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Joint ITER-IAEA-ICTP Advanced Workshop on Fusion and Plasma Physics

3 - 14 October 2011

Transport properties of strongly coupled magnetized plasmas

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Transport properties of strongly coupled magnetized plasmas

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Joint ITER-IAEA-ICTP Advanced Worksohp
on „Fusion and Plasma Physics“
Triest, 10 October 2011

Kiel: at Baltic Sea coast, 80km North of Hamburg

Physics Department of Kiel University

Birth place and first professorhip position of Max Planck

Chair Statistical Physics - Research Directions

C | A | U

Strongly correlated Coulomb systems

Classical Coulomb systems

Dusty plasmas
Coulomb liquids
Coulomb crystals
Anomalous transport
Plasma-surface interaction

Quantum Coulomb systems

Warm Dense matter
Astrophysical plasmas
Correlated bosons, excitons
Dense matter interacting with
Lasers and x-rays
Quark-gluon plasma

Quantum Kinetic Theory
First principle simulations

Co-workers and Collaborations

Torben Ott: First-principle molecular dynamics simulations
Hanno Kählert, Alexi Reynolds (Birmingham): collective excitations, wave spectra
Patrick Ludwig: simulations of classical and quantum plasmas

Collaborations: A. Piel (Kiel), A. Melzer (Greifswald): dusty plasma experiments
J. Dufty (Florida): theory of quantum plasmas
Z. Donko, P. Hartmann (Budapest): strongly coupled plasmas

Outline

1. Introduction: Strongly correlated plasmas

- properties and examples

2. Diffusion in magnetized plasmas

- a brief reminder

3. Diffusion in strongly correlated plasmas

- first principle Molecular dynamics approach

4. Superdiffusion in plasma layers

- role of dimensionality, transient effects

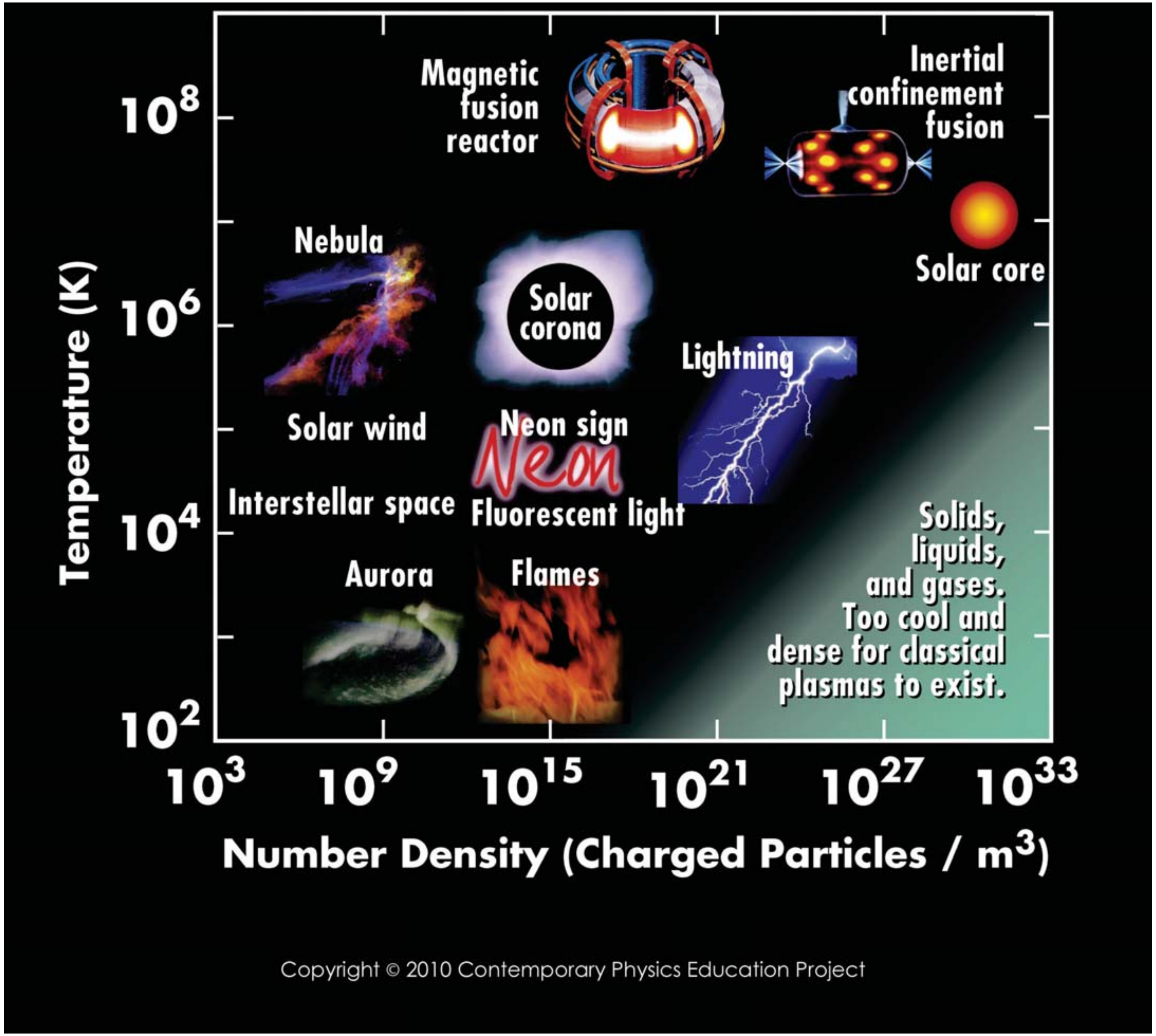
5. Collective modes in magnetized 2D plasmas

6. Strongly correlated 3D plasmas

- Diffusion coefficient perpendicular and parallel to **B**

7. Conclusion and outlook

Examples of plasmas

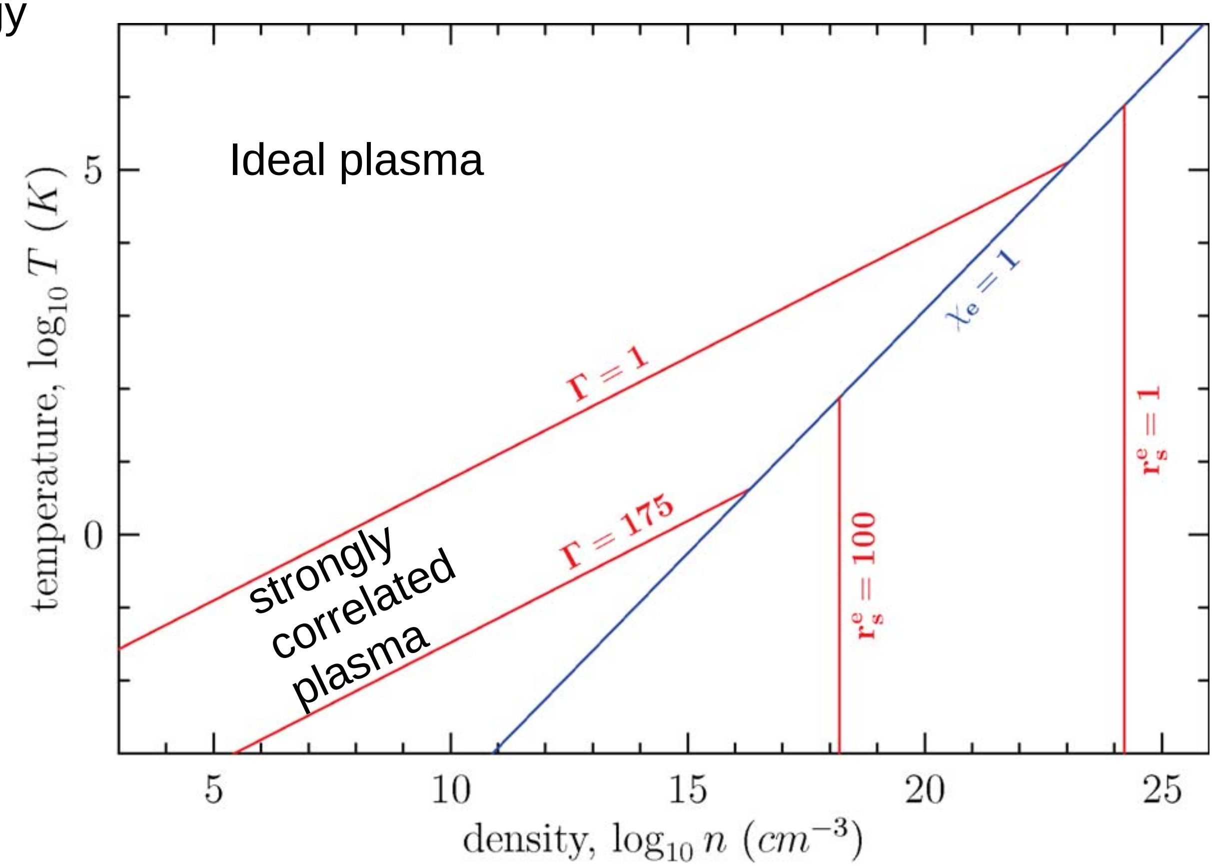


?

Nonideal (correlated) plasmas

Strength of Coulomb correlations: coupling parameter $\Gamma = \frac{U}{T}$

U: mean interaction energy
T: mean kinetic energy



Increase of coupling:
transition from
ideal plasma (gas)
→ liquid
→ solid

→ Plasmas with same coupling: same (qualitative) behavior

Overview: MB et al., *Classical and quantum Coulomb crystals*, Physics of Plasmas **15**, 055704 (2008)

Coulomb (Wigner) crystals in OCP

Ground state of the electron gas in metals

E. Wigner, Physical Review ***46,*** *1002 (1934):*

computed exchange and correlation energy of the electron gas in metals

„If the electrons had no kinetic energy, they would settle in configurations which correspond to the absolute minima of the potential energy. These are close-packed lattice configurations, with energies very near to that of the body-centered lattice....“

Crystallization of one-component plasma in trap:

a) temperature reduction

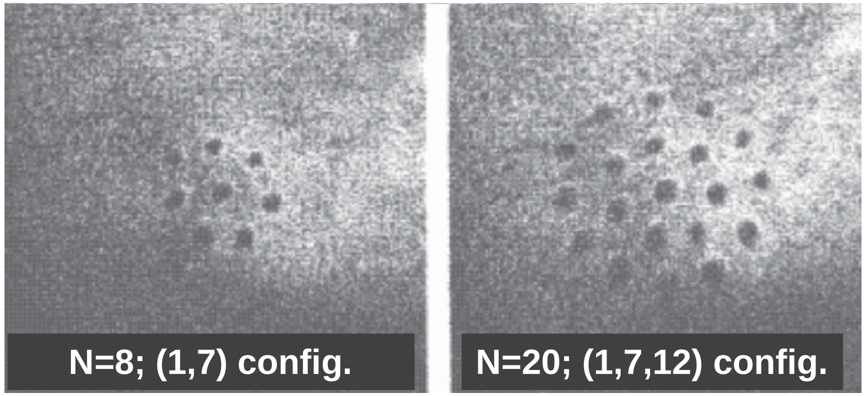
b) density increase

c) charge increase

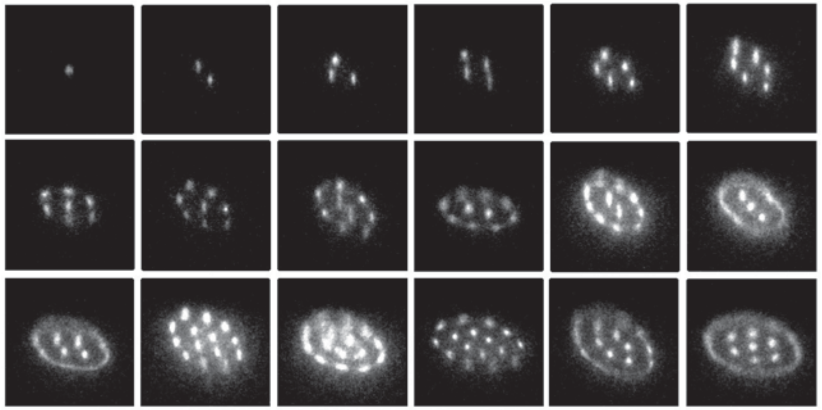
$$\Gamma = \frac{1}{k_B T} \frac{Q^2}{4\pi \epsilon_0 a_{ws}}$$

Overview: MB et al., *Classical and quantum Coulomb crystals*, Physics of Plasmas **15**, 055704 (2008)

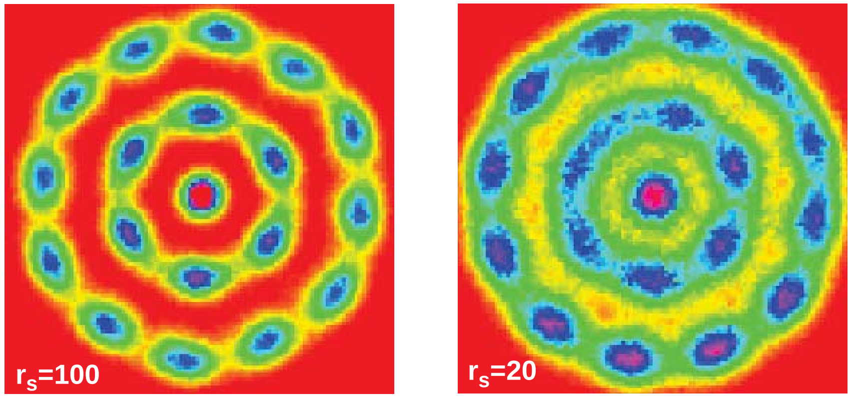
Coulomb crystals: 2D Examples



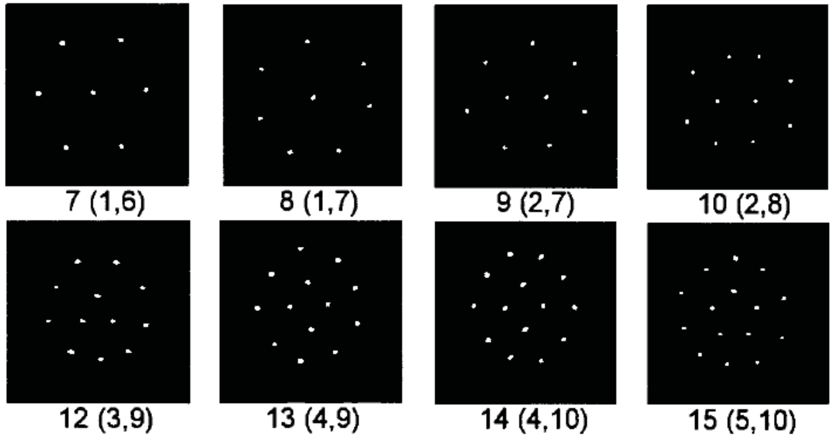
2D Electron Dimples on the Surface of Liquid ^{4}He
Leiderer *et al.*, Surface Science **113**, 405 (1982)



2D Ca$^+$ Ion Crystals in a Paul Trap
Werth *et al.*, University Mainz, Germany (2000)



2D Electron 'Wigner' Crystals in Quantum Dots
A. Filinov, M. Bonitz, Yu.E. Lozovik, PRL (2001)



2D Finite Dust Crystals in a rf Plasma Trap
Lin I *et al.*, Taiwan (1999)

Review: MB, C. Henning, and D. Block, *Complex plasmas – a laboratory for strong correlations*, Physics Reports **73**, 066501 (2010)

Correlations of quantum plasmas

$$\text{crystal: } \Gamma^Q = r_s \propto \frac{\bar{r}}{a_B} \geq r_s^{cr}$$

$$\Gamma^Q = \frac{|\langle U \rangle|}{\langle E_{KIN} \rangle} \sim \frac{e^2}{E_F\, \bar{r}} \propto \frac{e^2 n^{1/3}}{n^{2/3}} \propto n^{-1/3}$$

Quantum degeneracy

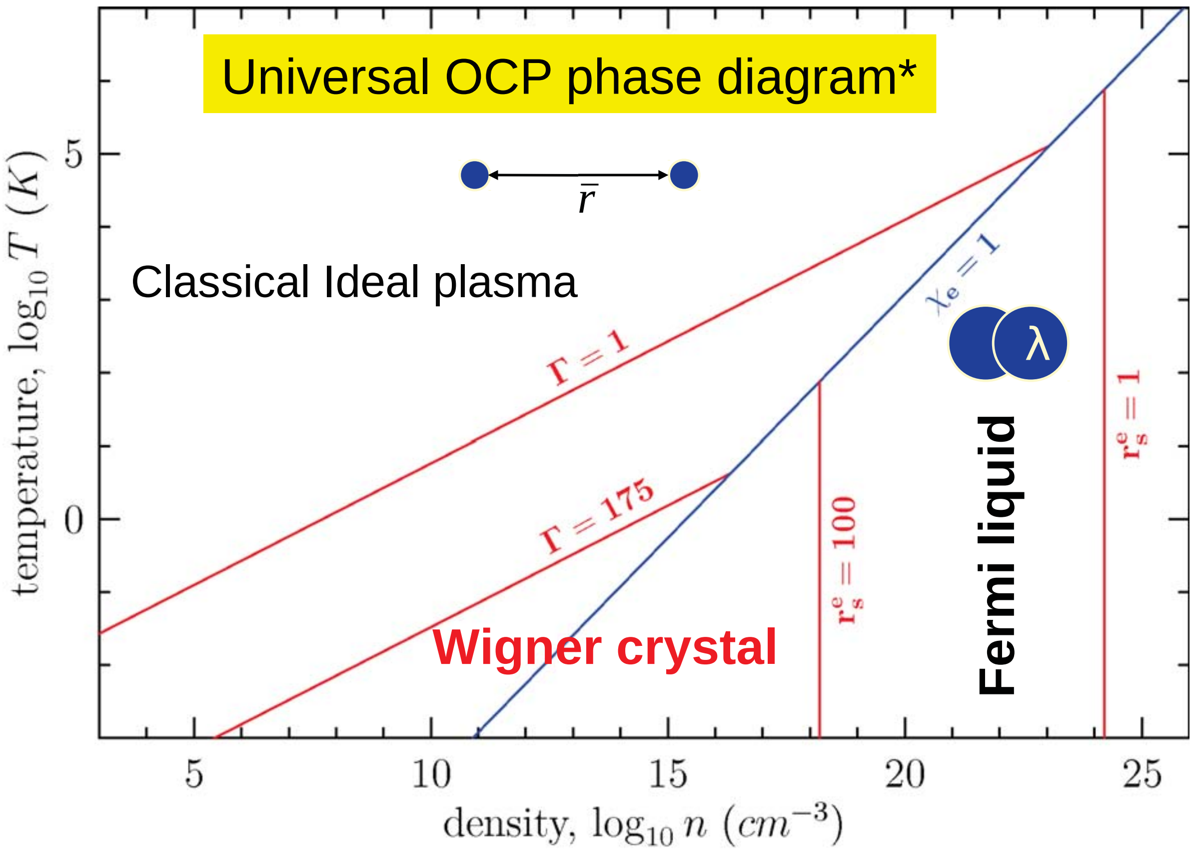
$$\chi = n\lambda^3$$

DeBroglie
wave length

$$\lambda = h / \sqrt{2\pi m k_B T}$$

$$r_s^{cr} \approx 100/37 \; (3D/2D)$$

Ceperley et al.,
A. Filinov, MB



Fermi gas

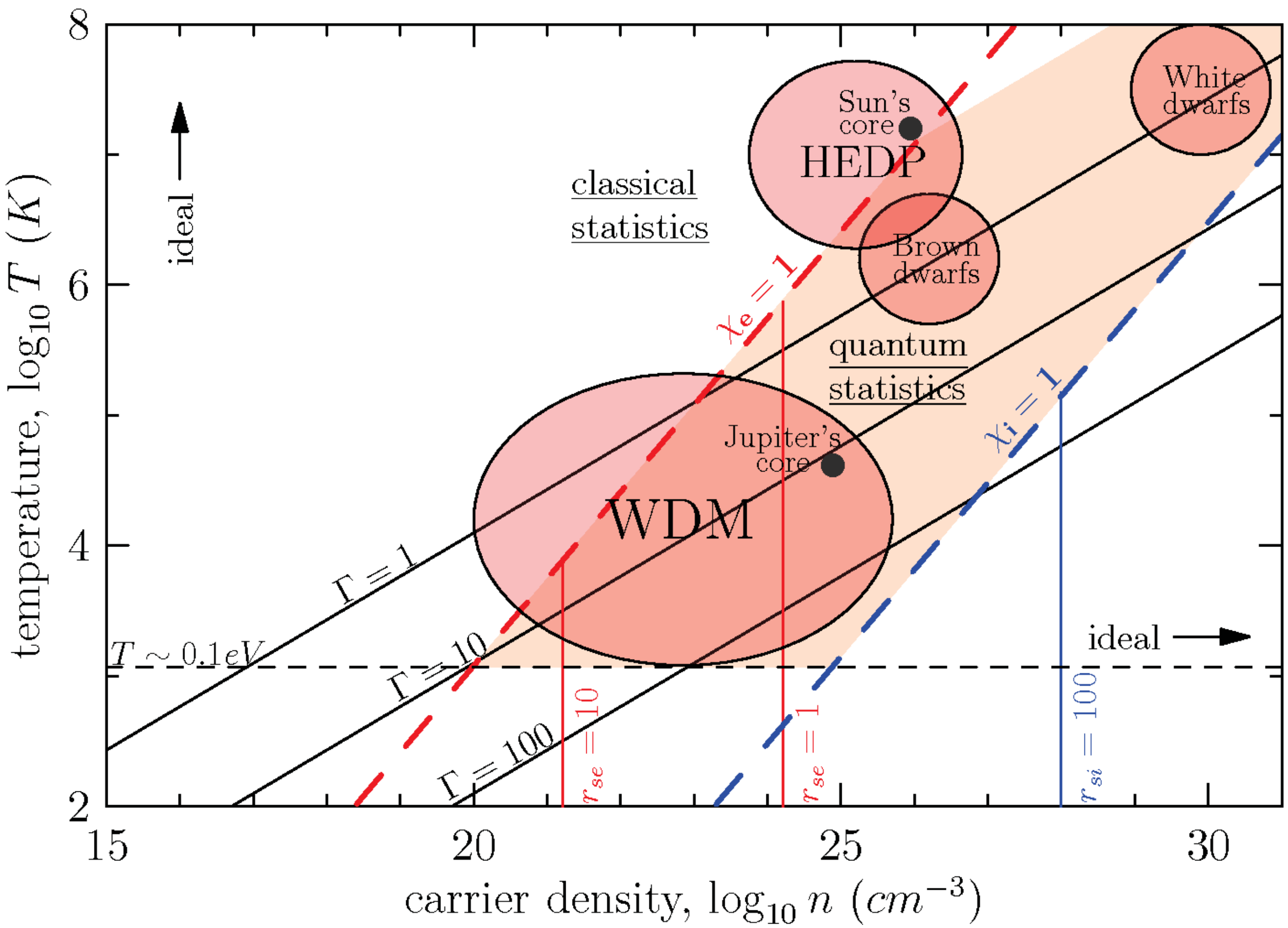
* M. Bonitz, Physik Journal 7/8 2002
MB et al. In: „Introduction to Complex Plasmas“, Springer 2010

Multi-component plasma

$$\chi_a = n_a \Lambda_a^3 \sim \frac{n_a}{m_a^{3/2}}$$

$$r_{sa} = \bar{r}_a / a_{Ba} \sim \bar{r}_a m_a Z_a^2$$

figure:
Hydrogen



WDM: „warm dense matter", **HEDP**: high energy density plasmas

MB, V. Filinov, V. Fortov, P. Levashov, and H. Fehske, PRL ***95****, 235006 (2005)*

Theory of strongly correlated plasmas

Challenge: selfconsistent treatment of
- Coulomb correlations
- external fields
- collective effects, plasma oscillations

Properties: Equation of state, transport, optics...

Equilibrium
- fluid theory, QLCA
- Monte Carlo
- Molecular dynamics

Nonequilibrium
- kinetic theory
- Molecular dynamics

Text books:
- M. Bonitz, D. Semkat (eds.), „*Computational Methods for Many-Body Physics*", Rinton Press, Princeton 2006
- M. Bonitz, N. Horing, P. Ludwig (eds.), „*Introduction to Complex Plasmas*", Springer 2010

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- properties and examples

2. Diffusion in magnetized plasmas

- a brief reminder

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- first principle Molecular dynamics approach

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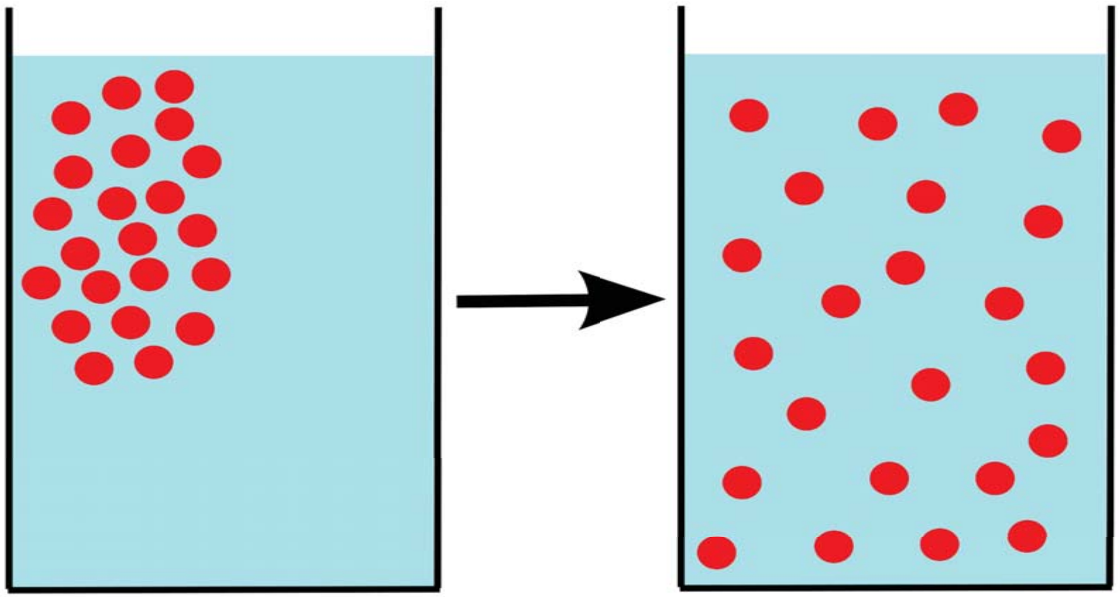
Diffusion



- Key transport property: mass transport
- Closely related to mobility, conductivity etc.
- Depends on interactions in system
  → sensitive to correlations, liquid behavior etc.
- Collective system property – linked to single-particle behavior

Diffusion – governs low-velocity stopping power

- Dufty and Berkovsky[1]: relation ship between diffusion coefficient and stopping power S for low-velocity projectiles:

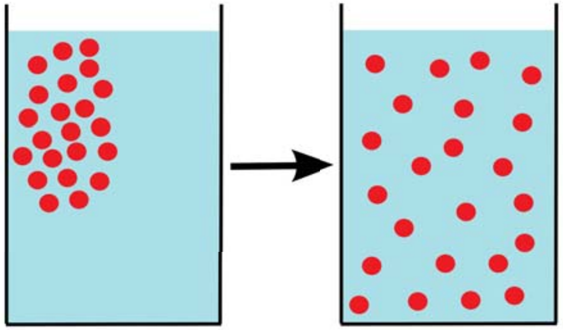
$$\lim_{V_{\mathrm{b}\to 0}} \frac{S(V_\mathrm{b})}{V_\mathrm{b}} = k_\mathrm{B} T_\mathrm{e} D^{-1}$$

projectile velocity

- Deutsch and Popoff[2]: application to magnetized target fusion and inertial confinement fusion → parallel and perpendicular geometries relevant

1 J.W. Dufty, M. Berkovsky, *Electronic stopping of ions in the low velocity limit,* Nucl. Instrum. Methods Phys. Res., Sect. B **96**, 626 (1995).
2 C. Deutsch, R. Popoff, *Low ion velocity slowing down in a strongly magnetized plasma target* Nucl. Instrum. Methods Phys. Res., Sect. A **606**, 212 (2009).

How to measure Diffusion?



$$D = \lim_{t \to \infty} \frac{\langle |\mathbf{r}(t) - \mathbf{r}(t_0)|^2 \rangle}{2dt}$$

Number of dimensions

- **Einstein 1905** $$\langle |\mathbf{r}(t) - \mathbf{r}(t_0)|^2 \rangle \propto t^\alpha$$

- **Cases:**

$\alpha = 1$: Normal Diffusion
$\alpha < 1$: Subdiffusion
$\alpha > 1$: Superdiffusion

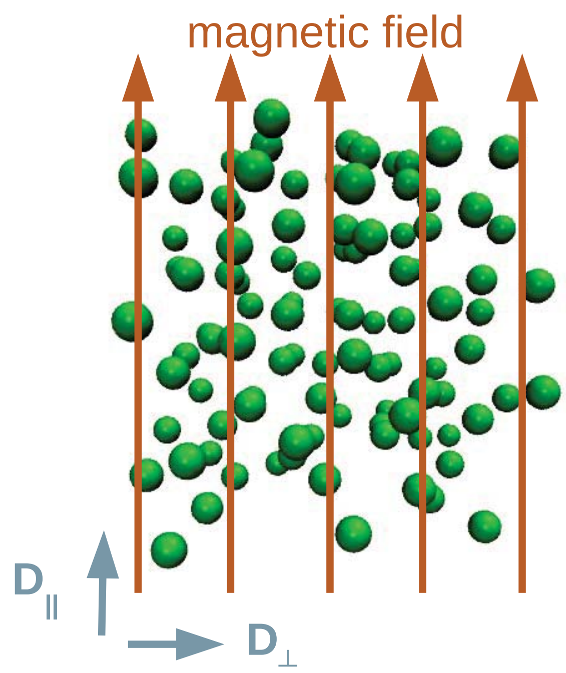
Examples: turbulent fluids, fractral geometries, certain two-dimensional systems, ...

→ The diffusion coefficient characterizes *how fast* particles diffuse with time
→ The diffusion exponent characterizes *according to which law* the particles diffuse with time

Diffusion in weakly correlated magnetized plasmas

Predicted scaling laws:

	$\beta \leq 1$	$\beta >> 1$
$D_\perp$	$b_0 B^0 + b_2 B^{-2}$ [3]	$\gamma_0 k_B T(qB)^{-1}$ [1,2] $\sim B^{-2}$ [3,4]
$D_\parallel$	$\sim B^0$ [3]	$\sim B^0$ [5]



units

$$\Gamma = q^2/(ak_B T)$$

$$\beta = \omega_c/\omega_p$$

$$\omega_c = QB/mc$$

[1] Bohm 1949
[2] Marchetti et al. 1984
[3] Lifshitz, Pitaevski 1981
[4] Dubin 1997
[5] Cohen, Suttorp 1984

Cross-field Diffusion in <u>strong</u> magnetic fields

„Bohm diffusion“: linear decrease of D with B

Bohm: $D_\perp = 1/16 \quad k_B T/(eB)$ [1]

Spitzer: $D_\perp = 0.21 \quad k_B T/(eB)$ [2]

[1] *The characteristics of electrical discharges in magnetic fields*, ed A. Guthrie and R. K. Wakerling (New York: McGraw-Hill) (1949).

[2] Phys. Fluids 3, 659 (1960)

Spitzer's result for diffusion in strong magnetic fields

Particle Diffusion across a Magnetic Field

LYMAN SPITZER, JR.
Project Matterhorn, Princeton University, Princeton, New Jersey
(Received May 18, 1960)

$$D \doteq 2\underbrace{K_1}_{\ll 1}{}^2\,\underbrace{K_2}_{\sim 1}\,\underbrace{K_3}_{>1}(ckT/eB)$$

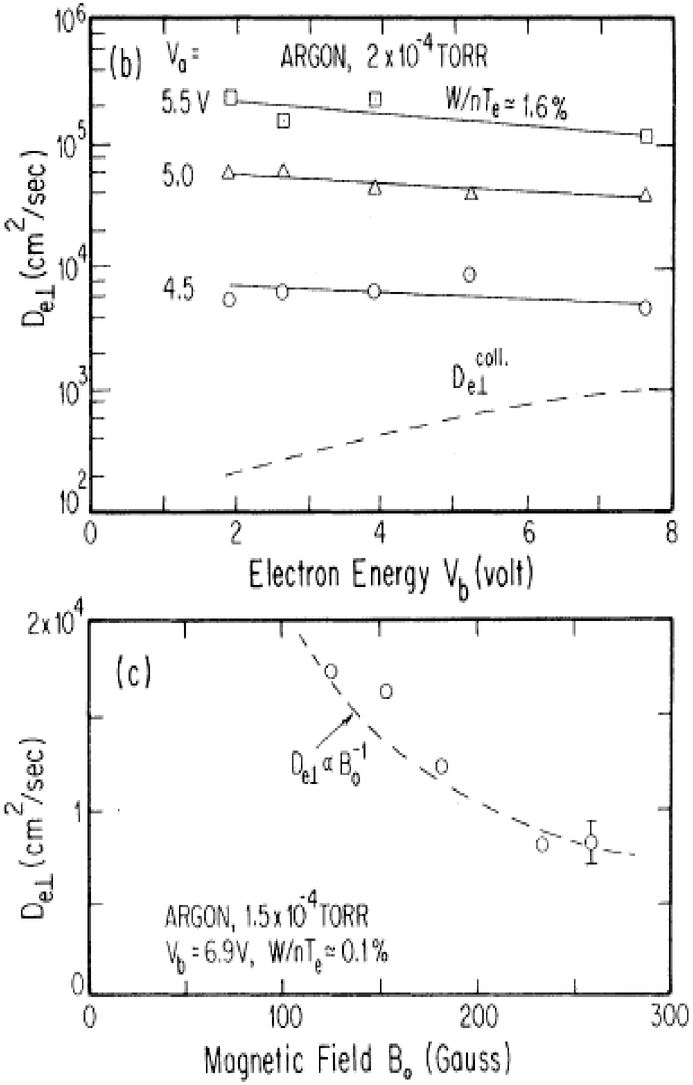
Phys. Fluids **3**, 659 (1960)

Bohm diffusion in turbulent plasma – experimental example

Stenzel, Gekelman,
PRL **40**, 550 (1978)

Experiment:
e-beam injected in magnetized plasma

excite current-driven
ion sound turbulence



W: wave energy density

V: amplitude of turbulent Field

← Normal collisional diffusion

Cross-field Diffusion in <u>strong</u> magnetic fields

Bohm: $D_\perp = 1/16 \; k_B T/(eB)$ [1]

Spitzer: $D_\perp = 0.21 \; k_B T/(eB)$ [2]

Explanation: anomalous transport (collective modes, instabilities, turbulence)

Marchetti *et al.*: derivation from coupled mode theory [3]
Relevant for magnetic fusion plasmas

[1] *The characteristics of electrical discharges in magnetic fields*, ed A. Guthrie and R. K. Wakerling (New York: McGraw-Hill) (1949).
[2] Phys. Fluids 3, 659 (1960)
[3] PRA 29, 2960 (1984)

Bohm Diffusion: Marchetti et al.

PHYSICAL REVIEW A VOLUME 29, NUMBER 5 MAY 1984

Anomalous diffusion of charged particles in a strong magnetic field

M. Cristina Marchetti, T. R. Kirkpatrick, and J. R. Dorfman
Institute for Physical Science and Technology and Department of Physics and Astronomy, University of Maryland, College Park, Maryland 20742
(Received 10 February 1984)

A self-consistent mode-coupling theory is used to calculate the coefficient of self-diffusion in a three-dimensional classical one-component plasma subjected to an external magnetic field. For asymptotically large fields a Bohm-like behavior is found for diffusion in the plane perpendicular to the magnetic field. The experimental consequences of these results are discussed.

Weak coupling theory, $\Gamma<1$:

- $D_\perp$ as an integral of the Velocity autocorrelation function [VACF] (Green-Kubo)
- decay of VACF *on long timescales* governed by hydrodynamic modes
- these modes are computed via the mode-coupling theory
- the B-dependence in the thermodynamic limit contributes to the VACF

Phys. Rev. A **29**, 2960 (1984)

Bohm Diffusion: Marchetti et al.

Field dependence of $D_\perp$:

Weak coupling:

Many particles in Deby sphere

$$N_D = 4\pi n_{0e}\lambda_e^3 = 1/\epsilon \gg 1$$

(a) a classical region where $D_\perp \sim B^{-2}$ for

$$\nu_c/\omega_p < \omega_B/\omega_p < 0.4(\epsilon_p \nu_c/\omega_p)^{-1/2} \;;$$

(b) a plateau region where $D_\perp \sim B^0$ for

$$0.4(\epsilon_p \nu_c/\omega_p)^{-1/2} < \omega_B/\omega_p < (\epsilon_p \nu_c/\omega_p)^{-1/2} \;;$$

collision frequency:

$$\nu_c = \omega_p \epsilon_p \ln \epsilon_p^{-1}$$

(c) a Bohm region where $D_\perp \sim B^{-1}$ for

$$\omega_B/\omega_p > 4(\epsilon_p \nu_c/\omega_p)^{-1/2} \;.$$

Phys. Rev. A **29**, 2960 (1984)

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   - role of dimensionality, transient effects

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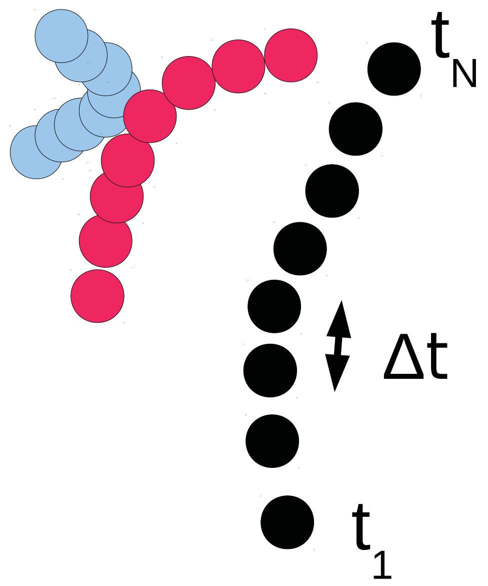
6. **Strongly correlated 3D plasmas**

   - Diffusion coefficient perpendicular and parallel to **B**

7. **Conclusion and outlook**

First-principle simulations: molecular dynamics

solve N coupled equations of motion
Variable boundary conditions, fields, pair forces
Equilibrium or non-equilibrium
Microcanonic or canonic ensemble



Equation of motion for particles in a constant magnetic field with friction:

$$\boxed{m\ddot{\boldsymbol{r}}_i = \boldsymbol{F}_i + q\,\dot{\boldsymbol{r}}_i \times \boldsymbol{B} + \boldsymbol{S}_i}$$

$$\boldsymbol{F}_i = -\frac{q^2}{4\pi\varepsilon_0} \sum_{j=1}^{N}{}' \left(\nabla \frac{e^{-r/\lambda_D}}{r} \right) \Bigg|_{\boldsymbol{r}=\boldsymbol{r}_i-\boldsymbol{r}_j}$$

$$\boldsymbol{B} = B \cdot \boldsymbol{e}_z$$

$$\boldsymbol{S}_i = -m_i \bar{\nu} \dot{\boldsymbol{r}}_i + \boldsymbol{y}_i \qquad \langle y_{\alpha,i}(t_0) y_{\beta,j}(t_0+t) \rangle = 2k_B T \bar{\nu} \delta_{ij} \delta_{\alpha\beta}\, \delta(t)$$

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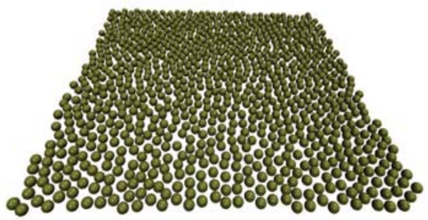

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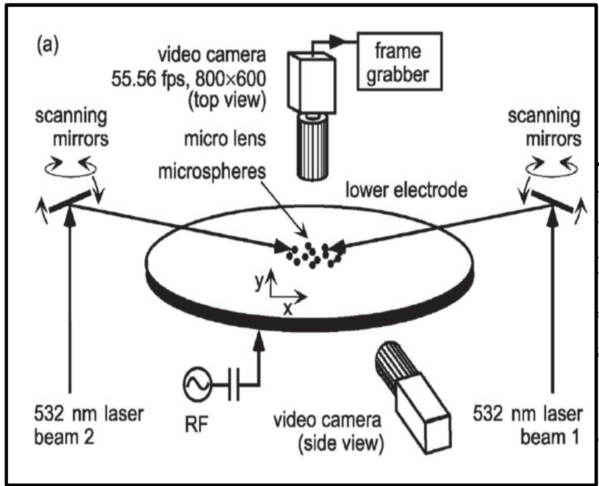
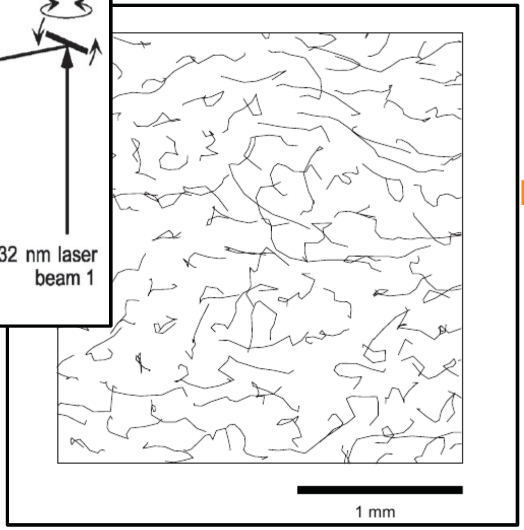
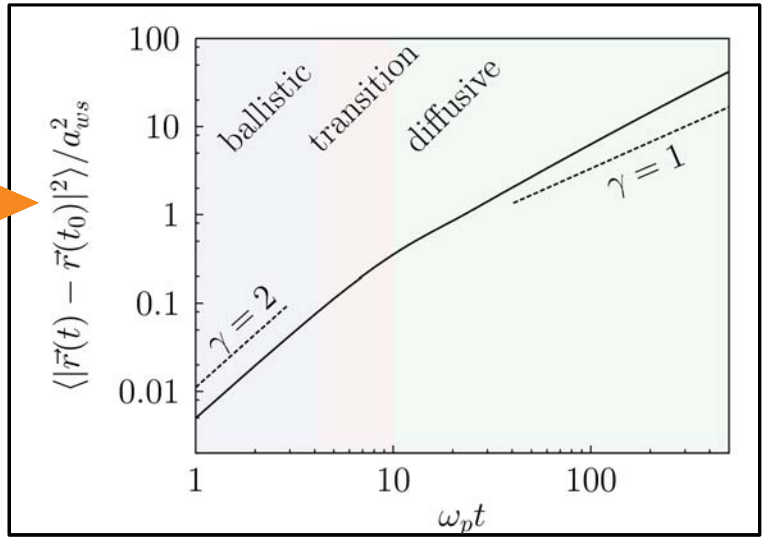
Diffusion experiments in 2D dusty plasmas



*Liu, Goree, PRL **100**, 05503 (2008)*





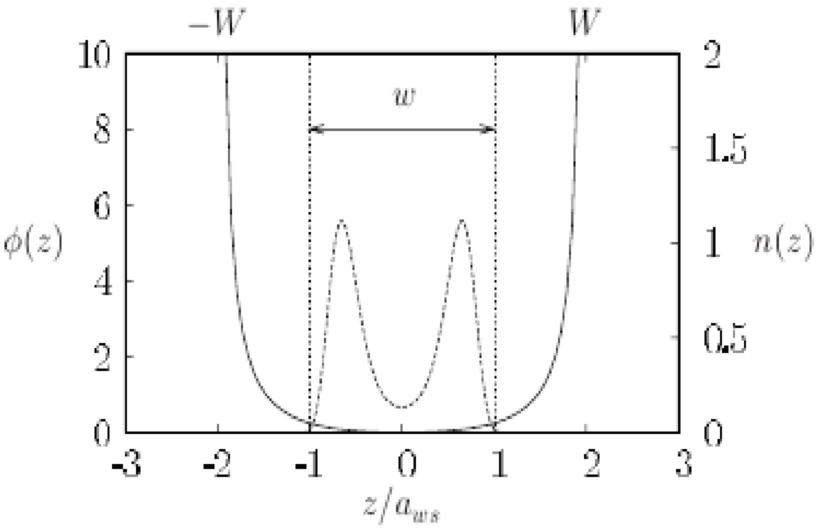
Diffusion exponent α	Authors	Year
1.31	Juan and Lin I	1998
1.1 . . . 1.3	Juan, Chen and Lin I	2001
1.0 . . . 1.3	Lai and Lin I	2002
1.3	Quinn and Goree	2002
1.35	Ratynskaia et al	2006
1.009 . . . 1.183	Liu and Goree	2008

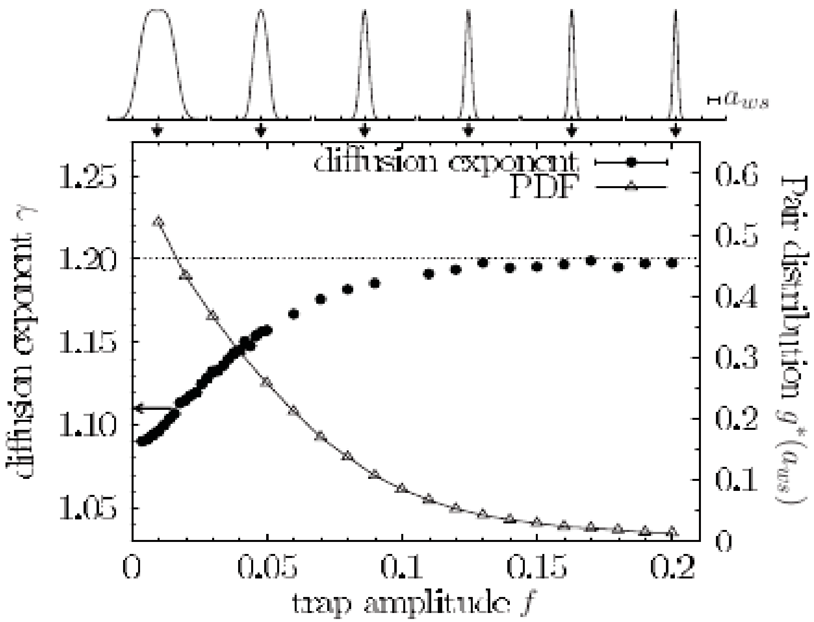
α>1: Superdiffusion

Similar scatter in theory papers...

Influence of dimensionality on Diffusion

Is observed superdiffusion a 2D geometry effect?





T. Ott, Z. Donko, P. Hartmann, and M. Bonitz, *Superdiffusion in quasi-two-dimensional Yukawa liquids,* Phys. Rev. E **78**, 026409 (2008)

Diffusion in 2D dusty plasmas

<u>Open Questions:</u>

- Influence of plasma parameters: screening, coupling, friction
- Origin of large scatter of α?

Diffusion exponent α	Authors	Year
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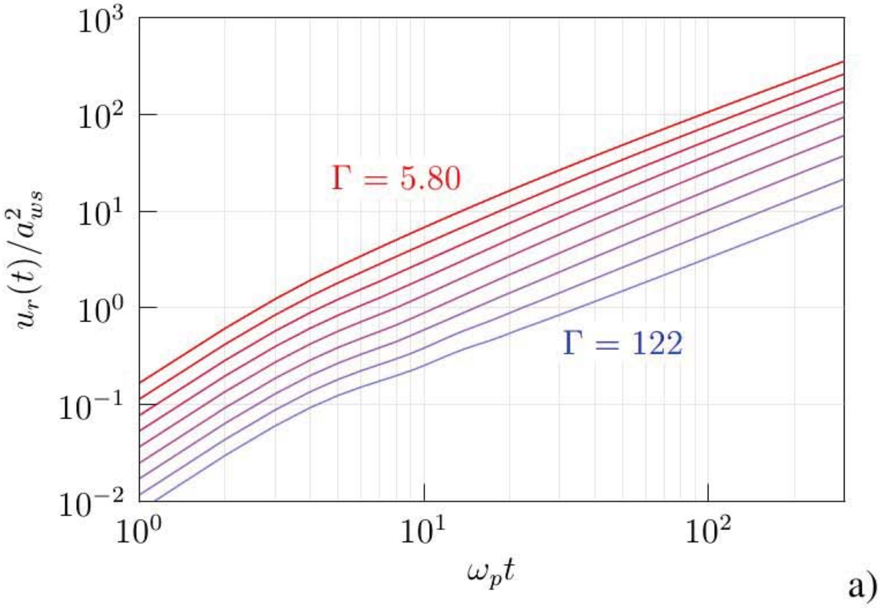
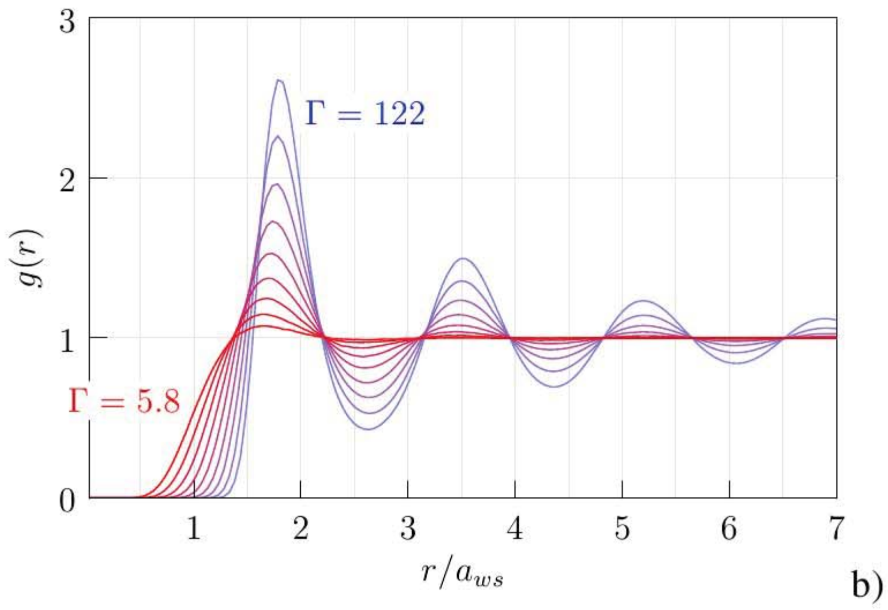
$\alpha>1$: Superdiffusion

Influence of order (coupling)

κ=1.0, mean relative particle displacement versus time

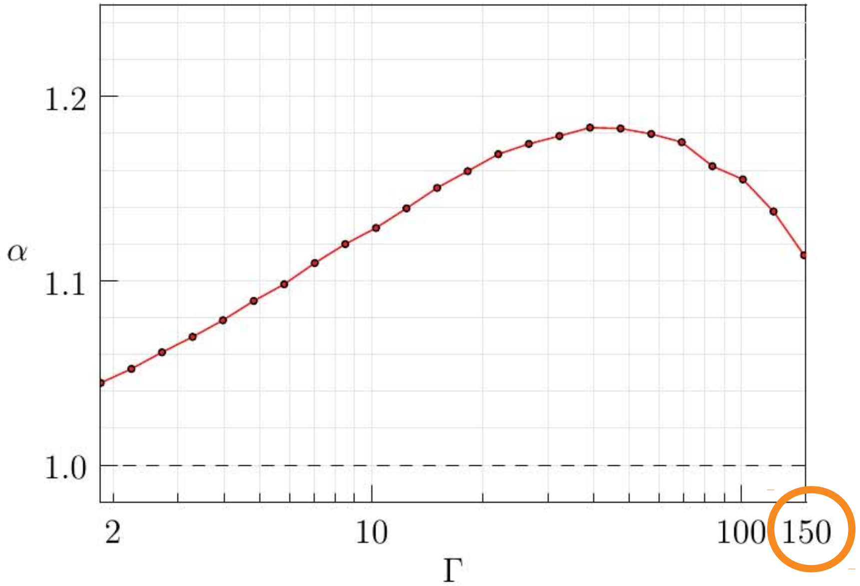
pair distribution function for different coupling strenths



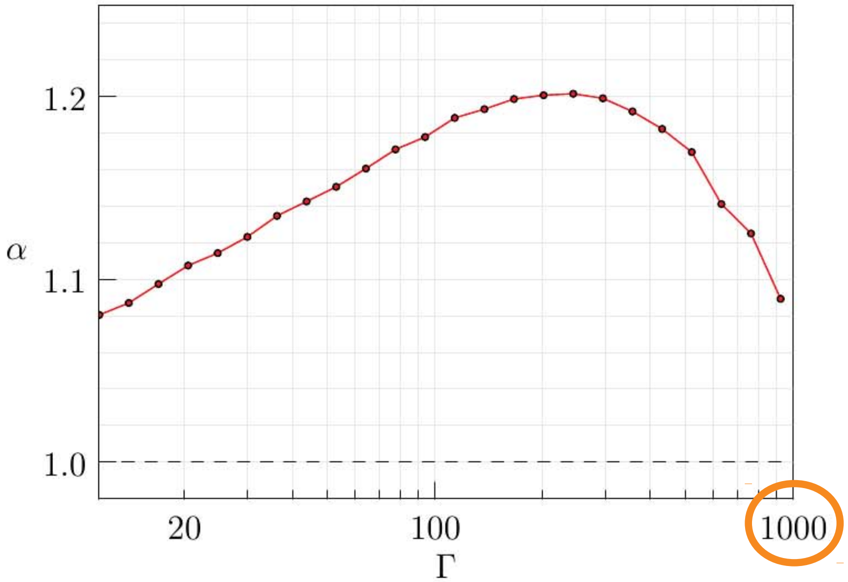


Influence of order and screening

κ=1.0



κ=3.0



$$\langle |\mathbf{r}(t) - \mathbf{r}(t_0)|^2 \rangle \propto t^\alpha$$

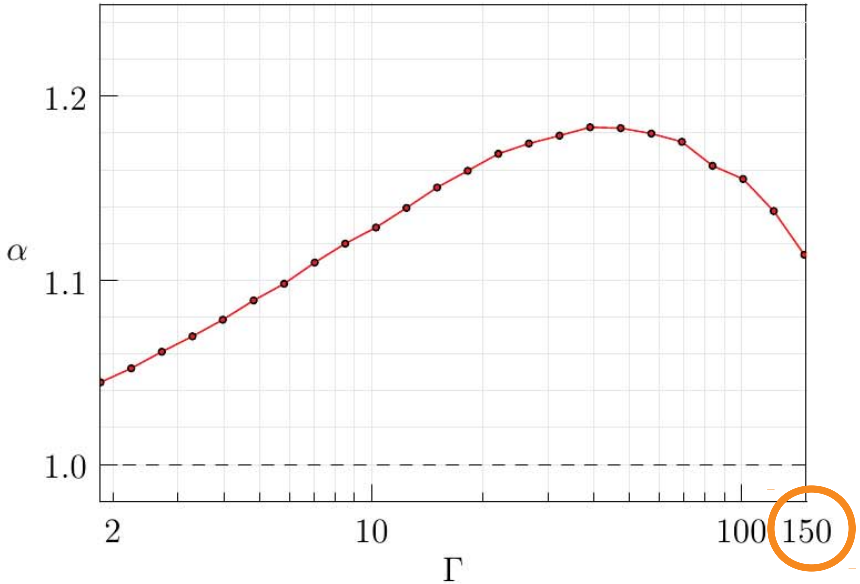
$\alpha = 1$: Normal Diffusion
$\alpha < 1$: Subdiffusion
$\alpha > 1$: Superdiffusion

$$\phi(r) = \frac{Q}{4\pi\varepsilon_0} \frac{e^{-r/\lambda_D}}{r}$$

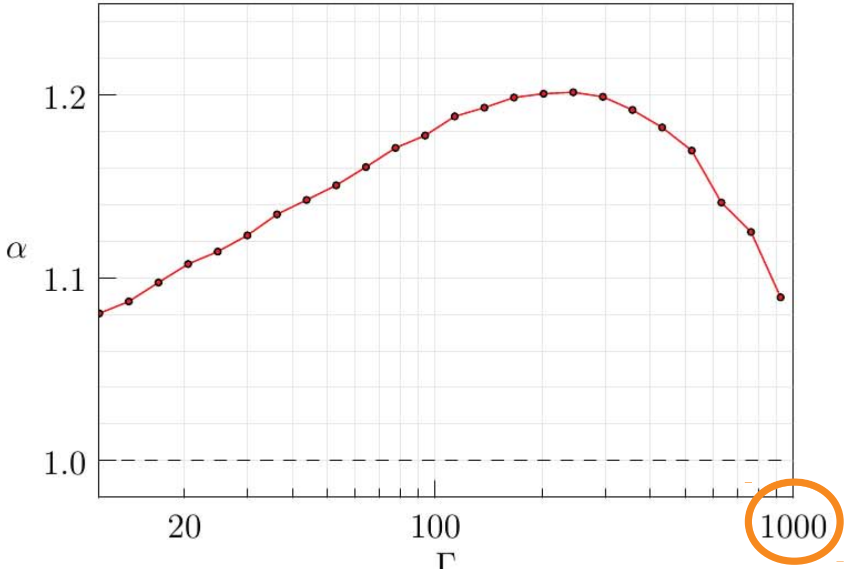
$$\kappa = a_{ws}/\lambda_D$$

Influence of order – (nearly) universal scaling

κ=1.0



κ=3.0



$$\langle |\mathbf{r}(t) - \mathbf{r}(t_0)|^2 \rangle \propto t^\alpha$$

$\alpha = 1$: Normal Diffusion
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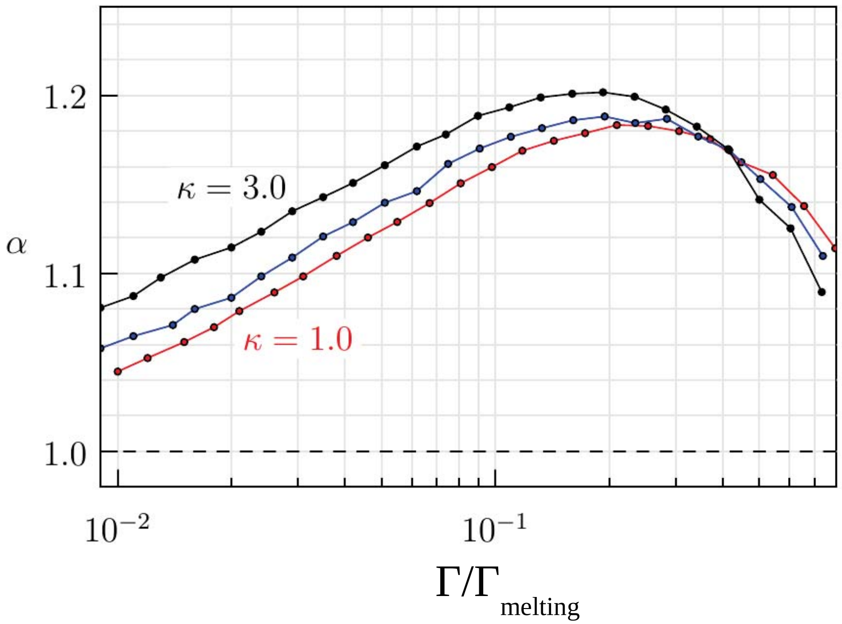
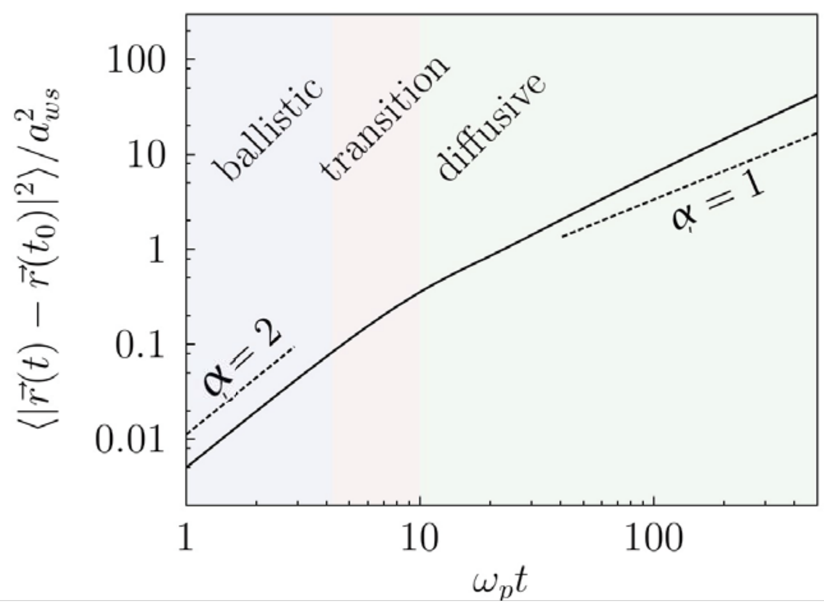
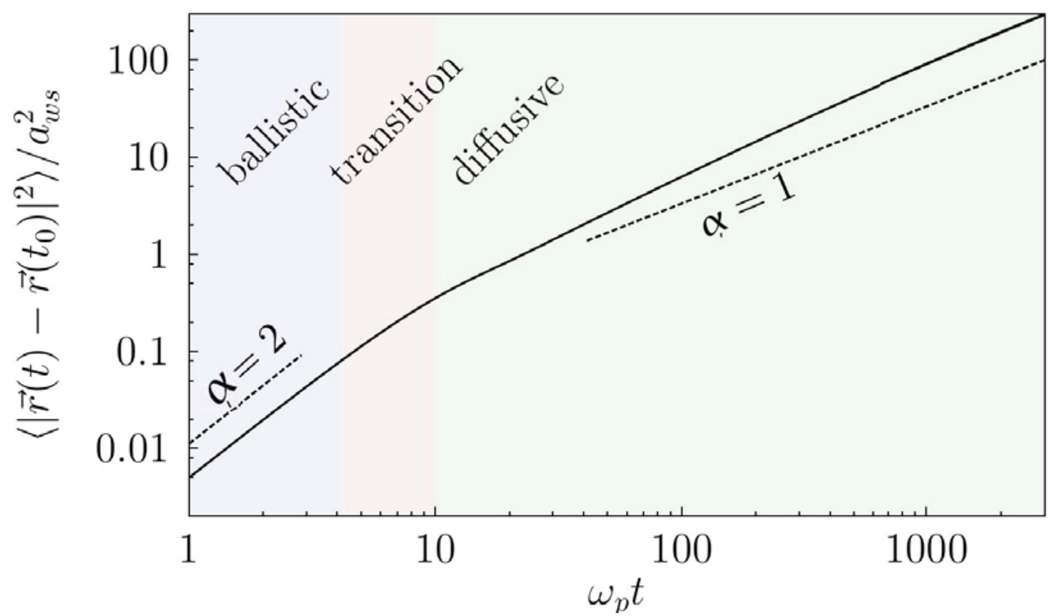


observe systematic behavior for arbitrary screening (interaction range)

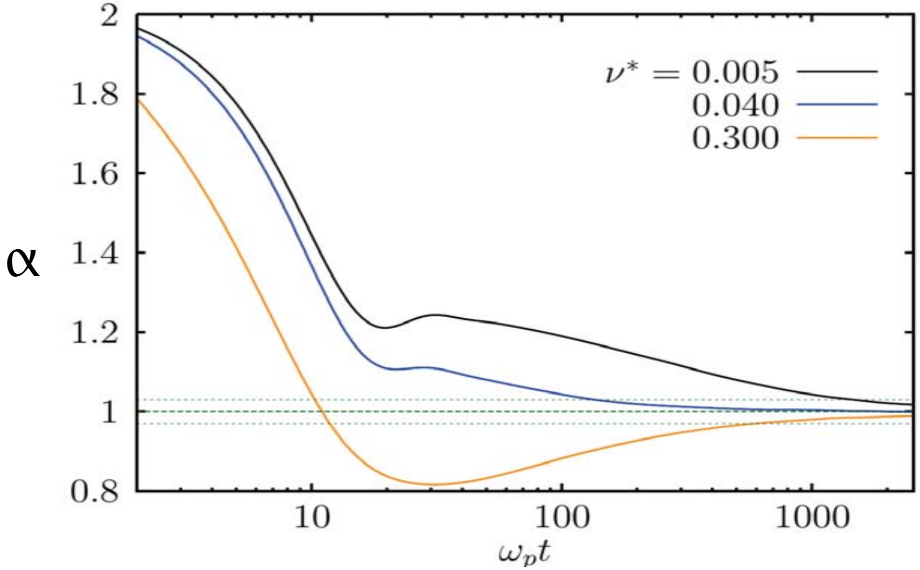
T. Ott, MB, P. Hartmann, PRL ***103****, 099501 (2009)*

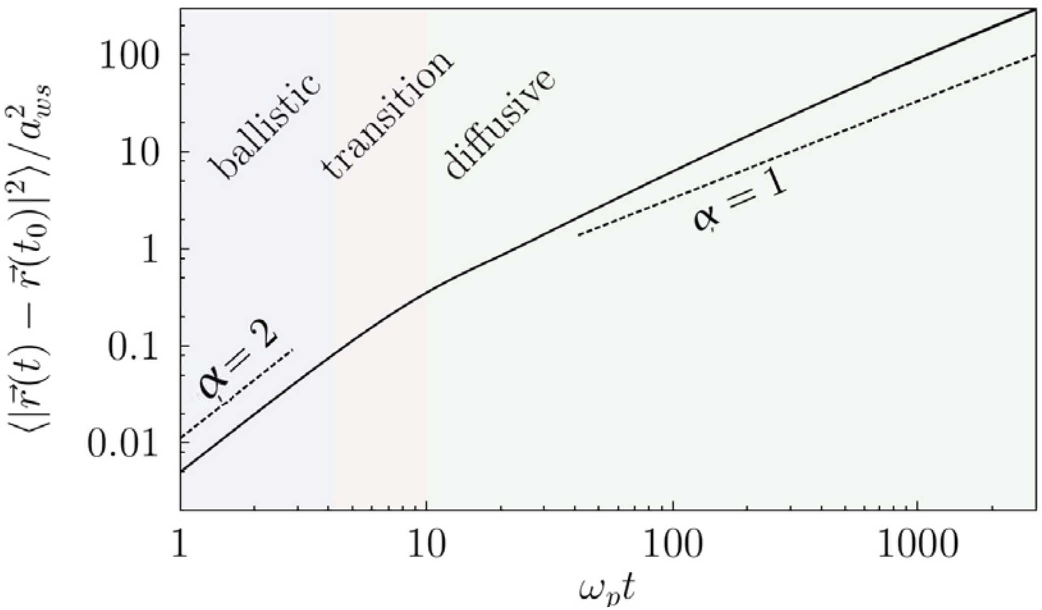
Anomalous diffusion - influence of friction





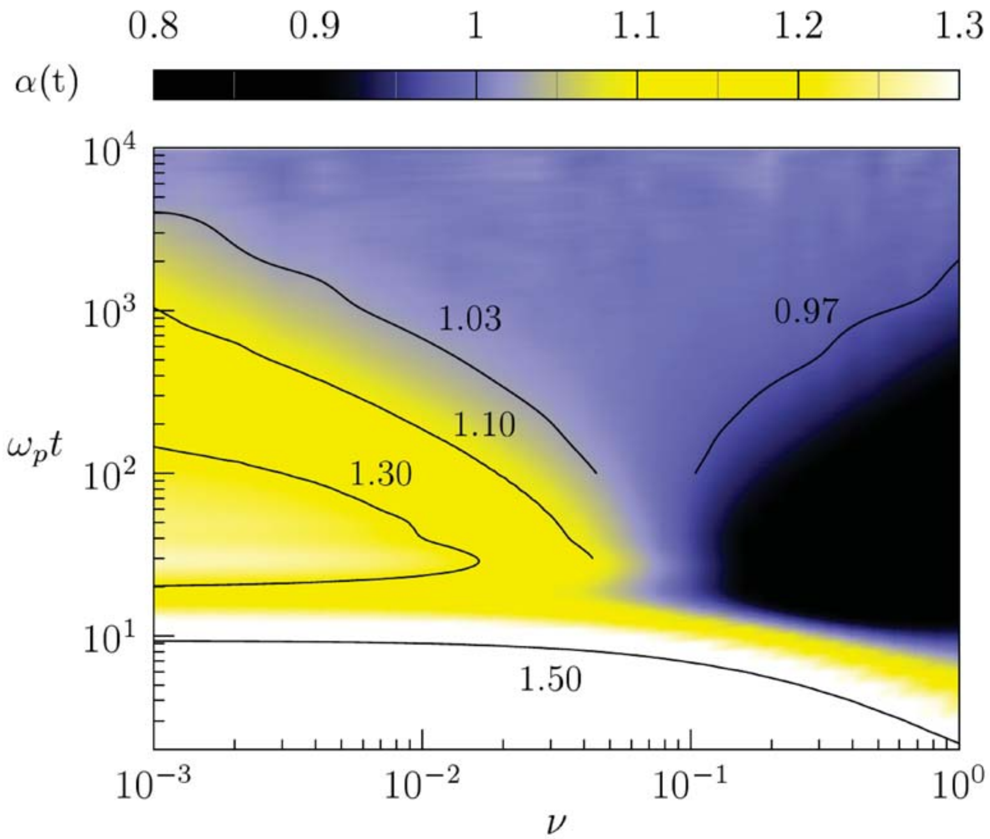
Anomalous diffusion - influence of friction

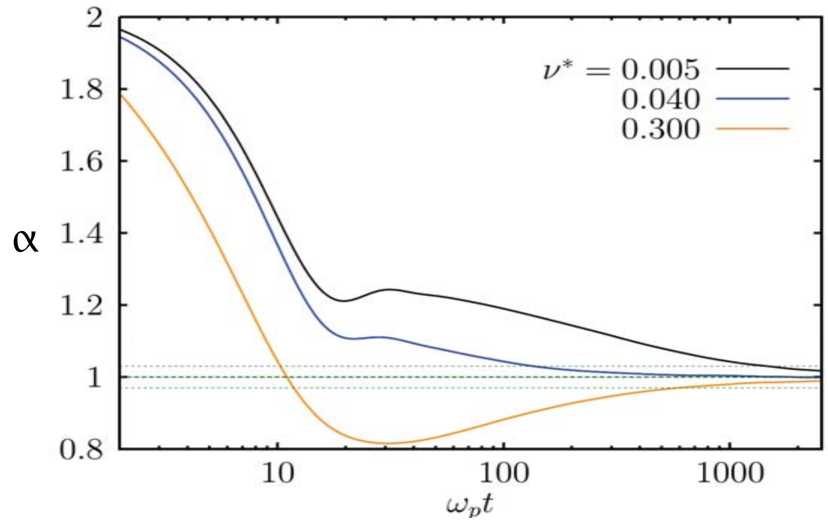




$$\partial_{\ln t} \ln[u_r(\ln t)]$$

Transient character of anomalous diffusion





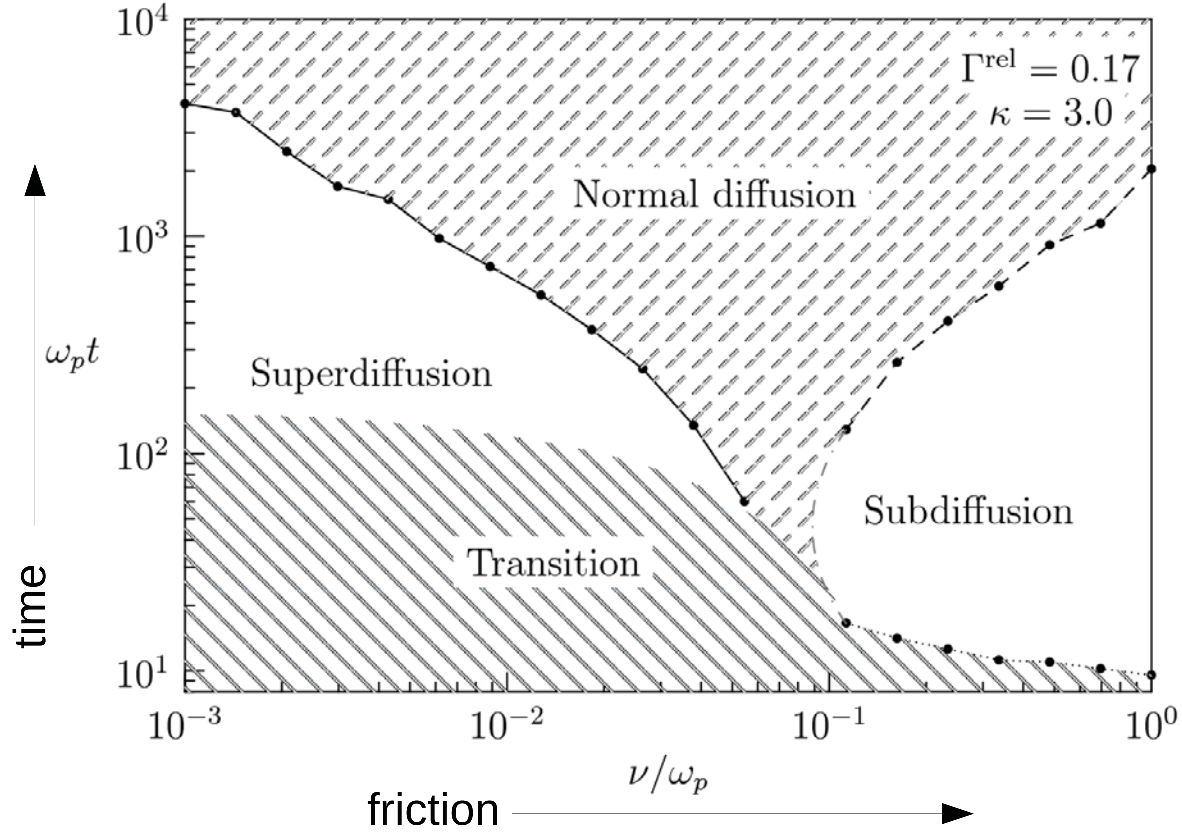
*T. Ott, M. Bonitz, PRL **103,** 195001 (2009)*

Transient character of anomalous diffusion

Non-trivial friction dependence:

strong friction: subdiffusion
weak friction: superdiffusion

anomalous diffusion does not extend to arbitrary timescales



T. Ott, M. Bonitz, PRL ***103,*** *195001 (2009)*

Outline

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- a brief reminder

3. Diffusion in strongly correlated plasmas

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4. Superdiffusion in plasma layers

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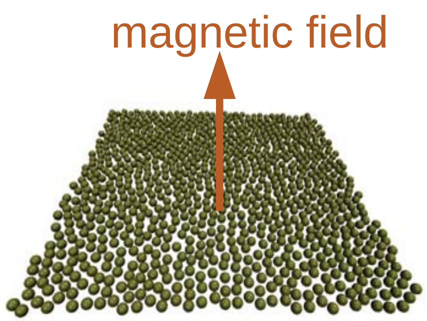
5. Collective modes in magnetized 2D plasmas

6. Strongly correlated 3D plasmas

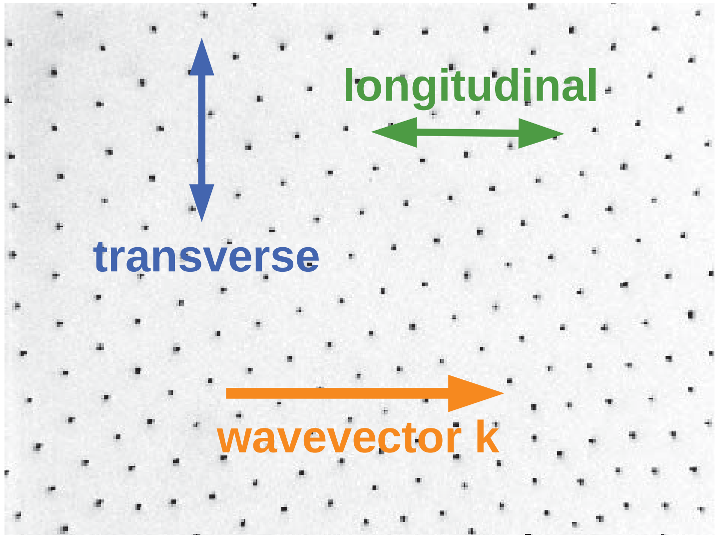
- Diffusion coefficient perpendicular and parallel to **B**

7. Conclusion and outlook



MD approach to plasma wave spectra (exact)

strongly coupled (Γ>1) systems support longitudinal and transverse waves



calculate autocorrelation function of microscopic fluxes:

$$\rho(\boldsymbol{k},t) = \sum_{j=1}^{N} e^{i\boldsymbol{k}\cdot\boldsymbol{r}_i(t)}$$

Fourier components of density

ACF

$$F(\boldsymbol{k},t) = \frac{1}{N}\langle\rho(\boldsymbol{k},t)\rho^*(\boldsymbol{k},t_0)\rangle$$

FT

$$S(\boldsymbol{k},w) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{i\omega t}F(\boldsymbol{k},t)$$

intermediate scattering function

dynamical structure factor

MD approach to plasma wave spectra (exact)

<u>Shortcut:</u>

$$\rho(\boldsymbol{k},t) = \sum_{j=1}^{N} e^{i\boldsymbol{k}\cdot\boldsymbol{r}_i(t)}$$

Fourier components of density

Wiener-Khinchine

$$S(k,w) = \frac{1}{2\pi N} \lim_{T\to\infty} \frac{1}{T} \left|\mathcal{F}_t\{\rho(k,t)\}\right|^2$$

dynamical structure factor

$$\left(F(\boldsymbol{k},t) = \frac{1}{N}\langle \rho(\boldsymbol{k},t)\rho^*(\boldsymbol{k},t_0)\rangle \right)$$

intermediate scattering function

Peaks of dynamical structure factor reveal dispersion relation for *longitudinal* waves

Wiener-Khinchine theorem:
The power spectral density of a (...) random process is the Fourier-transform of the corresponding autocorrelation function.

Current autocorrelation functions

$$j(k,t) = \sum_{j=1}^{N} v_j(t) \exp[ikx_j(t)]$$

Fourier components of currents

**projection along k
and (1-k)**

$$\lambda(k,t) = \sum_{j=1}^{N} v_{jx} \exp[ikx_j]$$

longitudinal currents

$$\tau(k,t) = \sum_{j=1}^{N} v_{jy} \exp[ikx_j]$$

transverse currents

$$L(k,\omega)$$
$$T(k,\omega)$$

Relation between S(k,ω) and L(k,ω):

using $d^2/dt^2 \langle A(t) B^*(t_0)\rangle = \langle \dot{A}(t) \dot{B}^*(t_0)\rangle$

and the continuity equation, one can show that

$$S(k,\omega) = \frac{k^2}{\omega^2} L(k,\omega)$$

→ S(k,ω) and L(k,ω) contain the same physical information

Magnetooscillations of 2d plasma – known results

magnetized strongly coupled

Yukawa plasma has 2 modes:

• magnetoplasmon

• magnetoshear

Theoretical description via e.g. [*]

• QLCA [Ke Jiang *et al.*, PoP 2007]

• Harmonic approximation

[Kalman *et al.* , PRL 2000]

MD simulations: see fig.

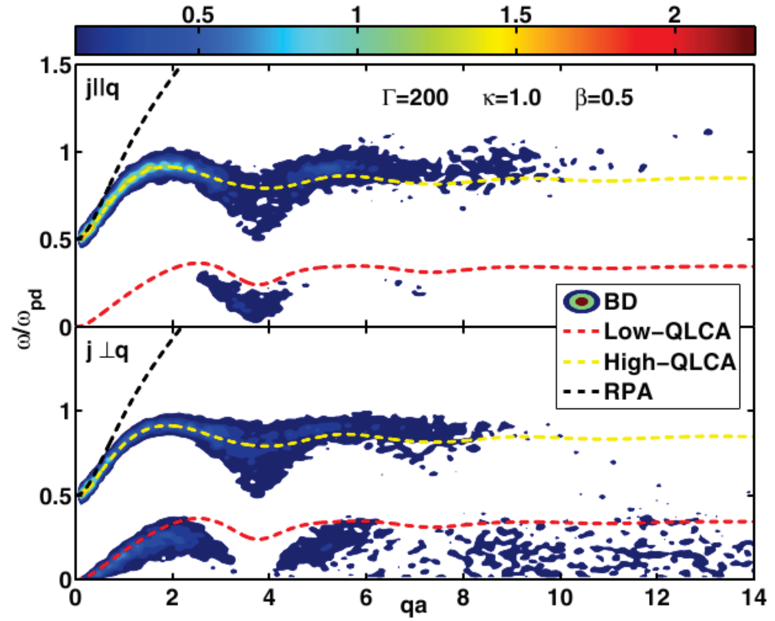


FIG. 1. (Color online) (Color bar in arbitrary units) The wave spectra in both the $\mathbf{j} \parallel \mathbf{q}$ and $\mathbf{j} \perp \mathbf{q}$ directions with $\beta=0.5$, $\kappa=1$, and $\Gamma=200$. Dashed lines are the dispersion relations determined by the RPA and QLCA.

Hou, Shukla, Piel, Mišković, PoP ***16*** *073704 (2009)*

Is this the complete spectrum?

[*] Quasi-localized charge approximation, G. Kalman and K. Golden, 1990

Additional magnetooscillations – high harmonics

Variation of
Field strength

β=0

β=0.5

β=1.0

β=1.5

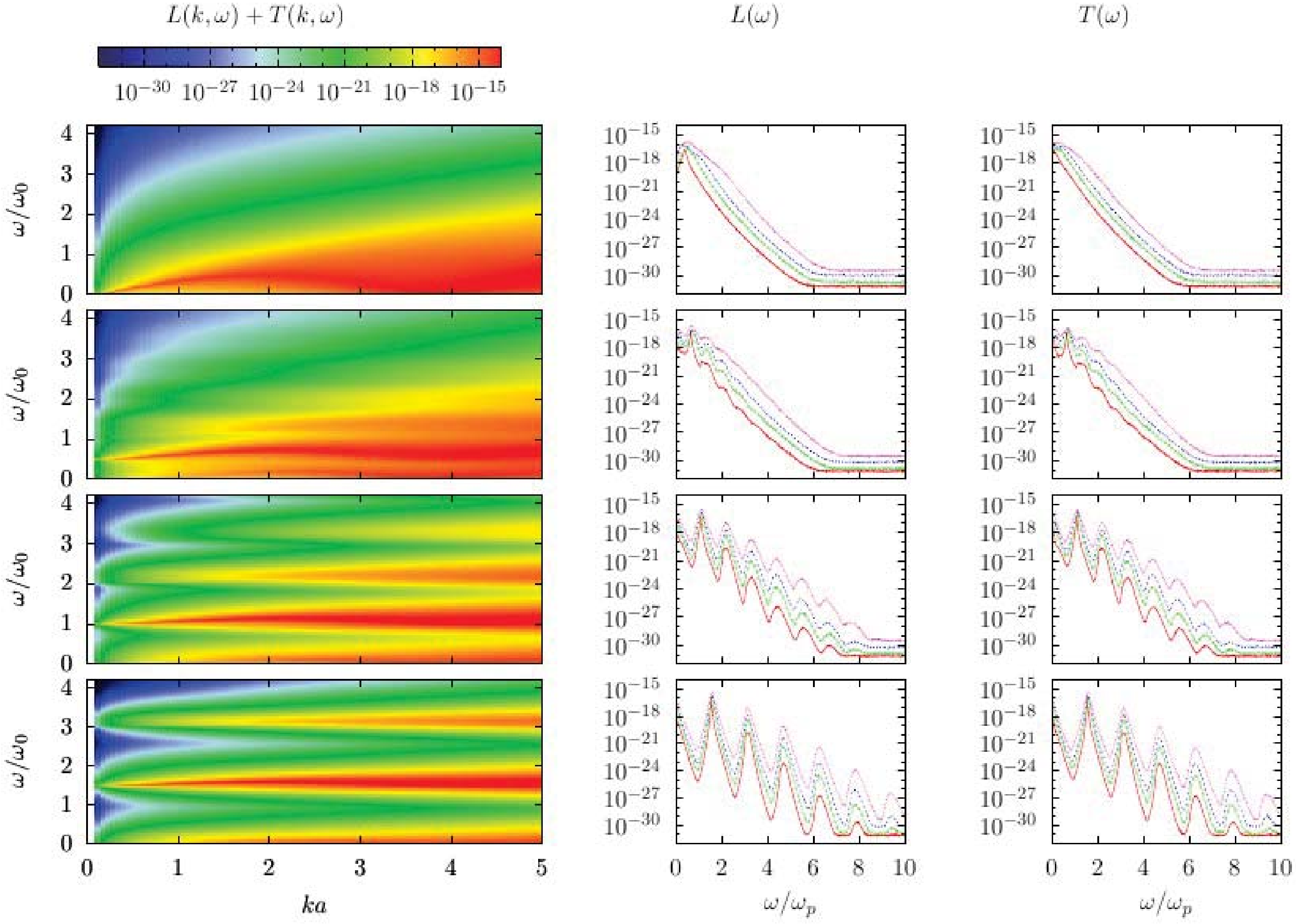


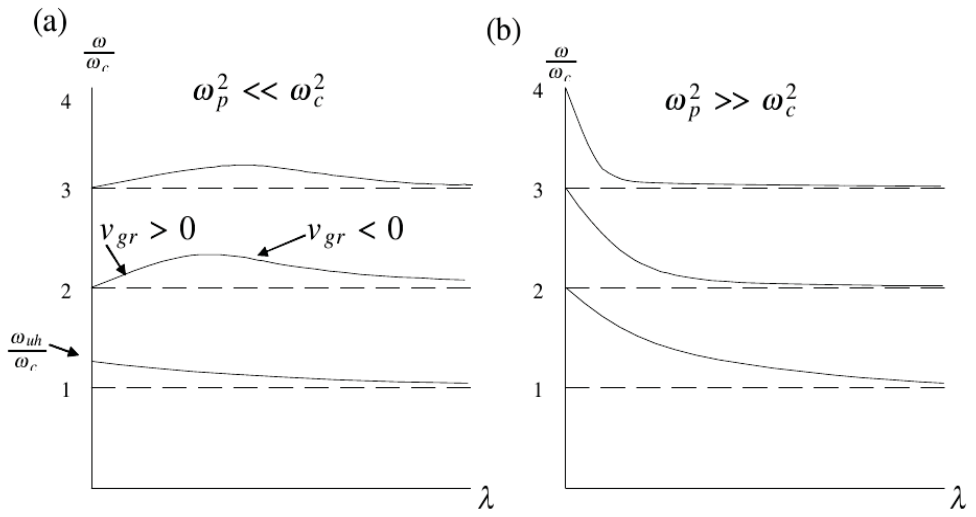
FIG. 1: (Color) $\Gamma = 200$, $\beta = 0.0, 0.5, 1.0, 1.5$ (from top to bottom) Left: Collective excitation spectra, $L(k,\omega) + T(k,\omega)$, Right: $L(\omega)$ and $T(\omega)$ at four values of k ($ka = 1, 2, 3, 5$, lowest to highest curve).

Bernstein modes?

Bernstein waves in ideal classical plasmas:

$$\omega_n^2 = n^2 \omega_c^2$$

magnetic field effect



Bellan: *Fundamentals of Plasma Physics,* Cambridge Univ. Press (2006)

Analogous behavior in ideal quantum plasmas
(e.g. semiconductor quantum wells)

Try Dielectric theory beyond QLCA

Correlated longitudinal dieletric function

$$\boxed{\epsilon^l(\omega,k) = \frac{k_i k_j}{k^2}\epsilon_{ij}(\omega,k) = 1 - V(k)\Pi^l(\omega,k)}$$

$$V(k) = 2\pi Q^2/(k^2+\kappa^2)^{1/2}$$

$$\Pi^l(\omega,k;B) = \frac{\Pi_0^l(\omega,k;B)}{1+V(k)\Pi_0^l(\omega,k;B)G(k,\omega;B)}$$

$$G(k,\omega;B) \approx -D(k)/\omega_p^2(k)$$

2D Vlasov DF with B-field

$$\boxed{\Pi_0^l(\omega,k;B) = \frac{2n_0}{k_B T}e^{-z}\sum_{n=1}^{\infty}\frac{n^2\omega_c^2}{\omega^2-(n\omega_c)^2}I_n(z)}$$

Dispersion relation yields Bernstein modes
Familiar from ideal plasmas[1]

$$z = \frac{k v_T}{\omega_c^2} = \frac{ka}{\beta^2\Gamma}$$

$$\tilde{\omega}_P^2 = \omega_P^2 + D(K),\ D = D_L + D_T$$

$$\boxed{\begin{aligned} \omega_1^2(k) &\approx \omega_c^2 + \tilde{\omega}_p^2(k) \\ \omega_n^2(k) &\approx (n\omega_c)^2 + [n\tilde{\omega}_p(k)]^2\,\frac{1}{n!}\left(\frac{z}{2}\right)^{n-1} \end{aligned}}$$

[1] 3D:. B. Bernstein, Phys. Rev. **109**, 10 (1958)
2D: H. Totsuji (1977); N. J. M. Horing, and M. M. Yildiz, Ann. Physics **97**, 216 (1976).

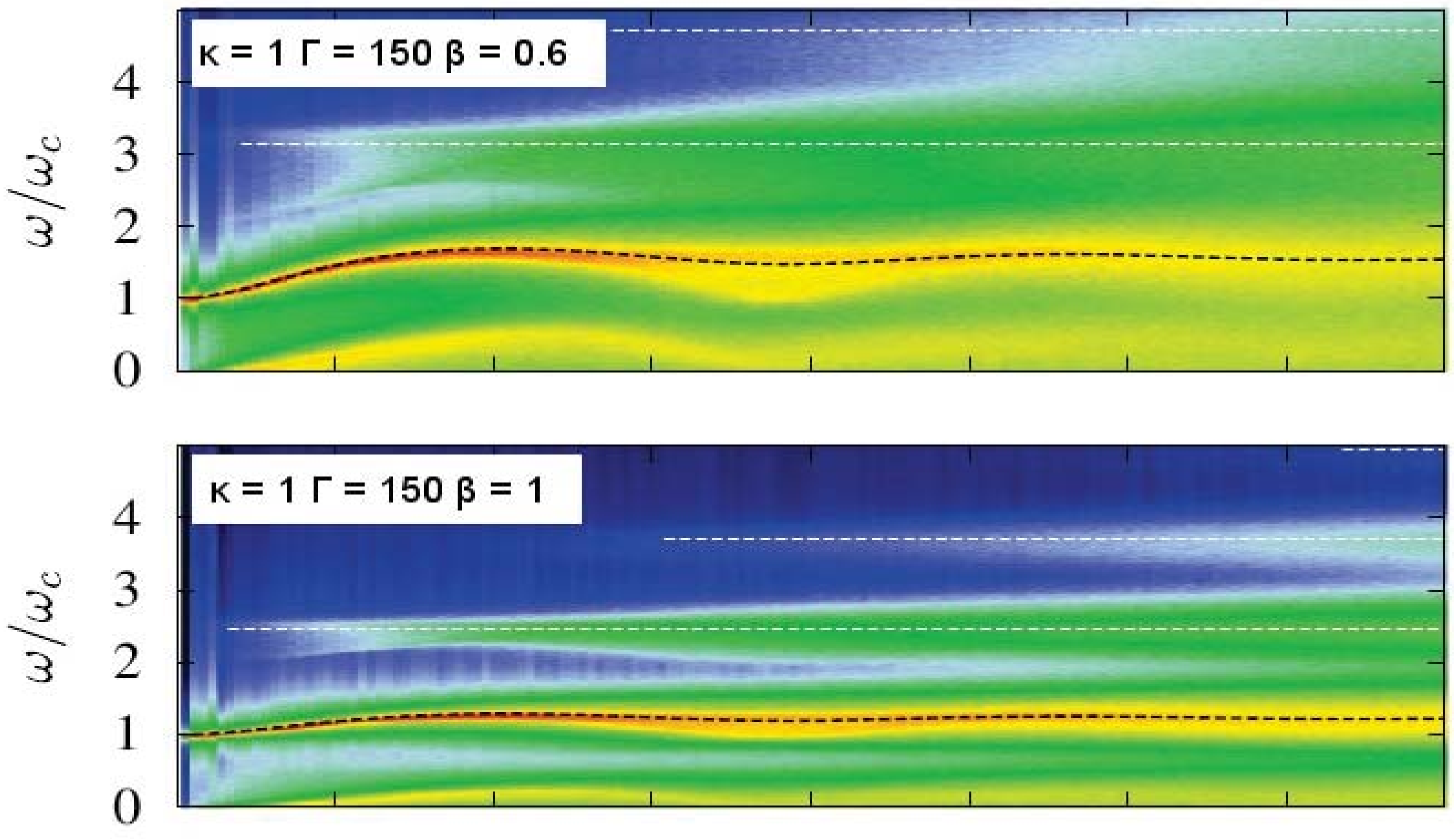
Bernstein modes in strongly correlated 2D Yukawa plasmas?

$$\omega_1^2(k) \approx \omega_c^2 + \tilde{\omega}_p^2(k) \qquad \tilde{\omega}_P^2 = \omega_P^2 + D_L(k) + D_T(k)$$

$$\omega_n^2(k) \approx (n\omega_c)^2 + [n\tilde{\omega}_p(k)]^2 \frac{1}{n!} \left(\frac{z}{2}\right)^{n-1}$$

Vlasov+QLCA predicts:

- wrong frequencies
- no damping
- unlimited k-range



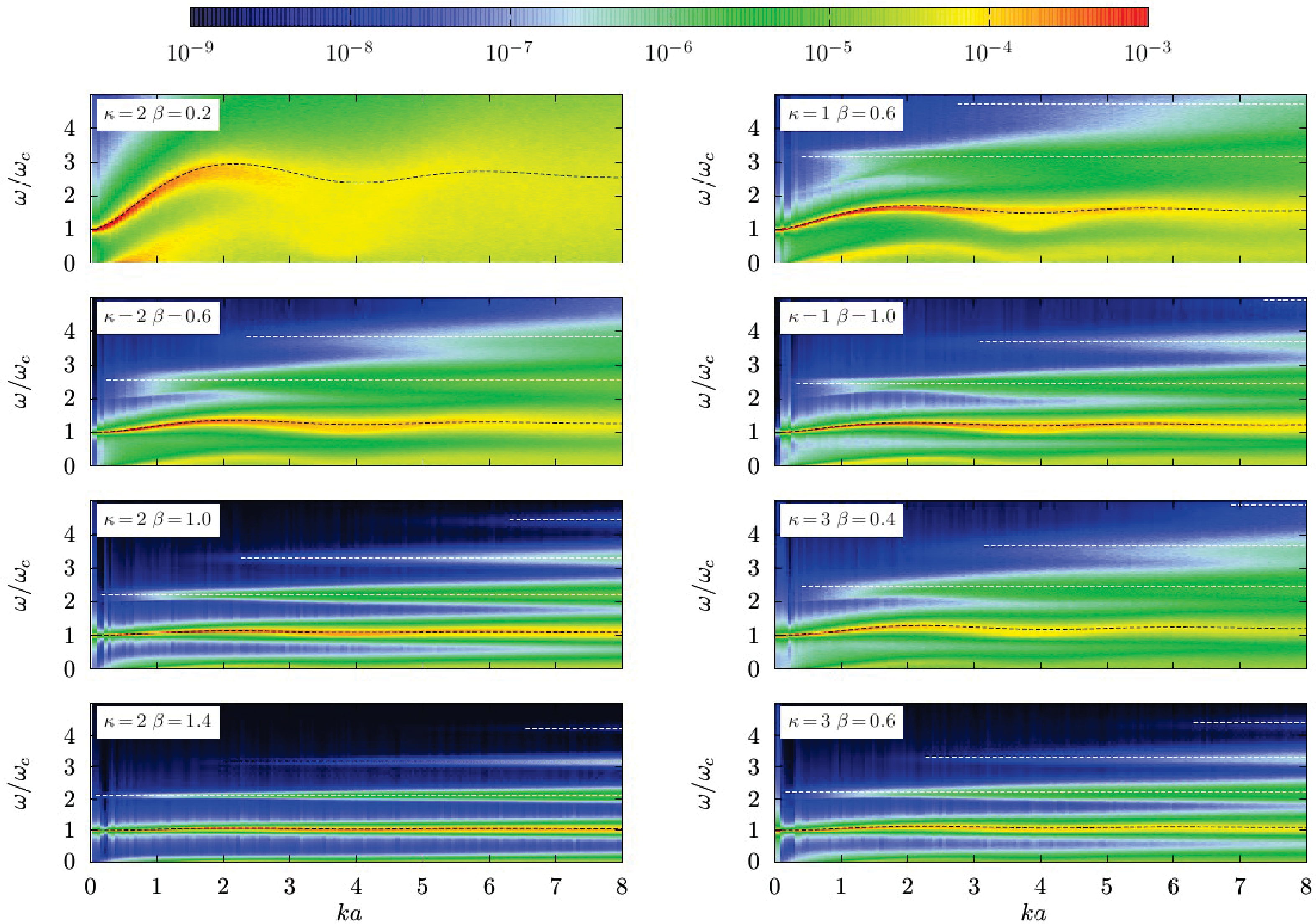
MD collective excitation spectra, L(k, ω) + T (k, ω)

frequencies scaled in units of ω_c .

Dressed Bernstein modes in strongly correlated 2D plasma

$$\boxed{\omega_n^2(k) \approx n^2\omega_{1,\infty}^2, \qquad \omega_{1,\infty}^2 = \omega_c^2 + 2\omega_E^2}$$

ω_E: Einstein frequency



Color: MD spectra, L(k, ω) + T (k, ω)

Lines: theory

Left: Γ = 200, κ=2

Right:
Γ = 150, κ=1 (top 2)
Γ = 600, κ=3 (bot 2)

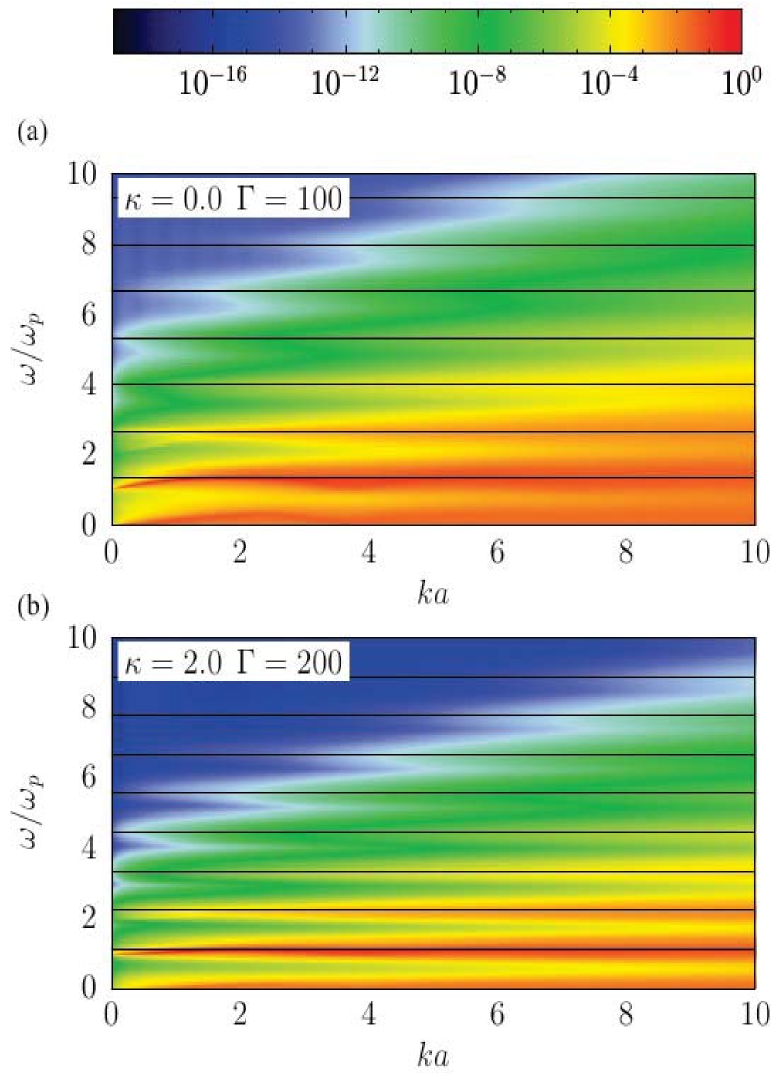
Bonitz, Donkó, Ott, Kählert, Hartmann, PRL **105**, 055002 (2010)

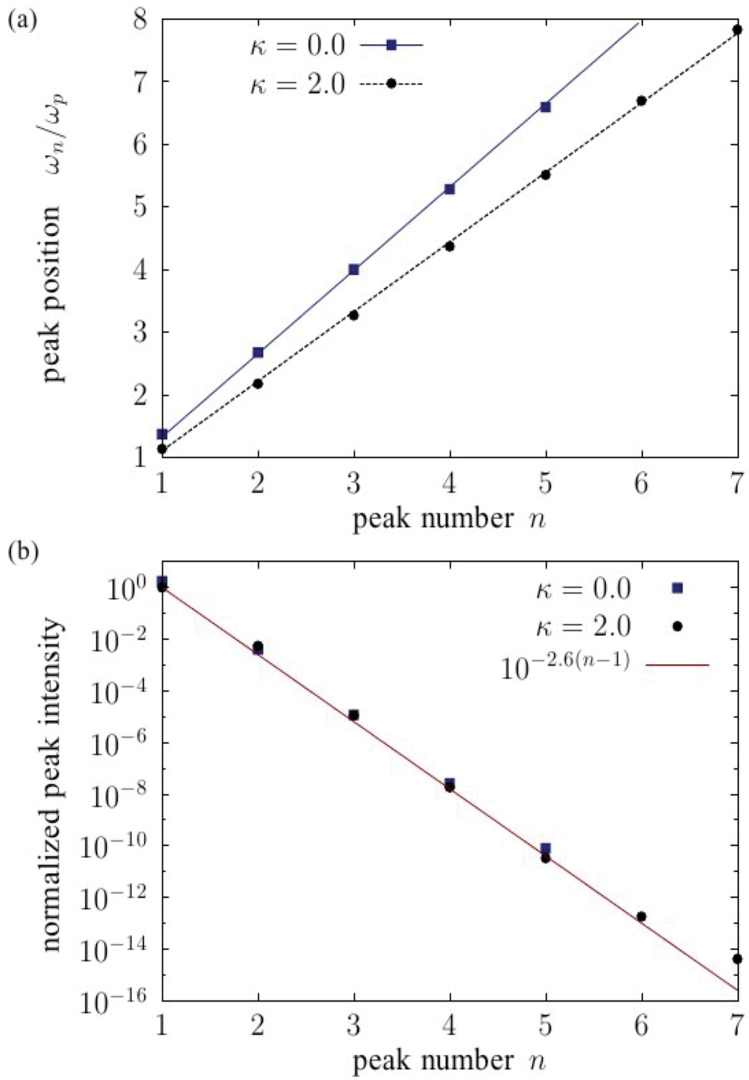
Dressed Bernstein modes in strongly correlated 2D plasma (β=1)

$$\boxed{\omega_n^2(k) \approx n^2\omega_{1,\infty}^2, \qquad \omega_{1,\infty}^2 = \omega_c^2 + 2\omega_E^2}$$

ω_E: Einstein frequency

Theory (black lines)





Ott et al., Phys. Rev. E **83**, 046403 (2011)

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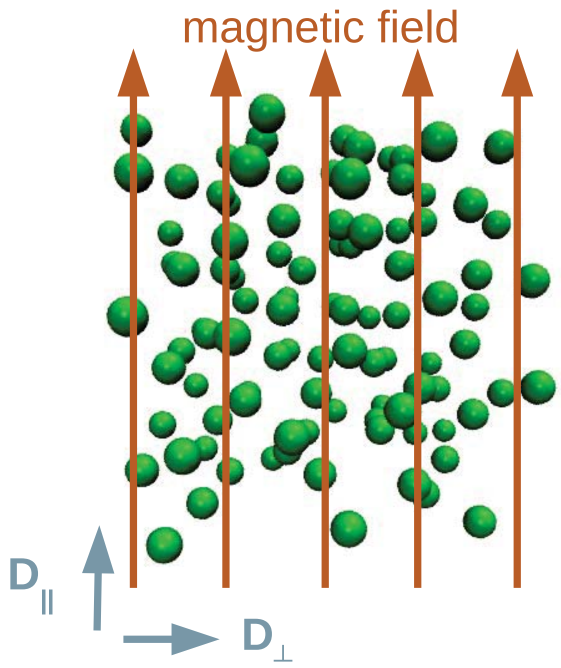
   - Diffusion coefficient perpendicular and parallel to **B**

7. **Conclusion and outlook**

Diffusion in magnetized 3D plasmas

Weakly coupled plasma

	$\beta \leq 1$		$\beta >> 1$	
$D_\perp$	$b_0 B^0 + b_2 B^{-2}$	[3]	$\gamma_0 k_B T (qB)^{-1}$ $\sim B^{-2}$	[1,2] [3,4]
$D_\parallel$	$\sim B^0$	[3]	$\sim B^0$	[5]



[1] Bohm 1949
[2] Marchetti et al. 1984
[3] Lifshitz, Pitaevski 1981
[4] Dubin 1997
[5] Cohen, Suttorp 1984

units

$$\Gamma = q^2/(a k_B T)$$
$$\beta = \omega_c/\omega_p$$
$$\omega_c = QB/mc$$

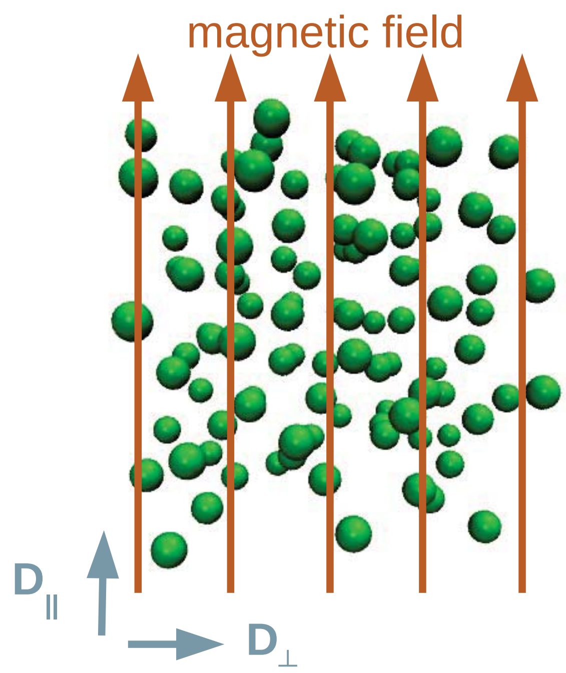
Diffusion in magnetized 3D plasmas

Weakly coupled plasma

	$\beta \leq 1$	$\beta >> 1$
$D_\perp$	$b_0B^0 + b_2B^{-2}$ [3]	$\gamma_0 k_B T(qB)^{-1}$ [1,2] $\sim B^{-2}$ [3,4]
$D_\parallel$	$\sim B^0$ [3]	$\sim B^0$ [5]

Strongly coupled plasma

$D_\perp$	?	$\rightarrow 0$ [6]
$D_\parallel$	$(c\, B^0 + d\, B^2)^{-1}$ [5]	$\rightarrow 0$ [6] $\sim B^0$ [5]



<u>units</u>

$$\Gamma = q^2/(ak_BT)$$
$$\beta = \omega_c/\omega_p$$
$$\omega_c = QB/mc$$

[6] Ranganathan et al., Phys. Chem. Liq. (2003)
[5] Cohen, Suttorp 1984

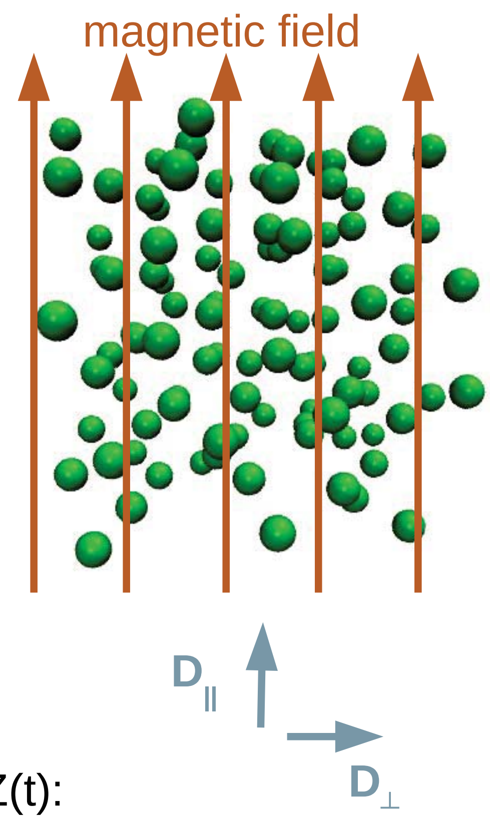
Diffusion in magnetized 3D plasmas - Numerics

Computational details

$$\ddot{\vec{r}}_i = \vec{F}_i/m + \omega_c \dot{\vec{r}}_i \times \hat{\vec{e}}_z, \quad i = 1 \dots N$$

- $N = 8196$
- cubic simulation box, periodic boundary conditions
- microcanonical ensemble

Efficient integration scheme, includes effects of magnetic field self-consistently [1]



Diffusion coefficients ($D_{\perp}$ $D_{\|}$) via Einstein or Green-Kubo relation from velocity ACF $Z(t)$:

$$D = \lim_{t \to \infty} \frac{\langle |\mathbf{r}(t) - \mathbf{r}(t_0)|^2 \rangle}{2\mathcal{D}t} \qquad D = \int_0^\infty Z(t)\, dt$$

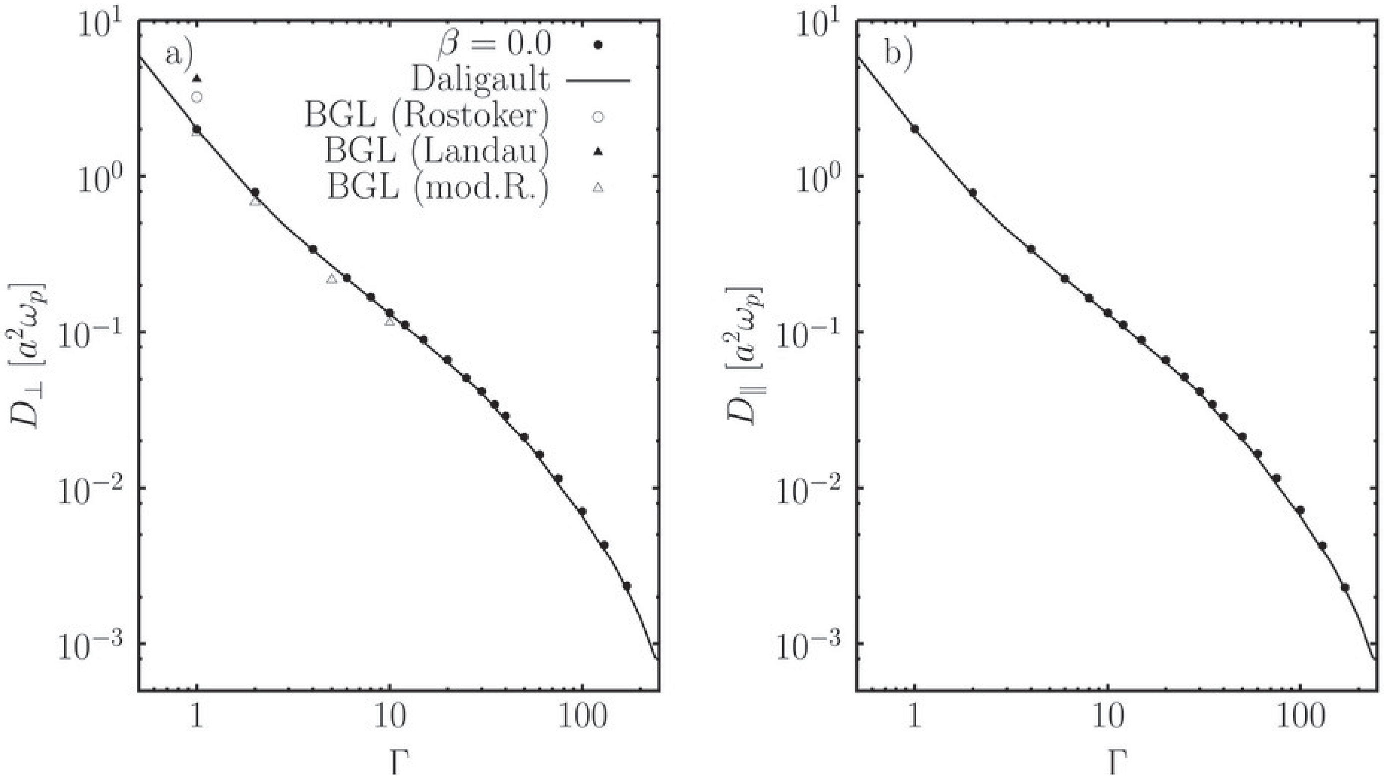
---

[1] Spreiter, Walter: J. Comp. Phys **152**, 102 (1999)

Diffusion in 3D plasmas versus coupling strength, B=0

MD results: filled dots

→ Coupling reduces diffusion

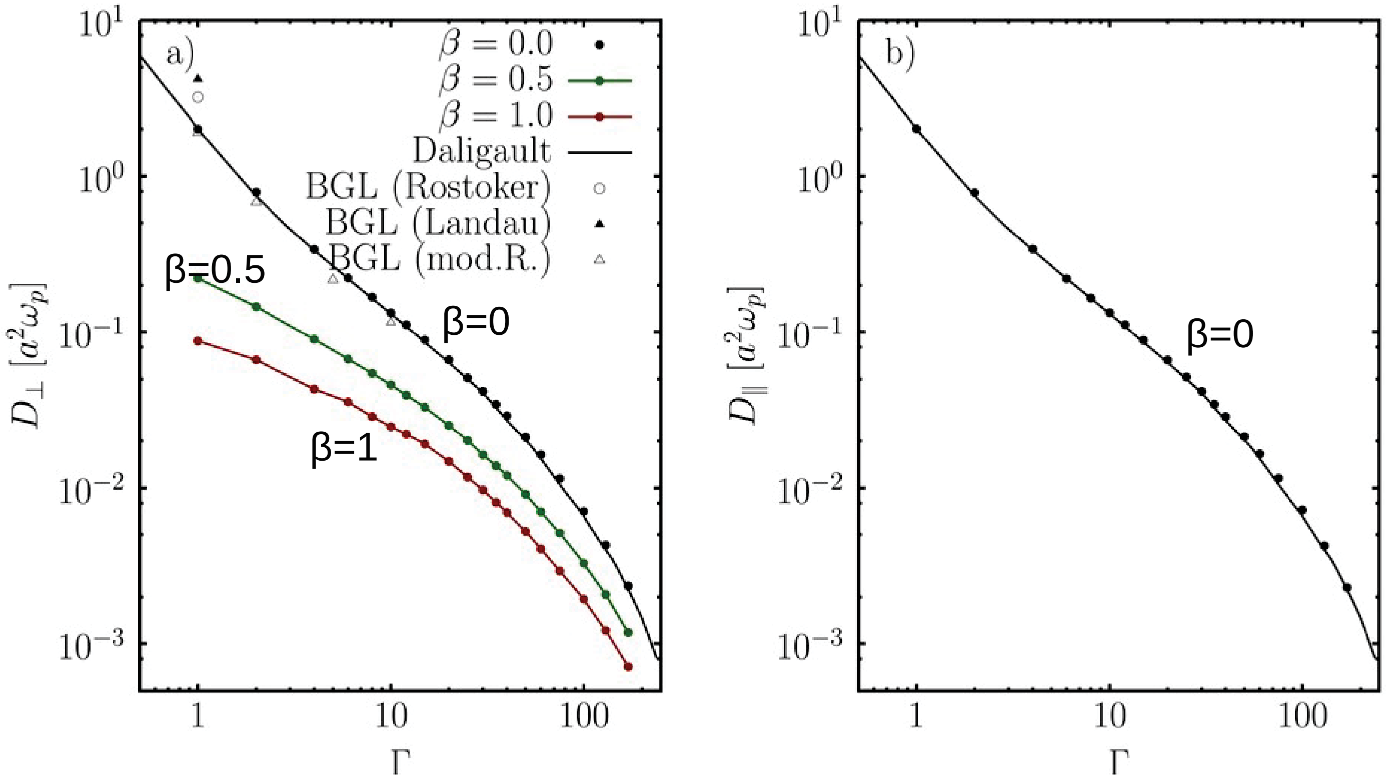


Daligault, PRL 2006
Cohen, Suttorp, Physica A 1984

Diffusion vs. Coupling: finite B-field

MD results: filled dots

→ Coupling reduces diffusion
→ B field reduces diffusion

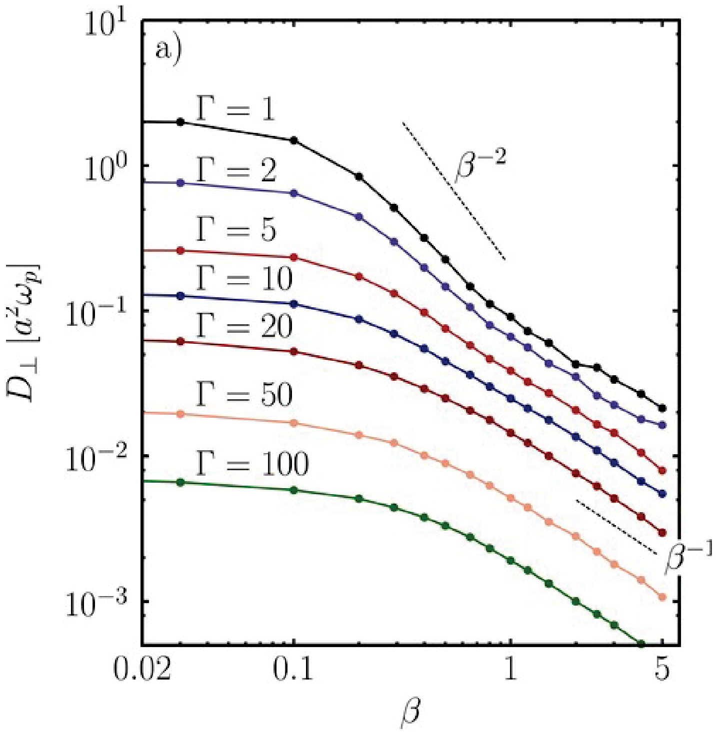


Daligault, PRL 2006
Cohen, Suttorp, Physica A 1984

Transverse Diffusion vs. B-field

MD results

→ Coupling reduces diffusion
→ B field reduces diffusion

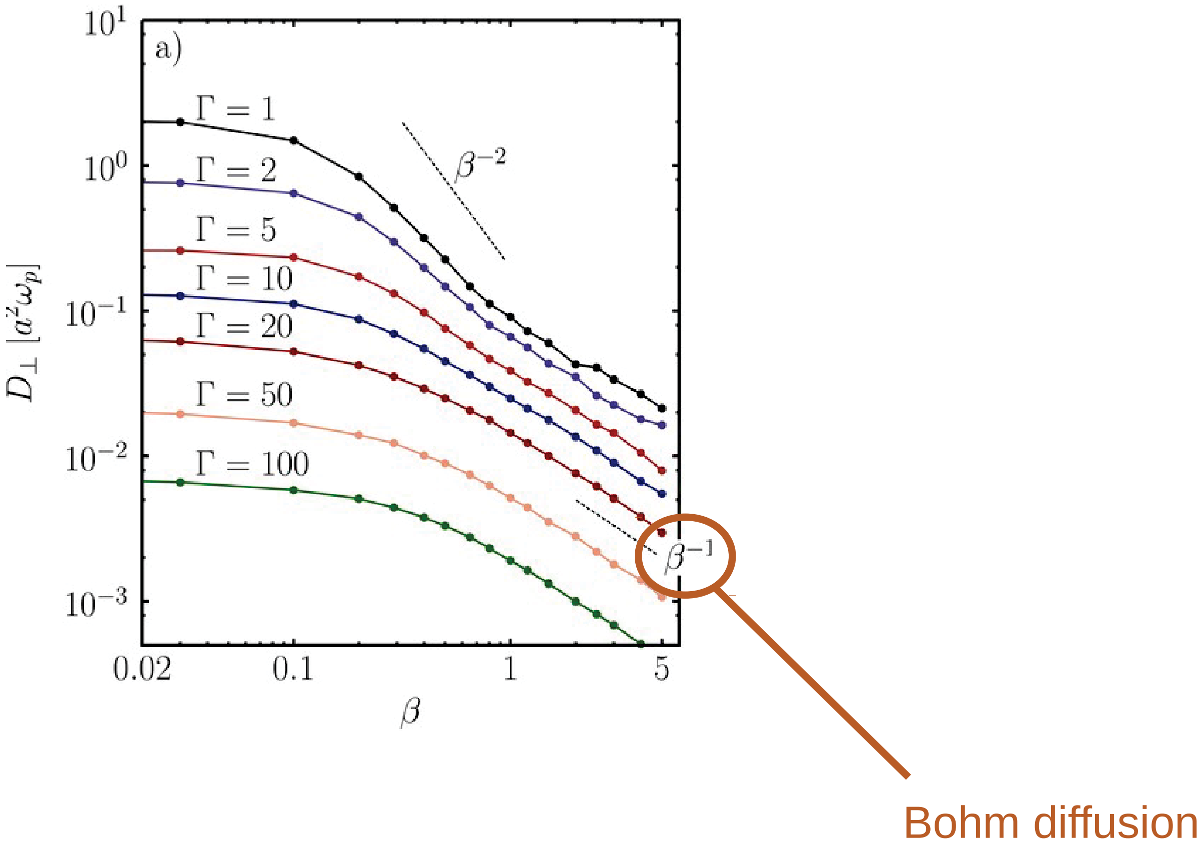


T. Ott, M. Bonitz, *Diffusion in a strongly coupled magnetized plasma,* Phys. Rev. Lett. **107**, 135003 (2011)

Transverse Diffusion vs. B-field

MD results

→ Coupling reduces diffusion
→ at strong B: recover Bohm diffusion



T. Ott, M. Bonitz, *Diffusion in a strongly coupled magnetized plasma,* Phys. Rev. Lett. **107**, 135003 (2011)

Field-parallel Diffusion at strong coupling

Physica 123A (1984) 560–576
North-Holland, Amsterdam

SELF-DIFFUSION IN A DENSE MAGNETIZED PLASMA

L.G. SUTTORP and J.S. COHEN

Instituut voor Theoretische Fysica, Universiteit van Amsterdam, Valckenierstraat 65, 1018 XE Amsterdam, The Netherlands

Received 26 July 1983

Self-diffusion through dense classical one-component plasmas in a uniform magnetic field is studied by means of renormalized kinetic theory. Extensions of the Landau and the Rostoker equations to plasmas of high density are derived. The coefficient of self-diffusion along the magnetic field is evaluated from the 1-Sonine approximation of the Landau kernel. The results show how the diffusion process is gradually impeded as the magnetic field strength increases.

Inclusion of dynamical screening effects: J.S. Cohen and L. G. Suttorp, Physica **126A** 308 (1984)

Field-parallel Diffusion at strong coupling: Cohen and Suttorp

B-Field dependence of parallel diffusion, D(B)/D(0):

includes dynamical screening

TABLE IV

The reduced self-diffusion coefficient $R_\parallel(b, \Gamma)$, as found from the modified Rostoker memory kernel in the 1-Sonine approximation.

b \ Γ	0.1	0.2	0.5	1	2	5	10
0.1	1.00	1.00	1.00	0.999	0.999	0.999	0.995
0.2	1.00	0.999	0.999	0.998	0.997	0.992	0.983
0.5	0.997	0.996	0.994	0.989	0.980	0.949	0.882
1	0.990	0.986	0.973	0.958	0.926	0.837	0.726
2	0.965	0.951	0.919	0.873	0.808	0.712	0.669
5	0.889	0.853	0.787	0.733	0.693	0.673	0.668
10	0.819	0.776	0.715	0.684	0.672	0.669	0.668
20	0.759	0.719	0.681	0.670	0.669	0.668	0.667
50	0.705	0.681	0.668	0.668	0.667	0.667	0.667

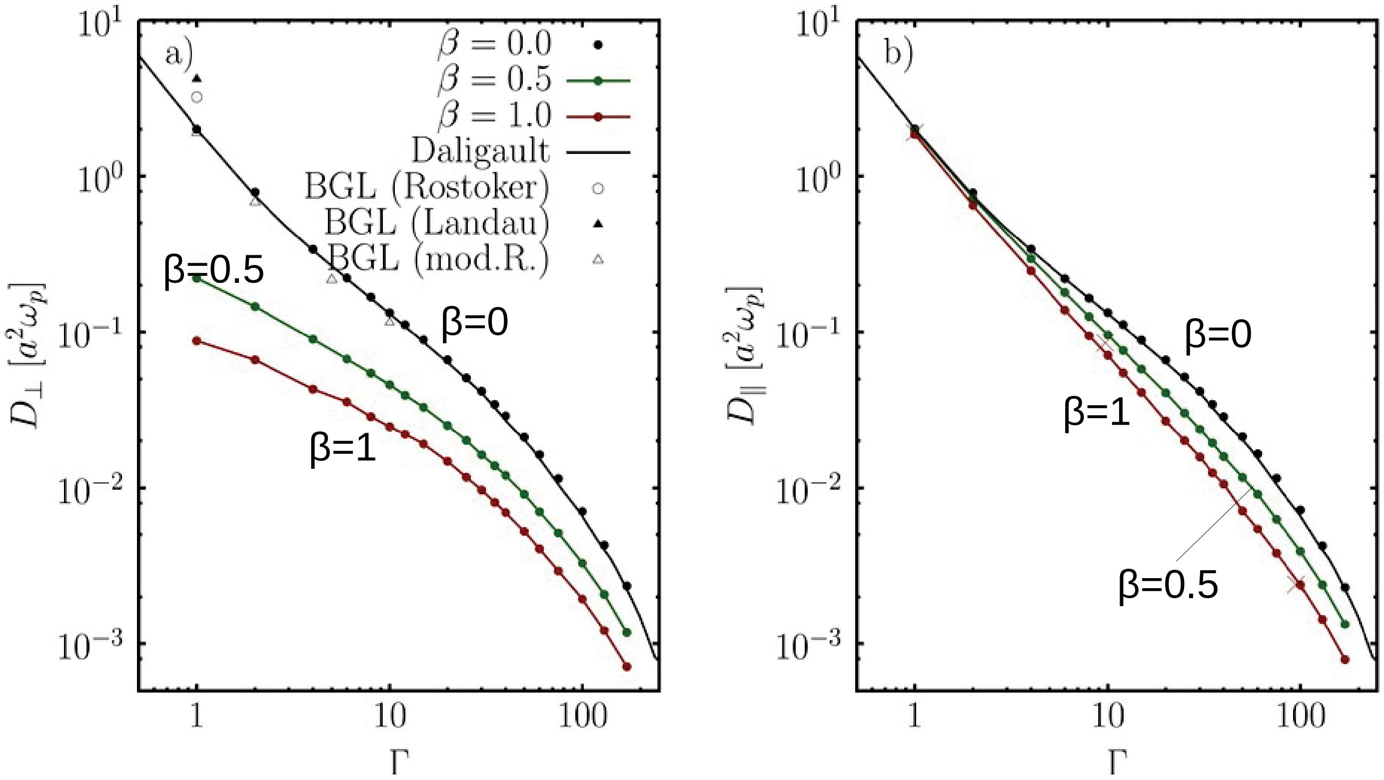
faster reduction for large Γ

Saturation at 2/3 of field-free value
for all coupling values and all models

J.S. Cohen and L. G. Suttorp, Physica **126A** 308 (1984)

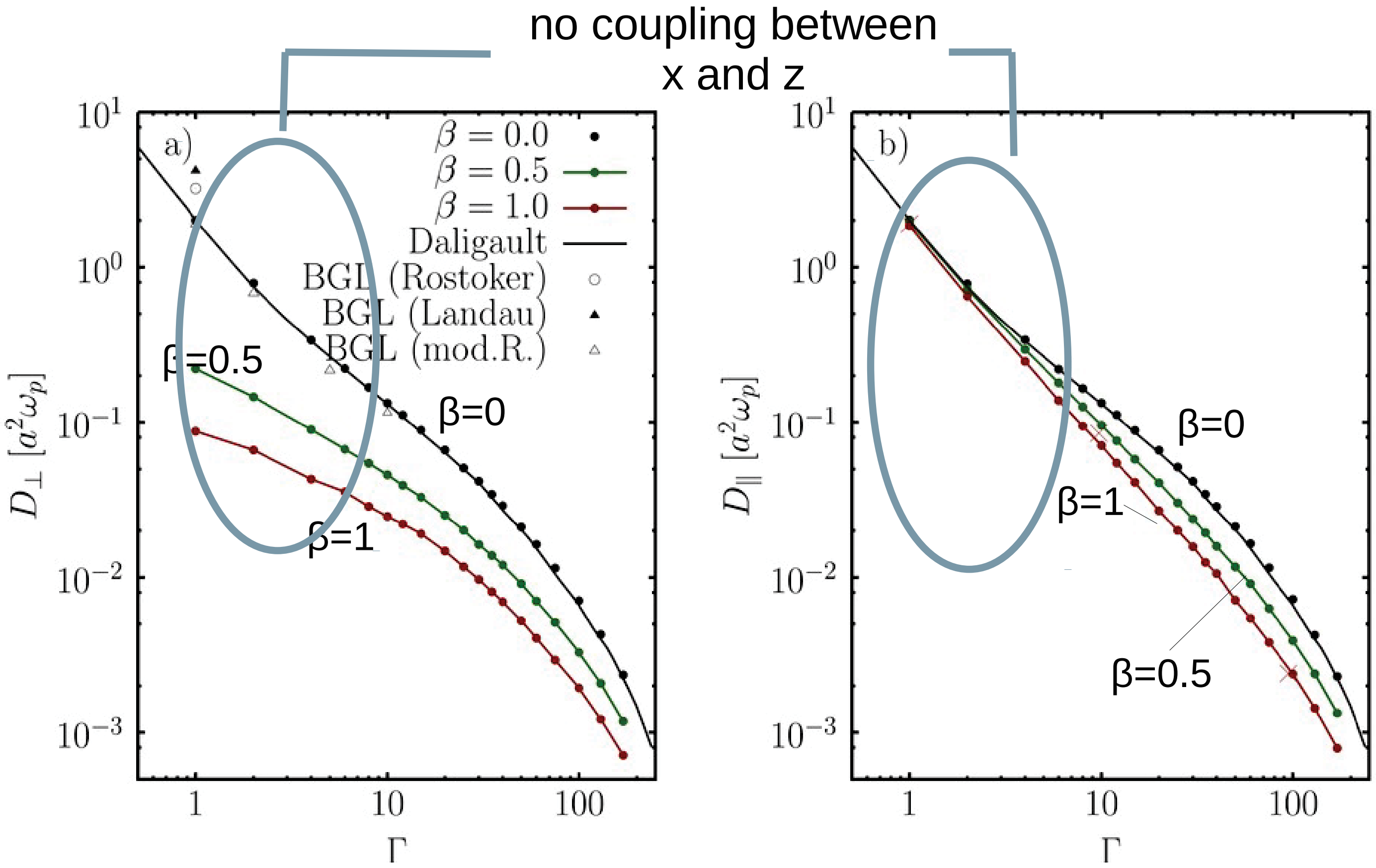
Field-parallel Diffusion vs. B-field: MD results

→ parallel diffusion also decreases with coupling and field



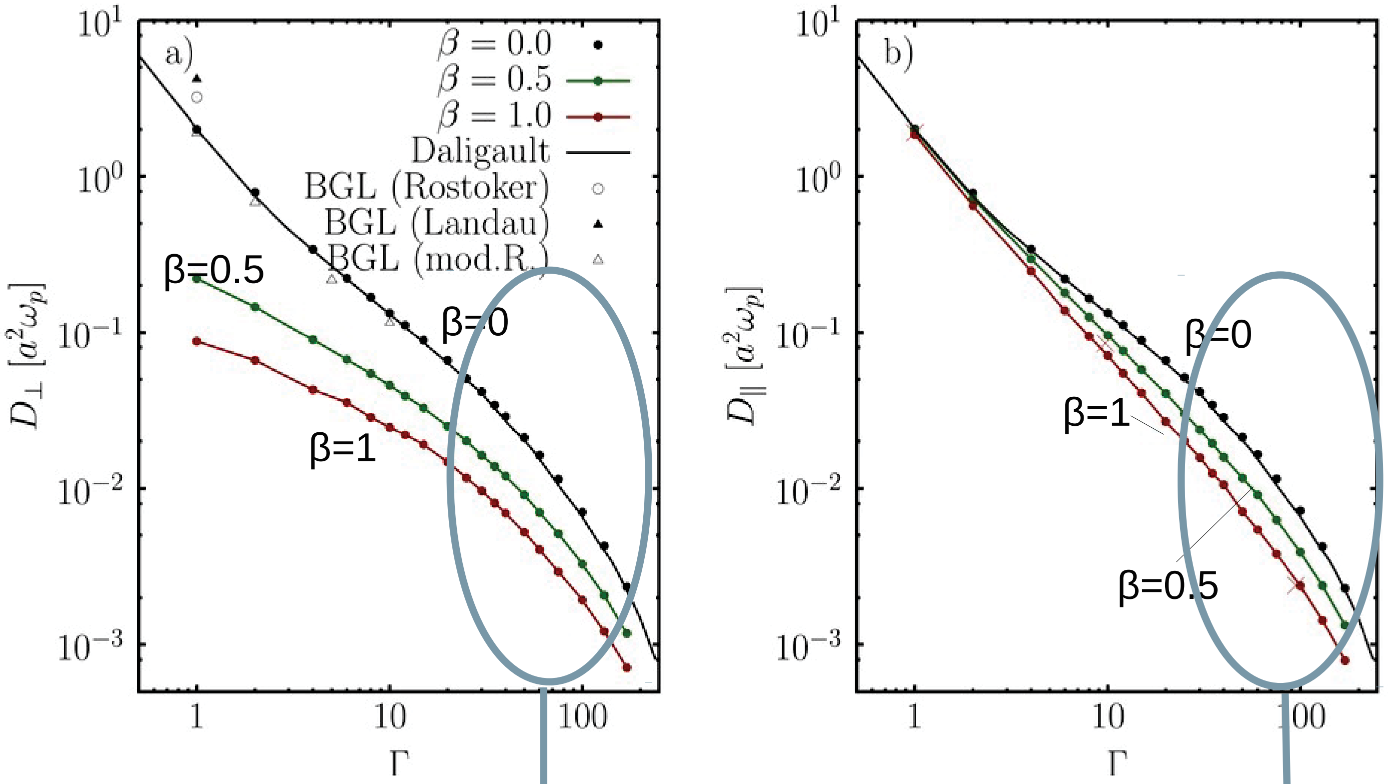
Field-parallel Diffusion vs. B-field

MD results
Moderate coupling: weak B-dependence of parallel diffusion



Field-parallel Diffusion vs. B-field

MD results
Strong coupling: strong B-dependence of parallel diffusion

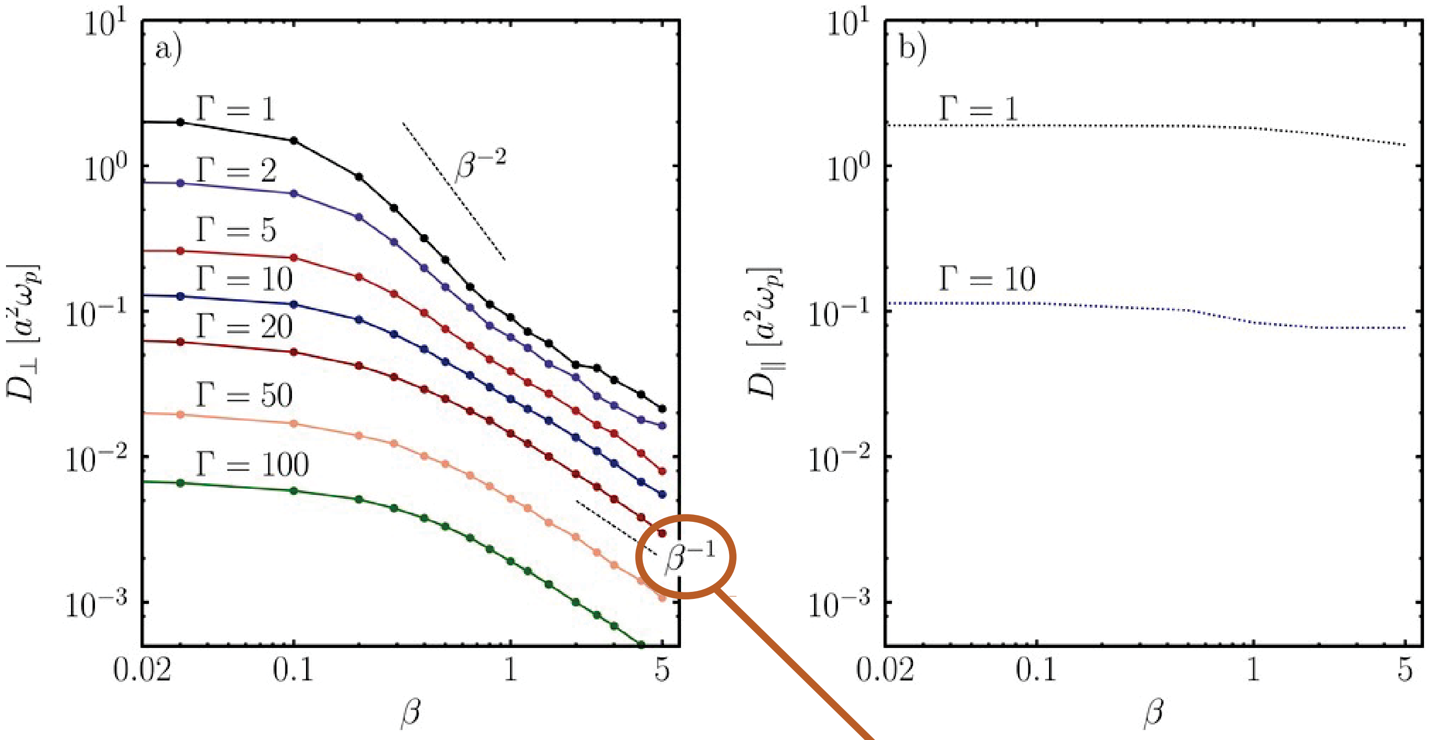


reduction of $D_\perp$
affects $D_\parallel$
↔ strong coupling
between x and z

Field-parallel Diffusion vs. B-field

MD results

Saturation to 2/3 of D(B=0)   [1]

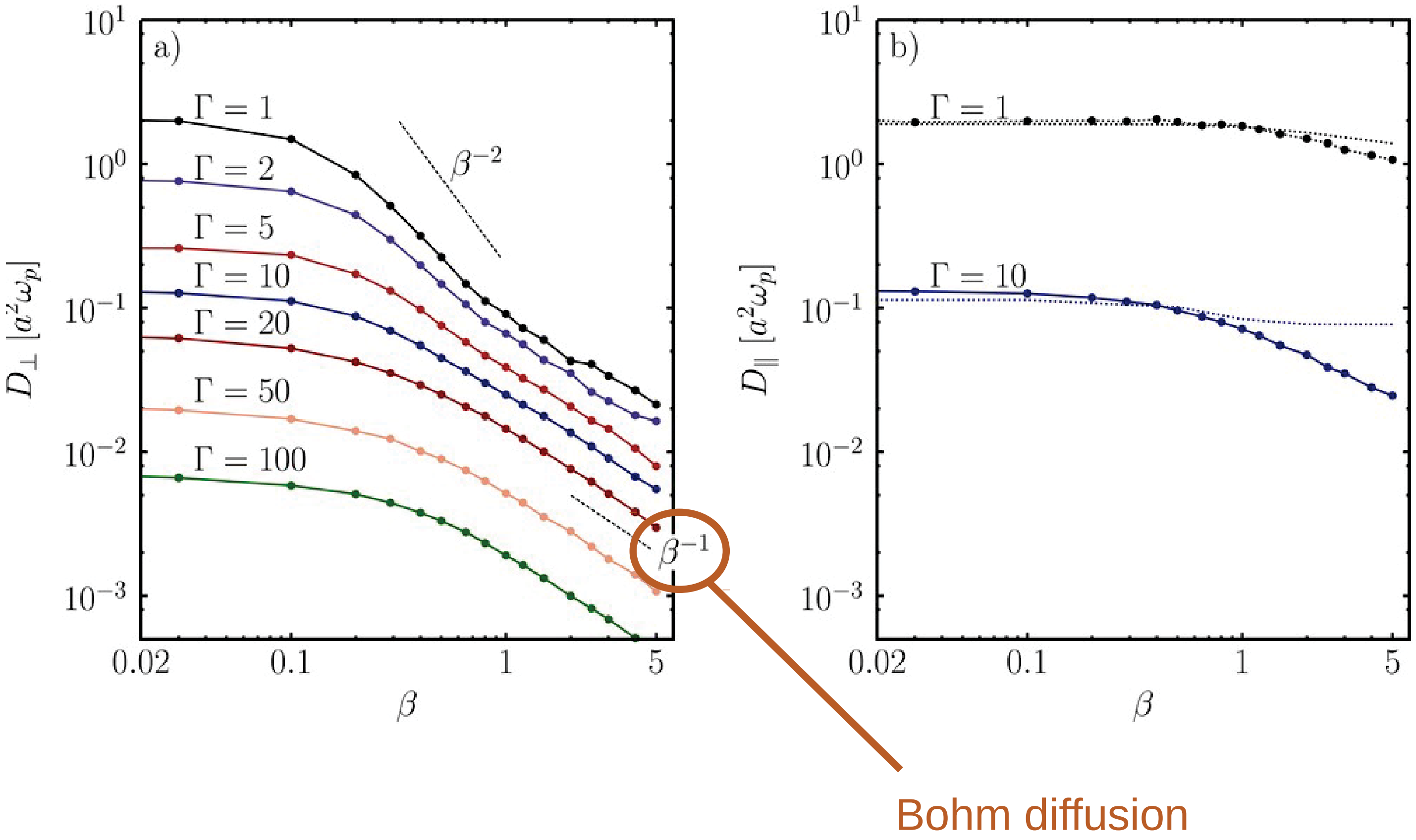


Bohm diffusion

[1] Cohen, Suttorp, Physica A 1984: D becomes B-independent (dotted lines), prediction based on Balescu-Lenard kinetic theory

Field-parallel Diffusion vs. B-field

MD results: lines with dots

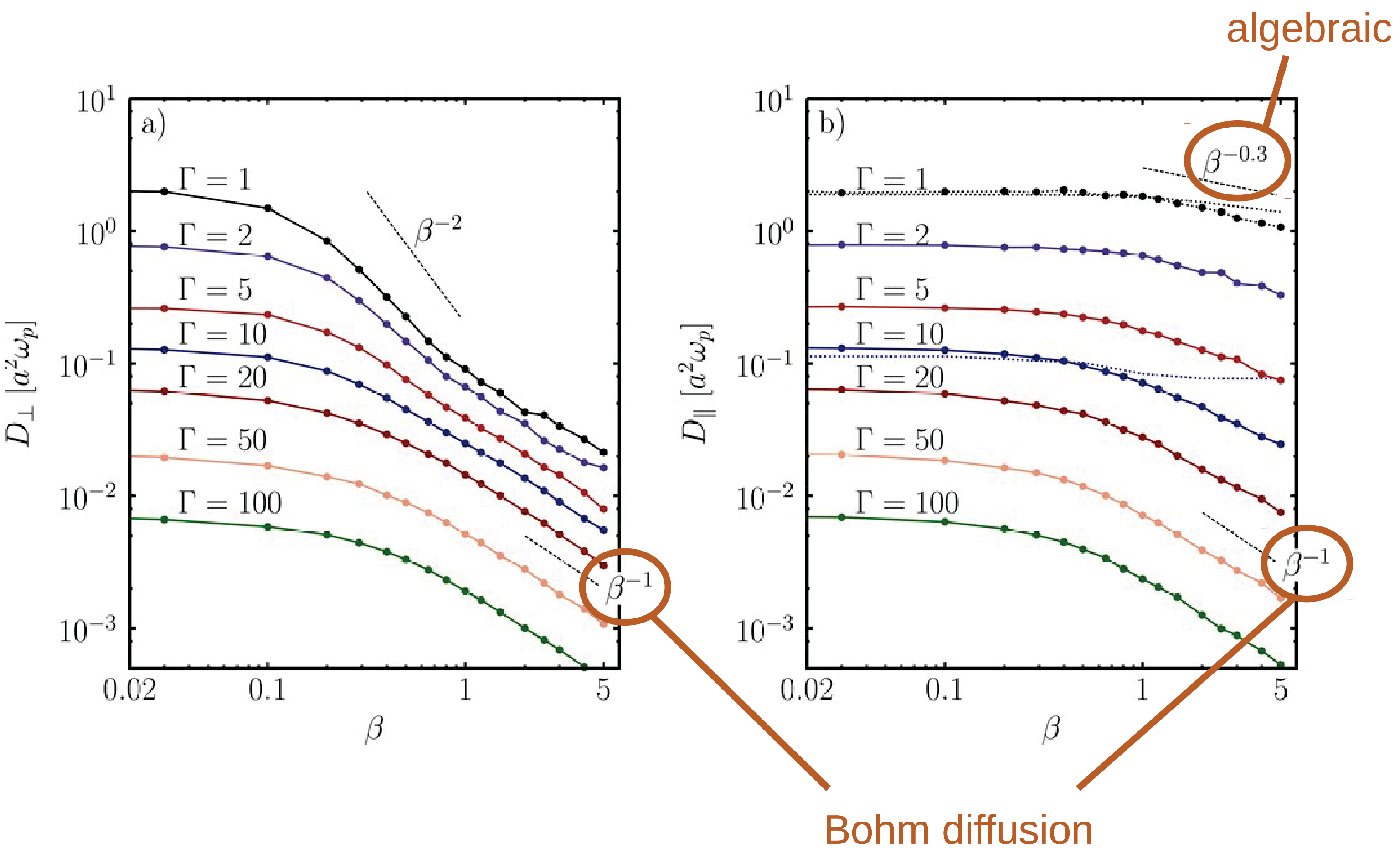


[1] Cohen, Suttorp, Physica A 1984: D becomes B-independent (dotted lines), prediction based on Balescu-Lenard kinetic theory

Field-parallel Diffusion vs. B-field

MD results: lines with dots

Strong B-field: recover Bohm diffusion in perp and longitudinal direction



[1] *Cohen, Suttorp, Physica A 1984*

T. Ott, M. Bonitz, *Diffusion in a strongly coupled magnetized plasma,* Phys. Rev. Lett., **107**, 135003 (2011)

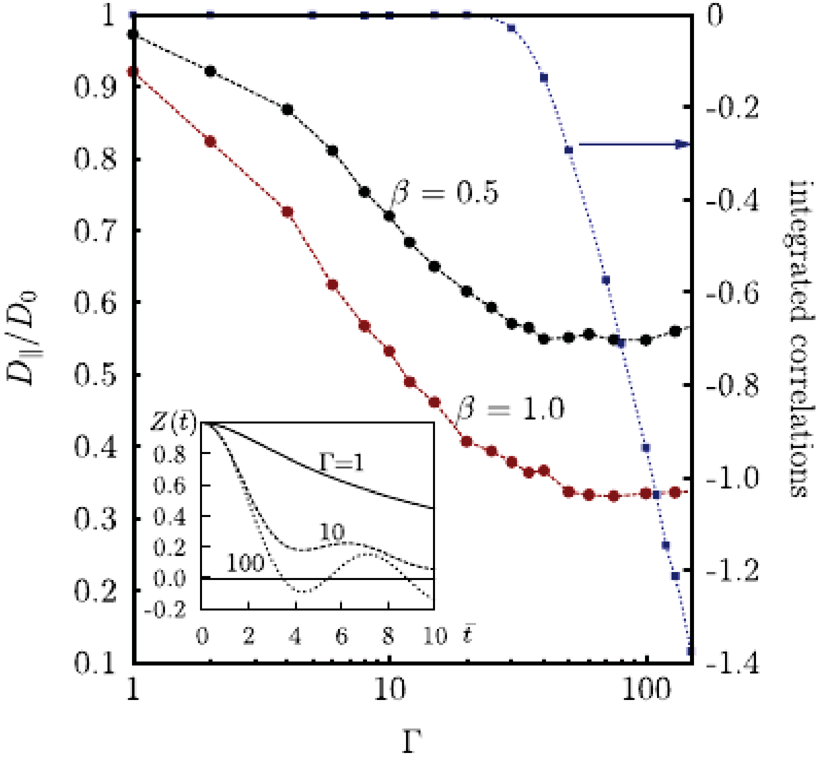
Bohm Diffusion – relevance of collective modes

Z: velocity autocorrelation function: $Z(t) \doteq \langle \vec{v}(t) \cdot \vec{v}(t_0) \rangle / 3$

$$D = \int_0^\infty dt \left\langle \mathbf{v}_{xi}(t) \cdot \mathbf{v}_{xi}(0) \right\rangle$$

- oscillations related to collective modes
- negative values indicate trapping („caging“) of particles in local potential minima

- caging dominates for Gamma > 30
- integral over negative areas gives „caging time“ T_c, with D ~ 1/T_c [1]
- B-field enhances caging [2]



D normalized to field-free value for current Gamma, from [2]

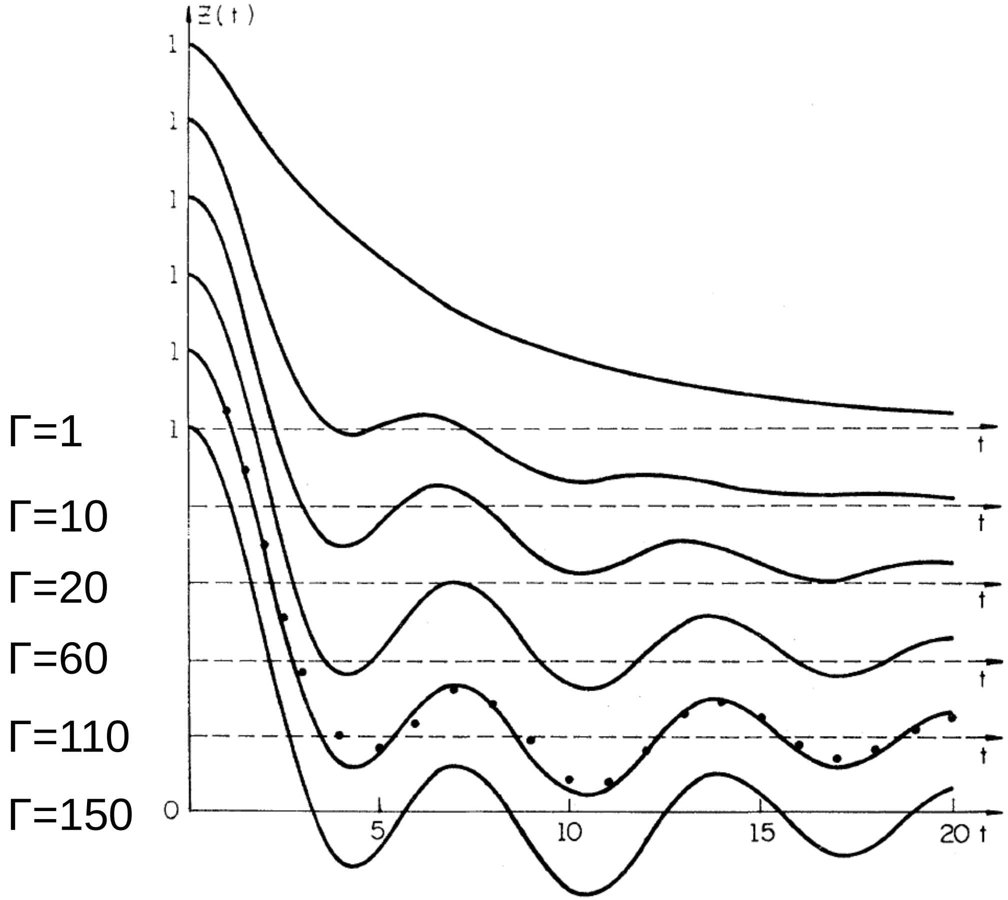
[1] Z. Donko, G. Kalman, and K. Golden, Phys. Rev. Lett., **88**, 225001 (2002)
[2] T. Ott, M. Bonitz, *Diffusion in a strongly coupled magnetized plasma,* Phys. Rev. Lett., **107**, 135003 (2011)

Velocity autocorrelation function and collective modes

Z: velocity autocorrelation function: $Z(t) = \langle \vec{v}(t) \cdot \vec{v}(t_0) \rangle / 3$ $\qquad D = \int_0^\infty dt \left\langle \mathbf{v}_{xi}(t) \cdot \mathbf{v}_{xi}(0) \right\rangle$

- oscillations related to collective modes

- negative values indicate trapping („caging“) of particles in local potential minima



1) J.P. Hansen *et al.* Physical Review A **11**, 1025 (1075)

Bohm Diffusion – ideal versus strongly coupled plasma

Large B → Bohm diffusion: $\boxed{D = y \; k_B T/(eB)}$

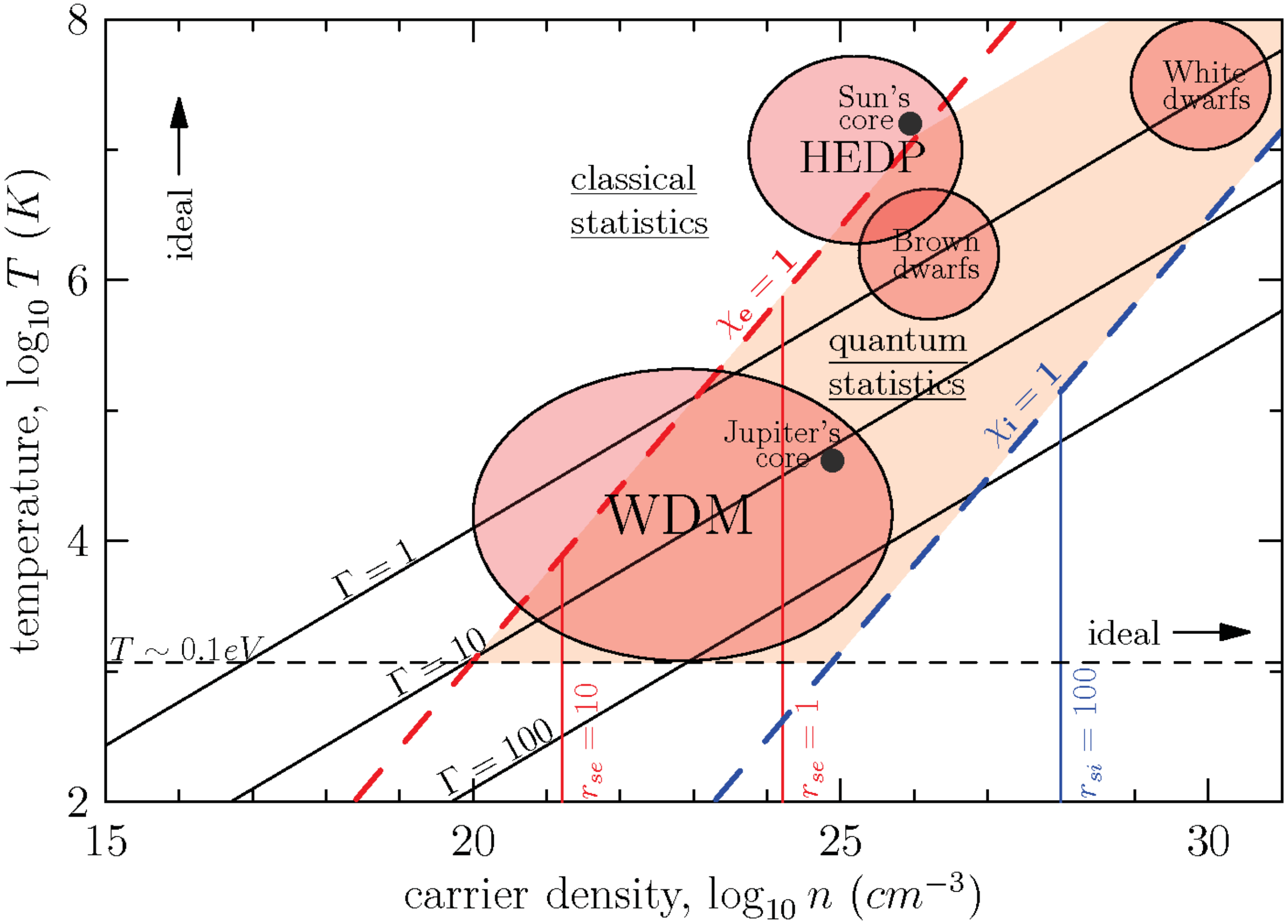
	Ideal plasma	Strongly coupled plasma
$D_\perp$	y = 1/16 [1] y = 0.21 [2]	y = 2/3, for Gamma=100 [3]
$D_\parallel$	B-independent	Bohm diffusion at large Gamma y ~ 0.9 [3]
Mechanism	4 magnetoplasmons [4]	6 magnetoplasmon and shear modes [5]

[1] *The characteristics of electrical discharges in magnetic fields*, ed A. Guthrie and R. K. Wakerling (New York: McGraw-Hill) (1949).
[2] L. Spitzer, Phys. Fluids **3**, 659 (1960)

[3] T. Ott, M. Bonitz, Phys. Rev. Lett., **107**, 135003 (2011)
[4] C. Marchetti et al., PRA **29**, 2960 (1984)
[5] T. Ott, H. Kählert, and MB, to be published

Implications

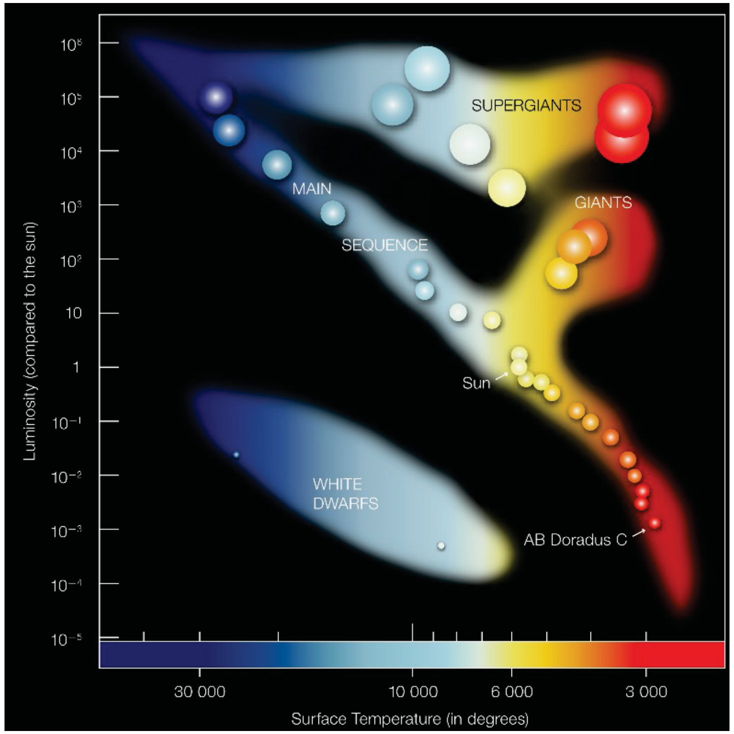
White dwarf and neutron star evolution



Dusty plasmas

Ions in traps

Implications: white dwarf star evolution



- Contain core with carbon and oxygen ion crystal
  no nuclear fusion, chemical energy from latent heat
  10% of white dwarfs carry a strong magnetic field

- white dwarf cooling observations used to date stellar systems

- gravitational energy released through <u>sedimentation</u>
  → strongly depend on diffusion properties

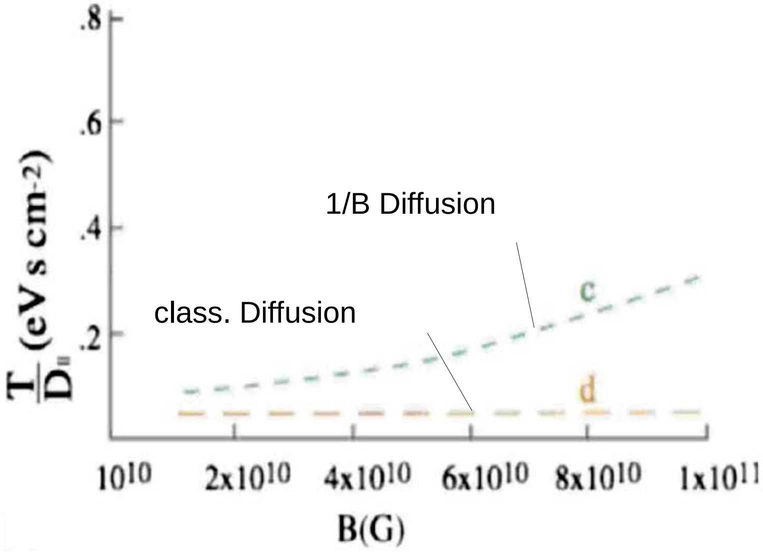
Source: European Southern Observatory

Diffusion – governs low-velocity stopping power

- Stopping power S for low-velocity projectiles[1]:

$$\lim_{V_{\mathrm{b}\to 0}} \frac{S(V_{\mathrm{b}})}{V_{\mathrm{b}}} = k_{\mathrm{B}} T_{\mathrm{e}} D^{-1}$$

projectile velocity



- Deutsch and Popoff[2]: application to magnetized target fusion and inertial confinement fusion
  → parallel and perpendicular geometries relevant

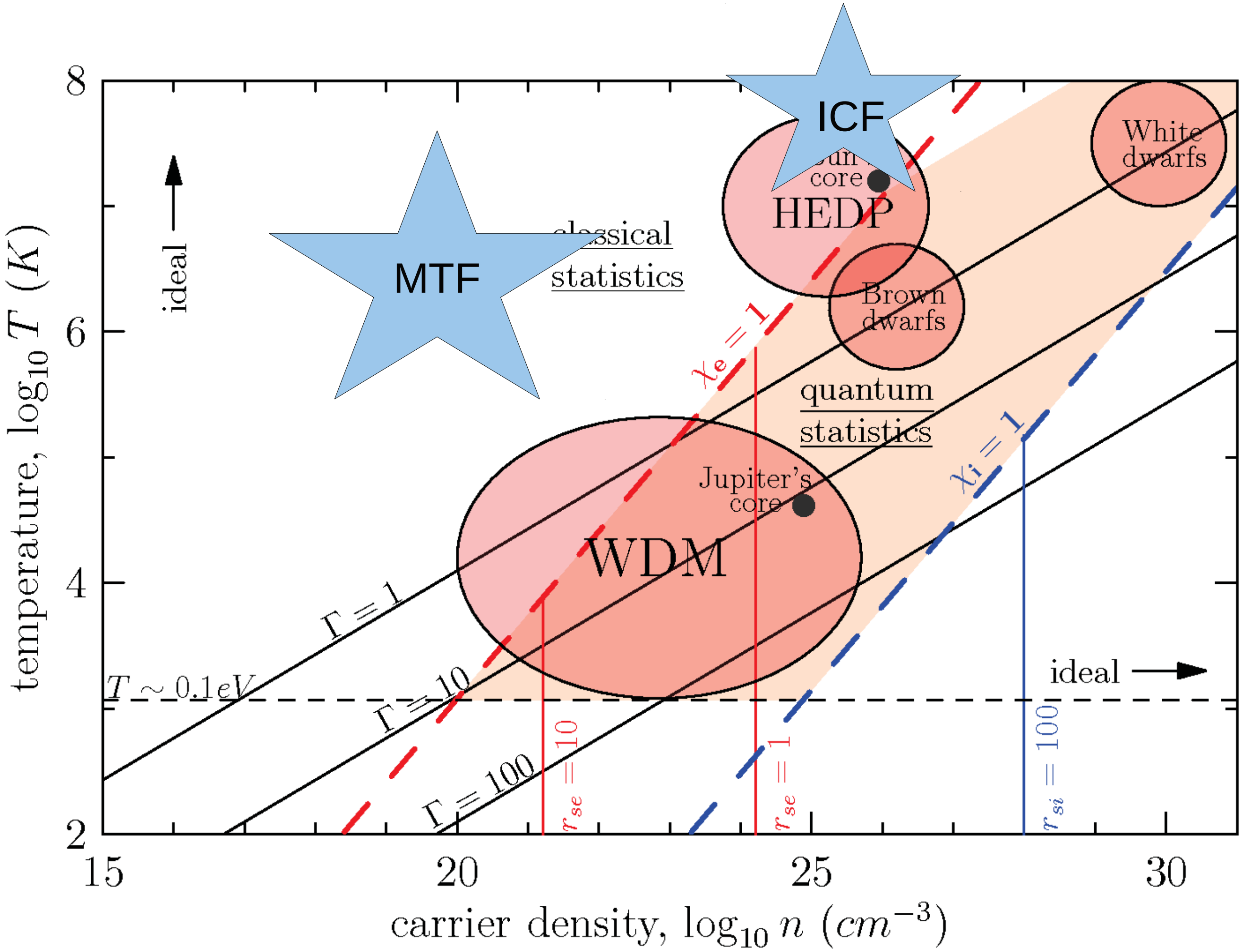
- Here: extension to strong coupling

1 J.W. Dufty, M. Berkovsky, *Electronic stopping of ions in the low velocity limit,* Nucl. Instrum. Methods Phys. Res., Sect. B **96**, 626 (1995).
2 C. Deutsch, R. Popoff, *Low ion velocity slowing down in a strongly magnetized plasma target* Nucl. Instrum. Methods Phys. Res., Sect. A **606**, 212 (2009).

Implications for fusion (?)

Inertial confinement Fusion (ICF)

Magnetized target fusion (MTF)



Summary and outlook

1. Strongly correlated plasmas:

- of increasing relevance in many fields
- strong similarities for same coupling

2. Anomalous diffusion in 2D plasma:

- systematic screening and coupling dependence, transient effect

3. Nonlinear magnetoplasmon: dressed Bernstein modes

- combined action of correlations and B-field

4. Diffusion in magnetized correlated 3D plasma

- Bohm diffusion at strong B across field
- Bohm diffusion at strong coupling parallel to B
- useful for correlated plasmas and possible applications