

Lecture 4: strongly interacting Fermions



thermodynamic properties

Measuring the EoS of the Homogeneous Gas

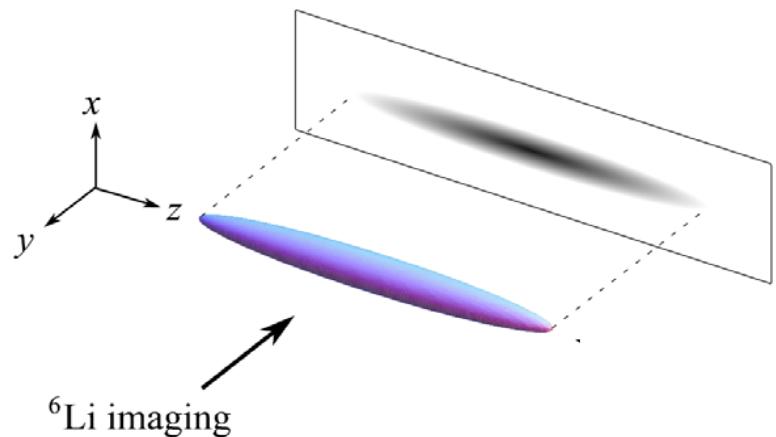


Extraction of the pressure from *in situ* images

$$P(\mu_{1z}, \mu_{2z}, T) = \frac{m\omega_r^2}{2\pi} (\bar{n}_1(z) + \bar{n}_2(z))$$

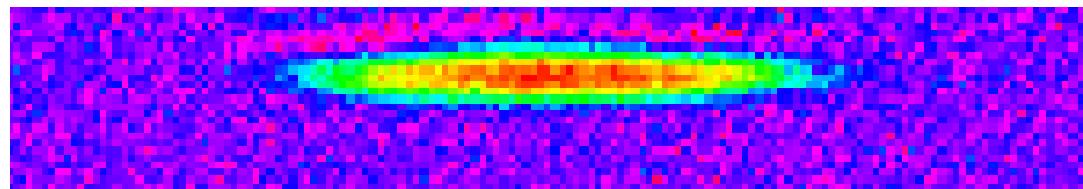
Ho, T.L. & Zhou, Q.,
Nature Physics, 09
S. K Yip, 07

- $\bar{n}_i(z) = \int dx dy n_i(x, y, z)$
doubly-integrated density profiles
equation of state measured for
all values of (μ_{1z}, μ_{2z}, T)



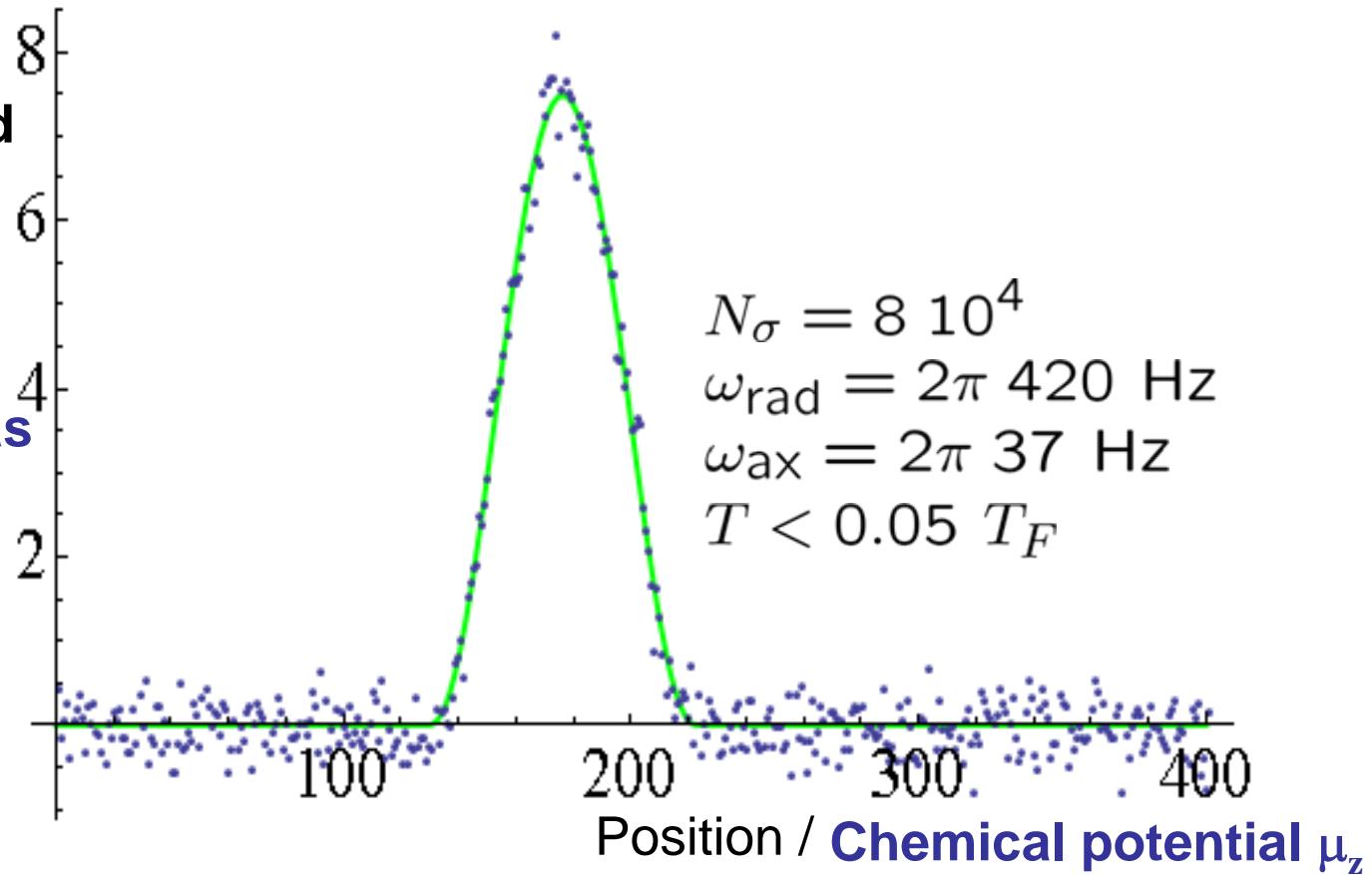
Unitary Fermi Gas

$a = \infty$



Doubly integrated
Density

Pressure of the
locally
homogeneous gas



Ground state of a tunable Fermi gas; T=0

- Pressure of single-component Fermi gas:

$$P_0(\mu) = \frac{1}{15\pi^2} \left(\frac{2m}{\hbar^2} \right)^{\frac{3}{2}} \mu^{5/2}$$

- Balanced two-component Fermi gas with adjustable a

$$\tilde{\delta} = \frac{\hbar}{\sqrt{2m\tilde{\mu} a}}$$

$\tilde{\delta}$: grand-canonical
analog of $1/k_F a$

with $\tilde{\mu} = \mu$ for $a < 0$

and $\tilde{\mu} = \mu - E_B$ for $a > 0$

$$P(\tilde{\mu}, a) = 2P_0(\tilde{\mu})h_S(\tilde{\delta})$$

The Equation of State in the BEC-BCS crossover

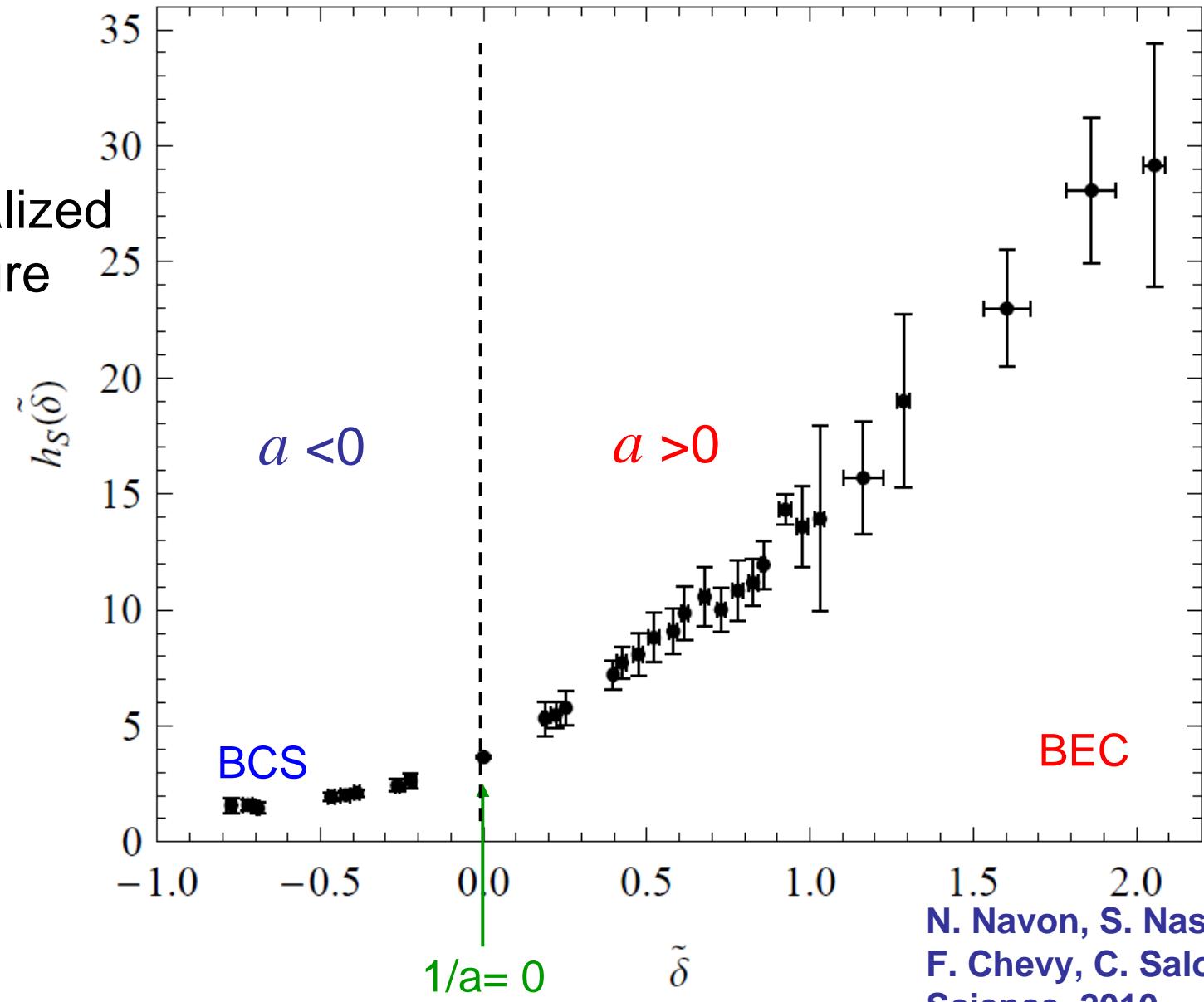
$$1/k_F a \neq 0$$

The ground state: T=0

N. Navon, S. Nascimbene, F. Chevy and C. Salomon,
Science 328, 729 (2010)

Superfluid Equation of State in the Crossover

Normalized pressure

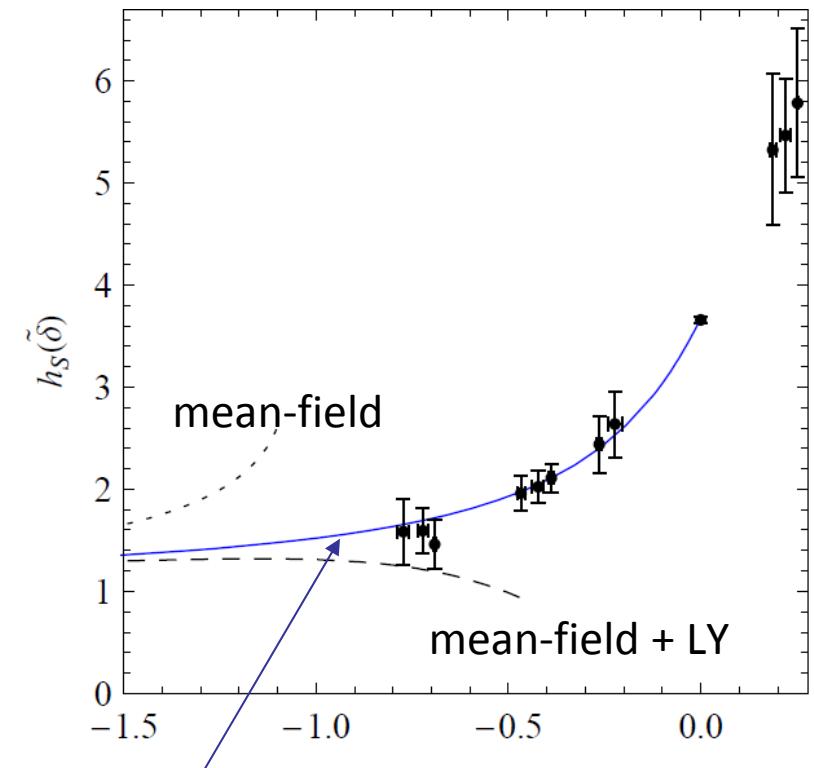


N. Navon, S. Nascimbène,
F. Chevy, C. Salomon
Science, 2010

Asymptotic behaviors: BCS limit

$$E = \frac{3}{5} N E_F \left(1 + \frac{10}{9\pi} k_F a + \frac{4(11 - 2 \log 2)}{21\pi^2} (k_F a)^2 \dots \right)$$

mean-field Lee-Yang correction, 1957



$$h_S^{\text{BCS}}(\tilde{\delta}) = \frac{\tilde{\delta}^2 + \alpha_1 \tilde{\delta} + \alpha_2}{\tilde{\delta}^2 + \alpha_3 \tilde{\delta} + \alpha_4}$$

Padé approximant with mean-field value used as a constraint

From $h_S^{\text{BCS}}(\tilde{\delta})$: we get the

Lee-Yang coefficient: 0.18(2)

theory: $\frac{4(11 - 2 \log 2)}{21\pi^2} \simeq 0.186$

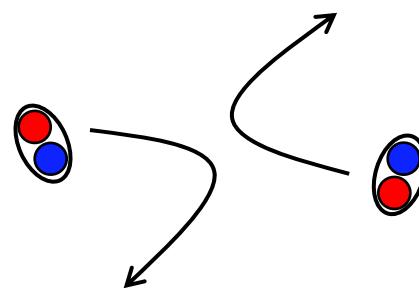
Asymptotic behavior (2): BEC limit

$$E = \frac{N}{2} E_b + N \frac{\pi \hbar^2 a_{dd}}{2m} n \left(1 + \frac{128}{15\sqrt{\pi}} \sqrt{n a_{dd}^3} + \dots \right)$$

molecular
binding
energy

mean-field

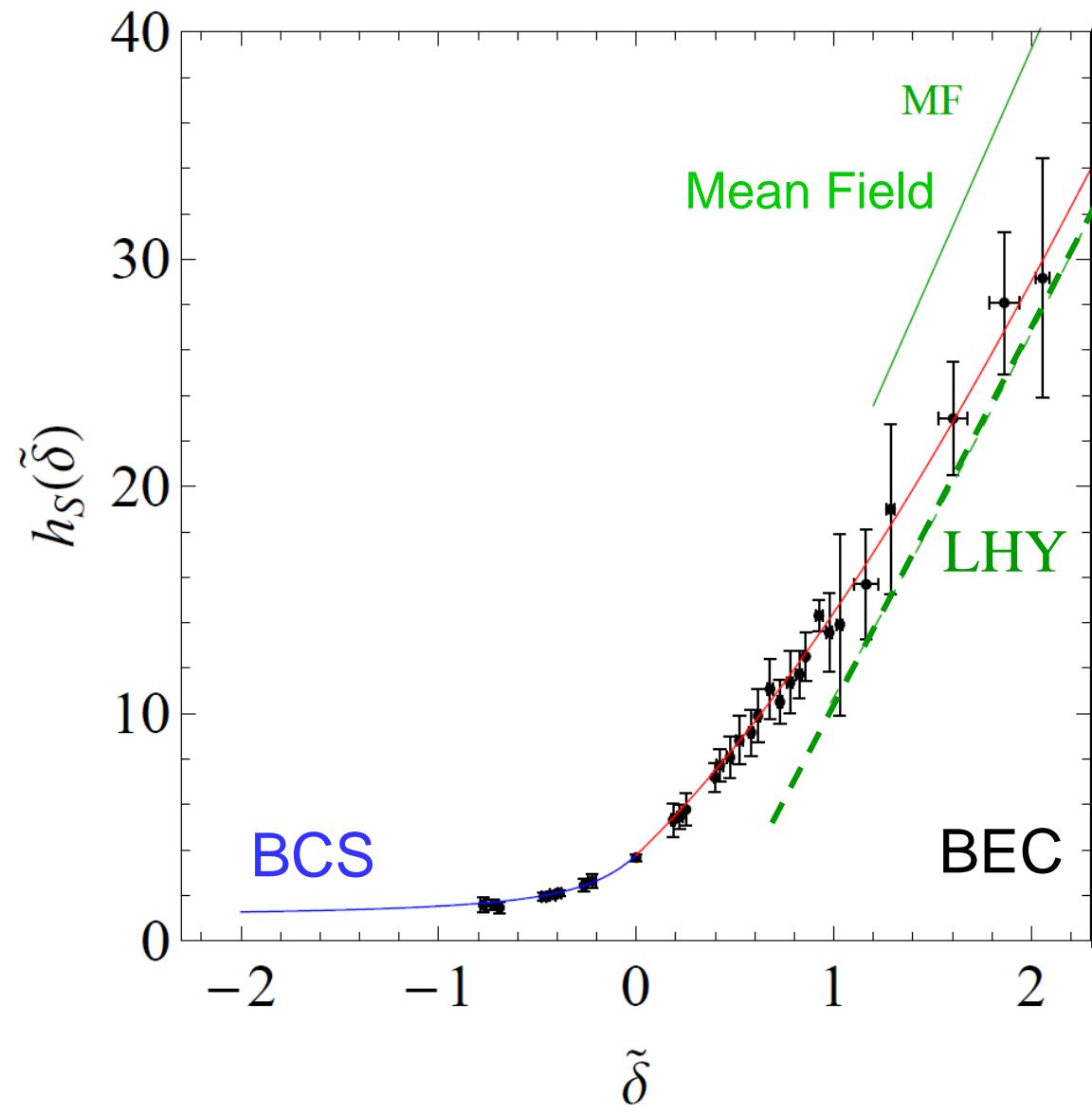
Lee-Huang-Yang
correction, 1957



$$a_{dd} = 0.6a$$

D. Petrov, C. Salomon, G. Shlyapnikov, 2004

Measurement of the Lee-Huang-Yang correction



Fit of the LHY coefficient:
4.4(5)

theory: $\frac{128}{15\sqrt{\pi}} \simeq 4.81$

LHY valid for composite
bosons

X. Leyronas *et al*,
PRL 99, 170402 (2007)

Difference between
composite bosons and
point-like bosons
only at next order

Asymptotic behavior (3):unitary limit $a = \infty$

$$E = \frac{3}{5} N E_F \left(\xi_s - \zeta \frac{1}{k_F a} + \dots \right)$$

$$\mu = (1 + \beta) \frac{\hbar^2}{2m} (6\pi^2 n)^{2/3} = \xi_s E_F$$

ξ_s is a universal number

We get: $\xi_s = 0.41(1)(2)$

In excellent agreement with Monte Carlo calculations

Astrakharchik et al.: 0.42(1)

Carlson et al. : 0.41(1)

$\zeta = 0.93(5)$ is related to the contact coefficient C
introduced by S. Tan

The Equation of State at unitarity: temperature dependence

$$1/k_F a = 0$$

Thermodynamics is universal

J. Ho, E. Mueller, '04

Pressure depends only on $\mu/k_B T$

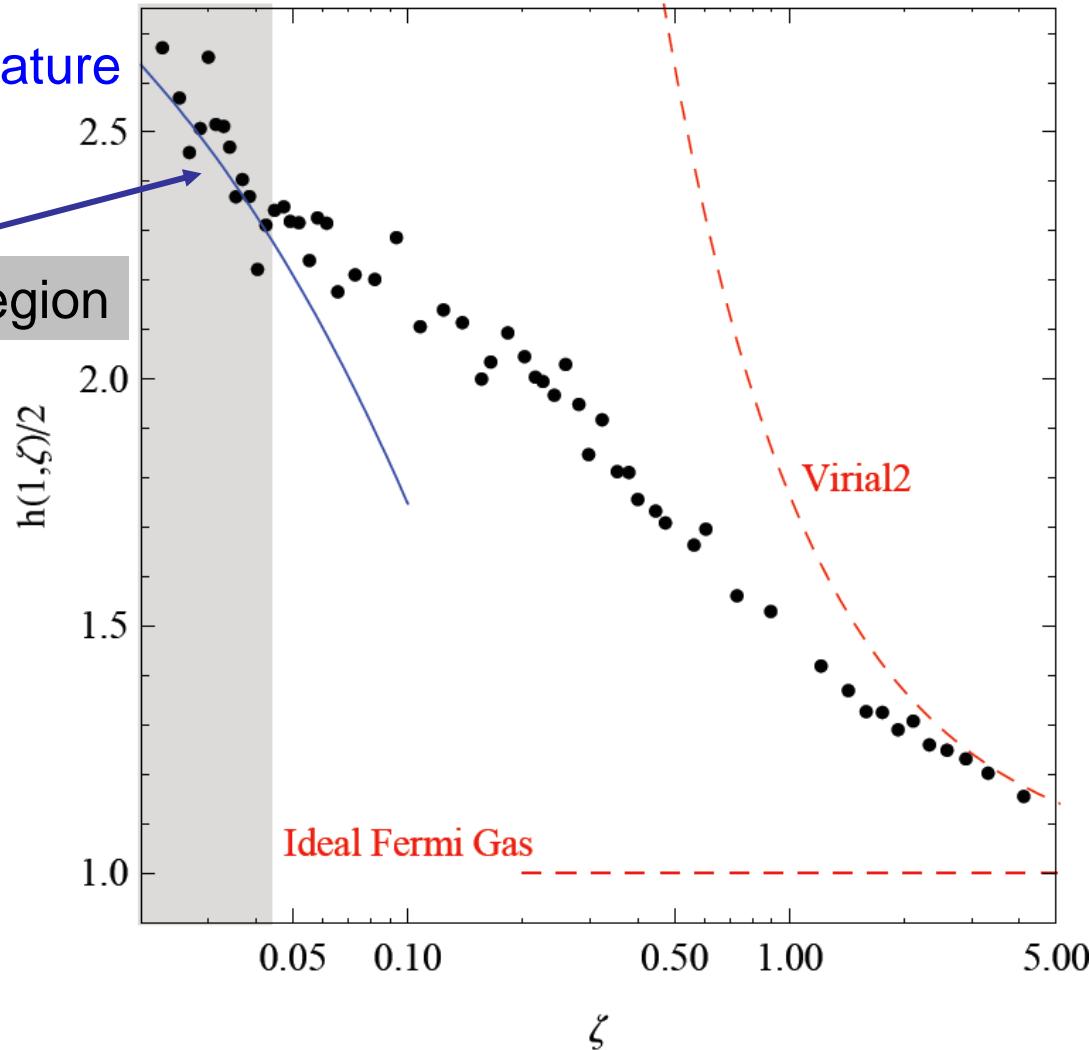
S. Nascimbène et al., Nature, 463, 1057, (2010)

Equation of state of balanced gas

$$P(\mu, T) = P_1(\mu, T)h(1, \zeta)$$

Low temperature

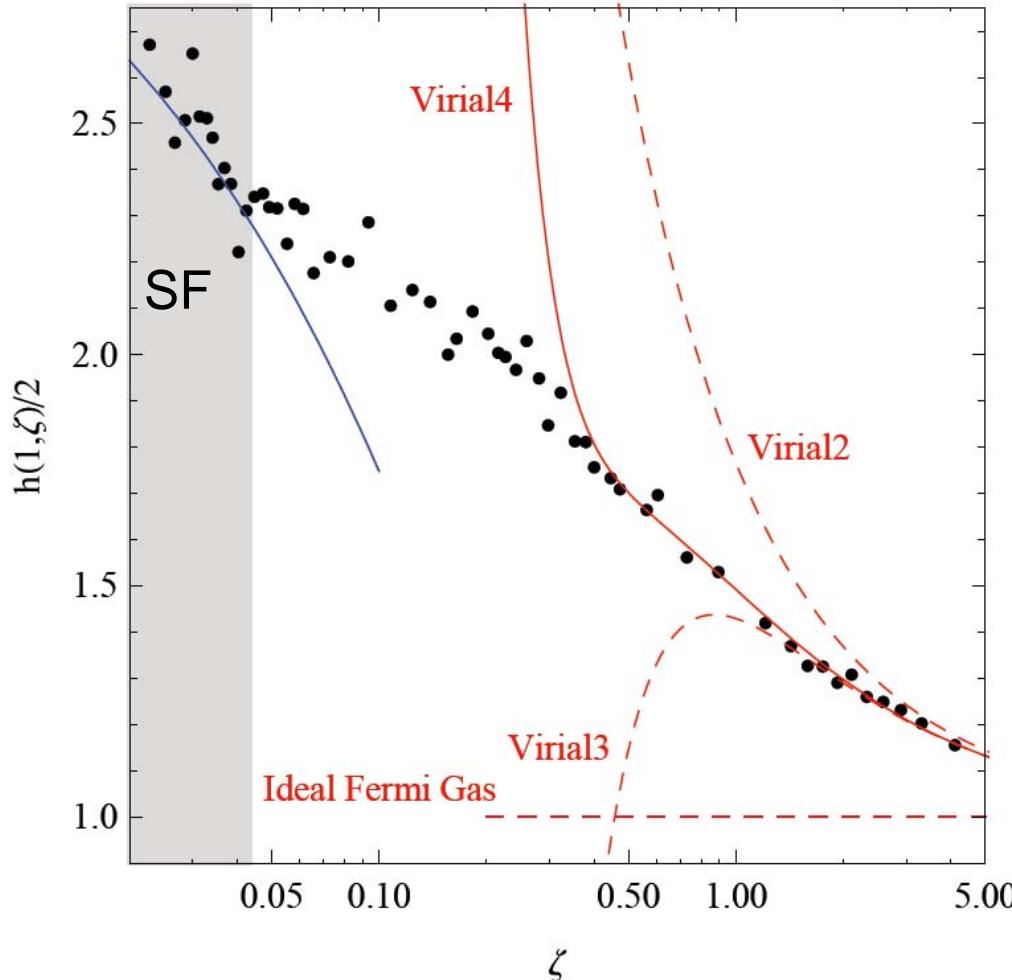
Superfluid region



$$\zeta = \exp\left(-\frac{\mu_1}{k_B T}\right)$$

High T : virial expansion

$$\frac{h(1, \zeta)}{2} = \frac{\sum_{n=1}^{\infty} ((-1)^{n+1} n^{-5/2} + b_n) \zeta^{-n}}{\sum_{n=1}^{\infty} (-1)^{n+1} n^{-5/2} \zeta^{-n}}$$



$$b_3 = -\boxed{0.35}(2)$$

$$b_3^{\text{th}} = -0.355$$

X. Liu et al., PRL 102, 160401 (2009)

$$b_3^{\text{th}} = 1.05$$

G. Rupak, PRL 98, 90403 (2007)

$$b_4 = \boxed{0.096}(15)$$

No theoretical prediction
→ 4-body problem

Comparison with Many-Body Theories

Diagram. MC

E. Buровски et. al., PRL 96, 160402 (2006)



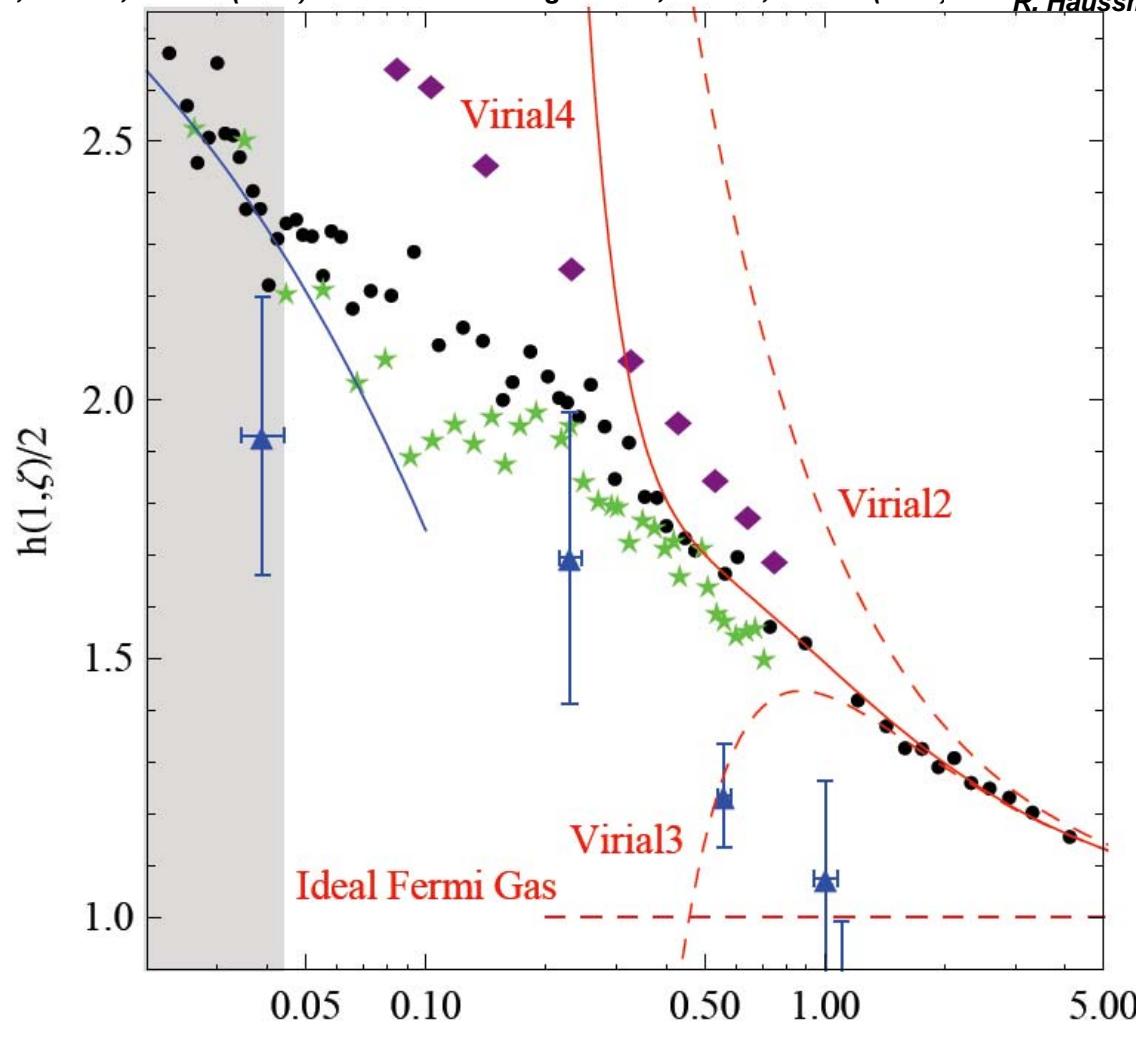
QMC

A. Bulgac et al., PRL 99, 120401 (2006)

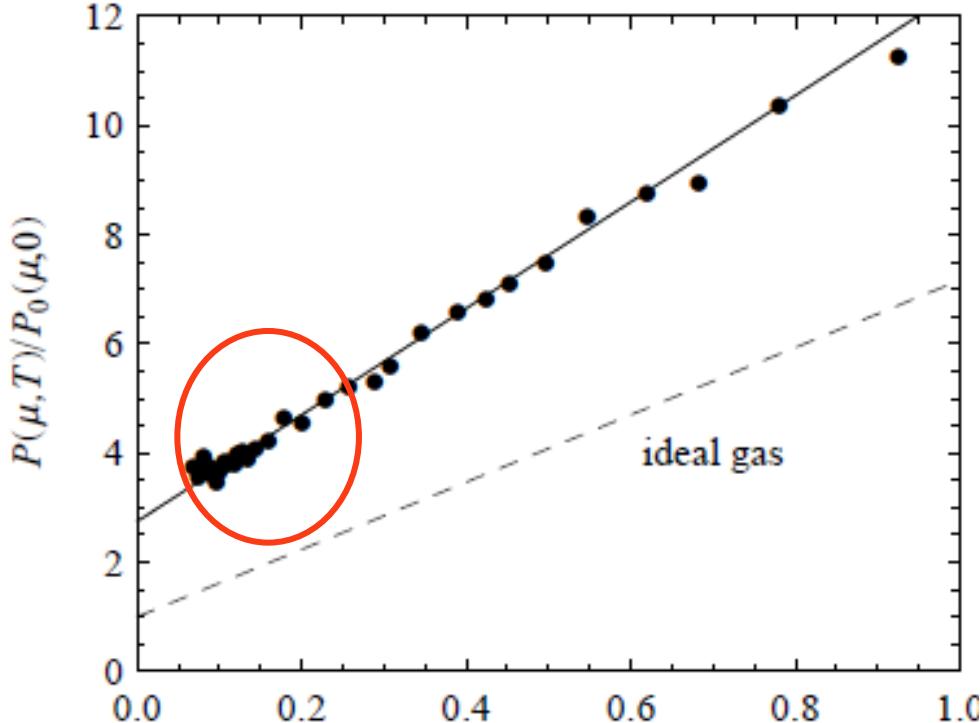


Diagram.+analytic

R. Haussmann. et al., PRA 75, 023610 (2007)



Low Temperature



$$(k_B T / \mu)^2$$

Normal phase : Fermi liquid behavior

$$P(\mu, T) = 2P_1(\mu, 0) \left(\xi_n^{-3/2} + \frac{5\pi^2}{8} \xi_n^{-1/2} \frac{m^*}{m} \left(\frac{k_B T}{\mu} \right)^2 \right)$$

we find : $\xi_n = 0.51(2)$ $\xi_n^{\text{th}} = 0.56$

$$m^* / m = 1.13(3)$$

C. Lobo et al., PRL 97, 200403 (2006)

Normal-Superfluid phase transition

We find the critical parameters

$$(k_B T / \mu)_c = 0.32(3)$$

0.32(2)  *E. Burovski et al., PRL 96, 160402 (2006)*

0.24  *K.B. Gubbels and H.T.C Stoof, PRL 100, 140407 (2008)*

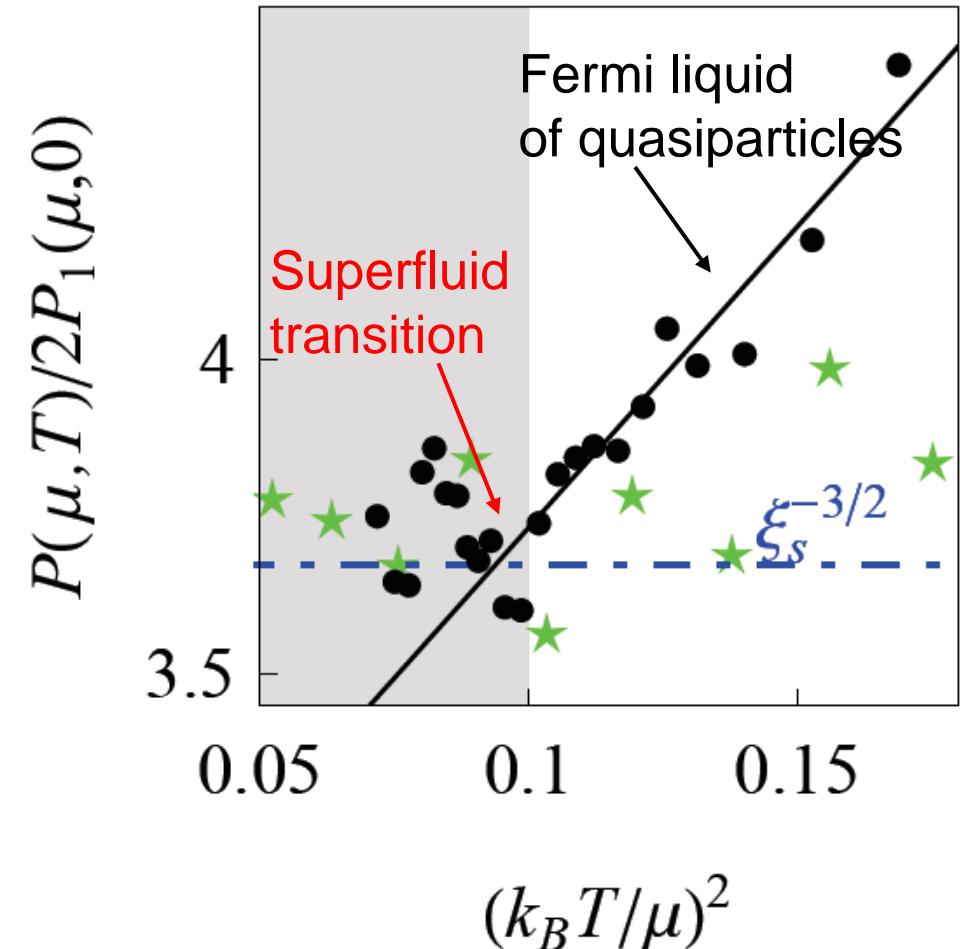
0.32  *A. Bulgac et al., PRA, 78, (2008)*

0.41  *R. Haussmann. et al., PRA 75, 023610 (2007)*

$$(\mu/E_F)_c = 0.49 (2)$$

also

$$T_c = 0.157(15)T_F$$



Good agreement with theory, with Riedl et al.,
and with M. Horikoshi, et al. *Science* **327**, 442 (2010);

What happens to superfluidity with imbalanced Fermi Spheres ?

Superconductors: apply an external magnetic field
but Meissner effect

Cold Atoms: change spin populations

A question discussed extensively since the BCS theory
and more than 30 papers in the last 3 years

Definition : $P = \frac{N_1 - N_2}{N_1 + N_2}$ $\delta\mu = \mu_1 - \mu_2$

P: spin polarization

MIT '06,: 3 phases, RICE '06: 2 phases, ENS '09: 3 phases

Clogston-Chandrasekhar limit

- Naive argument using BCS picture :

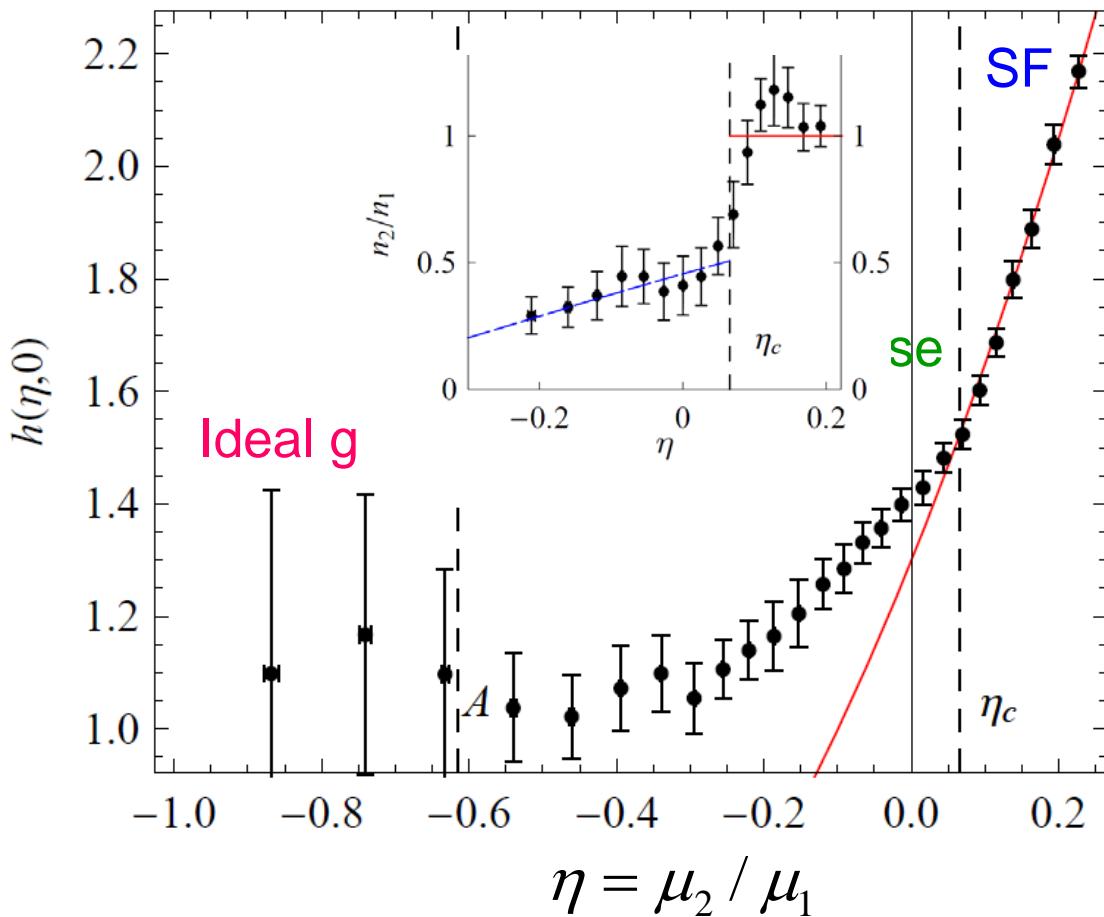
the energy of excess particles must be compared with
« robustness » of the fermion pairs :

$$\delta\mu \lesssim \Delta : \text{SF is stable with equal densities}$$
$$\delta\mu \gtrsim \Delta \quad ?$$

- Relation predicted by BCS theory :

$$\delta\mu_c = \frac{\Delta}{\sqrt{2}}$$

Equation of state $h(\eta, 0)$ i.e.(T=0)



$$h_s(\eta, 0) = \frac{1}{(2\xi_s)^{3/2}} (1 + \eta)^{5/2}$$

Deviation from h_s at

$$\eta_c = 0.065(20)$$

T=0 SF-Normal Phase Transition

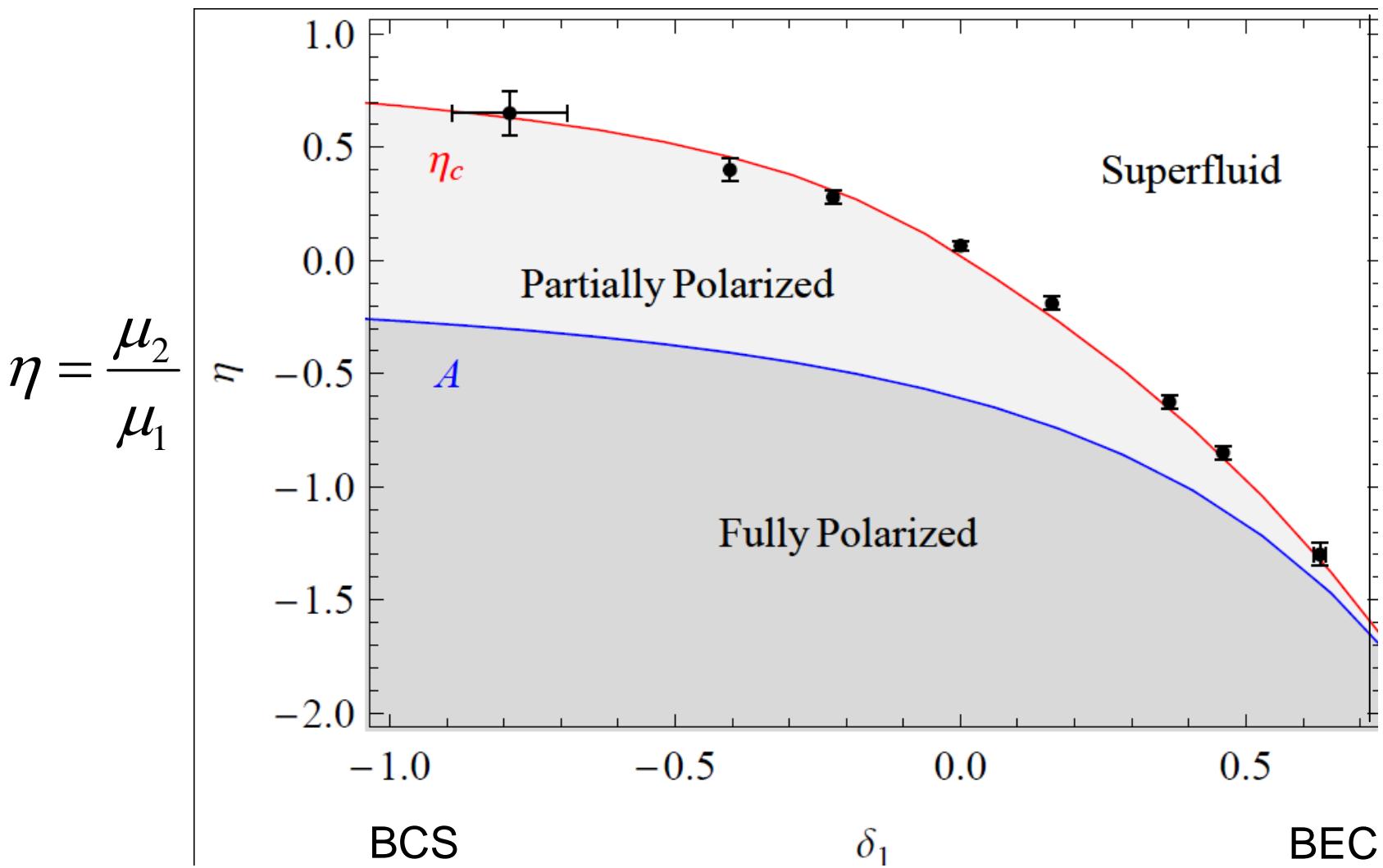
$$\eta_c = 0.02 \quad \text{Fixed-Node}$$

$$\eta_c = 0.03(2)$$

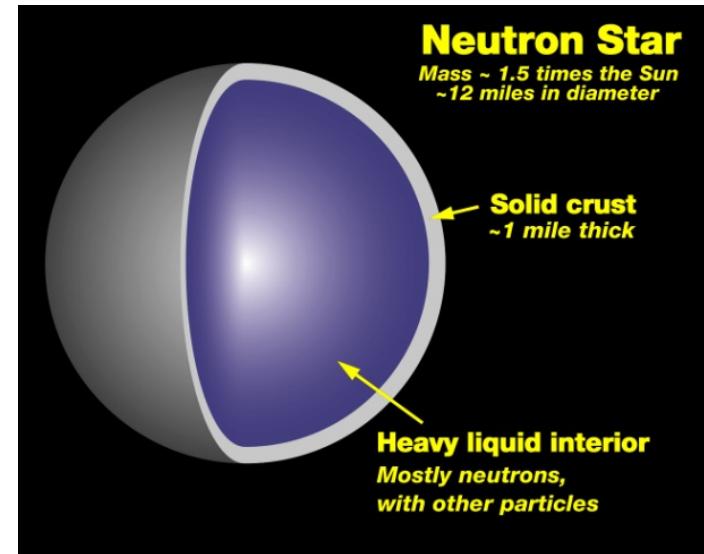
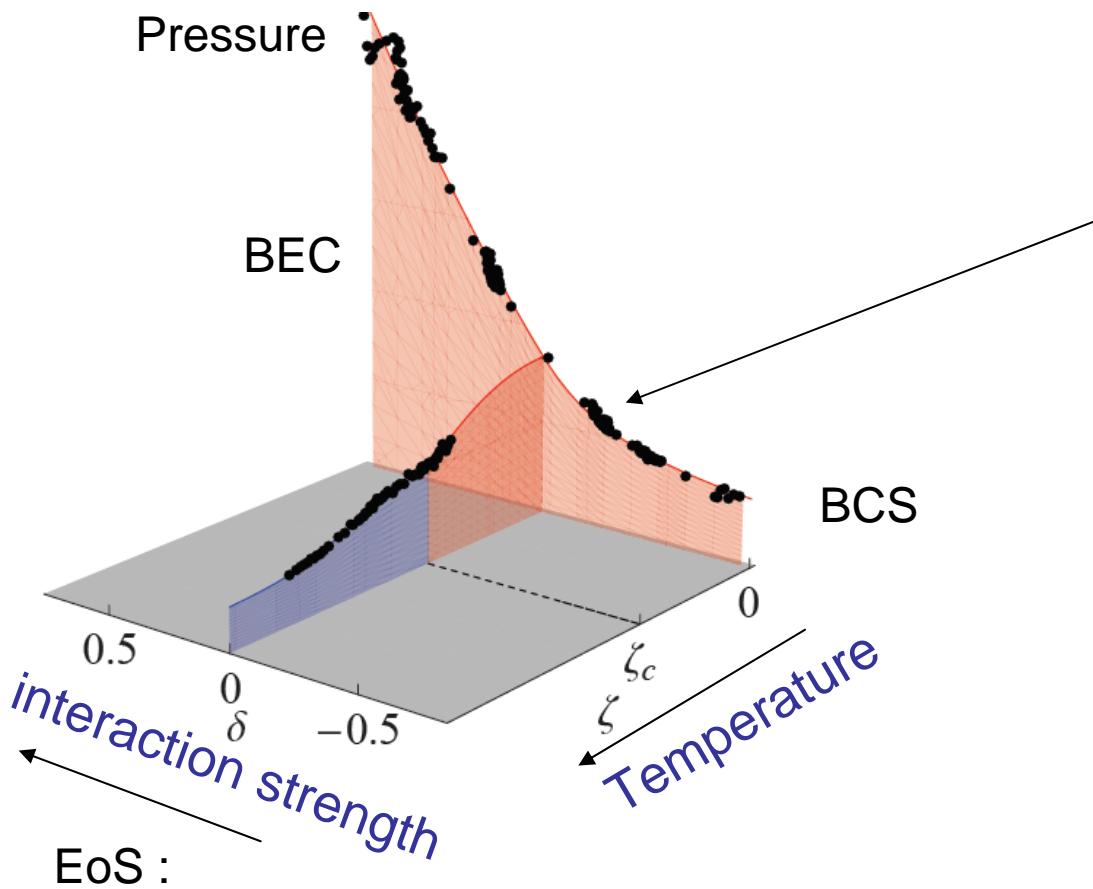
$$h(\eta, 0) = \begin{cases} \frac{1}{(2\xi_s)^{3/2}} (1 + \eta)^{5/2} & \text{if } \eta > \eta_c \\ h_n(\eta, 0) & \text{if } A < \eta < \eta_c \\ 1 & \text{if } \eta < A \end{cases}$$

MIT: Y. Shin, PRA 08

Phase diagram



Equation of State of a cold Fermi Gas $P(\mu, T)$



Neutron star, Spin $\frac{1}{2}$

$a = -18.6 \text{ fm}$, $n \sim 2 \cdot 10^{36} \text{ cm}^{-3}$

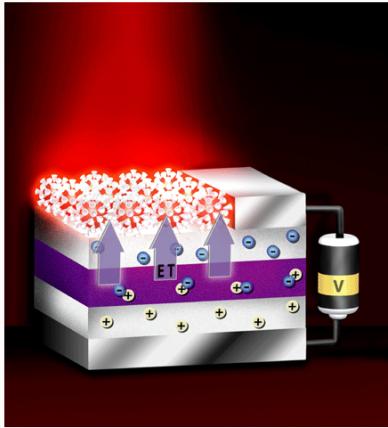
- $T_c = 10^{10} \text{ K}$, $T = T_F/100$
- $k_F a \sim -4, -10, \dots$

- $T=0$, vs interaction strength, beyond mean-field effects, Lee-Yang and Lee-Huang-Yang quantum corrections, Tan relations,...
- Unitary gas vs temperature

Quantitative N-body Physics: benchmark for theories

3: Cold gases in 2 dimensions

Two-dimensional Quantum Physics

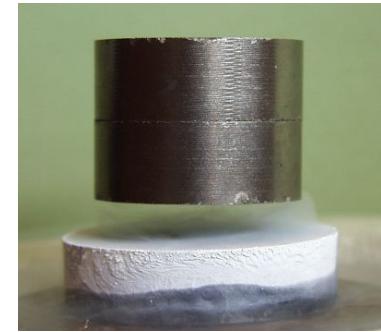


Quantum wells and MOS structures

High T_c superconductivity
Graphene,...
films of superfluid helium,

...

also Quantum Hall effect,

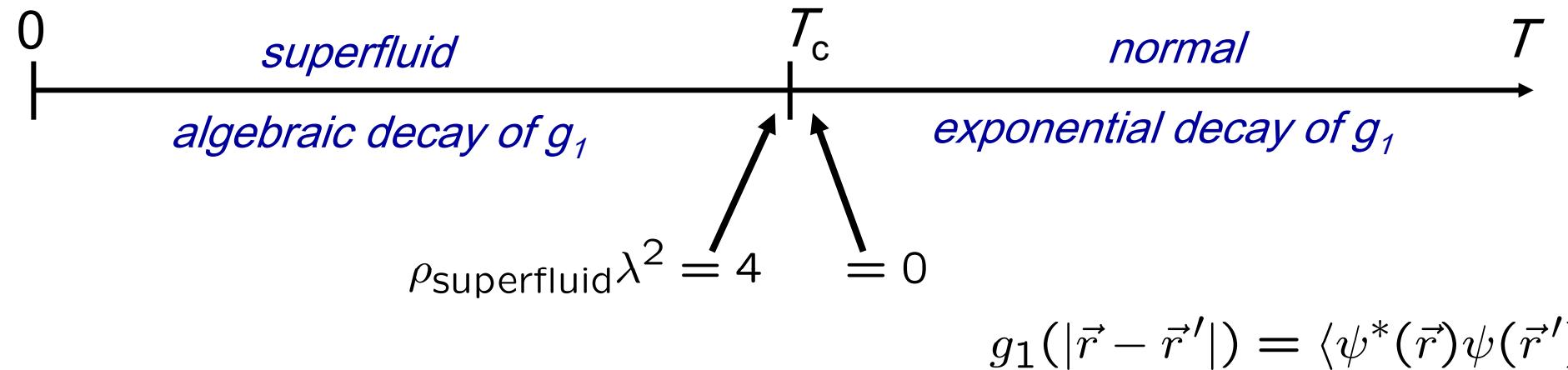


Key words of two-dimensional physics:

- absence of true long range order (no BEC stricto sensu)
- existence of a new kind of phase transition (Kosterlitz-Thouless)
- No spin-statistics theorem, and existence of parastatistics: any-ons
- Non abelian physics: towards topological quantum computing ??

The Berezinski-Kosterlitz-Thouless mechanism

1971-73, 2D gas of bosons



Microscopic origin of this phase transition: quantized vortices

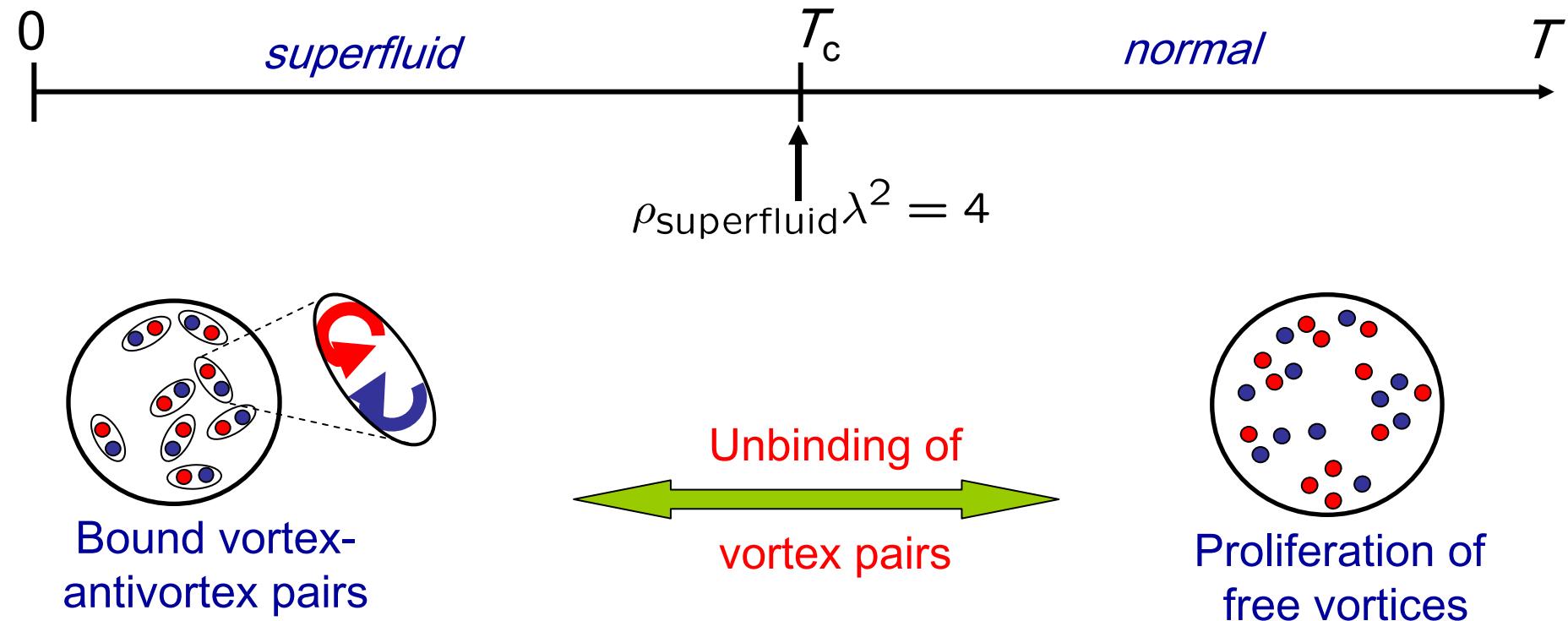
$$\psi(\vec{r}) = \sqrt{\rho(\vec{r})} e^{i\phi(\vec{r})}$$

Vortex: point where $\rho(\vec{r}) = 0$, around which $\phi(\vec{r})$ rotates by $\pm 2\pi$

Around a vortex: $\oint \vec{v} \cdot d\vec{r} = \pm \frac{h}{m}$

Superfluidity in 2D: Berezinski-Kosterlitz-Thouless Mechanism

Berezinski and Kosterlitz –Thouless 1971-73

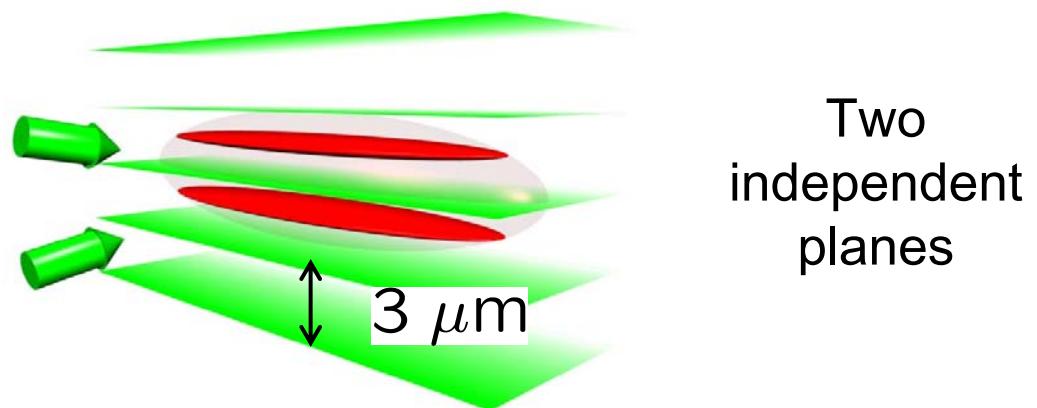


Superfluid transition observed with
liquid helium films by Bishop-Reppy, 1978

Producing a 2 Dimensional Cold Gas

2D experiments at MIT, Innsbruck, Oxford, Florence, Boulder,
Heidelberg, Gaithersburg, Paris (Dalibard's group)

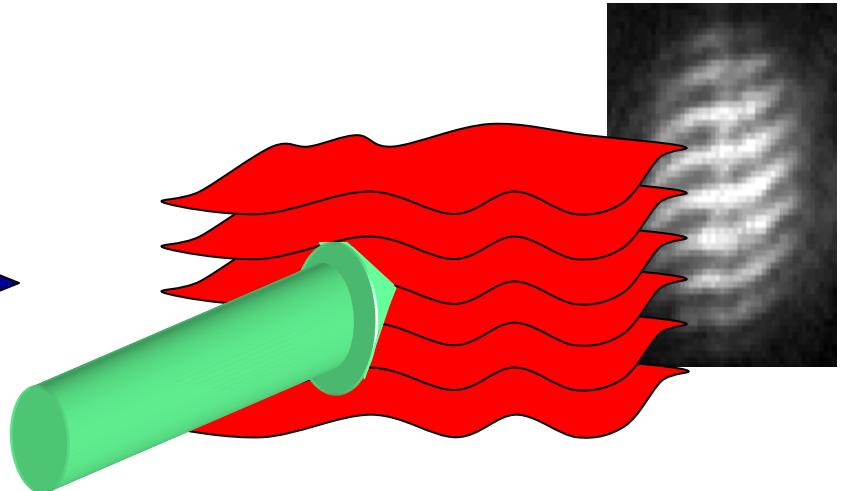
Paris: superposition of a
harmonic magnetic potential
+
periodic potential of
a laser standing wave



Two
independent
planes

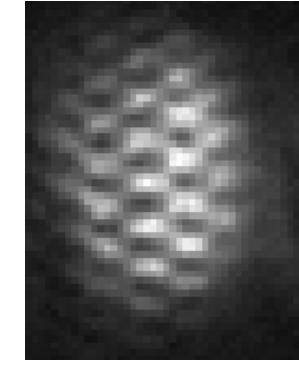
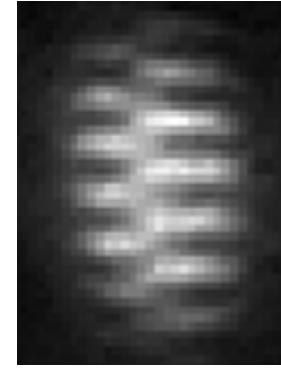
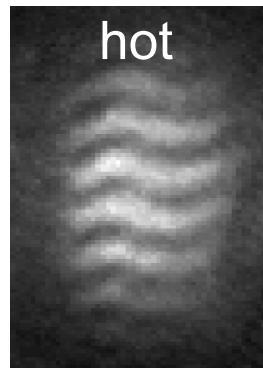
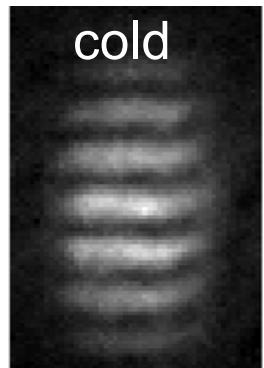
$$\begin{aligned} \psi_a(x, y) \\ \psi_b(x, y) \end{aligned}$$

Time of
flight



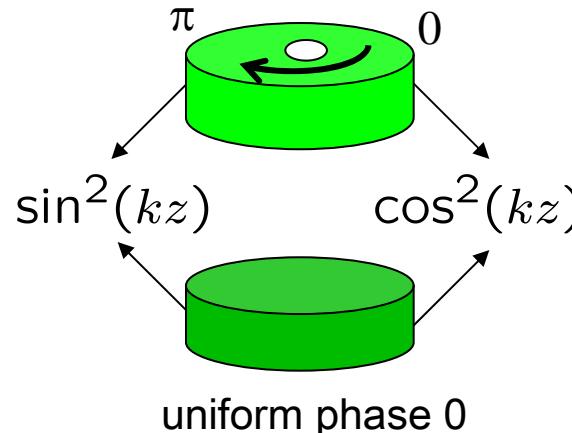
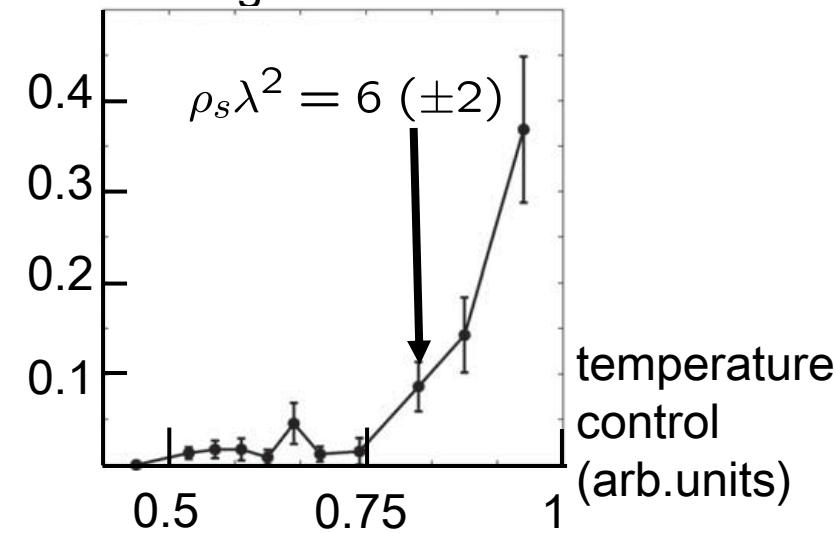
2D physics revealed by matter-wave interferometry

ENS Dalibard group



sometimes: dislocations!

fraction of images
showing a dislocation



Dislocation
=
evidence for
free vortices

The equation of state of a 2D Bose gas

The equation of state of a 2D Bose gas

General form (uniform system): $P = \mathcal{F}(\mu, T)$

P is a pressure in 3D (J/m^3), a surface tension in 2D (J/m^2), and a force in 1D (J/m)

Once P is known, all other thermodynamics quantities can be obtained:

$$\text{spatial density: } n = \left(\frac{\partial P}{\partial \mu} \right)_T$$

It is convenient to introduce dimensionless quantities :

$$3D: \quad \tilde{P} = \frac{P \lambda_{dB}^3}{k_B T}$$

$$2D: \quad \tilde{P} = \frac{P \lambda_{dB}^2}{k_B T}$$

The pressure of a 2D Bose gas

Muller, Ho-Zhou, 2D version: Castin

Atoms in a 2D harmonic potential $V(r) = \frac{1}{2}m\omega^2 r^2$ $r^2 = x^2 + y^2$

Number of atoms in the trap: $N = \int n(x, y) dx dy = 2\pi \int_0^{+\infty} n(r) r dr$

Local density approximation
→ relate to homogeneous case $N = 2\pi \int_0^{+\infty} n_{\text{hom.}} \left(\mu - \frac{1}{2}m\omega^2 r^2, T \right) r dr$

$$N = \frac{2\pi}{m\omega^2} \int_{-\infty}^{\mu} n_{\text{hom.}} (\mu', T) d\mu'$$

$$n_{\text{hom.}} = \left(\frac{\partial P}{\partial \mu} \right)_T$$

$$N = \frac{2\pi}{m\omega^2} P(\mu, T)$$

*Just measure
the atom number!*

scale invariance of a 2D Bose gas

In 3D, interactions are described by a length a , the so-called *scattering length*

An energy scale is associated to this length: $U = \hbar^2 / (ma^2)$

Equation of state $P = \mathcal{F}(\mu, T)$ with dimensionless variables:

$$\tilde{P} = \mathcal{F}\left(\frac{\mu}{U}, \frac{k_B T}{U}\right) \quad \tilde{P} = \frac{P \lambda_{dB}^3}{k_B T}$$

In 2D, interactions are described by a dimensionless number \tilde{g}

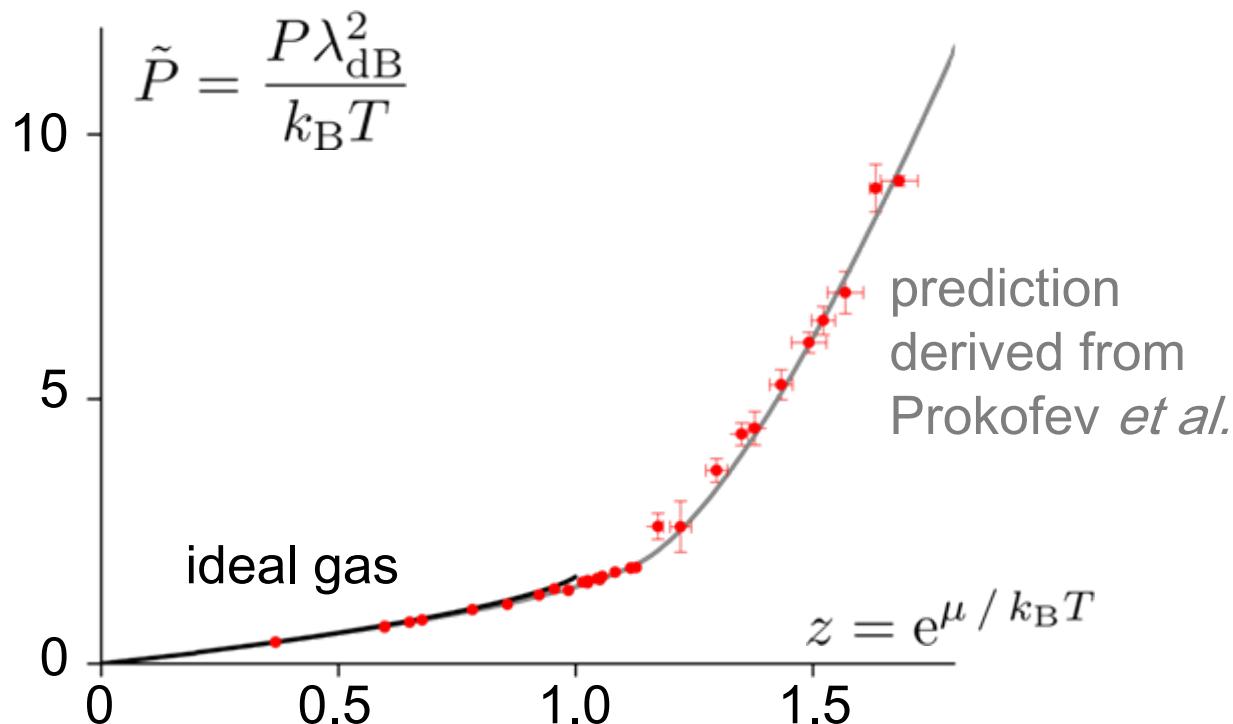
The equation of state with dimensionless variables must read

$$\tilde{P} = \mathcal{F}\left(\frac{\mu}{k_B T}, \tilde{g}\right) \quad \tilde{P} = \frac{P \lambda_{dB}^2}{k_B T} \quad \text{scale invariance}$$

scale invariance of a 2D Bose gas: experiment

Procedure:

- Take an image of the gas
- Fit the mean-field prediction to the wings of the distribution: T , μ
- Measure the atom number N and deduce the pressure $P = m\omega^2 N / 2\pi$



ENS group,
to be published

Confirmation of
scale invariance:

*Points at different
 μ and T fall on a
“universal” curve*

Validation of the use of trapped gases to extract
thermodynamic properties of bulk systems

Prospects

With cold atoms, one can simulate several many-body Hamiltonians

- **Bosons, fermions, and mixtures**
- **Pairing with mismatched Fermi spheres, exotic phases**
- **Periodic potential or disordered (Anderson localization)**
- **Gauge field with rotation or geometrical phase**
- **Non abelian Gauge field for simulating the Hamiltonian of strong interactions in particle physics**
- **Quantum Hall physics and Laughlin states**

New experimental methods:

- **Image a many-body wavefunction with micrometer resolution**
- **Measure correlation functions**
- **Photoemission spectroscopy to measure Fermi surface and single particle excitations**
- **Time-dependent phenomena in 1, 2, and 3 D**

