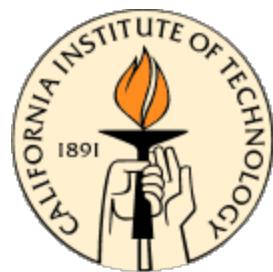


Putting Floquet theory to work: topological phases with time dependent Hamiltonians

Netanel Lindner (Caltech, IQI)

Victor Galitski (UMD)

Gil Refael (Caltech)



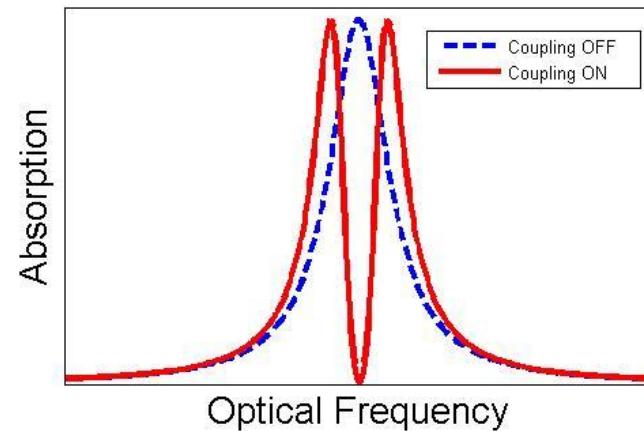
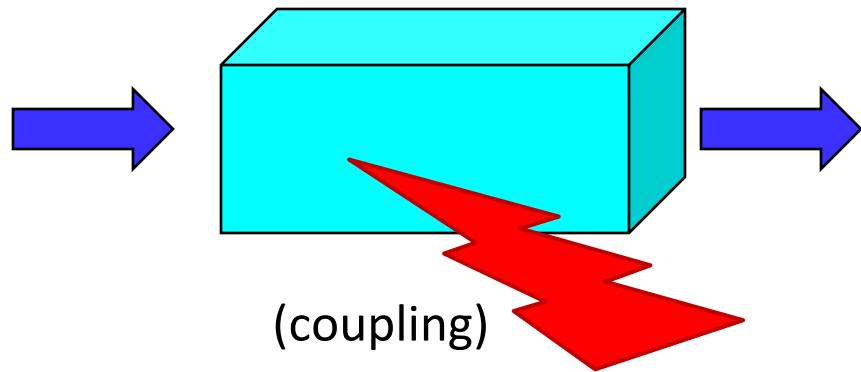
Funding:
Packard Foundation,
Sloan Foundation,
NSF

Outline

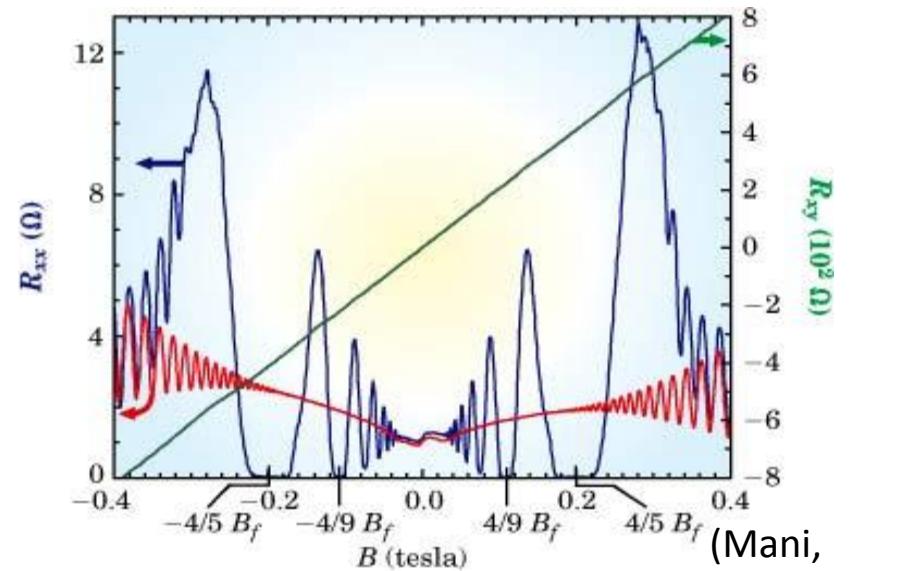
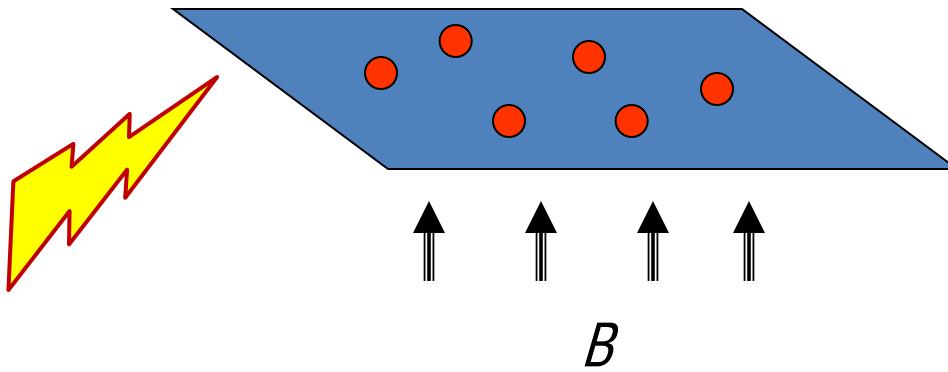
- Motivation: photo-induced phases.
- 2d Topological phases crash course.
- Light-induced topological insulator – could it be done?
- Experimental realization (a theorist's perspective)

Photo-induced phases: Examples

- photo-induced transparency:



- photo-induced zero resistance states in 2DEG in a magnetic field:

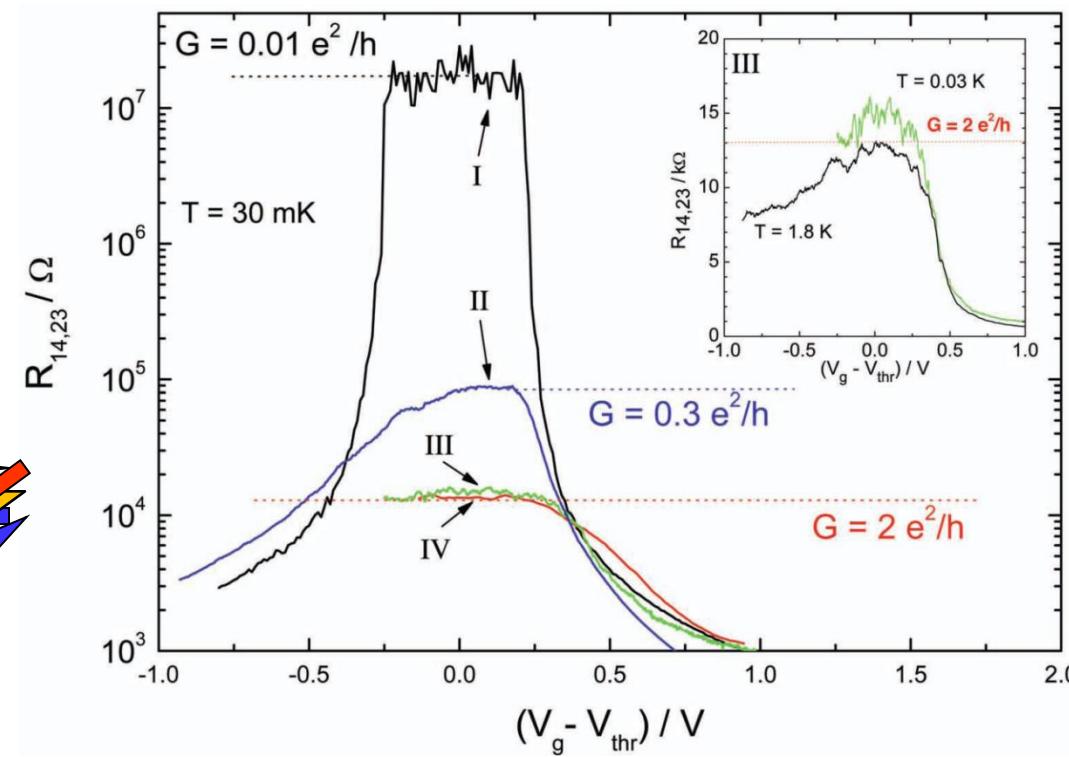
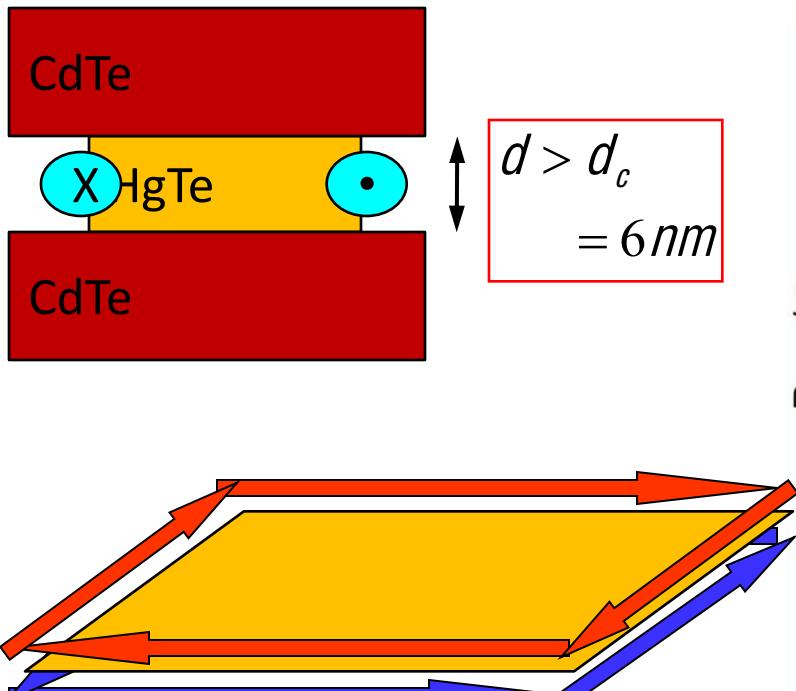


(Mani,
et al, 2003)

Can we have a photo-induced topological insulator?

Topological phases: An introduction

First observed topological insulator: CdTe, HgTe heterostructures



Bernevig, Hughes, Zhang (2006)

Koenig,.., Molenkamp.., Zhang (2007)

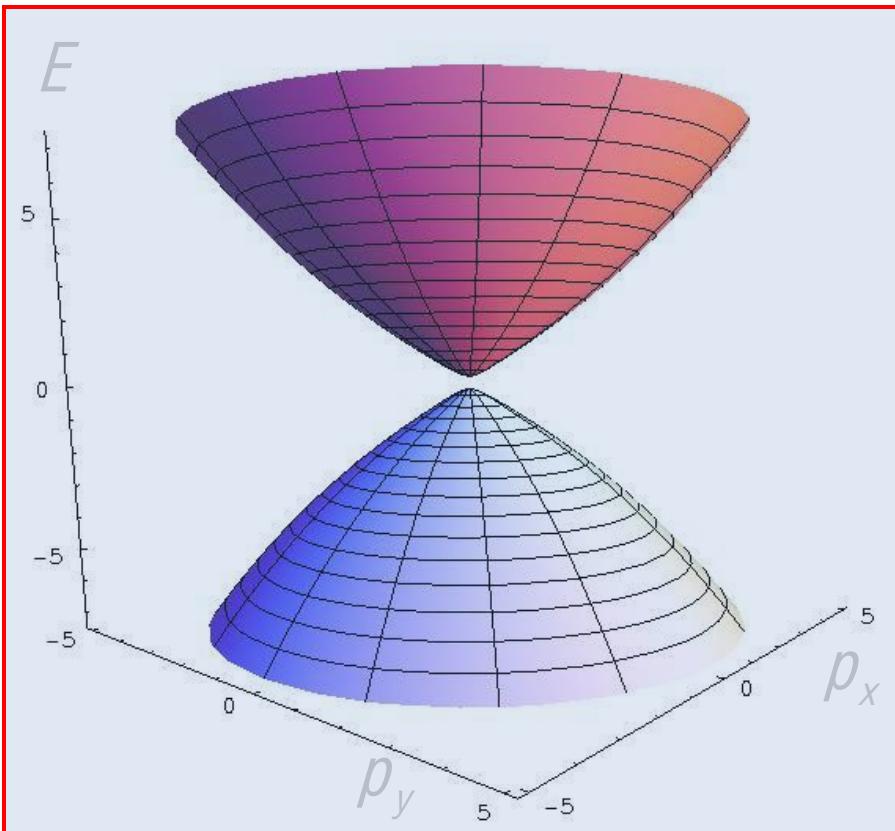
First observed topological insulator: HgTe

- HgTe dispersion (per spin): Dirac cone with small band gap

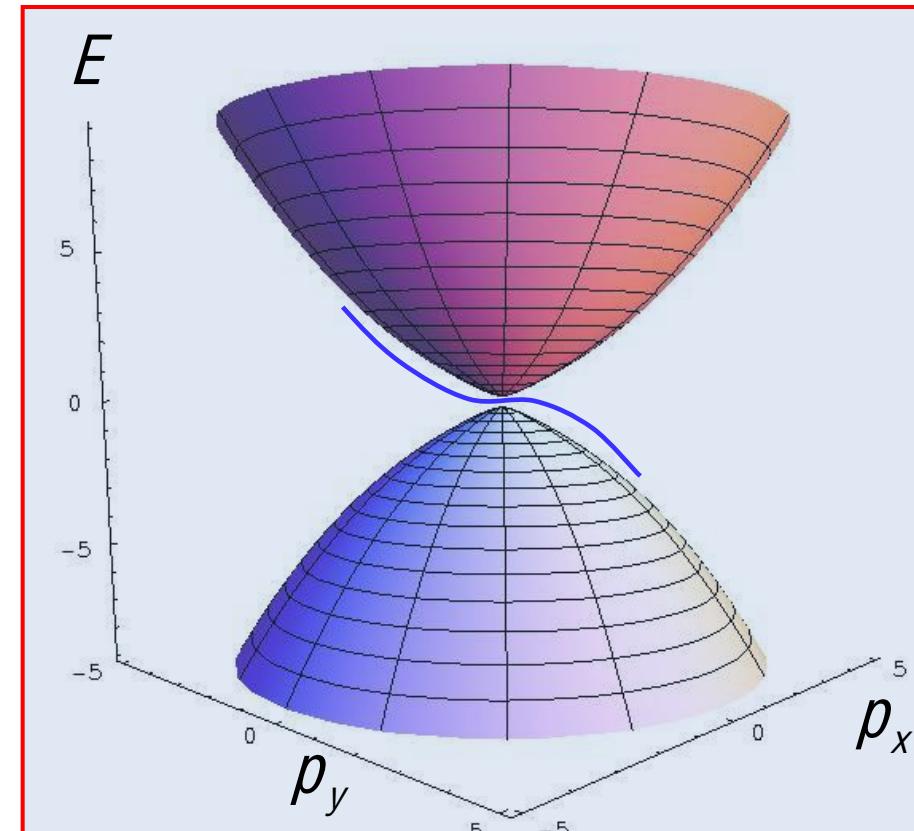
$$H = \varepsilon_0 + \vec{d} \cdot \hat{\tau}$$

$$\vec{d} = p_x \cdot \hat{x} + p_y \cdot \hat{y} + \left(m + b(p_x^2 + p_y^2) \right) \cdot \hat{z}$$

$m > 0, b > 0$ (Trivial)



$m > 0, b < 0$ (Topological)



What makes a phase topological?

$$H = \varepsilon_0 + \vec{\tau} \cdot \vec{d}$$

$$\boxed{\vec{d} = (p_x, p_y, m + b(p_x^2 + p_y^2))}$$

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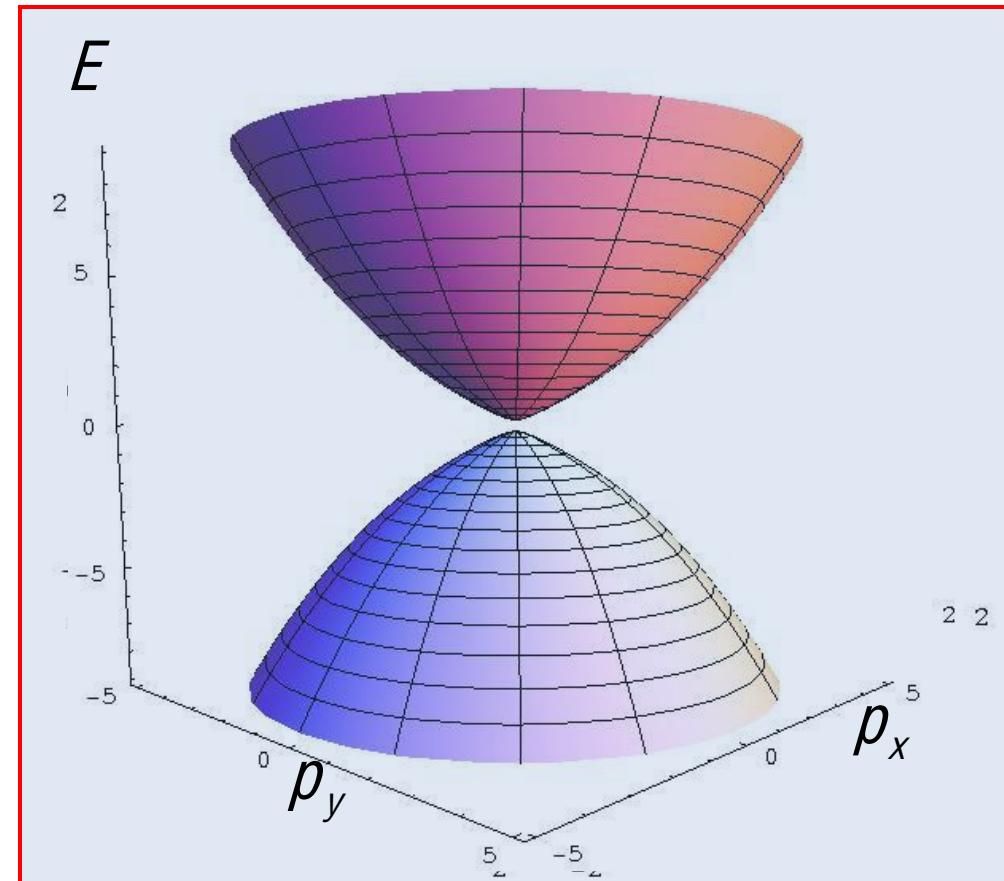
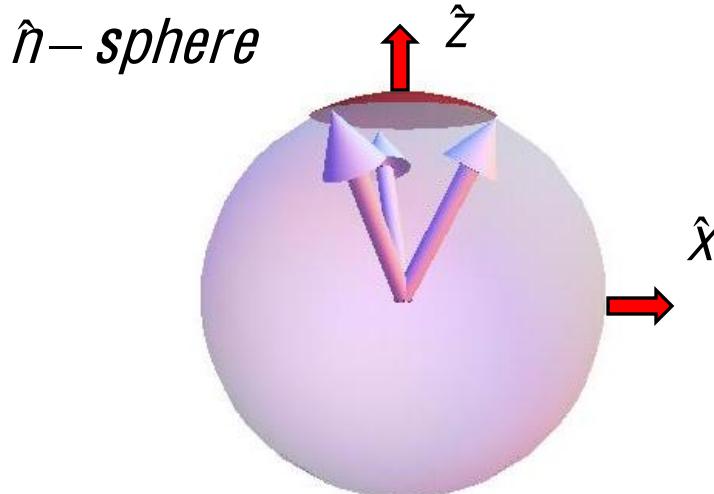
(map BZ **to** unit sphere)

- TKNN invariant:

$$\sigma_{xy} = \frac{e^2}{4\pi\hbar} \int_{p \in BZ} dp_x dp_y \hat{n} \cdot \left(\frac{\partial \hat{n}}{\partial p_x} \times \frac{\partial \hat{n}}{\partial p_y} \right)$$

$$m=0.2, \quad b=0.3 \quad (\textit{non-topo})$$

Wrapping the unit sphere?



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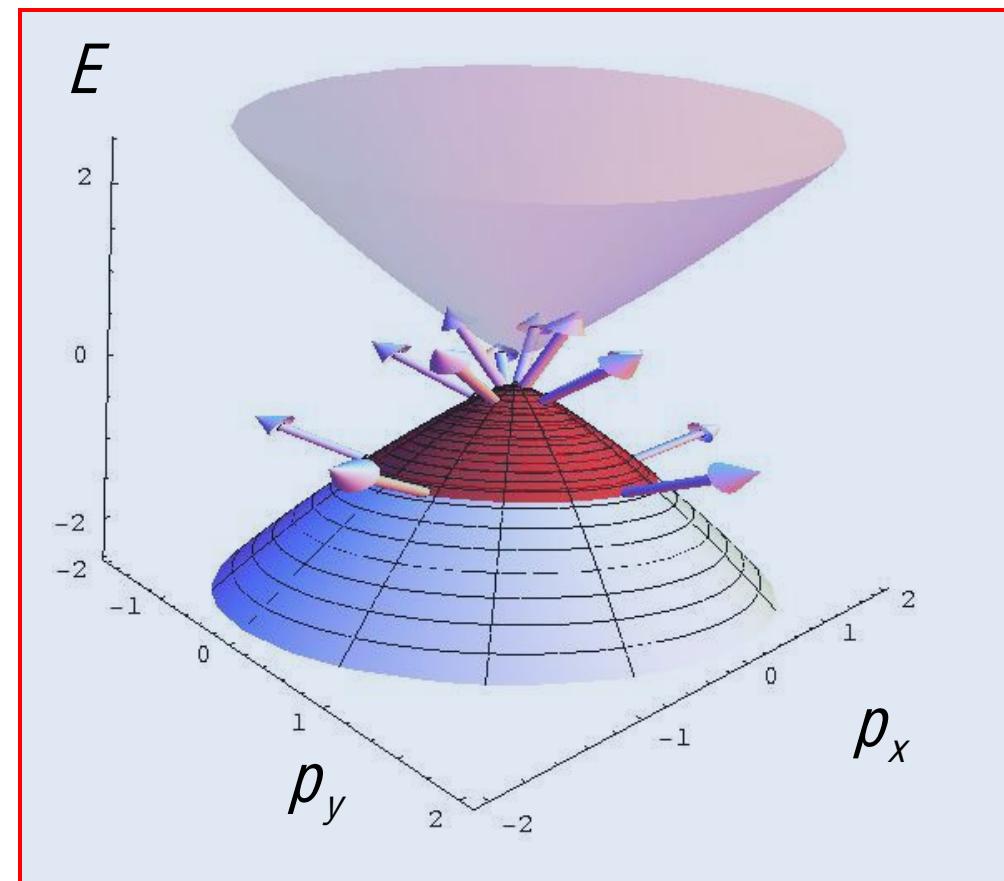
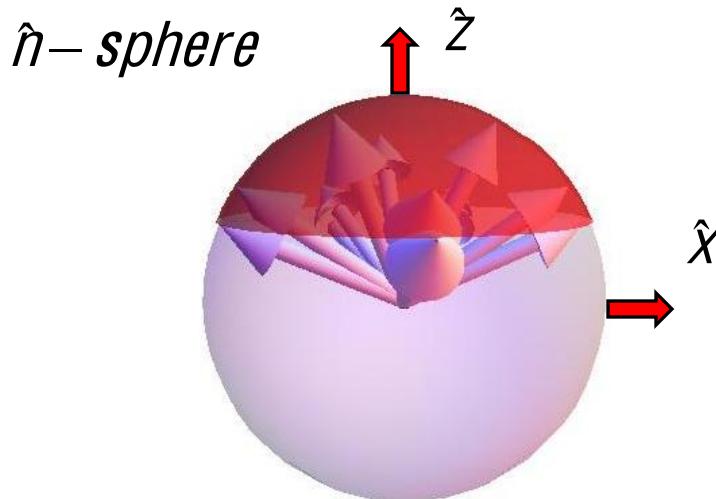
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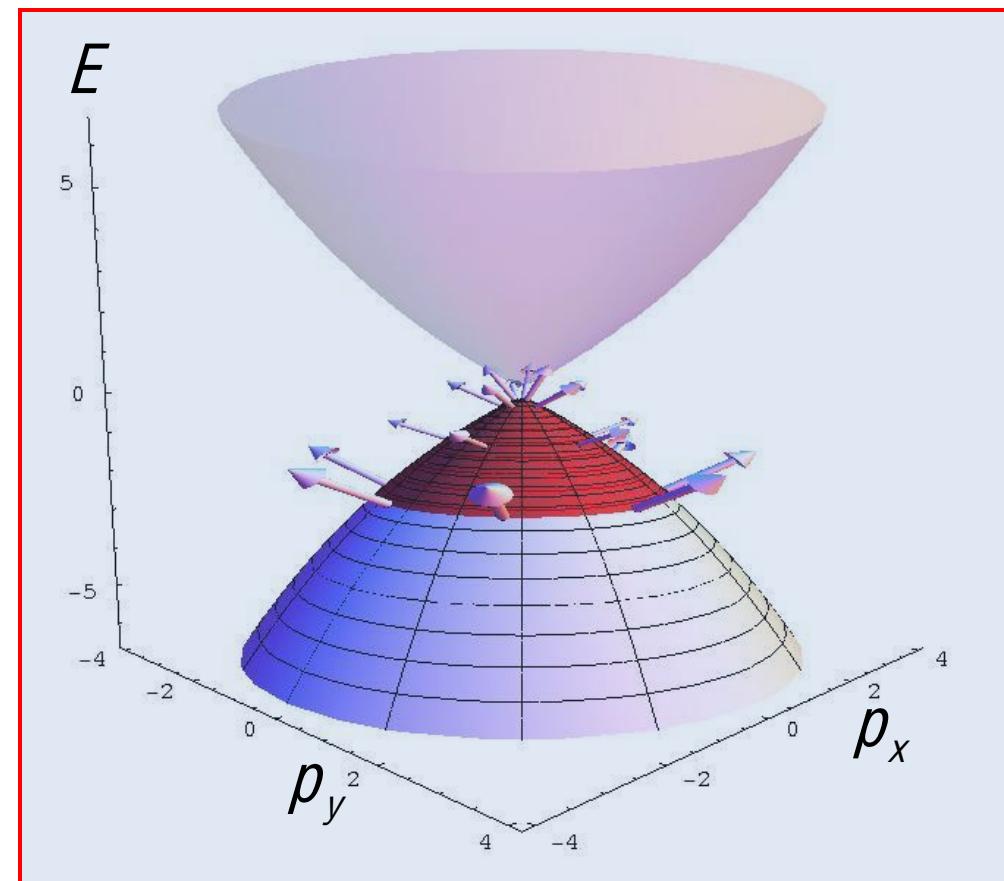
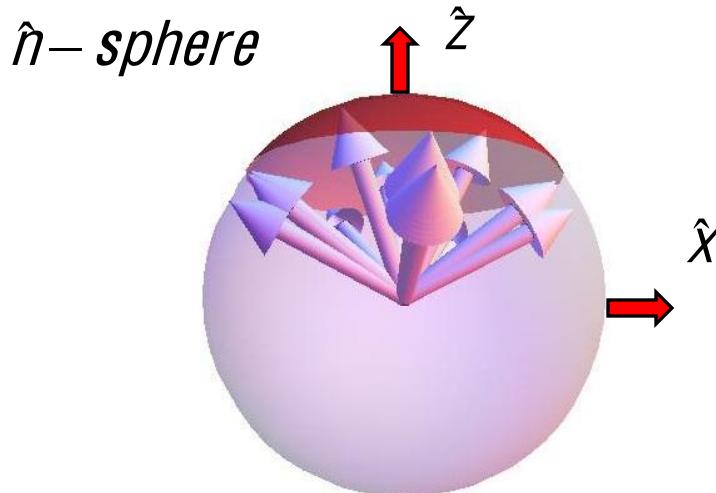
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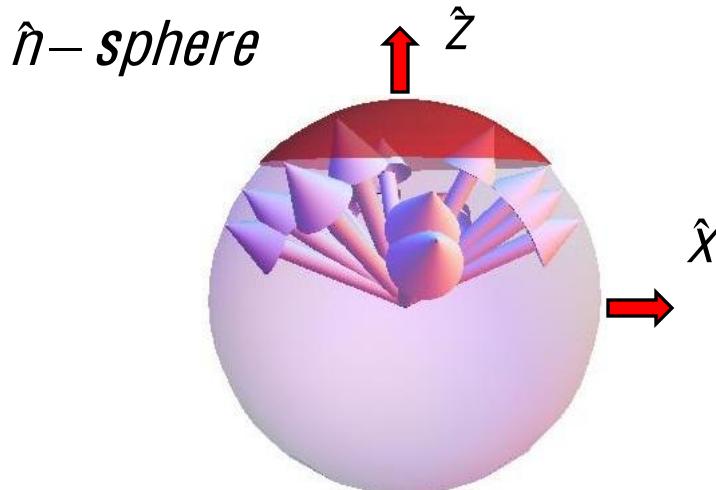
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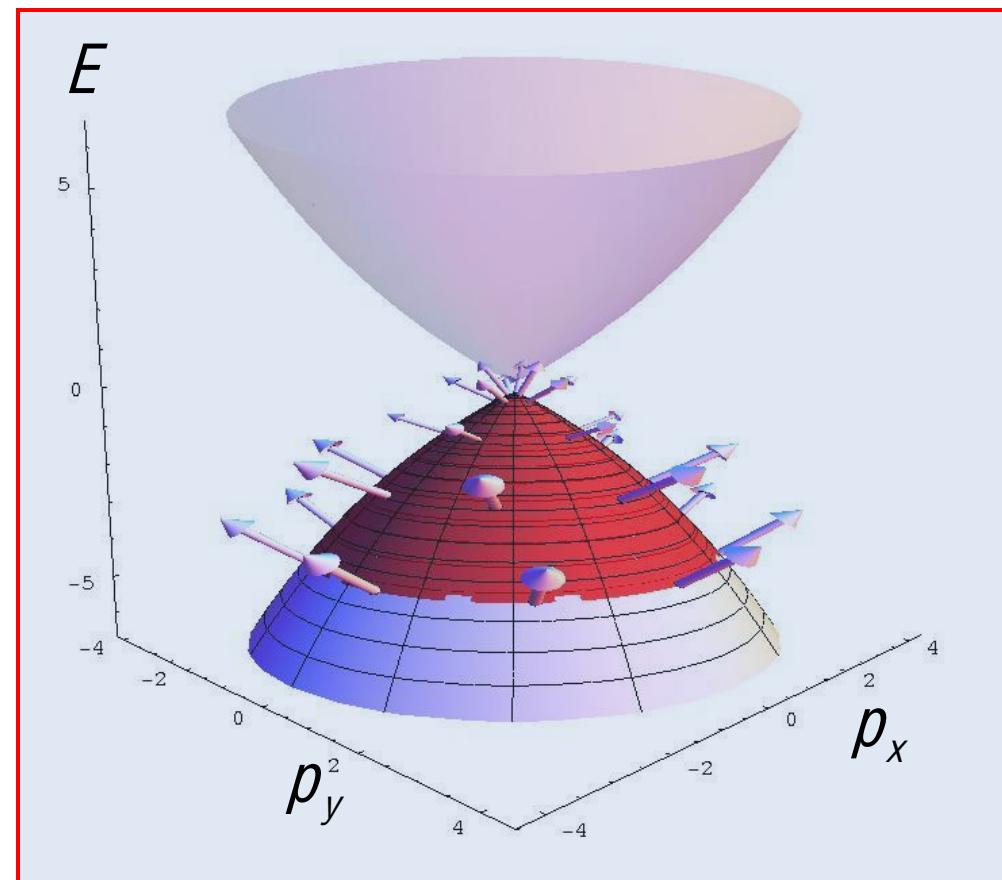
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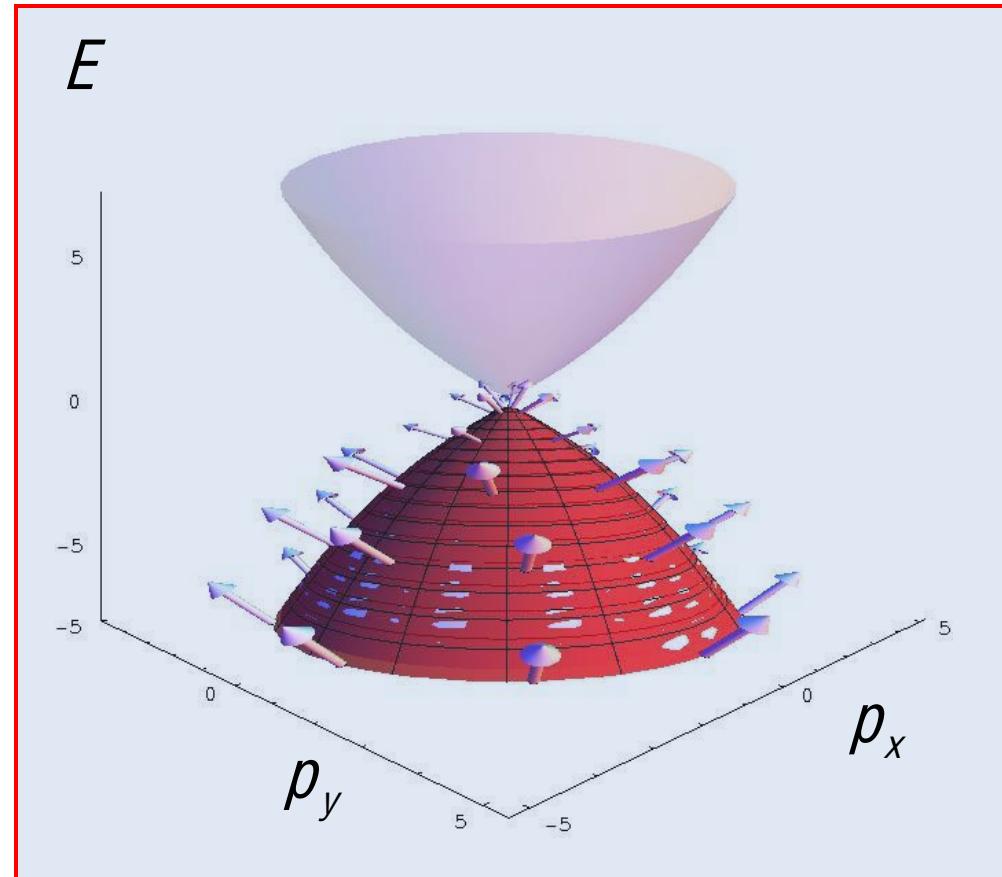
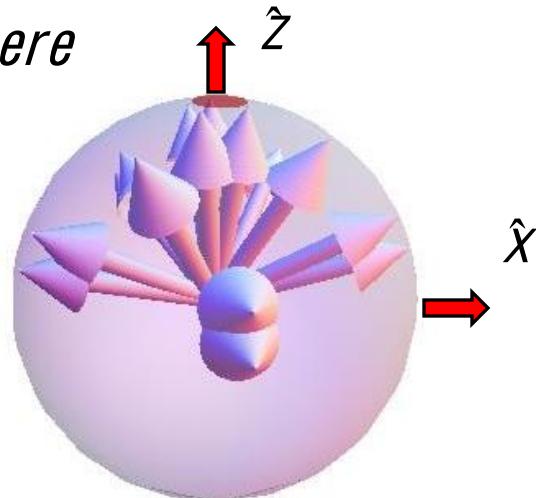
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Wrapping the unit sphere?

\hat{n} -sphere



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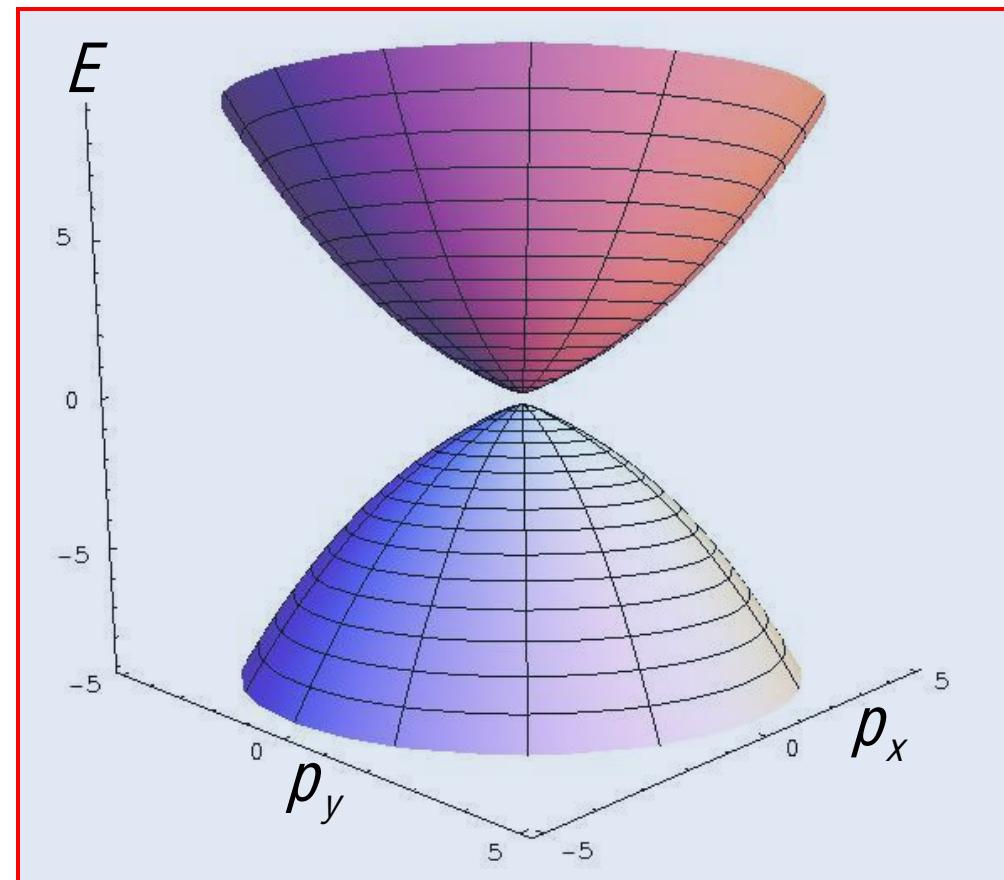
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Wrapping the unit sphere?

$m=0.2, \ b=-0.3 \ (\text{topo})$



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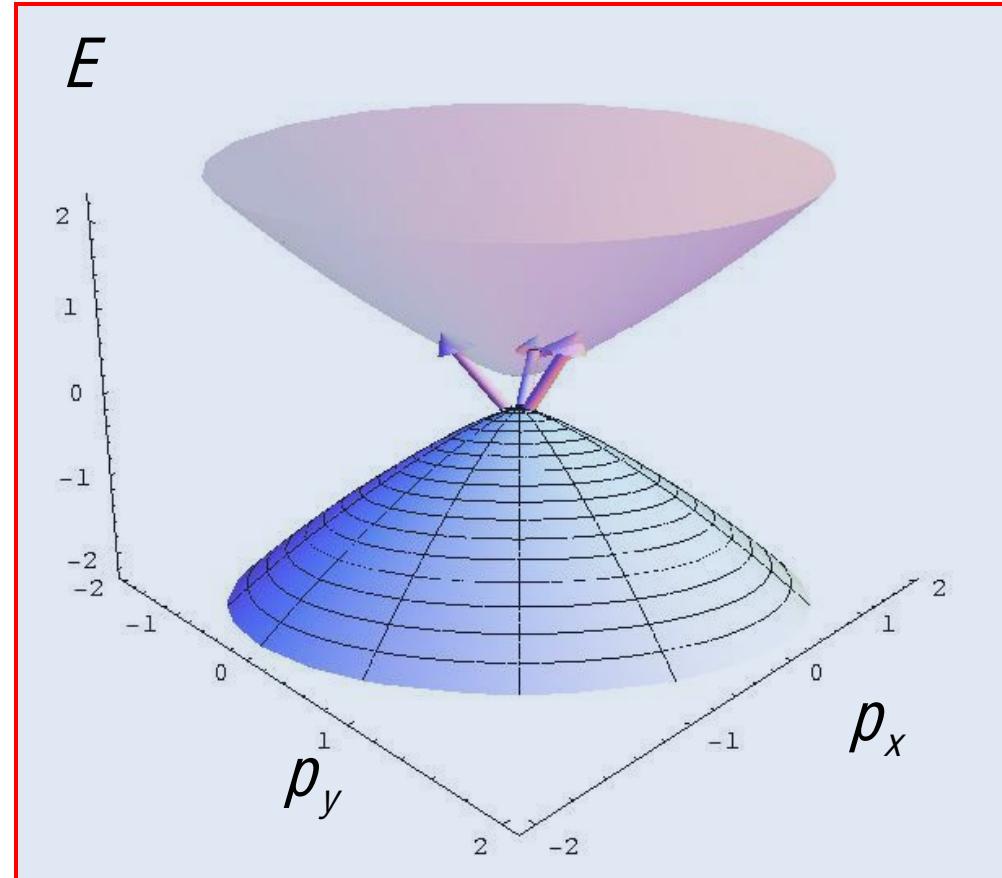
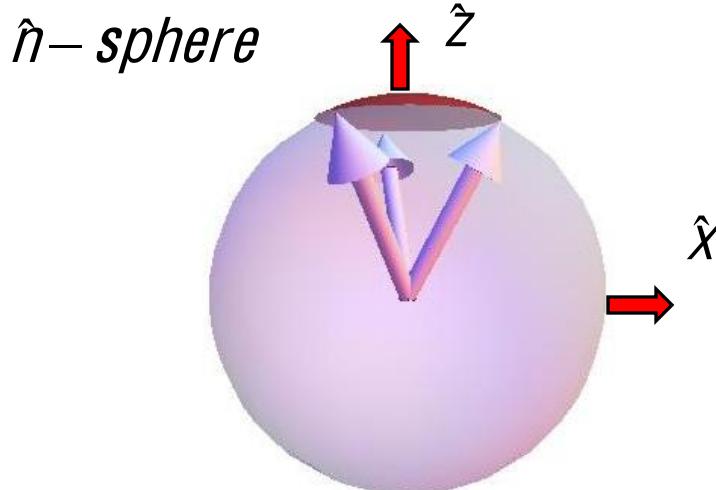
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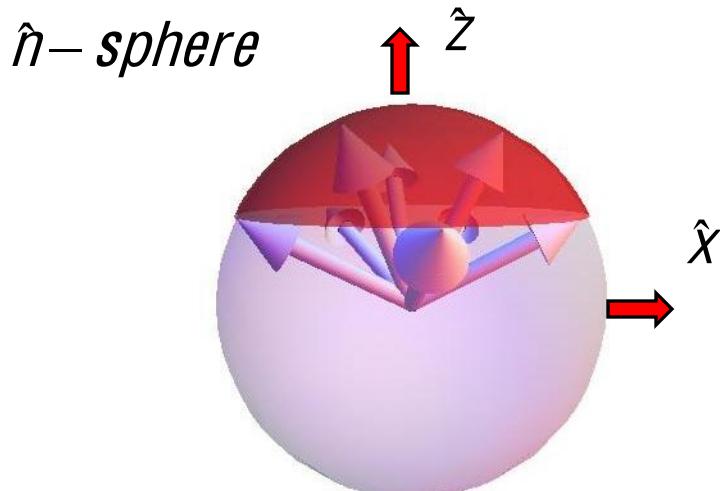
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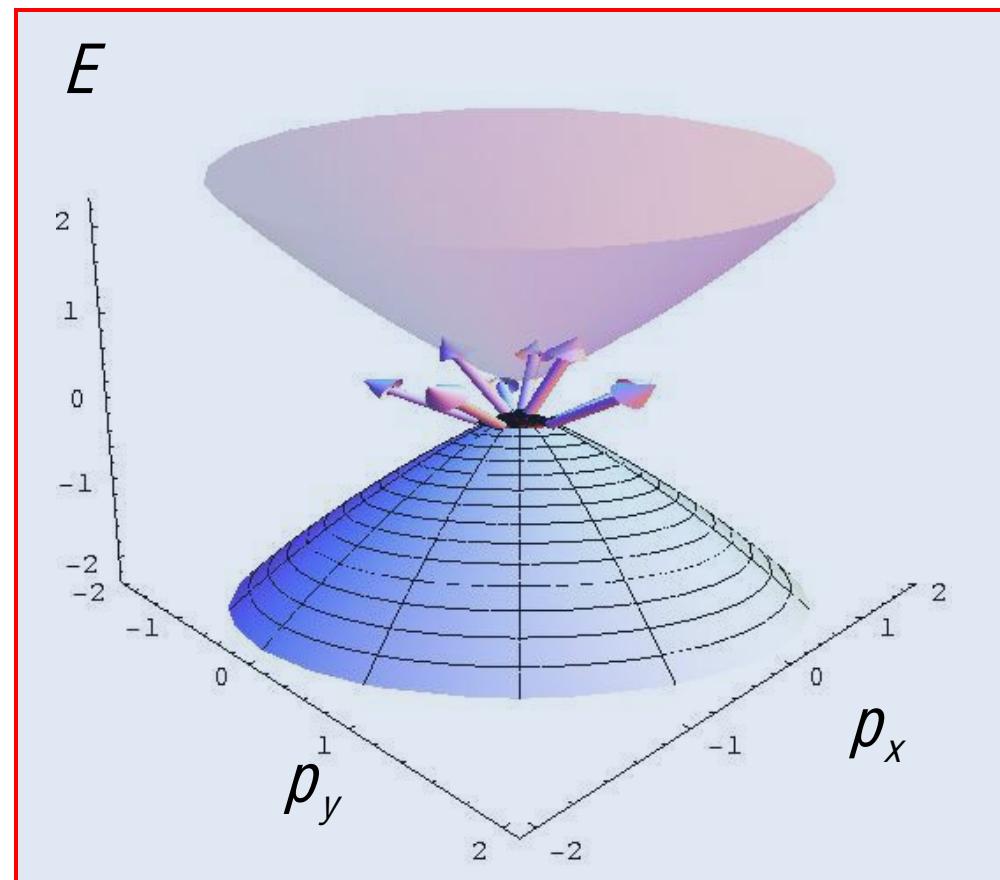
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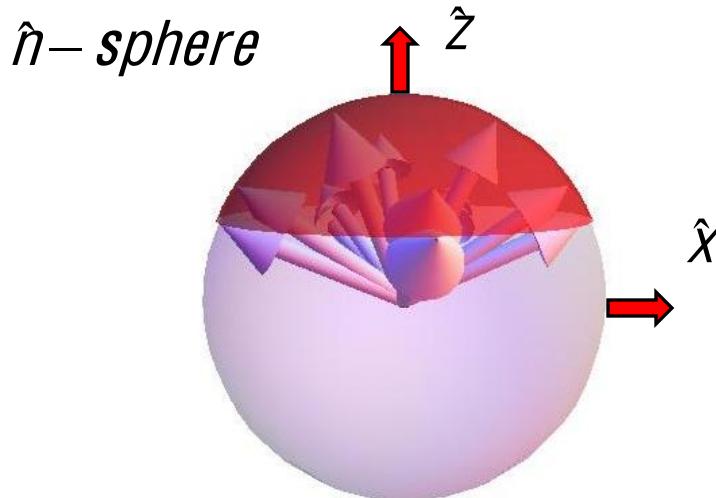
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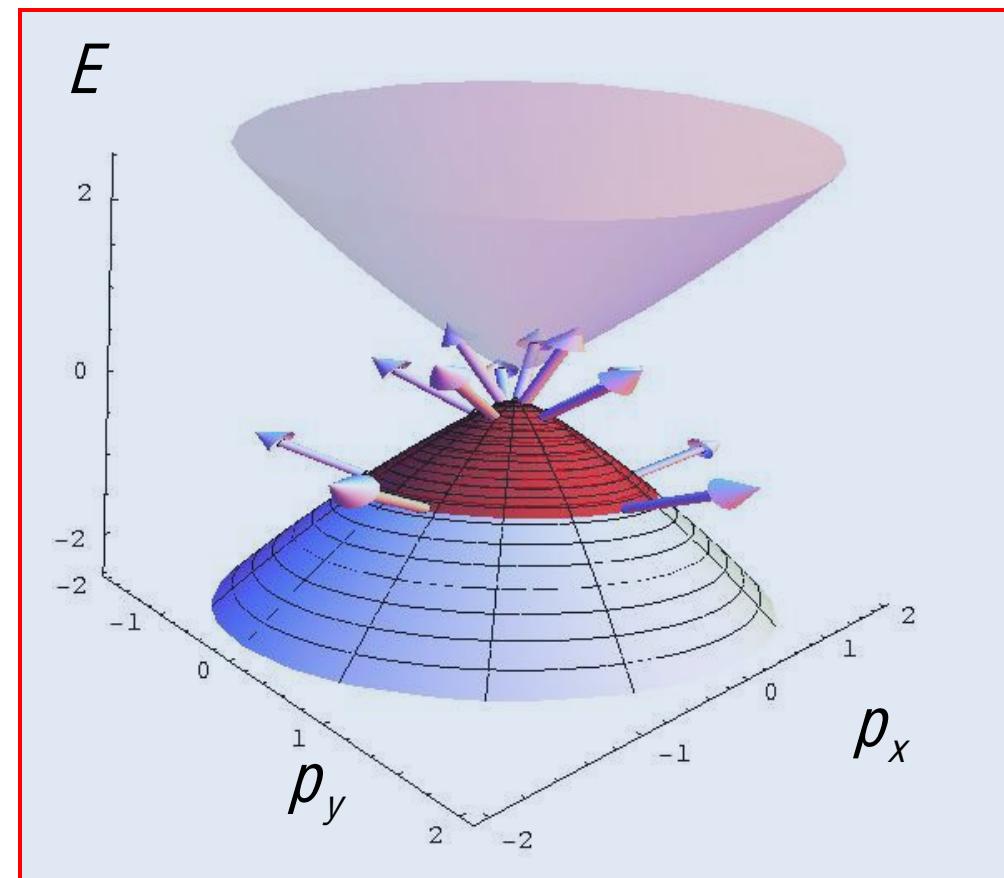
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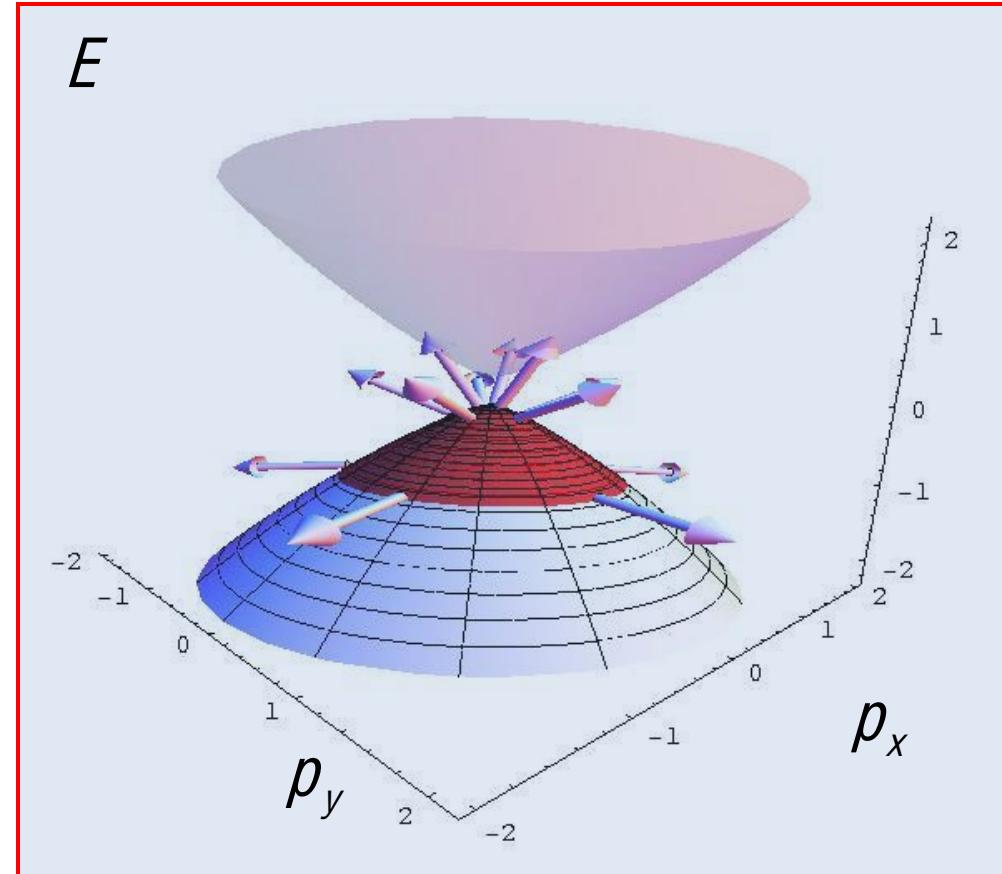
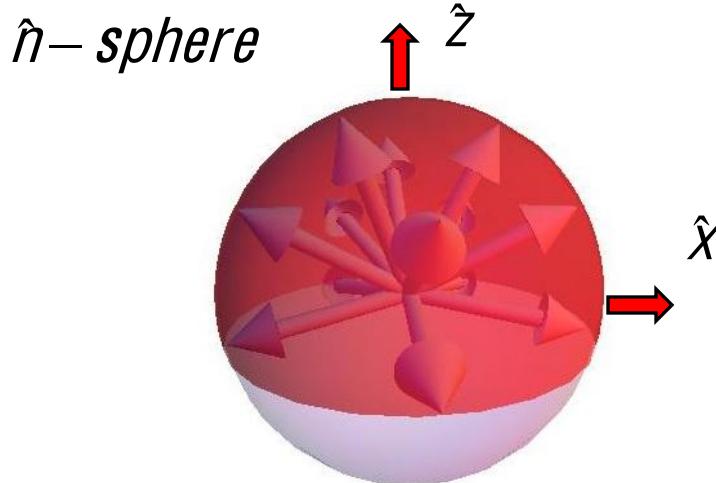
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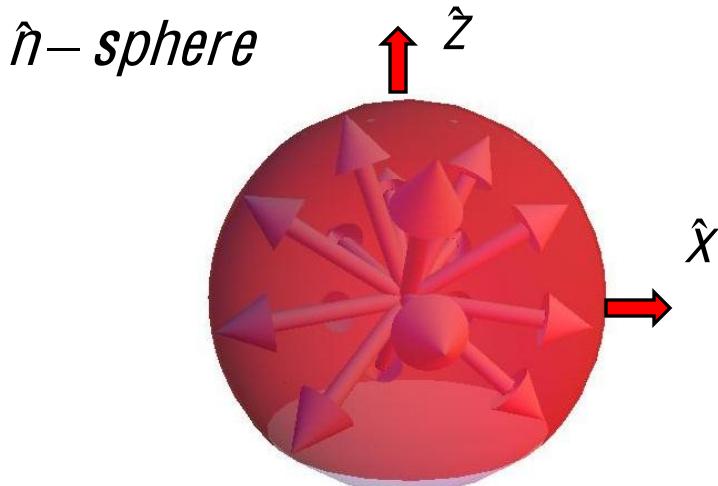
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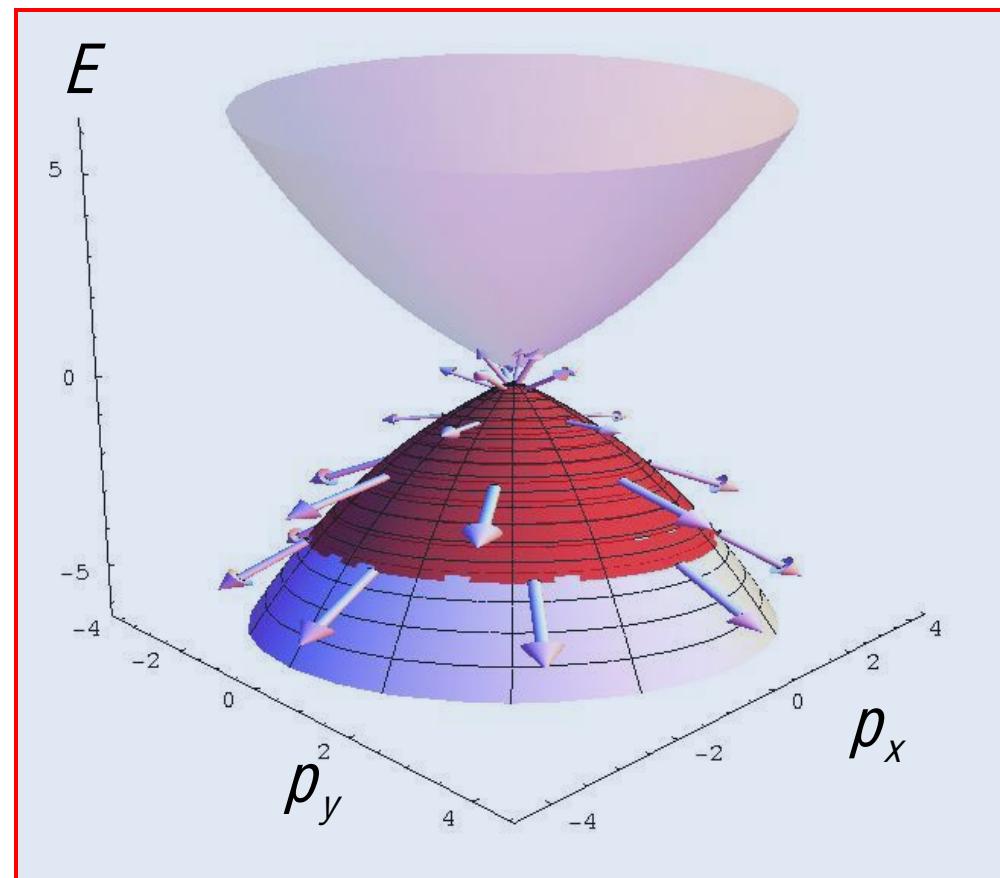
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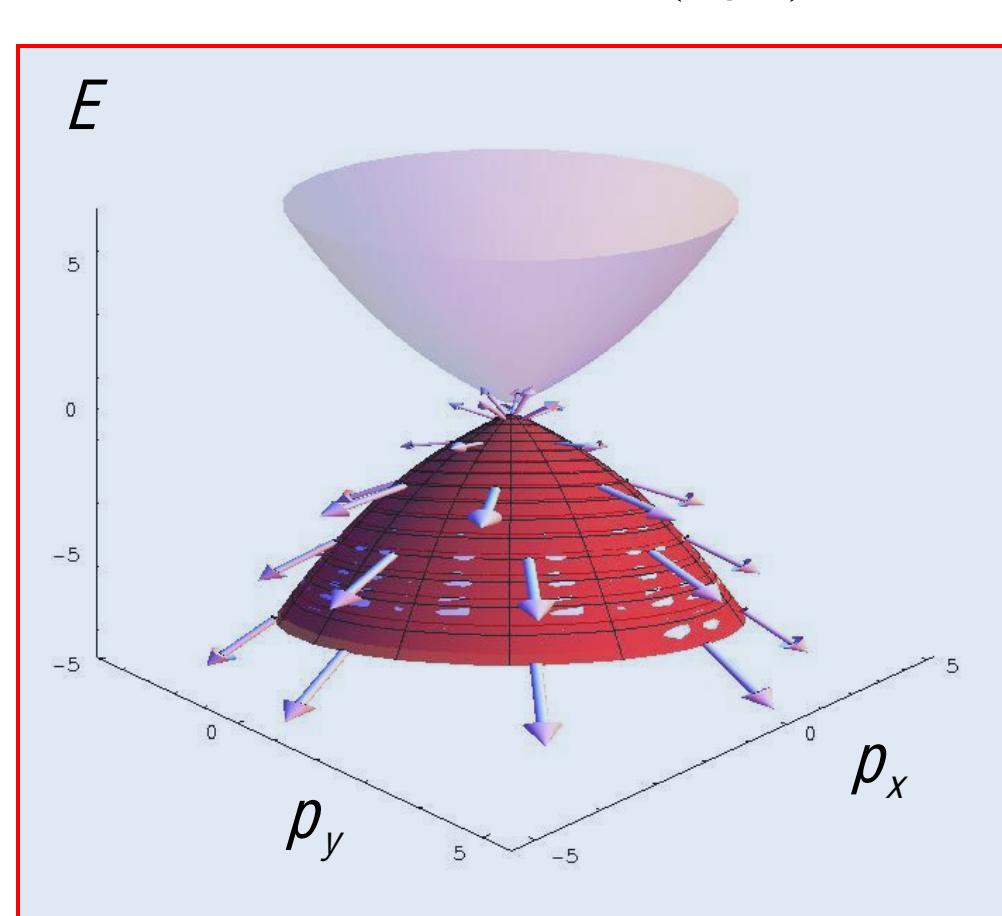
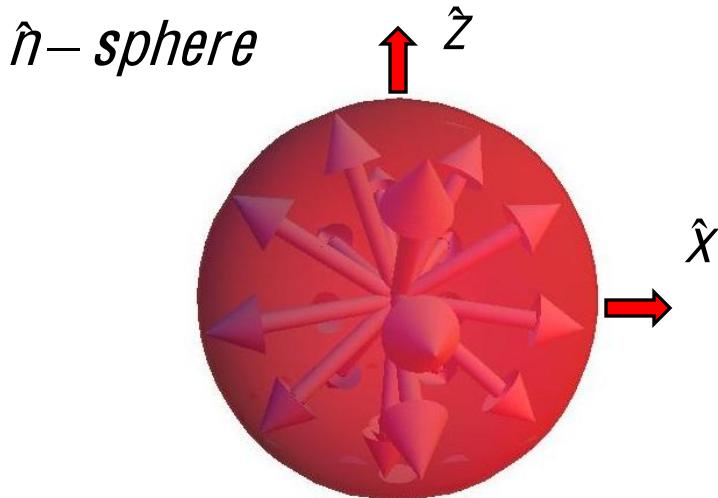
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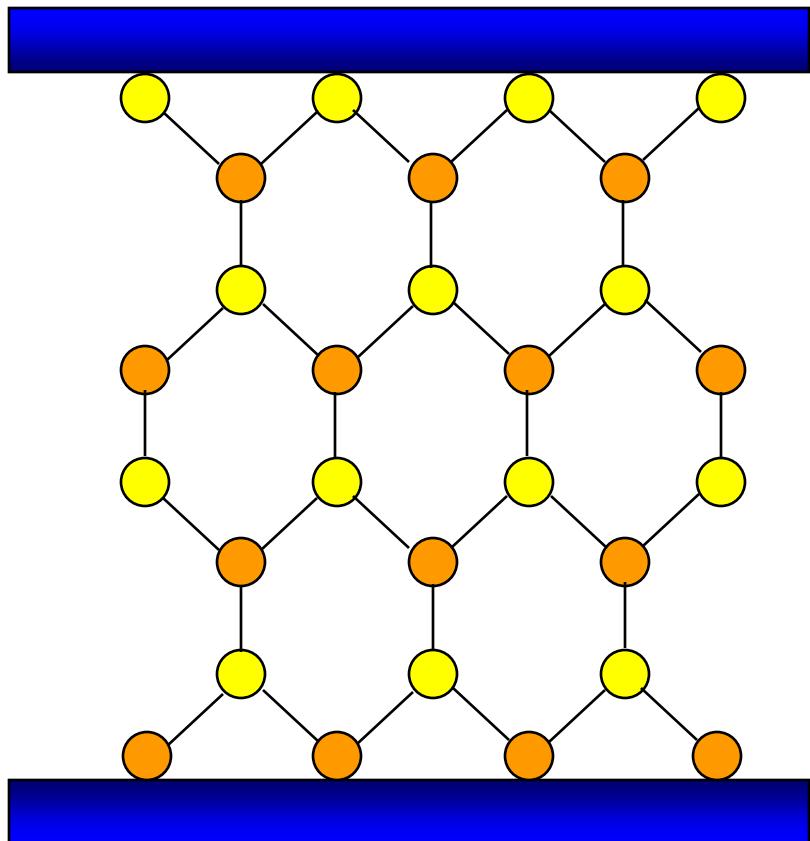
Wrapping the unit sphere?



The simplest model – Kane-Mele honeycomb lattice

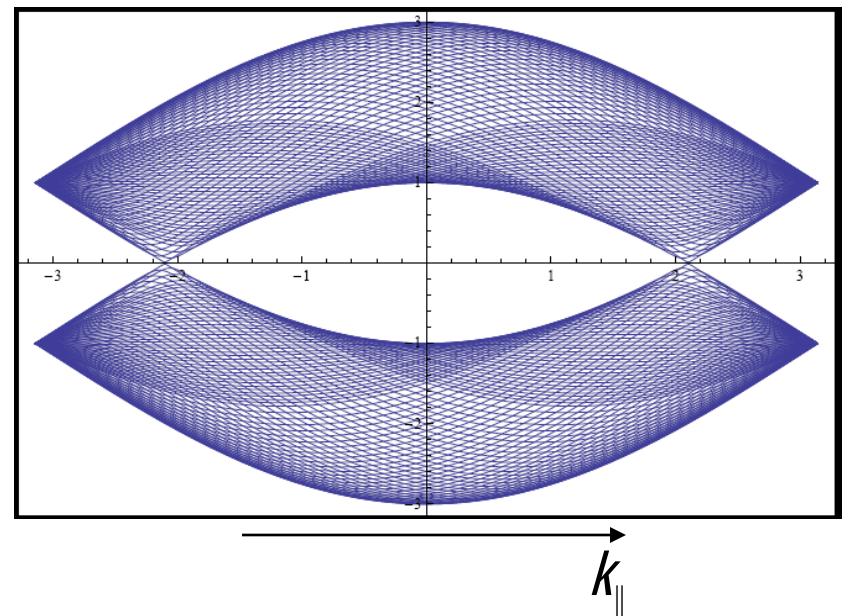
Kane-Mele model:

Honeycomb lattice with 2nd nearest neighbor hopping



\vec{k}_{\parallel}

$$H = -\sum_{\langle ij \rangle} t c_{i,s}^+ c_{j,s}$$

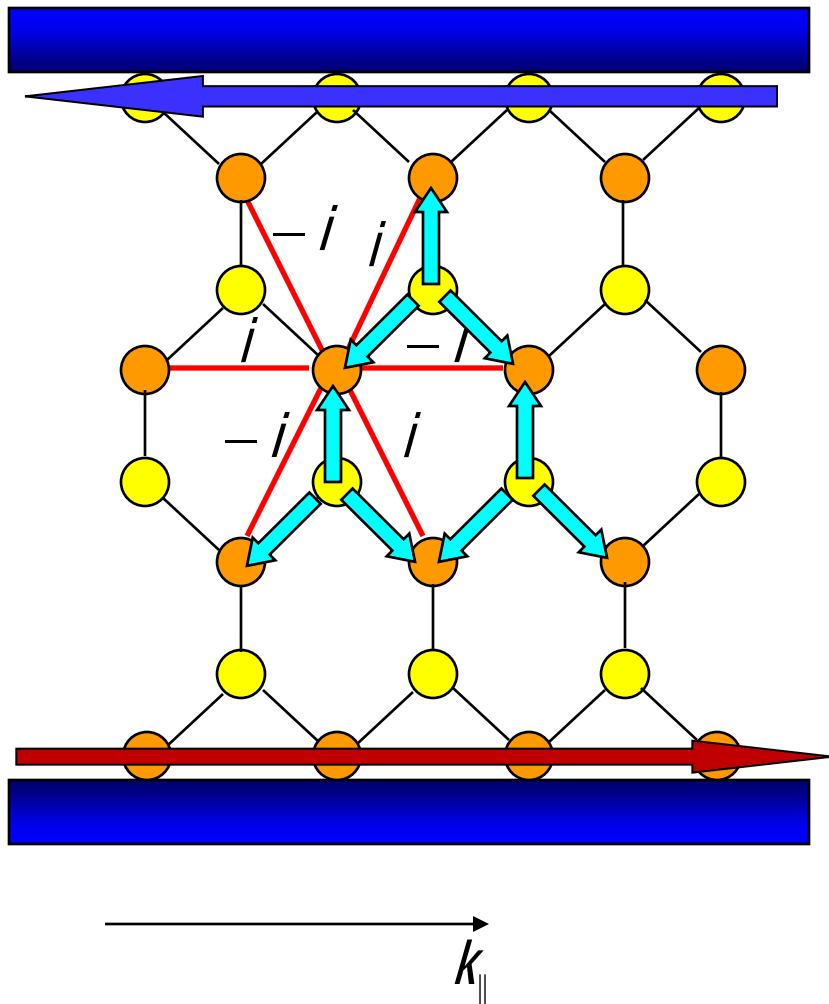


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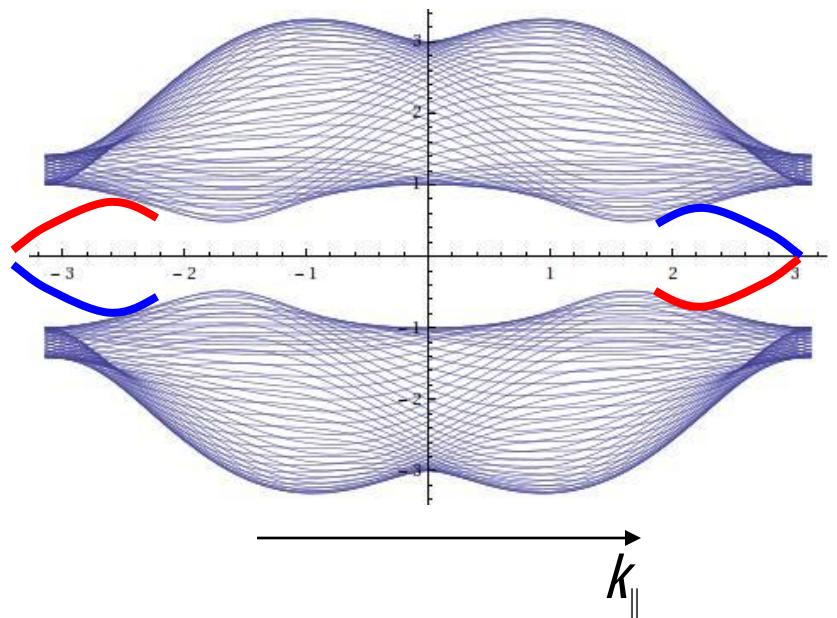
Kane-Mele model:

Honeycomb lattice with 2nd nearest neighbor hopping



$$H = - \sum_{\langle ij \rangle} t c_{i,s}^+ c_{j,s}$$

$$H = -i \sum_{\langle\langle ij \rangle\rangle} \lambda (\vec{d}_1 \times \vec{d}_2) c_{i,s}^+ c_{j,s} \sigma^z$$



The simplest model – Kane-Mele honeycomb lattice

Nearest neighbor:

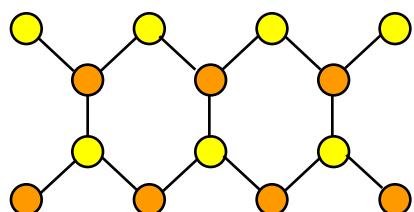
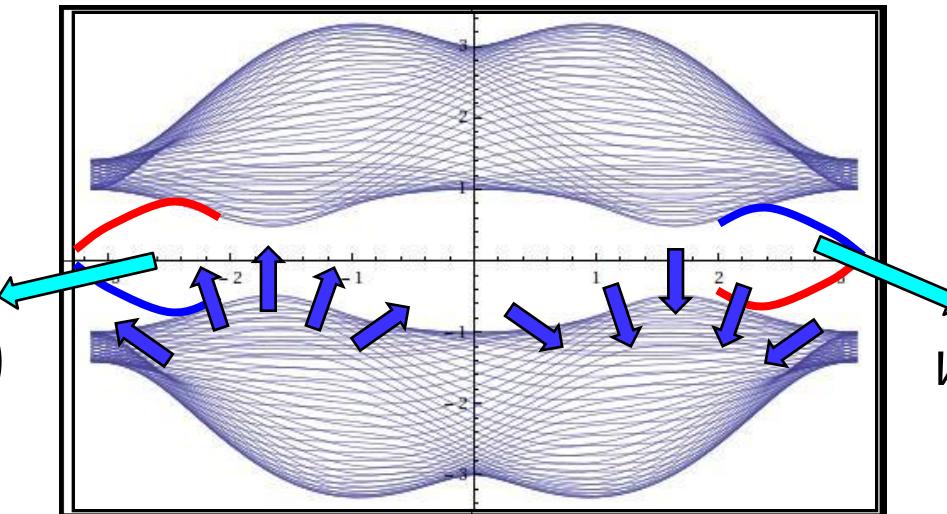
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2nd nearest neighbor:

$$H = -i \sum_{\langle\langle ij \rangle\rangle} \lambda (\vec{d}_1 \times \vec{d}_2) c_{i,s}^+ c_{j,s} \sigma^z$$

$$\nu_F (\delta k_x \hat{\tau}_x + \delta k_y \hat{\tau}_y) \\ + \hat{\tau}^z \lambda \cdot \hat{\sigma}^z$$

$$\nu_F (-\delta k_x \hat{\tau}_x + \delta k_y \hat{\tau}_y) \\ - \hat{\tau}^z \lambda \cdot \hat{\sigma}^z$$

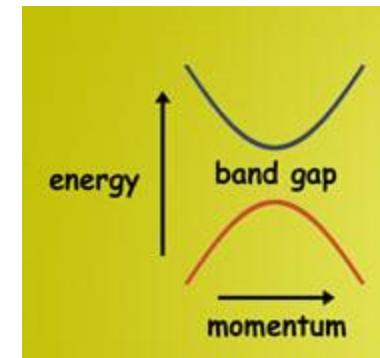
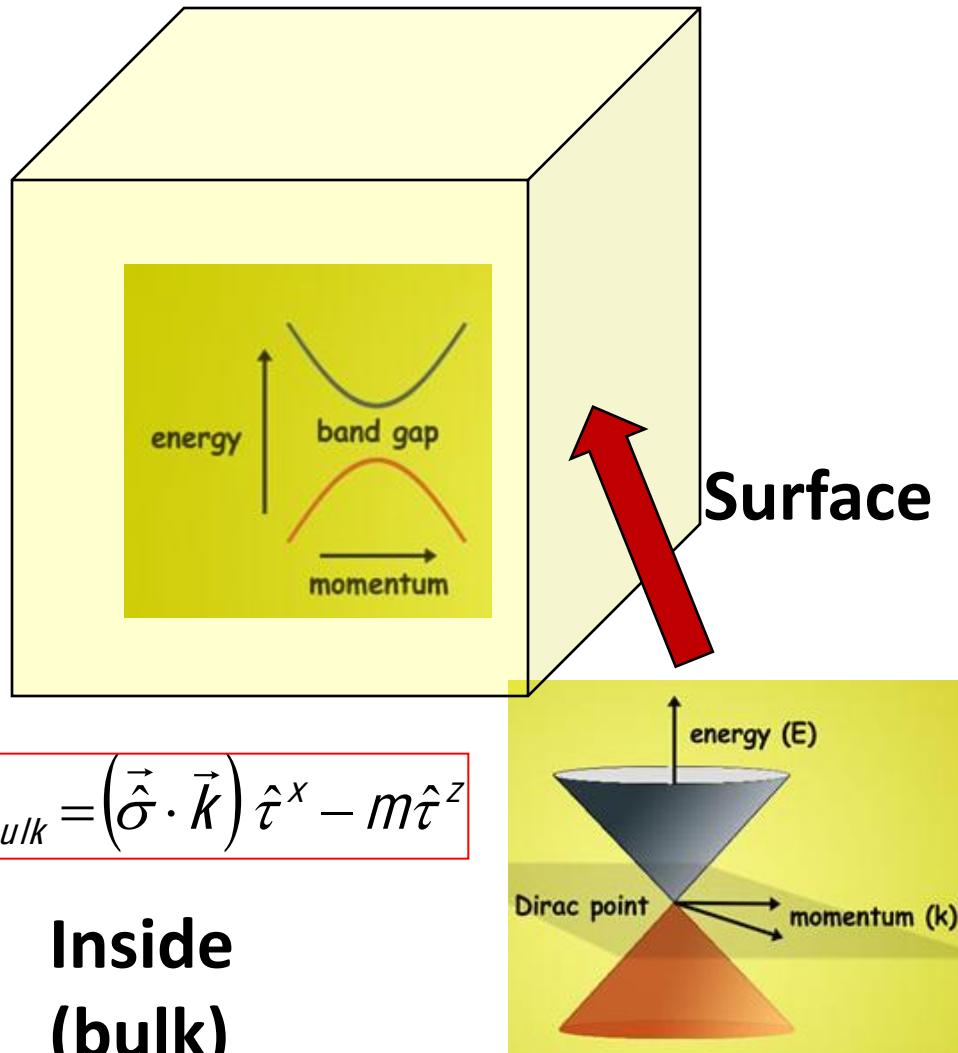


$$H = \varepsilon_0 + \vec{\tau} \cdot \vec{d} \\ \hat{n} = \vec{d} / |\vec{d}|$$

$$k_{\parallel}$$

3D Topological Insulators

Dirac cones on the edge planes:



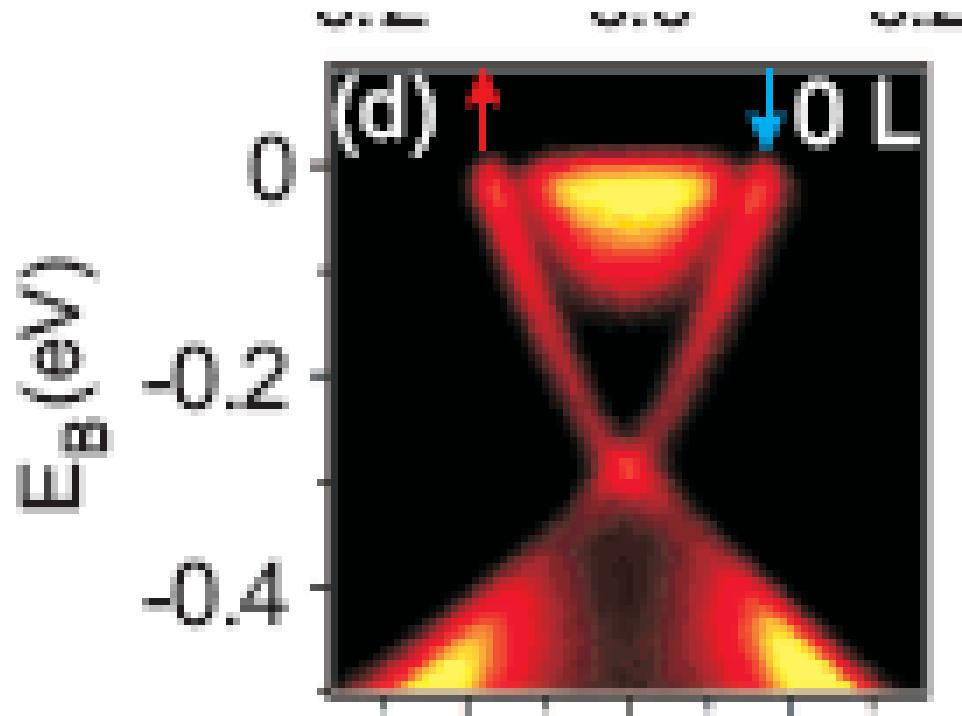
$$H_{bulk} = (\vec{\sigma} \cdot \vec{k}) \hat{\tau}^x + m \hat{\tau}^z$$

Outside

$$m=0$$

Topological (non)-insulators

Experimental overview: Topological (non)-insulators

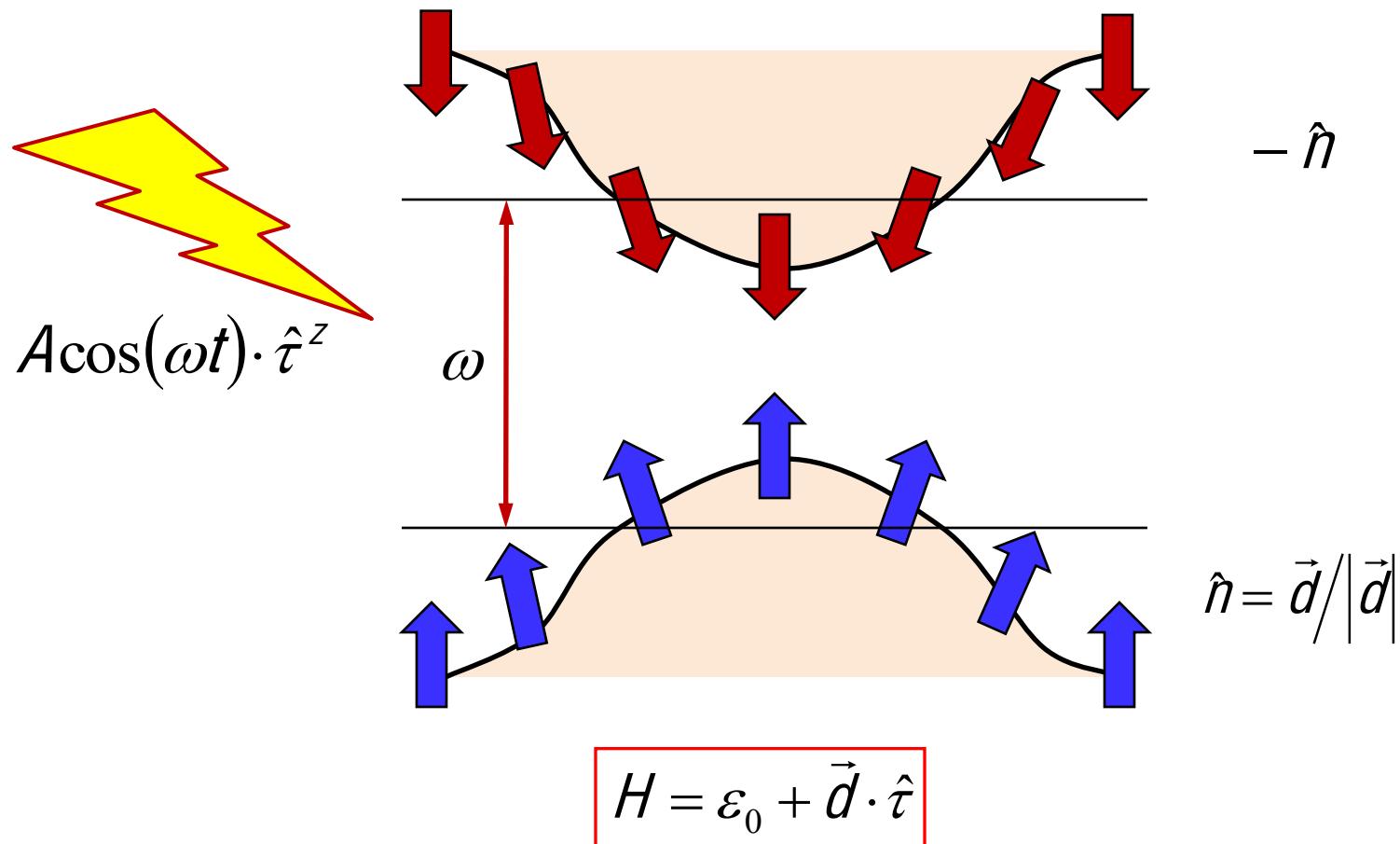


Bi_2Se_3

Hsieh...Hasan (2010)

Can we induce a topological phase with radiation effects?

- Start with non-topological HgTe wells:

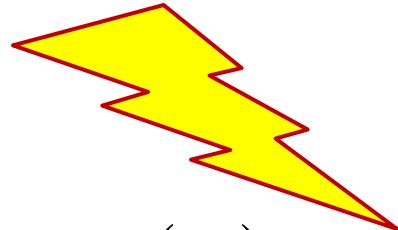


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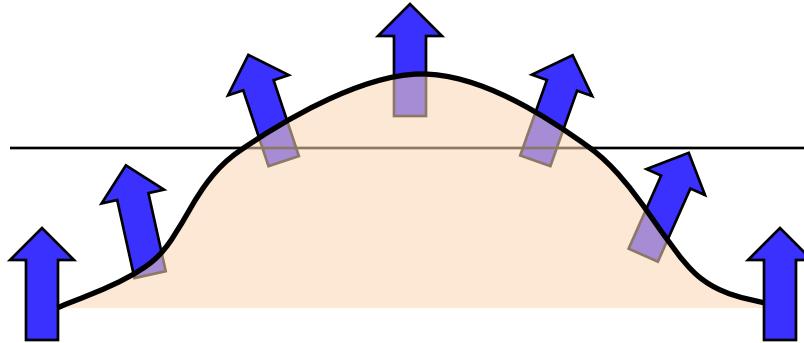
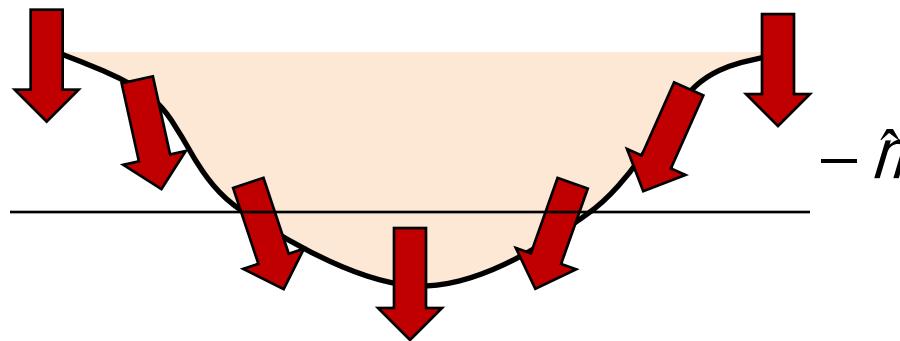
$$m, b > 0$$

Irradiation effects

- Start with non-topological HgTe wells:



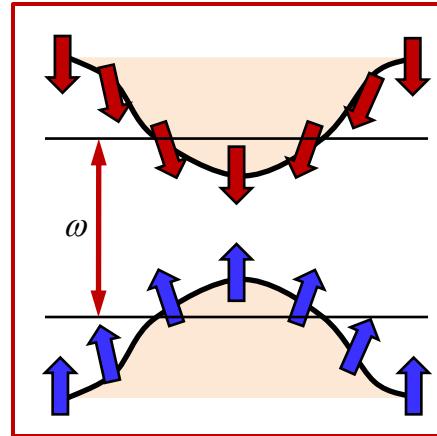
$$A \cos(\omega t) \cdot \hat{\tau}^z$$



$$\hat{n} = \vec{d} / |\vec{d}|$$

$$H = \varepsilon_0 + \vec{d} \cdot \hat{\tau}$$

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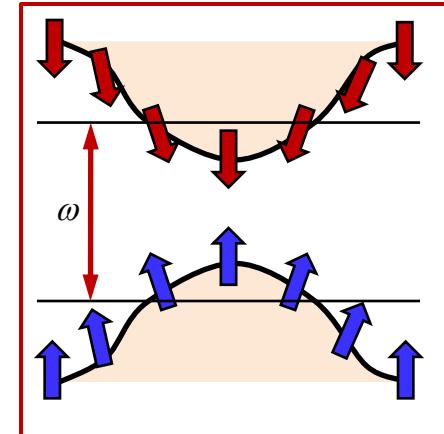
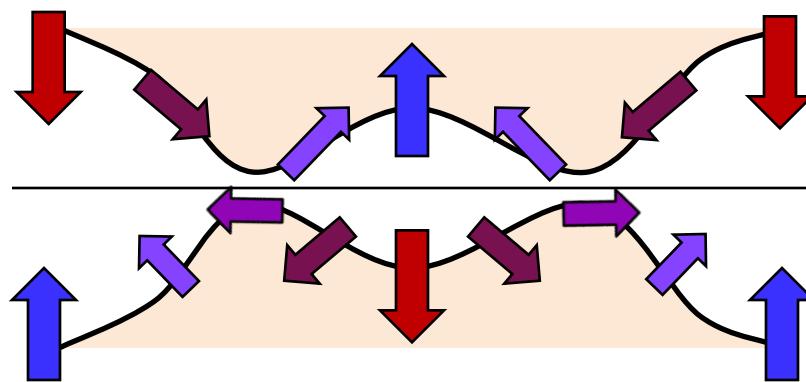


$$m, b > 0$$

Irradiation effects

- Start with non-topological HgTe wells:

$$A \cos(\omega t) \cdot \hat{\tau}^z$$



More honest analysis: Floquet Hamiltonians

- Periodically varying Hamiltonian:

$$H = H_0 + V(t)$$

with

$$V(t) = V(t+T)$$

and:

$$\int_0^T V(t) dt = 0.$$

- Hamiltonian  Floquet operator:

$$U(T) = \exp\left(-i \int_0^T dt H(t)\right)$$

- Energy eigenstates  Quasi-energy states:

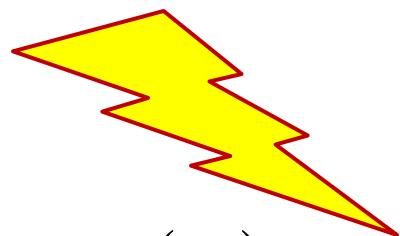
$$U(T) \psi_n = e^{-i\varepsilon_n T} \psi_n$$

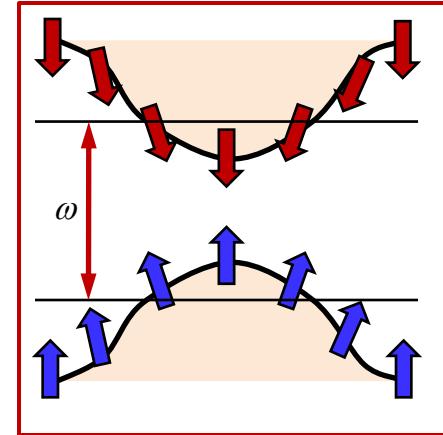
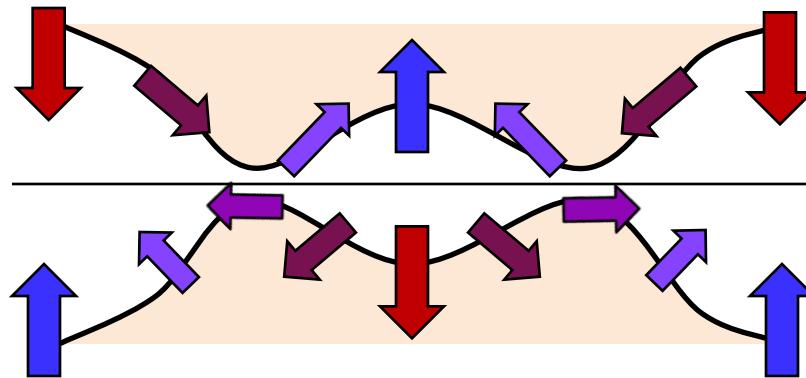
$$\varepsilon_n \in [0, \omega] + \varepsilon_0$$

$$\omega = \frac{2\pi}{T}$$

Irradiation effects

- Start with non-topological HgTe wells:


$$A \cos(\omega t) \cdot \hat{\tau}^z$$

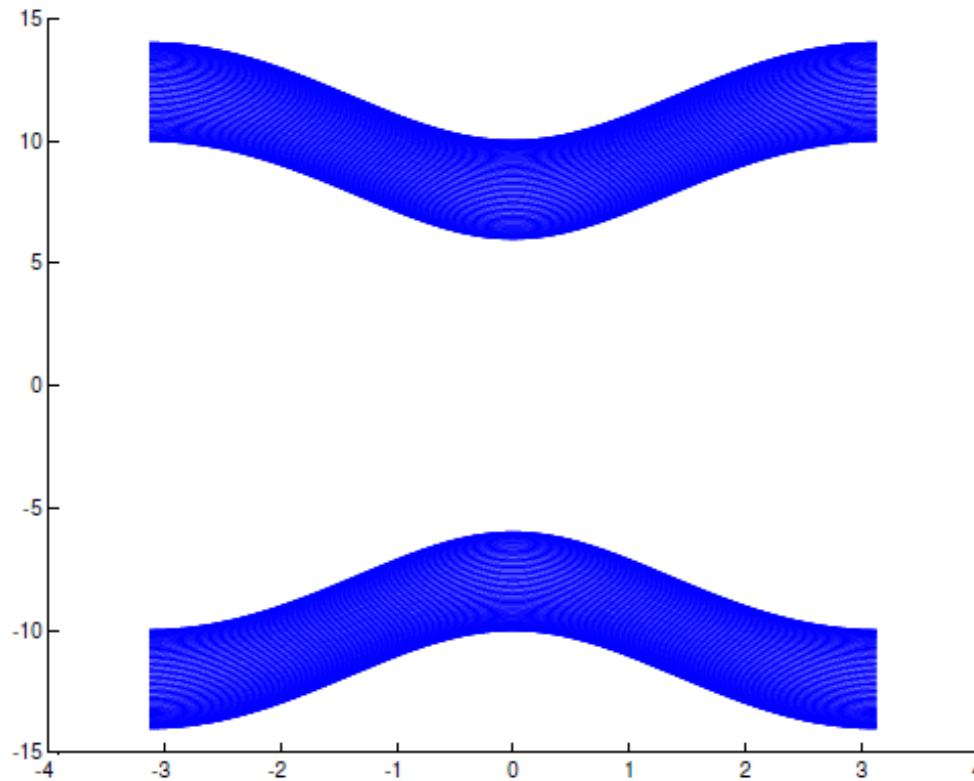


$$S = \exp\left(-i \int_0^T dt H\right) = S_1 + S_2 \hat{n}_S(\vec{p}) \cdot \hat{\tau}$$

Floquet topological phase

$$H = \varepsilon_0 + \vec{d} \cdot \hat{\tau}$$

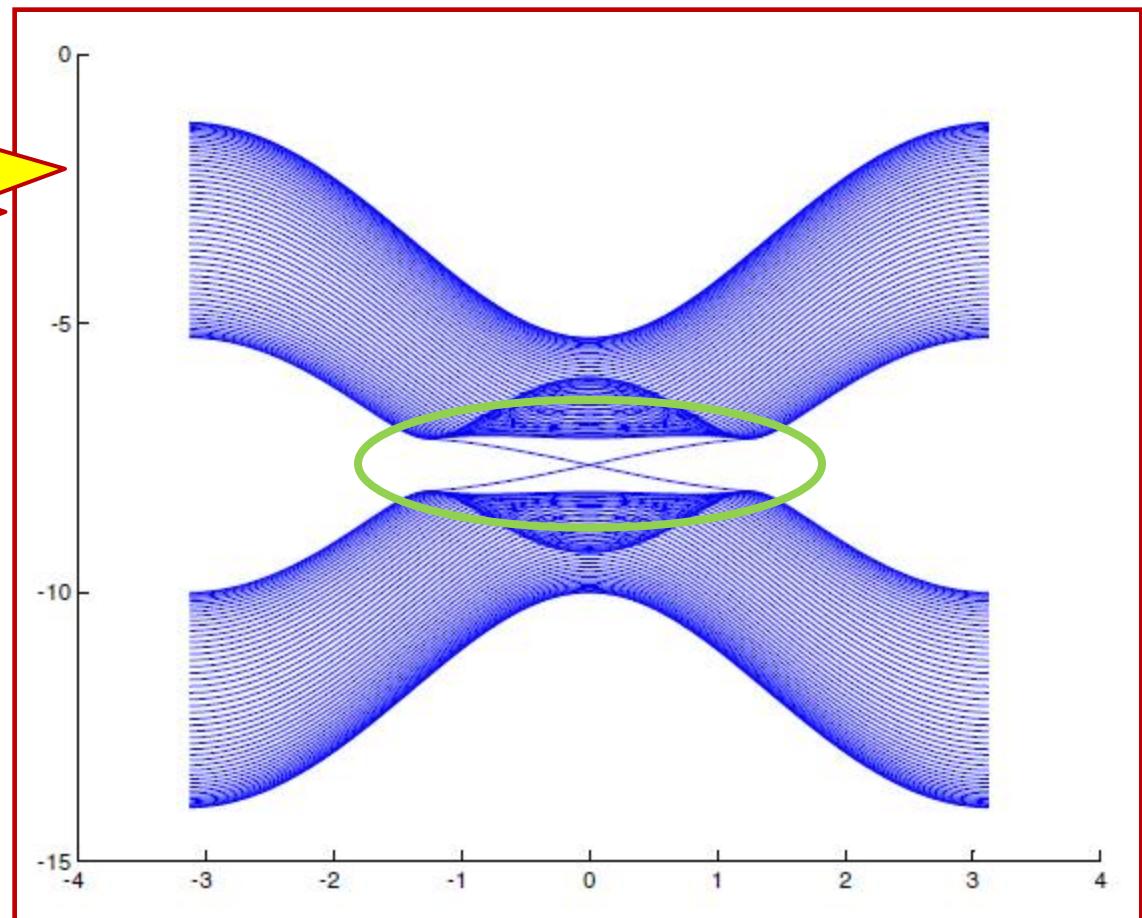
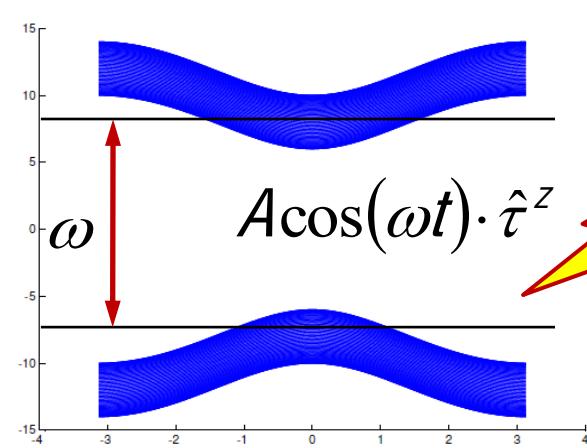
$$\vec{d} = p_x \cdot \hat{x} + p_y \cdot \hat{y} + \left(m + b(p_x^2 + p_y^2) \right) \cdot \hat{z}$$



Floquet topological phase

$$H = \varepsilon_0 + \vec{d} \cdot \hat{\tau}$$

$$\vec{d} = p_x \cdot \hat{x} + p_y \cdot \hat{y} + \left(m + b(p_x^2 + p_y^2) \right) \cdot \hat{z}$$

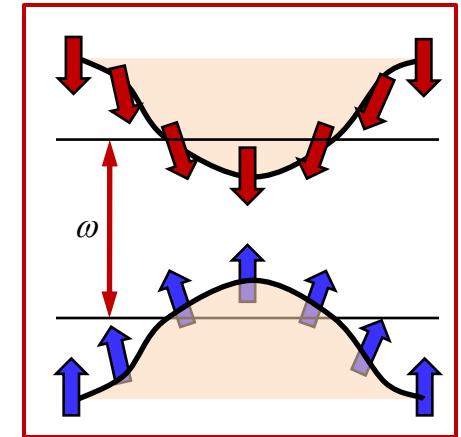


Geometrical considerations

$$H = \varepsilon_0 + \varepsilon_k (\vec{\tau} \cdot \hat{n}_k) + \hat{V} \cos \omega t$$



$$H = \sum_k \varepsilon_k \hat{P}_{k+} - \varepsilon_k \hat{P}_{k-} + \vec{V} \cdot \vec{\tau} \cos \omega t$$



- Rotating wave approximation:

$$H_k = (\varepsilon_k - \omega) \hat{P}_{k+} - \varepsilon_k \hat{P}_{k-}$$

$$+ \hat{P}_{k+} \vec{V} \cdot \vec{\tau} \hat{P}_{k-} + \hat{P}_{k-} \vec{V} \cdot \vec{\tau} \hat{P}_{k+}$$



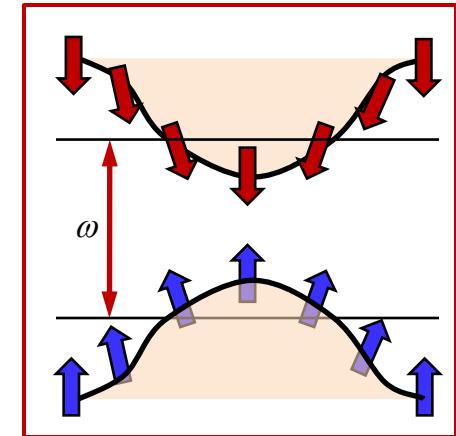
$$\vec{V} \cdot \vec{\tau} - (\hat{n}_k \cdot \vec{\tau}) \vec{V} \cdot \vec{\tau} (\hat{n}_k \cdot \vec{\tau}) \rightarrow (\vec{V} - (\hat{n}_k \cdot \vec{V}) \hat{n}_k) \cdot \vec{\tau} = \vec{V}_\perp \cdot \vec{\tau}$$

Geometrical considerations

$$H = \varepsilon_0 + \varepsilon_k (\vec{\tau} \cdot \hat{n}_k) + \hat{V} \cos \omega t$$



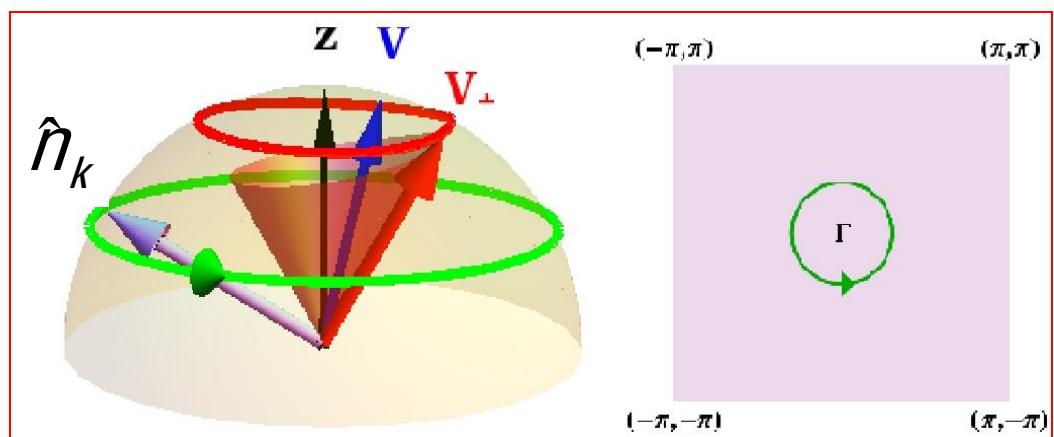
$$H = \sum_k \varepsilon_k \hat{P}_{k+} - \varepsilon_k \hat{P}_{k-} + \vec{V} \cdot \vec{\tau} \cos \omega t$$



- Rotating wave approximation:

$$H_k = (\varepsilon_k - \omega) \hat{P}_{k+} - \varepsilon_k \hat{P}_{k-}$$

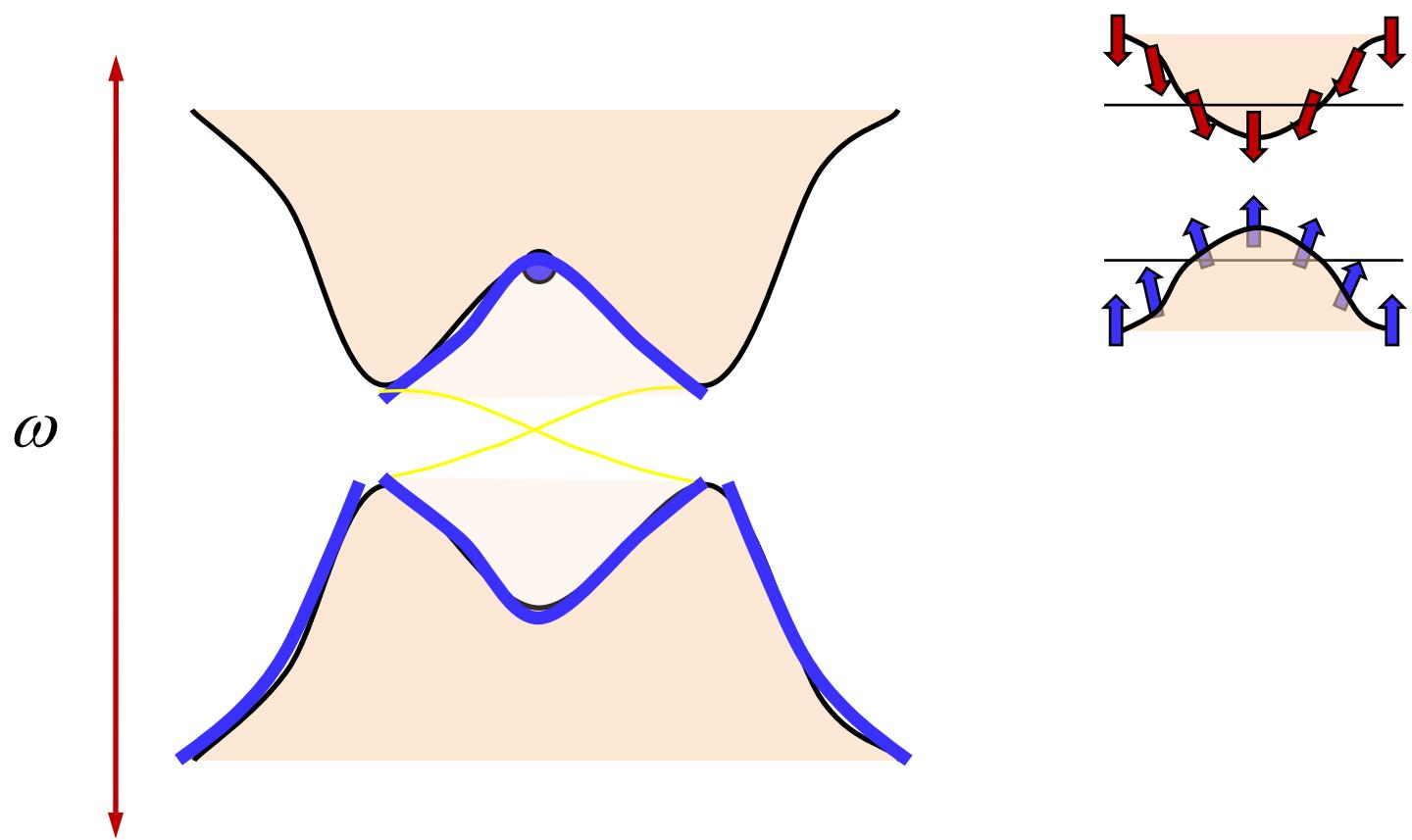
$$+ \vec{V}_\perp \cdot \vec{\tau}$$



Equilibration in a Floquet topological phase?

(Work in progress)

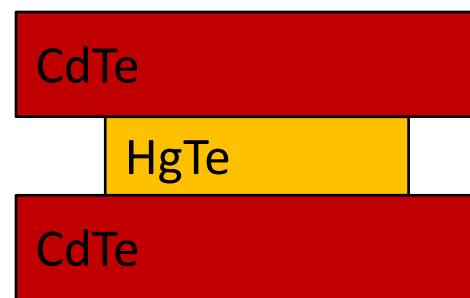
- Phonons can provide bath for low-energy, low momentum relaxation.



Experimental realization In (Hg,Cd)Te wells?

$\alpha B \cos(\omega t) \cdot \hat{\tau}^z$

(Tera-Hertz frequencies)



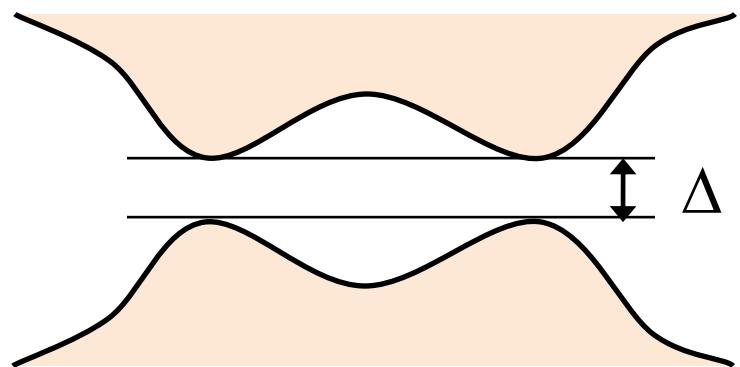
(Initially Non-
Topological)

- Using Zeeman coupling:

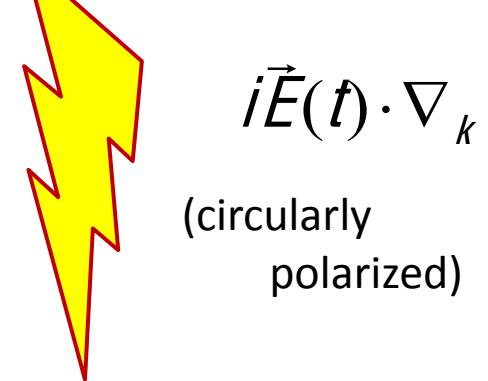
$$B \sim 10 \text{ mT}$$



$$\Delta \sim 0.1 \text{ K}$$



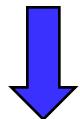
Stark effect magic



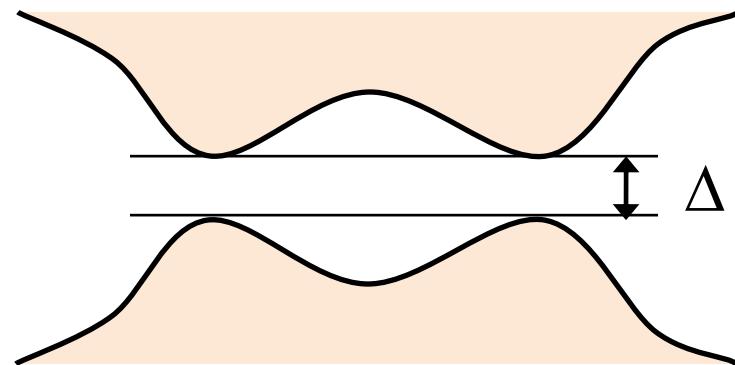
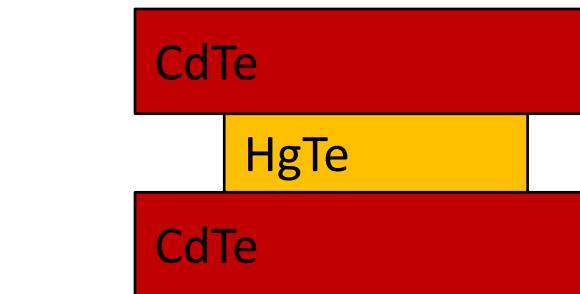
- Electric field:
Linear Stark effect enhancement

$$\Delta \sim \frac{\nu_F \Omega^2}{m^3} |E|$$

$$E \sim 1 \text{ V/m}$$



$$\Delta \sim 10 \text{ K}$$



Summary and conclusions

- Periodically modulated systems could be used to stabilize unique quantum states.
- Topological insulator on demand: *modulate the subband gap.*
- Open questions:
Relaxation into a steady state – will the unique properties shine out?