

Autumn College on Non-Equilibrium Quantum Systems  
May 2-13, 2011  
Buenos Aires, Argentina

Norman Birge\*, Michigan State University

## Lecture II: Nonequilibrium Experiments in S/N Hybrid Systems (Distribution function engineering)

\* Work supported by NSF DMR

# Collaborators

*Michigan State University*



J. Huang



F. Pierre



M. Crosser

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T. Heikkila



P. Virtanen

*Waterloo*



F. Wilhelm

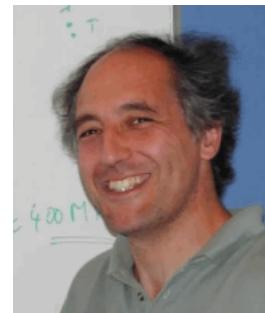
I started working  
in this field with:



S. Gueron



H. Pothier



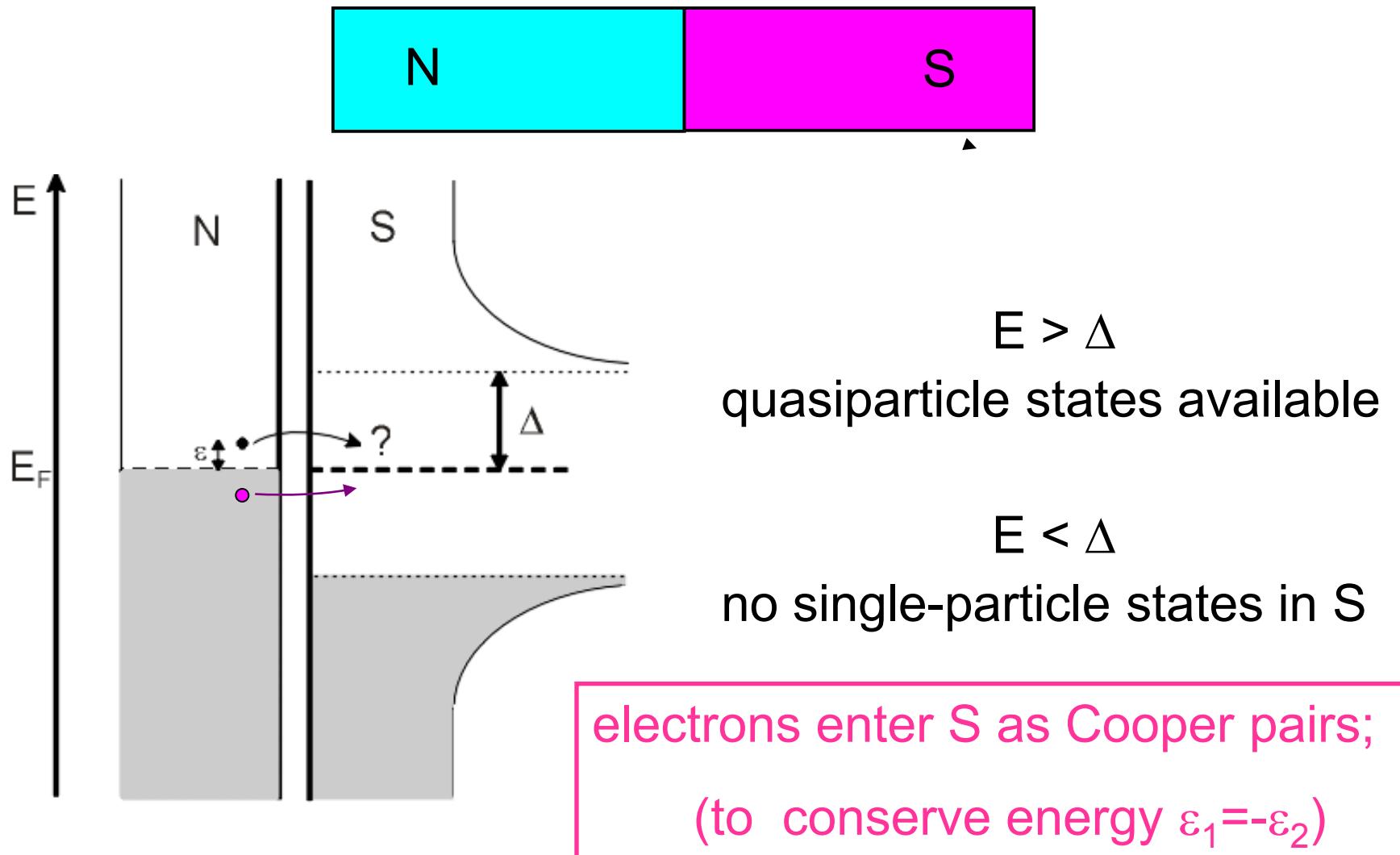
D. Esteve



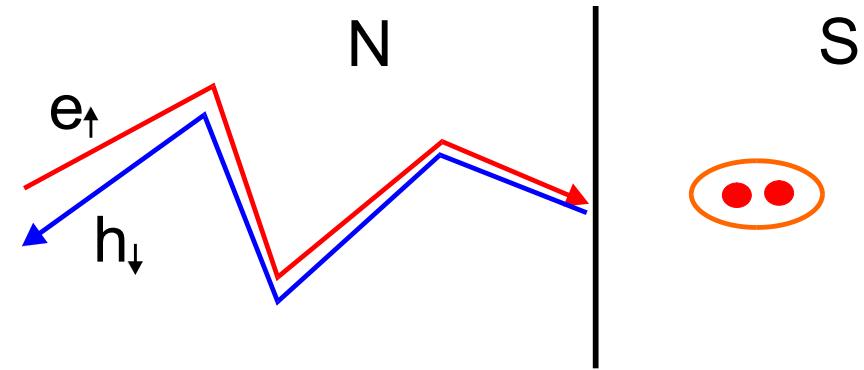
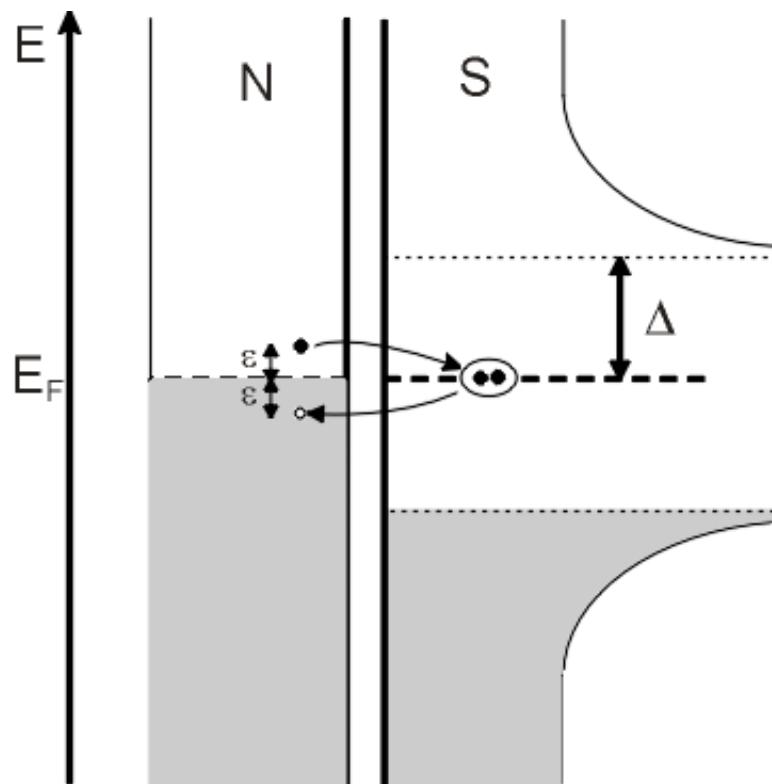
M. Devoret

# Introduction to Andreev Reflection

## How do electrons pass from N to S?



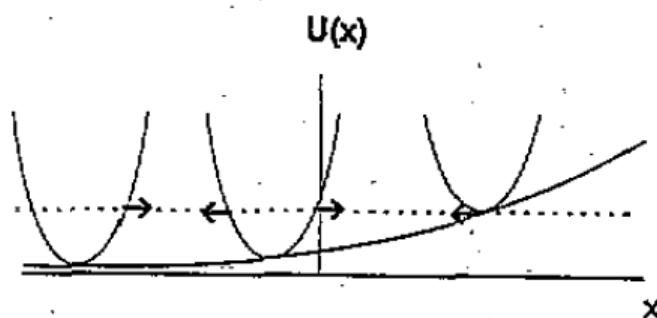
# Alternative view: Andreev reflection: hole is retro-reflected from NS interface



At  $E=0$ , reflected hole follows time-reversed path of incident electron

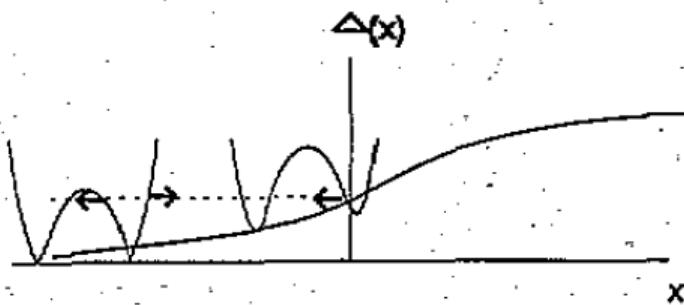
# Andreev reflection in k-space:

(a)



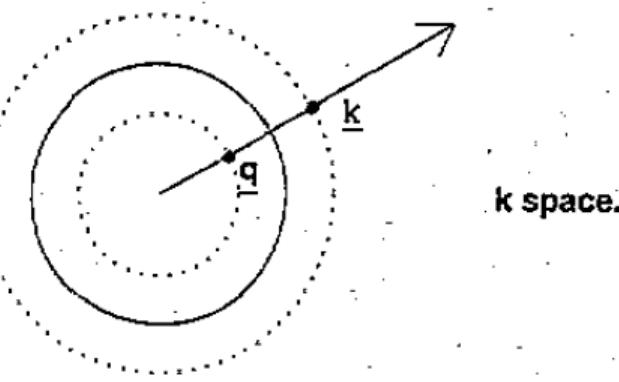
Semiclassical view of electron scattering from potential barrier

(b)



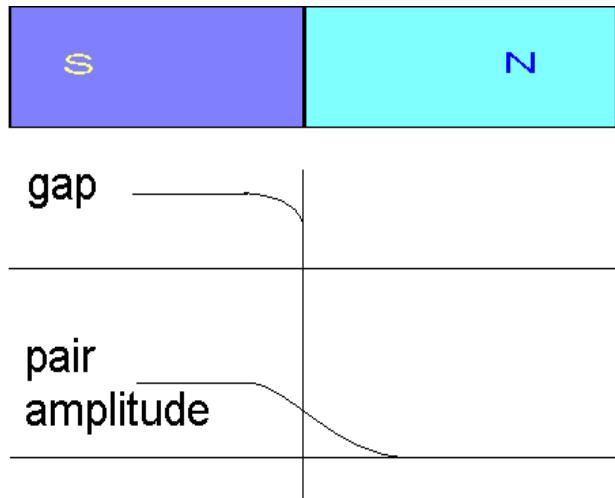
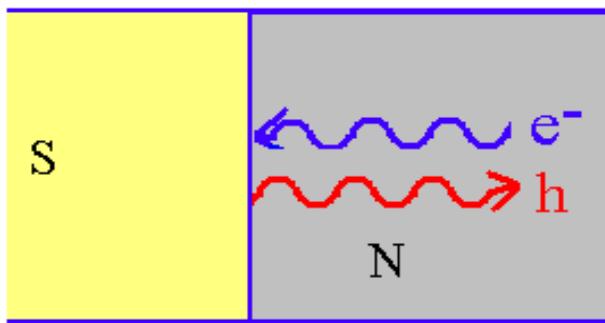
Semiclassical view of electron undergoing Andreev reflection at a normal/superconductor interface.  
(Dispersion curve drawn in the excitation picture.)

(c)



From Lambert, Hui, and Robinson, J. Phys. Condens. Matt. **5**, 4187 (1993).

# Andreev Reflection and Proximity Effect



Electron and reflected hole stay in phase for time:

$$\tau_\varepsilon = \frac{\hbar}{2\varepsilon}$$

Length scale:

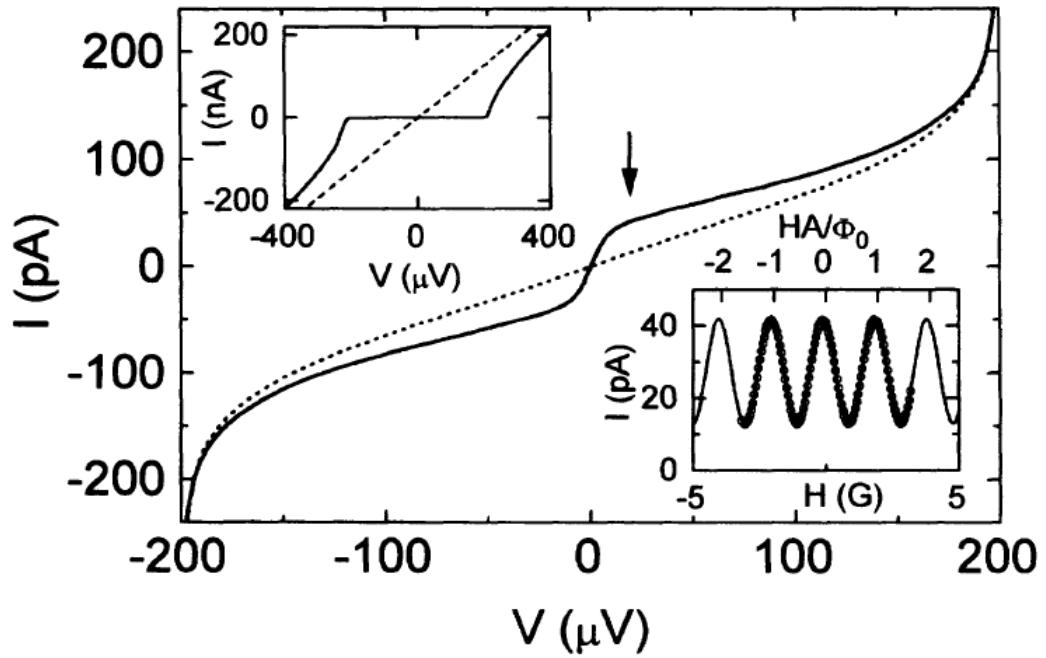
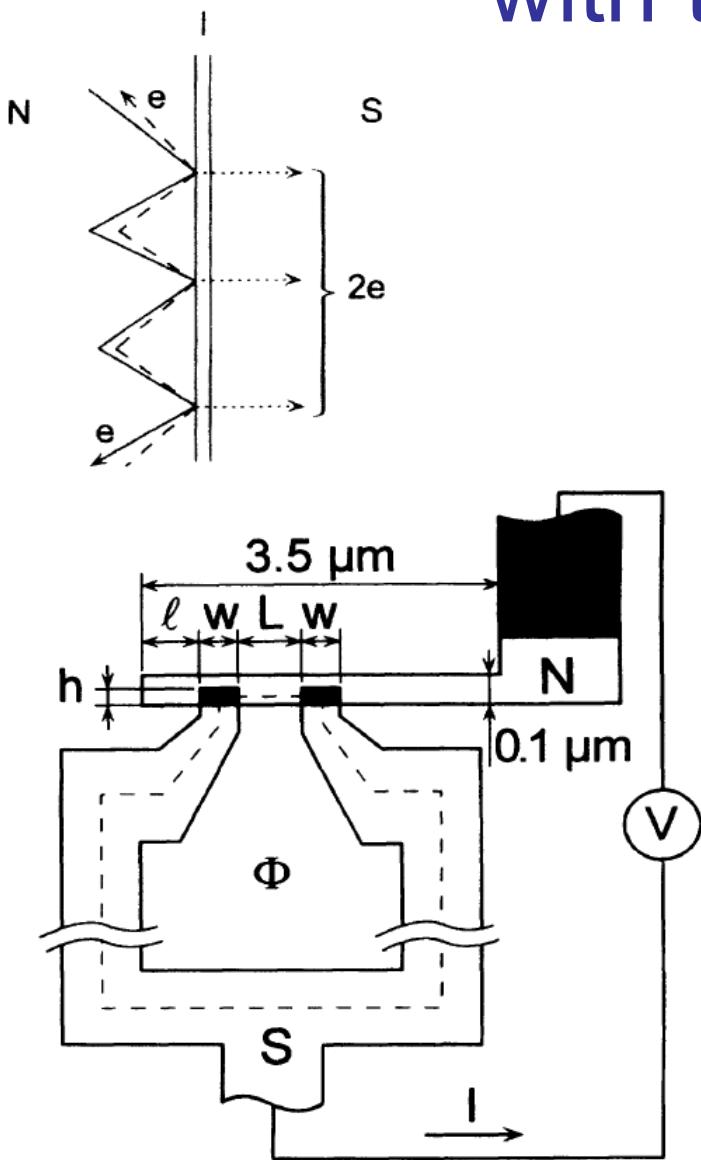
$$\begin{cases} L_\varepsilon = \frac{\hbar v_F}{\varepsilon} & \text{ballistic} \\ L_\varepsilon = \sqrt{\frac{\hbar D}{\varepsilon}} & \text{diffusive} \end{cases}$$

Finite T:  $\Psi \propto e^{-x/\xi_N}$ ,  $\xi_N = \sqrt{\frac{\hbar D}{2\pi k_B T}}$

(assumes  $\xi_N < L_\phi$ )

$\xi_N$  is “normal metal coherence length”

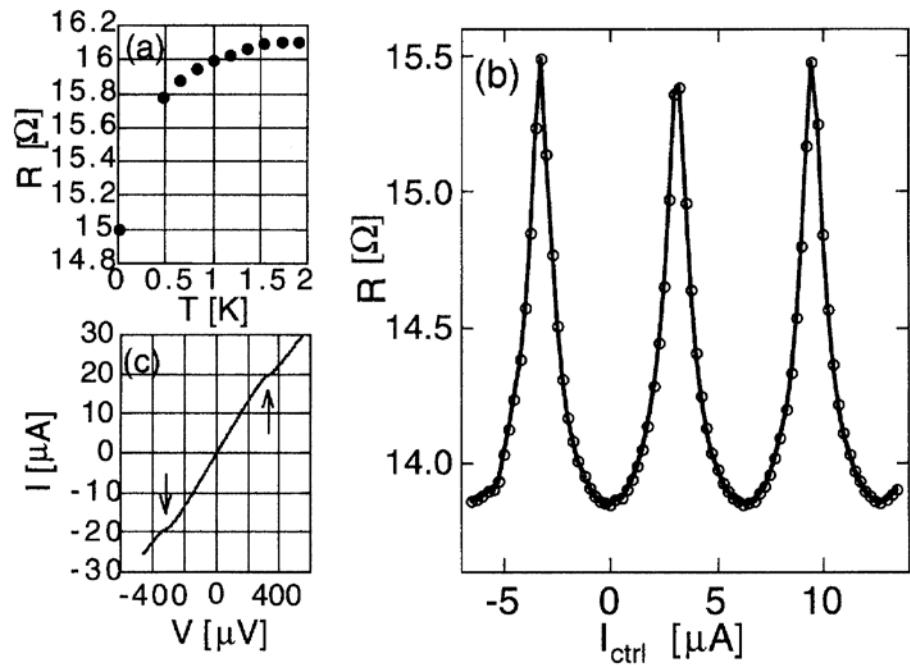
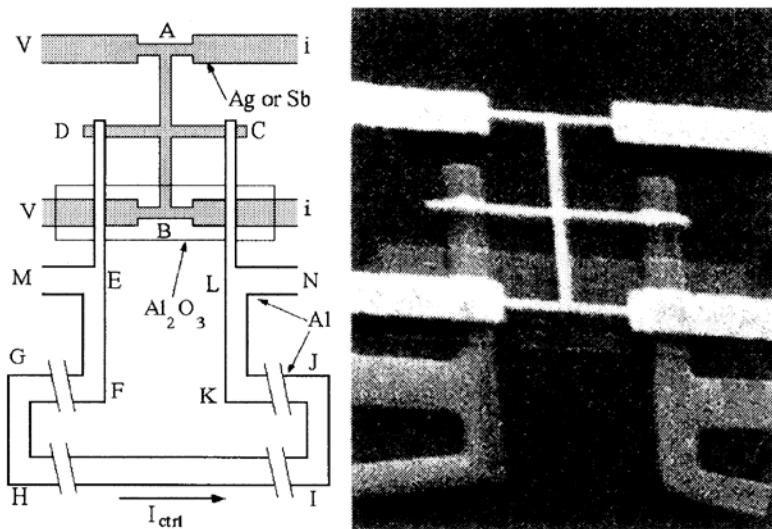
# Andreev Reflection Experiment with tunnel contacts



Pothier, Gueron, Esteve, & Devoret,  
PRL 73, 2488 (1994).

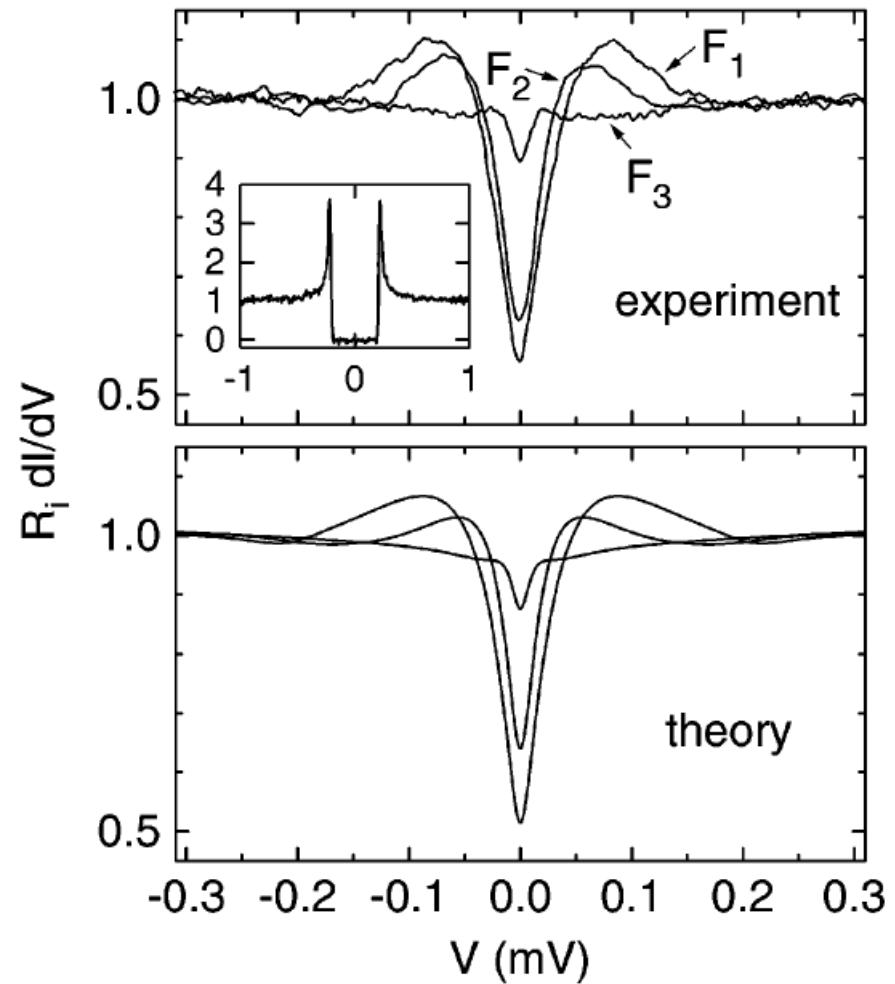
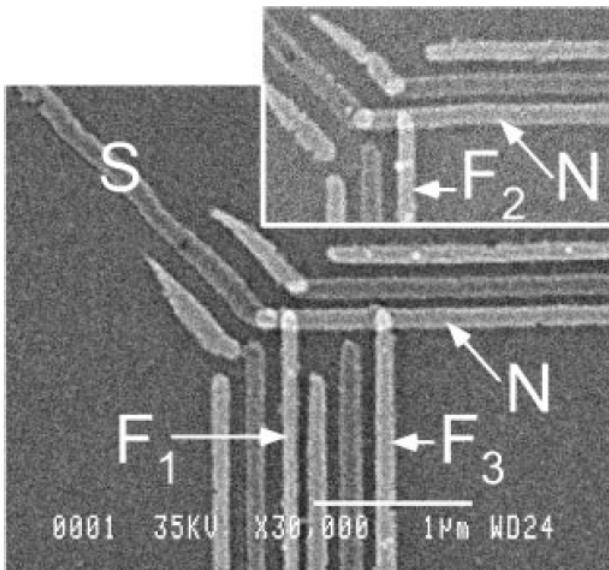
Theory by Hekking & Nazarov

# Proximity Effect Experiment with transparent contacts



Petrashov, Antonov, Delsing, & Claeson,  
PRL 74, 5268 (1995).

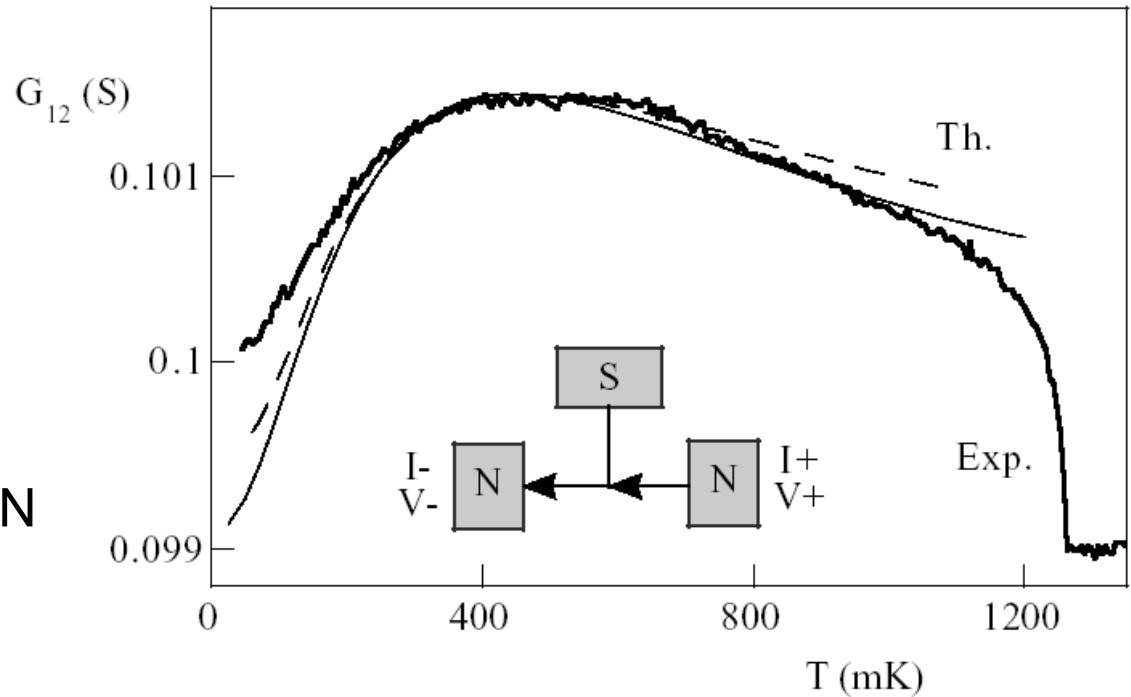
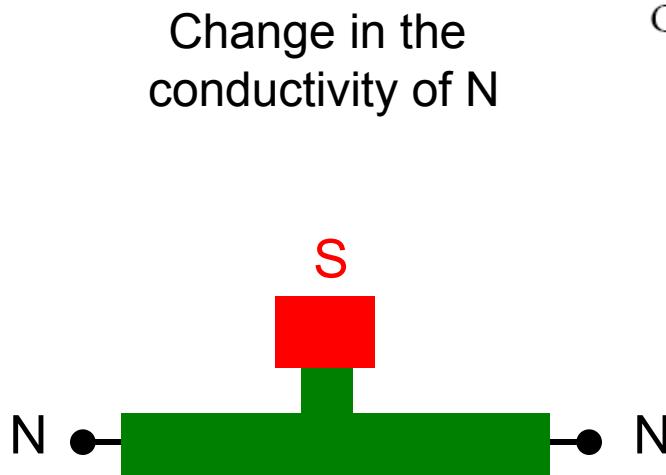
# Proximity Effect Experiment: density of states



Gueron, Pothier, Birge, Esteve, & Devoret, PRL 77, 3025 (1996).

# Proximity Effect – Re-entrant resistance

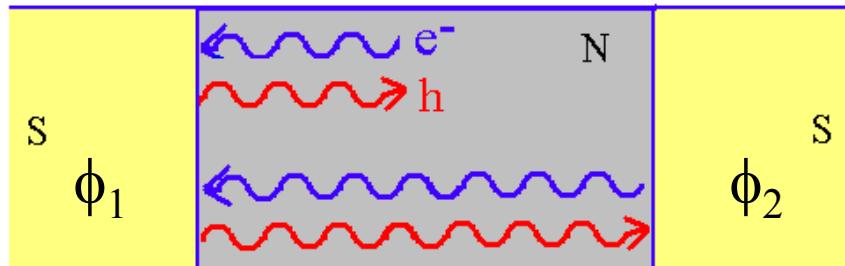
Courtois et al., J. Low Temp. Phys. 116, 187 (1999)



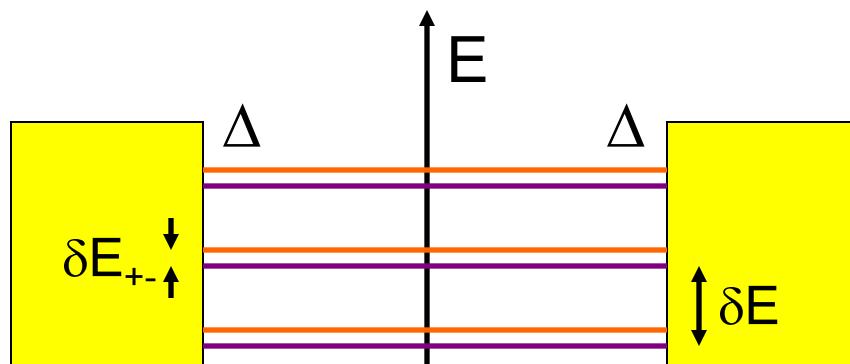
Resistance decreases upon cooling below  $T_c$ , but then increases again at lower T!

# Andreev bound state picture for S/N/S Josephson junction

## 1D single-channel ballistic wire



Supercurrent carried by  
Andreev bound states in N  
(I.O. Kulik 1970)



$$E_n^{+-} = \frac{\hbar v_F}{2L^*} [2\pi(n + \tfrac{1}{2}) \pm \phi]$$

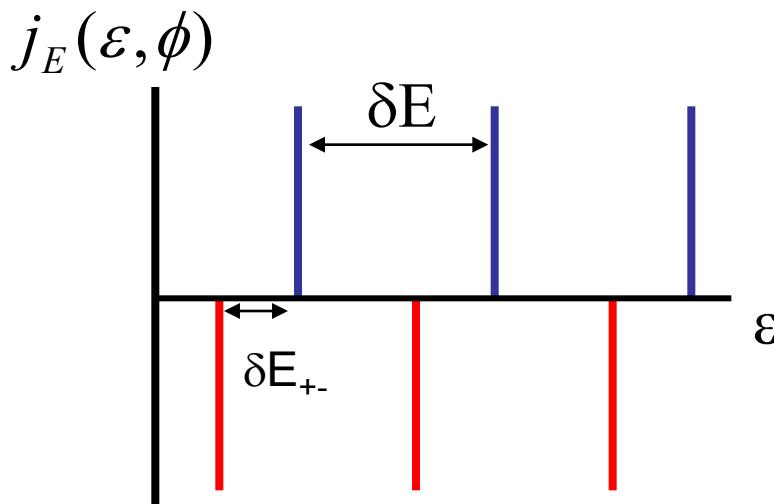
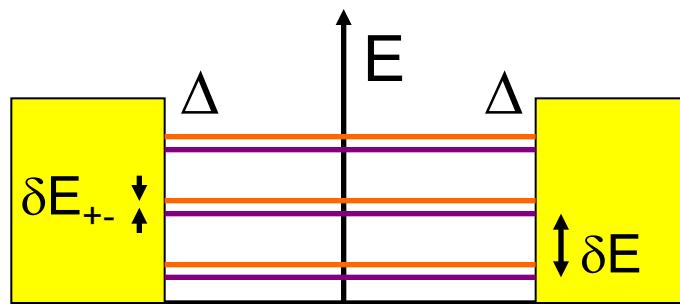
$$L^* = L + \frac{\hbar v_F}{\Delta_0}$$

pairs of states are degenerate when  $\phi=0$       no supercurrent

# Spectral supercurrent density: $j_E(\varepsilon)$

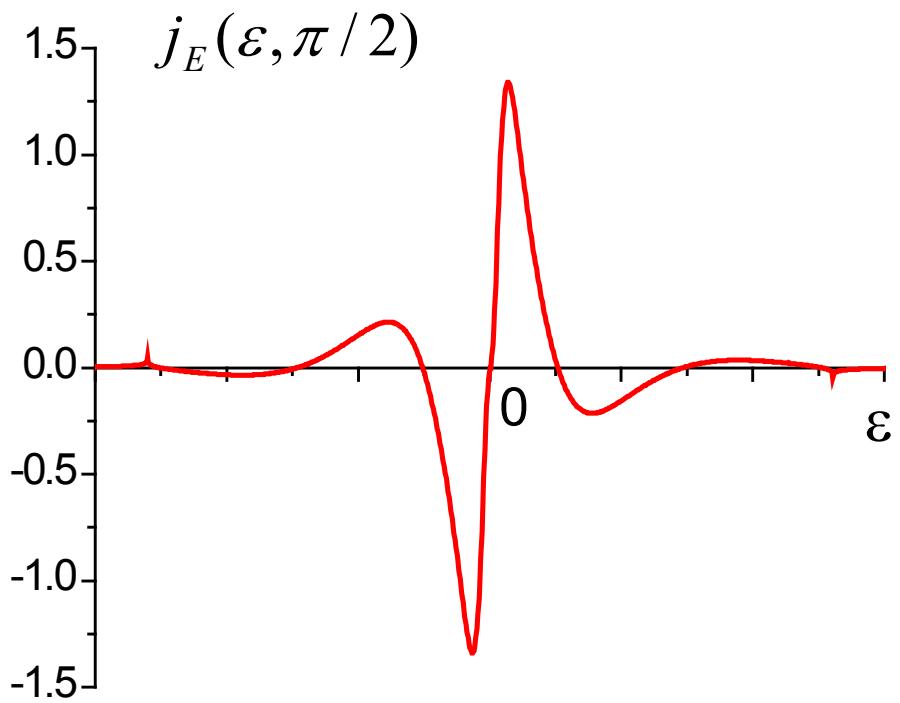
(left and right-moving Andreev bound states)

ballistic



energy scale:  $\frac{\hbar v_F}{L}$

diffusive



$j(\varepsilon)$  retains oscillatory behavior

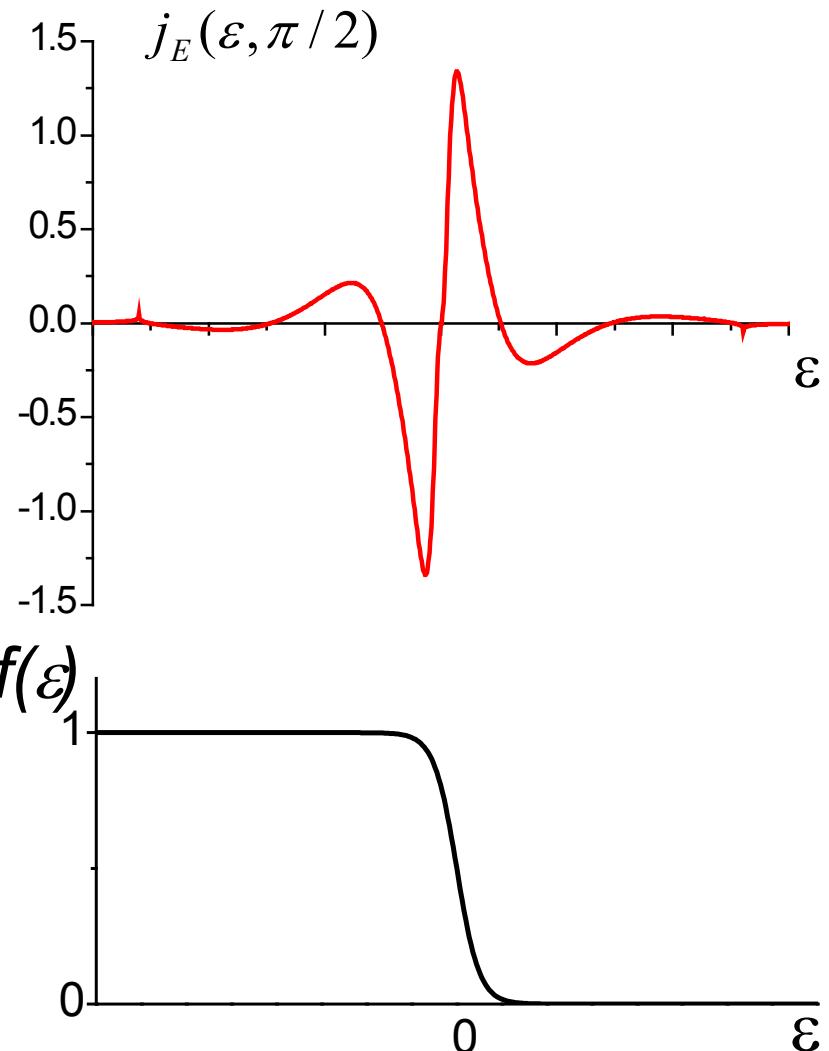
energy scale:  $E_{Th} = \frac{\hbar D}{L^2}$

# Supercurrent: filling of Andreev states

$$I_s = -\sigma_N A \int_{-\infty}^{\infty} d\varepsilon j_E(\varepsilon) f(\varepsilon)$$

Semiconductor picture:

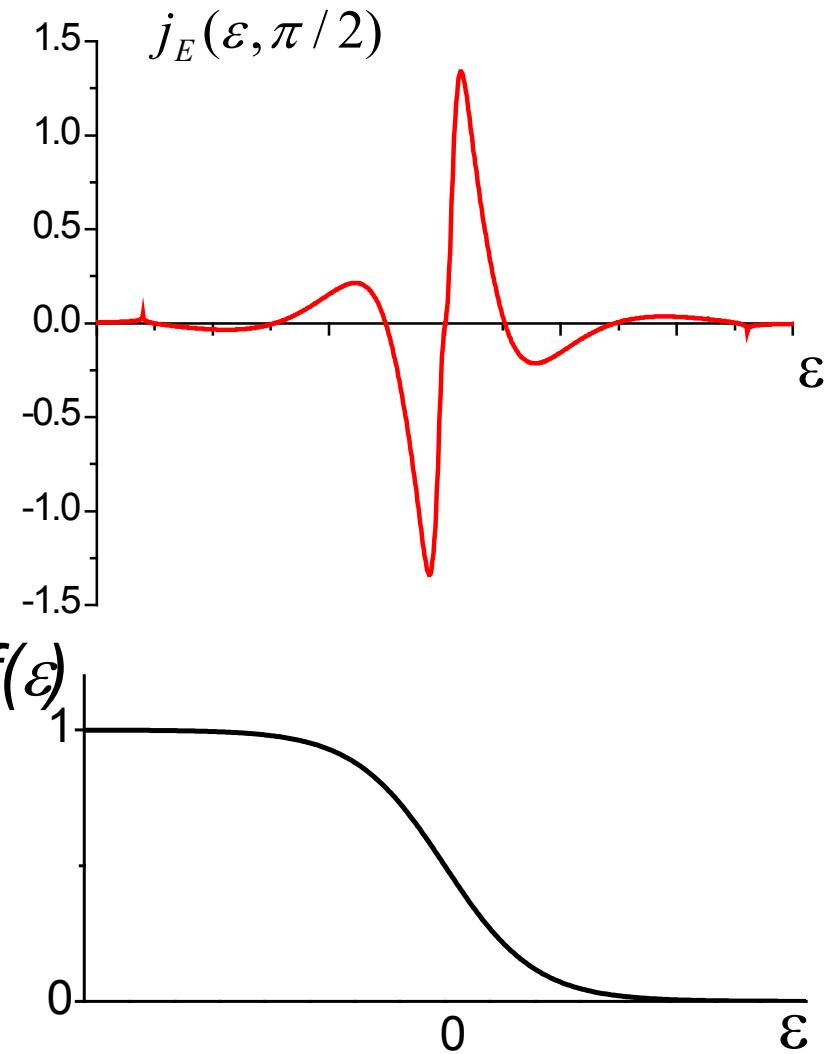
$T = 0 \Leftrightarrow$  states at  $\varepsilon < 0$  are filled



# Supercurrent: filling of Andreev states

$$I_s = -\sigma_N A \int_{-\infty}^{\infty} d\varepsilon j_E(\varepsilon) f(\varepsilon)$$

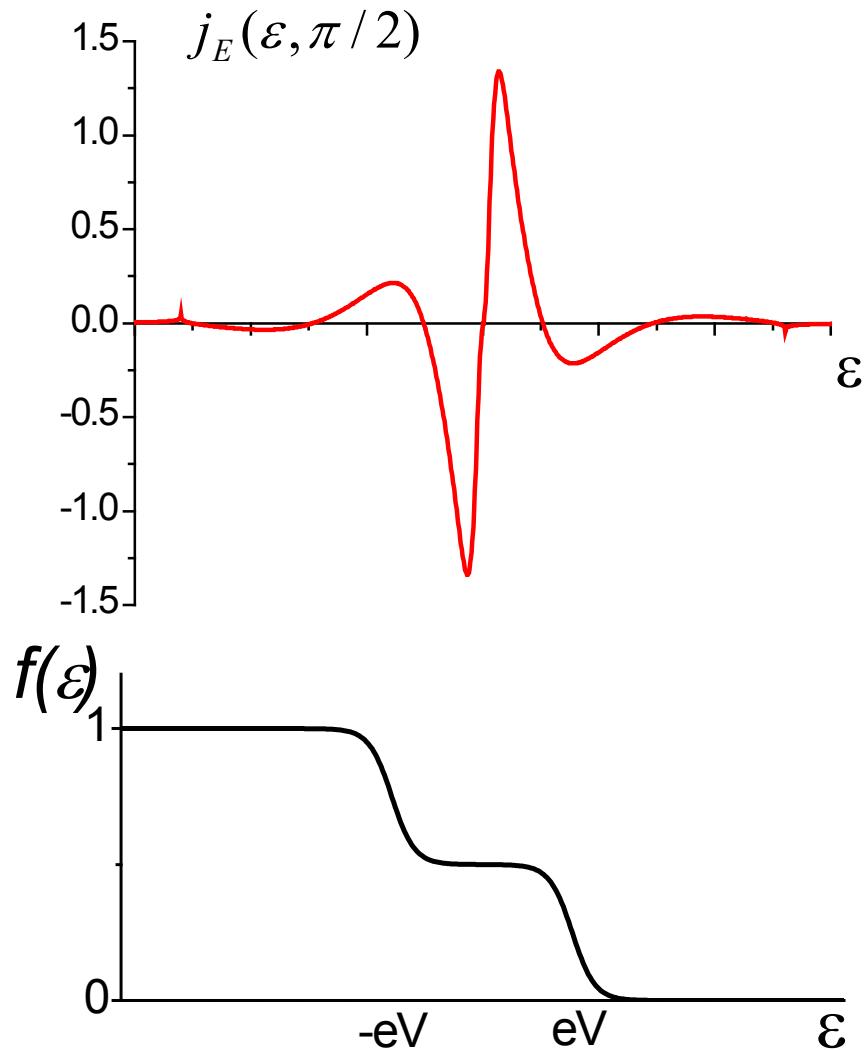
As  $T$  increases,  $I_s$  decreases



# Supercurrent: filling of Andreev states

$$I_S = -\sigma_N A \int_{-\infty}^{\infty} d\varepsilon j_E(\varepsilon) f(\varepsilon)$$

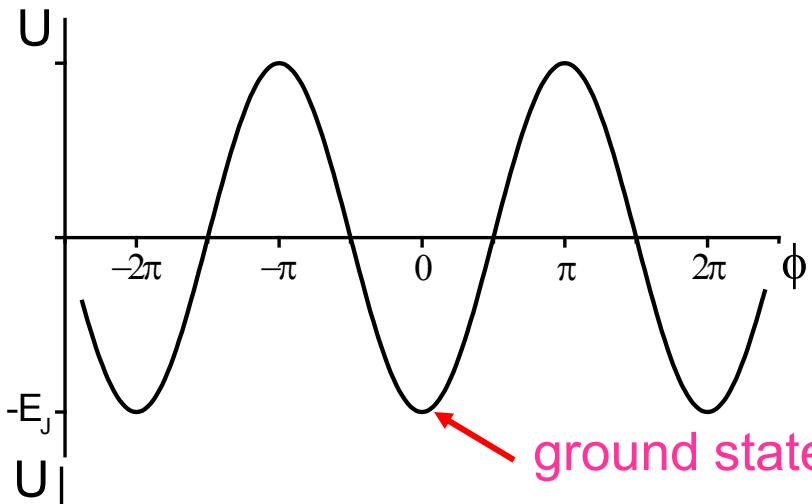
To get a  $\pi$ -junction, fill the Andreev states that carry negative supercurrent



S.-K. Yip, PRB 58, 5803 (1998).

F.K. Wilhelm, G. Schon, and A.D. Zaikin, PRL 81, 1682 (1998).

# What is a $\pi$ junction?



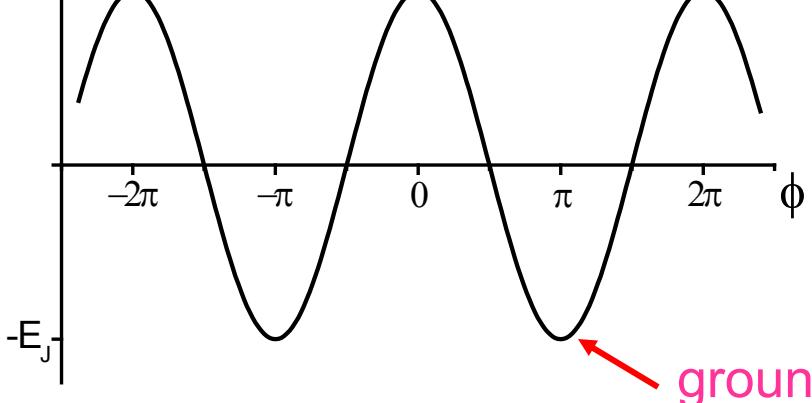
Josephson energy:  $E_J = \frac{\hbar I_c}{2e}$

Supercurrent:  $I_s(\phi) = \frac{2e}{\hbar} \frac{dU}{d\phi}$

standard Josephson junction

$$U = -E_J \cos(\phi)$$

$$I_s = I_c \sin(\phi)$$



$\pi$ -junction

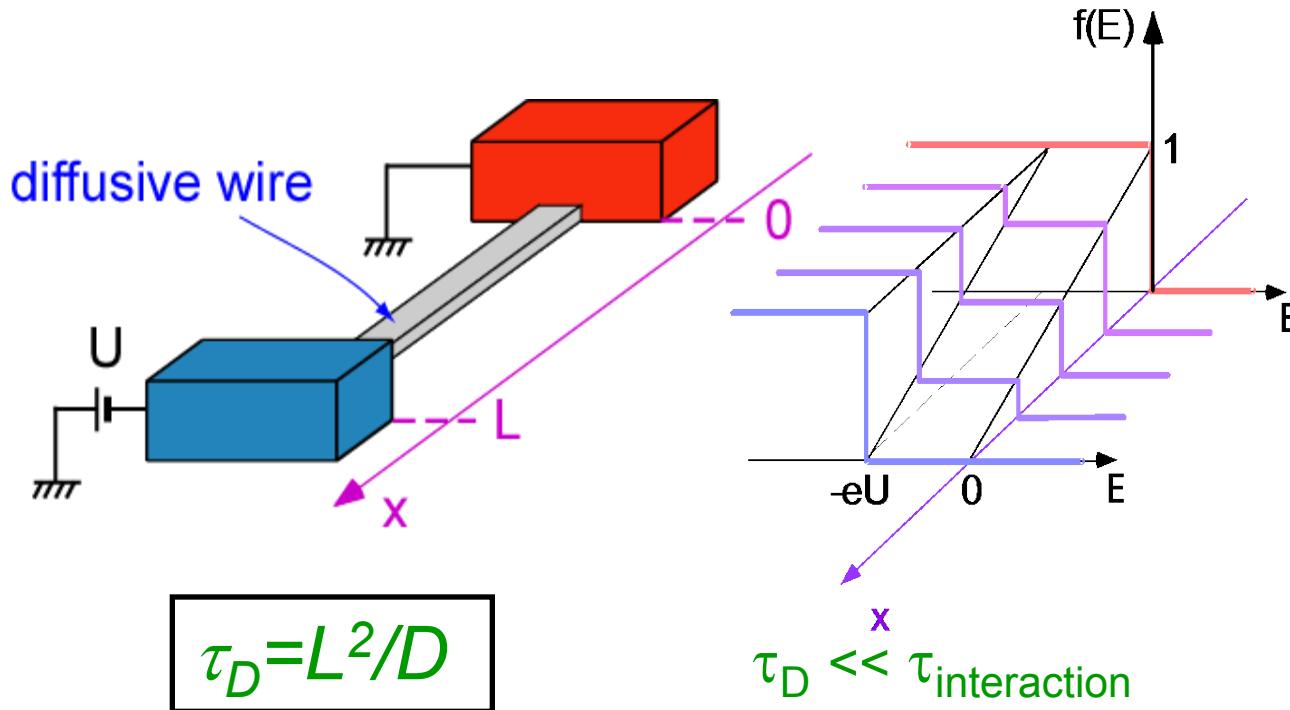
$$U = -E_J \cos(\phi + \pi)$$

$$I_s = I_c \sin(\phi + \pi)$$

# How to get double-step shape of $f(\varepsilon)$ ?

Nagaev (1994); Kozub and Rudin (1994); Pothier *et al.* (1997)

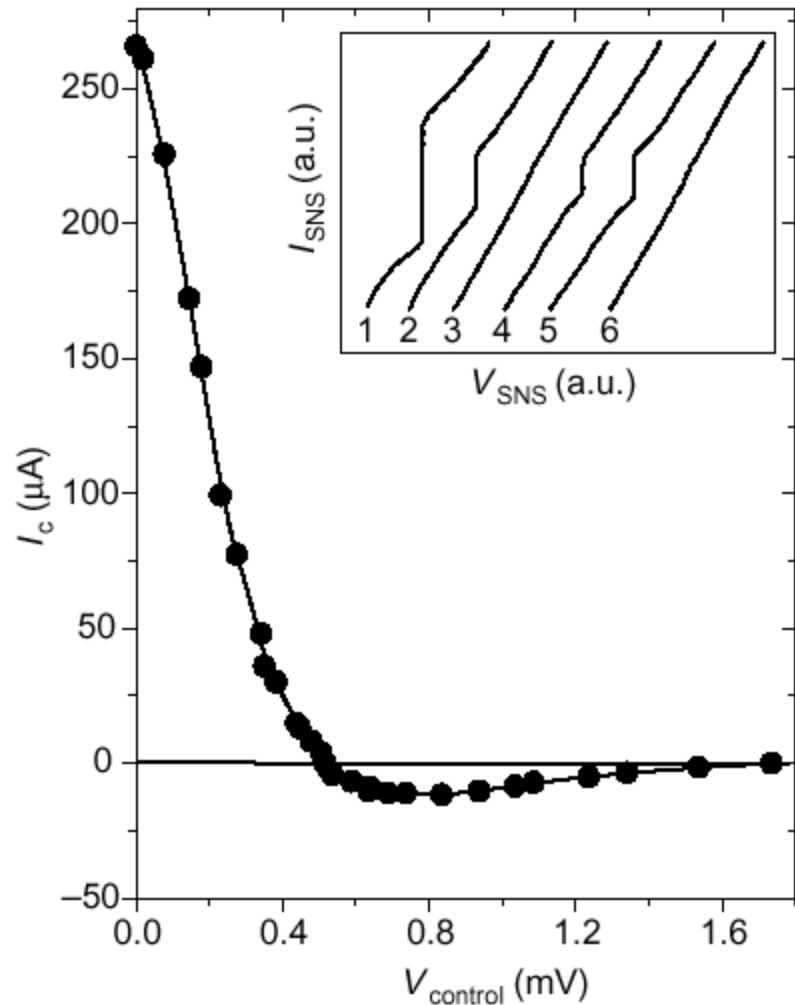
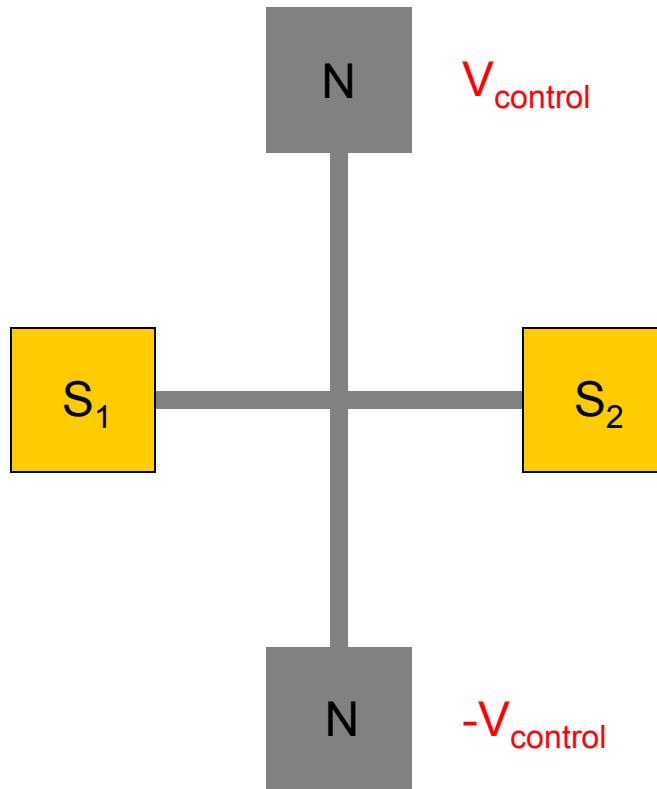
From my previous lecture:



$f(x, E)$  has double-step shape when  $\tau_D \ll \tau_{\text{interaction}}$

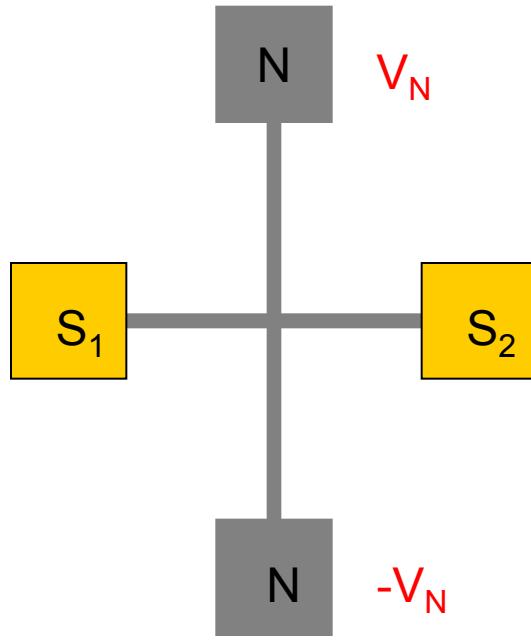
# Experimental observation of nonequilibrium $\pi$ junction in S/N/S system

Baselmans, Morpurgo, van Wees,  
Klapwijk, Nature 397, 43 (1999)



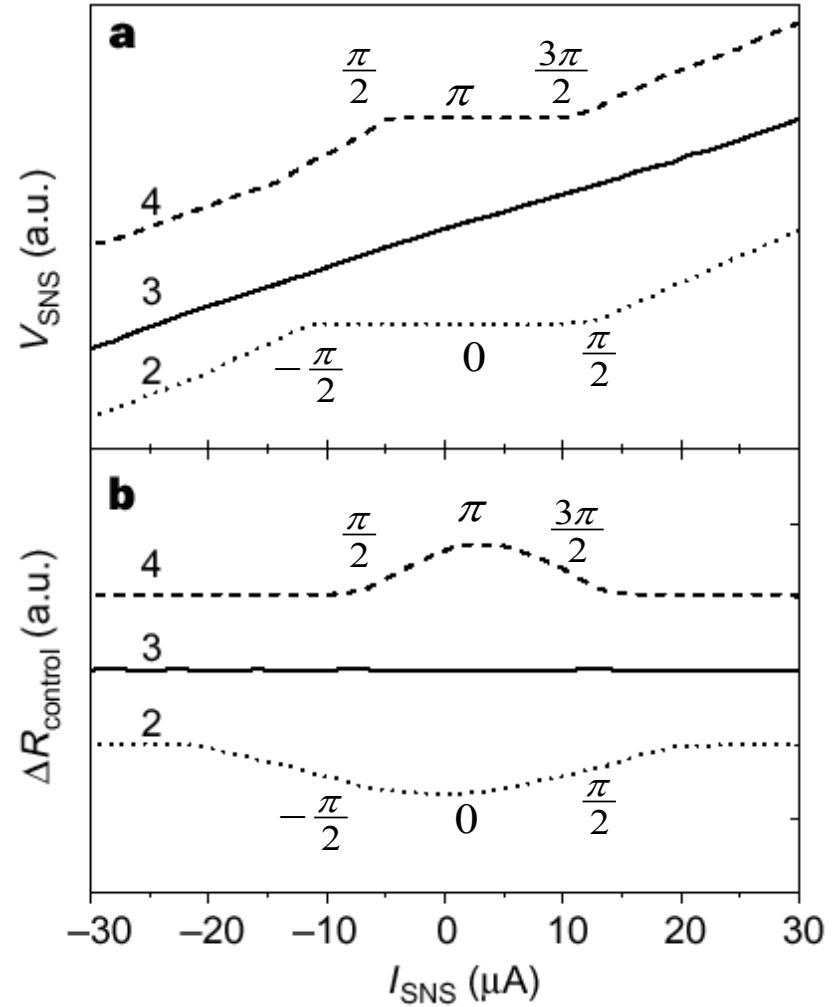
# Experimental proof of $\pi$ junction

Baselmans *et al.*, Nature 397, 43 (1999)



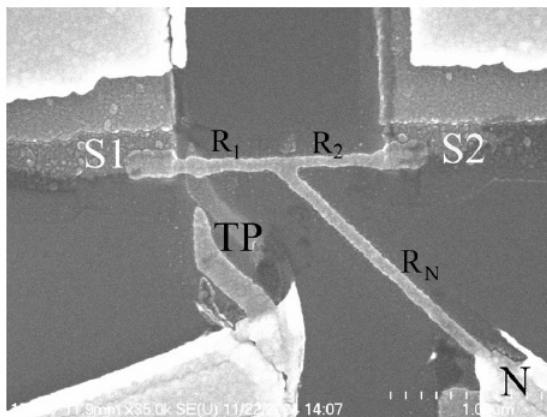
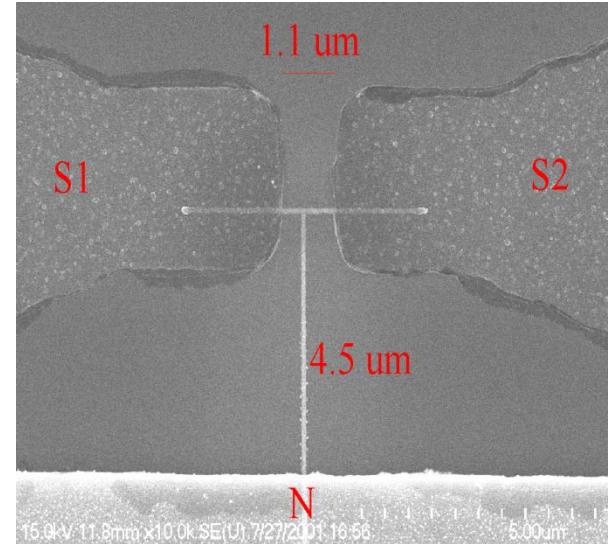
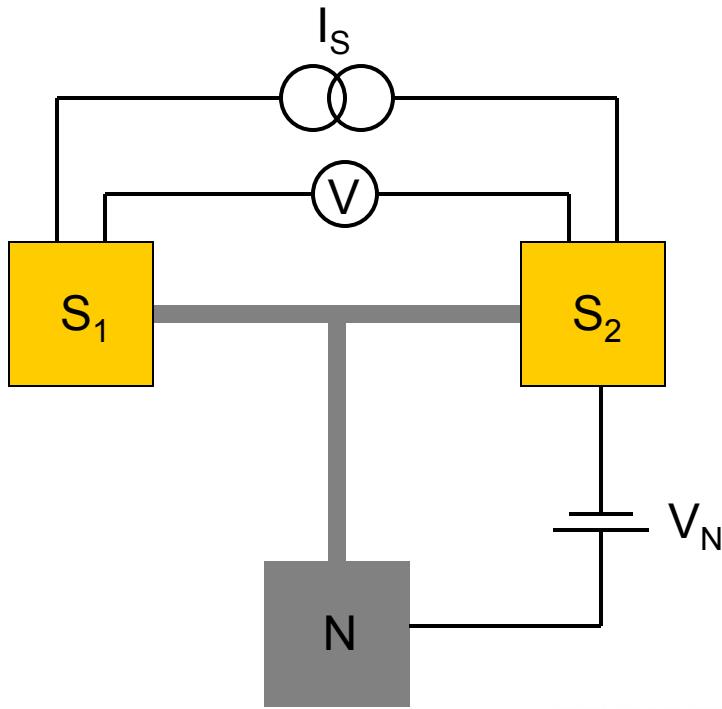
Resistance of N wire varies  
with  $\phi$  due to proximity effect:

$$R = R_0 - \delta R \cos(\phi)$$

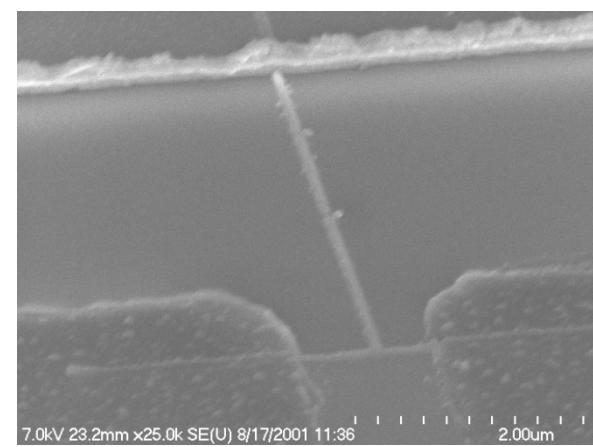


# $\pi$ junction in 3-terminal S/N/S sample

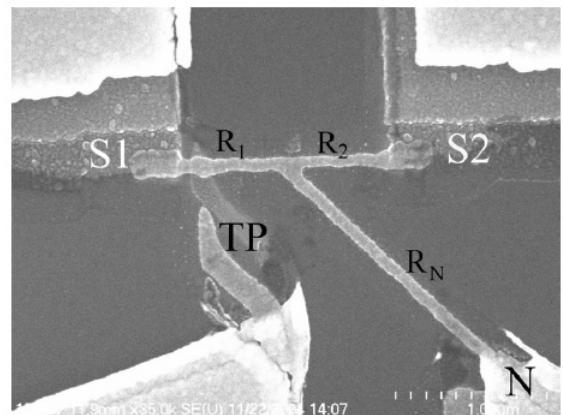
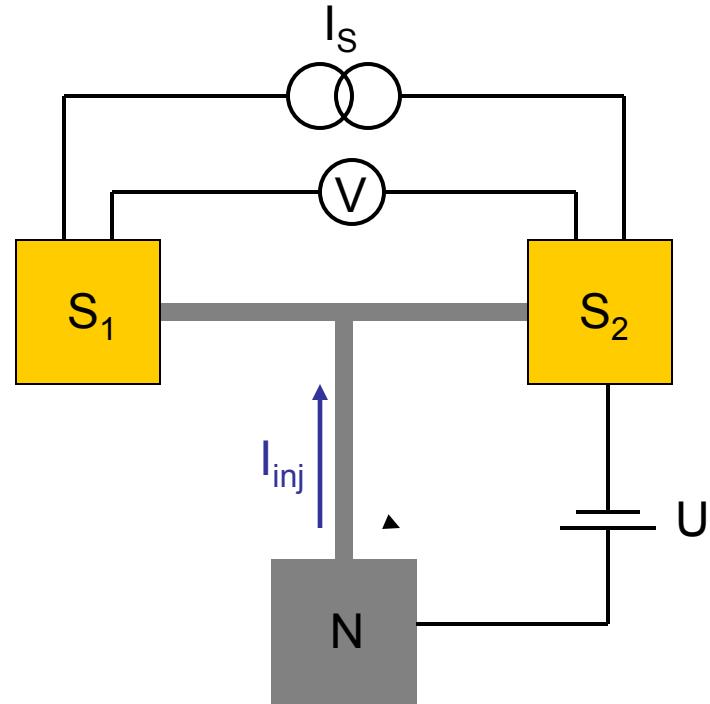
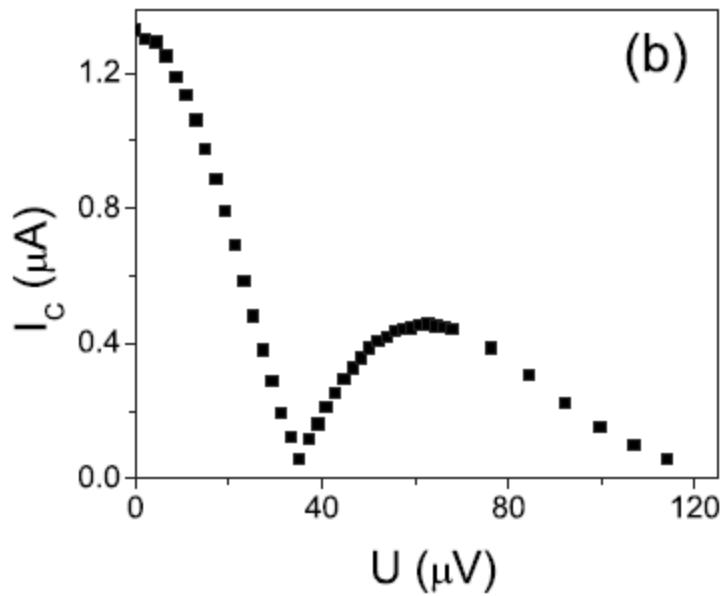
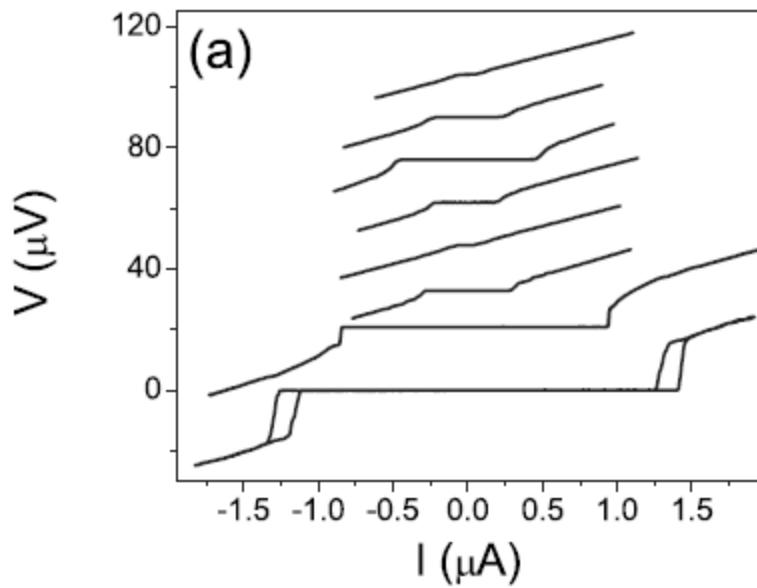
J. Huang *et al.*, PRB 66, 020507 (2002)



M.S. Crosser *et al.*,  
PRB 77, 014528 (2008).

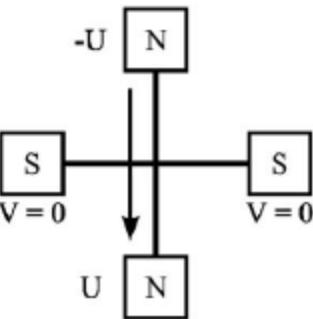


# SNS V-I curves vs. U

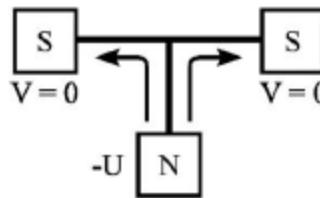


# Compare $f(E)$ in 3-terminal and 4-terminal samples

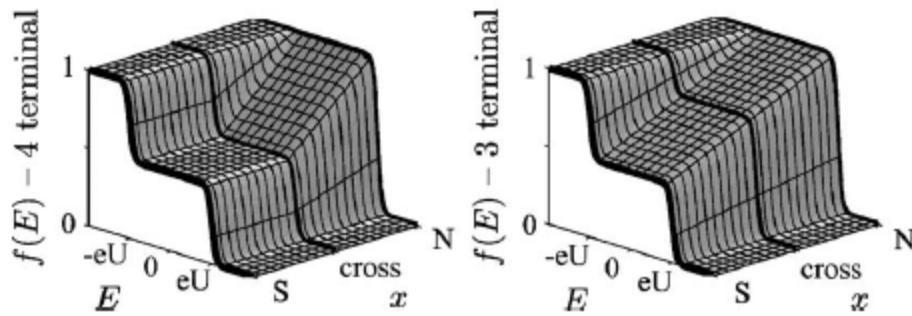
(a)



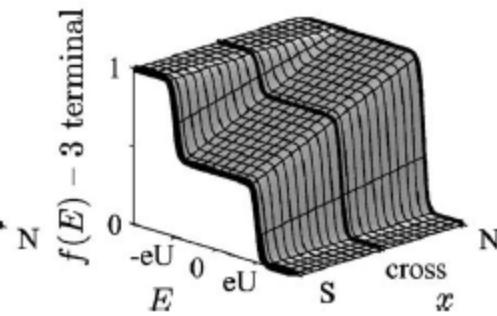
(b)



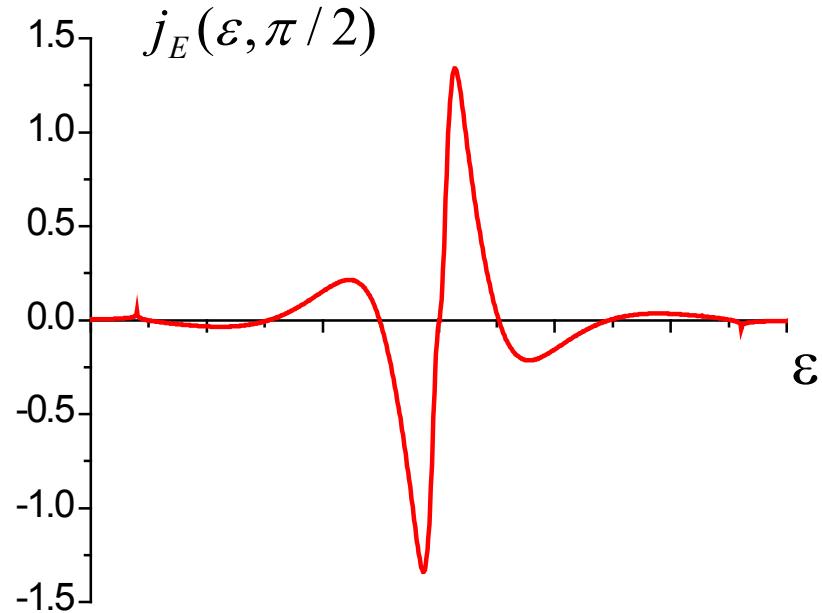
(c)



(d)



# Can we directly measure $j_E(\varepsilon)$ ?



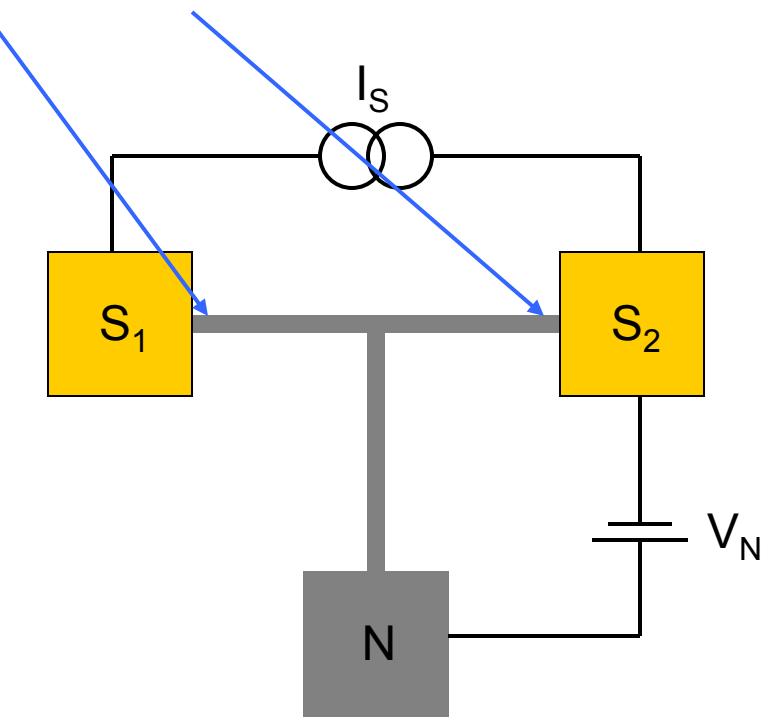
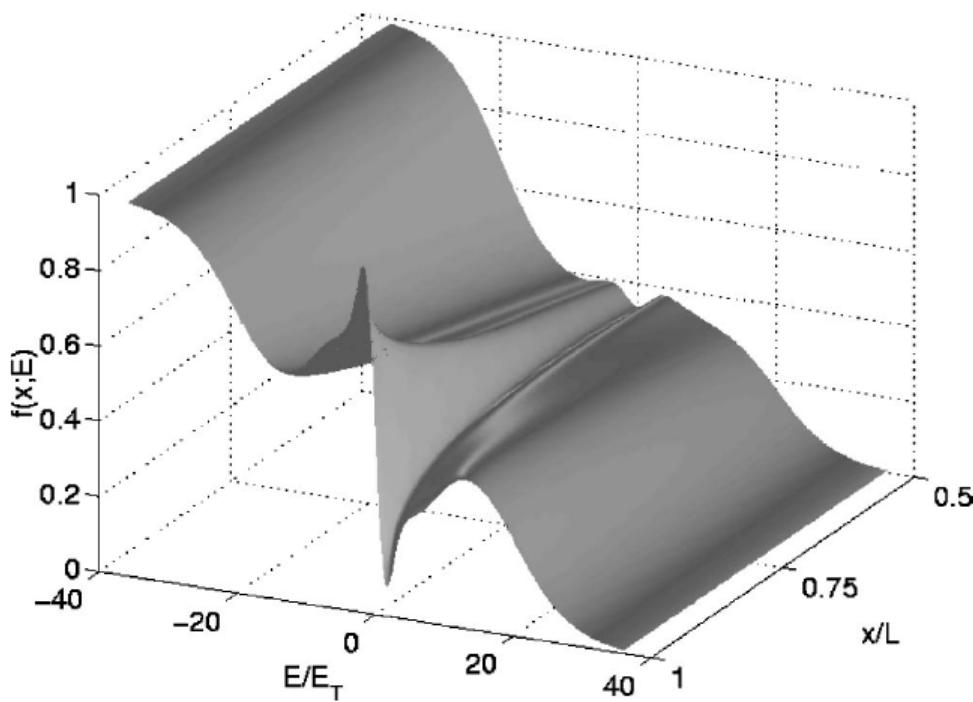
Almost ...

# “Peltier-like” effect

T. Heikkila, T. Vanska, and F.K. Wilhelm, PRB 67, 100502 (2003)

Prediction:

nonequilibrium  $f(\varepsilon)$  + supercurrent  $\Rightarrow f(\varepsilon, x) \neq f(\varepsilon, -x)$  or  $T_{\text{eff}}(x) \neq T_{\text{eff}}(-x)$

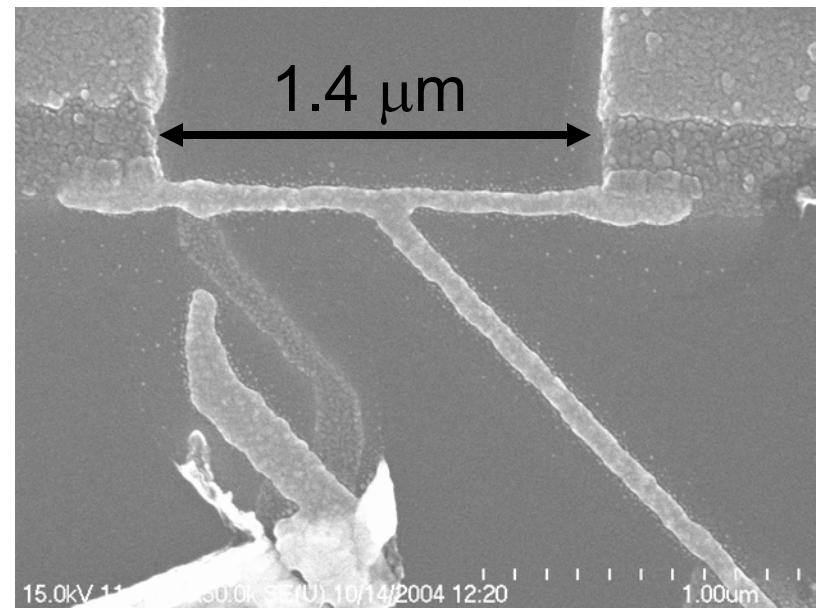
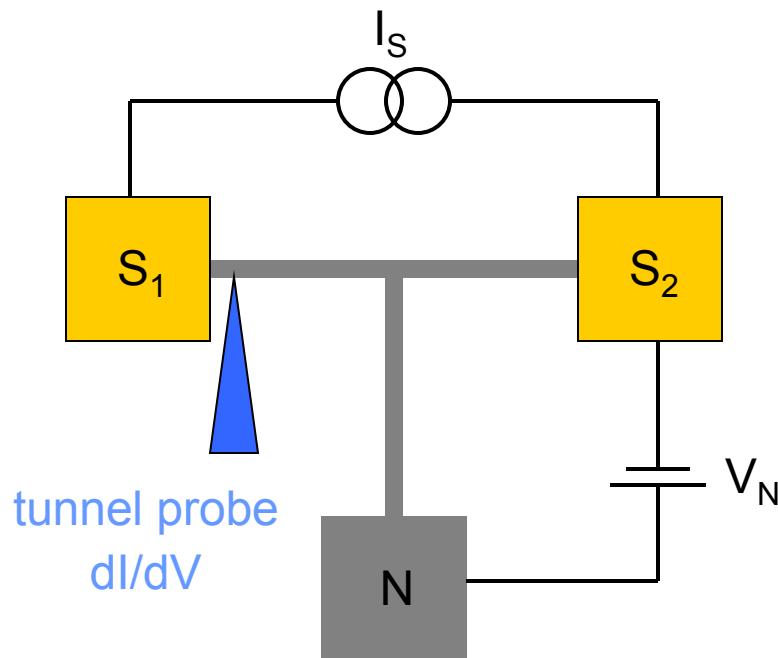


# How to measure “Peltier-like” effect

Local thermometers measure

$$\frac{\pi^2}{6} k_B^2 T_{eff}^2 = \int_{-\infty}^{\infty} d\varepsilon \varepsilon (f(\varepsilon) - f^0(\varepsilon))$$

Measure full shape of  $f(\varepsilon)$  using **Tunneling Spectroscopy**



Note:

In my lecture, I skipped slides 27 - 45

# Origin of “Peltier-like” effect

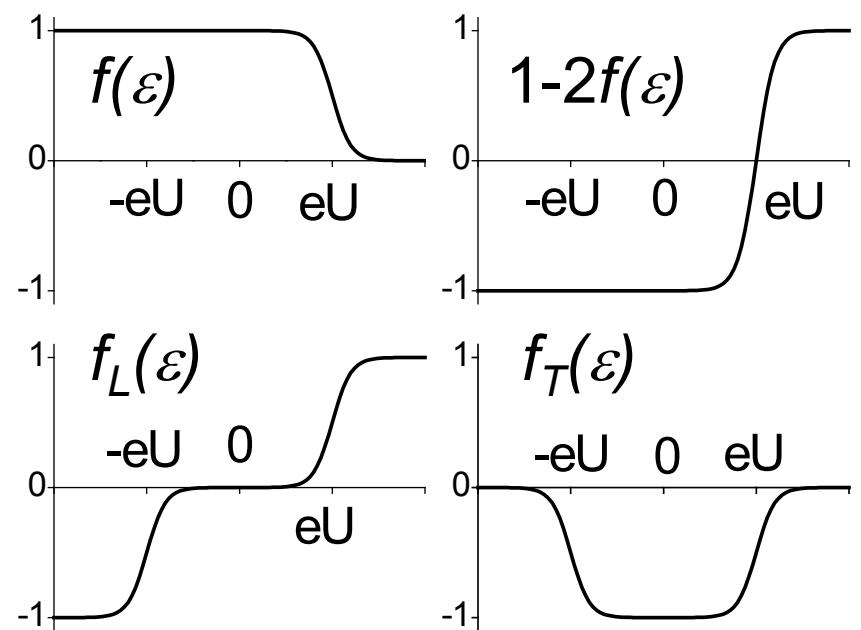
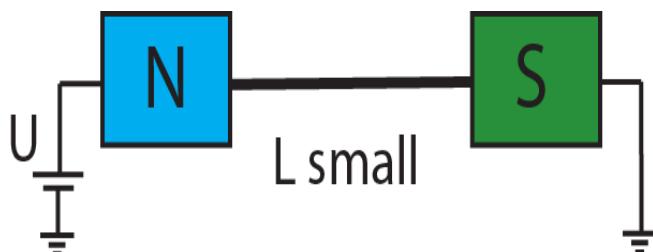
Aside: Charge and energy components of  $f(\varepsilon)$

$$1-2f(\varepsilon) = f_L(\varepsilon) + f_T(\varepsilon)$$

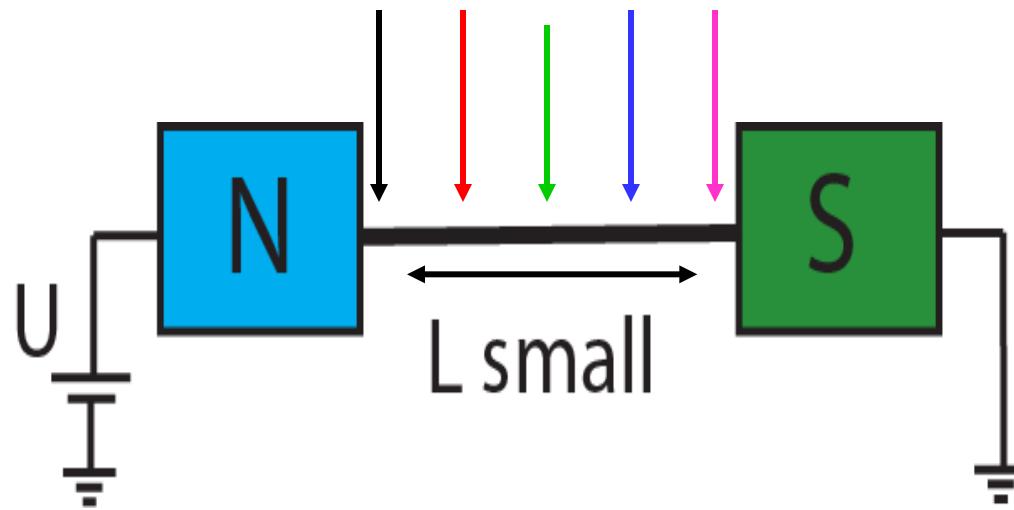
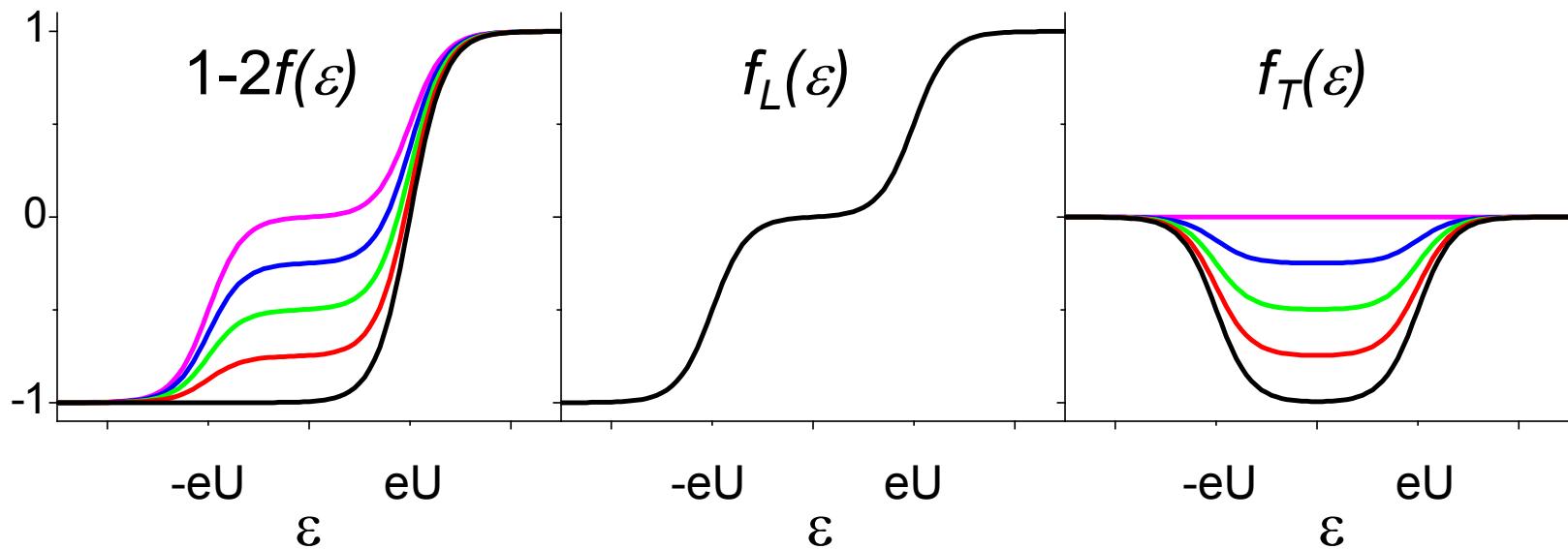
$f_L(\varepsilon)$ : odd function of  $\varepsilon$ , excess energy is  $\varepsilon f_L(\varepsilon)$

$f_T(\varepsilon)$ : even function of  $\varepsilon$ , charge imbalance

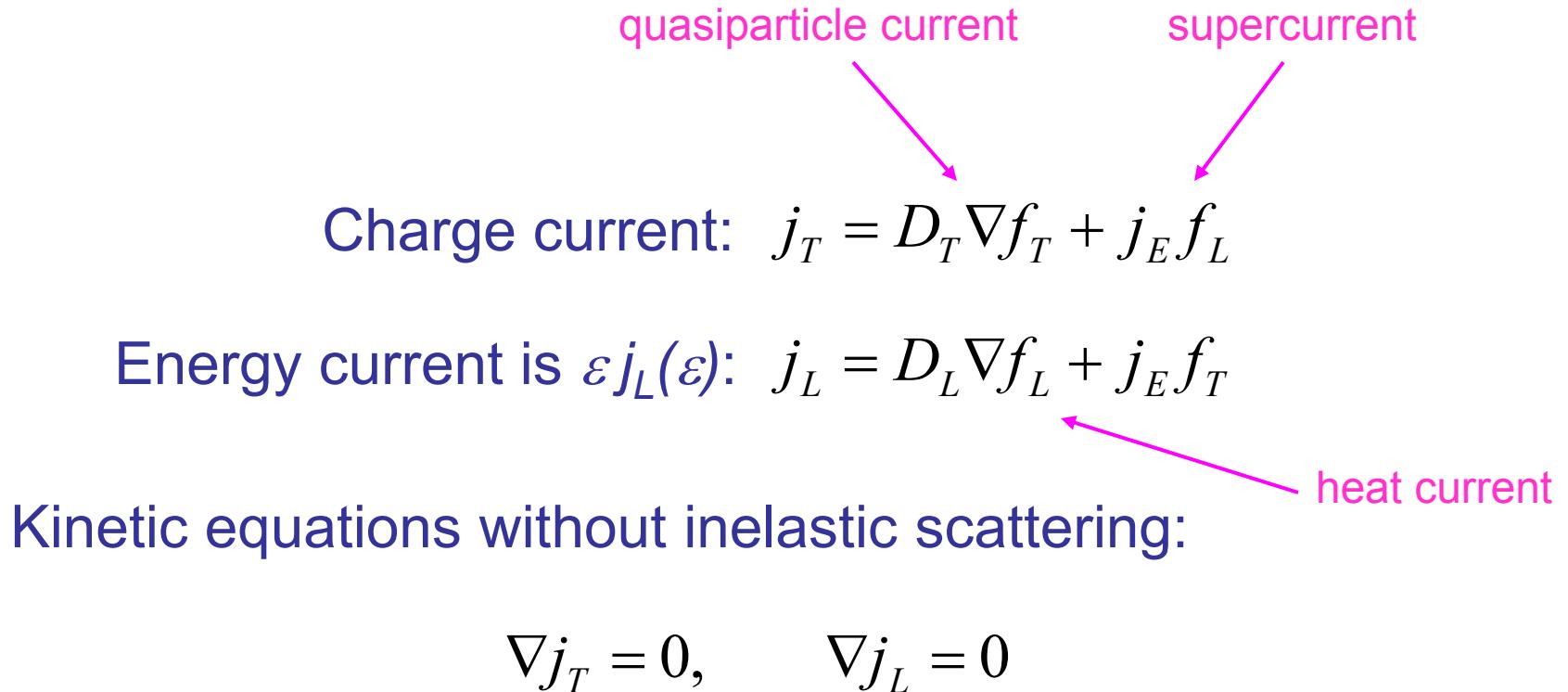
Example: a normal reservoir  
at potential  $-U$



# $f_L(\varepsilon)$ and $f_T(\varepsilon)$ in N-S system

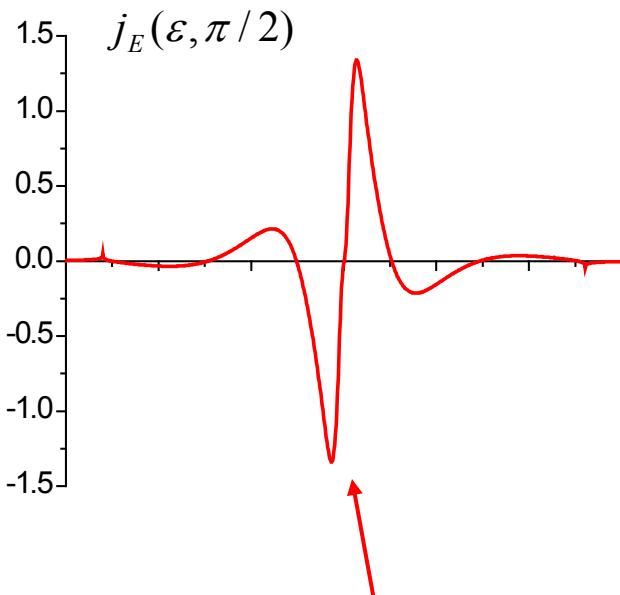


# Kinetic equations with $f_T$ and $f_L$



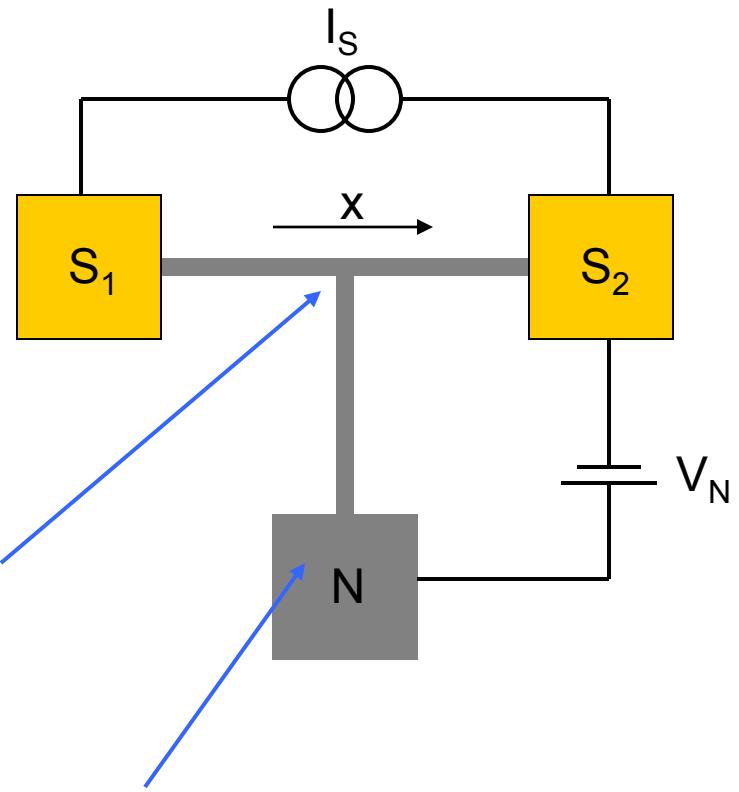
Spectral supercurrent,  $j_E \neq 0$  mixes  $f_T$  and  $f_L$

# Approximate solution without proximity effect, inelastic collisions



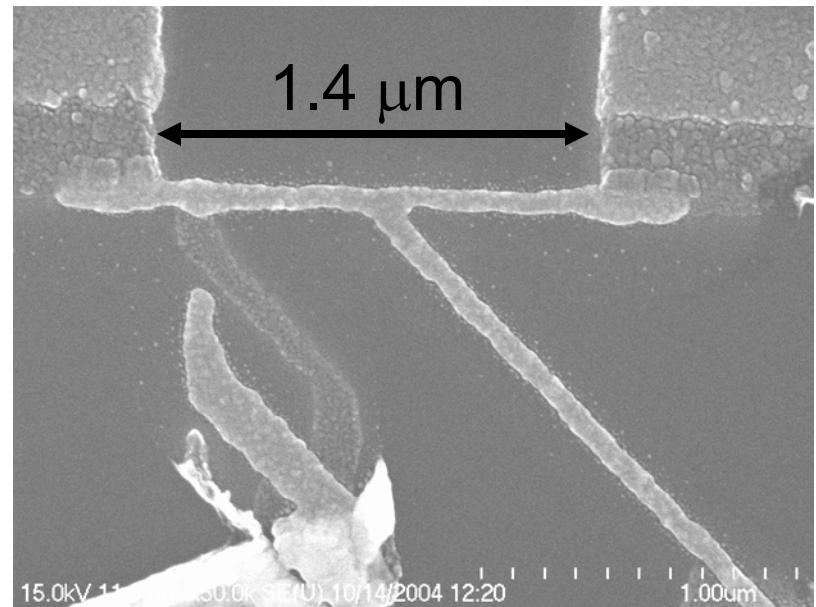
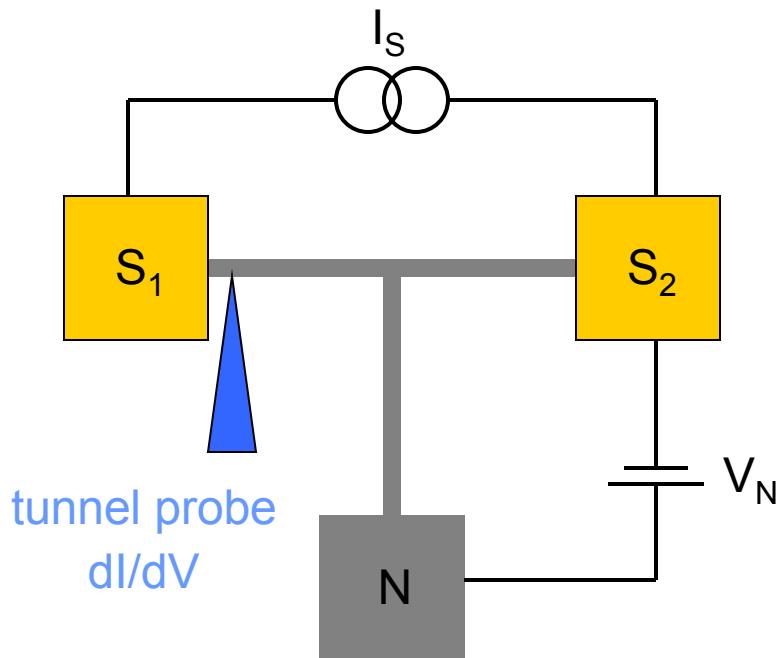
$$f_L(x, \varepsilon) = -j_E(\varepsilon) x \left(1 - \frac{x}{2L_s}\right) f_T(x=0, \varepsilon)$$

$$f_T(x=0, \varepsilon) = \frac{A_N L_S}{A_N L_S + 2 A_S L_N} f_T^0(\varepsilon)$$



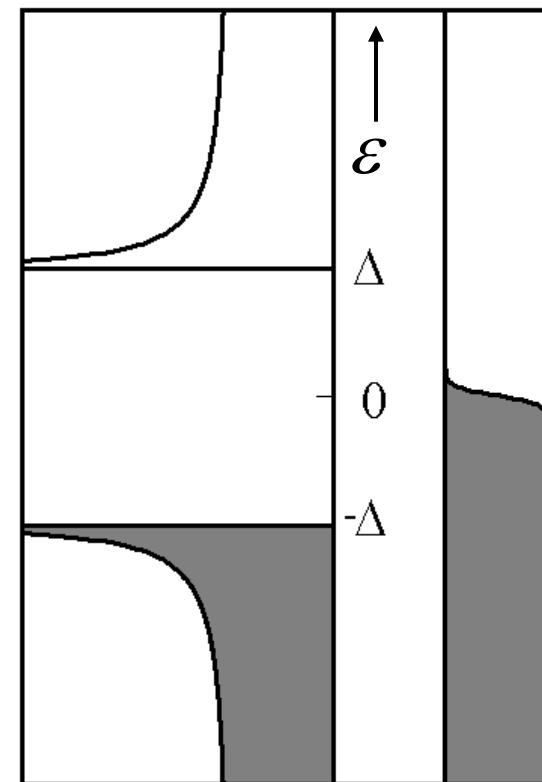
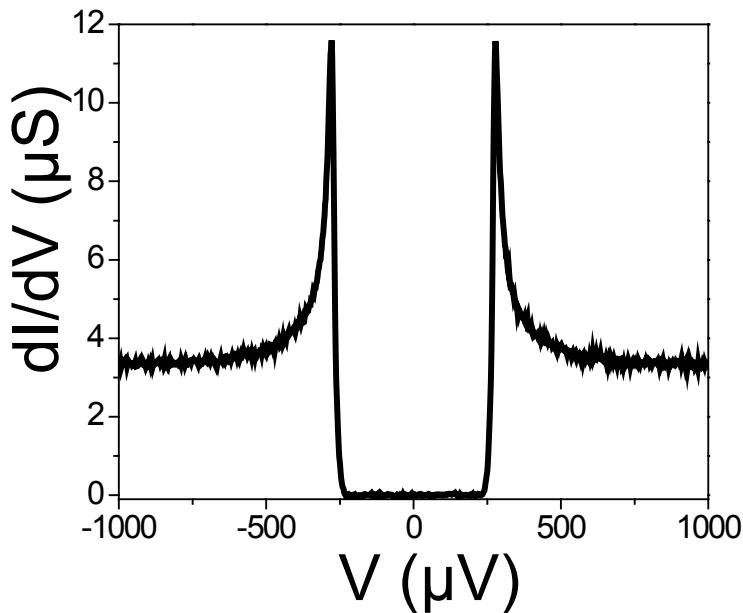
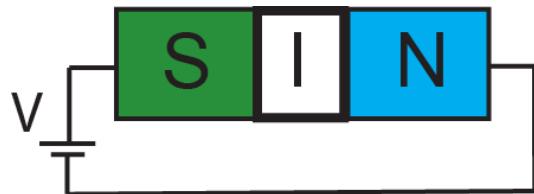
# Experiment

Measure full shape of  $f(\varepsilon)$  using **Tunneling Spectroscopy**



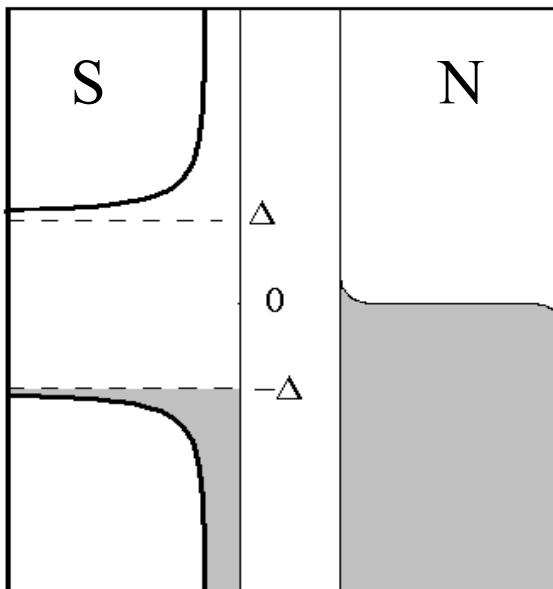
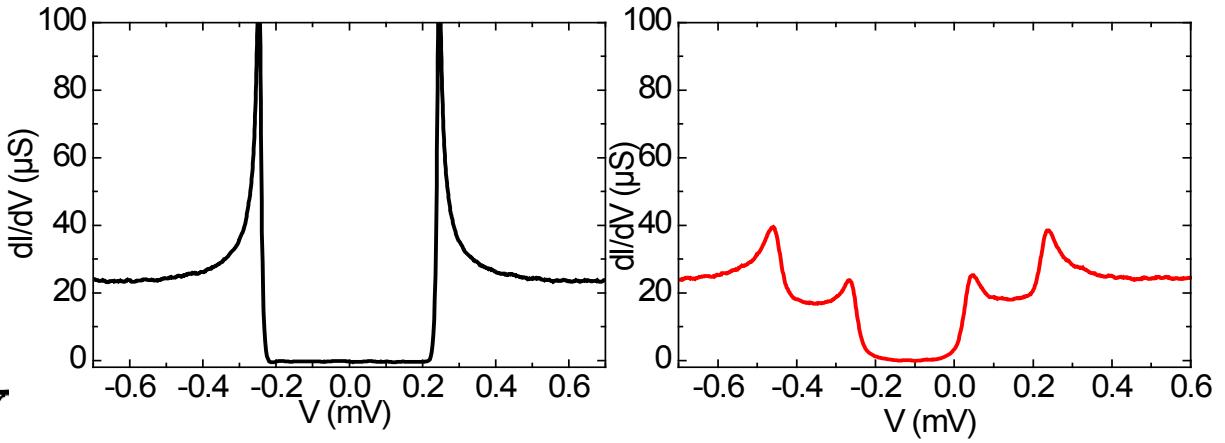
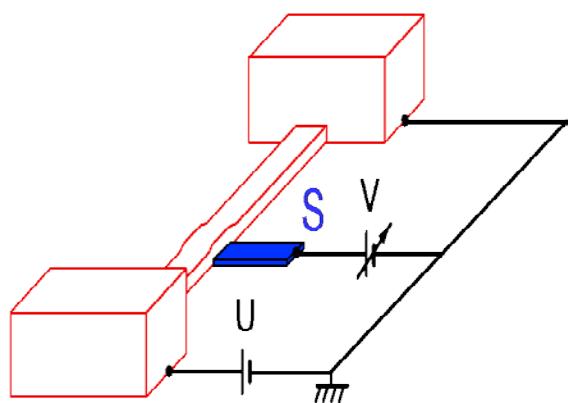
# Tunneling Spectroscopy: SIN junction

$$I = \frac{1}{eR_T} \int_{-\infty}^{\infty} d\varepsilon n_2(\varepsilon) n_1(\varepsilon + eV) [f_2(\varepsilon) - f_1(\varepsilon + eV)]$$

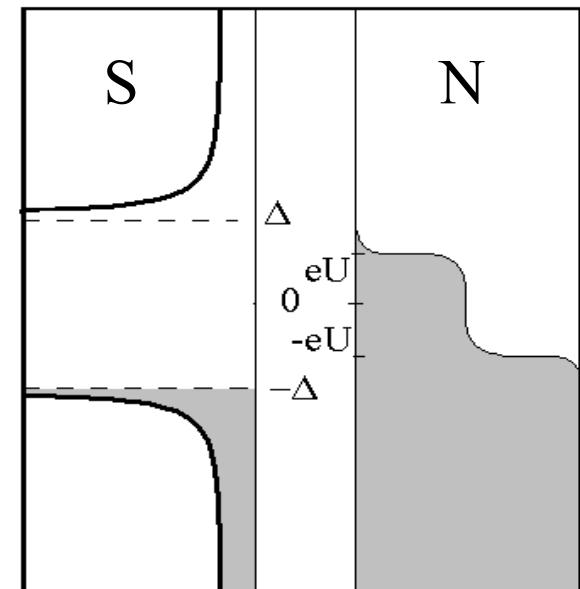


# Method: measure $f(\varepsilon)$ in normal wires

Pothier *et al.*, PRL 79, 3490 (1997).



$U=0$  mV



$U=0.2$  mV

# Measure $f(\varepsilon)$ : general case

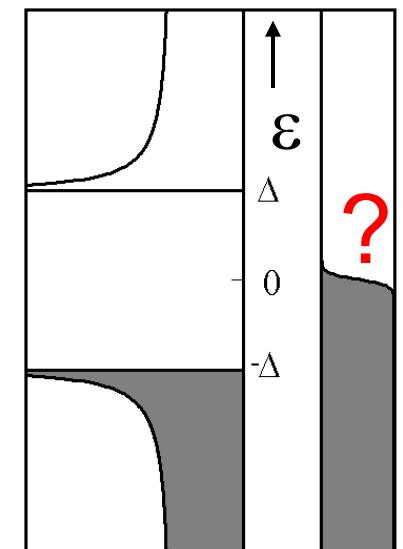
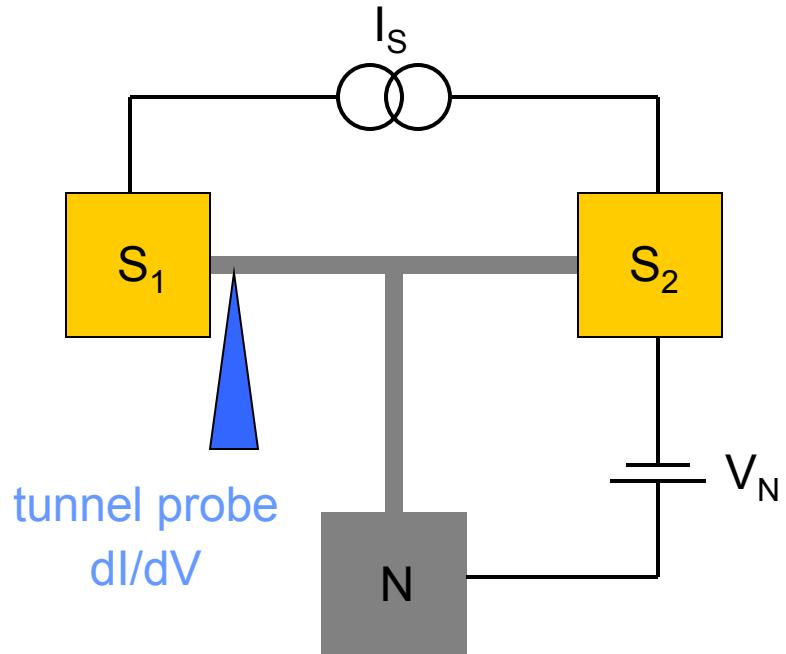
$$I = \frac{1}{eR_T} \int_{-\infty}^{\infty} d\varepsilon n_2(\varepsilon) n_1(\varepsilon + eV) [f_2(\varepsilon) - f_1(\varepsilon + eV)]$$

$n_1(\varepsilon)$  = BCS DOS

$f_1(\varepsilon)$  = Fermi-Dirac

$n_2(\varepsilon)$  = Normal w/ Proximity effect

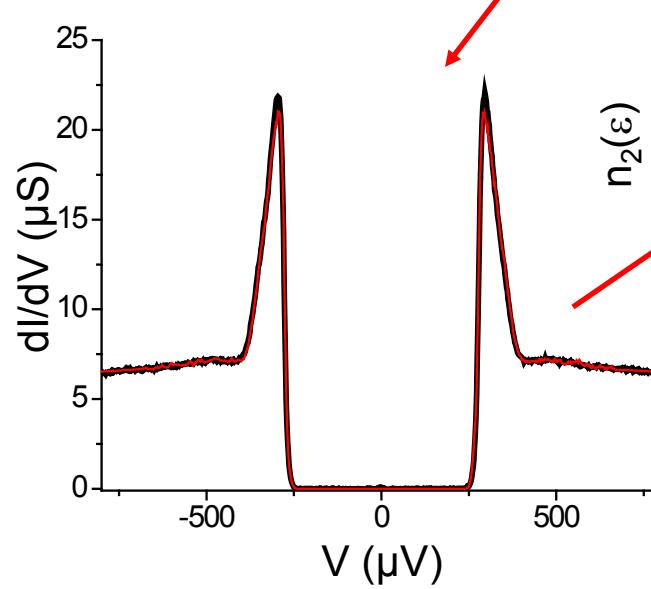
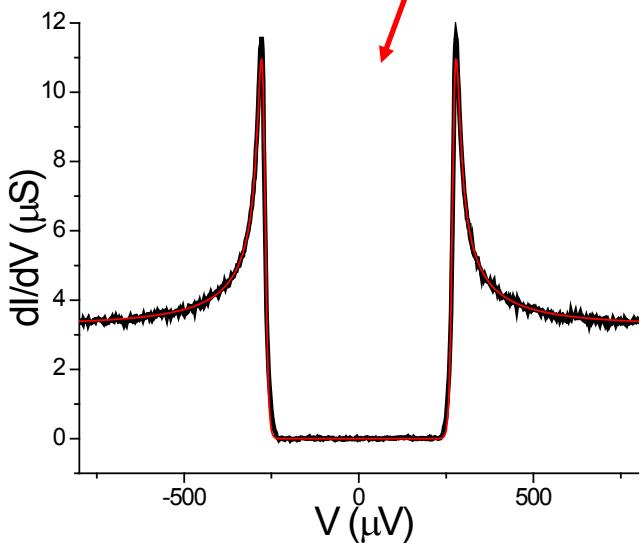
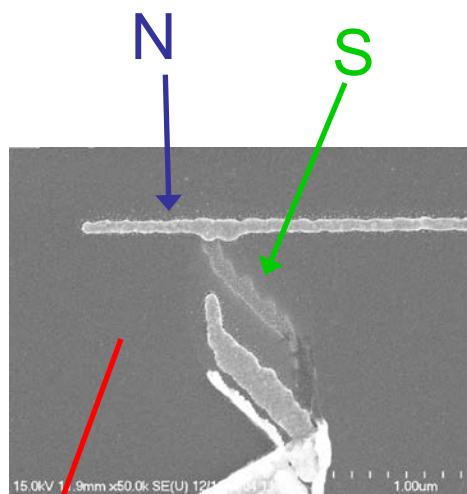
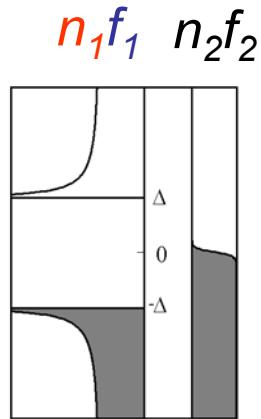
$f_2(\varepsilon)$  = Unknown



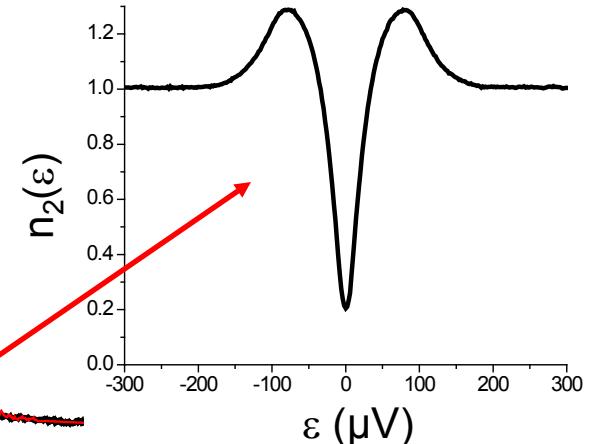
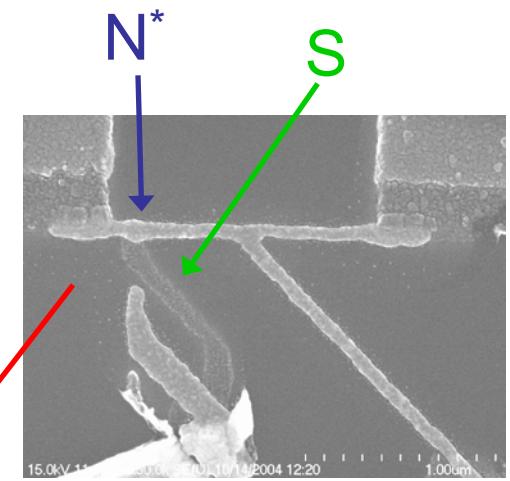
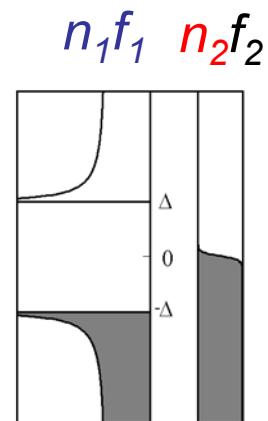
$n_1 f_1$

$n_2 f_2$

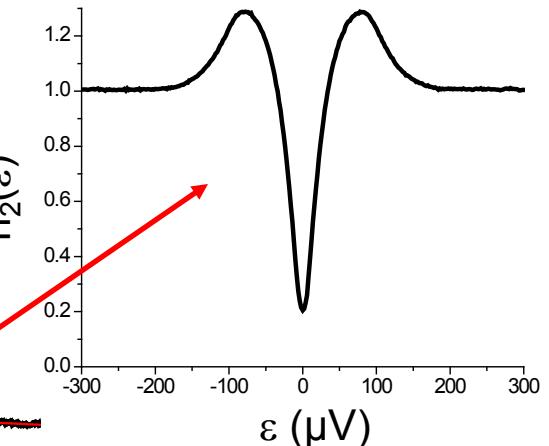
# Step 1: Reference S/I/N to find $\Delta$



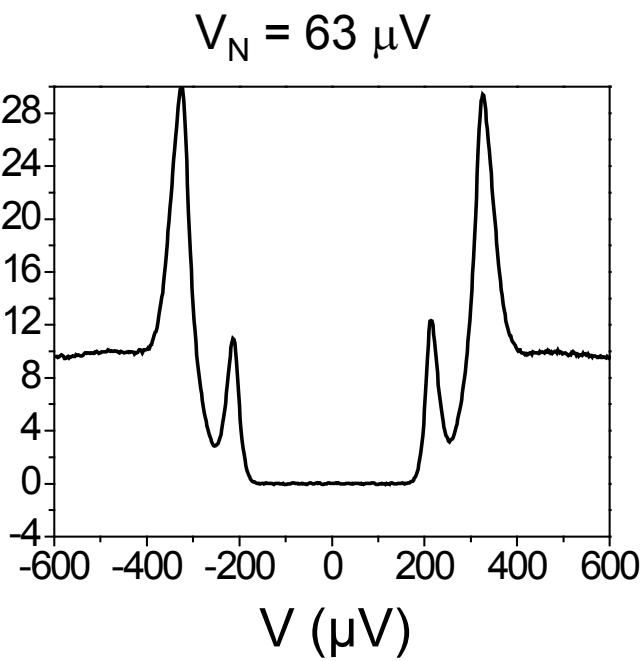
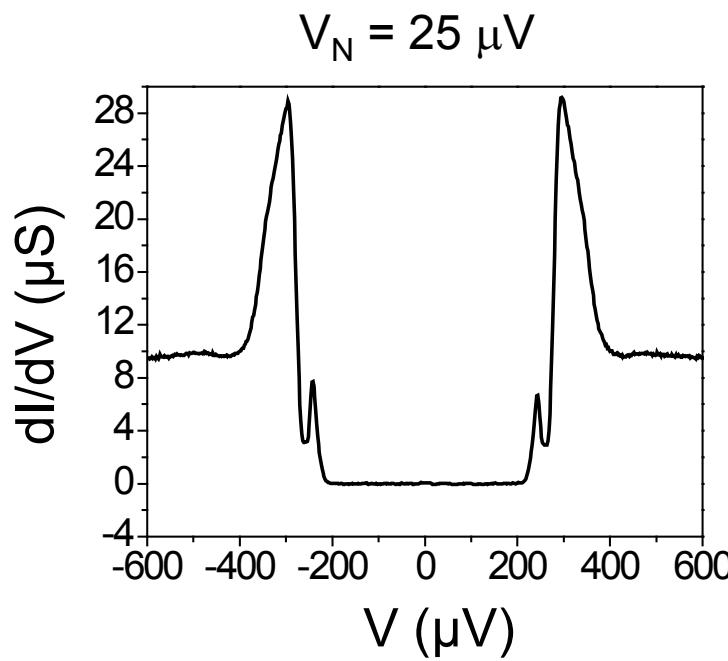
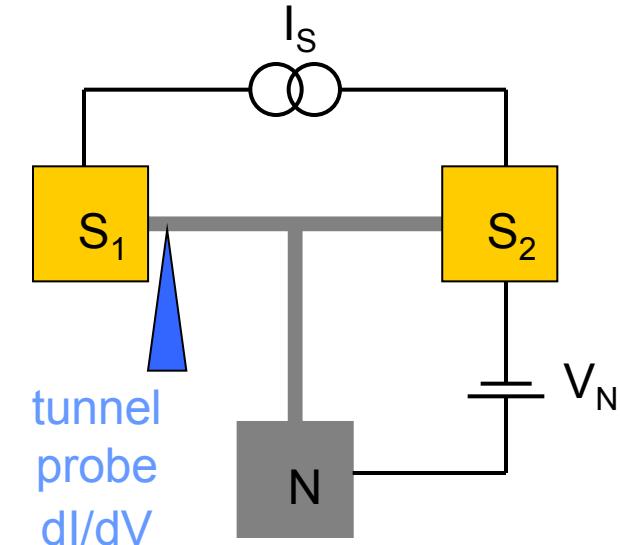
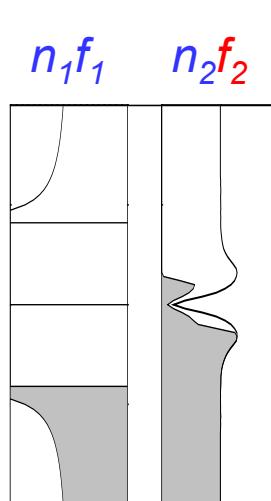
# Step 2: Find $n_2(\varepsilon)$ in sample with proximity effect



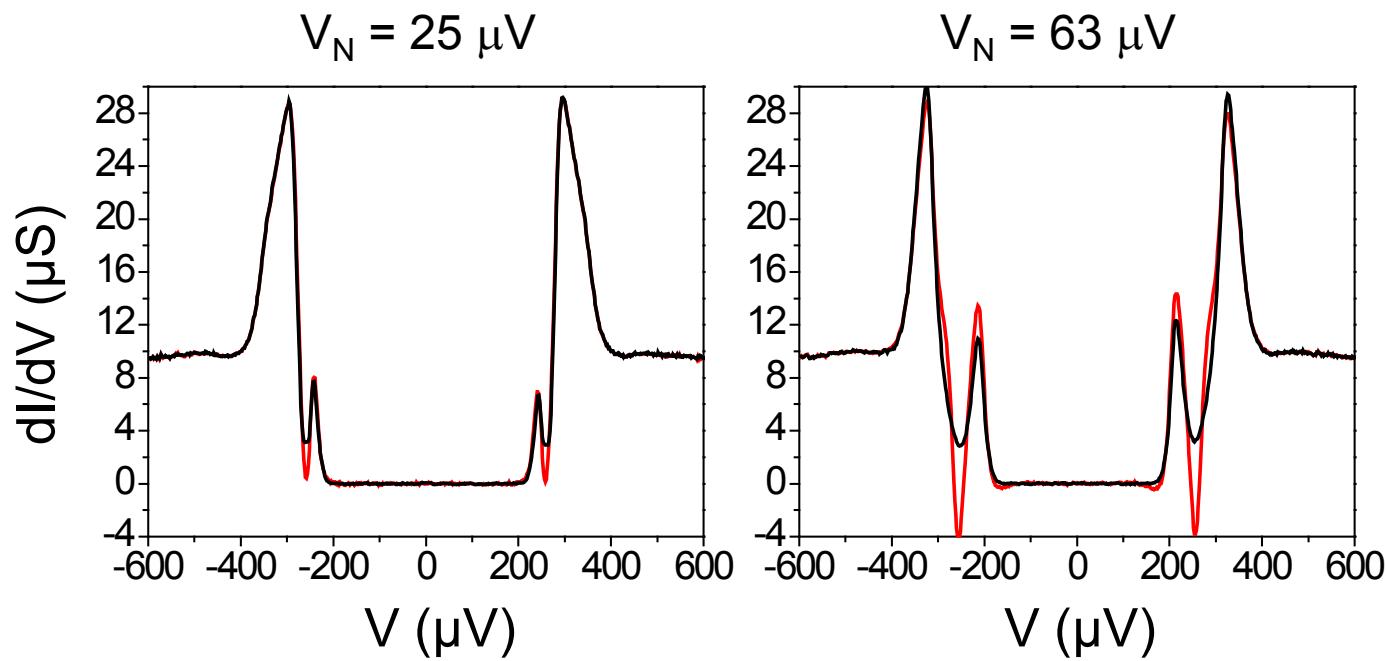
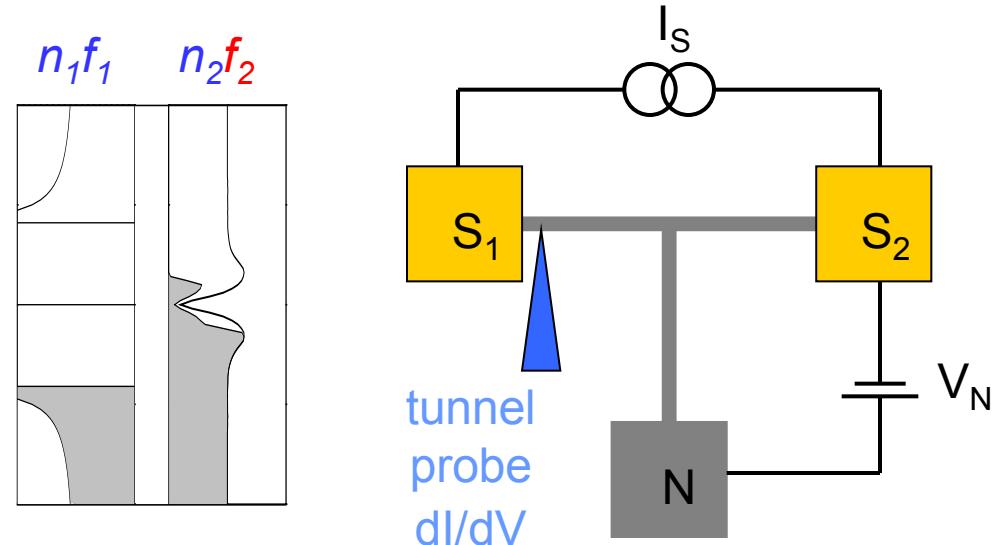
$$n_2(\varepsilon)$$



# Step 3: Find $f_2(\varepsilon)$ with $V_N \neq 0$



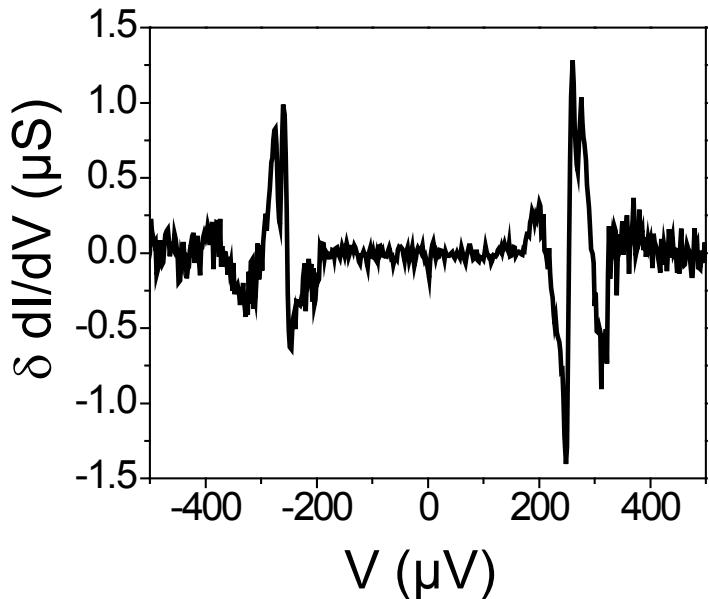
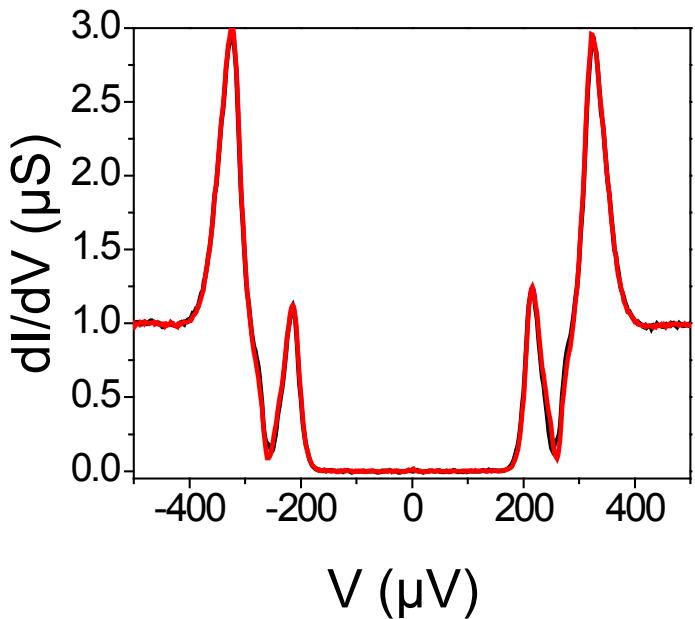
# Step 3: Find $f_2(\varepsilon)$ with $V_N \neq 0$



Cannot fit data unless we allow  $n_2(\varepsilon)$  to change slightly with  $V_N$

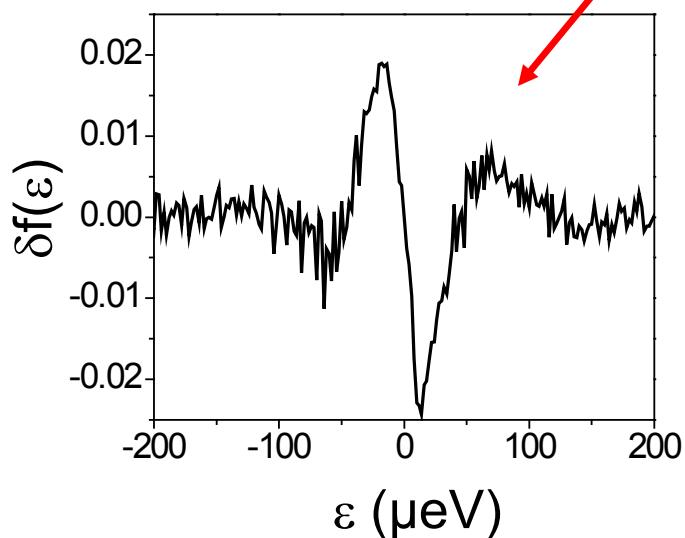
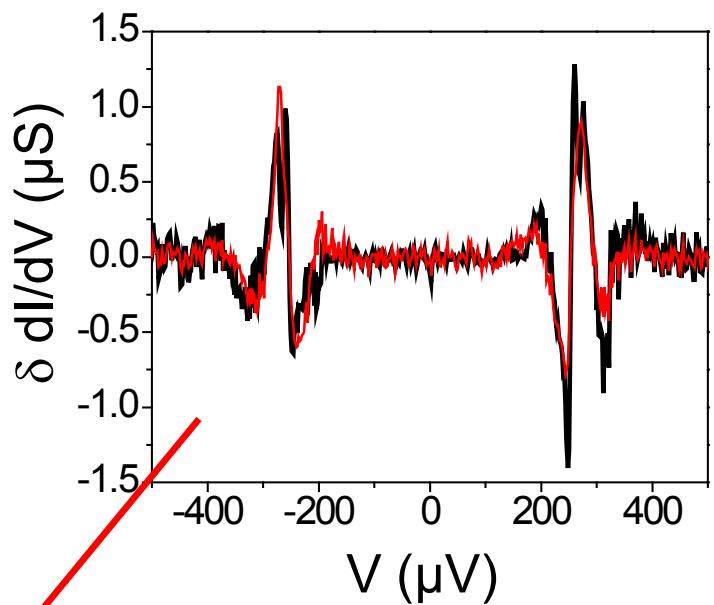
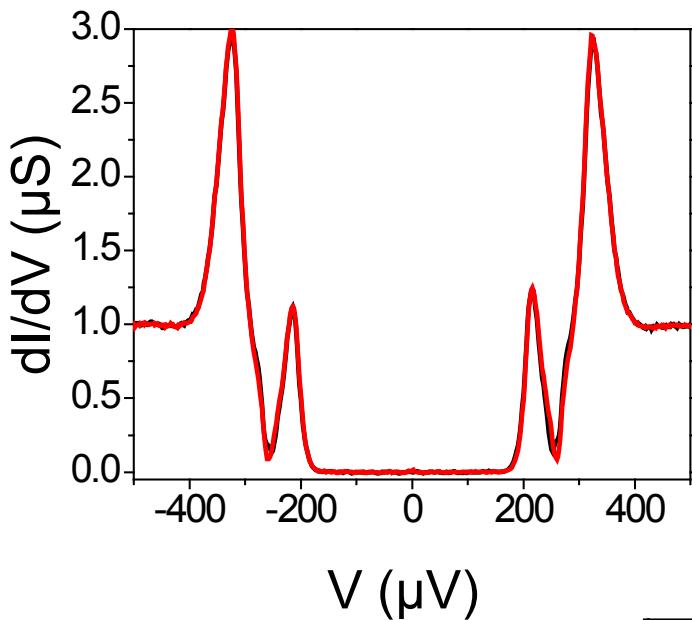
# What to do:

If  $n_2(\varepsilon, I_S) = n_2(\varepsilon, -I_S)$ , analyze:  $\delta \frac{dI}{dV}(V, U, I_S) \equiv \frac{dI}{dV}(V, U, I_S) - \frac{dI}{dV}(V, U, -I_S)$



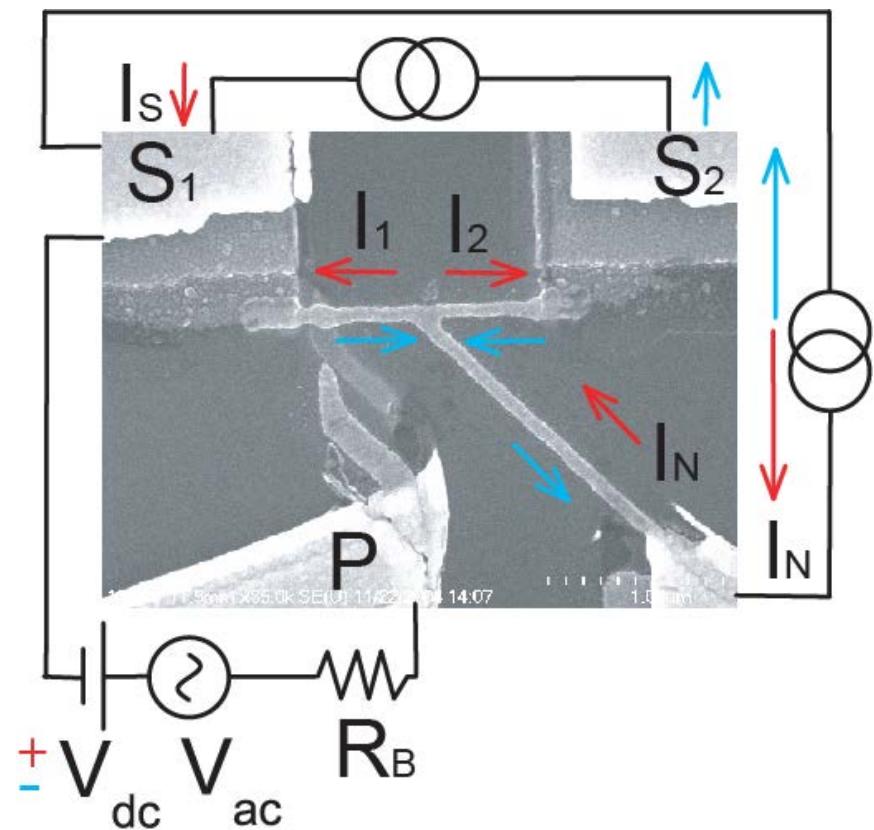
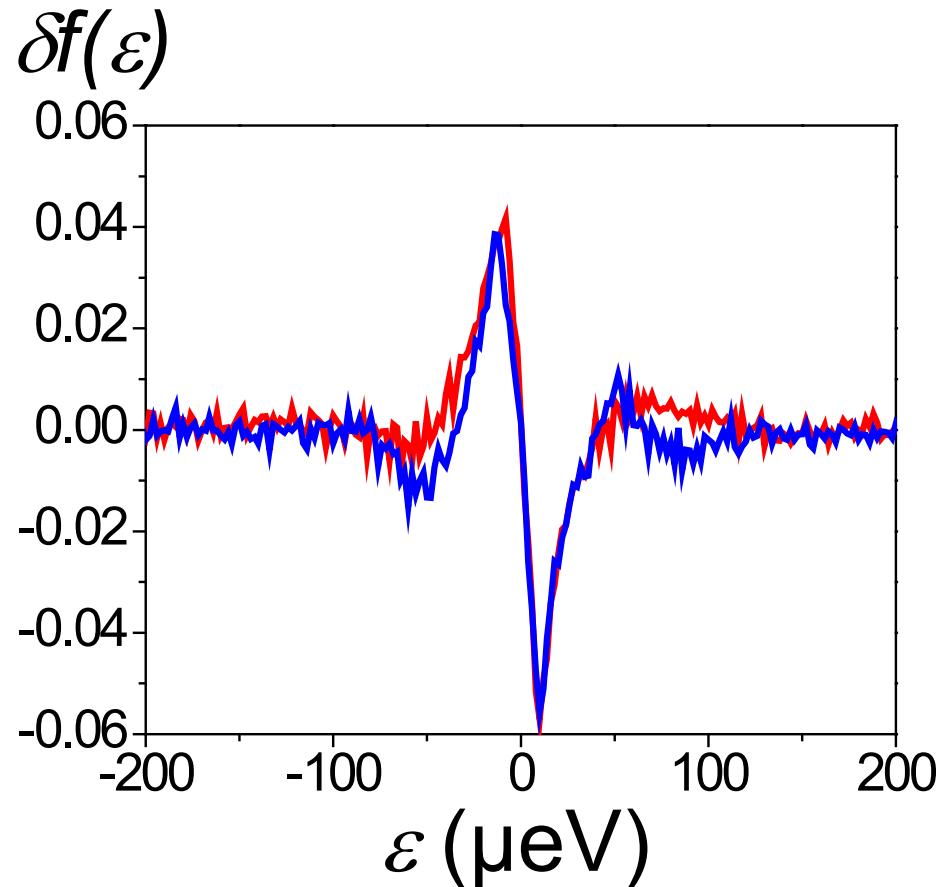
# What to do:

If  $n_2(\varepsilon, I_S) = n_2(\varepsilon, -I_S)$ , analyze:  $\delta \frac{dI}{dV}(V, U, I_S) \equiv \frac{dI}{dV}(V, U, I_S) - \frac{dI}{dV}(V, U, -I_S)$



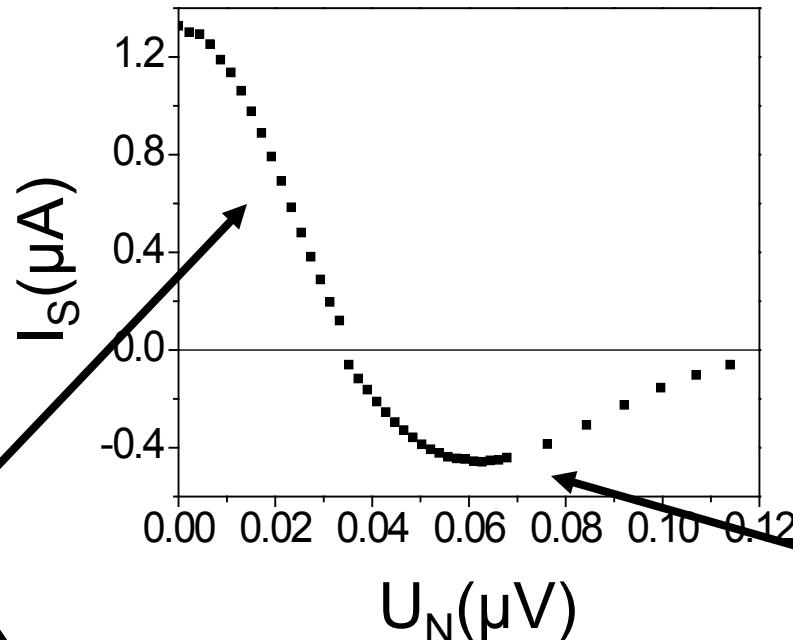
This is our result!

$\delta f(\varepsilon)$  unaffected by reversing all currents



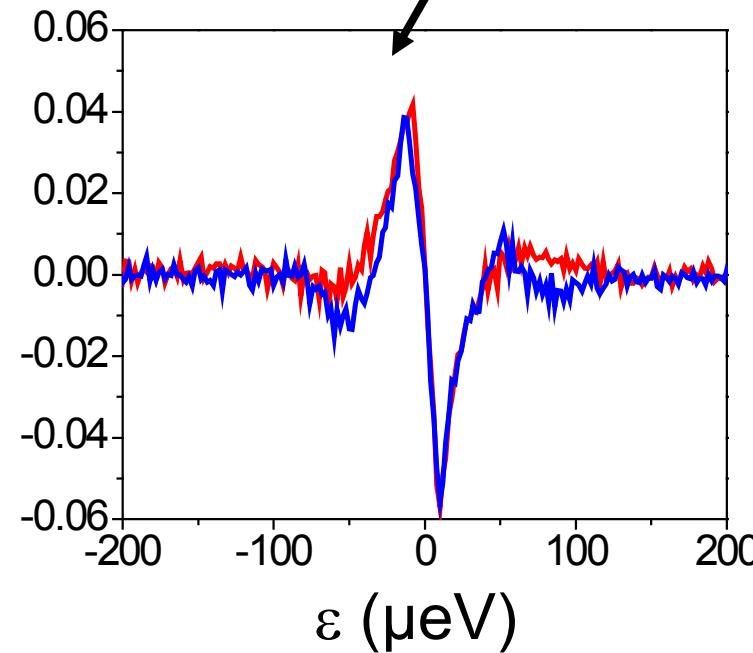
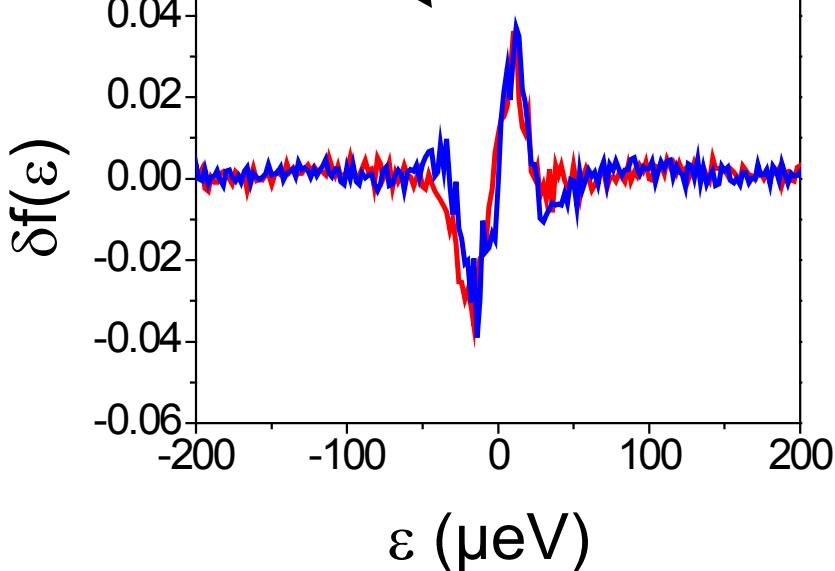
Sign of  $\delta f(\varepsilon)$   
reverses with  
 $j_E(\varepsilon)$

0-state



$U_N(\mu\text{V})$

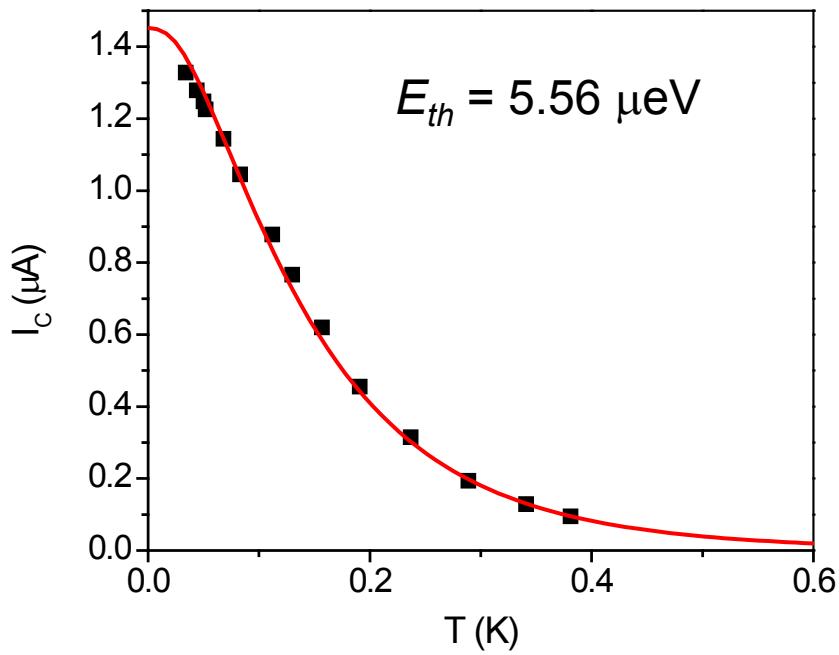
$\pi$ -state



# Fit theory to data:

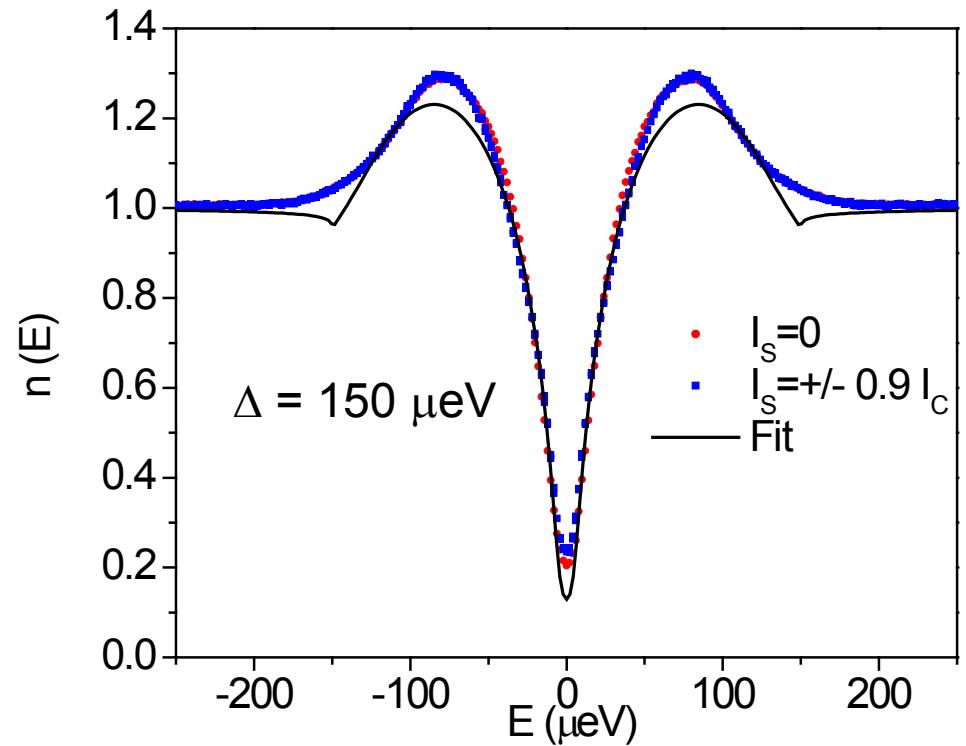
(Tero Heikkila & Pauli Virtanen)

1. Find  $E_{Th}$  from fit to  $I_c$  vs.  $T$



$$E_{th} \text{ consistent with } E_{Th} = \frac{\hbar D}{L_S^2}$$

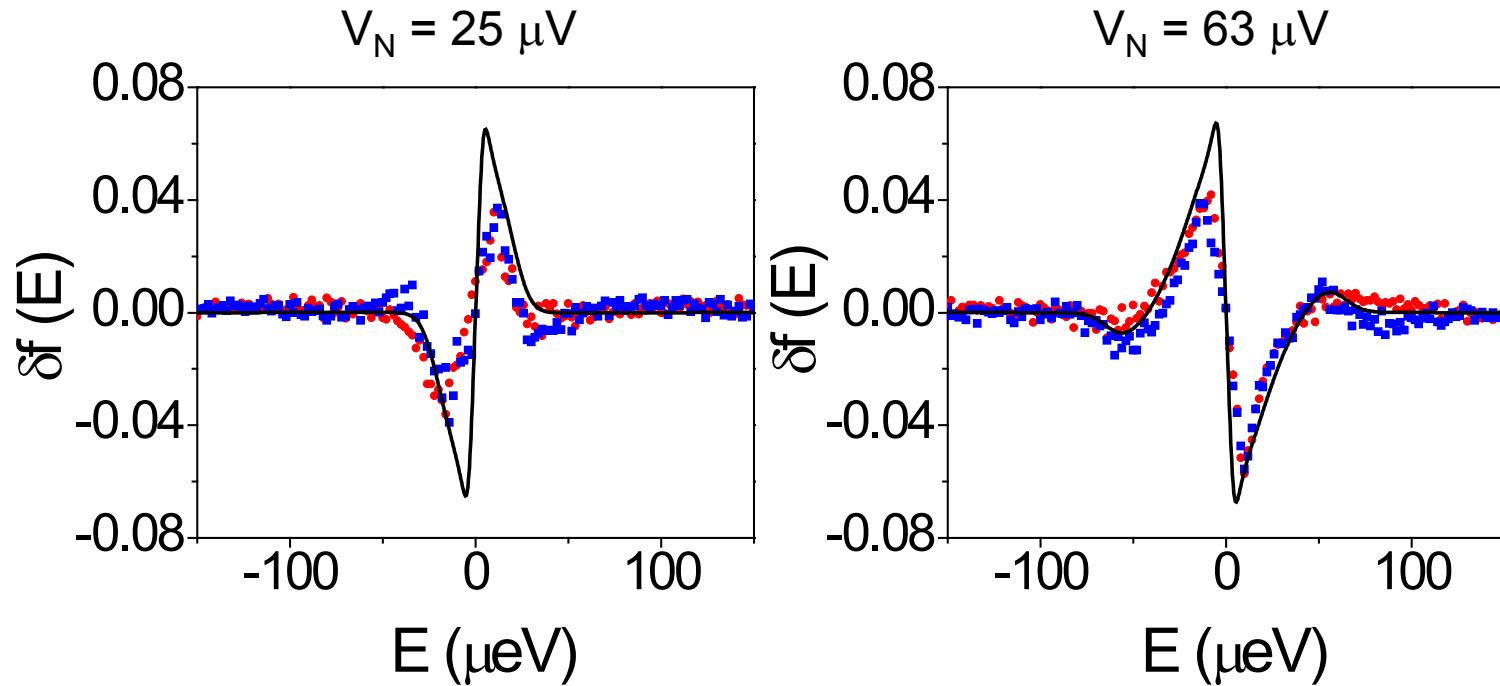
2. Find  $\Delta$  in S leads from fit to  $n(E)$



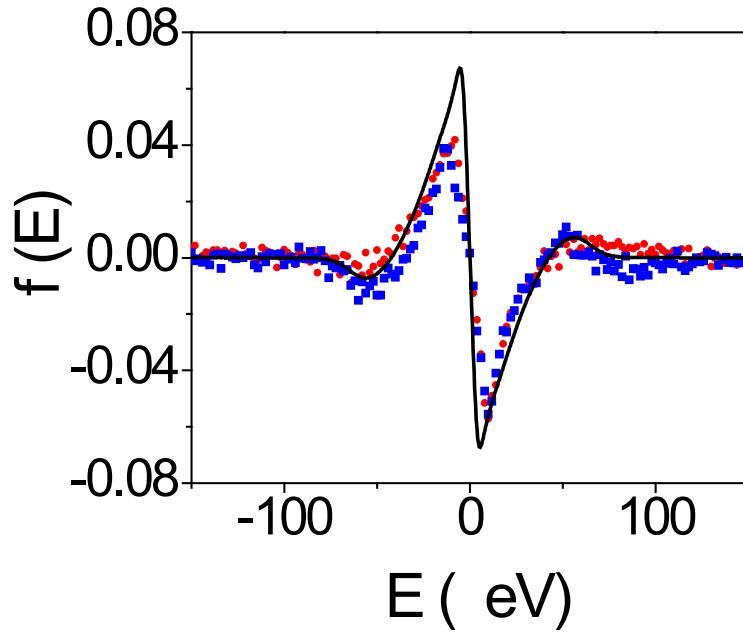
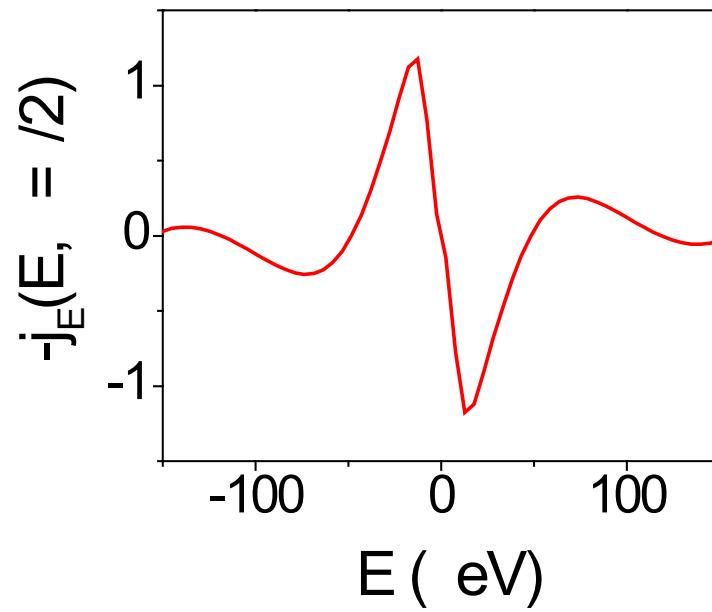
$\Delta$  suppressed due to S/N overlap region

# No further fitting parameters for $\delta f(E)$

(fit neglects inelastic scattering in Ag wire)



# Compare results with bare $j_E(E)$



Energy scale modified by proximity effect in diffusion coefficients,  $D_L$   $D_T$

# Related work: Thermopower of Andreev interferometers

Experiment:

Dikin, Jung, and Chandrasekhar, Europhys. Lett. **57**, 564 (2003)

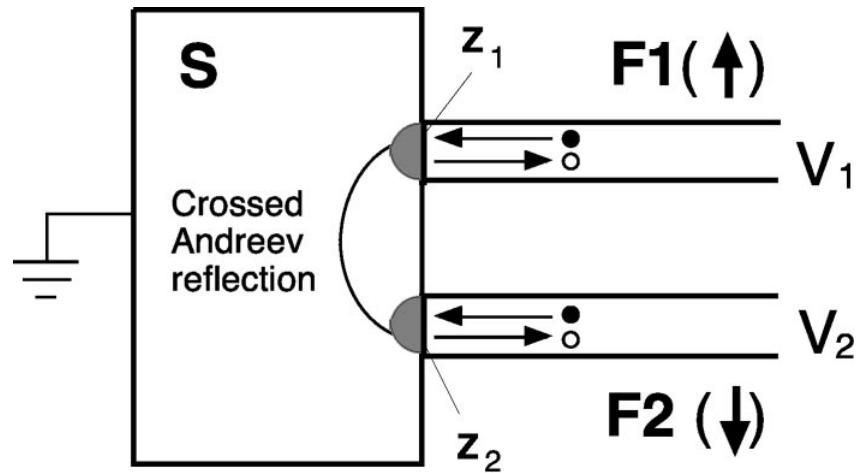
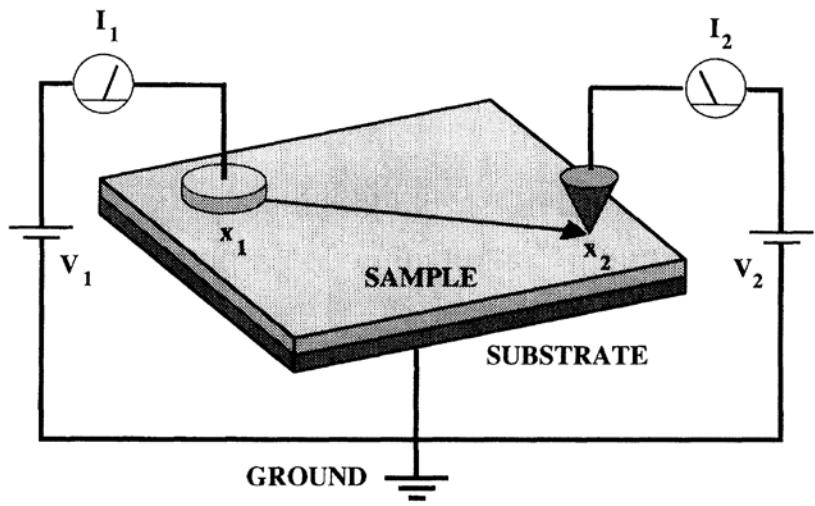
Parsons, Sosnin, and Petrushov, Phys. Rev. B **67**, 140502 (2003)

Theory:

Seviour and Volkov, Phys. Rev. B 62, 6116 (2000)

Virtanen and Heikkila, Phys. Rev. Lett. 92, 177004 (2004)

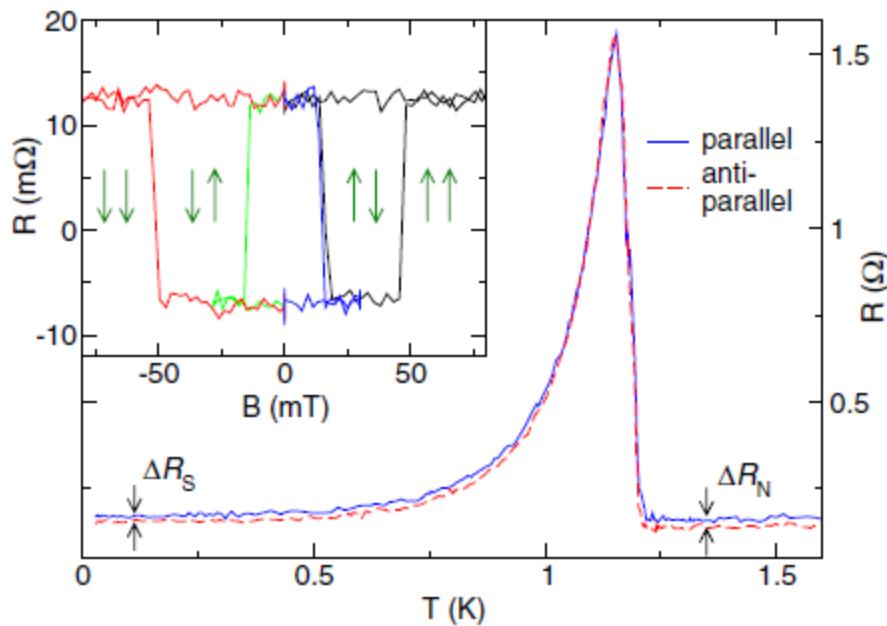
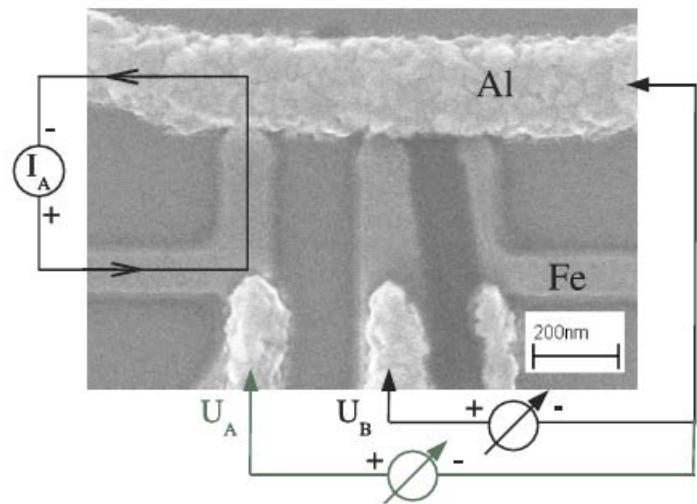
# A topic of current interest: Crossed Andreev Reflection



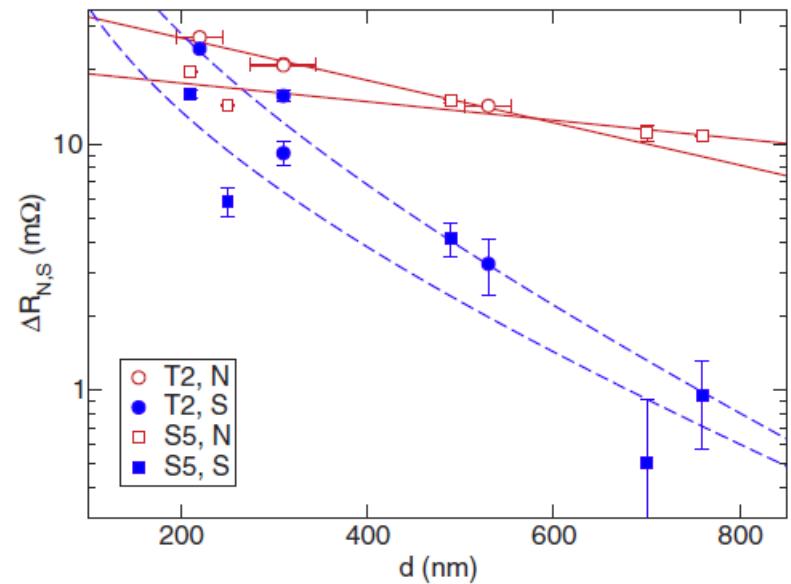
Byers & Flatte, PRL 74, 306 (1995).

Deutscher & Feinberg, APL 76, 487 (2000)

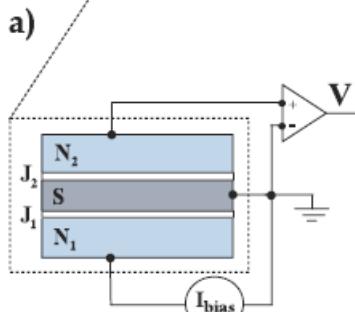
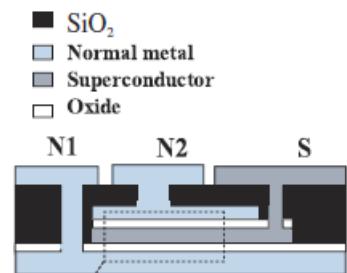
# Crossed Andreev Reflection



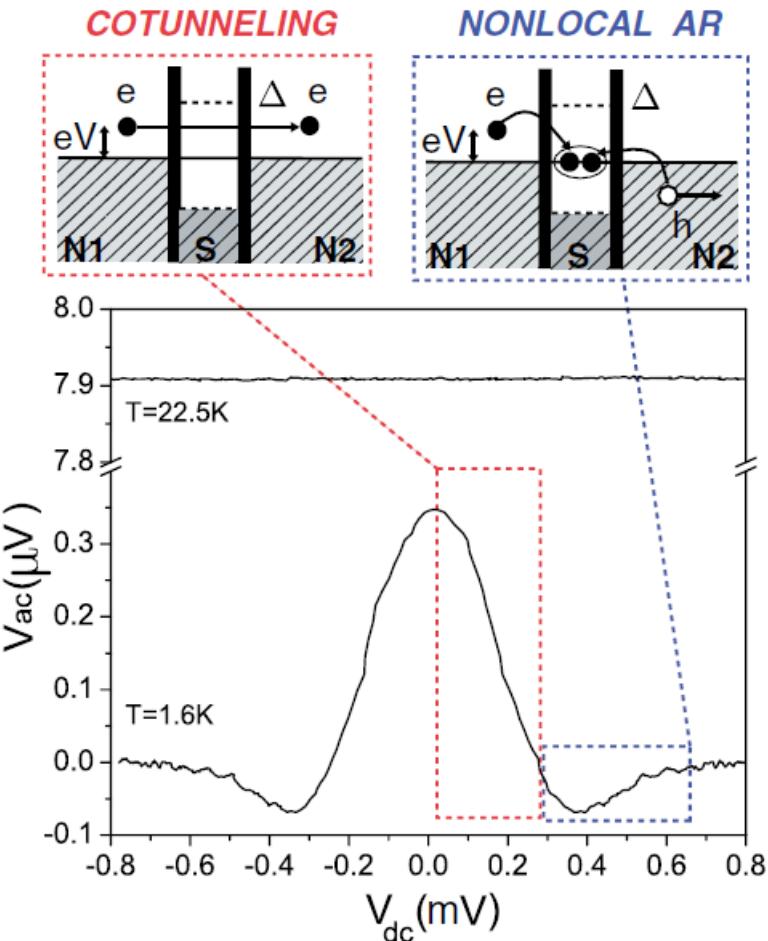
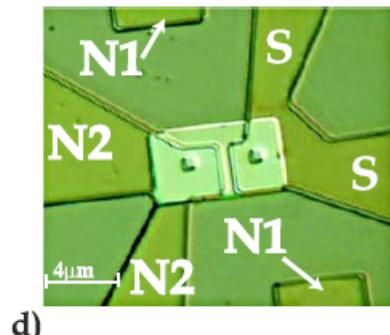
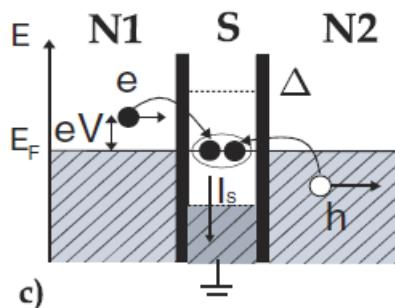
Beckmann, Weber, & Lohneysen,  
PRL 93, 197003 (2004).



# Crossed Andreev Reflection



b)



Russo, Kroug, Klapwijk, & Morpurgo,  
PRL 95, 027002 (2005).

# Entangling electrons with CAR

PRL 104, 026801 (2010)

 Selected for a Viewpoint in Physics  
PHYSICAL REVIEW LETTERS

week ending  
15 JANUARY 2010



## Carbon Nanotubes as Cooper-Pair Beam Splitters

L. G. Herrmann,<sup>1,2,5</sup> F. Portier,<sup>3</sup> P. Roche,<sup>3</sup> A. Levy Yeyati,<sup>4</sup> T. Kontos,<sup>1,2,\*</sup> and C. Strunk<sup>5</sup>

nature

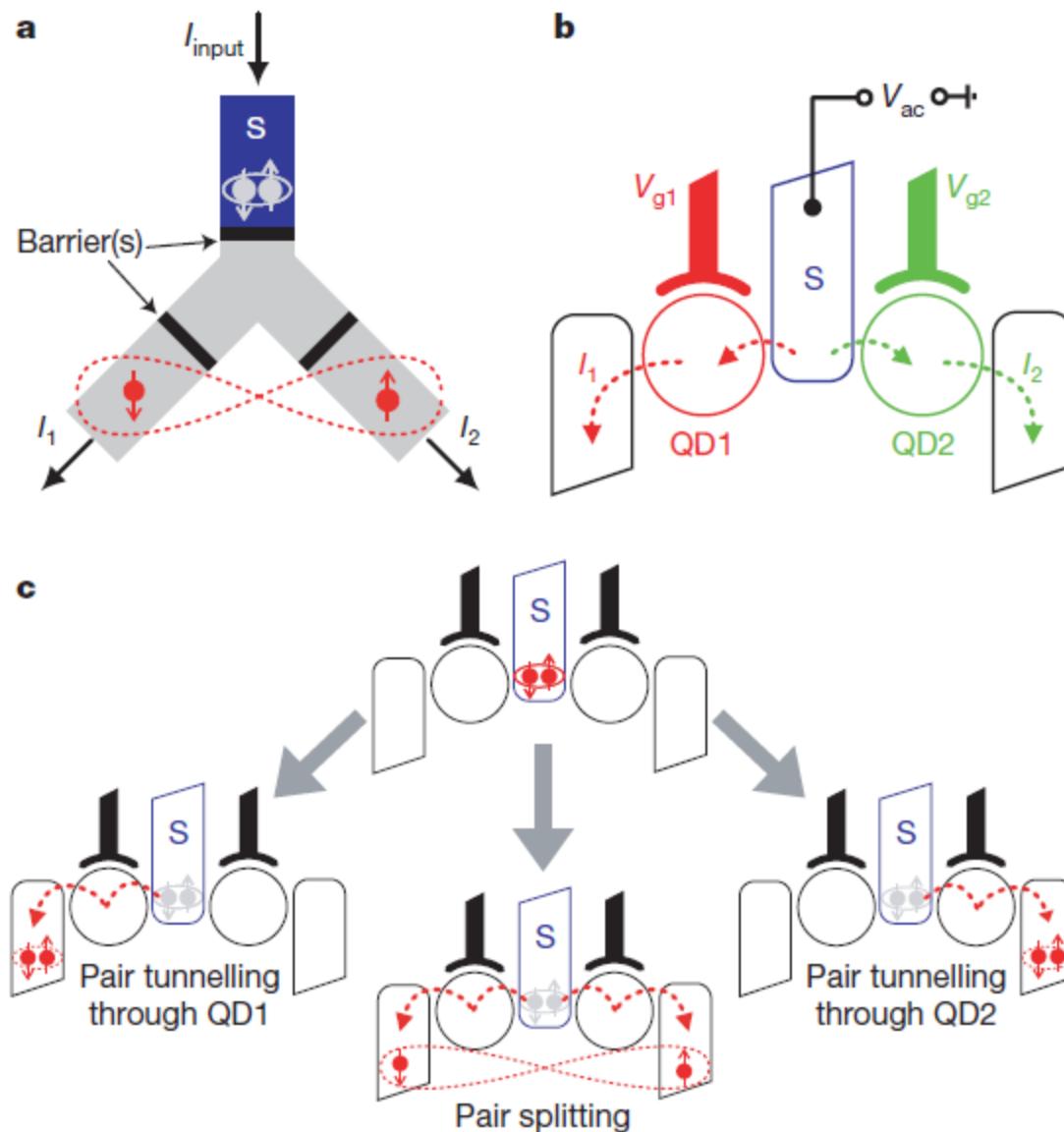
Vol 461 | 15 October 2009 | doi:10.1038/nature08432

LETTERS

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## Cooper pair splitter realized in a two-quantum-dot Y-junction

L. Hofstetter<sup>1\*</sup>, S. Csonka<sup>1,2\*</sup>, J. Nygård<sup>3</sup> & C. Schönenberger<sup>1</sup>



from Hofstetter et al.

# Resolving Andreev Bound States

nature  
physics

LETTERS

PUBLISHED ONLINE: 14 NOVEMBER 2010 | DOI: 10.1038/NPHYS1811

## Andreev bound states in supercurrent-carrying carbon nanotubes revealed

J-D. Pillet<sup>1</sup>, C. H. L. Quay<sup>1†</sup>, P. Morfin<sup>2</sup>, C. Bena<sup>3,4</sup>, A. Levy Yeyati<sup>5</sup> and P. Joyez<sup>1\*</sup>

LETTERS

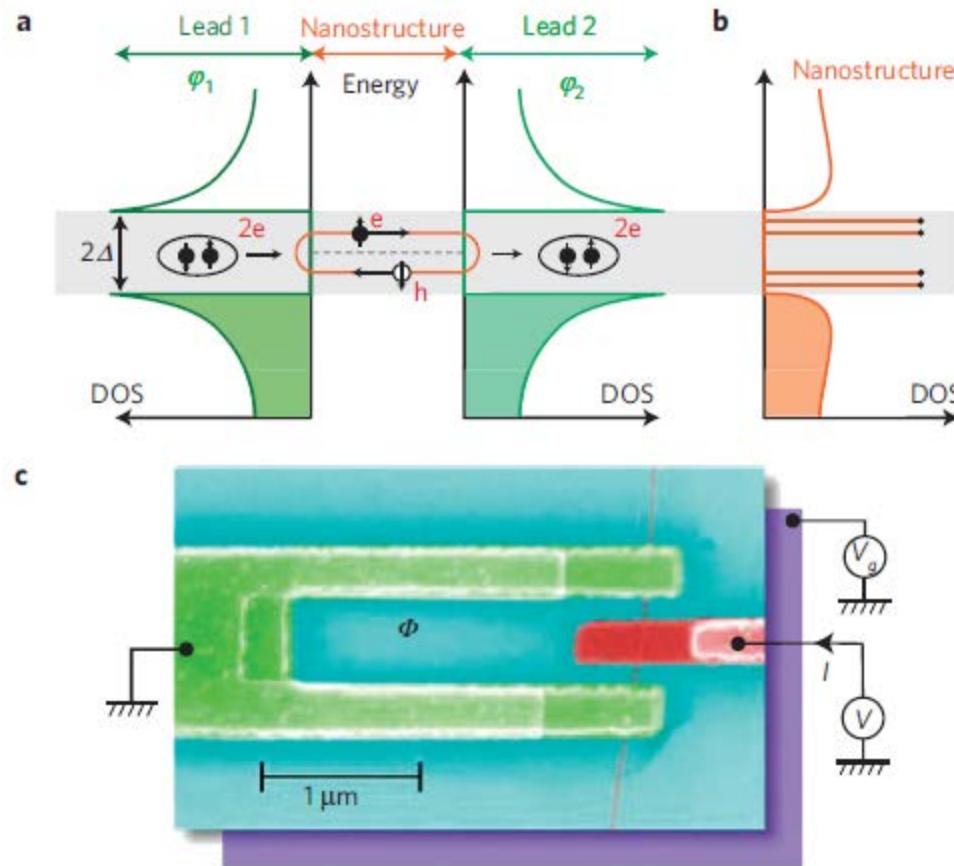
PUBLISHED ONLINE: 6 FEBRUARY 2011 | DOI:10.1038/NPHYS1911

nature  
physics

## Transport through Andreev bound states in a graphene quantum dot

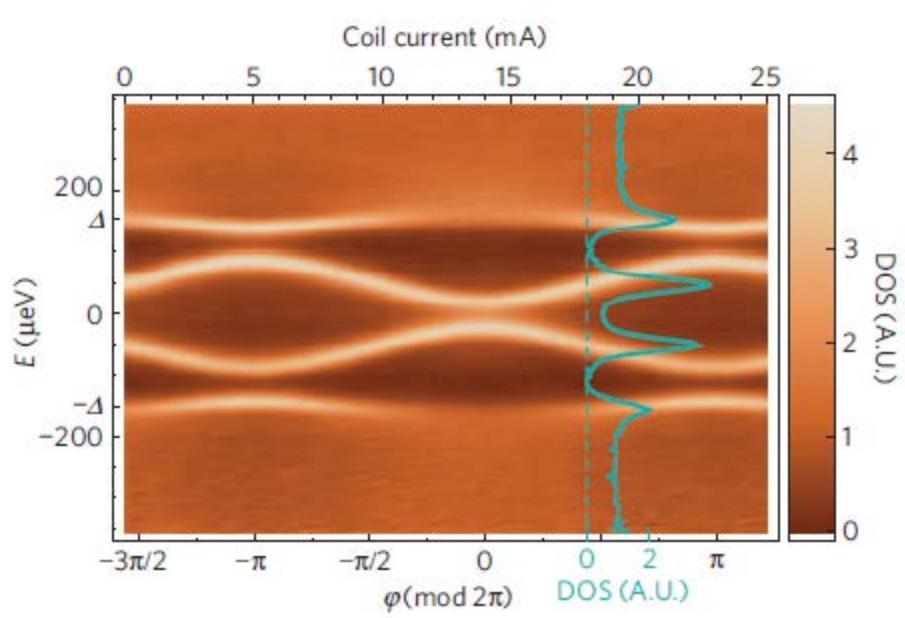
Travis Dirks, Taylor L. Hughes, Siddhartha Lal<sup>†</sup>, Bruno Uchoa, Yung-Fu Chen, Cesar Chialvo, Paul M. Goldbart<sup>†</sup> and Nadya Mason<sup>\*</sup>

# Andreev Bound States

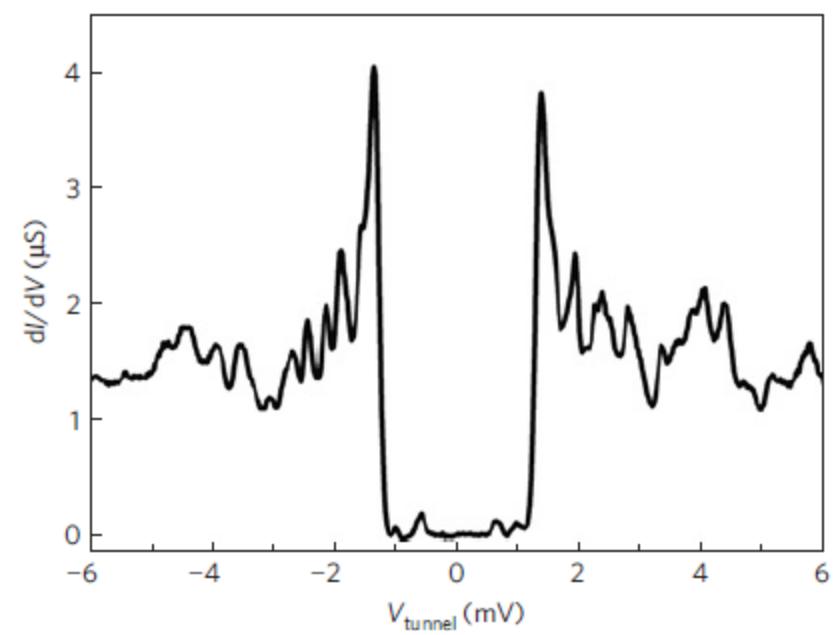


from Pillet et al.

# Andreev bound states revealed by tunneling spectroscopy

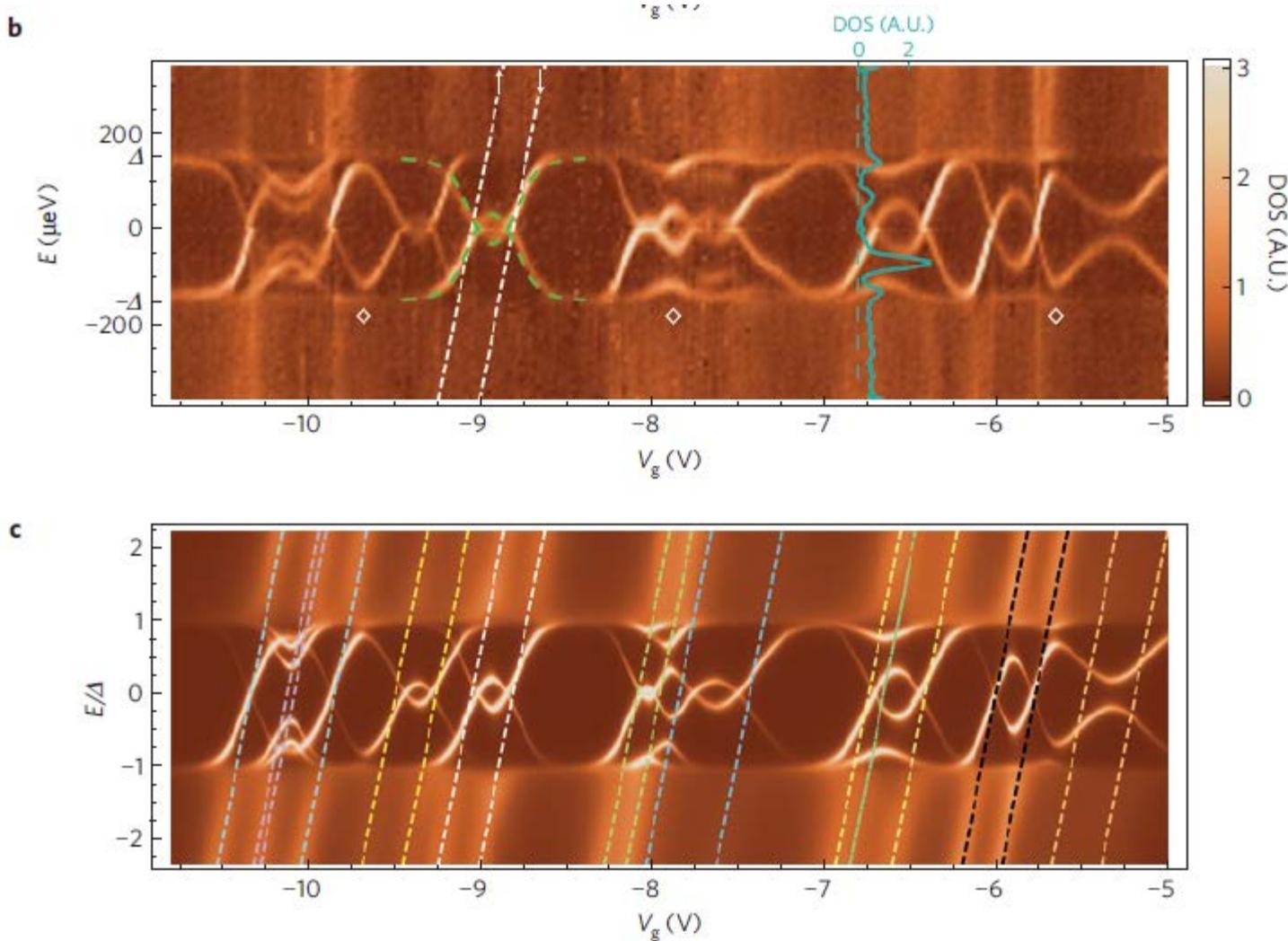


Flux dependence  
(from Pillet et al.)



From Dirks et al.

# Gate dependence



from Pillet et al.