The Abdus Salam International Centre
for Theoretical Physics

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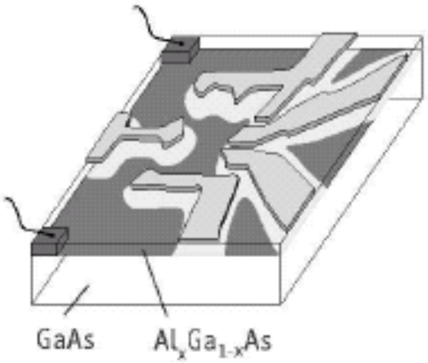
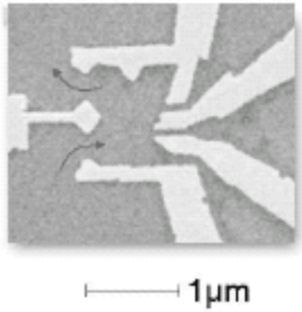
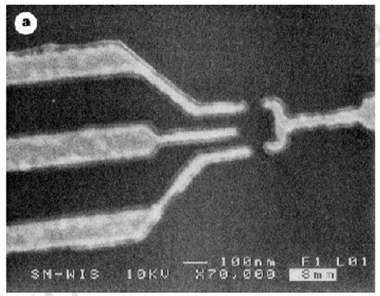
Thermoelectric transport through a quantum dot: Interplay between FL and NFL behavior

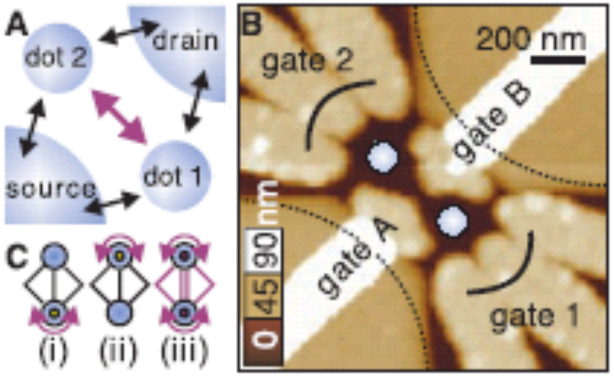
Autumn College on Non-Equilibrium Quantum Systems
2 - 13 May 2011, Buenos Aires, Argentina

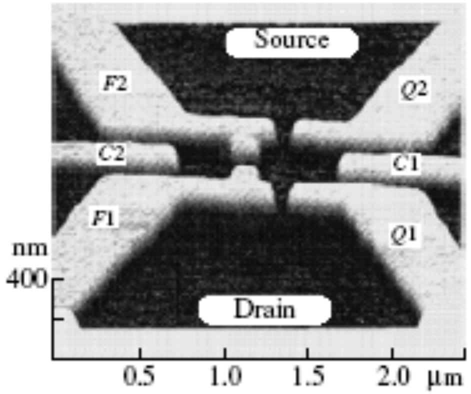
Quantum dots: from simple to complex

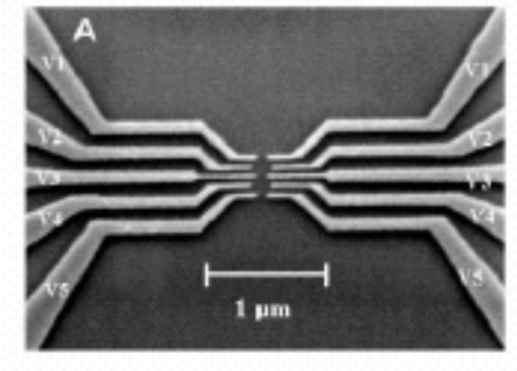












D.Goldhaber-Gordon et al (1998)

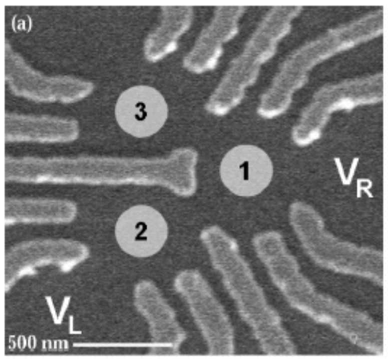
J.P.Kotthaus (1995)

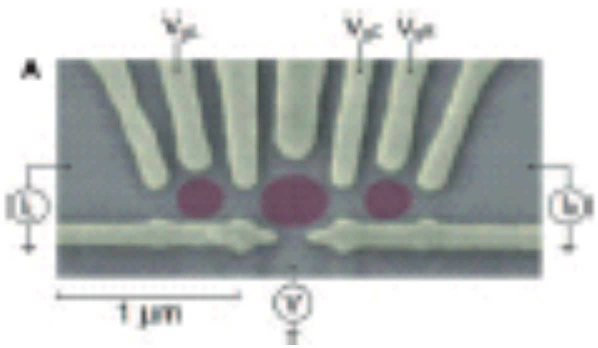
A.Holleitner et al (2002)

L.W.Molenkamp et al (1995)

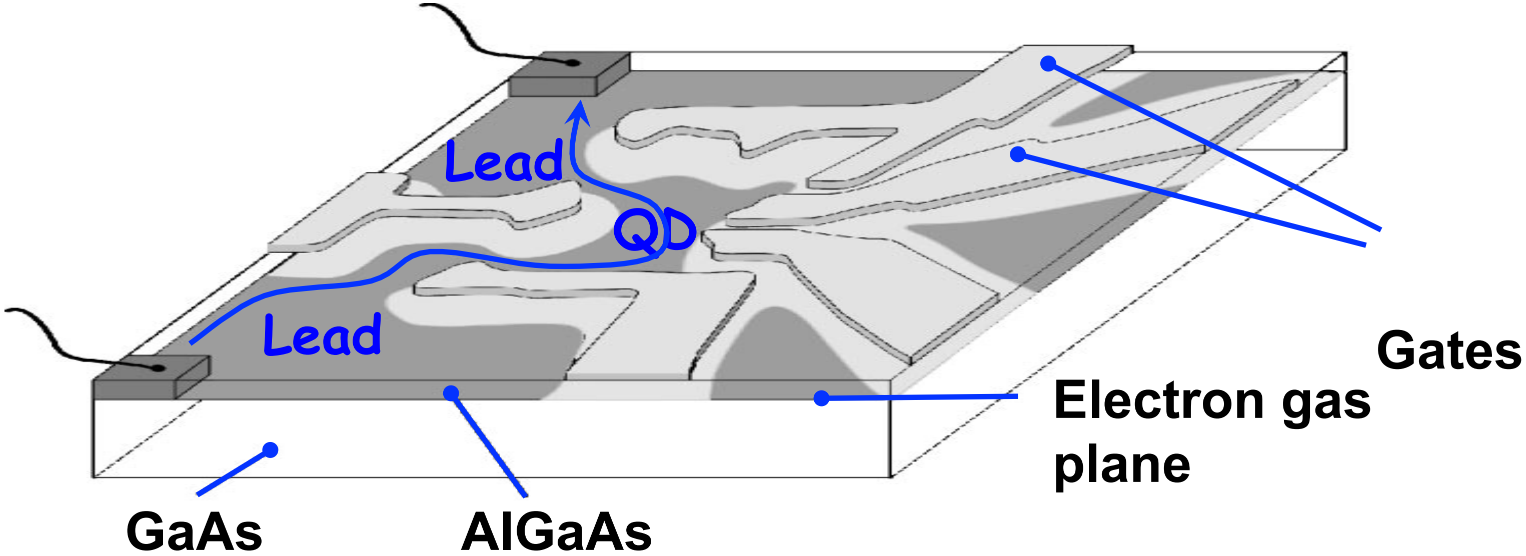
H.Jeong et al (2001)

C.Marcus et al (2003)





Quantum dots



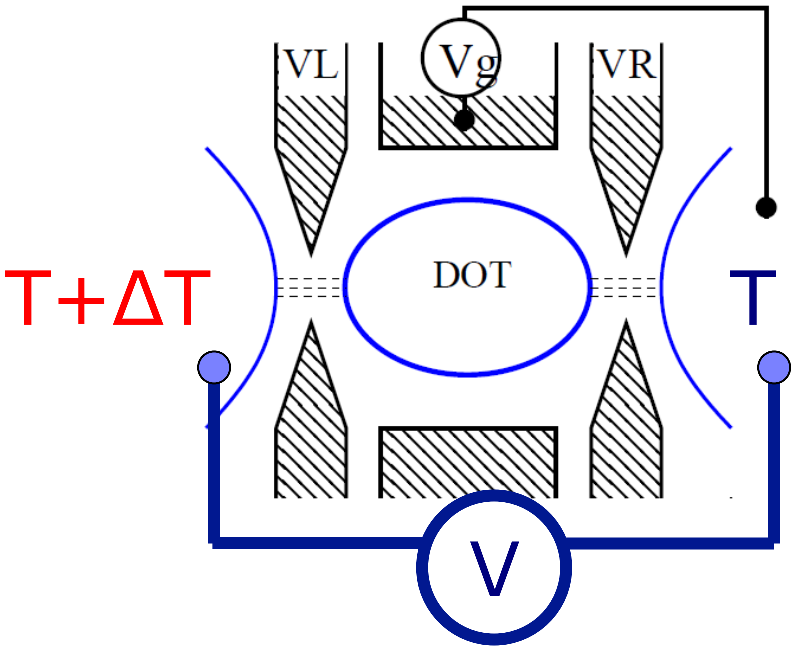
- Tune: gate potentials, temperature, field…
- Measure: I-V curves, conductance G…
- Aharonov-Bohm interferometry, dephasing, coherent state manipulation…

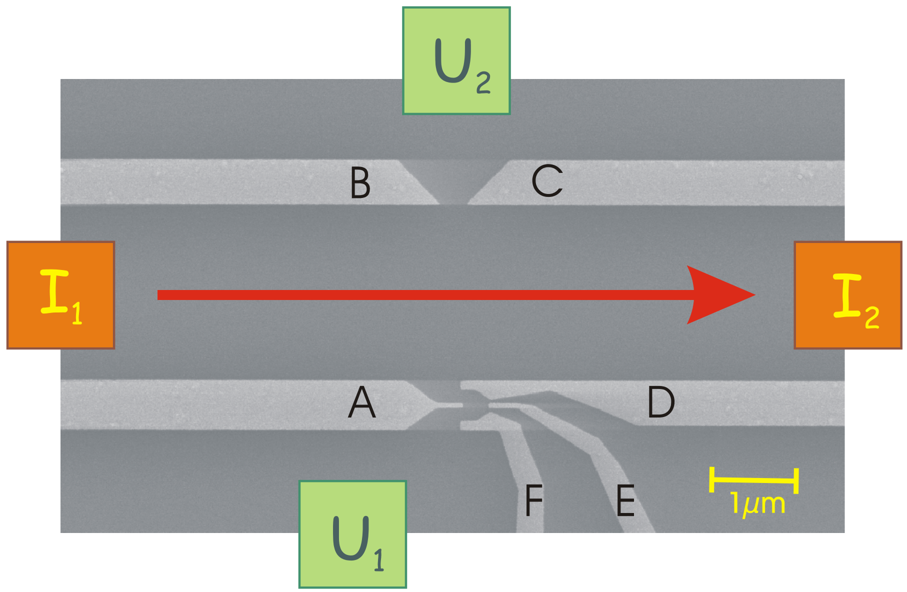
Thermoelectric transport through nanostructures

thermopower

$$S = -\frac{V}{\Delta T}$$

thermovoltage





Thermoelectric transport: FL description

Bulk metals (Fermi Liquid Theory):

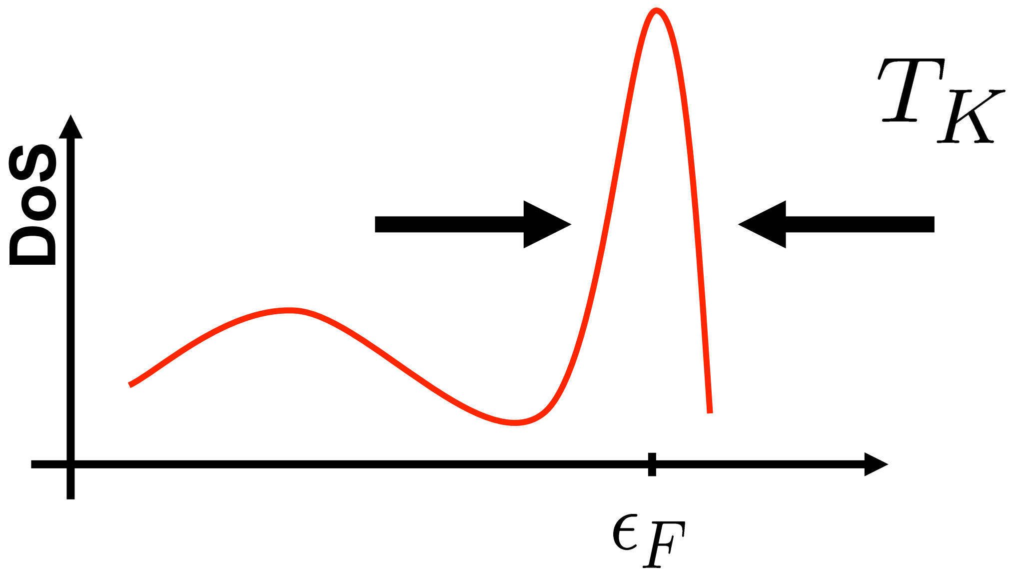
$$S \sim T/\epsilon_F \xrightarrow{\text{strongly correlated metals}} S \sim T/T^*$$

$$T^* \ll \epsilon_F$$

strong electron-electron interaction

resonance scattering effects

Example: Kondo effect

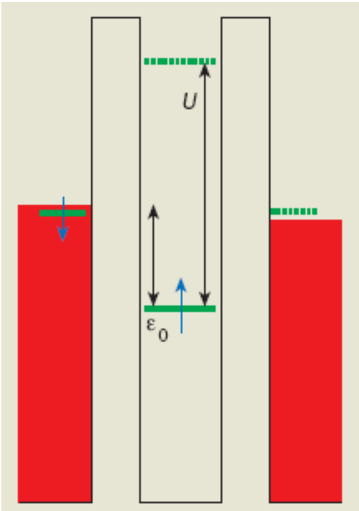


$$S \sim T/T_K$$

Single orbital level coupled to two leads

$$H = H_{leads} + H_{tun} + H_{dot}$$

$$H_{leads} = \sum_{k,\sigma\alpha=L,R} [\epsilon_k - \mu_\alpha] c^\dagger_{k,\sigma\alpha} c_{k,\sigma\alpha}$$



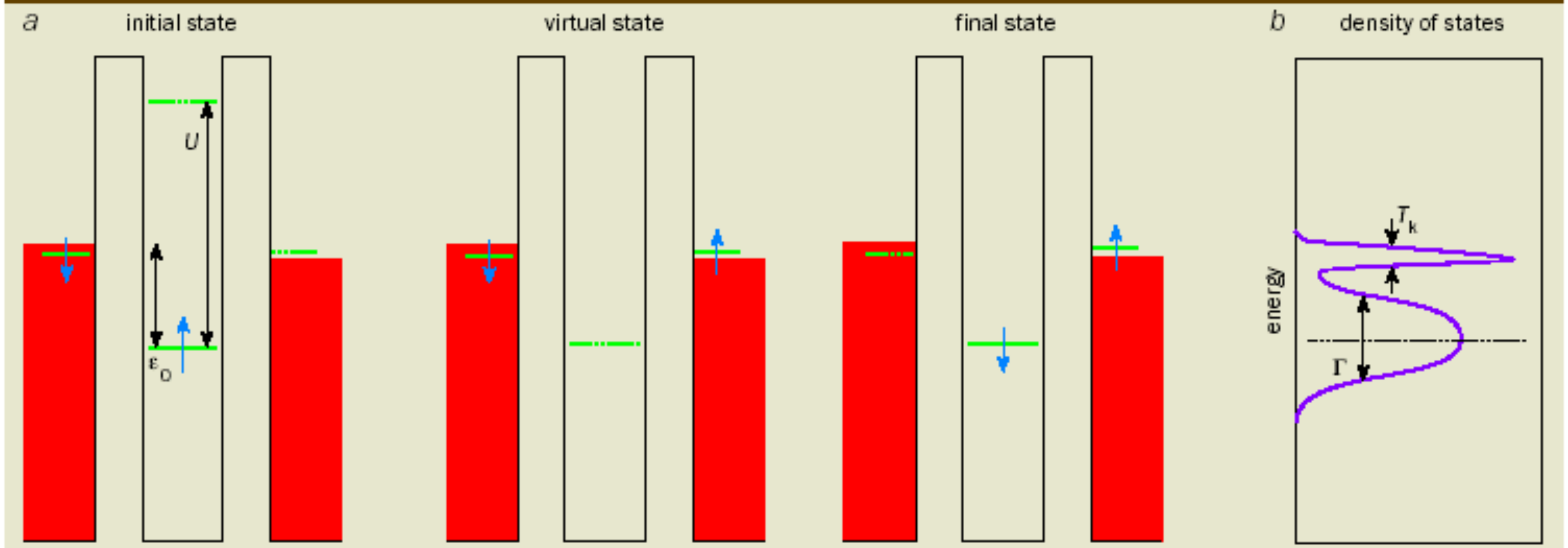
$$H_{tun} = \sum_{k,\sigma\alpha} [V_\alpha c^\dagger_{k,\sigma\alpha} d_\sigma + H.c.]$$

$$H_{dot} = \sum_\sigma \varepsilon_0 d^\dagger_\sigma d_\sigma + U(n-N)^2$$

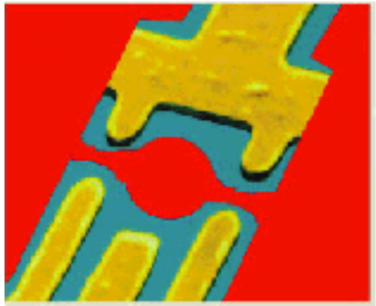
Tunneling width $\qquad \Gamma_\alpha = \pi\rho|V_\alpha|^2$

Kondo Effect in Quantum Dots

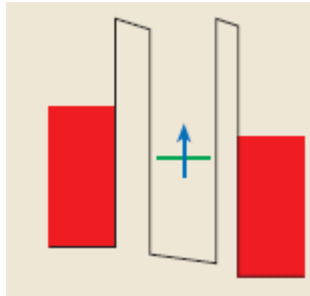
2 Spin flips

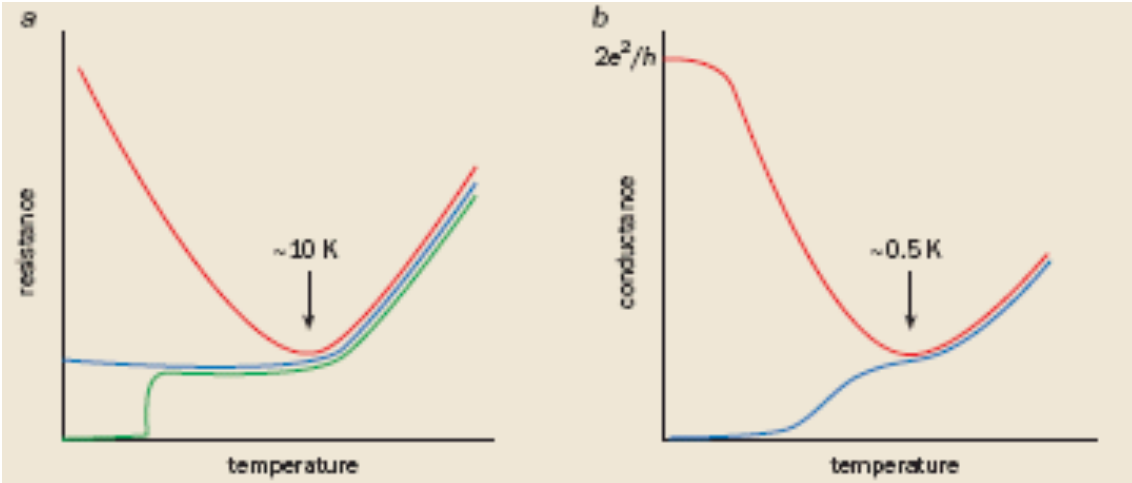


(a) The Anderson model of a magnetic impurity assumes that it has just one electron level with energy ε_0 below the Fermi energy of the metal (red). This level is occupied by one spin-up electron (blue). Adding another electron is prohibited by the Coulomb energy, U, while it would cost at least $|\varepsilon_0|$ to remove the electron. Being a quantum particle, the spin-up electron may tunnel out of the impurity site to briefly occupy a classically forbidden "virtual state" outside the impurity, and then be replaced by an electron from the metal. This can effectively "flip" the spin of the impurity. (b) Many such events combine to produce the Kondo effect, which leads to the appearance of an extra resonance at the Fermi energy. Since transport properties, such as conductance, are determined by electrons with energies close to the Fermi level, the extra resonance can dramatically change the conductance.

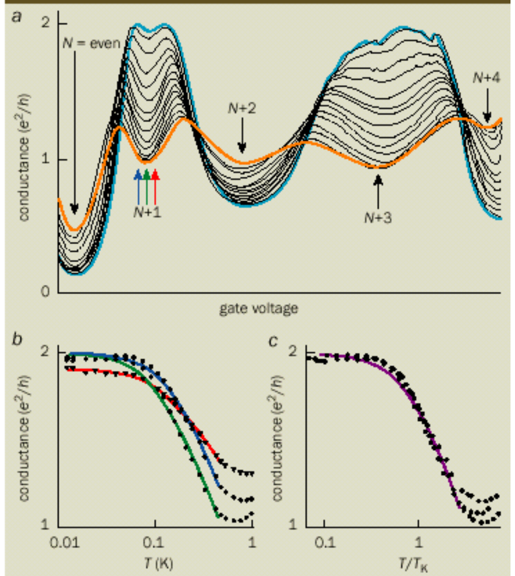


Universal Scaling





5 Universal scaling



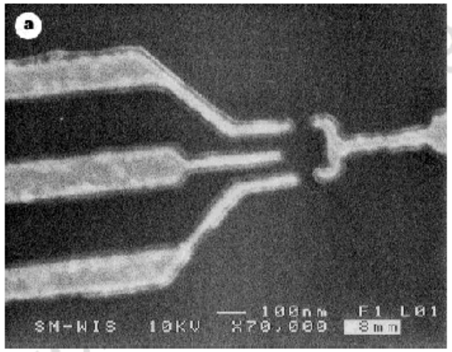
(*a*) The conductance (*y*-axis) as a function of the gate voltage, which changes the number of electrons, *N*, confined in a quantum dot. When an even number of electrons is trapped, the conductance decreases as the temperature is lowered from 1 K (orange) to 25 mK (light blue). This behaviour illustrates that there is no Kondo effect when *N* is even. The opposite temperature dependence is observed for an odd number of electrons, i.e. when there is a Kondo effect. (*b*) The conductance for *N* + 1 electrons at three different fixed gate voltages indicated by the coloured arrows in (*a*). The Kondo temperature, T_K, for the different gate voltages can be calculated by fitting the theory to the data. (*c*) When the same data are replotted as a function of temperature divided by the respective Kondo temperature, the different curves lie on top of each other, illustrating that electronic transport in the Kondo regime is described by a universal function that depends only on T/T_K.

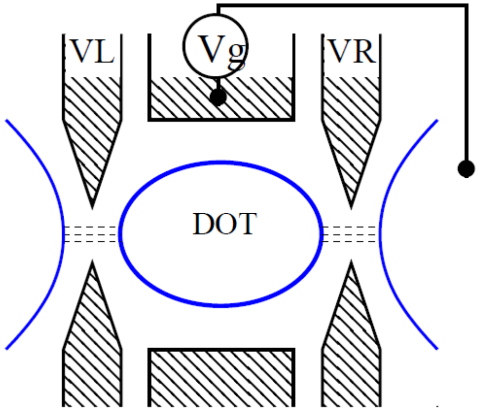
$$G/G_0 \propto \ln^{-2}\left(\max[T/T_K]\right)$$

$$T_K = \frac{1}{2}(\Gamma U)^{1/2}\exp\left(\pi\varepsilon_0\frac{\varepsilon_0+U}{\Gamma U}\right)$$

Realization of Kondo-effect in nanostructures I

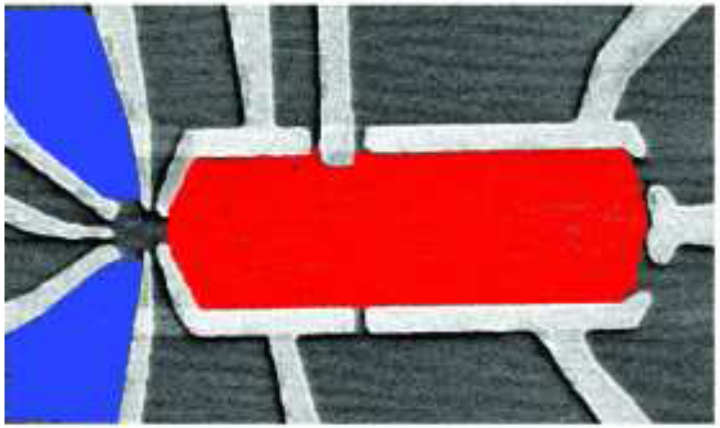
1CK

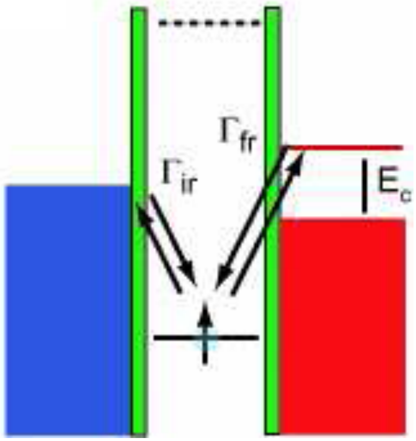




D.Goldhaber-Gordon et al, Nature, 1998

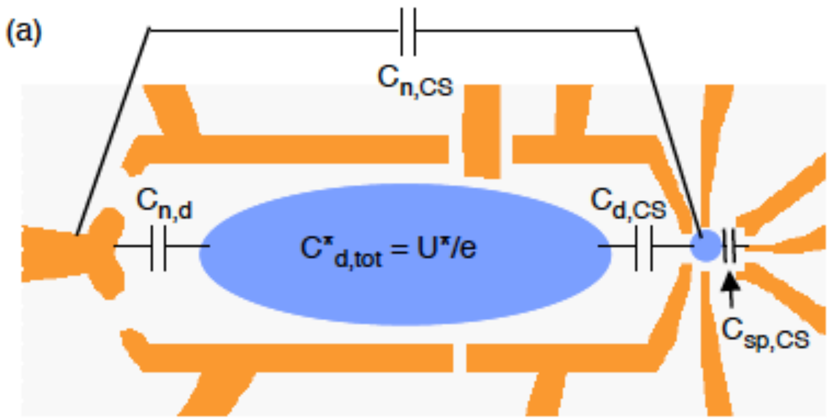
2CK

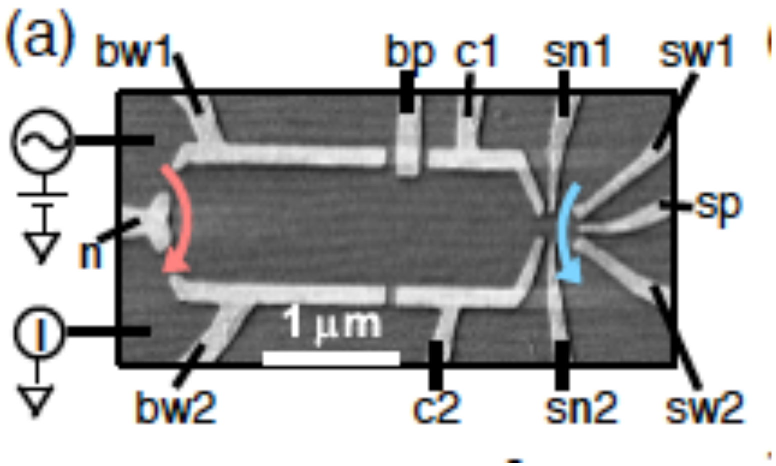


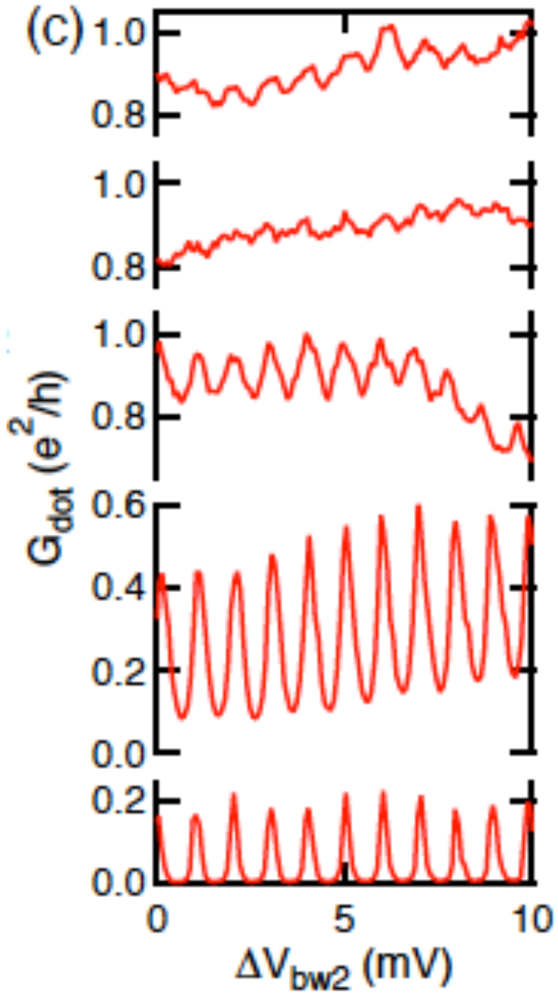


R.M. Potok et al, Nature, 2007

Realization of Kondo-effect in nanostructures II







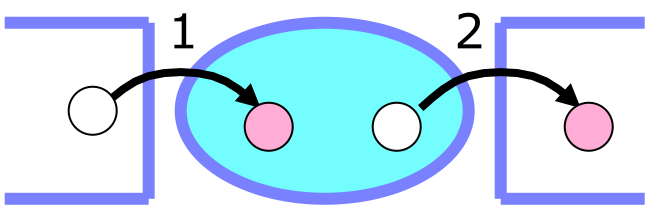
S.Amasha et al, arXiv: 1009.5348

Q: How do the effects of strong electron correlations manifest themselves in the thermoelectric transport through the nanostructures?

Q: What are possible mechanisms for enhancement of the thermoelectric power?

Q: Is the thermo-transport through nanostructures always characterized by the Fermi-Liquid concept?

Sequential tunneling at Coulomb blockade

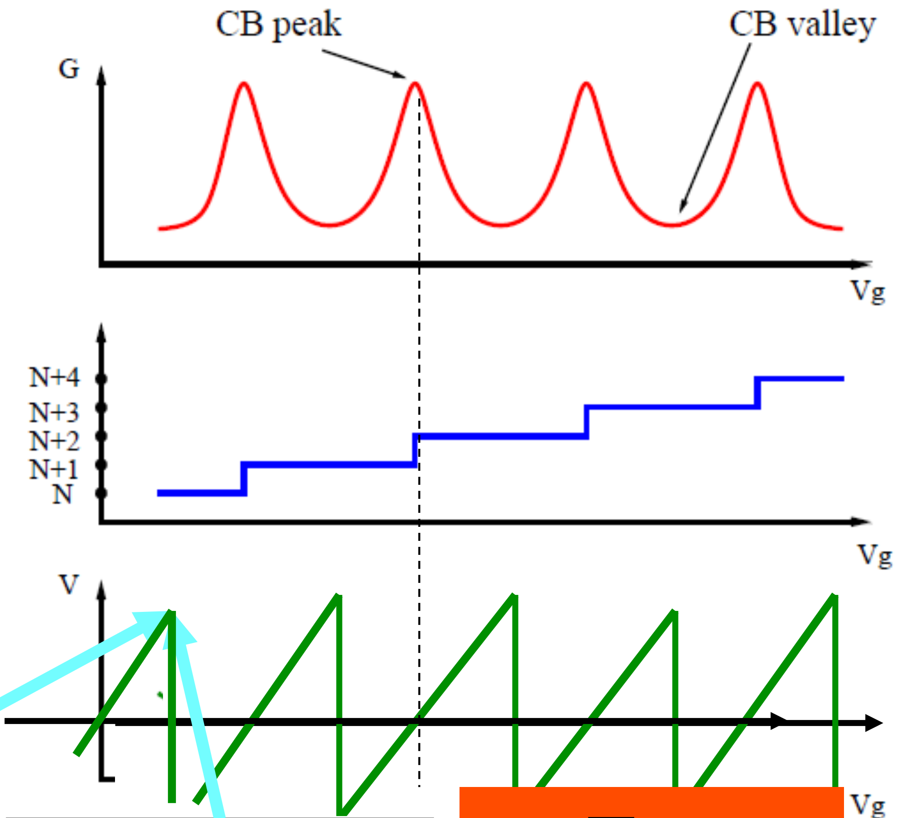


$$S_{\mathrm{Mott}} \equiv -\frac{\pi^2}{3}\frac{k}{e}kT\frac{d\ln G}{dE_F}$$

For a bulk metal
$eS \sim T/E_F << 1$

Mott's rule would give for sequential tunneling
$eS \sim 1$

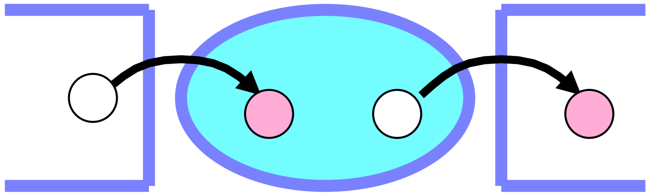
Beenakker & Staring 1992



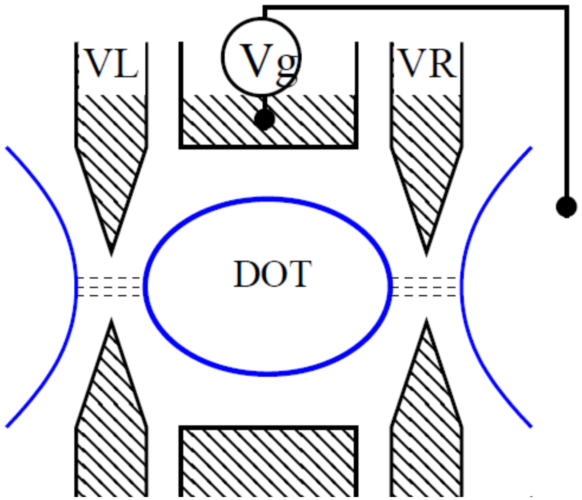
Beenakker & Staring
$eS \sim E_C / T >> 1$

Too large??

Effect of co-tunneling at weak coupling



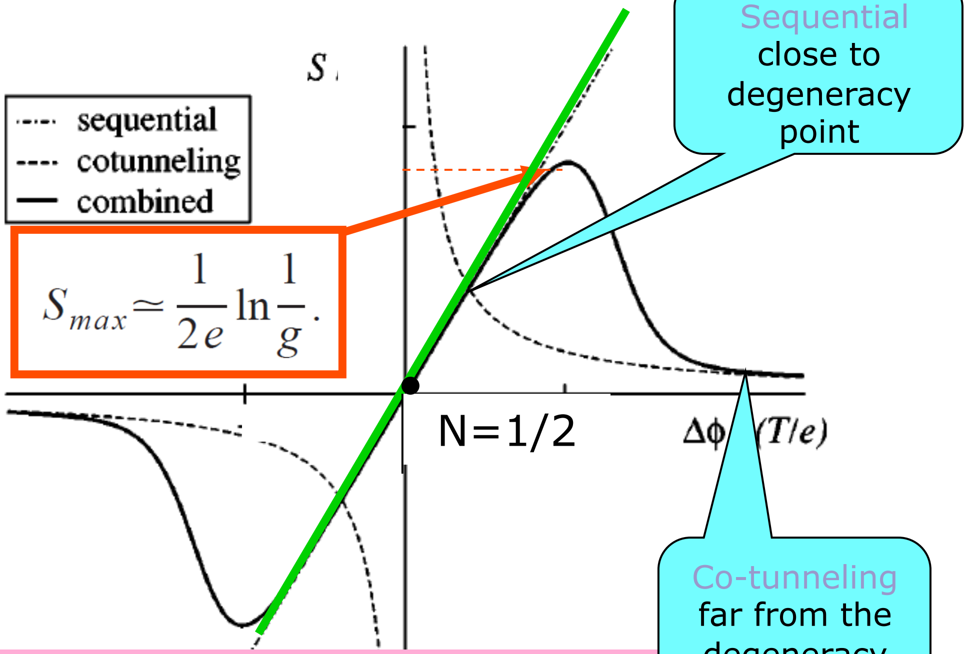
Turek & Matveev, 2002



Weak coupling

$$g = \frac{\hbar(G_l + G_r)}{2\pi e^2} << 1$$

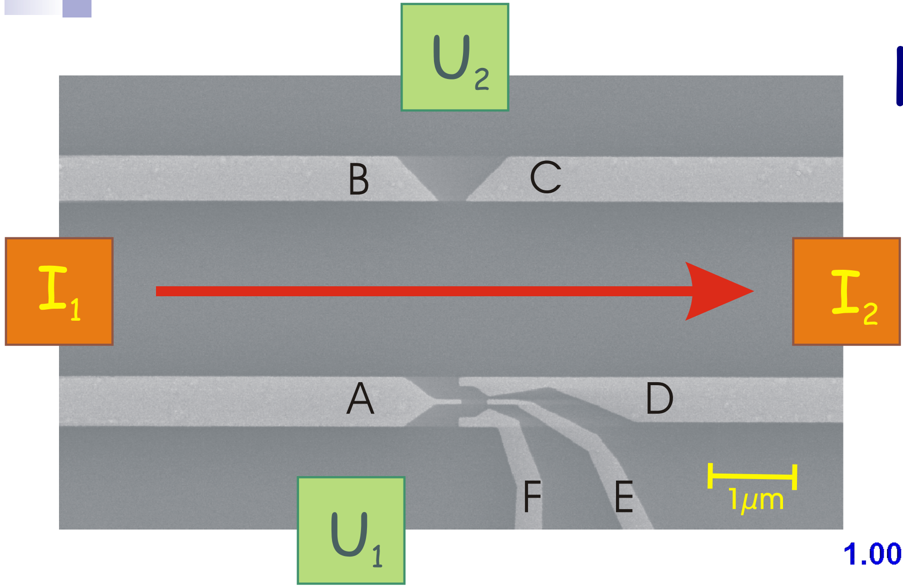
No Coulomb energy is payed

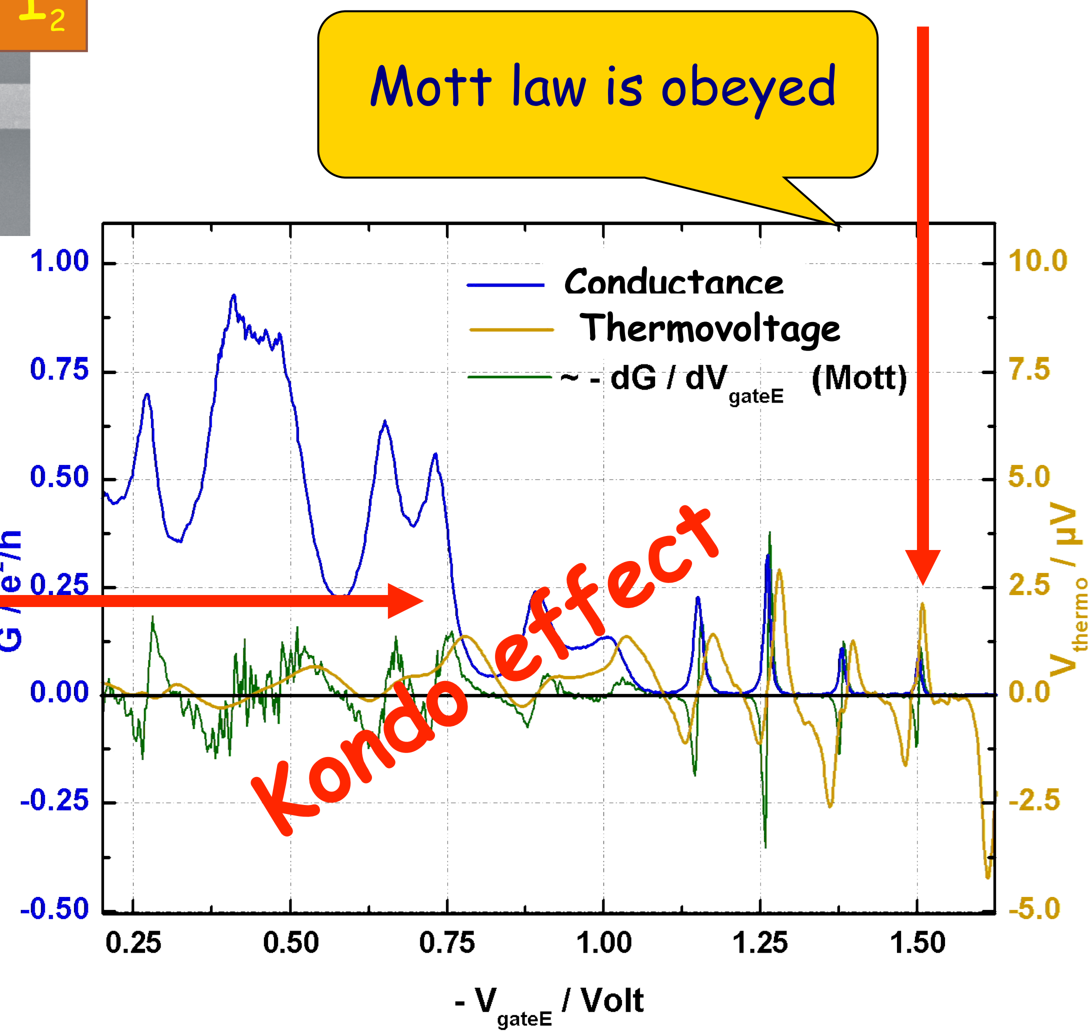


$$S_{max} \simeq \frac{1}{2e} \ln \frac{1}{g}.$$

S_{max} is much smaller than Beenakker&Staring estimation, more consistent with the Mott's rule result eS~1 but enhanced compared to bulk eS~T/E_F<<1

From weak to strong coupling





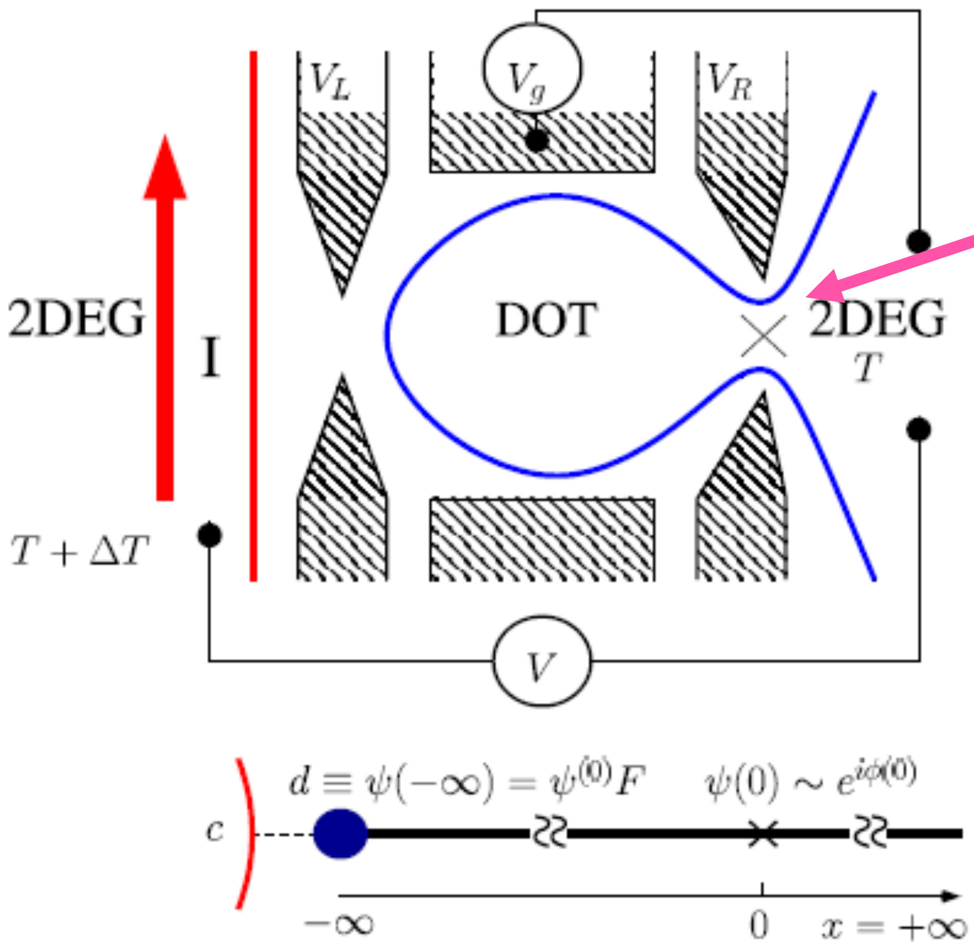
Molenkamp et al, 2005

Q1: How does the Kondo effect influence a thermoelectric transport through nano-structures?

Q2: What are the manifestations of Kondo effect in the thermoelectric transport through nanostructures?

Q3: Is there a room for NFL enhancement of thermopower in nanostructures?

Matveev's suggestion for realization of Kondo effect



QPC

$$|r| \ll 1$$

1CK - 2CK

$$T_K \sim E_C$$

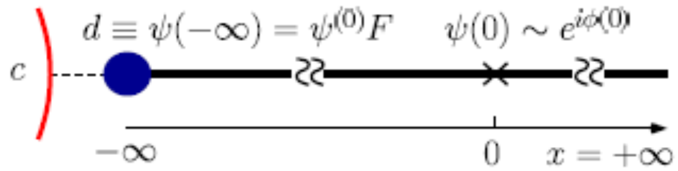
Matveev 1995, Furusaki, Matveev 1996

Model

$$H = H_0 + H_L + H_R + H_C$$

$$H_0 = \sum_{k,\alpha} \epsilon_{k,\alpha} c^\dagger_{k,\alpha} c_{k,\alpha} + \sum_{\alpha} \epsilon_\alpha d^\dagger_\alpha d_\alpha +$$

$$+ \sum_\alpha \frac{v_{F,\alpha}}{2\pi} \int_{-\infty}^{\infty} \left\{ [\Pi_\alpha(x)]^2 + [\partial_x \phi_\alpha(x)]^2 \right\} dx$$



$$H_L = \sum_{k,\alpha} (t_{k,\alpha} c^\dagger_{k,\alpha} d_\alpha + hc)$$

$$H_R = -\frac{D}{\pi} \sum_\alpha |r_\alpha| \cos[2\phi_\alpha(0)]$$

$$H_C = E_C \left[\hat{n} + \frac{1}{\pi} \sum_\alpha \phi_\alpha(0) - N(V_g) \right]^2$$

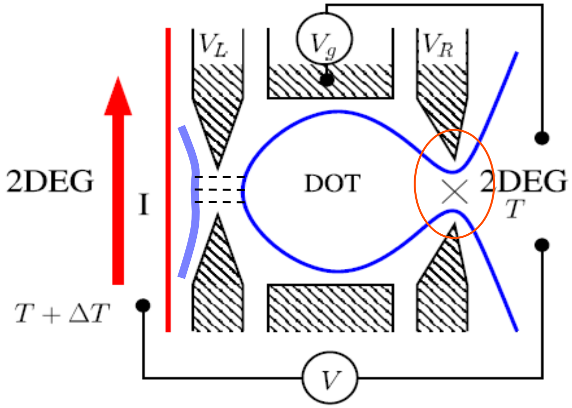
Assumptions:

Strong coupling regime $\qquad T \ll E_C$

Weak Coulomb Blockade $\qquad |r_\alpha| \ll 1$

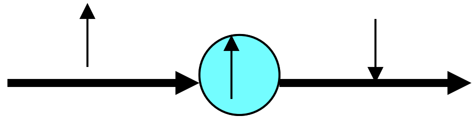
Metallic regime $\qquad \delta \ll T$

Strong coupling and the Kondo physics
(Matveev & Andreev, 2002)



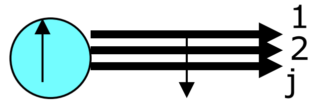
Ordinary (one-channel) Kondo

$$J c_\sigma^+ \vec{\sigma}_{\sigma\sigma'} c_{\sigma'} \vec{S}$$

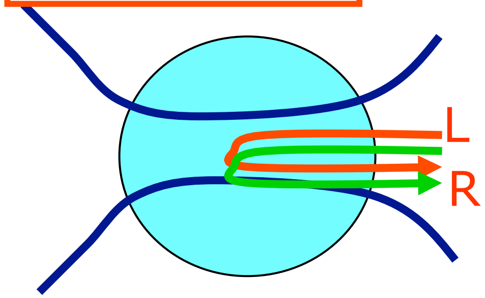


N-channel Kondo: spin-1/2 impurity +N orbital channels

$$J \sum_j c_{j\sigma}^+ \vec{\sigma}_{\sigma\sigma'} c_{j\sigma'} \vec{S}$$



Reflection plays a role of spin-flip

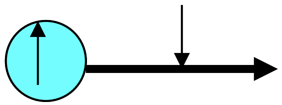


QPC is a quasi-spin-1/2 orbital "impurity" + two spin ↑ and ↓ channels: 2-channel Kondo. Symmetry of channels is protected by TRS

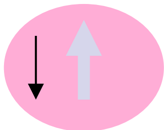
FL and non-FL behavior

Enhancement by non-Fermi-liquid effects

$$J c_{\sigma}^{+} \vec{\sigma}_{\sigma\sigma'} c_{\sigma'} \vec{S}$$



Kondo screening of the impurity spin



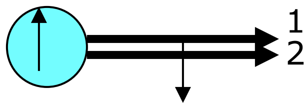
Complete screening below Tk

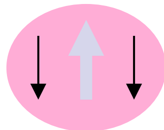
Fermi-liquid behavior

Magnetic susceptibility
χ=const as T->0

$eS \sim T/T_k$

$$\sum_j J_j c_{j\sigma}^{+} \vec{\sigma}_{\sigma\sigma'} c_{j\sigma'} \vec{S}$$





overscreening

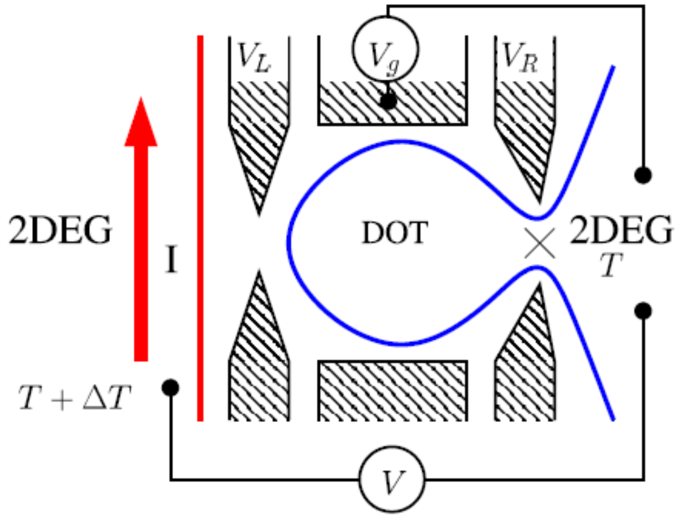
Non-Fermi-liquid behavior

Magnetic susceptibility
χ~ln(Tk/T)

$eS \sim (T/T_k)^{1/2} \ln(T_k/T)$

Matveev&Andreev, 2002

Two Kondo regimes



Spinless fermions:
QPC is fully spin-polarized: 1CK
Fermi liquid behavior:

$$S \propto -\frac{T}{E_C}|r|\sin(2\pi N)$$

Enhancement of thermopower by electron-electron interaction !

Spinful fermions:
QPC is non-polarized:
isotropic 2CK
Non Fermi liquid behavior:

$$S \propto -|r|^2 \frac{T}{\Gamma} \ln \frac{E_C}{T+\Gamma} \sin(2\pi N) F\left(\frac{T}{\Gamma}\right)$$

$$\Gamma \propto E_C |r|^2 \cos^2(\pi N)$$

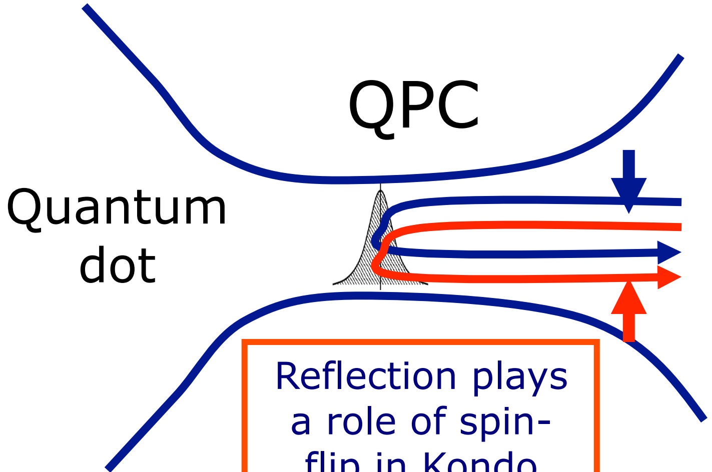
Enhancement by non-Fermi-liquid effects

Q1: How does one regime crossover to another one?

Matveev, Andreev, 2001-2002

How does magnetic field influence two Kondo regimes?

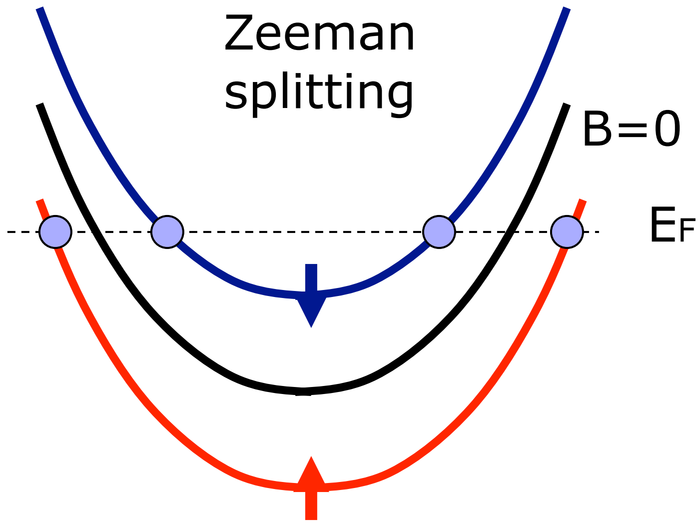
Parallel to the plane magnetic field



Reflection plays a role of spin-flip in Kondo problem

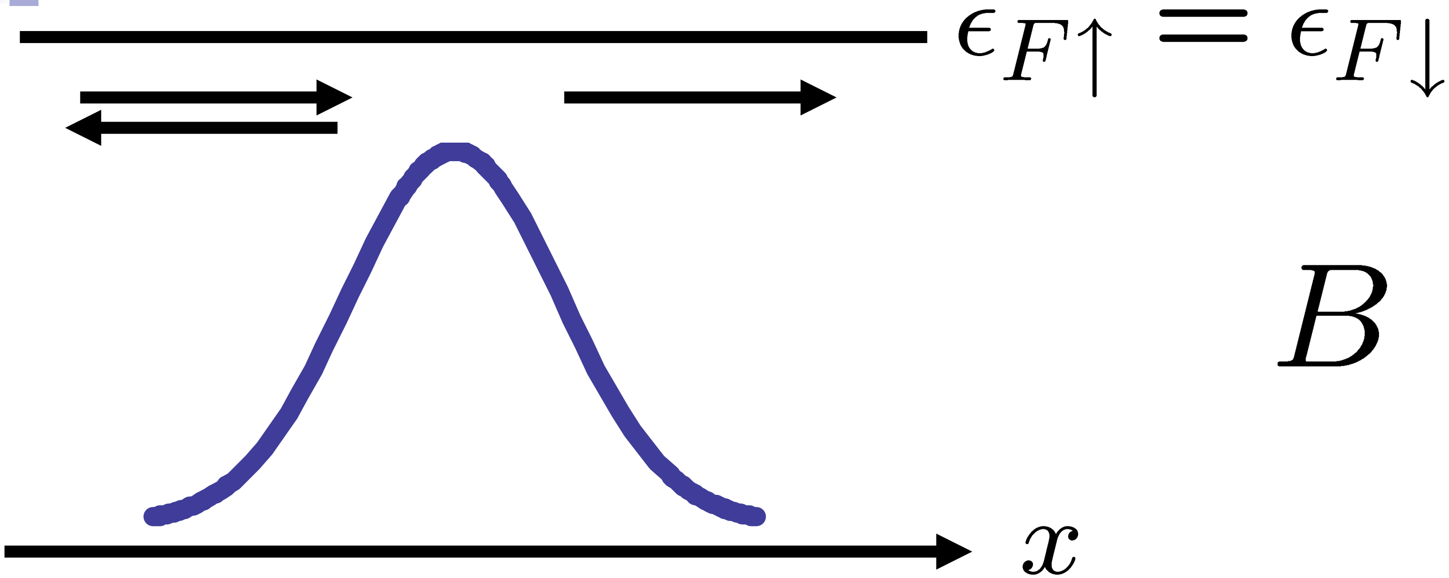
$|r| \Leftrightarrow J$

$T_{\max} \sim |r|^2 E_C, \quad E_C = e^2/(2C)$

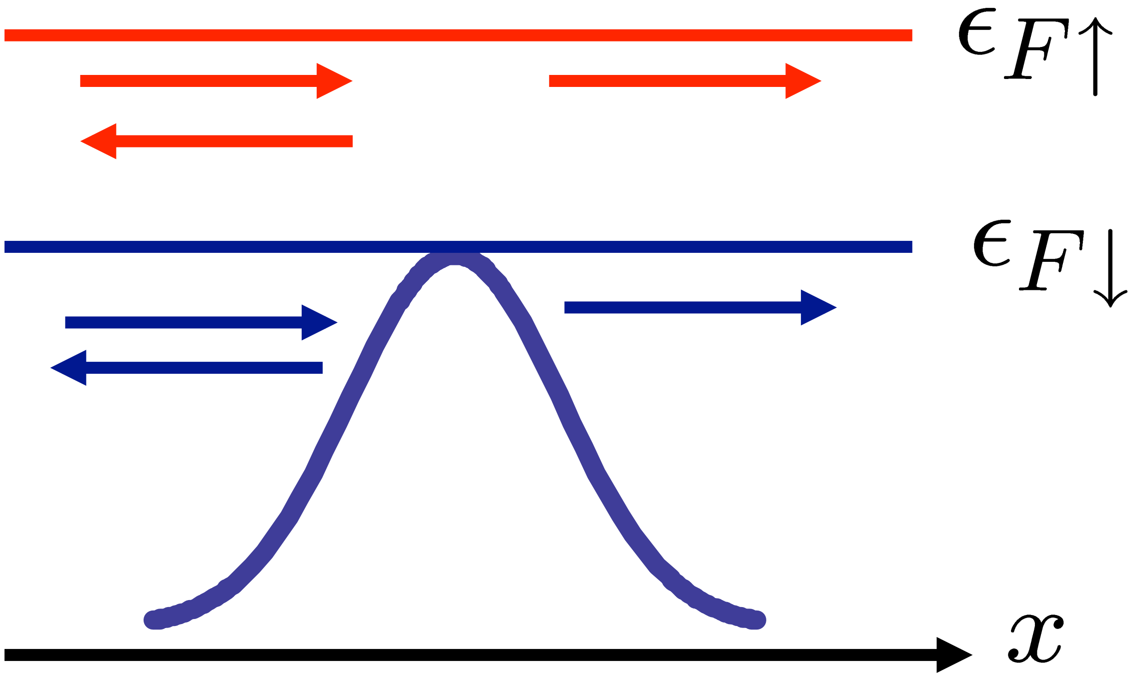


$$B \neq 0: \quad k_\uparrow \neq k_\downarrow \Rightarrow r_\uparrow \neq r_\downarrow$$

$$J_1 \neq J_2$$

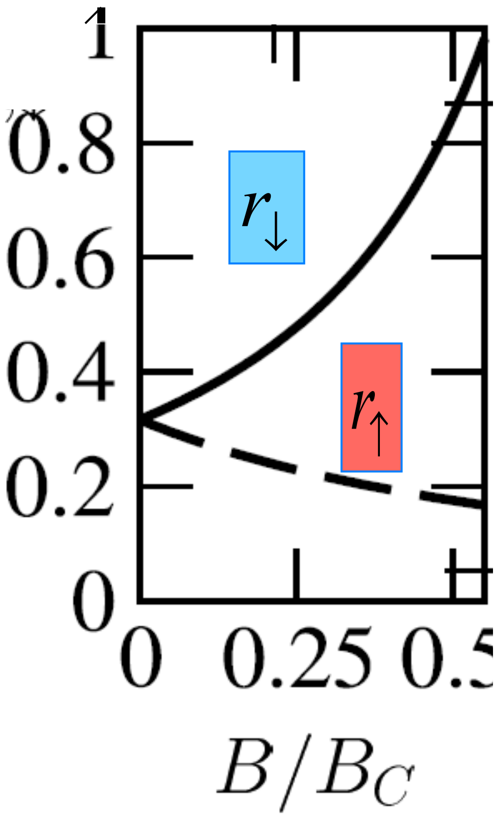

$\epsilon_{F\uparrow} = \epsilon_{F\downarrow}$
x

$$B = 0$$


$\epsilon_{F\uparrow}$
$\epsilon_{F\downarrow}$
x

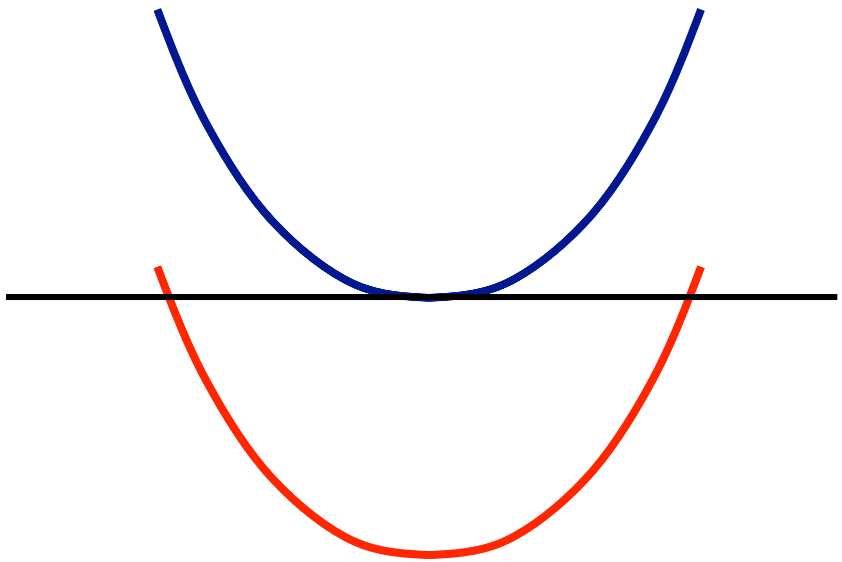
$$B = B^*$$

Characteristic scales of magnetic field



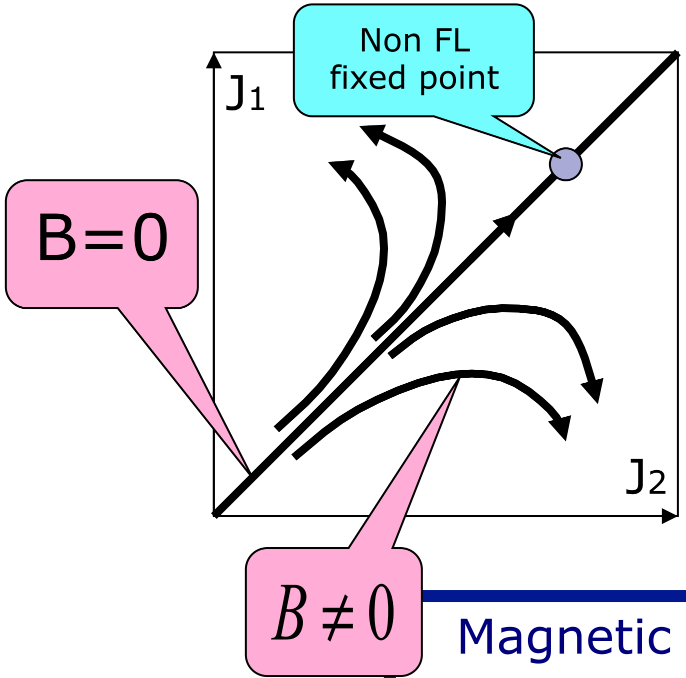
Field $B^* < B_C$ where spin-down electron is fully reflected

(model dependent)



Field of full polarization B_C

Instability of non-FL fixed point



$$\sum_j J_j c^{+}_{j\sigma} \vec{\sigma}_{\sigma\sigma'} c_{j\sigma'} \vec{S}$$

The symmetric state $J_1=J_2$ and the non-FL fixed point is only stable if protected by the basic symmetry (Time Reversal Symmetry)

Magnetic field breaks TRS and drives system to the 1-channel Kondo with decreasing the temperature T

Suppression of thermopower by magnetic field

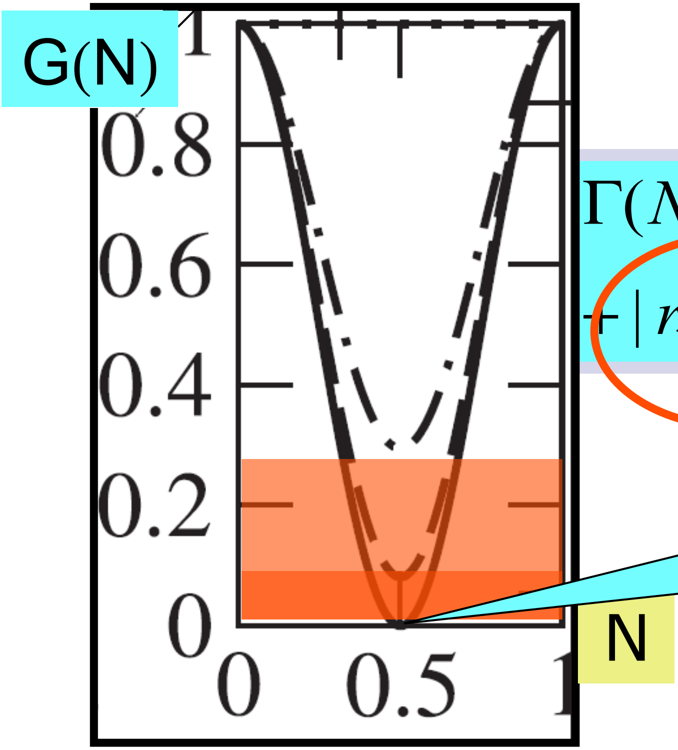
The main result: B<<B

At a finite B a gap in G(N) opens up at the degeneracy point N=1/2

$$eS \sim -|r_\uparrow r_\downarrow| \left(\frac{T}{\Gamma(N)}\right) \ln\left(\frac{E_C}{T+\Gamma(N)}\right) \sin(2\pi N) F\left(\frac{\Gamma(N)}{T}\right)$$

$$F(x) = \begin{cases} x, & x << 1 \\ const, & x >> 1 \end{cases}$$

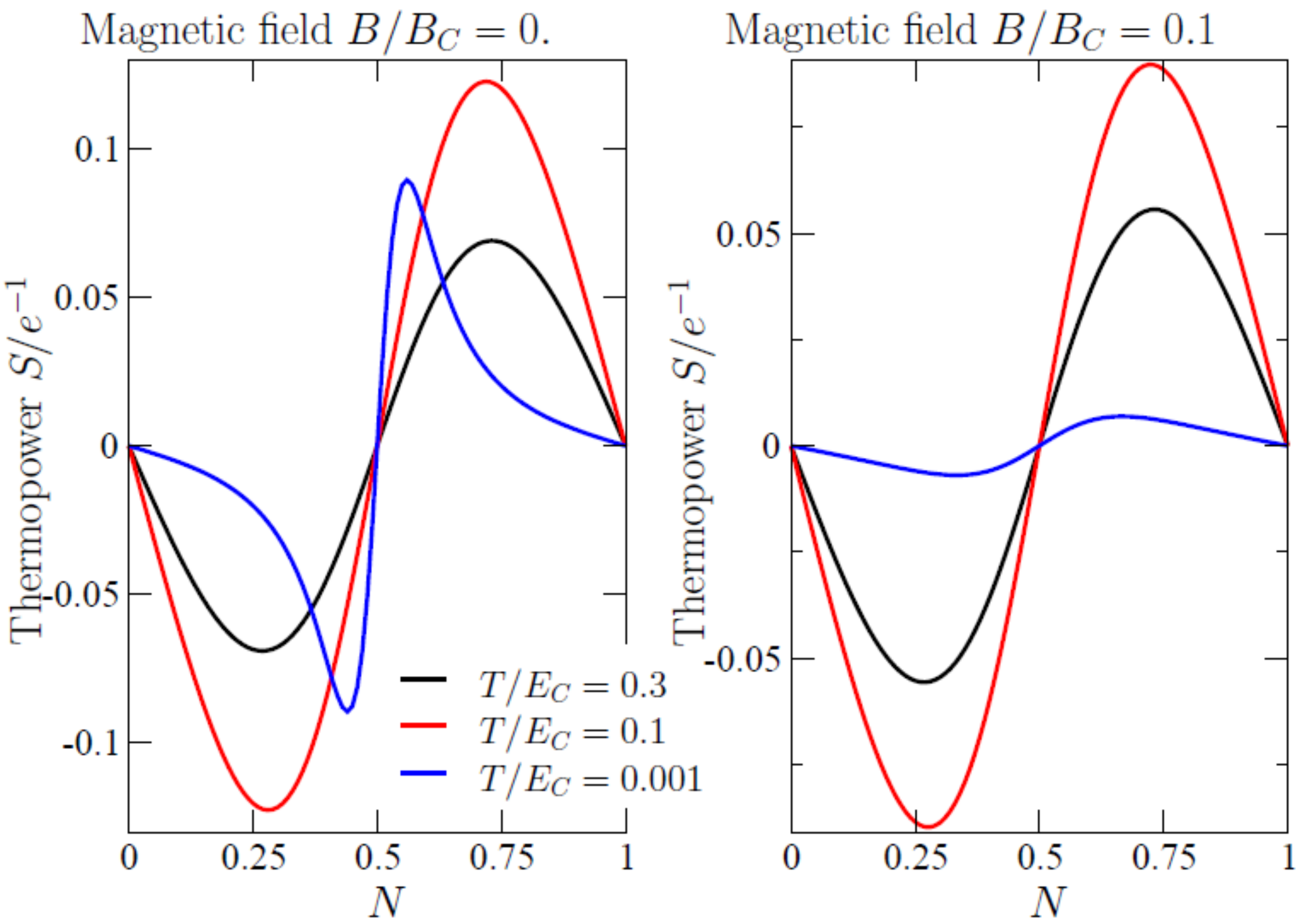
$$\Gamma(N) \sim E_C[|r_\uparrow + r_\downarrow|^2 \cos^2(\pi N) + |r_\uparrow - r_\downarrow|^2 \sin^2(\pi N)]$$



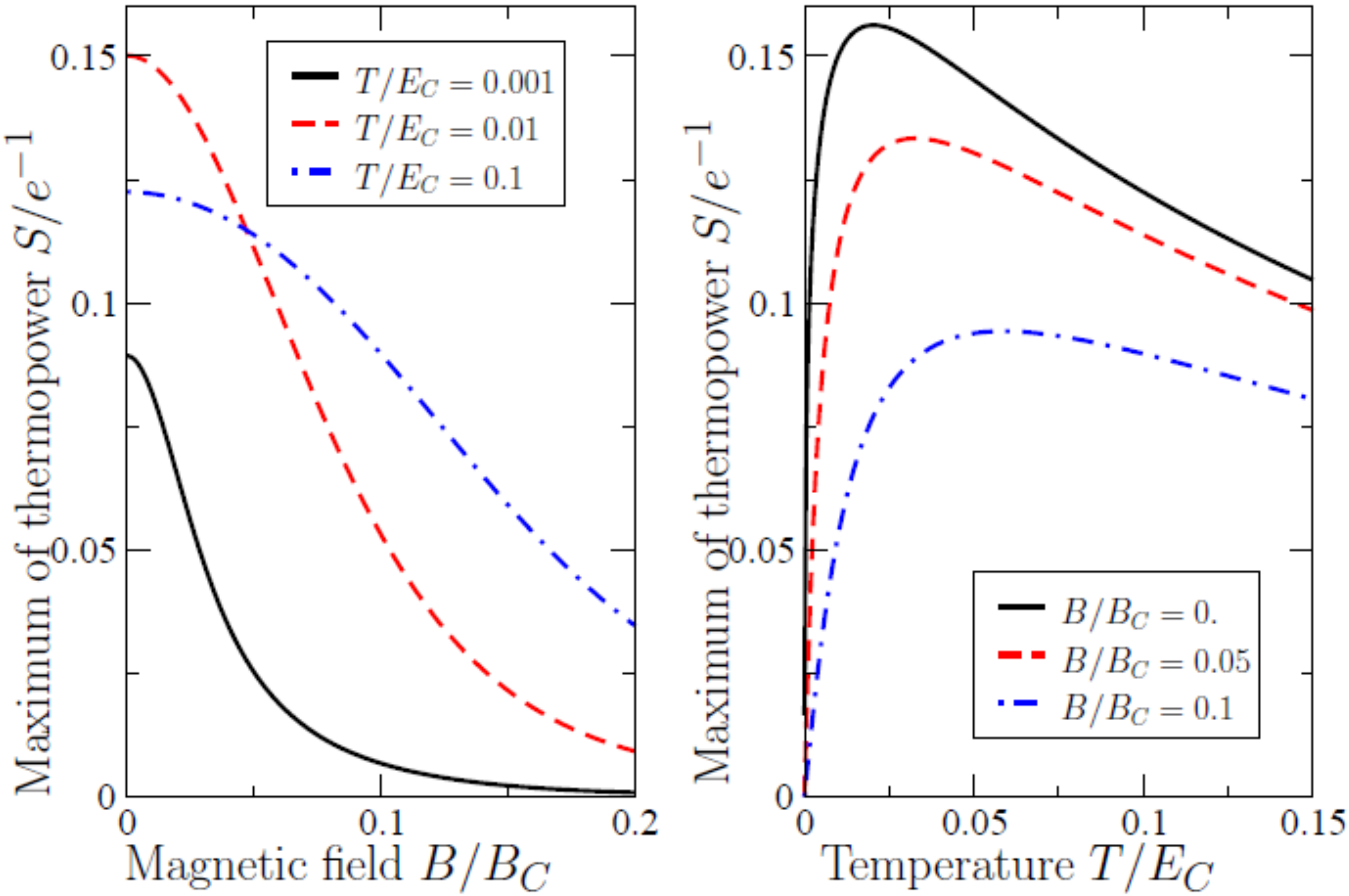
Coulomb blockade peak (degeneracy) point

Effect of B

Theoretical predictions: gate voltage dependence

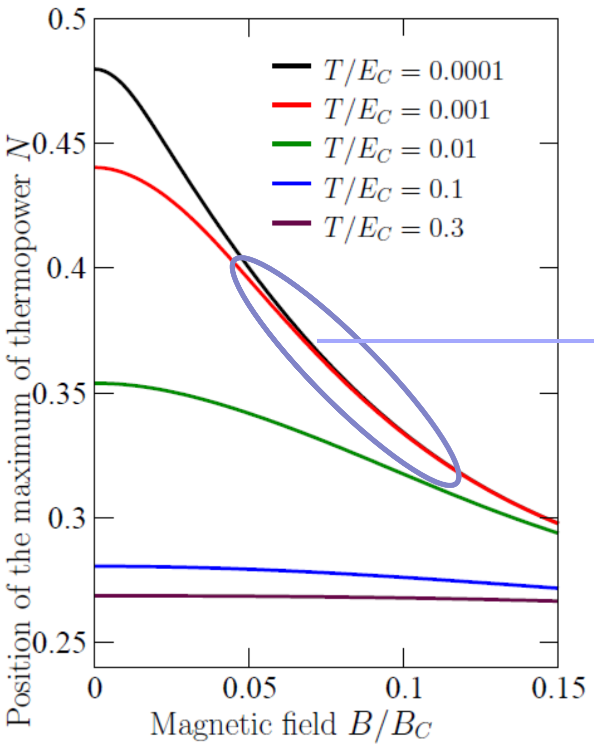


Theoretical predictions: B and T -dependences



Effects of magnetic field on thermopower

$$|r_\downarrow r_\uparrow| \xrightarrow{B\uparrow} |r_\uparrow|$$



For $T << T_{\min}$:

$$S \propto -\frac{1}{e}|r_\uparrow r_\downarrow| \frac{T}{\Gamma(N)} \ln\frac{E_C}{\Gamma(N)} \sin(2\pi N)$$

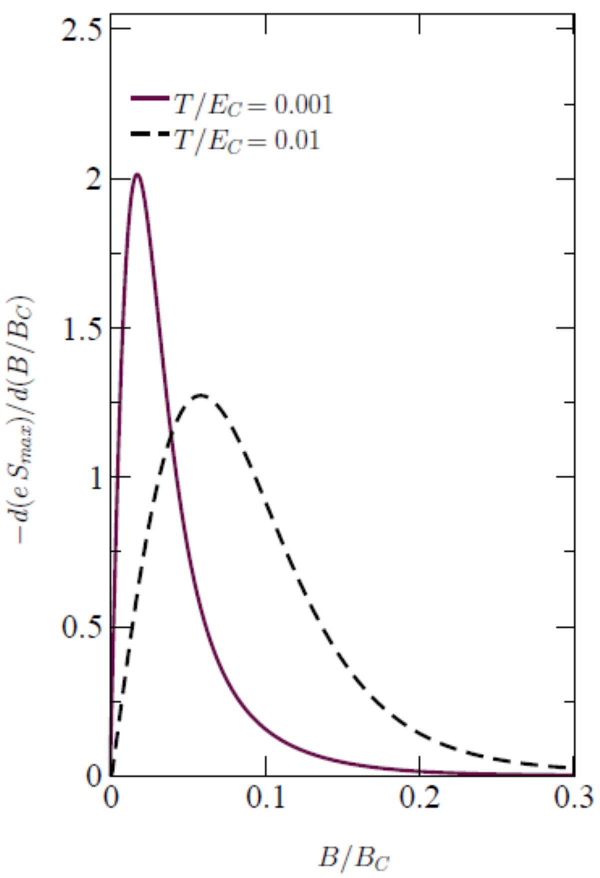
$$eS_{\max} \propto \frac{T}{E_0}$$

$$E_0 = E_C \left|\frac{B}{B_C}\right| \ln^{-1}\left[\frac{B_C}{B|r|}\right]$$

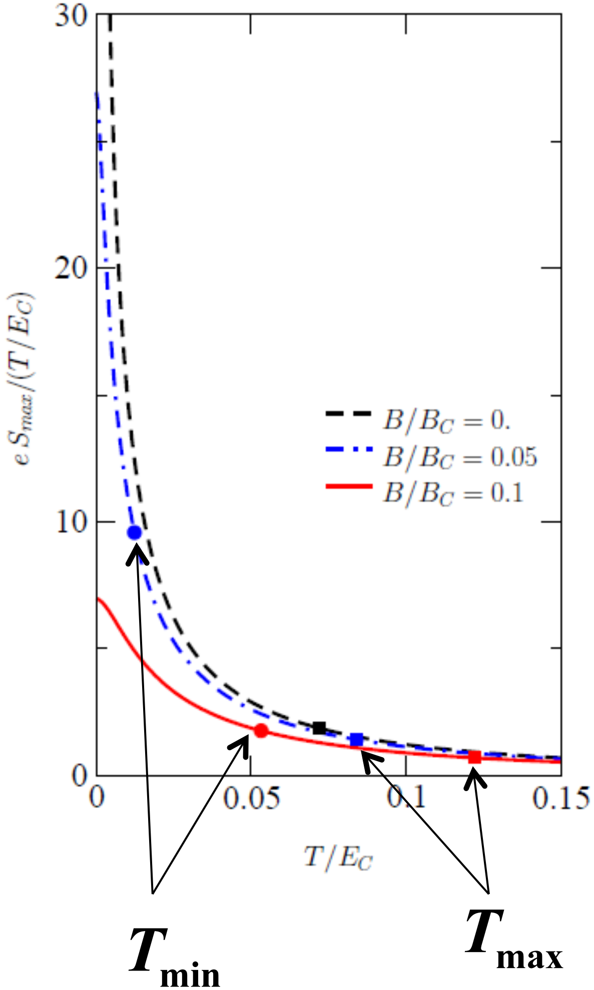
"Giant Fermi energy"

Giant Fermi-liquid behavior in magnetic field

Theoretical predictions: derivatives



Existence of maximum in dS/dB



B=0: Smax/T diverges at T = 0;
Finite B: Smax/T saturates below Tmin

Message to take home

0 — $T_{\min} \sim (|r_\downarrow| - |r_\uparrow|)^2 E_C$ — $T_{\max} \sim (|r_\downarrow| + |r_\uparrow|)^2 E_C$ — E_C — E_F — **T**

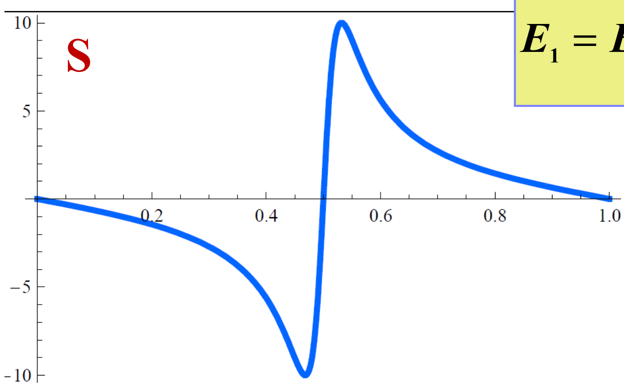
"Giant FL", 1CK

$$eS^{\max} \sim \frac{T}{E_0}$$

Non-perturbative, strong NFL, 2CK

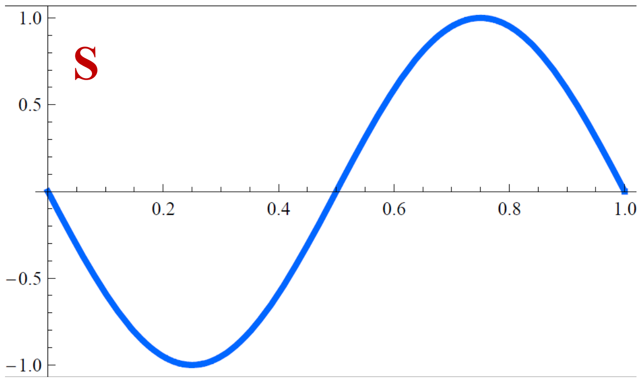
$$eS^{max} \sim \sqrt{\frac{T}{E_1}} \ln\left(\frac{E_C}{T}\right)$$

$$E_1 = E_C\left(\frac{1}{r_\uparrow} + \frac{1}{r_\downarrow}\right)^2$$



Perturbative CB, weak NFL, 2CK

$$eS^{\max} \sim |r_\uparrow r_\downarrow| \ln\left(\frac{E_C}{T}\right)$$

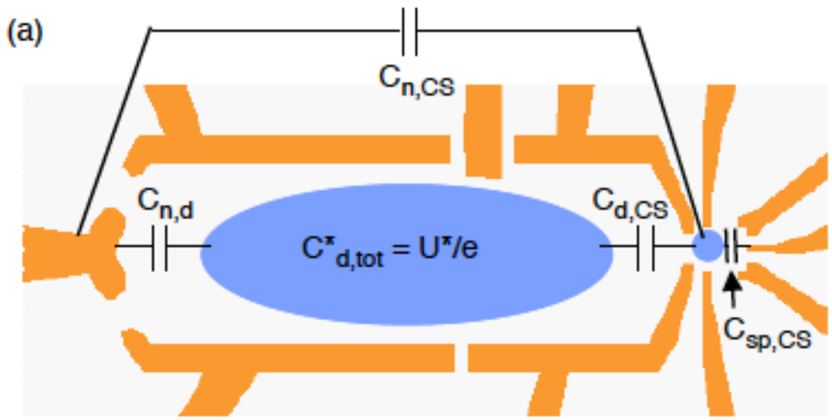


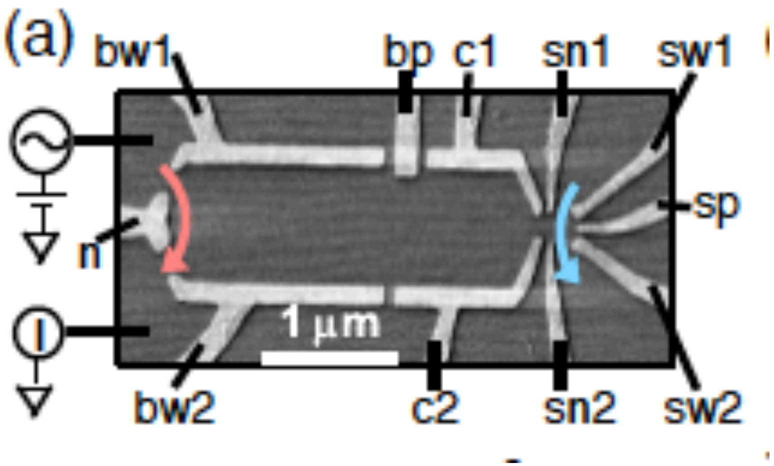
BULK

$$S \sim \frac{T}{E_F}$$

T.K.T. Nguyen, MK and V.E. Kravtsov, PRB 82, (2010)

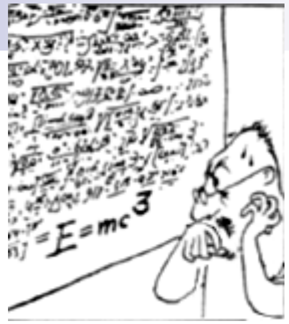
Perspectives:





- Multi-channel Kondo effect
- Influence of noise
- Influence of spin-orbit
- Influence of finite s-d voltage
- Quenches with the gate and s-d voltage
- Quench with magnetic field
- Multy-dot setup: Bell inequalities?

Conclusions



- Thermopower of a quantum dot can be much larger than in the bulk $eS_{BULK} \sim T/E_F$

- For closed dot ($g \ll 1$) the maximum thermopower $eS \sim \ln(1/g) > 1$; for open dot the maximum thermopower $eS \sim r < 1$.

- Kondo physics in thermopower of an open dot; magnetic field leads to crossover from 2CK to 1CK

- Magnetic field suppresses thermopower and restores (non-perturbative) FL behavior at $T < E_C\,|r|\ (B/B)$ with "E_F" $\sim E_C\,(B/B_C)$