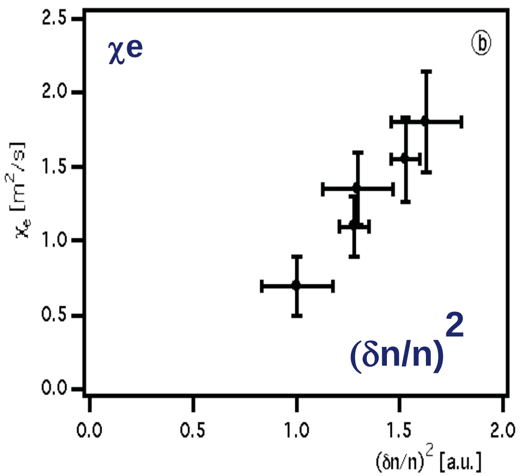
*Equipe **TFR** & A. Truc, NF (1986)*

Brower NF (1987) TEXT

*Paul et al, PoF (1992) **TFTR*** *R=2.5m, a=0.89m*

Durst et al, PRL (1993)

Local confinement also scales with turbulence level



Tore Supra

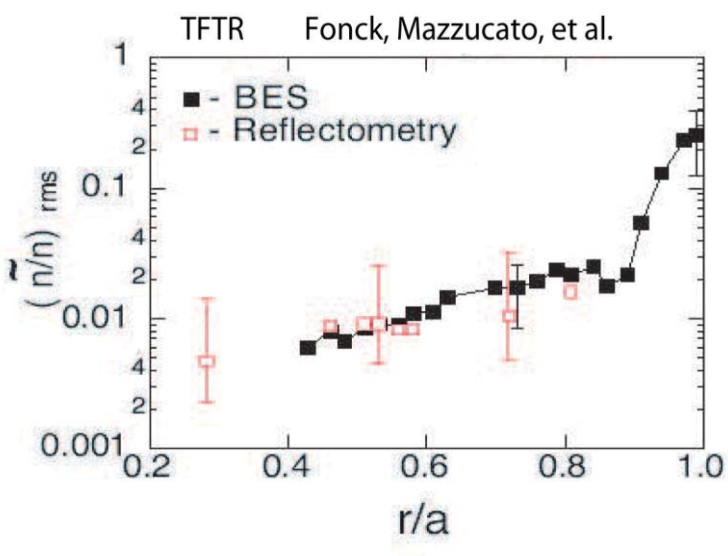
R=2.4m, a=0.7m

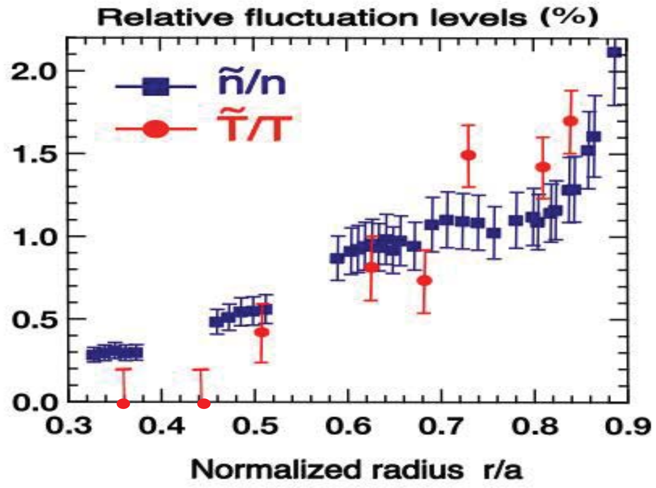
Laviron et al., IAEA (1996)

Zou et al., PRL (1995)

Hoang et al, Nuc. Fus. (1998)

Amplitude of Tokamak Microturbulence



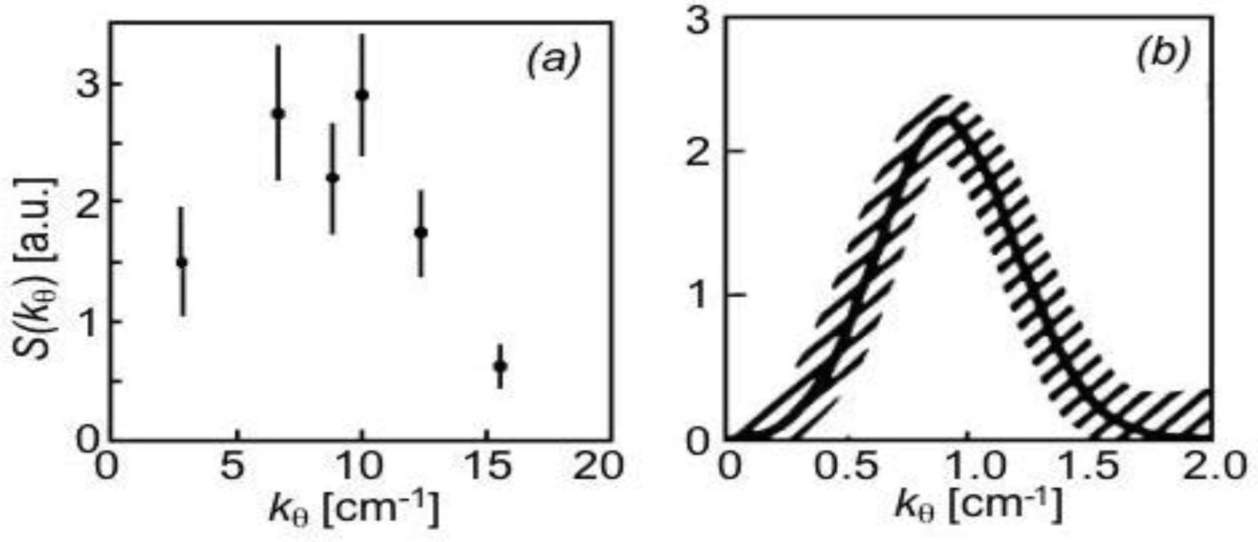


- Relative fluctuation amplitude $\delta n / n_0$ at core typically less than 1%

- At the edge, it can be greater than 10%

- Confirmed in different machines using different diagnostics

k-spectra of tokamak micro-turbulence



$k_\theta \, \rho_i \sim 0.1 - 0.2$

-from Mazzucato et al., PRL '82 (μ-wave scattering on ATC)
Fonck et al., PRL '93 (BES on TFTR)

-similar results from

TS, ASDEX, JET, JT-60U and DIII-D

Properties of Tokamak Core Microturbulence

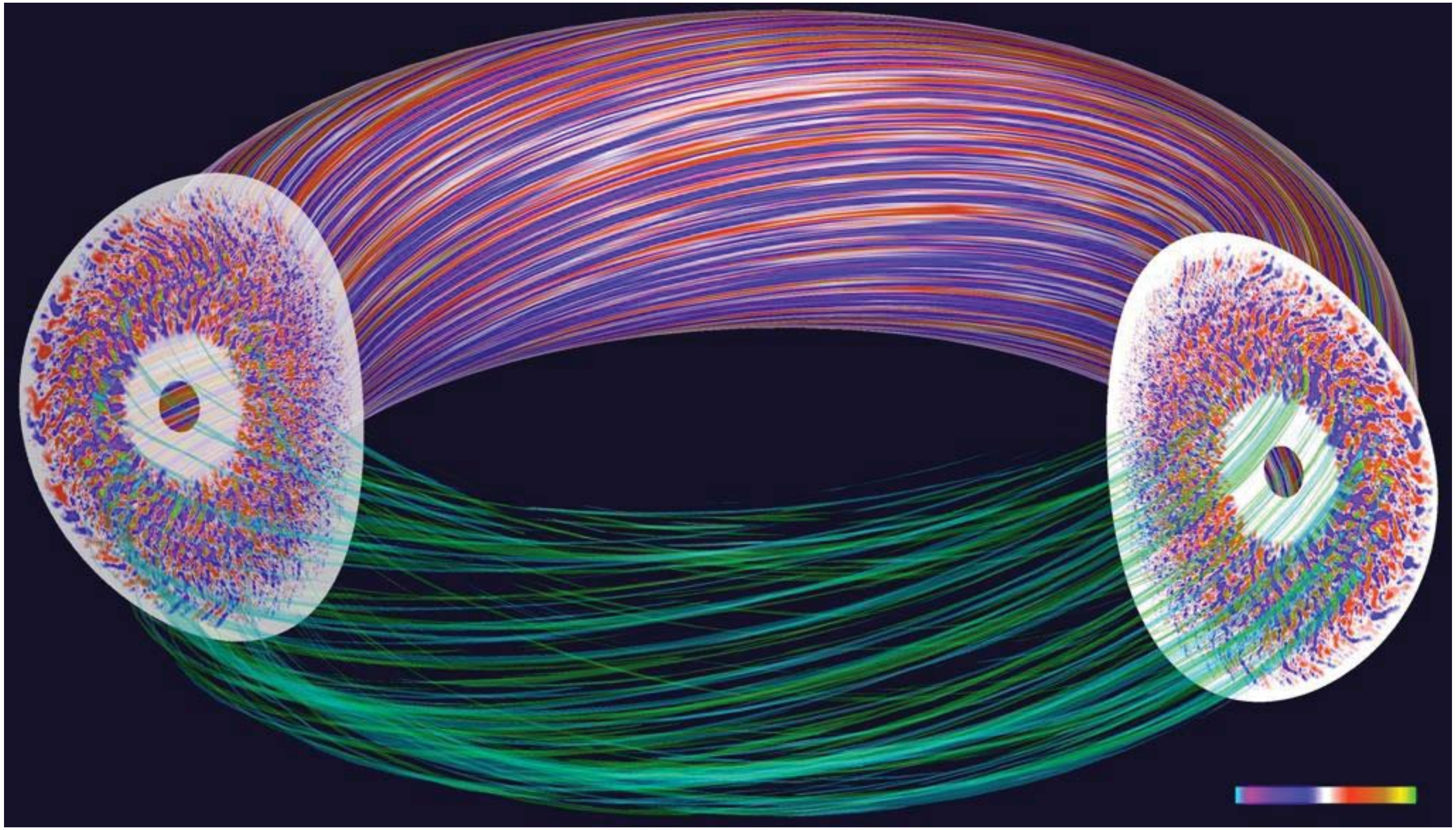
from Measurements

- $\delta n / n_0 \sim 1\%$
- $k_r \rho_i \sim k_\theta \rho_i \sim 0.1 - 0.2$
- $k_{||} < 1/qR << k_\perp$: Rarely measured
- $\omega - \mathbf{k} \bullet \mathbf{u}_E \sim \Delta\omega \sim \omega_{*pi}$:

Broad-band $\Rightarrow$ Strong Turbulence

Sometimes Doppler shift dominates in rotating plasmas

Contours of Density Fluctuations Exhibit Turbulence Structure



Fully Developed Ion Temperature Gradient (ITG) Driven Turbulence:

from Gyrokinetic Particle Simulations by S. Ethier, W. Wang et al.,

Outline

Properties of Tokamak Core Turbulence

Implications on Tokamak Confinement Scaling with respect to Machine Size

Kinetic Description of Micro Instabilities

⊛ Linear Threshold Condition for ITG Mode

From Fluid Theory, we learned that for flat density, uniform $\vec{\mathbf{B}}$,

★ITG linear growth rate $\propto |\omega_{*Ti}|^{1/3}(k_\parallel C_s)^{2/3}$, for $\omega_{*Ti} > \omega >> k_\parallel v_{Ti}$

This prediction from fluid picture $\rightarrow \gamma_{ITG} > 0$ for any value of $|\nabla T_i|$

However, as $|\nabla T_i| \searrow$, $\omega \searrow$ so that $\omega >> k_\parallel v_{Ti}$ (fluid approx.) breaks down. $\rightarrow$ To accurately predict the onset condition for ITG, one must do kinetic theory, which is valid for $\dfrac{\omega}{k_\parallel} \sim v_\parallel$ so that wave-particle resonant interaction (Landau damping) is properly described.

From NLGK for uniform $\vec{\mathbf{B}}$,

$$\left(\frac{\partial}{\partial t}+v_{\|}\nabla_{\|}+\frac{c}{B}\hat{b}\times\nabla\langle\phi\rangle\cdot\nabla-\frac{q}{m}\nabla_{\|}\langle\phi\rangle\frac{\partial}{\partial v_{\|}}\right)\langle f\rangle=0 \qquad \text{— (3.1)}$$

Linearize it, i.e., $\langle f\rangle = F_0+\delta f$, $\phi=\cancel{\phi_0}+\delta\phi$

and drop nonlinear terms

By further separating nonadiabatic part δh from adiabatic part $\left(-\frac{q\phi}{T_i}F_0\right)$

$$\delta f=-\frac{|e|\delta\phi}{T_i}F_0+\delta h$$

$$\boxed{\left(\frac{\partial}{\partial t}+v_{\|}\nabla_{\|}\right)\delta h+\left(-\frac{|e|}{T_i}\frac{\partial}{\partial t}\langle\delta\phi\rangle+\frac{c}{B}\hat{b}\times\vec{\nabla}\langle\delta\phi\rangle\cdot\nabla\right)F_0=0 \qquad \text{— (3.2)}}$$

$$\langle\delta\phi\rangle=J_0(k_\perp\rho_i)\delta\phi$$

$$\boxed{\delta h=\frac{\omega_{*i}\left\{1+\eta_i\left(\frac{v^2}{2v_{Ti}^2}-\frac{3}{2}\right)\right\}+\omega}{\omega-k_{\|}v_{\|}}\frac{|e|\delta\phi}{T_i}J_0F_0}$$

— For Maxwellian F_0

— (3.3)

where $\eta_i\equiv\frac{d\ln T_i}{d\ln n_0}$, $\omega_{*_i}\equiv-\left(\frac{T_i}{T_e}\right)\omega_{*_e}=-\frac{k_y\rho_i v_{Ti}}{L_n}$

Dispersion Relation can be obtained from quasi-neutrality condition :

$$\boxed{\frac{|e|\delta\phi}{T_e} = -\frac{|e|\delta\phi}{T_i} + \int d^3\vec{v} J_0 \delta h} \quad \text{—— (3.4)}$$

adiabatic e$^-$-response   adiabatic part   nonadiabatic part   of ion response

Fluid limit of ITG (Lecture I) can be obtained by taking $\omega >> k_{\|} v_{Ti}$ and $k_{\perp}\rho_i \to 0$ limit (for $\nabla n = 0$, $\Gamma = 0$).

Stability is determined by Imaginary part of Eq. (3.4).

In kinetic regime, with strong ion Landau damping, $\frac{\omega}{k_{\|}} < v_{Ti}$ such that $\omega << \omega_{*Ti}$ at marginality. So the usual “ion heating” term coming from $\nabla_{\|}\langle\phi\rangle \frac{\partial}{\partial v_{\|}} F_0$ is negligible. (The last term in Eq. (3.3))

Tracing back the origin of the rest in Eq. (3.3),

$$\frac{T_i}{T_e}\omega_{*e}\left\{1+\eta_i\left(\frac{v^2}{2v_{Ti}^2}-\frac{3}{2}\right)\right\} = -v_{Ti}\rho_i k_y \frac{\partial}{\partial x}\ln F_0$$

➡ Then, $\mathrm{Im}\int d^3v\left\{\frac{-v_{Ti}\rho_i k_y \frac{\partial}{\partial x}\ln F_0}{\omega-k_{\|}v_{\|}}\right\}=0$

i.e., marginality condition

corresponds to near "zero" relaxation of the <u>velocity-dependent free energy</u> due to wave-particle resonance.

$$\omega_{*e}\left\{1+\eta_i\left(\frac{v^2}{2v_{Ti}^2}-\frac{3}{2}\right)\right\} \qquad\qquad \omega-k_{\|}v_{\|}$$

$$\lim_{\omega\to 0}\mathrm{Im}\int_{-\infty}^{\infty}dv_{\|}\int_0^{\infty}dv_{\perp}v_{\perp}\frac{1}{\omega-k_{\|}v_{\|}}\frac{\partial}{\partial r}\left(\frac{n_0}{T_i^{3/2}}J_0^2(k_{\perp}\rho_i)e^{-v^2/2v_{Ti}^2}\right)$$

$$=\lim_{\omega\to 0}\mathrm{Im}\int_{-\infty}^{\infty}dv_{\|}\frac{1}{\omega-k_{\|}v_{\|}}\frac{\partial}{\partial r}\left\{\left(\frac{n_0}{T_i^{3/2}}\right)\int_0^{\infty}dv_{\perp}v_{\perp}e^{-v^2/2v_{Ti}^2}J_0^2\left(k_{\perp}\frac{v_{\perp}}{\Omega}\right)\right\}$$

$$=-\frac{\pi}{|k_{\|}|}\lim_{\omega\to 0}\frac{\partial}{\partial r}\left[e^{-v^2/2v_{Ti}^2}\frac{n_0(r)}{T_i^{1/2}(r)}\Gamma_0(k_{\perp}\rho_i)\right]=0 \qquad \text{— (3.5)}$$

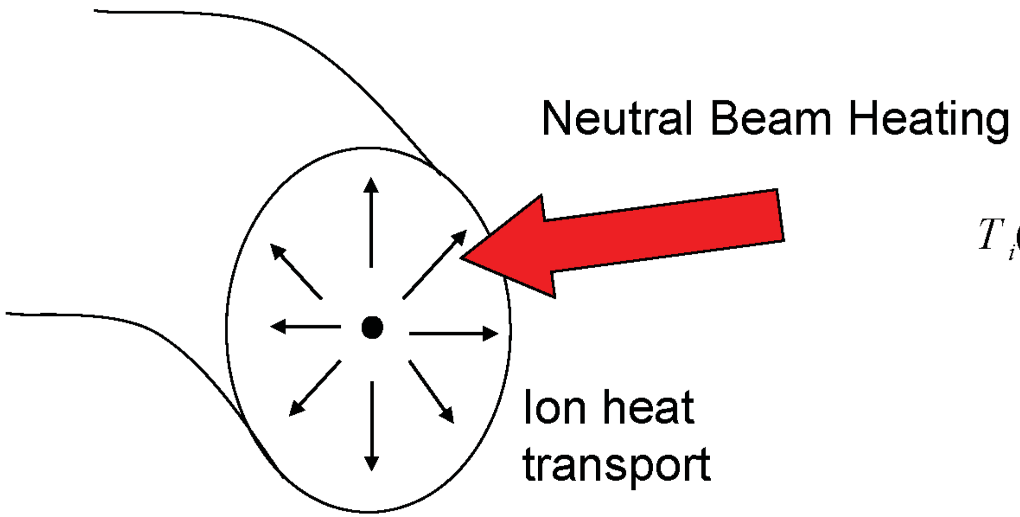
Therefore, for $\dfrac{\omega}{k_{\|}} << v_{Ti}$,

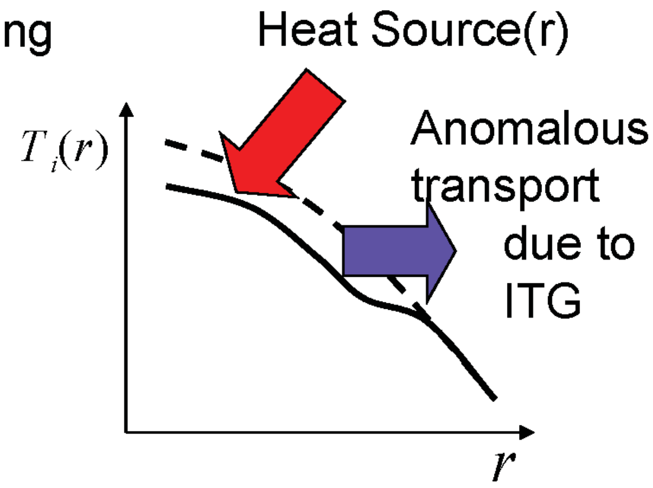
$$\frac{\partial}{\partial r}\left[\frac{n_0(r)}{T_i^{1/2}(r)}\Gamma_0\left(k_\perp \rho_i\right)\right] = 0 \qquad \text{— (3.6)}$$

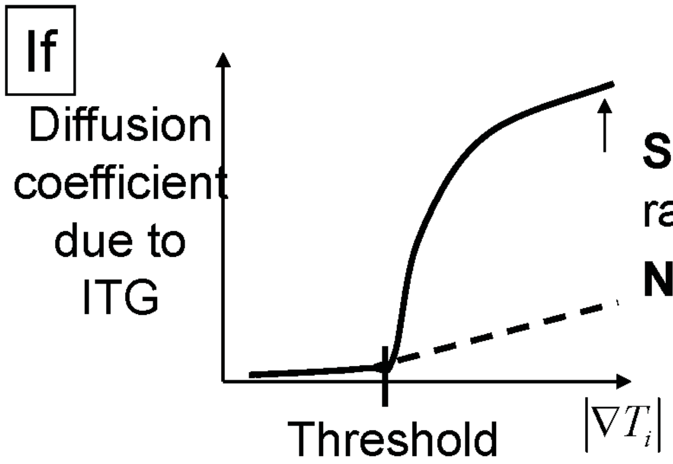
defines the linear threshold of ITG instability. $\left(\Gamma_0(b) \equiv I_0(b)e^{-b}\right)$

For $k_\perp \rho_i \to 0$, $\dfrac{n_0(r)}{T_i^{1/2}(r)} = \text{"const."}$ is the marginality profile.

So if ITG instability were very violent enough to throw out ion heat very rapidly once it's excited, one can imagine that the ion temperature profiles in tokamak plasmas are approximately near the linear marginal profile. Of course, it's an oversimplified and pessimistic viewpoint, but has an interesting implication and sometimes a useful rough guidance.







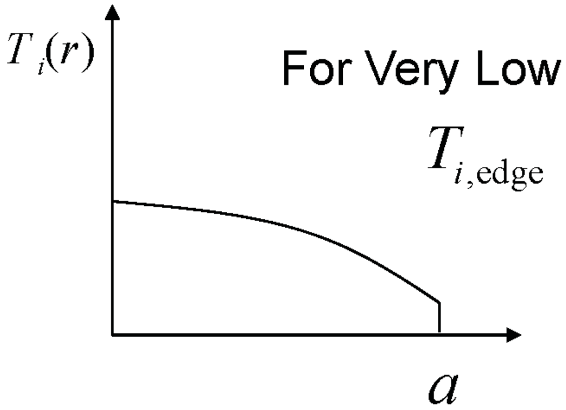
Stiff
rather than
Non-stiff

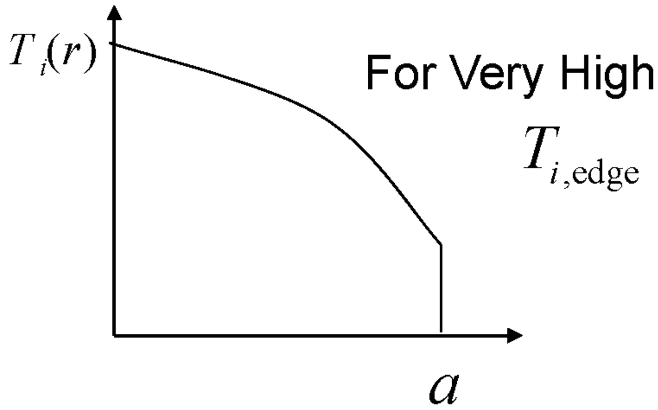
$$|\nabla T_i| \approx |\nabla T_i|_{\mathrm{Threshold}}$$

or

$$\eta_i \approx \eta_i^{\mathrm{critical}}$$

So if $T_i(r) \propto \mathrm{const} \cdot n_0(r)^2$ is the marginality profile, and n_0 is given, $T_i(0)$ is almost uniquely determined by $T_{i,\mathrm{edge}}$





- ❖ In this extreme limit, $T_i(0)$ (better be high for fusion) is mostly determined by $T_i(a)$, not much by transport in the core (since it's so rapid, throws away excess heat which will raise $T_i(r)$ above marginality.)
- ❖ This is (very simplified) reason why ITER needs to achieve H-mode plasmas in which $T_{i,\mathrm{edge}}$ (pedestal) is high.

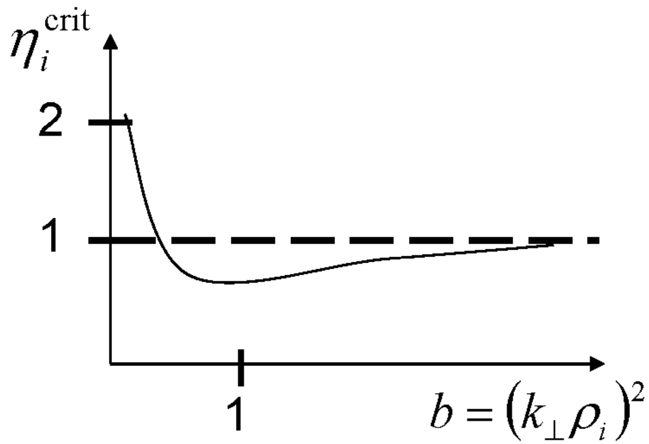
Back to $\frac{\partial}{\partial r}\left[\frac{n_0(r)}{T_i^{1/2}(r)}\Gamma_0(k_\perp \rho_i)\right]=0$ In Eq. (3.6),

If we know $n_0(r)$ and the wavelength of ITG, the marginally unstable ion temperature profile is given by $T_i(r)=\mathrm{const}\cdot\left(n_0(r)\Gamma_0(k_\perp\rho_i)\right)^2$,

or $\frac{T_i(r)}{T_i(a)}=\left(\frac{n_0(r)}{n_0(a)}\right)^2\left(\frac{\Gamma_0(k_\perp\rho_i(r))}{\Gamma_0(k_\perp\rho_i(a))}\right)^2.$

Eq. (3.6) implies that the onset condition for ITG instability in uniform plasma is given by

$$\eta_i=\frac{d\ln T_i}{d\ln n_0}=\frac{L_n}{L_{Ti}}\geq\frac{2}{1+2b_i\left(1-\frac{I_1(b_i)}{I_0(b_i)}\right)} \qquad (3.7)$$



B.B. Kadomtsev and O.P. Pogutse, Review of Plasma Physics Vol. 5. Pg. 249 (1970)

Spatial Structure of Microturbulence

A. Role of Magnetic Geometry

So far in this lecture series, we've discussed microinstabilities in the context of "local theory", i.e., for given values of Macroscopic Parameters, $n_0(x = x_0)$, $\dfrac{dn_0}{dx}(x = x_0)$, $\vec{B}_0$ (uniform), etc.

with $$\delta\phi(x, y, z, t) = \sum_{\mathbf{k},\omega} \underline{\underline{\delta\phi_{\mathbf{k},\omega}}} e^{i(k_x x + k_y y + k_\| z) - i\omega t}$$

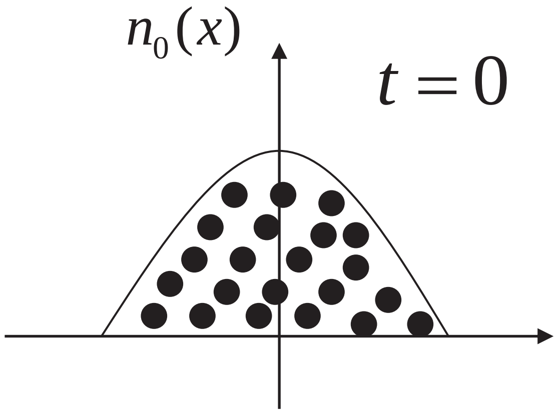
Independent of x, while there are $n_0(x)$, $T_i(x)$ profiles so on.

A challenge is to find more realistic and relevant representation of tokamak microinstabilities.

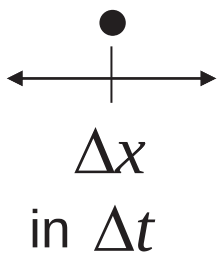
Very rough estimation of the anomalous transport coefficient

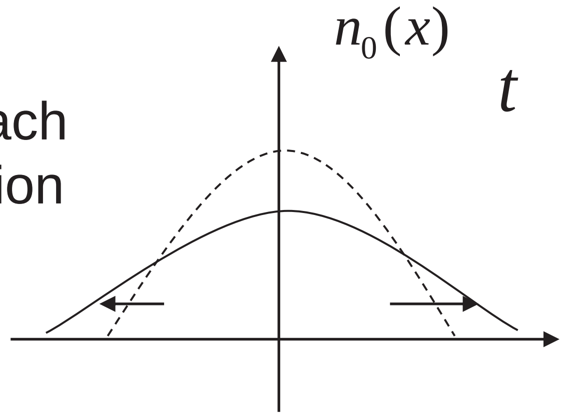
D_{Turb} using dimensional analysis based on

"Random Walk" argument



Each particles walk randomly in each direction





Diffuse with a coefficient ~ $\Delta x^2 / \Delta t$

Since anomalous transport is caused by fluctuating δv_x due to microinstabilities in plasmas, we can argue

$$\Delta x \sim \frac{1}{k_x}\ , \ \Delta t \sim \omega_{\text{decorrelation time}}^{\text{Turb}\ -1} \sim \gamma_{\text{linear}}^{-1}$$

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Then,

$$D_{\text{Turb}} \sim \frac{\Delta x^2}{\Delta t} \sim \frac{\gamma_{\text{lin}}}{k_x^2} \sim \frac{\omega_*}{k_x^2} \sim \frac{k_y}{k_x^2 \rho_i} \frac{\rho_i}{L} \left(\frac{cT_i}{eB} \right)$$

$L \sim a$, L_n for drift waves, L_{Ti} for ITG turbulence, …so on

It's obvious that depending on the choice of k_x and k_y,

D_{Turb} scaling has many possibilities.

If one takes a practical approach of using values of k_x and k_y from experimental measurements, $k_x, k_y \propto \rho_i^{-1}$ (where the spectrum peaks)

Then

$$D_{\text{Turb}} \sim \left(\frac{\rho_i}{L} \right) \left(\frac{cT_i}{eB} \right) \quad : \quad \propto \frac{cT}{eB}$$ is called the "Bohm" scaling.

Since it's reduced by a factor $\left(\frac{\rho_i}{L} \right) << 1$, "gyroBohm" scaling

While it's more common to get "gyroBohm" scaling from simple local theory, most experiments in tokamaks exhibited results which are closer to "Bohm" scaling rather than "gyroBohm" scaling, especially for ion thermal transport (χ_i) in L-mode plasmas. It's very important to achieve a thorough understanding of "size-scaling" of D_{Turb} , for prediction to larger devices in the future.

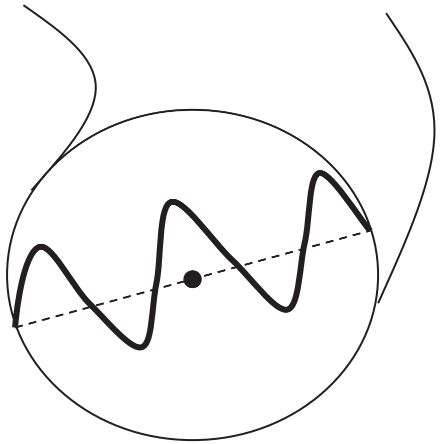
$$D_{\text{Bohm}} \propto \left(\frac{cT_i}{eB}\right)$$

or

$$D_{\text{gyroBohm}} \propto \left(\frac{\rho_i}{a}\right)\left(\frac{cT_i}{eB}\right) \quad ?$$

Then, what scales of k_x, and k_y can give us D_{Bohm} ?

"Bohm" came from experimental observations on very early basic devices (i.e., small). Then, even drift wave type instabilities have relatively low mode numbers.



Eg., Quantization condition

$$k_x a \sim N_x \pi$$

$$k_y a \sim M_y \pi$$

$(N_x, M_y \sim O(1)$ integer)

$$\longrightarrow \quad D_{\mathrm{Turb}} \sim \frac{k_y}{k_x^2 \rho_i} \frac{\rho_i}{L} \left(\frac{cT_i}{eB} \right) \sim \left(\frac{cT_i}{eB} \right) \quad !$$

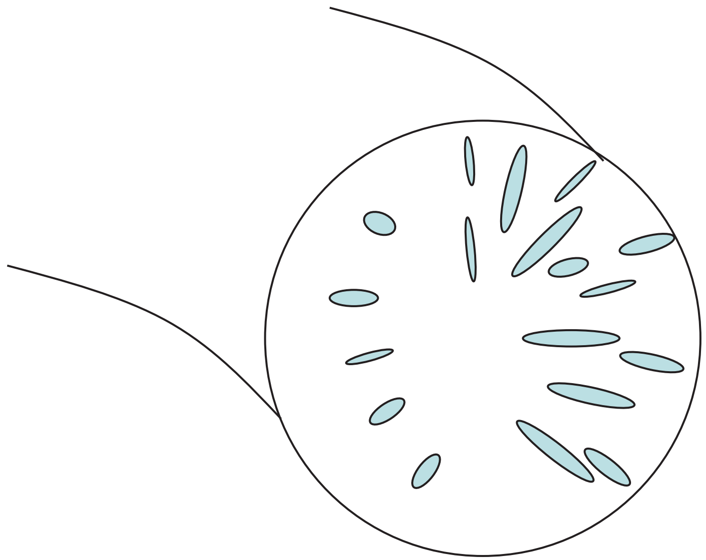
We learn that if $\lambda_x, \lambda_y \propto a$ (system size)

one can get "Bohm" scaling of transport.

Then, what happens to present day larger tokamaks? say $a \gtrsim 100\rho_i$

- From B.E.S. Microwave Scatt. etc. ➡ Eddy size ~ λ_x, λ_y ~ several ρ_i

- From Nonlinear Gyrokinetic Simulations



$$\lambda_x, \lambda_y << a$$

So we want to know what physics mechanisms determine

dominant λ_x and λ_y (eddy size to be more precise).

➡ "Nonlocal Analysis" is required to find

"spatial structure of micro-turburbulence."

Linear theory limit

"eigenmode structure of microinstabilities"

❖ Toroidal Geometry, taking into account of

$$\left|\vec{B}\right| \sim B_\phi \propto \frac{1}{R} = \frac{1}{R_0 + \underline{\underline{r\cos\theta}}}$$

❖ In the end, Self-Organization or Self-Regulation

determines the spatial structure of tokamak micro-turbulence

So far, in slab

$$x \longrightarrow r$$
$$y \longrightarrow r\theta$$
$$z \longrightarrow R\phi$$

but $\vec{B}$ has both

B_ϕ (toroidal)
B_θ (poloidal)
components

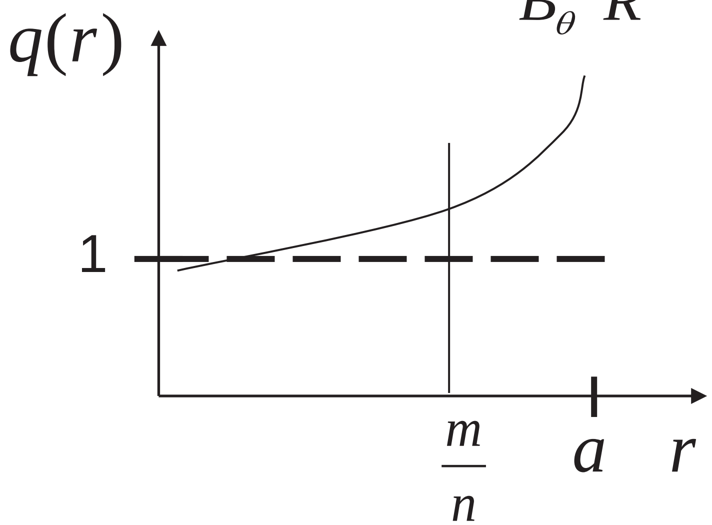
$$\delta\phi(r,\theta,\phi) = \sum_{n.m} \delta\phi_{n,m}(r) e^{i(m\theta - n\phi)}$$

"pitch of fluctuation"

$$q(r) \simeq \frac{B_\phi}{B_\theta}\frac{r}{R}$$

determines pitch of $\vec{B}$

$$n, m \in \mathbf{Z}_+$$



If two coincide, $q(r_s) = \dfrac{m}{n}$.

This radial location $r = r_s$ is called the "mode rational surface."

$$e^{i(m\theta - n\phi)} \quad \Rightarrow \quad k_\theta = \frac{m}{r}, \quad k_\phi = -\frac{n}{R}$$

$$\wr$$

$$e^{i(k_y y + k_z z)}$$

$$\vec{B} = B_\phi \hat{\phi} + B_\theta \hat{\theta}$$

$$\therefore k_\parallel = \frac{\vec{k}\cdot\vec{B}}{\left|\vec{B}\right|} = \frac{m}{r}\frac{B_\theta}{B} - \frac{n}{R}\frac{B_\phi}{B} = \frac{B_\theta}{rB}(m - nq(r))$$

★ $k_\parallel = 0$ at $r = r_s$, ($q(r_s) = \dfrac{m}{n}$).

m, n fixed but $q(r)$ & therefore $k_\parallel$ varies with r .

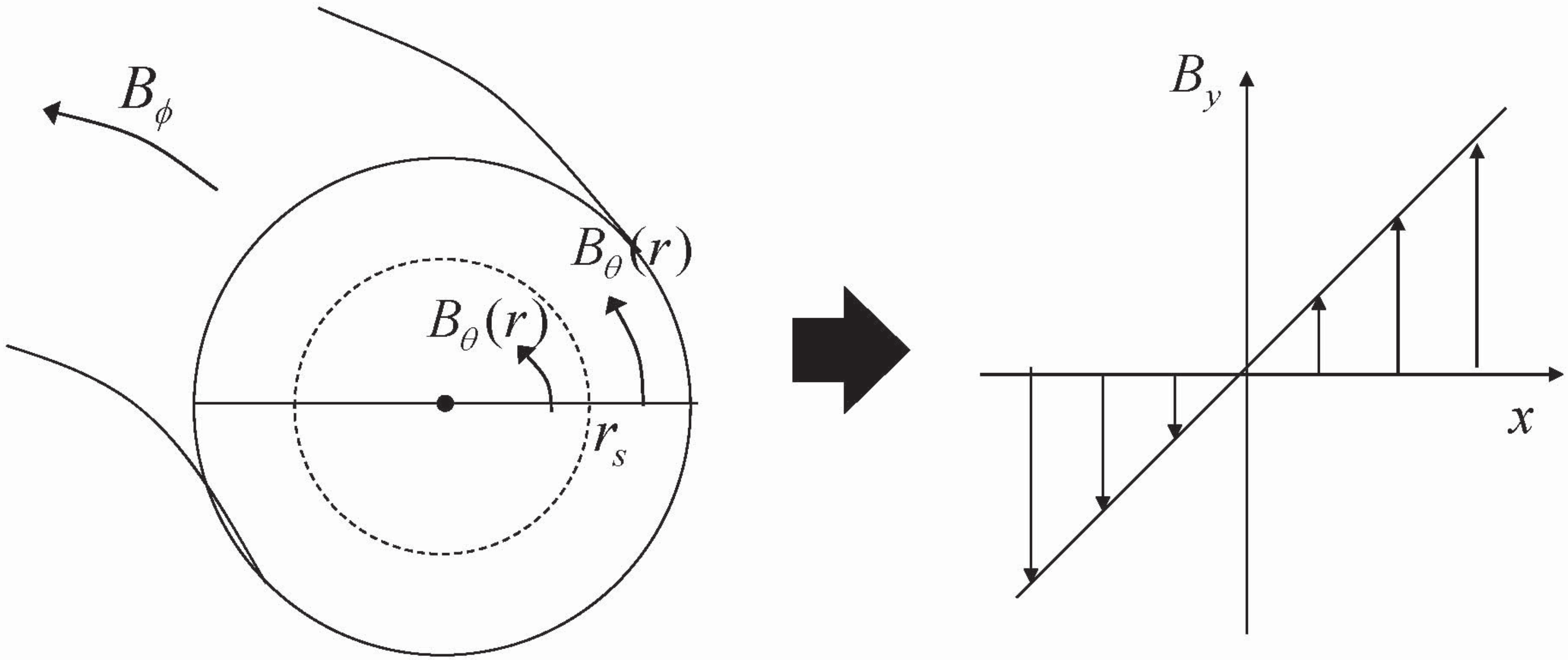
Expanding $q(r) = q(r_s) + (r - r_s)\left(\dfrac{\partial q}{\partial r}\right)(r_s) + \cdots$

$k_\parallel(r - r_s) = k_\parallel(x)$ increases with x,

flips sign across r_s (or $x = 0$).

➡ Near mode rational surface $\vec{B} = B(\hat{z} + \frac{x}{L_s}\hat{y})$ (3.9)

where $L_s = \frac{qR}{\hat{s}}$, $\hat{s} = \frac{r}{q}\frac{dq}{dr}$ magnetic shear.



"Sheared Slab Geometry"

In sheared slab geometry (with one rational surface),

$$k_{\parallel} = \frac{k_y}{L_s} x \quad , \qquad k_x^2 \rightarrow -\frac{\partial^2}{\partial x^2}$$

k_z has been shifted away.

" $\delta\phi(\vec{x},t) = \sum_{k_y} \delta\phi_{k_y,\omega}(x) e^{i(k_y y - \omega t)}$ " : This mode refers to "a single helicity fluctuation" (n,m) with $\frac{m}{n} \rightarrow q(r_s)$

Radial Mode Structure of Drift Wave

Lecture I. Local theory $\longrightarrow$

$$\left(1 + \rho_s^2 k_\perp^2 - \frac{\omega_{*e}}{\omega} - \frac{k_\parallel^2 C_s^2}{\omega^2}\right) \delta\phi_{\vec{k},\omega} = 0$$

In sheared slab :

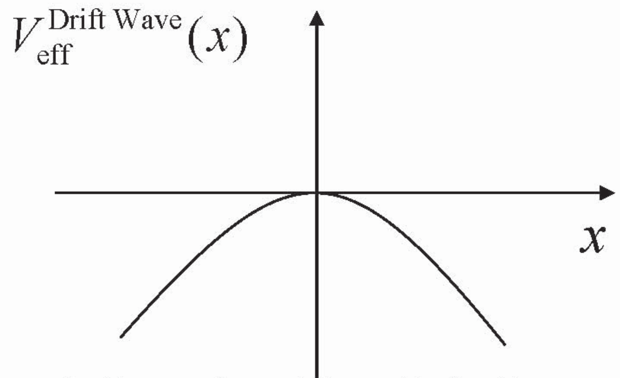
$$\left(1 + \rho_s^2 k_y^2 + \rho_s^2 \frac{\partial^2}{\partial x^2} - \frac{\omega_{*e}}{\omega} - \frac{C_s^2}{\omega^2} \frac{k_y^2}{L_s^2} x^2\right) \delta\phi_{\vec{k},\omega} = 0 \qquad (3.10)$$

★ Weber Eqn. : familiar from Single Harmonic Osc. In Q.M.

$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi = (E - V_{\mathrm{SHO}}(x))\psi = \left(E - \frac{1}{2}m\omega^2 x^2\right)\psi \qquad (3.11)$$

➡ We know how get Eigenfunctions and Eigenvalues of this eqn.

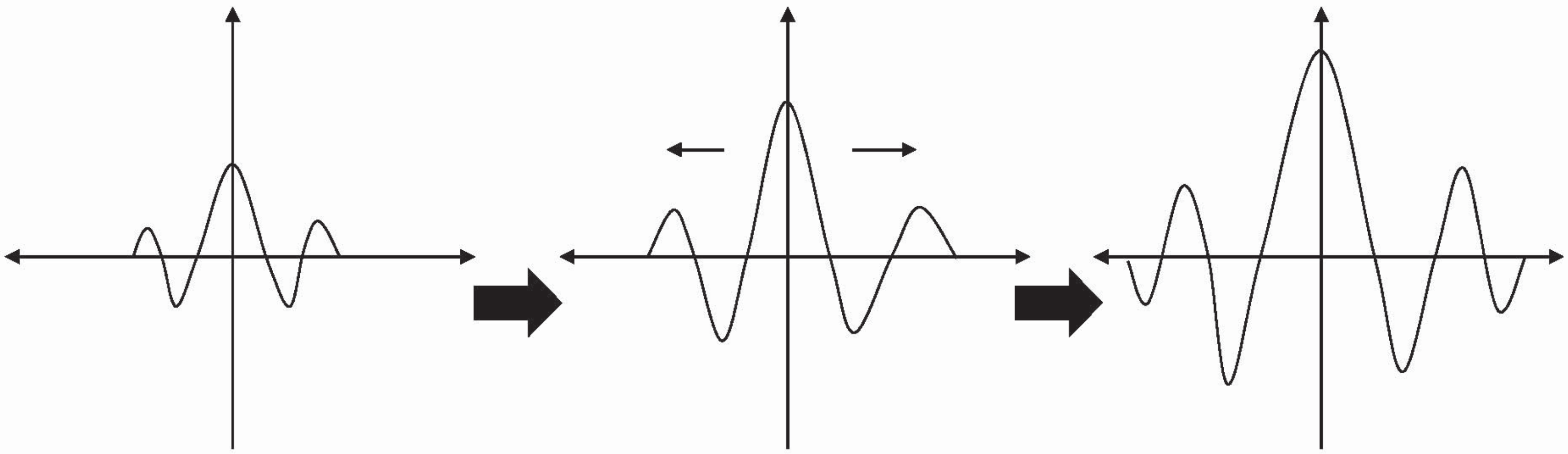
One tricky point is that we have an antiwell potential (hill) rather than potential well, for $|\mathrm{Re}(\omega)| >> |\mathrm{Im}(\omega)|$.



➡ Physically meaningful solution should satisfy the causality condition.

i.e., $\lim_{|x|\to\infty}|\delta\phi(x)| \to 0$ for $\mathrm{Im}(\omega) > 0$

unstable solution.



for unstable solution

while fluctuation grows locally in time,

at a given time, it should decay in radius as $|x| \to \infty$

Group Velocity : $v_{gx} \equiv \dfrac{\partial \omega}{\partial k_x}$ As $|x| \to \infty$

$$\frac{\partial \omega}{\partial k_x} > 0 \quad \text{for} \quad x > 0$$

$$\frac{\partial \omega}{\partial k_x} < 0 \quad \text{for} \quad x < 0$$

Out of two (mathematically) possible solutions

we should choose the upper one!

$$\sim \exp\left(-\frac{ik_y C_s}{2\omega L_s \rho_s} x^2 \right)$$

$$\sim \exp\left(+\frac{ik_y C_s}{2\omega L_s \rho_s} x^2 \right)$$

Eigenvalue : $$\omega = \omega_{*e}\left(\frac{1}{1+k_y^2\rho_s^2} - \underbrace{\frac{i(2\ell_x+1)}{1+k_y^2\rho_s^2}\frac{L_n}{L_s}}_{\text{magnetic shear-induced damping}} \right)$$

$\ell_x = 0,1,\cdots$: radial quantum number

$$\Delta x(\sim \lambda_x) \sim \sqrt{\frac{L_s \omega_{*e} \rho_s}{k_y C_s}} \sim \sqrt{\frac{L_s}{L_n}}\rho_s$$

$\hat{s} \nearrow \;\Rightarrow\; \Delta x \searrow$: i.e., magnetic shear localizes the mode within the device (not determined by B.C. at the WALL).

➡ get “gyroBohm” scaling. (if it were unstable by additional mechanism, eg., trapped electrons)

One can also extend local theory of "ITG" to sheared slab geometry.

(negative compressibility
acoustic mode)

Analysis is slightly more complicated than e^- DW, but can be reduced to Weber-Eqn. *[Coppi, Rosenbluth & Sagdeev, PF* ***10****, 582 (1967)].*

It's noteworthy that an elaborate nonlinear mode coupling theory in sheared slab yielded (rather than from dimensional analysis we're discussing).

$$\chi_i^{\mathrm{ITG}} \propto \text{ gyroBohm}$$

[G.S. Lee & P.H. Diamond, Phys. Fluids ***29****, 3291 (1986)]*

"Nonlocal " kinetic theory can also be pursued in sheared slab geometry :

- Local kinetic theory predicted [K&P] Eq. (3.7)

$$\eta_i \geq \frac{2}{1+2b_i\left(1-\frac{I_1(b_i)}{I_0(b_i)}\right)} \qquad \text{for ITG excitation.}$$

Since $\eta_i \equiv \frac{L_n}{L_{Ti}}$, it predicts instability for very weak $\frac{1}{L_{Ti}}$ for flat density profile ($L_n \to \infty$)

NO GOOD for that limit.

I don't recall a credible analytic ITG onset condition in the flat density limit.

• For sheared slab, flat density case,

$$\frac{L_s}{L_{Ti}} \geq 1.9\left(\frac{T_i}{T_e}+1\right)$$ is the onset condition

[Hahm & Tang, PFB '89]

" $\frac{T_i}{T_e} > 1$ strong magnetic shear favorable for ITG stability !"

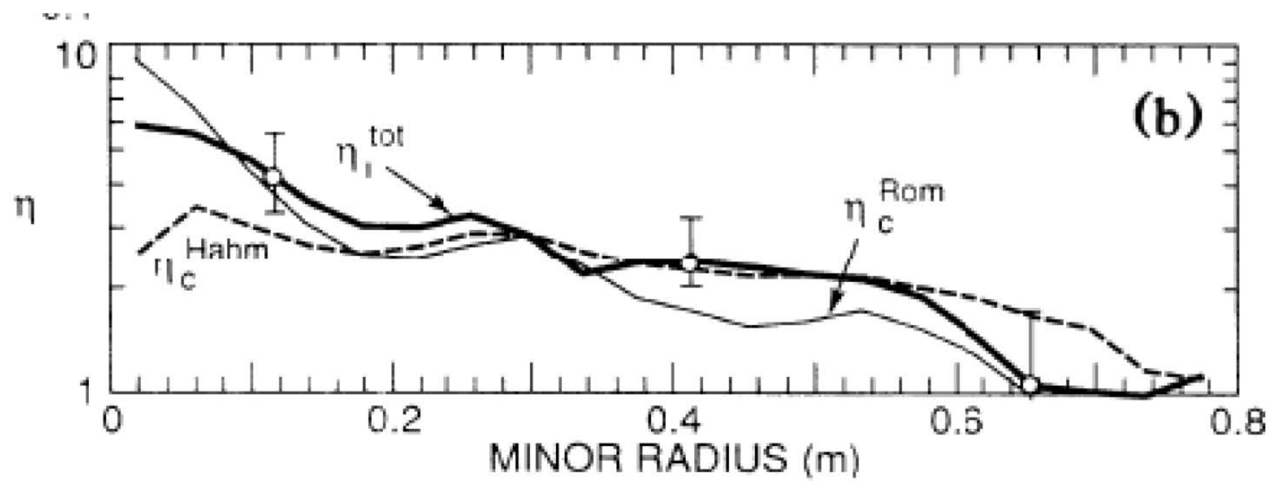
• In toroidal geometry : $\omega - k_{\parallel} v_{\parallel}$ resonance should be generalized to $\omega - k_{\parallel} v_{\parallel} - \omega_{di}$ resonance (ω_{di} is from ∇B & Curvature drift)

If one keeps only $\omega - \omega_{di}$ resonance, (ignoring $k_{\parallel} v_{\parallel}$)

$$\frac{R}{L_{Ti}} \geq \frac{4}{3}\left(\frac{T_i}{T_e}+1\right)$$

[Romanelli, PFB '89]

→ Comparisons to TFTR



Comparison of measured η_i^{tot}

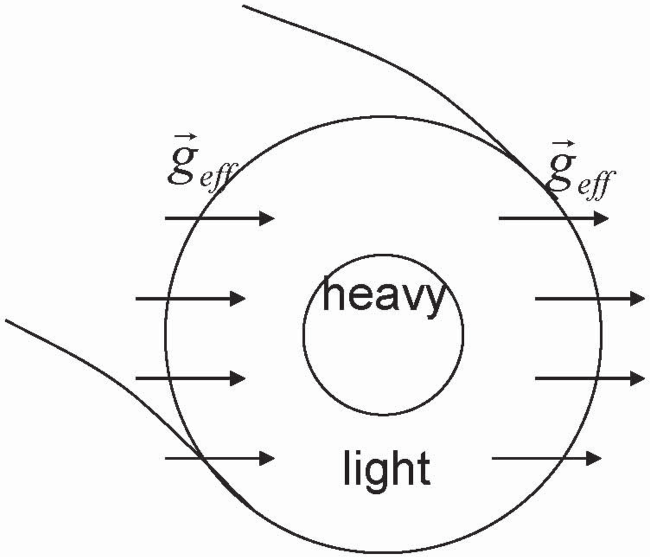
with the theoretical estimates η_c^{Rom} and η_c^{Hahm}

[S. Scott et al., PRL ***29****, 531 (1990)]*

Role of Toroidal Geometry

Uniform $\vec{B}$ model $\longrightarrow$ Sheared Slab Model $\longrightarrow$ "Toroidal Geometry"

$$\vec{B} = B_0(\hat{z} + \frac{x}{L_s}\hat{y})$$



$\hat{y}$ still a symmetry direction : $\frac{1}{R}$ dependence

m : $\longrightarrow$ poloidal angle

No longer symmetry direction

n : a good quantum #

For given n ,
many " m " harmonics
should couple
(even in linear theory)

$$\left|\vec{B}\right| \propto \frac{1}{R} = \frac{1}{R_0 + r \cdot \cos"\theta"}$$

Before looking for a good representation of fluctuation decomposition in torus, let's consider a simplest nontrivial example of ITG mode in torus (with : can be unrelated to "negative compressibility acoustic" ITG.)

It has an "interchange" or "Rayleigh-Taylor" character.

➡ must be localized at bad curvature (low B field) side.

⟶ motivate a "local" theory at bad curvature side.

$\left|\vec{B}\right| \propto \dfrac{1}{R}$ ➡ particles drift in vertical direction (∇B & curvature drift)

We'll get to details later, but

$$v_{\nabla B,\mathrm{Curv}} \propto \bar{v}_{\nabla B,\mathrm{Curv}} \frac{v_{\|}^2 + \mu B}{v_{Ti}^2}$$

An important consequence of this "energy dependent" particle drift

$\Rightarrow$ δn couples to δT_i !

Uniform $\vec{B}_0$ $\Rightarrow$ $\partial_t \delta n + \delta \vec{u}_E \cdot \vec{\nabla} n_0 + n_0 \nabla_{\|} \delta u_{\|} \simeq 0$

Now with nonuniform $\vec{B}_0$: take moments of linearized GK eqn in torus.

$$\left(\frac{\partial}{\partial t} + v_{\|} \nabla_{\|} + \underline{\underline{\vec{v}_d \cdot \vec{\nabla}}} \right) \delta f + \left(\frac{c}{B} \vec{\nabla} \langle \delta \phi \rangle \times \hat{b} \cdot \nabla - \frac{q}{m} \nabla_{\|} \langle \delta \phi \rangle \frac{\partial}{\partial v_{\|}} \right) F_0 = 0$$

most obvious addition due to toroidal effects

Apply

$\int d^3 \vec{v}$: noting that $\vec{v}_d = \vec{v}_{d,th} \left(\frac{v_{\|}^2 + \mu B}{v_{Ti}^2} \right)$

Then,

$$\frac{\partial}{\partial t}\delta n_i + \delta\vec{u}_E \cdot \vec{\nabla} n_0 + \frac{n_0}{T_i}\overline{\omega}_{di}\delta T_i + n_0 \nabla_{\|}\delta u_{\|} + \cdots = 0$$

To focus on "interchange" physics, take $k_{\|} \to 0$ (as done by Romanelli in kinetic regime)

and take a simple "flat density" limit.

$$\Rightarrow \quad \frac{\partial}{\partial t}\delta n_i + \frac{n_0}{T_i}\overline{\omega}_{di}\delta T_i \simeq 0 \qquad \overline{\omega}_{di} \equiv -\frac{cT_i}{eBR}k_y$$

(at bad curvature side)

&

Take simplest ∇T_i evolution eqn.

$$\frac{\partial}{\partial t}\delta T_i + \delta u_E \cdot \nabla T_0 \simeq 0$$

(assumed $\omega > \overline{\omega}_{di}$, but $|\omega_{*Ti}| > \omega$)

(2x2) $\Rightarrow$

$$\boxed{\omega^2 = -\frac{T_e}{T_i}\left|\overline{\omega}_{di}\omega_{*Ti}\right|}$$

— (simplified) Toroidal ITG

bad-curvature coupled to ∇T_i

This is another very illuminating limiting (but relevant) case.

Further readable physical discussion in M.A. Beers et al., Ph.D Thesis

Princeton U. ’95.

Both in this fluid limit & analytic kinetic derivation of Romanelli, PFB’89

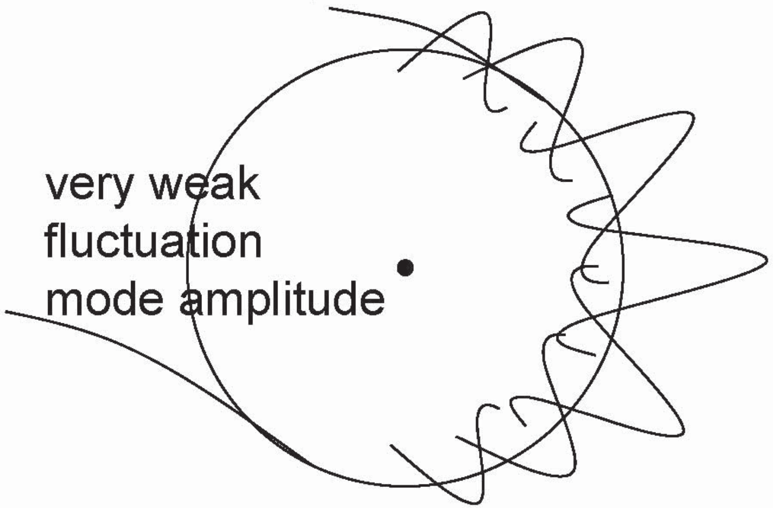
$k_\parallel \to 0$ has been assumed.

$$\omega - \omega_{di} - \cancel{k_\parallel v_\parallel}$$

resonance

This is incompatible with fluctuation localized in bad-curvature side

i.e., “ballooning” mode structure



➡ “ $k_\parallel \gtrsim \dfrac{1}{qR}$ “

Recap :

★ In sheared slab : $k_{\parallel} = \frac{k_y}{L_s} x$, some fluctuations can be localized in radius

(small $k_{\parallel}$ $\rightarrow$ minimize magnetic shear-induced damping

ion-Landau damping)

& Extended along $\vec{B}$

➡ "flute-like fluctuations"

★ But "ballooning" fluctuations localized at bad curvature side.

➡ $k_{\parallel} \gtrsim \frac{1}{qR}$

What's their radial extent? $\longrightarrow$ Next Lecture.

Radially Elongated Eddy is a Natural Structure

Since

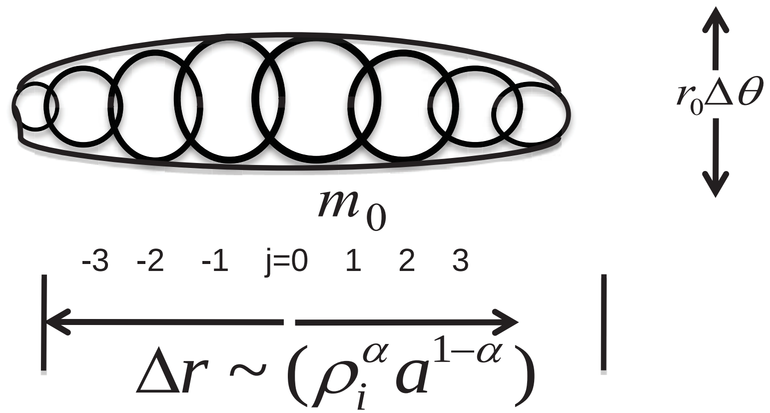
- Poloidal direction no longer symmetric in torus.
- Poloidal harmonics couple to form a Global Eigenmode.

Radially elongated eddy

➡

"Streamers"

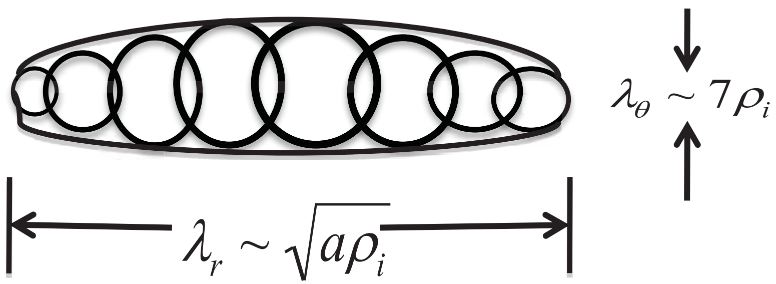
Cf. This is a linear theory-based
simple illustration.
Some strongly prefer "nonlinear" explanation.



Radially Elongated Eddys extract free energy efficiently, and minimize convective (vector) nonlinearity which increases with k_r

From
$$D_{turb} \sim \frac{\Delta x^2}{\Delta t} \sim \frac{\gamma}{k_r^{\;2}} \sim \frac{\omega_*}{k_r^{\;2}} \sim \left(\frac{k_\theta}{k_r^{\;2}\rho_i}\right) \frac{\rho_i}{L} \left(\frac{cT_i}{eB}\right)$$

Radially Elongated Eddys transport heat very efficiently ! :

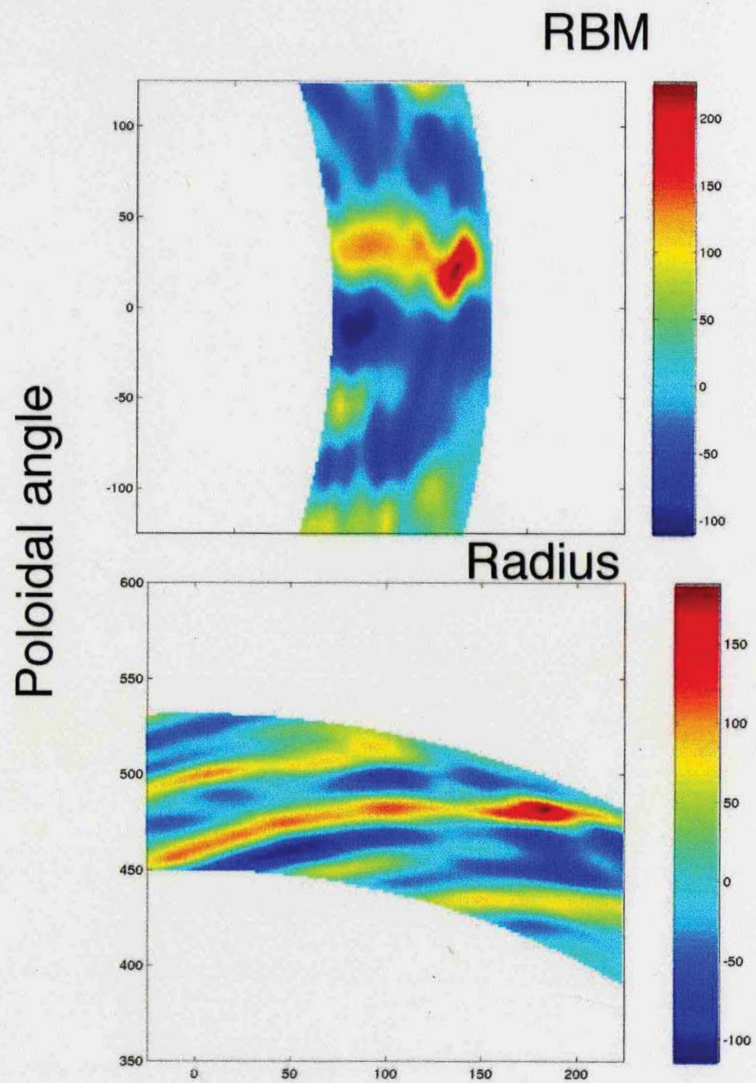


$$k_\theta \sim \rho_i^{\;-1}$$
$$k_r \sim (\rho_i a)^{-1/2}$$

$$\boxed{D_{turb} \sim \left(\frac{cT}{eB}\right)}$$: Bohm !

3D Structure of Streamers

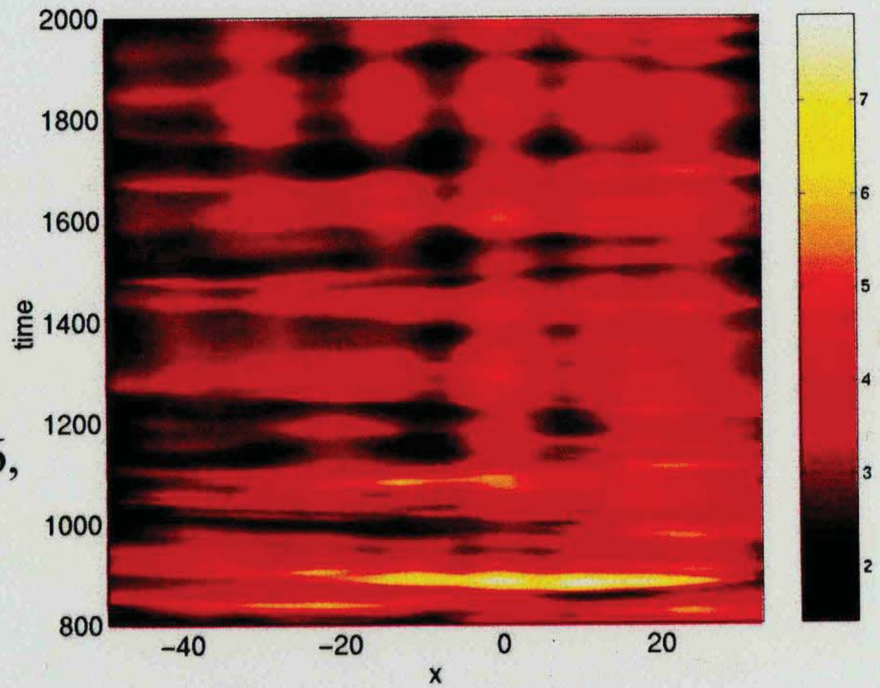
- Maps of the flux in poloidal planes.
- Elongated structures in the radial direction: **streamers**.
- Aligned with the direction of field lines.



Bursty Transport

- Diamond and Hahm 95: profile relaxations at all spatial and time scales (avalanches).
- Observed in many turbulence simulations (Carreras 96, Sarazin and Gendrih 98 Garbet and Waltz 98, Beyer et al. 99,...)

Beyer et al 99



Flux vs. r and t

Partial Summary

Radially Elongated Eddys (Streamers) can be formed in toroidal geometry and transport heat efficiently.

---> Bohm Scaling of Confinement ~ Experimental Trends

Why not sufficient ?

Recall that from experimental measurements:

Eddy size ~ λ_x, λ_y ~ several ρ_i