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Physics of TAE Modes in Tokamaks

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Physics of TAE Modes in Tokamaks*

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Motivation

- The challenge of understanding fast particle collective behaviors in burning plasmas of fusion interest is to develop a predictive capability for describing energetic particle confinement and its link to the dynamic evolution of thermal plasma profiles.
- Theory and simulation must play fundamental roles:
  - fusion plasmas are complex systems in which long time scale behaviors will be determined by cross-scale couplings of phenomena occurring on micro- and meso-spatiotemporal scales
  - existing experiments can look at these issues separately, since cross-scale couplings are not those of burning plasmas
  - mutual positive feedbacks are necessary between theory, simulations and experiments for V&V of present predictive capabilities

Why TAE Modes?

- Toroidal Alfvén Eigenmodes (TAE) [Cheng, Chen, Chance AP85] are the prototype of all Alfvénic Eigenmodes (AE):
  - Nearly undamped plasma eigenmodes located in the frequency gaps of the shear Alfvén continuous spectrum
  - Can be resonantly excited by supra-thermal particles and fusion products, for their parallel group velocity in toroidal devices of fusion interest is typically of the same order as parallel particle speed
  - Potentially dangerous for particle transport, since their $\perp$ group velocity is negligible and resonant condition can be effectively maintained

- Modes of the continuous shear Alfvén spectrum can also be excited at the characteristic frequency of the supra-thermal particle motions, when the resonant drive is strong enough to overcome continuum damping: Energetic Particle Modes (EPM) [Chen POP94]

- There is a one-on-one correspondence between TAE/EPM and diamagnetic [Coppi and Porcelli PRL86]/precessional [Chen et al PRL84] fishbones.

- This is a general result, which allows us to identify fluctuations of the Alfvén branch that are most important for causing wave-induced fast ion transports in burning toroidal plasmas of fusion interest.

- Within the optimal wavelength and frequency ordering for studying collective Alfvén mode excitations in burning plasmas one can demonstrate two fundamental results [Zonca and Chen PPCF06]):

  1. Both resonant and non-resonant wave-particle interaction with fast ions are determined by the magnetic drift curvature coupling
  2. All shear Alfvén waves can be described within the unified framework of one single *fishbone-like* dispersion relation

- The general fishbone-like dispersion relation (GFLDR) is the most elementary framework for discussing the profound connection of shear Alfvén wave fluctuations with MHD and e.m. micro–turbulence.

Generalized FL dispersion relation: properties

- In general, demonstrate that the mode dispersion relation can be always written in the form of a *fishbone-like* dispersion relation [Chen et al PRL84]

$$-\mathrm{i}\Lambda + \delta W_f + \delta W_k = 0 \quad ,$$

where δW_f and δW_k play the role of fluid (core plasma) and kinetic (fast ion) contribution to the potential energy, while Λ represents a generalized inertia term.

- The generalized fishbone-like dispersion relation can be derived by asymptotic matching the regular (ideal MHD) mode structure with the general (known) form of the SA wave field in the singular (inertial) region, as the spatial location of the shear Alfvén resonance, $\omega^2 = k_\parallel^2 v_A^2$, is approached.

- Examples are : $\Lambda^2 = \omega(\omega - \omega_{*pi})/\omega_A^2$ for $|k_\parallel q R_0| \ll 1$ and $\Lambda^2 = (\omega_l^2 - \omega^2)/(\omega_u^2 - \omega^2)$ for $|k_\parallel q R_0| \approx 1/2$, with ω_l and ω_u the lower and upper accumulation points of the shear Alfvén continuous spectrum toroidal gap [Chen POP94].

- δW_f is real and freq. independent, whereas freq. dependent δW_k has complex values, the real part accounting for non-resonant and the imaginary part for resonant wave particle interactions with energetic ions.

- The fishbone-like dispersion relation demonstrates the existence of two types of modes (note: $\Lambda^2 = k_\parallel^2 q^2 R_0^2$ is SAW continuum):
  - a discrete gap mode, or Alfvén Eigenmode (AE), for $\mathbb{R}e\Lambda^2 < 0$;
  - an Energetic Particle continuum Mode (EPM) for $\mathbb{R}e\Lambda^2 > 0$.

- For EPM, the iΛ term represents continuum damping. Near marginal stability [Chen et al PRL84, Chen POP94]

$$\mathbb{R}e\delta W_k(\omega_r) + \delta W_f = 0 \quad \text{determines } \omega_r$$

$$\gamma/\omega_r = (-\omega_r \partial_{\omega_r} \mathbb{R}e\delta W_k)^{-1} (\mathbb{I}m\delta W_k - \Lambda) \quad \text{determines } \gamma/\omega_r$$

- For AE, the non-resonant fast ion response provides a real frequency shift, *i.e.* it removes the degeneracy with the continuum accumulation point, while the resonant wave-particle interaction gives the mode drive. Causality condition imposes

  - $\delta W_f + \mathbb{R}\mathrm{e}\delta W_k > 0$ when AE frequency is above the continuum accumulation point: inertia in excess w.r.t. field line bending

$$\Lambda^2 = \lambda_0^2(\omega_\ell - \omega) \; ; \qquad \omega > \omega_\ell \;\Rightarrow\; \Lambda \to -\mathrm{i}\sqrt{-\Lambda^2}$$

  - $\delta W_f + \mathbb{R}\mathrm{e}\delta W_k < 0$ when AE frequency is below the continuum accumulation point: inertia is lower than field line bending

$$\Lambda^2 = \lambda_0^2(\omega - \omega_u) \; ; \qquad \omega < \omega_u \;\Rightarrow\; \Lambda \to \mathrm{i}\sqrt{-\Lambda^2}$$

- For AE, $\mathrm{i}\Lambda$ represents the shift of mode frequency from the accum. point

- For both AE and EPM, the SAW accumulation point is the natural gateway through which modes are born at marginal stability
- For EPM, ω is set by the relevant energetic ion characteristic frequency and mode excitation requires the drive exceeding a threshold due to continuum damping. However, the non-resonant fast ion response is crucially important as well, since it provides the compression effect that is necessary for balancing the positive MHD potential energy of the wave.
- The GFLDR consistently treats SAW continuum, for which wave decay by phase mixing on a time scale $\propto (\Delta r \partial\omega_A(r)/\partial r)^{-1}$ [Chen and Hasegawa 74]
- As a consequence, AE form a dense population of eigenmodes (lighthouses) with unique (equilibrium-dependent) frequencies and locations [Chen NF07], nearby positions where $\partial\omega_A(r)/\partial r = 0$ and characteristic properties depending on the peculiar equilibrium that is being considered via δW_f and δW_k.

TAE: special case I of fishbone-like D.R.

- Toroidal Alfvén Eigenmodes (TAE) are the canonical example of AE in toroidal plasmas

- The generalized inertia term reads

$$\Lambda^2 = (\omega_l^2 - \omega^2)/(\omega_u^2 - \omega^2)$$

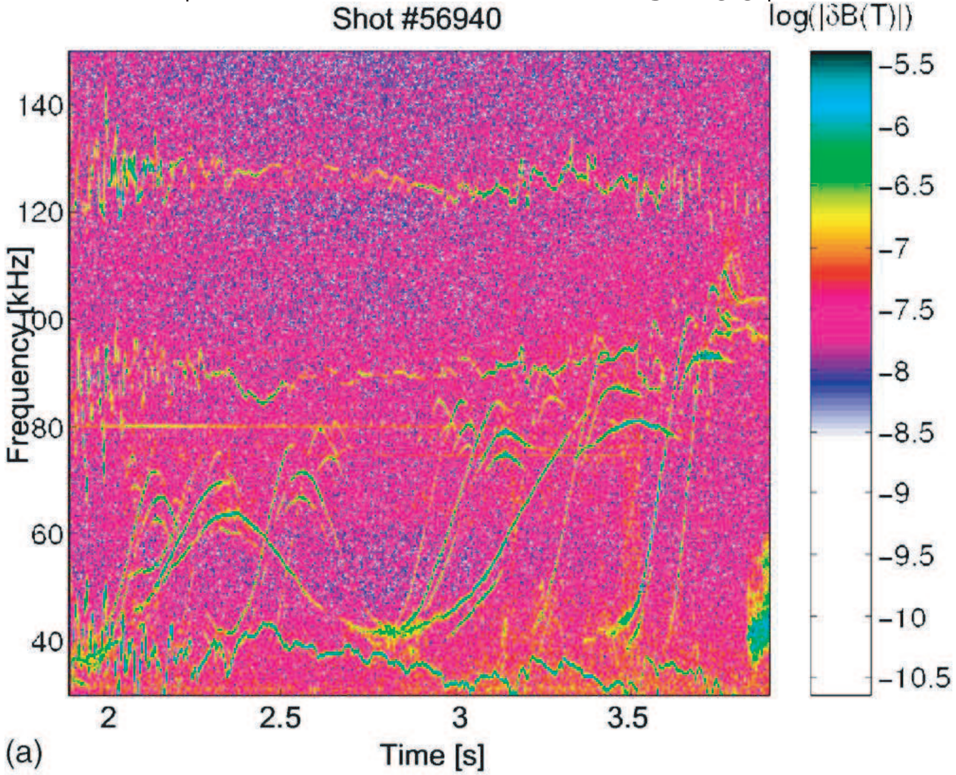
  with $\omega_{l,u} = (v_A/2qR_0)(1 \mp \epsilon_0/2)$ the lower and upper accumulation points of the shear Alfvén continuous spectrum toroidal gap and $\epsilon_0 = 2(r/R_0 + \Delta')$.

- At $s, \alpha = -R_0 q^2 \beta' \ll 1$ $\delta W_f = (s\pi/4)(1 - \alpha/\alpha_c)$ and $\omega \gtrsim \omega_l$, consistent with the existence condition of discrete AE gap mode just above the lower accumulation point, $\delta W_f + \mathbb{R}\mathrm{e}\delta W_k > 0$. TAE stabilization for $\alpha > \alpha_c$.

- As α increases above α_c, another TAE mode can exist for $\omega \lesssim \omega_u$ provided the appropriate existence condition for discrete AE gap mode is satisfied, i.e. $\delta W_f + \mathbb{R}\mathrm{e}\delta W_k < 0$. (ballooning unstable).

Alfvén Cascades: special case II of GFLDR

- Original theoretical interpretation of Alfvén Cascade (AC) excitation by large orbit trapped energetic ion tails generated by Ion Cyclotron Resonant Heating (ICRH) on JET [Sharapov etal PLA01] and [Berk etal PRL01] [Breizman et al PPCF05]



- AC can be interpreted as AE gap modes in the natural frequency gap that arises in the Alfvén continuous spectrum at the radial location, r_0, where $q(r_0) = q_0$ has a minimum, *i.e.* $S^2 \equiv r_0^2 q''(r_0)/q_0^2 > 0$.

- Consistently with theoretical prediction, AC are described by the fishbone-like dispersion relation with:

$$\Lambda^2 = (nq_0 - m)^{-1}(\omega^2 - k_\parallel^2 v_A^2)/\omega_A^2 \quad , \qquad \delta W_f = -(\pi/4)(S/n^{1/2})$$

- Condition for a discrete AE gap mode, $\mathbb{R}\mathrm{e}\Lambda^2 < 0$ with $\omega^2 > k_\parallel^2 v_A^2$, it is necessary to have $nq_0 - m < 0$ and:

$$\delta W_f + \mathbb{R}\mathrm{e}\delta W_k > 0 \quad .$$

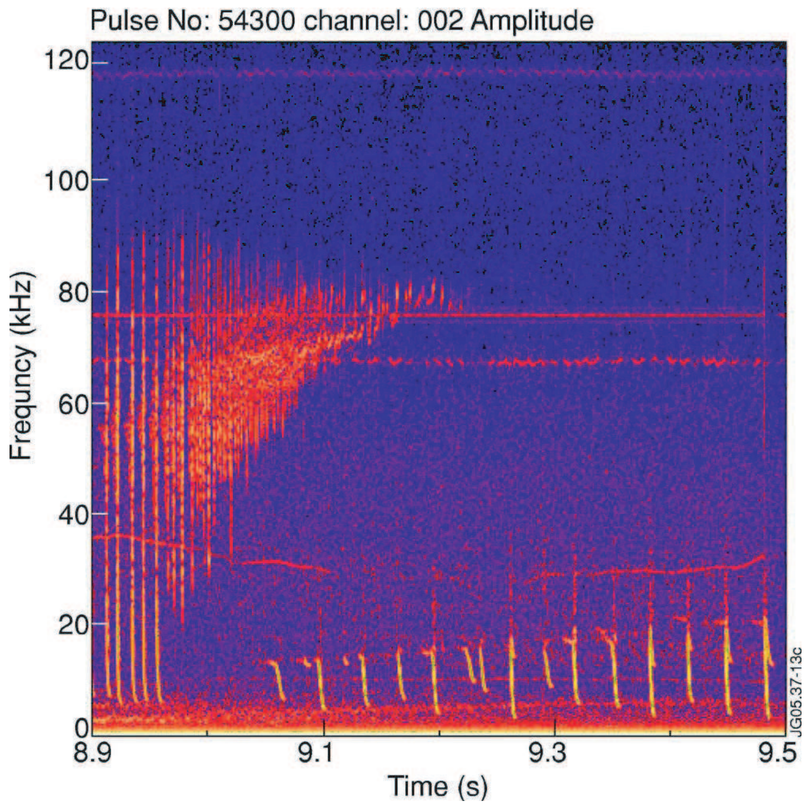
- This condition can be provided both by fast ions as well as by core plasma equilibrium effects ([Zonca et al. POP02]; [Fu and Berk POP06])

The roles of low frequencies

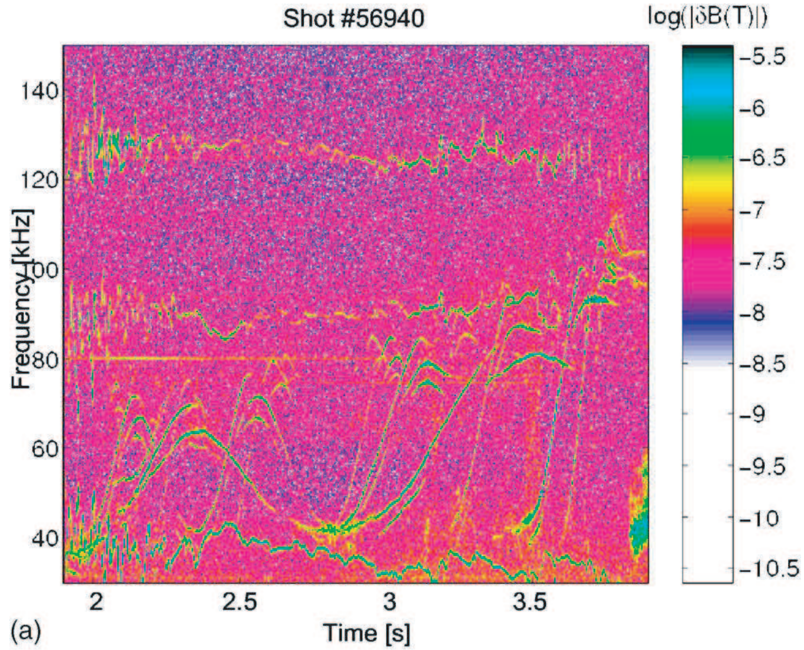
- So far: assessed the important role of SAW for energetic particle transport in burning plasmas magnetically confined in toroidal geometry
- Constructed the general theoretical framework for investigating SAW fluctuations based on the general fishbone like dispersion relation
- Next: look at specific applications, favoring low frequencies, since they are the natural ones on which both energetic as well as thermal particles can resonantly excite collective modes characterized by the respective scale lengths: fast ions $\Rightarrow$ mesoscales, thermal particles $\Rightarrow$ microscales.
- Similar temporal scales of disparate phenomena facilitates their interplay and dictates long time-scale nonlinear dynamic response (importance for the fusion burn)

Experimental observations: JET

- Observation of finite frequency fishbone oscillations at the GAM frequency ([Nabais etal POP05]; interpretation based on GFLDR [Zonca etal NF09]) and "low-frequency feature" of Alfvén Cascades [Breizman etal PPCF05].



$$\Lambda^2 = k_{\parallel 0}^2 q^2 R_0^2$$



Micro- and meso- scale excitation of low-frequency AE/EPM

- With the general expression of Λ (F, G, N, D given functions [Zonca NF09])

$$\Lambda^2 = \frac{\omega^2}{\omega_A^2}\left(1-\frac{\omega_{*pi}}{\omega}\right) + q^2\frac{\omega\omega_{ti}}{\omega_A^2}\left[\left(1-\frac{\omega_{*ni}}{\omega}\right)F(\omega/\omega_{ti}) - \frac{\omega_{*Ti}}{\omega}G(\omega/\omega_{ti}) - \frac{N_m(\omega/\omega_{ti})}{2}\left(\frac{N_{m+1}(\omega/\omega_{ti})}{D_{m+1}(\omega/\omega_{ti})} + \frac{N_{m-1}(\omega/\omega_{ti})}{D_{m-1}(\omega/\omega_{ti})}\right)\right]$$

  low frequency AE/EPM can be excited by both thermal ions (micro-scale) and energetic ions (meso-scales)

$$i\Lambda = \delta W_f + \delta W_k \qquad \text{(fast ions included)}$$

- Global hybrid GK simulations of BAE with eXtended HMGC (XHMGC) [Wang etal IAEA10 and POP11].

- Excitation of low-frequency AE by thermal ions is most easily seen using the simplified expression of Λ:

AITG excitation mechanism

$$\Lambda^2 = \frac{1}{\omega_A^2}\left[\omega^2 - \left(\frac{7}{4} + \frac{T_e}{T_i}\right) q^2 \omega_{ti}^2\right] + i\sqrt{\pi} q^2 e^{-\omega^2/\omega_{ti}^2} \frac{\omega^2}{\omega_A^2}\left(\frac{\omega_{ti}}{\omega} - \frac{\omega_{*Ti}}{\omega_{ti}}\right)\left(\frac{\omega^2}{\omega_{ti}^2} + \frac{T_e}{T_i}\right)^2 .$$

- When $\omega\omega_{*Ti} < \omega_{ti}^2$ BAE/EPM [Zonca and Chen PPCF96, 98] and high-frequency fishbones [Zonca et al NF09].
- When $\omega\omega_{*Ti} > \omega_{ti}^2$ accumulation point becomes unstable! The unstable continuum is not a concern [Zonca and Chen PPCF96, 98].
- When $\omega\omega_{*Ti} > \omega_{ti}^2$ and equilibrium effects localize the AE, the Alfvénic ITG mode is excited (AITG) [Zonca and Chen PPCF96, 98].

Structures of the low-frequency SAW spectrum

- Low-frequency Shear Alfvén Wave (SAW) gap: $\omega \sim \omega_{*i} \sim \omega_{ti}$; $\Lambda^2(\omega) = k_\parallel^2 v_A^2$

  $\Rightarrow$ (ideal MHD) accumulation point (at $\omega = 0$) shifted by thermal ion kinetic effects [Zonca et al PPCF96]

  $\Rightarrow$ new low-freq. gap! Kinetic Thermal Ion (KTI) gap [Chen NF07]
  - Diamagnetic drift: KBM [Biglari et al PRL91]
  - Thermal ion compressibility: BAE [Heidbrink et al PRL93]
  - ∇T_i and wave-part. resonances: AITG [Zonca POP99]

  $\Rightarrow$ unstable SAW accumulation point

  $\Rightarrow$ "localization" $\Rightarrow$ unstable discrete AITG mode

  $\Rightarrow$ Excitation of BAE/EPM/AITG at all scales, form micro (thermal ions) to meso (fast ions)

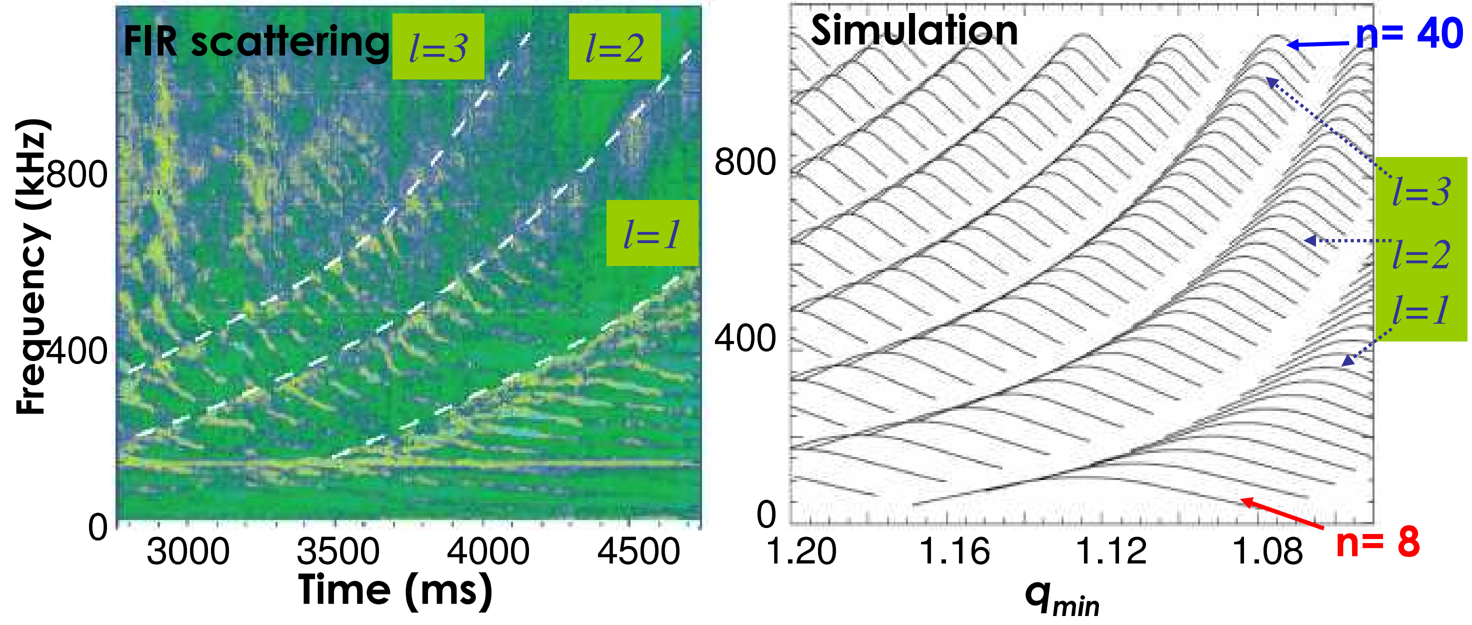
- For physics analogy: BAE – GAM degeneracy [Zonca et al PPCF06; Chen et al NF07, EPL08]. $\Rightarrow$ Implications on cross-scale couplings mediated by zonal structures [Chen and Zonca ASICTP08; Varenna08].

From micro- to meso-scales: transport processes

- Drift wave plasma turbulence is well known and turbulent plasma transport is widely studied (ITG/TEM/ETG).
- Alfvénic turbulence has been addressed mostly for plasma edge conditions [Scott PPCF97], but much less studied in the tokamak core [Chen et al NF78, Tang et al NF80].
- The investigation of Alfvénic ITG activity [Zonca and Chen PPCF96] is more recent. It is of relevance to burning plasmas of fusion interest since it can be excited at acoustic frequencies over a broad range of scale-lengths, from thermal ion Larmor radius to the typical fast ion orbit width [Zonca and Chen POP99], with a smooth transition to MHD modes [Zonca and Chen PPCF06, NF07, NF09].
- There is a wide observation database of these phenomena, accumulated in the recent years after the first observations in DIIID [Nazikian et al PRL06]. Recent review is given by [Heidbrink POP08].

A "Sea of Core Localized Alfvén Eigenmodes" Observed in DIII-D Quiescent Double Barrier (QDB) plasmas

R. Nazikian, *et al.* 06, *PRL* **96**, 105006



- Bands of modes $m=n+l$, $l=1, 2, \ldots$ $\boxed{\omega_{n+1}-\omega_n \approx \omega_{rot}\ \text{(CER)}}$
- Neutral beam injection opposite to plasma current: $V_{||} \approx 0.3V_A$

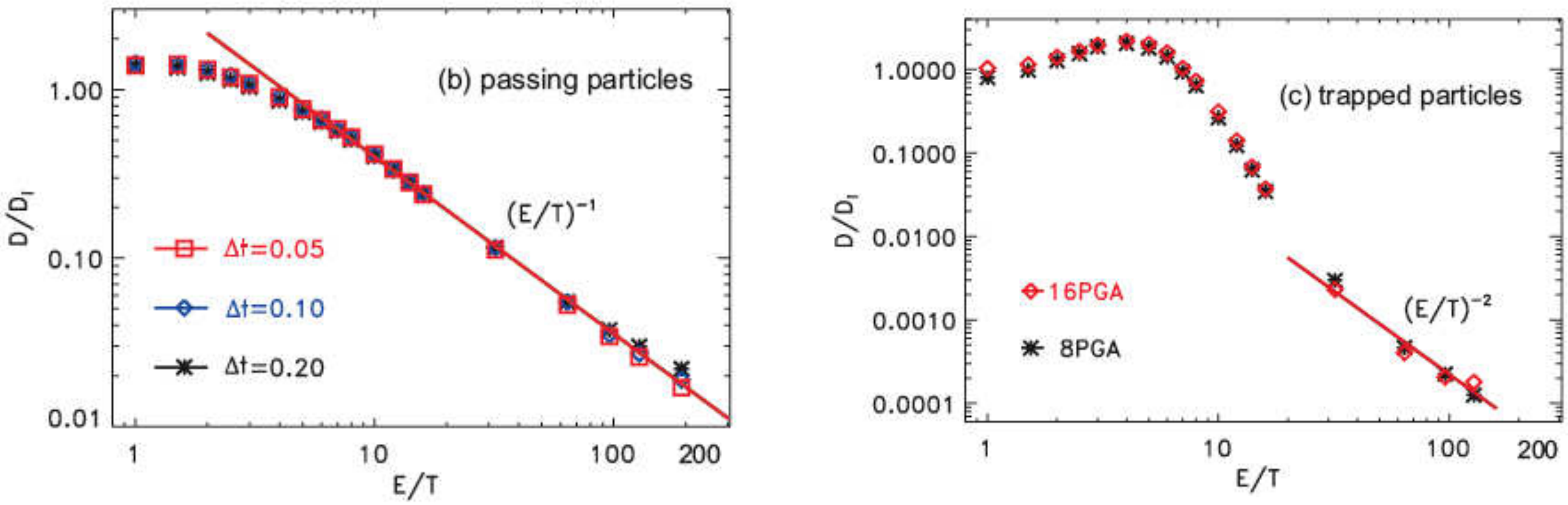
DIII-D NATIONAL FUSION FACILITY

R. Nazikian, *et al.* 06, *PRL* **96**, 105006

Fast particle transport by plasma turbulence

- Significant interest on this topic was triggered by recent AUG [Günter et al NF07], JT-60U [Suzuki et al NF08] and DIIID [Heidbrink et al PRL09] experimental results with NBI, showing evidence of anomalies in fast ion transport (clarified now: [Zhang et al POP10])

From Zhang et al POP **17** 055902 (2010)



- Diffusivity behaviors are consistent with theoretical predictions based on quasi-linear theory [Chen JGR99]
- Intrinsic interest is mostly connected with explanation of present day experiments, with low characteristic values of E/T; e.g., evidence of ITG induced transport of NBI supra-thermal ions in DIIID [Heidbrink et al PPCF09].
- Results show that fast ion transport by micro/turbulence above the critical energy is negligible (reactor relevant conditions). Effects are expected on He ashes or medium energy supra/thermal tails: possible good news?
- Fairly complete reconstruction of original theoretical works, e.g. [White and Mynick PFB89], theoretical issues, and experimental evidence in recent paper [Zhang et al POP10]. See also Heidbrink et al 2010 EPS invited talk [PPCF10].
- Recent detailed investigations [Albergante, PhD Thesis CRPP 2011; Albergante et al, POP09, NF10, PPCF11], with applications to present day experiments, ITER and DEMO (possible problems due to lower E/T_e).

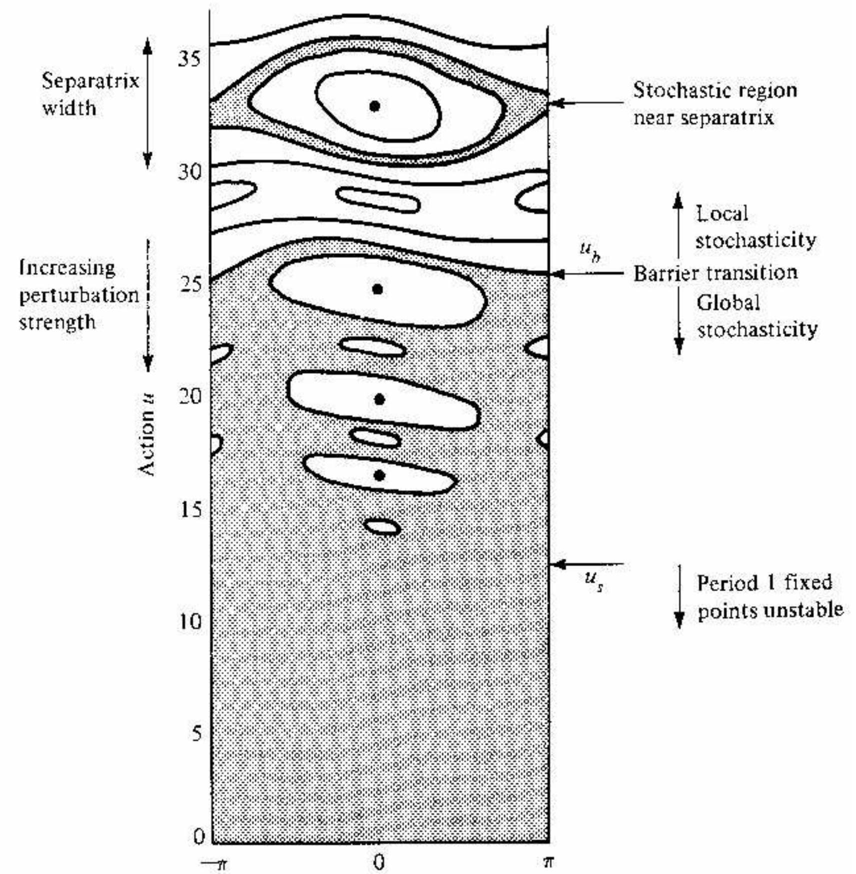
Fast ion transports in burning plasmas

- Alfvén Eigenmodes (AE) modes are predicted to have small saturation levels and yield negligible transport unless stochastization threshold in phase space is reached [Berk and Breizman, PFB90; Sigmar et al PFB92].
- Strong energetic particle redistributions are predicted to occur above the Energetic Particle Modes (EPM) excitation threshold in 3D Hybrid MHD-Gyrokinetic simulations [Briguglio et al POP98].
- Nonlinear Dynamics of Burning Plasmas: energetic ion transport in burning plasmas has two components:
  - slow diffusive processes due to weakly unstable AEs and a residual component possibly due to plasma turbulence [Vlad et al PPCF05, Estrada-Mila et al POP06].
  - rapid transport processes with ballistic nature due to coherent nonlinear interactions with EPM and/or low-frequency long-wavelength MHD: fast ion avalanches & experimental observation of Abrupt Large amplitude Events (ALE) on JT60-U [Shinohara et al PPCF04].

Phase space structures: fast ion resonant interactions with AE

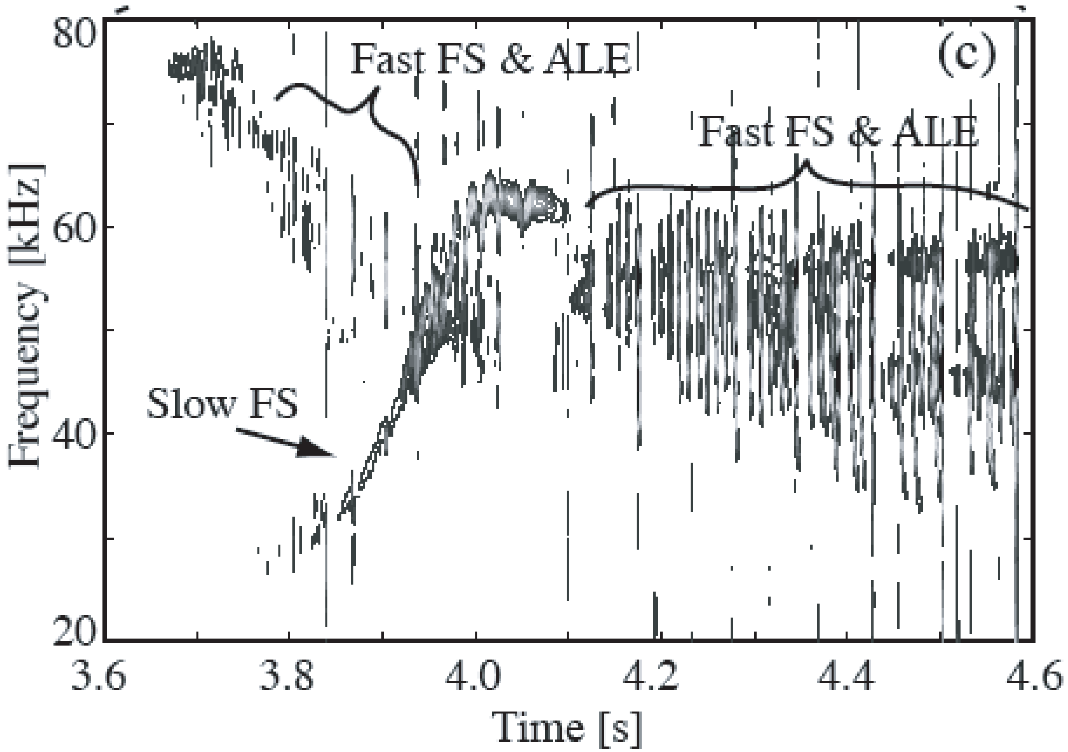
D.J. Sigmar, *et al.* 1992, *PFB* **4**, 1506 ; C.T. Hsu and D.J. Sigmar 1992, *PFB* **4**, 1492

- Transient losses $\approx \delta B_r/B$: resonant drift motion across the orbit-loss boundaries in phase space
- Diffusive losses $\approx (\delta B_r/B)^2$ above a stochastic threshold, due to stochastic diffusion in phase space across orbit-loss boundary
- Uncertainty in the stoch. threshold: $(\delta B_r/B) \lesssim 10^{-4}$ in the multiple mode case. Possibly reached via phase space explosion: "domino effect" [Berk et al POP96]



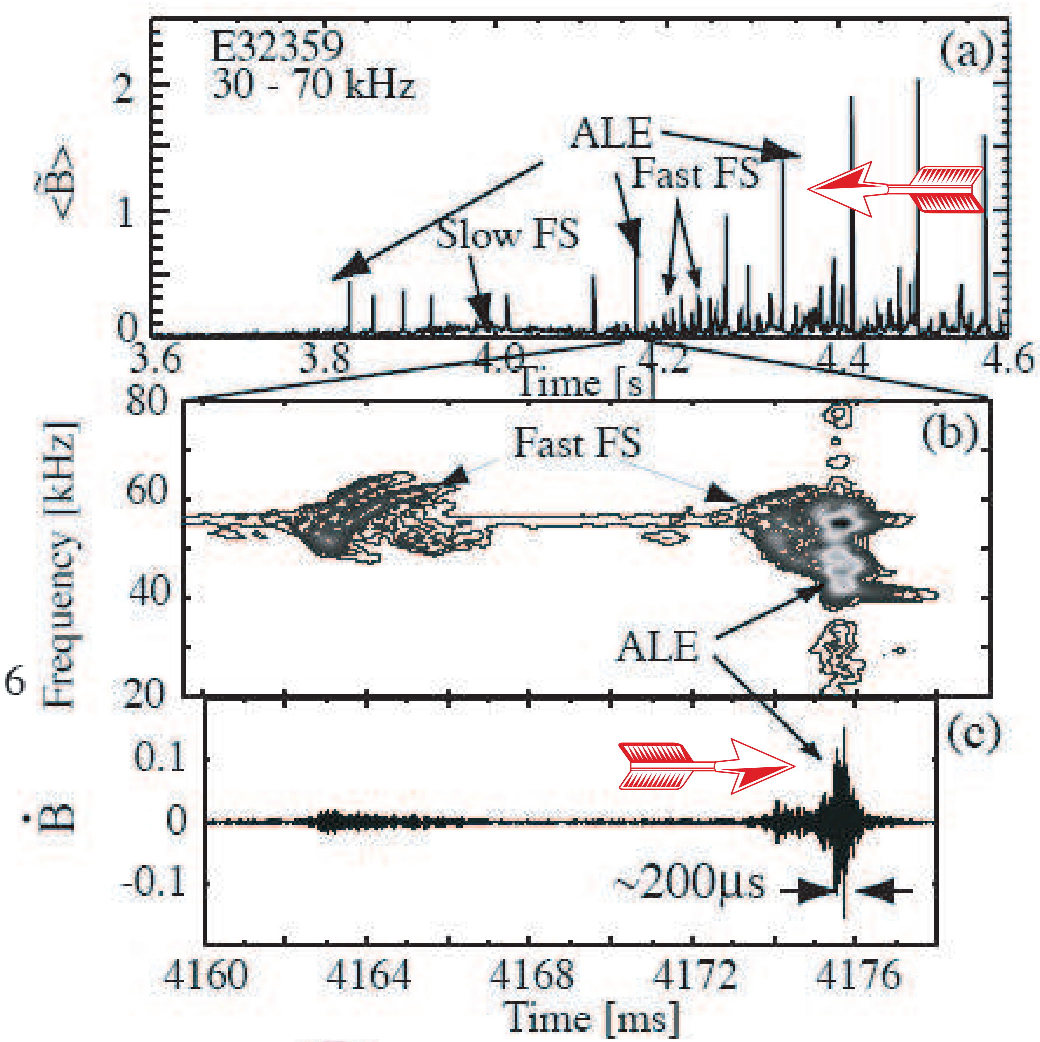
Lichtenberg & Lieberman 1983, Sp.-Ver. NY

ALE on JT-60U [Shinohara et al. NF01]



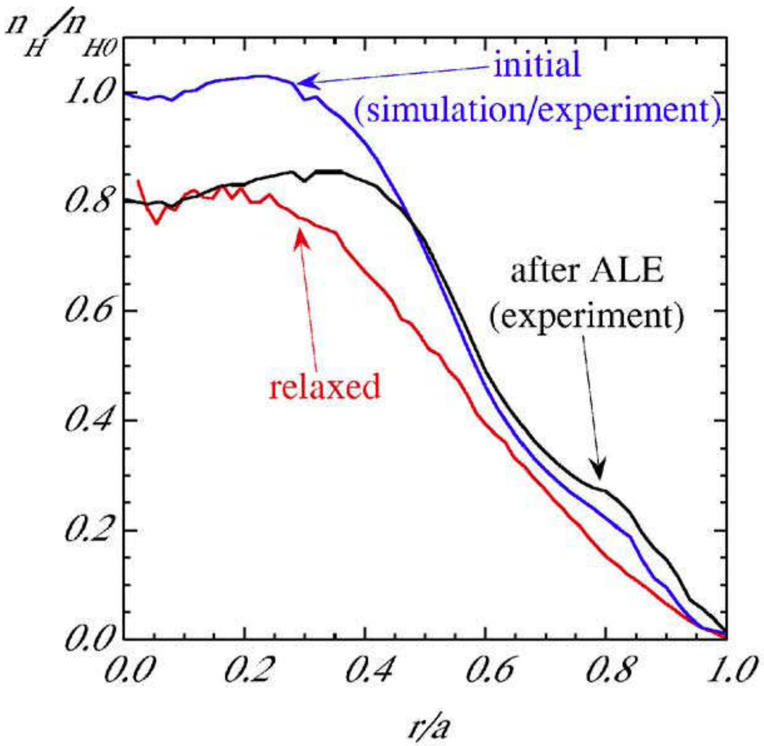
ALE = Abrupt Large amplitude Event

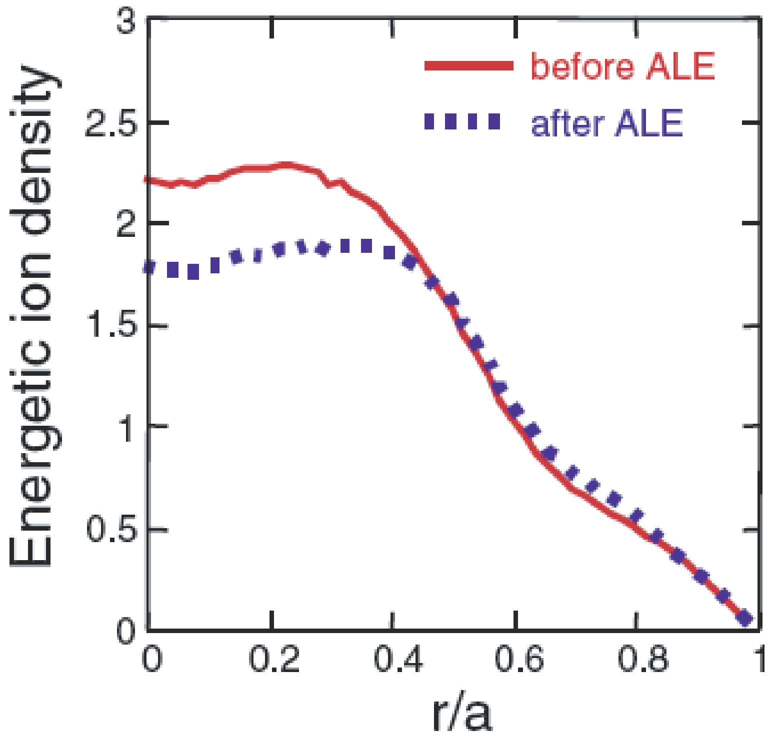
Courtesy of K. Shinohara and JT-60U



Fast ion transport: simulation and experiment

- □ Numerical simulations show fast ion radial redistributions, qualitatively similar to those by ALE on JT-60U.



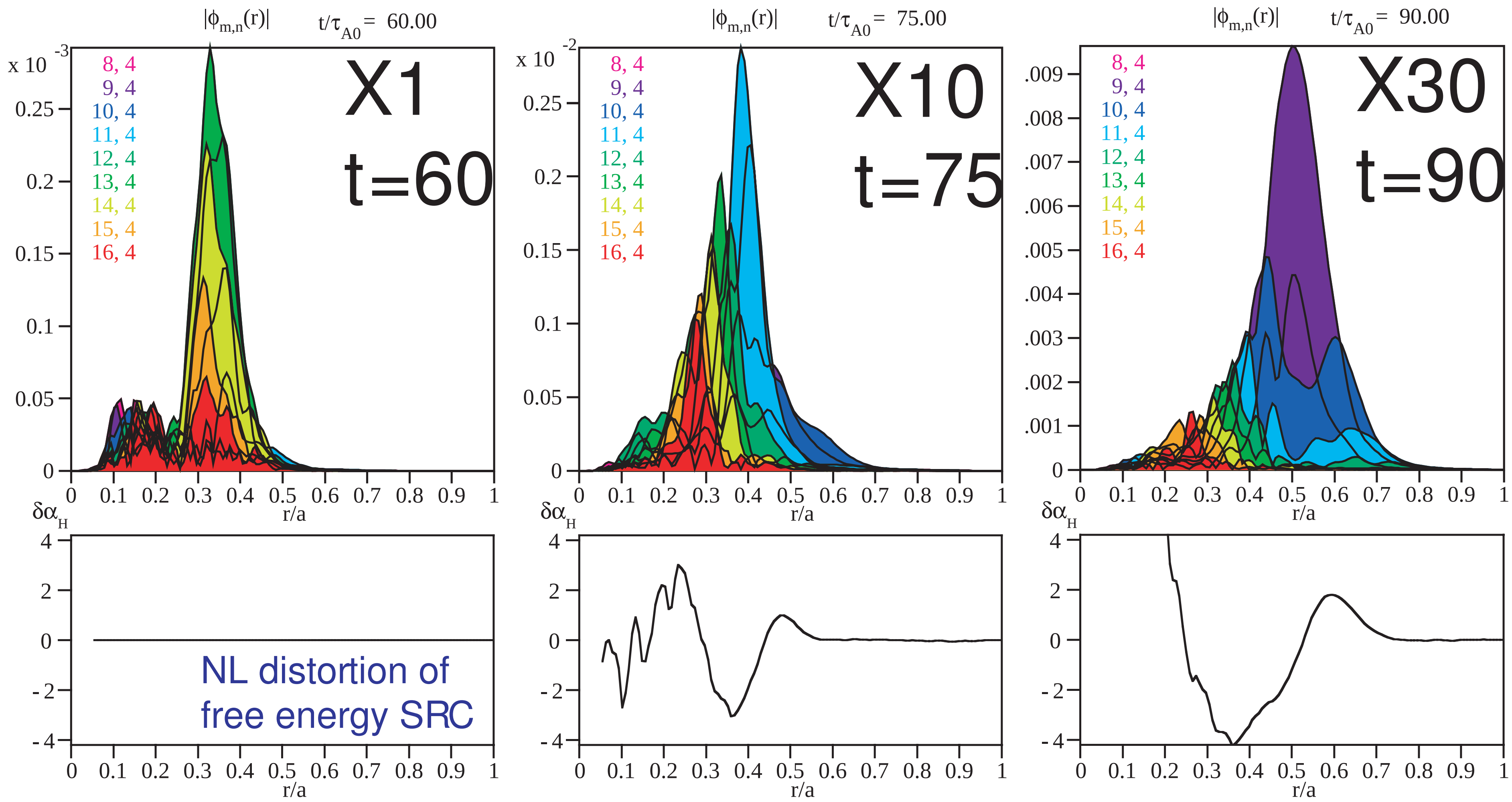


S. Briguglio et al POP 14, 055904, (2007)

K. Shinohara et al PPCF 46, S31 (2004)

Zonca et al. IAEA, (2002)

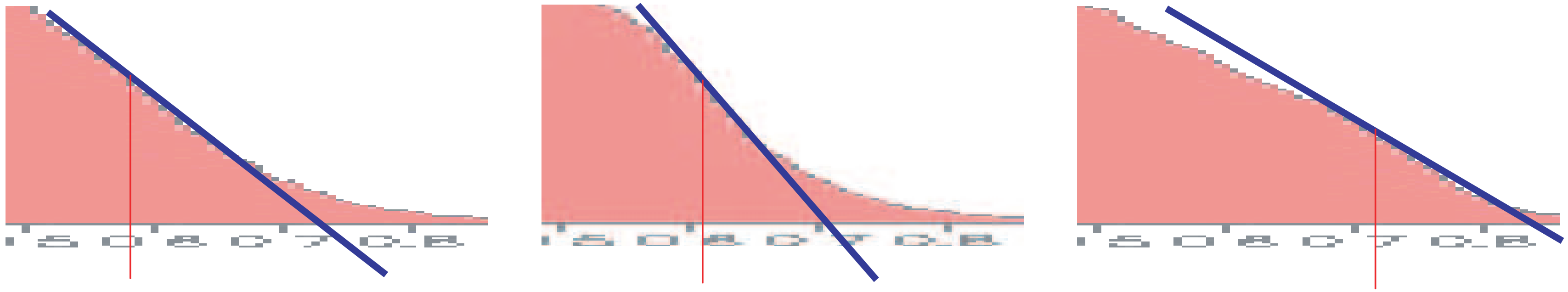
Avalanches and NL EPM dynamics

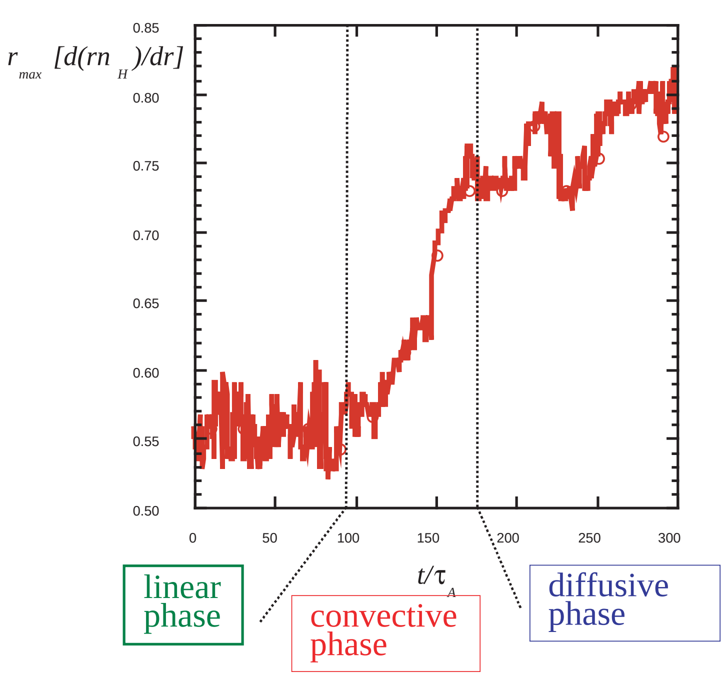


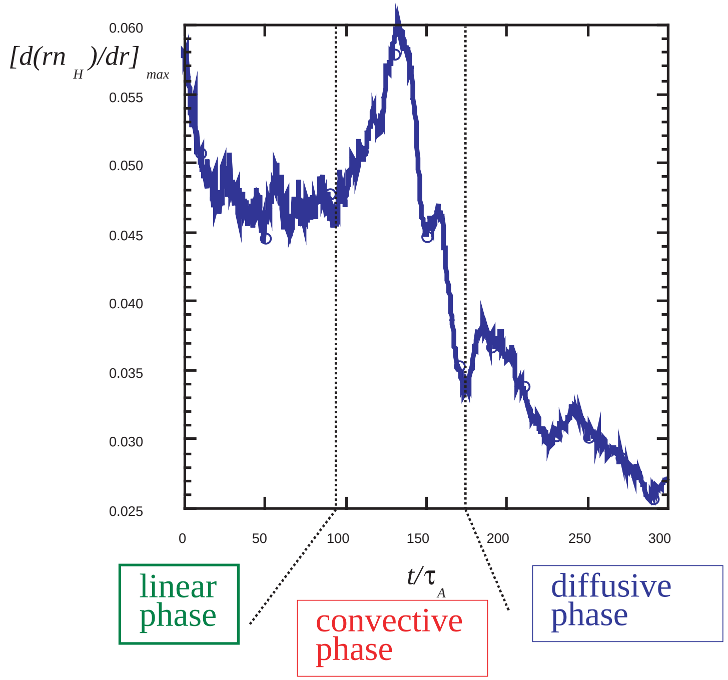
- Importance of toroidal geometry on wave-packet propagation and shape

Vlad et al. IAEA-TCM, (2003)

Propagation of the unstable front





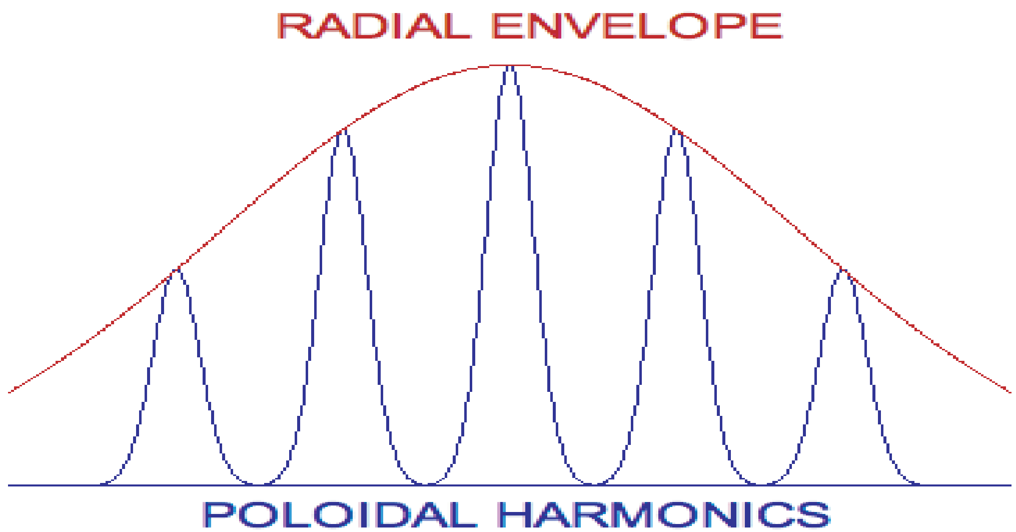


□ Gradient steepening and relaxation: spreading ... similar to turbulence

The form of nonlinear interactions in a torus

- Channels for nonlinear interactions can be described via the three degrees of freedom of mode structures: the toroidal mode number n, the parallel mode structure reflecting the radial width of a single poloidal harmonic m, and radial mode envelope $A(r) = \exp i \int \theta_k d(nq)$.

- The notion of radial envelope: both linear and nonlinear interactions are affected by peculiar mode structures in toroidal geometry and by equilibrium non-uniformities



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- Correspondingly, nonlinear interactions could occur via the following three channels: mode coupling between two n's, distortion of the parallel mode structure, and modulation of the radial envelope.
  - radial envelope modulations: ZF, GAM, zonal structures
  - mode coupling: spectral transfer (cascade) via 3-wave interactions

- Radial envelope modulation via generation of zonal flows dominates in ITG turbulence. Zonal structures (profile relaxation etc.) and/or GAM/ZF influence AE/EPM dynamics depending on the strength of mode drive.

- ETG turbulence is regulated by nonlinear toroidal mode couplings. What effect of nonlinear toroidal mode coupling on Alfvénic turbulence?

Three-wave interactions and spectral transfer

- This is still an open problem, largely unexplored:
  - Alfvénic fluctuations in toroidal plasmas of fusion interest cover all scales, from thermal ion Larmor radius up to characteristic energetic ion orbit widths (meso-scale AE/EPM) and macro-scales (MHD modes).
  - All scales involved: no unique approach to investigate spectral transfer processes (local vs non-local in $\mathbf{k}$, anisotropy, density of states).
  - In realistic conditions, these dynamics are influenced by non-uniformities and toroidal geometries: wave-particle power exchange $\langle \mathbf{v} \cdot \delta\mathbf{E} \rangle = \mathbf{v_d} \cdot \langle \delta\mathbf{E} \rangle$; gyro-averaged forces $\langle \mathbf{v} \times \delta\mathbf{B} \rangle = -i\mathbf{v_d} \cdot \mathbf{k} \left\langle \delta\mathbf{A}_\parallel \right\rangle$
- Geometry and non-uniformity influence three-wave interactions (spectral transfer) via density of quasi-modes and scattering cross-section [Chen et al PPCF05] $\Rightarrow$ Filament-like structures ($|k_\parallel|qR \ll 1 \approx \mathrm{O}(\mathrm{n}^{-1/2})$) are naturally generated in toroidal systems.

- Spectral transfer must be evaluated in competition with generation of ZF, GAM, zonal structures: e.g., phase-space holes and clumps if adiabatic local processes dominate [Berk et al PLA97, Breizman et al POP97, Berk et al POP99].

- More generally, ZF, GAM and phase space zonal structures (non-adiabatic) play dominant roles, where mode structures, toroidal geometries, equilibrium non-uniformities with respective evolutions and nonlinear dispersive properties are crucial. [Chen et al POP00, Zonca et al Varenna00 & NF05, Chen and Zonca NF07, Bass and Waltz POP10] (more shortly ...).

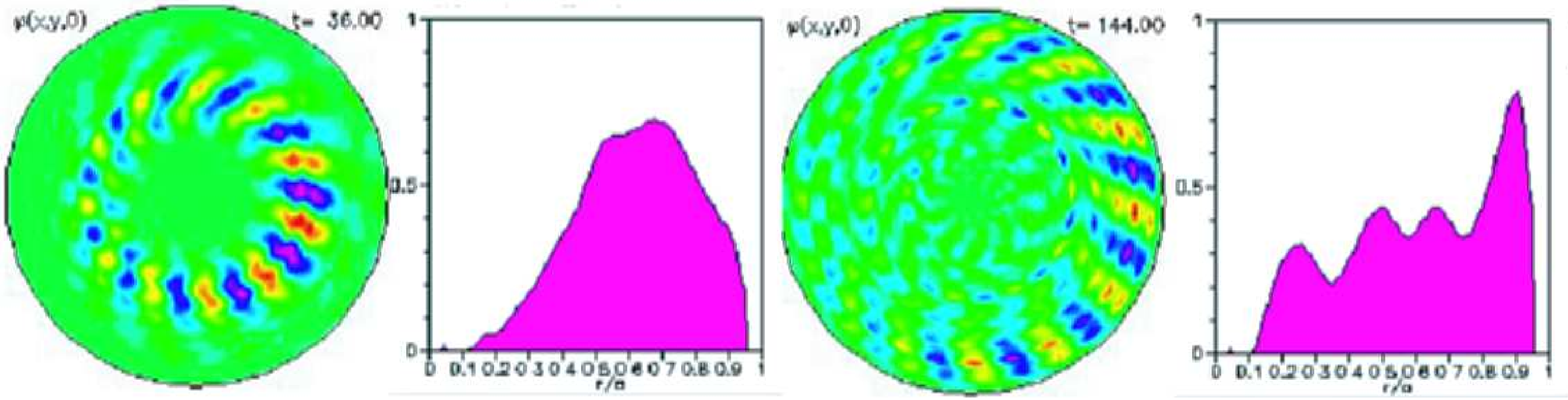
- Most relevant open issue: determine hierarchy of relevant non-linear time scales in realistic conditions, where cross-scale couplings are reproducing those expected to occur in plasmas of fusion interest.

Zonal Flows and Zonal Structures

- Very disparate space-time scales of AE/EPM, MHD modes and plasma turbulence: complex self-organized behaviors of burning plasmas will be likely dominated by their nonlinear interplay via zonal flows and fields [Chen and Zonca ASICTP08]:
  - Effects of different ZF and ZS (ω, k_r) spectra in cross/scale couplings
  - Peculiar roles of Alfvénic fluctuations in the acoustic frequency range: AITG, AE/EPM and MHD with similar frequencies
- Crucial role of toroidal geometry for Alfvénic fluctuations: fundamental importance of magnetic curvature couplings in both linear and nonlinear dynamics [Scott NJP05; Naulin et al POP05]
- Long time scale behaviors of zonal structures are important for the overall burning plasma performance: generators of NL (time varying) "equilibria"
- The corresponding stability determines the dynamics underlying the dissipation of zonal structures in collision-less plasmas and the nonlinear up-shift of thresholds for turbulent transport [Chen et al NF07].

Long time scale behaviors

- Depending on proximity to marginal stability, AE and EPM nonlinear evolutions can be predominantly affected by
  - spontaneous generation of zonal flows and fields [Chen et al NF01, Guzdar et al PRL01]
  - radial modulations in the fast ion profiles: for sufficiently strong fast ion drive, ZF effects are negligible – $(\beta_H/\beta_e)(R/L_{pH}) \gg \epsilon^{3/2}(T_H/q^2T_e)$; [Zonca et al Varenna00, Bass and Waltz POP10]



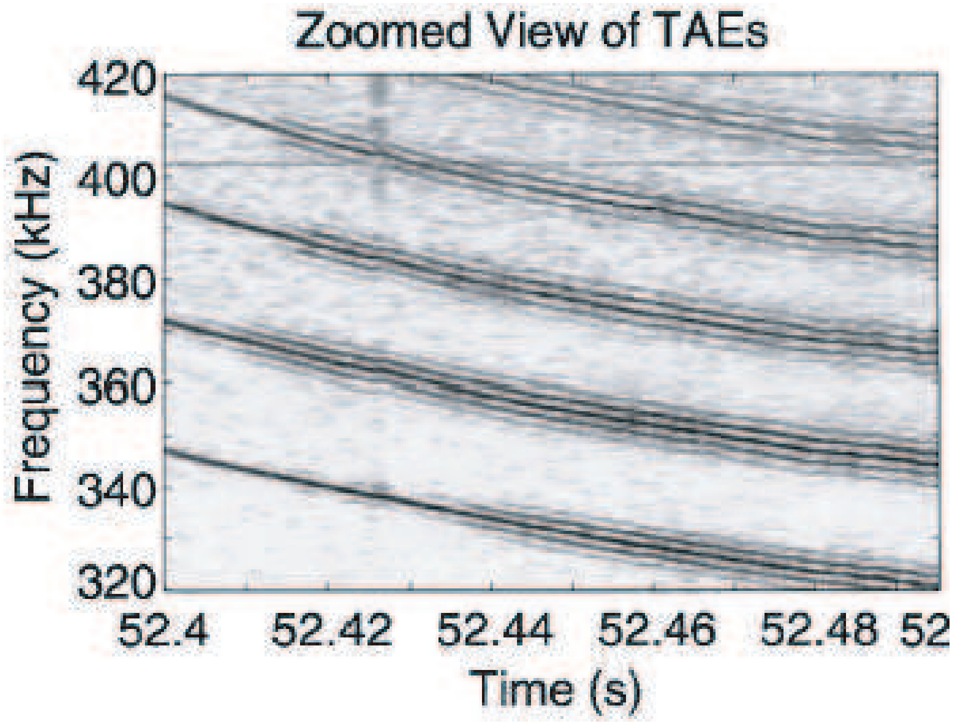
- AITG and strongly driven MHD modes behave similarly

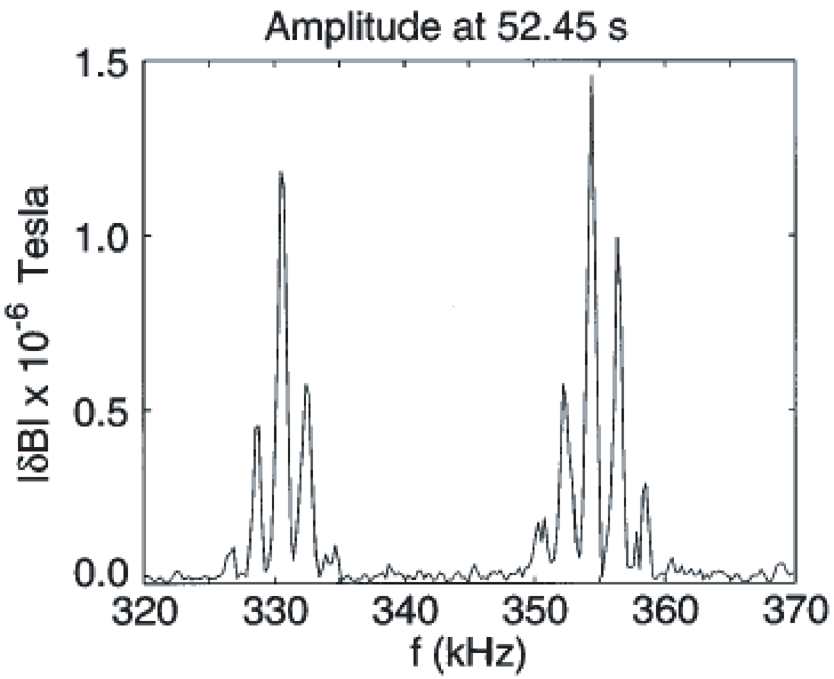
Nonlinear Dynamics: local vs. global processes

- Alfvénic fluctuation spectrum in burning plasma is dense and consists of mode with characteristic frequencies and radial locations [Chen NF07]. However, nonlinear dynamics has been investigated so far mostly for one single mode.
- Mode saturation via wave-particle trapping [Berk et al PFB90, PLA97] has been successfully applied to explain pitchfork splitting of TAE spectral lines [Fasoli et al PRL98]: local distortion of the fast ion distribution function because of quasi-linear wave-particle interactions.
- Negligible transport by AE is expected because of these processes, unless stochastization threshold is reached in phase space possibly via "domino effect" [Berk et al POP96].

Pitchfork splitting of TAE in JET

Fasoli, Phys. Rev. Lett. **81**, 5564, (1998)





High resolution MHD spectroscopy: Pinches et al, PPCF **46**, S47, (2004)

- Such analyses generally assume proximity to marginal stability:
  - One single low amplitude wave, such that linear mode structures can be assumed and drop out of the problem: radial uniform problem
  - Finite background dissipation does not depend on the finite amplitude wave (no continuum damping)
  - Wave dispersiveness is fixed and due to the background plasma (no beam mode/EPM)
  - Frequency sweeping is absent or adiabatic (no mode-particle-pumping [White et al PF83], crucial for explaining, e.g., the nonlinear fishbone and EPM cycles.)

- These properties delineate an analogy between single Alfvén Eigenmode behaviors in burning plasmas with those of Langmuir waves (1D beam-plasma problem) [Lesur PhD Thesis 2010]. However they show a strong contrast between dynamics of beam-mode in 1D systems and Energetic Particle Modes in toroidal devices [Zonca IFTS Lecture Notes Spring 2010].

- Next: focus on the transition from weak to strong NL dynamic regime, where intrinsic EPM resonant nature plays a crucial role, with plasma non-uniformity and toroidal geometry. Control parameter is power density input: proximity to marginal stability does not hold.

Nonlinear initial value problem for EPM

[Chen et al POP00 & NF01 & PRL04, Zonca et al Varenna00 & NF05]

- It is possible to systematically generate standard NL equations in the form (expand wave-packet propagation about envelope ray trajectories):

$$\left\{ \underbrace{\omega^{-1}\partial_t - \frac{\gamma}{\omega}}^{\text{drive/damping}} - \underbrace{\frac{\xi}{nq'\theta_k}\partial_r}_{\text{group vel.}} + \overbrace{i(\lambda+\xi)}^{\text{potential well}} + \underbrace{i\frac{\lambda}{(nq'\theta_k)^2}\partial_r^2}_{\text{(de)focusing}} \right\} A(r,t) \;\; = \;\; \underbrace{\text{NL TERMS}}_{\text{renormalized } F_0(\hat{F}_0(\omega))}$$

- θ_k solution of $D_R(r,\omega,\theta_k) = 0$ and

$$\lambda = \left(\frac{\theta_k^2}{2}\right)\frac{\partial^2 D_R/\partial\theta_k^2}{\omega\partial D_R/\partial\omega}\,; \quad \xi = \frac{\theta_k(\partial D_R/\partial\theta_k) - \theta_k^2(\partial^2 D_R/\partial\theta_k^2)}{\omega\partial D_R/\partial\omega}\,; \quad \gamma = \frac{-D_I}{\partial D_R/\partial\omega}$$

- Equations admit the well-known local limit, which is readily recovered.

The renormalized $\hat{F}_0$ expression

- The F_0 expression is obtained from the solution of the nonlinear GKE [Frieman and Chen PF82] with a source term S (collisions are added trivially; note that only trapped particles are considered for simplicity, $\bar{v}_\parallel = 0$).

$$\frac{\partial F_0}{\partial t} = S - \frac{c}{B_0}\frac{i}{r}\frac{\partial}{\partial r}\left(\delta K_{-k} J_{0k} \delta\phi_k - \delta K_k J_{0-k} \delta\phi_{-k}\right)$$

- Decompose fluctuating particle responses into adiabatic and non-adiabatic

$$\delta F_k = \frac{e}{m}\delta\phi_k \frac{\partial}{\partial v^2/2} F_0 + \sum_{\mathbf{k}_\perp} \exp\left(-\mathrm{i}\mathbf{k}_\perp \cdot \mathbf{v} \times \mathbf{b}/\omega_c\right) \overline{\delta H}_k$$

$$\overline{\delta H}_k = \delta K_k - \frac{e}{m}\frac{QF_0}{\omega_k} J_{0k} \delta\psi_k \qquad \delta A_{\parallel k} \equiv -\mathrm{i}\left(\frac{c}{\omega}\right)\mathbf{b}\cdot\nabla\delta\psi_k$$

- Definition: $QF_0 = (\omega\partial_{\mathcal{E}} + \hat{\omega}_*)F_0$, $\hat{\omega}_* F_0 = (mc/eB)(\mathbf{k}\times\mathbf{b})\cdot\nabla\mathbf{F_0}$.

- Solve the problem in the Laplace transform space;
  $F_0(t) = \int_{-\infty}^{\infty} e^{-i\omega t}\hat{F}_0(\omega)d\omega,\ \hat{F}_0(\omega) = (2\pi)^{-1}\int_0^{\infty} e^{i\omega t}F_0(t)dt.$

$$\delta\hat{K}_k(\omega) = \int_{-\infty}^{\infty} \frac{e}{m}\frac{\bar{\omega}_{dk}}{\bar{\omega}_{dk}-\omega}\frac{Q_{k,x}}{x}\hat{F}_0(\omega-x)J_{0k}J_{0dk}^2\delta\phi_k(x)dx$$

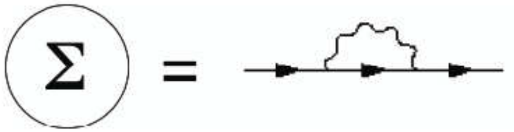
$$\hat{F}_0(\omega) = \frac{i}{\omega}\hat{S}(\omega)+\frac{i}{2\pi\omega}F_0(0)-\frac{ck_\theta}{\omega B_0}\frac{\partial}{\partial r}\int_{-\infty}^{\infty}[J_{0k}\delta\phi_k(x)\delta K_{-k}(\omega-x)-J_{0-k}\delta\phi_{-k}(x)\delta K_k(\omega-x)]dx$$

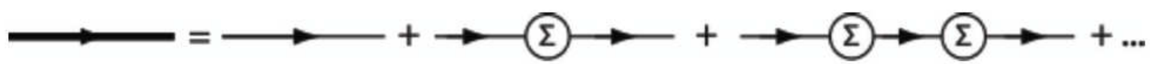
- Assuming an implicit summation on k

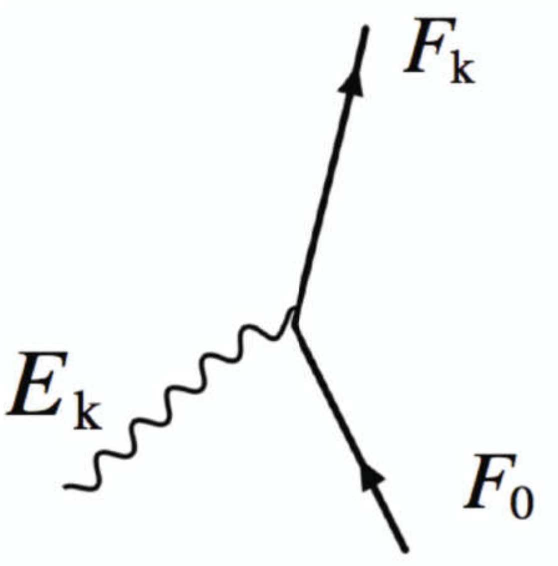
$$\hat{F}_0(\omega) = \frac{i}{\omega}\hat{S}(\omega)+\frac{i}{2\pi\omega}F_0(0)-\frac{ck_\theta}{\omega B_0}\frac{e}{m}\frac{\partial}{\partial r}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}J_{0k}^2J_{0dk}^2\left[\delta\phi_k(x)\frac{\bar{\omega}_{d-k}}{\bar{\omega}_{d-k}+x-\omega}\frac{Q_{-k,x'}}{x'}\right.$$

$$\left.\times\ \hat{F}_0(\omega-x-x')\delta\phi_{-k}(x')-\delta\phi_{-k}(x)\frac{\bar{\omega}_{dk}}{\bar{\omega}_{dk}+x-\omega}\frac{Q_{k,x'}}{x'}\hat{F}_0(\omega-x-x')\delta\phi_k(x')\right]dxdx'$$

- This solution is valid for strong distortions of the equilibrium distribution function and is the analogue of the Dyson equation ($G = G_0 + G_0 \Sigma G$, with G_0/G the bare/dressed propagators), as noted, e.g., by [Al'tshul' and Karpman 65].







- The description does not include power spectrum generation and spatial bunching and accurate treatment of phase mixing on scales much longer than the wave-particle trapping time. Not a limitation for EPM nonlinear dynamics $\Rightarrow$ convective amplification of unstable front.

- Monochromatic EPM: reasonable assumption, since EPM spectrum is peaked for optimizing resonance condition. $\delta\phi(t) = \delta\bar{\phi}(\tau)\exp(-i\omega(\tau)t)$, $|\dot{\omega}(t)| \ll |\gamma(t)\omega(t)|$.

$$\delta\hat{\phi}_k(\omega) = \frac{i\delta\bar{\phi}_k(\tau)}{2\pi(\omega-\omega(\tau))} \qquad \delta\hat{\phi}_{-k}(\omega) = \frac{i\delta\bar{\phi}_{-k}(\tau)}{2\pi(\omega+\omega^*(\tau))}$$

$$\hat{F}_0(\omega) = \frac{i}{\omega}\hat{S}(\omega) + \frac{i}{\omega}\mathrm{St}\hat{F}_0(\omega) + \frac{i}{2\pi\omega}F_0(0) - \frac{ck_\theta}{\omega B_0}\frac{e}{m}\frac{\partial}{\partial r}J_{0k}^2 J_{0dk}^2\left[\frac{\bar{\omega}_{dk}}{\bar{\omega}_{dk}+\omega-\omega(\tau)}\frac{Q^*_{k,\omega(\tau)}}{\omega(\tau)^*}\right.$$
$$\left.-\frac{\bar{\omega}_{dk}}{\omega+\omega^*(\tau)-\bar{\omega}_{dk}}\frac{Q_{k,\omega(\tau)}}{\omega(\tau)}\right]\hat{F}_0(\omega-2i\gamma(\tau))|\delta\bar{\phi}_k(\tau)|^2$$

- For evanescent drive and flat envelope, this problem is reduced to the wave-particle trapping and to [Berk et al PLA97, POP99] for $\omega_B t \ll 1$.

- For increasing drive, EPM mode structures play a crucial role: particles are convected out efficiently since - with frequency locking - wave-particle resonances are de-correlated only in velocity space. $(nqs)^{-1} \lesssim (\gamma/\omega)(\omega_*/\omega) \lesssim (L_{pH}/nqr)^{1/2}$.

The EPM avalanche and convective EP transport

- Assume isotropic slowing-down and EPM NL dynamics dominated by precession resonance, $(\beta_H/\beta_e)(R/L_{pH}) \gg \epsilon^{3/2}(T_H/q^2T_e)$ [Zonca et al NF05].

$$[D_R(\omega,\theta_k;s,\alpha)+iD_I(\omega,\theta_k;s,\alpha)]\,\partial_t A = \frac{3\pi\epsilon^{1/2}}{4\sqrt{2}}\alpha_H \bar{J}^2\left[1+\frac{\omega}{\bar{\omega}_{dF}}\ln\left(\frac{\bar{\omega}_{dF}}{\omega}-1\right)\right.$$

$$\left.+i\pi\frac{\omega}{\bar{\omega}_{dF}}\right]\partial_t A + \underbrace{i\pi\frac{\omega}{\bar{\omega}_{dF}}\bar{J}^2 A\frac{3\pi\epsilon^{1/2}}{4\sqrt{2}}k_\theta^2\rho_H^2\frac{T_H}{m_H}\partial_r^2\partial_t^{-1}\left(\alpha_H \bar{J}^2|A|^2\right)} \ .$$

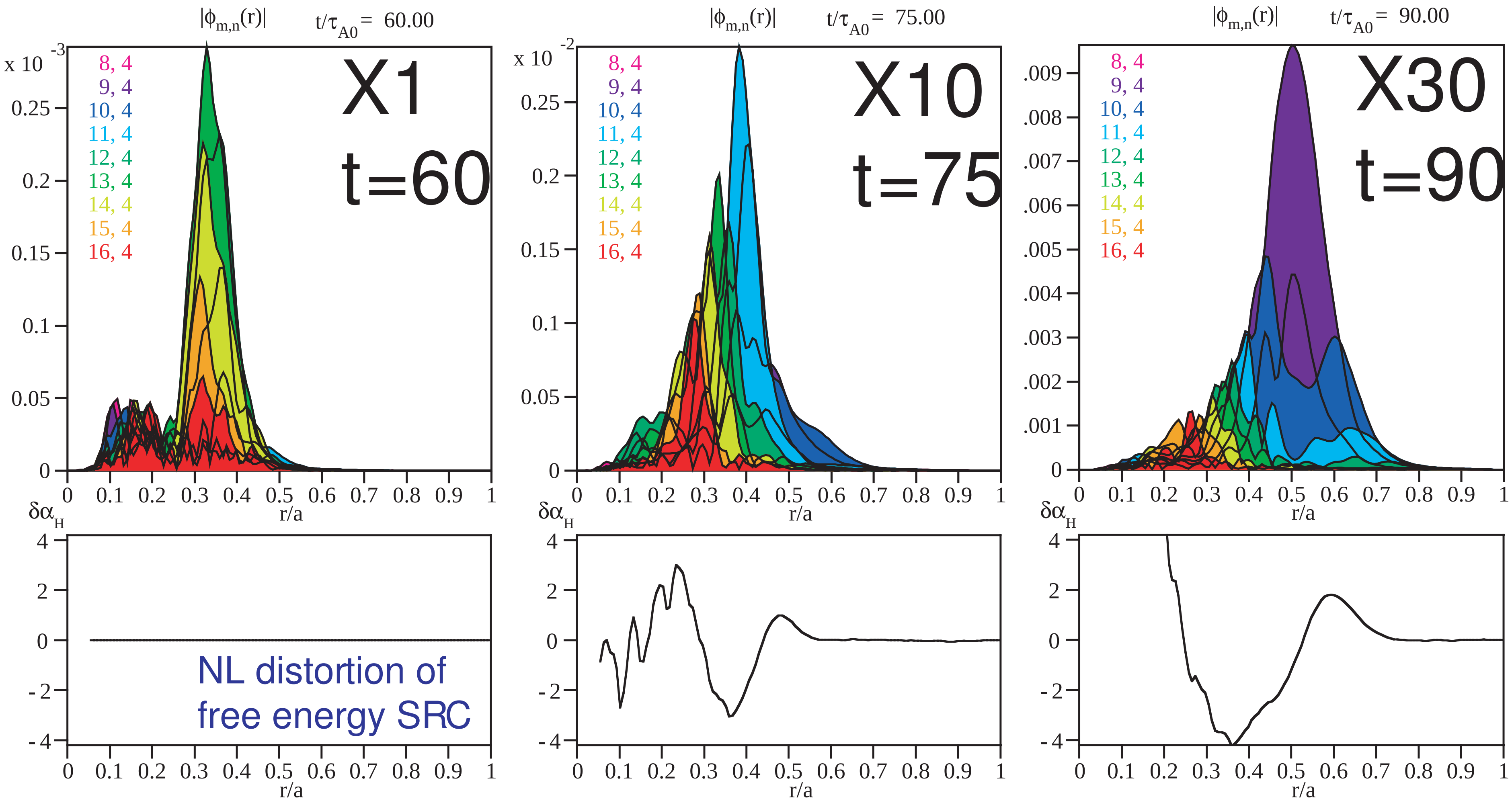
DECREASES DRIVE@ MAX $|A|$

INCREASES DRIVE NEARBY

- Assume $\alpha_H = \alpha_{H0}\exp(-x^2/L_p^2) \simeq \alpha_{H0}(1-x^2/L_p^2)$, with $x=(r-r_0)$. EPM structure has characteristic linear radial envelope width $\Delta \approx (L_p/k_\theta)^{1/2}$ and is nonlinearly displaced to maximize the drive as

$$(x_0/L_p) = \gamma_L^{-1}k_\theta\rho_H\left(T_H/M_H\right)^{1/2}\left(|A|/W_0\right) \ , \qquad W_0 = \text{NL EPMwidth}$$

Zonca et al. IAEA, (2002)



- Importance of toroidal geometry on wave-packet propagation and shape

Crucial points and Summary

- TAE modes are the prototype of all Alfvénic eigenmodes that may collectively excite fluctuations and have consequences on supra-thermal particle transports in burning plasmas.
- Linear dispersive properties of TAE and EPM are well understood and can be described in terms of a single fishbone like dispersion relation.
- Fluctuation induced fast particle transport in toroidal plasmas of fusion interest involves both micro- and meso-scales phenomena, as well as macro-scale MHD.
- Micro-turbulence induced transport of energetic particles is well described by quasi-linear theories, while meso-scale fluctuations exhibit both coherent and incoherent non-linear behaviors, corresponding to convective and diffusive transport events.

- Given a dense spectrum of Alfvénic fluctuations, spectral transfers (cascade) are strongly affected by non-uniformities and toroidal geometries. Meanwhile, three-wave couplings are in competition with zonal structure formations: determining an hierarchy of nonlinear time scales in realistic conditions remains an open problem.

- There is a relationship of MHD and shear Alfvén waves in the kinetic thermal ion frequency gap with micro-turbulence, Zonal Flows and Geodesic Acoustic Modes, which has importance in determining long time scale dynamic behaviors in burning plasmas.

- When drive is sufficiently strong, coherent nonlinear wave-particle interactions are the dominant processes, which determine energetic particle transport as avalanche phenomena, characterized by convective amplification of the unstable front and gradient steepening and relaxation, where plasma non-uniformities, mode structures and toroidal geometries are crucial elements.

Grazie – 謝謝您