The Abdus Salam
International Centre for Theoretical Physics

United Nations
Educational, Scientific and
Cultural Organization

IAEA
International Atomic Energy Agency

2267-4

Joint ITER-IAEA-ICTP Advanced Workshop on Fusion and Plasma Physics

3 - 14 October 2011

On the physics description of fusion plasmas 1

J. Weiland

Chalmers University of Technology
Gothenburg
Sweden

Strada Costiera 11, 34151 Trieste, Italy - Tel.+39 040 2240 111; Fax +39 040 224 163 - sci_info@ictp.it

On the physics description of fusion plasmas 1

J. Weiland[1], A. Zagorodny[2], V. Zasenko[2] and N. Shatashvili[3]

1. Chalmers University of Technology and EURATOM-VR Association, Gothenburg, Sweden
2. Bogoliubov Institute for Theoretical Physics, 03680 Kiev, Ukraine
3. Ivane Javahishvili Tbilisi State University , Tbilisi 028 and Andronikashvili Institute of Physics, Department of Plasma Physics,
Tbilisi,0177, Georgia

Joint ITER-IAEA –ICTP Advanced Workshop on "Fusion and Plasma Physics" October 3 – 14 2011

Contents

General properties of fusion plasmas

What characterizes fusion plasmas?

We need to take *inhomogeneities* into account

Mainly interested in low frequency phenomena, $\omega << \Omega_{ci}$
this applies to MHD and transport, (not to heating)

$$\chi_i = \frac{\gamma^3 / k_r^2}{\omega_r^2 + \gamma^2} \qquad (1.1)$$

Low frequency perturbations give more transport!

Thermodynamics

- Gradients of density, temperature etc always have to be present in a confined plasma
- This means that the system is not in thermodynamic equilibrium
- There will always be *free energy* avaliable that may drive *instabilities*
- Since we want to confine *density and temperature* we should look for instabilities driven by *gradients* in these
- The system prefers to relax with *comparable length scales* of density and temperature. If these are too different we may get *pinch effects* with a tendency to *equilibrate* these *length scales*

Stability and transport

Plasma confinement is usually divided into *Large scale stability and Transport*.

Large scale stability is usually described my *Magneto Hydro Dynamics* (MHD). Here one fluid equations are usually used.

Transport is usually due to small scale (Micro) instabilities. These usually require multi fluid or kinetic descriptions.

Multifluid – MHD

MHD instabilities generally require detailed geometry. However, their growthrate is so large that it is typically larger than the drift frequencies (which are different for different species) and, accordingly, we can use a *single fluid* description.

Microinstabilities are more localized. They can often be described by a **WKB** approximation and are thus not quite so sensitive to geometry. On the other hand growthrates are of the order of drift frequencies which are different for different species. Thus multifluid or kinetic descriptions are needed.

Multi fluid – MHD

Toroidal effects represent the *third dimension* in which particles are not confined by the magnetic field. Although microinstabilities are less sensitive to geometry, **toroidal effects** are very important, in particular in the core.

The interchange driving terms are linear in curvature, thus one fluid equations are only linear in curvature (other curvature effects give a real eigenfrequency)

There is, however, one toroidal effect entering together with the Alfve'n frequency which requires a two fluid description in the core

Different habits in using physics descriptions

A very strong fusion community dealing primarily with large scale instabilities (which are indeed the most dangerous) has made single fluid MHD equations one of the most used descriptions.

Unfortunately the difficulties with dealing with more detailed two fluid effects and the success of one fluid equations outside of their formal regime of applicability has led to a too strong focus on one fluid equations, some researcher using only one fluid or kinetic descriptions. As it turns out, a multi fluid description is usually the best for all types of low frequency phenomena

Different habits cont.

In addition to the efficiency of multi fluid descriptions they have also the advantage of describing physics in a clear way. This is particularly true in comparison with kinetric descriptions.

It is not unusual that too simplified two fluid equations are used for the intrepretation of kinetic results.

A fluid model for Ion Temperature Gradient (ITG) modes which includes only linear terms in the curvature, merely serves to show that there exist destabilizing terms in the temperature gradient!

Habits-cont.

- As it turns out, a multi fluid description is usually the best for all types of low frequency modes once you know how to deal with convective diamagnetic and stress tensor effects. This is so since the two fluid derivation of MHD type modes is not much more complicated than the one fluid derivation and advanced fluid closures make a kinetic treatment unnecessary for instabilities driven by gradients in configuration space (this excludes modes driven resonantly by fast particles). Unfortunately there are also risks of making mistakes with e.g. convective diamagnetic effects and this has sometimes lead researchers to use only one fluid or kinetic models.

Low ion temperature

In many cases also small ion temperature has been assumed. In this way it is possible to avoid most of the difficulties with the two fluid approach. However, one of the main goals with fusion plasmas is to get sufficiently high ion temperature for thermonuclear reactions!

Some outstanding research was made in the 1970's with the *Hasegawa – Mima equation* by using small ion temperature. This research reviled the *cascades both to lower and higher modenumbers and the generation of zonal flows*.

However, including ion temperature effects requires a considerably more involved description with several similar equations and today further work with small ion temperature equations are mainly mathematical exercises.

Expansions for weak curvature

- As will be shown later, a weak curvature expansion for ITG modes merely shows that there are destabilizing temperature gradient terms. While terms linear in the curvature are mainly destabilizing, effects that are quadratic in the curvature are stabilizing!
- In fact, the bulk of tokamaks are in the regime where quadratic terms in the curvature dominate! The most common stability condition $\frac{R}{L_T} < \left(\frac{R}{L_T}\right)_{crit}$ (1.2)

  where R is the major radius and L_T is the temperature scalelength , actually shows that curvature is stabilizing!

Multi fluid description

We will now show the low frequency fluid expansion which is commonly used for microinstabilities

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\cdot\nabla)\mathbf{v} = \frac{q}{m}(\mathbf{E} + \mathbf{v}\times\mathbf{B}) - \frac{1}{mn}(\nabla P + \nabla\cdot\boldsymbol{\pi}) = 0 \quad (1.3a)$$

$$\omega << \Omega_c \Rightarrow \quad (1.3b)$$

$$\mathbf{v}_\perp = \mathbf{v}_E + \mathbf{v}_p + \mathbf{v}_* \quad (1.3c)$$

$$\mathbf{v}_E = \frac{1}{B}(\mathbf{E}\times\hat{\mathbf{z}}) \quad (1.3d)$$

$$\mathbf{v}_* = \frac{\hat{\mathbf{z}}\times\nabla P}{\mathrm{qnB}} \quad (1.3e)$$

$$\mathbf{v}_p = \frac{1}{\Omega_c}\frac{\partial}{\partial t}(\hat{\mathbf{z}}\times\mathbf{v}) \approx \frac{1}{B\Omega_c}\frac{\partial \mathbf{E}}{\partial \mathrm{t}} \quad (1.3f)$$

Here $\mathbf{v}$ was approximated with $\mathbf{v}_E$ in $\mathbf{v}_p$. We note that $\mathbf{v}_E$ is the same for different particle species.

Fluid drifts

The fluid equations are in themselves exact but each equation couples to the next higher equation. The only approximation thus lies in using a finite number of fluid equations. The low frequency drifts, of course, have the additional approximation of assuming $\omega << \Omega_c$. The small parameter involved here is, however, typically less that 10^{-2}. Then the reason for including the ion polarisation drift at all is that in the continuity equation the divergence of the ExB and diamagnetic fluxes give inverse background lengthscales while the divergence of the polarisation drift gives us the perpendicular wave number and we assume: $k_\perp >> \frac{1}{L_n}, \frac{1}{R}$

Properties of fluid drifts

Since the fluid equations include the pressure force which is due to the simultaneous action of all particles, they lead to drifts that are pure fluid drifts in inhomogeneous plasmas.

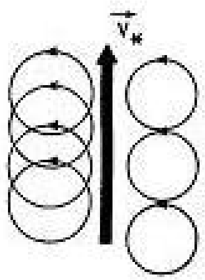


Fig 1. Diamagnetic drift

The fact that more particles from the higher density side go through a point leads to an average velocity. The same thing happens if the particles on one side rotate more rapidly (temperature gradient)

The magnetic drift

As we can see from the expansion (1.3), the magnetic drift is missing. This is because it is not a fluid drift.

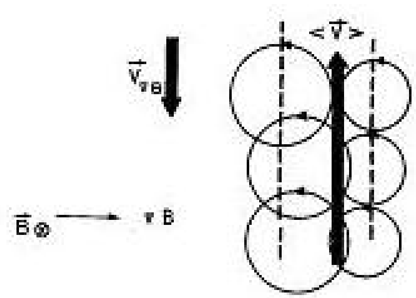


Fig 2. Magnetic drift

The particle drift is compensated by the fact that more particles contribute from the side with weaker magnetic field in such a way that there is no fluid drift.

Properties of drifts

In a homogeneous magnetic field the fact that the diamagnetic drift does not move particles is expressed by:

$$\nabla \cdot (n\mathbf{v}_*) = 0 \qquad (1.4)$$

Eq (1.4) is the lowest order consequence of the fact that the diamagnetic drift does not move particles. In the momentum equation the stress tensor cancel convective diamagnetic effects. Such effects are cancelled also in the energy equation as we will soon see.

Alfve'n waves

To demonstrate the derivation of MHD modes by using the two fluid expansion we will now derive the dispersion relation of Alfve'n waves. Quasineutrality gives:

$$\nabla \cdot \mathbf{j} = 0 \qquad (1.5)$$

$$\nabla \cdot (en\mathbf{v}_{\mathbf{p}i}) = -\hat{e}_{\|} \cdot \nabla j_{\|} = \frac{1}{\mu_0} \Delta A_{\|} \qquad (1.6)$$

Where we used the Ampe're law

$$\delta\mathbf{B} = \nabla \times (A_{\|}\,\hat{\mathbf{z}}) = -\hat{\mathbf{z}} \times \nabla A_{\|} \Rightarrow j_{\|} = -\frac{1}{\mu_0} \Delta A_{\|}$$

Now the MHD constraint

$$E_{\|} = 0 \Rightarrow A_{\|} = \frac{1}{\omega} \phi$$

Alfve'n waves cont.

in combination with 1.6 gives:

$$\omega^2 = k_\parallel^{\ 2} v_A^{\ 2} \qquad (1.7)$$

where

$$v_A = \frac{B}{\sqrt{\mu_0 n m_i}} \qquad (1.8)$$

This derivation is probably simpler than with the one fluid equations.

The Alfve'n frequency plays an important stabilizing role for MHD modes. The vanishing of $E_\parallel$ means that magnetic fieldlines are frozen into the plasma. Thus they get bent, with increasing magnetic energy,

due to pressure or current driven instabilities.

Interchange modes

Now adding magnetic curvature:

$$\nabla \cdot (n\mathbf{v}_{*j}) = \frac{1}{T}\mathbf{v}_{\mathbf{D}_j} \cdot \nabla P_j \qquad (1.9)$$

Where j indicates particle species and $\mathbf{v}_D$ is the magnetic drift

$$\mathbf{v}_{De} = -\frac{T_e}{T_i}\mathbf{v}_{\mathbf{D}i}$$

$$\nabla \cdot (n\mathbf{v}_{*i} - n\mathbf{v}_{*\mathbf{e}}) = \frac{1}{T_i}\mathbf{v}_{\mathbf{D}i} \cdot \nabla(\delta P_i + \delta P_e) \qquad (1.10)$$

Interchange modes cont.

- The critical point of the physics description is here how we treat the pressure perturbations.
- In single fluid MHD it is here conventional to use an adiabatic incompressional approximation. Thus we use only the convective perturbation: (ξ is the ExB displacement)

$$\delta P_j = -\xi \cdot \nabla P_j \qquad (1.11)$$

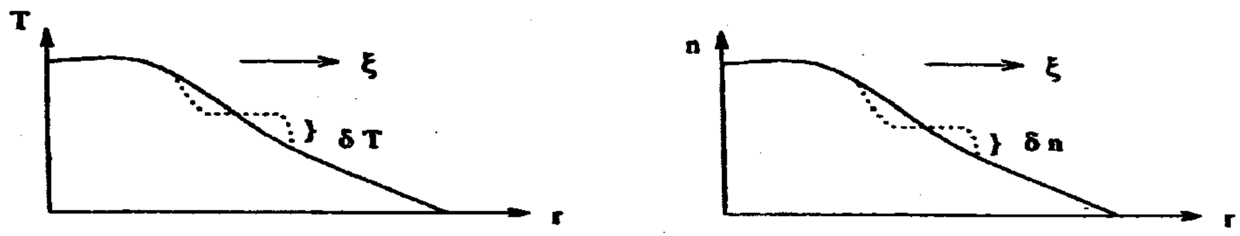


Fig3 Convective perturbations

Interchange modes cont.

Since v_{Di} is proportional to T_i we notice that (1.10) only depends on temperatures through the pressures. This is a particular property of the ideal MHD limit. We can now write the total dispersion relation as:

$$\omega^2 = k_{\|}^2 v_A^2 + \frac{\partial P}{P \partial r} \frac{T_e + T_i}{m_i R_c} \qquad (1.12)$$

$P = P_e + P_i$. Of course also (1.12) depends on temperature only through the sum of ion and electron temperatures. Since we use quasineutrality for these low frequency perturbations, the density n is just a multiplicative factor. We can say that the ideal MHD limit is degenerate with respect to temperatures.

Degeneracy of temperature dependence

regarding temperatures and pressures. Eq (1.12) is the dispersion relation for electromagnetic interchange modes where the last term is destabilizing (pressure typically decreases with r). In tokamak geometry we have to use an *eigenvalue equation* since both parallel modenumber and the curvature are space dependent. The curvature is destabilizing only on the outside of a tokamak so unstable modes tend to localise there and lead to ballooning like perturbations. Thus Eq (1.12) turns into the eigenvalue equation of the MHD ballooning mode in proper geometry.

Nonadiabatic nonisothermal compressional equations

As mentioned above the critical approximation, leading to the conventional treatment of ballooning modes in the MHD limit is (1.12). We will now extend this to a *nonadiabatic compressional* treatment. This is actually all we need for modes driven by gradients in configuration space! We use the energy equation:

$$\frac{3}{2}\left(\frac{\partial}{\partial t} + \mathbf{v}_i \cdot \nabla\right)\mathrm{T_i} + \mathrm{P_i}\nabla \cdot \mathbf{v}_i = -\nabla \mathbf{q}_{*i} \qquad (1.13a)$$

$$\mathbf{q}_{*i} = \frac{5}{2}\frac{\mathrm{P_i}}{\mathrm{m_i}\Omega_{\mathrm{c_i}}}(\mathbf{e}_{\parallel}\, x\nabla \mathrm{T_i}) \qquad (1.13b)$$

$$\nabla \cdot \mathbf{q}_{*i} = -\frac{5}{2}\mathrm{n}\mathbf{v}_{*\mathbf{i}} \cdot \nabla \mathrm{T_i} + \frac{5}{2}\mathrm{n}\mathbf{v}_{\mathbf{Di}} \cdot \nabla \mathrm{T_i} \qquad (1.13c)$$

Nonadiabatic, compressional··· cont

We now use (1.13c) in (1.13a) where we also have other convective diamagnetic effects. The last one is obtained by using the contimuity equation in the form:

$$\nabla \cdot \mathbf{v_i} = -\frac{1}{\mathrm{n}}\left(\frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla\right)\delta \mathrm{n} \qquad (1.14)$$

Then

$$\frac{\delta T_i}{T_i} = \frac{\omega}{\omega - \frac{5}{3}\omega_{D_i}}\left[\frac{2}{3}\frac{\delta n_i}{n_i} - \frac{\omega_{*e}}{\omega}\left(\frac{2}{3} - \eta_i\right)\frac{e\phi}{T_e}\right] \qquad (1.15)$$

$$\eta_i = \frac{L_n}{L_{T_i}} \qquad L_j = -\frac{1}{j}\frac{\partial j}{\partial r};\ j = \mathrm{n}, \mathrm{T_j}$$

Eq (1.15) is the principal result needed to express temperature perturbations in toroidal magnetized plasmas.

The Energy Equation

Eq (1.15) is compressional and makes a continous transition between adiabatic and isothermal regimes since it includes the fluid resonance. This leads to a model which is quadratic in curvature and thus includes the flat density regime where the stability limit is given by (1.2):

$$\frac{R}{L_T} < \left(\frac{R}{L_T}\right)_{crit} \qquad (1.2)$$

Although the motivation we will give for the closure (1.13) is nonlinear, the agreement with linear kinetic theory is usually also quite good. Thus the accuracy obtained for (1.2) when FLR and parallel ion motion are neglected is 5%!

Where the diamagnetic drift with subindex T includes the full pressure gradient. It appears as a convective diamagnetic effect but comes from ExB convective density and temperature perturbations.
Such perturbations are here substituted because the FLR term is assumed to be small but have, in practice, turned out to work better than expected. This is seen from a comparison with gyrokinetics.

Finite Larmor Radius effects

The correct energy equation is obviously a cornerstone in a theory that includes ion temperature effects. However we also have to deal with Finite Larmor Radius effects (FLR). These are avaliable through the stress tensor in fluid theory.

where

$$\mathbf{v}_p = \frac{1}{\Omega_c}\left(\frac{\partial}{\partial t} + \mathbf{v}\cdot\nabla\right)(\hat{\mathbf{z}}\times\mathbf{v})$$

$$\mathbf{v} = \mathbf{v}_{\mathbf{E}} + \mathbf{v}_*$$

gives

$$\nabla\cdot\left[\mathrm{n}(\mathbf{v}_{\mathbf{p}_i} + \mathbf{v}_{\Pi_i})\right] = -i\mathrm{n}\mathrm{k}^2\rho_s^{\,2}(\omega - \omega_{*iT})\frac{e\phi}{\mathrm{T_e}} \qquad (1.16)$$

Subindex T on the diamagnetic drift indicates full pressure gradient. The apparent convective diamagnetic drift is instead a convective ExB drift in combination with a convective pressure perturbation

The parallel electric field

We have already used the MHD limit **E•B** =0 in the derivation of Alfve'n waves. We will now see under which circumstances this is a good approximation. For this purpose it is useful to combine the parallel equation of motion of electrons (Ohms law) with the electron continuity equation. Ignoring electron inertia we get:

$$\frac{\delta n_{ef}}{n_{ef}} = \frac{e}{T_e}\left(\phi - \frac{\omega - \omega_{*eT}}{k_{\|}} A_{\|}\right) - \frac{\delta T_e}{T_e} + i\frac{\mathrm{v}_{\|e}}{k_{\|} D_e} \qquad (1.17)$$

where $D_e = \frac{T_e}{m_e \nu_e}$

We now need an equation of state of electrons. At these low frequencies electrons are isothermal. However they thermalize along magnetic field lines that are bent. Thus we get:

The parallel electric field cont.

We get

$$\delta T_e = \eta_e \frac{\omega_{*e}}{k_\parallel} e A_\parallel \qquad (1.18)$$

This is just radial convection in the background temperature gradient due to the bending of the fieldline. We then get a cancellation between the temperature perturbations and:

$$\frac{\delta n_{ef}}{n_{ef}} = \frac{e}{T_e}\left(\phi - \frac{\omega - \omega_{*e}}{k_\parallel} A_\parallel\right) + i\frac{v_{\parallel e}}{k_\parallel D_e} \qquad (1.19)$$

The electron continuity equation

The electron continuity equation now is:

$$\frac{\partial n_{ef}}{\partial t} + \nabla \cdot \left[n_{ef} \left(\mathbf{v}_{\mathbf{E}} + \mathbf{v}_{*\mathbf{e}} + \mathrm{V}_{\parallel 0} \frac{\delta \mathbf{B}_{\perp}}{\mathrm{B}} + \mathrm{v}_{\parallel} \widehat{\mathbf{e}}_{\parallel} \right) \right] = 0 \qquad (1.20)$$

Here we included a background electron current giving Kink effects. We now also use

$$\frac{\delta \mathbf{B}_{\perp}}{\mathrm{B}} \cdot \nabla \mathrm{J}_{\parallel 0} = \frac{d\mathrm{J}_{\parallel 0}}{\mathrm{dr}} \frac{1}{\mathrm{B}r} \frac{\partial A_{\parallel}}{\partial \theta}$$

Parallell electric field

- The relation between magnetic and electric potentials is then:

$$\frac{eA_{\|}}{T_e} = k_{\|} \frac{\omega - \omega_{*e}}{\omega(\omega - \omega_{*e}) + \omega_{De}(\omega_{*eT} - \omega) - \frac{k_{\|} k_\theta T_e}{e^2 B n_0} \frac{\partial J_{\|0}}{\partial r} - k_{\perp}^2 \rho_s^2 k_{\|}^2 v_A^2 (1 - i\delta)} \frac{e\phi}{T_e} \qquad (1.21)$$

where $\delta = \dfrac{\omega - \omega_{De}}{k_{\|}^2 D_e}$

and

$$E_{\|} = k_{\|}\phi - \omega A_{\|} = k_{\|}\phi \frac{\omega_{D_e}(\omega_{*eT} - \omega) - k_{\perp}^2 \rho_s^2 k_{\|}^2 v_A^2 - \frac{k_{\|} k_\theta T_e}{e^2 B n_0} \frac{\partial J_{\|0}}{\partial r}}{\omega(\omega - \omega_{*e}) + \omega_{De}(\omega_{*eT} - \omega) - \frac{k_{\|} k_\theta T_e}{e^2 B n_0} \frac{\partial J_{\|0}}{\partial r} - k_{\perp}^2 \rho_s^2 k_{\|}^2 v_A^2 (1 - i\delta)} \qquad (1.22)$$

The Parallel Electric field cont

- We note that FLR, curvature and current gradient contribute to a parallel electric field. Since the denominator is quadratic in ω and it contains the growthrate, which is larger than all drifts in ideal MHD we can see that the parallel electric field gets small in this limit. When the frequency is mich smaller than the Alfve'n frequency and FLR is not too small, the Alfve'n terms dominate in both numerator and denominator and the broken rational factor approaches 1. This is the electrostatic limit. The current gradient term also enters in (1.6) and gives the usual MHD kink mode by assuming vanishing parallel electric field. It enters mainly for low mode numbers and will usually be neglected in the following. We can use the parallel electric field to separate between MHD type ($E_{\|}$ small) and drift type ($A_{\|}$ small) modes.

Degeneracy of temperature and density gradients

The linear gyrokinetic equation is:

$$(\omega-\omega_D(\mathrm{v}_{\|}{}^2,\mathrm{v}_{\perp}{}^2)-k_{\|}\mathrm{v}_{\|})(f^{(1)}{}_{k,\omega}+\frac{q\phi_{k,\omega}}{T}f_0)=(\omega-\omega_{\bullet})\frac{q}{T}\phi_{k,\omega}e^{iL_k}J_0(\xi_k)f_0 \qquad (1.23)$$

It leads to the density response in 2d (no parallel motion)

$$\frac{\delta \mathrm{n}_{\mathrm{i}}}{\mathrm{n}_{\mathrm{i}}}=-\frac{e\phi}{T_i}\left[1-\frac{1}{n_0}\int_0^\infty\frac{\omega-\omega_{*i}\left[1+\eta_i(m_i\mathrm{v}^2/2T_i-3/2\right]}{\omega-\omega_{D_i}(\mathrm{v}_{\|}{}^2+\mathrm{v}_{\perp}{}^2/2)/\mathrm{v}_{\mathrm{th}}{}^2}J_0(\xi)^2 f_0 d^3\mathrm{v}\right] \qquad (1.24)$$

Expansion in FLR and curvature

- An expansion in FLR and ω_D/ω_* up to quadratic terms (which is only allowed at the edge) gives:

$$\frac{\delta n}{n} = \left[\frac{\omega_{*e}}{\omega} - \left(1 - \frac{\omega_{*iT}}{\omega}\right)(1 + \Gamma\frac{\omega_{Di}}{\omega})(\frac{\omega_{De}}{\omega} + k_\perp{}^2\rho^2) + \Gamma\eta_i\frac{\omega_{*i}\omega_{Di}\omega_{De}}{\omega^3}\right]\frac{e\phi}{T_e} \qquad (1.25)$$

where $\Gamma = 7/4$. The corresponding fluid expansion, using (1.15) , agrees except for that $\Gamma = 5/3$. The difference is only 5% and seems to be due to that the kinetic temperature perturbations not are isotropic. We here notice the presence of η_i in the last term. This is clearly a term that can not be recovered from ideal MHD. It actually is the only nonadiabatic term here and is due to the diamagnetic heatflow in the fluid model. In the electromagnetic ballooning mode, this term gives us the kinetic ballooning mode.

Kinetic ballooning mode

- The term containing the temperature gradient separately now gives the kinetic ballooning mode. A plot of growthrate versus pressure gradient is shown in Fig 4

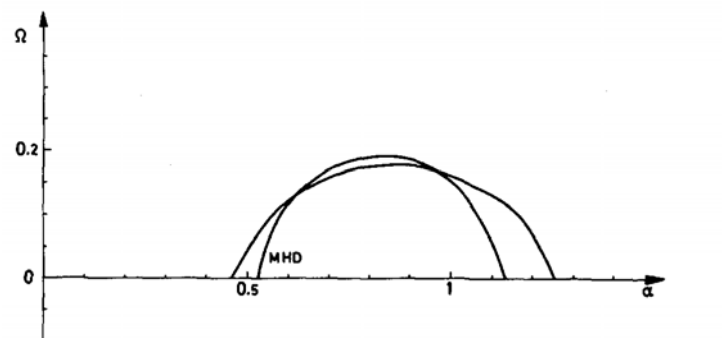


Fig 4 Growthrates of electromagnetic ballooning modes as a function of normalized β. The inner curve corresponds to ideal MHD while the outer includes Kinetic ballooning modes with a larger unstable region. Here $\varepsilon_n = 0.35$, η_i=2 and $k^2\rho^2$=0.01. The upper stability regime is due to the magnetic field geometry

Degeneracy of temperature and density gradients

- Although the effect of the broken degeneracy is rather modest here, it will be lager for larger η_i and ε_n and it was one of the main effects (together with peeling modes) that limited the slope of the H-mode barrier in recent simulations I have made. However, as we will see, using a single pressure gradient gives a dramatically worse approximation for Ion Temperature Gradient (ITG) modes. We write the continuity equation for ions: $\frac{\partial \delta n_i}{\partial t} + \mathbf{v_E} \cdot \nabla n_0 + n_0 \nabla \cdot \mathbf{v_E} + \nabla \cdot (n\mathbf{v}_{*i}) = 0$ (1.26)

Degeneracy of temperature and density gradients

Using (1.9) we now get:

$$\omega\frac{\delta n}{n} = (\omega_{*e} - \omega_{D_e})\frac{e\phi}{T_e} - \omega_{Di}\frac{\delta p_i}{P_i} \qquad (1.27)$$

Now using (1.11) in the form:

$$\frac{\delta \mathrm{p_i}}{\mathrm{P_i}} = \frac{\omega_{*\mathrm{iT}}}{\omega}\frac{e\phi}{T_i} = \frac{\omega_{*\mathrm{i}}(1+\eta_i)}{\omega}\frac{e\phi}{T_i} \qquad (1.28)$$

We get: $$\omega^2 - \omega(\omega_{*e} - \omega_{De}) = -\tau\omega_{Di}\omega_{*\mathrm{i}}(1+\eta_i) \qquad (1.29)$$

With the solution

Degeneracy of temperature and density gradients

$$\omega = \frac{1}{2}(\omega_{*e} - \omega_{De}) + \sqrt{\frac{1}{4}(\omega_{*e} - \omega_{De})^2 - \tau\omega_{Di}\omega_{*i}(1+\eta_i)}$$

- Here the stabilizing fist term under he root is often small due to the factor $\frac{1}{4}$ and, as it turns out, resistive instability would persist also if that term has stabilized the system. Thus the sign of the last term was sometimes given as the stability condition i.e. we get the stability limit: $\eta_i = -1$

$$(1.30)$$

Degeneracy of temperature and density gradients

- This result is, of course, completely wrong. It can be seen as due to an expansion

$$\frac{\omega_D}{\omega} << 1$$

Actually, if we use the convective perturbation only for temperature we get

$$\frac{\delta \mathrm{T_i}}{\mathrm{T_i}} = \eta_i \frac{\omega_{*\mathrm{iT}}}{\omega} \frac{e\phi}{T_i} \qquad (1.30)$$

and

$$\frac{\delta \mathrm{n_i}}{\mathrm{n_i}} = \frac{\omega_{*e} - \omega_{De} - \tau\omega_{Di} \dfrac{\eta_i \omega_{*i}}{\omega} \dfrac{\delta \mathrm{T_i}}{\mathrm{T_i}}}{\omega - \omega_{Di}} \frac{e\phi}{T_e} \qquad (1.31)$$

Spurious expansion in curvature

- We notice that the quadratic term in ω needed for instability vanishes for vanishing temperature gradient if (1.31) is combined with Boltzmann electrons if we myltiply with the denominator. However such a term can be obtained by expansion in ω_D/ω ! This connects in a nice way to the common problem of expansion in ω_D/ω which was routinely done until the end of the 1980's.

Threshold for ITG using only pressure gradient

- In Fig 5 we show qualitatively and semiquantitatively the threshold of ITG when we use only pressure gradient and when we consider temperature ırately.

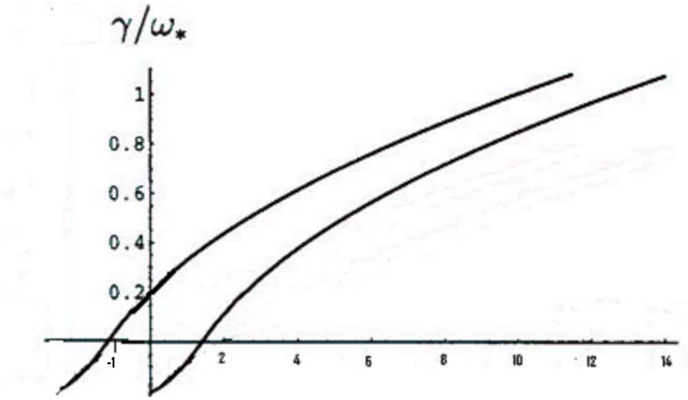


Fig 5 ITG threshold (−1) with convective pressure perturbation

The correct threshold is shown by the right curve.

Global stability – Transport

- Our example has shown us some general properties of instabilities associated with global stability and transport
- For **MHD-type modes**, *geometry* is more important than the physics description
- For **drift waves**: The *physics description* is more important than geometry.
- Nevertheless there are cases, in particular in enhanced confinement regimes where geometry can be quite important for drift waves. However, the very fact that we have reduced transport means that the largest stabilizing and destabilizing terms are almost balancing so that small effects become important. The corresponding thing happens with MHD type modes, i.e. *the physics description becomes important close to marginal stability*.

Radial growth of transport coefficients

- Another example of effect of the physics description on drift waves is the growth of transport coefficients with radius.

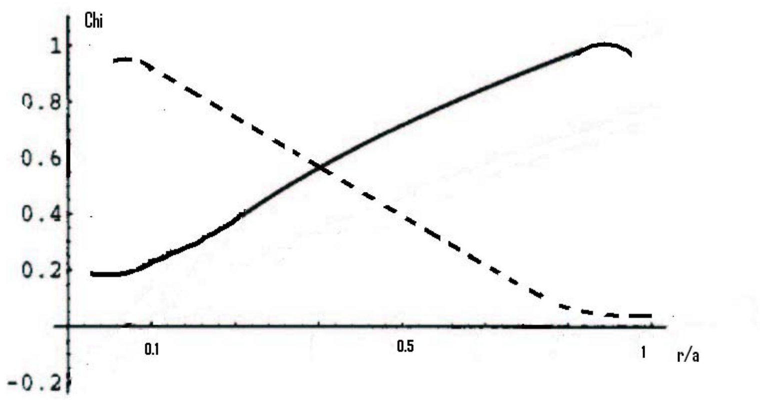


Fig 6 Radial profiles of χ for a fluid or kinetc model that is expanded in ω_D/ω (dashed) and an unexpanded model (full line). The experimental curve is also close to the full line.

Expansion in ω_D/ω

- It is instructive to look at the linear gyrokinetic density perturbation for ions.

$$\frac{\delta n_i}{n_i} = -\frac{e\phi}{T_i}\left[1-\frac{1}{n_0}\int_0^\infty \frac{\omega-\omega_{*i}\left[1+\eta_i(m_i v^2/2T_i - 3/2\right]}{\omega-\omega_{D_i}(v_\parallel^2+v_\perp^2/2)/v_{th}^2} J_0(\xi)^2 f_0 d^3v\right] \qquad (1.32)$$

- We have already shown an expansion of this expression (15) but that was just in order to explore the similarity to the fluid response. The question is now under what circumstances we may be allowed to expand (1.32) in ω_D/ω. If (1.32) for ions is combined with the Boltzmann response for electrons, the drift frequencies are the only frequencies that can generate ω. Thus we have to assume that ω will be of the order of the drift frequencies. Now we want to expand in ω_D so ω_* should be larger. Then assuming ω to be of order ω_*, the critical parameter to expand in is $\varepsilon_n = \omega_D/\omega_* = 2L_n/L_B$.

Expansion in ε_n

- In practice ε_n is of order 1 iun the bulk plasma at least out to r/a = 0.8. Close to the axis ε_n goes to infinity. Using (1.13) and Boltzmann electrons one arrives at:

$$\chi_i = \frac{1}{\eta_i}\left(\eta_i - \frac{2}{3} - \frac{10}{9\tau}\varepsilon_n\right)\frac{\gamma^3/k_r^{\,2}}{(\omega_r - \frac{5}{3}\omega_{Di})^2 + \gamma^2} \qquad (1.33)$$

- It is obvious that (9) leads to the type of thermal conductivity shown in Fig 6 for the unexpanded model at least close to the axis since both L_n and L_T become large towards the axis so that η remains finite. As it turns out, we need also electron trapping to recover the full curve in Fig 3. Both ε_n and ω_D in (1.33) are due to the curvature effect of the diamagnetic heatflow as given by (1.13c).

Ion thermal conductivity

- Thus the growth of Chi with radius in Fig (6) is due to both magnetic curvature and nonadiabaticity. Now, Eq (1.30) is directly connected to the fluid closure since at marginal stability:

$$\omega_r = \frac{5}{3}\omega_{D_i} \qquad (1.34)$$

- so we are at the fluid resonance at marginal stability. However, this works well, both in comparison with kinetic theory and experiment. Fig 7 compares qualitatively and semi-quantitatively the linear instability, nonlinear saturation and continued oscillations for a fluid model with nonlinear closure and a reactive fluid model according to Holod, Weiland and Zagorodny (Phys. Plasmas 9, 1217 (2002

Fluid closure

- and the Hammett Perkins Gyro – Landau fluid model according to Mattor and Parker Phys. Rev. Lett. **79**, 3419 (1997). The interaction is between two slab ITG modes and a zonal flow.

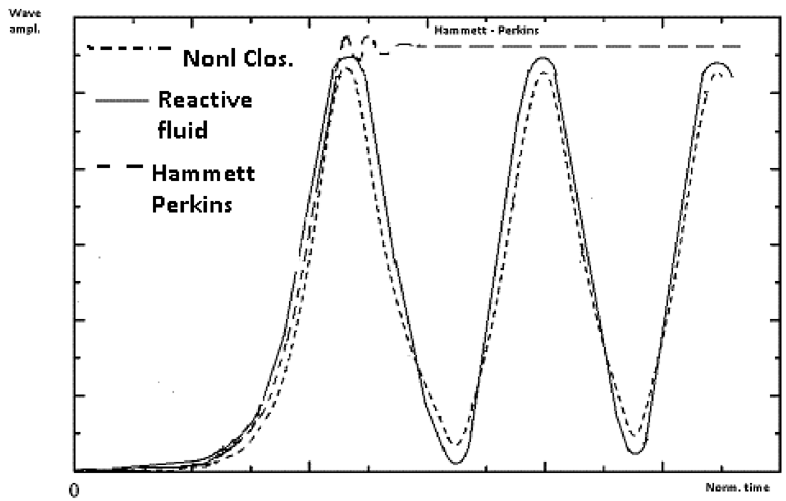


Fig 7 Development in time of three-wave interaction between two slab ITG modes and a zonal flow with different fluid descriptions including reactive fluid, fluid with nonlinear closure and the Hammett Perkins gyro-Landau fluid model.

Fluid closure

- The paper by Mattor and Parker showed that the fluid model with a nonlinear closure is quite close to the full kinetic model. The only difference between the reactive model and the model with nonlinear closure is the kinetic resonance. The velocity distribution is here Maxwellian and the highest moment (fifth) is expressed through the kinetic integral where a nonlinear frequency shift is included, i.e. $\omega = \omega_L + \delta\omega_{nl}$ (1.35)

The result of the nonlinear frequency shift in the kinetic integral may be difficult to see by inspection. However, in the expanded form, corresponding to the Universal instability (driven by inverse Landaudamping) it can be visualized.

Phase mixing due to nonlinear frequency shifts

$$\gamma = \left(\frac{\pi}{2}\right)^{1/2} \omega_{*e} \frac{\omega - \omega_{*e}}{k_{||} v_{te}} e^{-\omega^2/(k_{||} v_{te})} \qquad (1.36)$$

- As seen from (1.36), the dissipation (energy) changes sign when the frequency equals the diamagnetic drift frequency. Thus the nonlinear frequency shift could easily change the sign of the imaginary part ɣ. This is what happens in Fig 7. As can be seen the wave particle resonance is stabilizing near maxima and destabilizing near minima. It is the absence of this effect that causes the Hammett Perkins model to phase lock at an amplitude above the other models. Thus *kinetic resonances effectively vanish* although we keep the *Maxwellian* distribution function.

Fluid closure

- This particular case is coherent but turbulence can be seen as due to coupling of very many such systems. Already two waves lead to stochasticity of marginally trapped – detrapped particles. B.V. Chirikov, Phys. Rep. **52**, 263 (1979). Stochastic particles diffuse quasilinearly. However, in practice *nonlinear effects reintroduce correlations.* The first example of this is the *kinetic equation for waves, i.e.*

The **Random phase approximation**, Sagdeev and Galeev, *Nonlinear Plasma Theory*, Benjamin, New York 1969.

The next example is the nonlinear Fokker Planck equation:

Fokker–Planck equation

$$\left(\frac{\partial}{\partial t}+\mathrm{v}\frac{\partial}{\partial x}\right)f(x,\mathrm{v},t)=\frac{\partial}{\partial \mathrm{v}}\left[\beta\mathrm{v}+D^{\mathrm{v}}\frac{\partial}{\partial \mathrm{v}}\right]f(x,\mathrm{v},t) \qquad (1.37)$$

- When we consider turbulent collisions we are in a random phase situation where the friction coefficient β and diffusivity in velocity space D^{v} are proportional to sums of intensities of wave amplitudes (phase dependent terms have been averaged out). Clearly friction in this case gives a **nonlinear frequency shift** which is a strongly nonlinear feature, i.e. nonlinearities have reintroduced correlations. When the coefficient are constants, (1.37) has an analytical solution: (S. Chandrasekhar, *Stochastic Problems in Physics and Astronomy*, Rev. Modern Physics **15**, 1 (1943)). The shape of the solution is shown in Fig 8.

Fokker-Planck equation

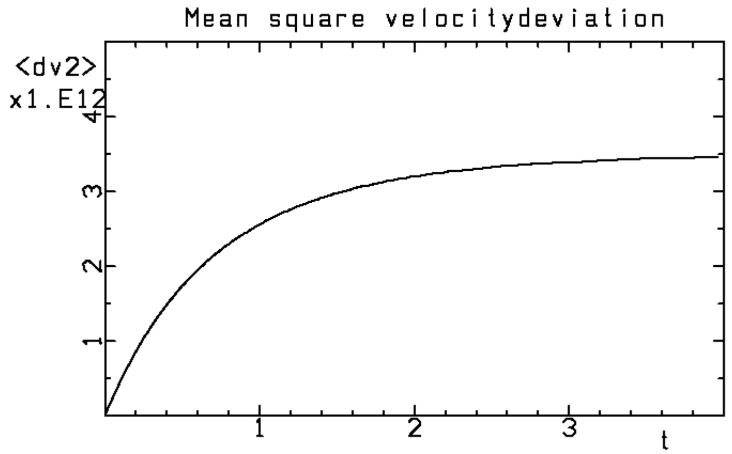


Fig 8. Mean square velocitydeviation $<(\Delta v)^2>$ as a function of time showing intitial quasilinear linear growth and later saturation due to strongly nonlinear effects.

Here the first linearly growing part corresponds to quasilinear diffusion while the asymptotic flat part is strongly nonlinear. This type of behavior can be obtained by renormalization . (*T.H. Dupree, Phys. Fluids 9, 1773 (1966), J. Weinstock, Phys. Fluids 12, 1045 (1969)).* An important aspect of the flat part is that there is no energy transfer between resonant particles and waves on the average. Thus linear wave-particle resonances have been averaged out! This is analogous to the coherent case we just discussed. In both cases the phase mixing of linear resonances is due to **strongly nonlinear effects.**

Particle pinches

- A sensitive test on the presence of linear kinetic resonances is the strength of particle pinches. In Fig 9 we show Chi and D as a function of temperature gradient for a reactive fluid model, and a fluid model with Landau damping.
-

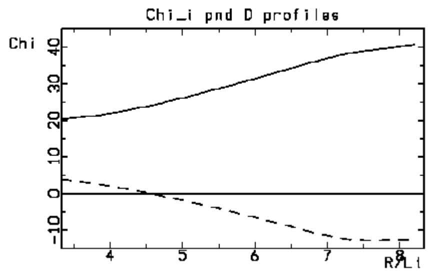


Fig 9a

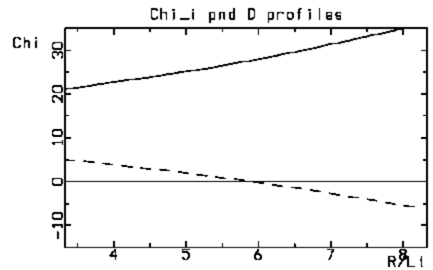


Fig 9c

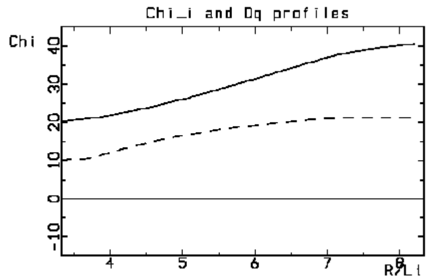


Fig 9b

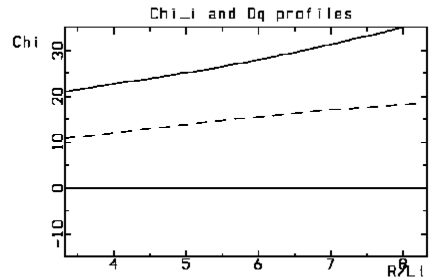


Fig 9d

Fig 9 Particle transport as a function of temperature gradient for a reactive fluid model a) and b) and for a fluid model where Landaudamping was added, c) and d).The full lines show χ_i for comparison while the dotted lines show particle diffusivities. Here a) and c) show that of the main ions (Hydrogen) while b) and d) show the diffusivities of Coal.

Quasilinear particle transport

- As it turns out quasilinear transport is more sensitive to particle pinches than fluid models where Landaudamping is added in the energy equation

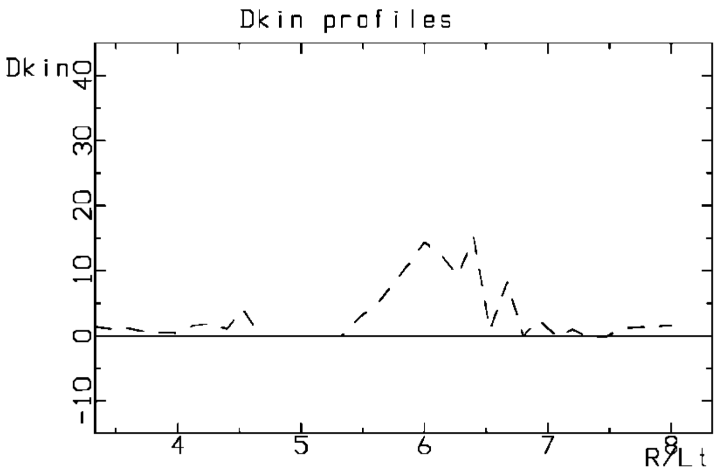


Fig 10. Particle diffusion in Quasilinear kinetic theory for the same parameters as in Fig 9

$k_\Theta\ \rho_s = 0.3$

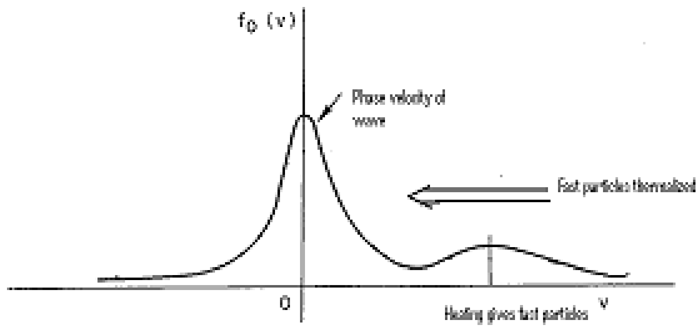
Effects of kinetic resonances on particle pinches

We here used $$k_\theta \rho_s = 0.3 \tag{1.38}$$

- As it turns out, the existence of the quasilinear particle pinch depends on modenumber. However, in order to obtain the total transport, we need to consider a modenumber corresponding to the *inverse correlation length* of the system. This is typically given by (1.38) but can vary due to different parameters, typically due to magnetic shear, flowshear and magnetic q. This variation would usually be between 0.2 and 0.4.
- The fact that the transport of coal is insensitive to kinetic resonances of the main ions means that the main ions are also insensitive to the heating which is much more distant in phase space

Nonresonant heating

Heating at much higher velocities than for typical waves. The results is that the fast particles have been thermalized when they reach the phase velocity of the wave. The wave experiences only an ideal heating.



- Fig 11. Drift waves are so far away from the region of heating in phase space so they are independent.

- If the ITG mode associated with Coal would be unstable, however, we would have to consider its resonance but the situation would be similar to that of the main ITG mode since it would be driven by gradients in real space. The situation is quite different for instabilities driven by fast particles.

Time scales

- Since the confinement time is limited in a reactor, we need to reach sufficiently high reaction rate to get out more energy than we put in during a confinement time. Since we, in a tokamak, have a pulsed operation, the pulse time is the time we need to control the plasma. It is instructive to compare the different timescales in a reactor as shown in Fig 12.

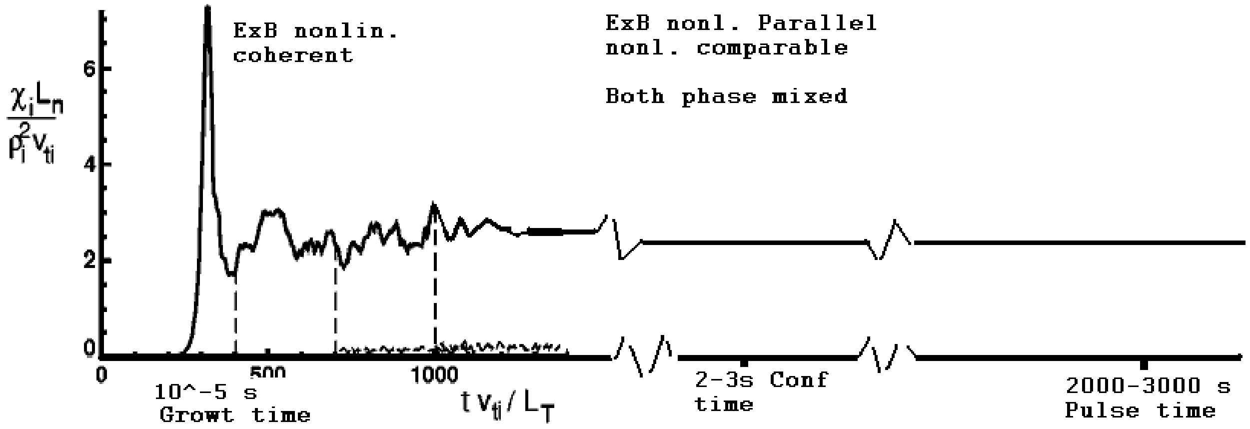


Fig 12 Time scales in a fusion reactor

Time scales cont.

- While the growth time is a few times 10^-5, the confinement time is typically 2 – 3 s. A quasistationary turbulent spectrum appears to exist after about 10^-3 s. A lot of discussions have concerned the relevance of the parallel nonlinearity. Several aspects were given in:
- J. Candy, R.E. Waltz, S.E. Parker and Y. Chen, *Relevance of the parallel nonlinearity in gyrokinetic simulations of tokamak plasmas*, Physics of Plasmas **13**, 074501 (2006).
- There it was pointed out that the parallel nonlinearity enters on the transport timescale. Since the main parallel nonlinearity is due to Nonlinear Landaudamping, which considers beating of waves with almost equal modenumbers, we conclude that this nonlinearity can be seen as nonlinear Landaudamping. (N.L. Shatashvili and N.L. Tsintsadze, Physica Scripta **T2:2**, 511 (1982).

Detuning of kinetic resonances

- The fact that the parallel nonlinearity enters on the confinement timescale was also noted by
- J. Weiland, A. Eriksson, H.Nordman and A. Zagorodny, *Progress on Anomalous Transport in Tokamaks, Drift Waves and Nonlinear Structures*, PPCF **49**, 1 (2007).
- As a result of this ordering, also the friction and diffusivity in the Fokker-Planck equation enter on the confinement timescale. *Thus in order to reach steady state in kinetic simulations we, in principle, have to rum our codes on the confinement timescale*.
- As it turns out, most gyrokinetic simulations appear to reach steady state on a much shorter timescale. We can actually see a possibility for that from Fig 7. There the kinetic resonance is averaged out on a few growth times. However, that system is coherent while tokamak turbulence is almost completely incoherent

Kinetic resonances

- Thus the possibility we can see is if the kinetic resonances are averaged out due to the much stronger ExB nonlinearity while the dynamics is still coherent. Since there is only one resonance (36), particles can be taken out of resonance by either parallel or perpendicular acceleration.

$$\omega = k_{\|} \mathrm{v}_{\|} + \omega_D \qquad (1.39)$$

- Thus if the stronger perpendicular resonance takes particles out of resonance during the coherent phase of the development, there will be no kinetic effect of the parallel nonlinearity. Nonlinear parallel fluid effects are usually ignorable.