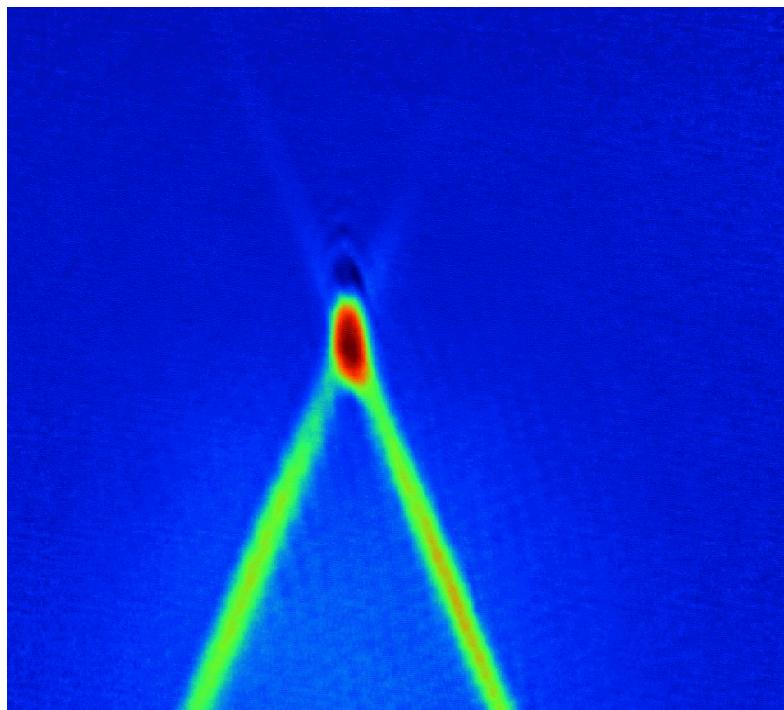


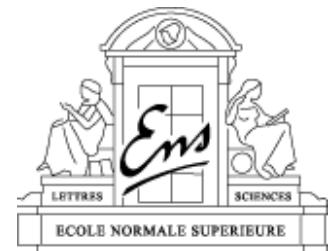
# Many-Body Physics with Quantum Gases



Christophe Salomon  
Buenos aires, May 10-14, 2011



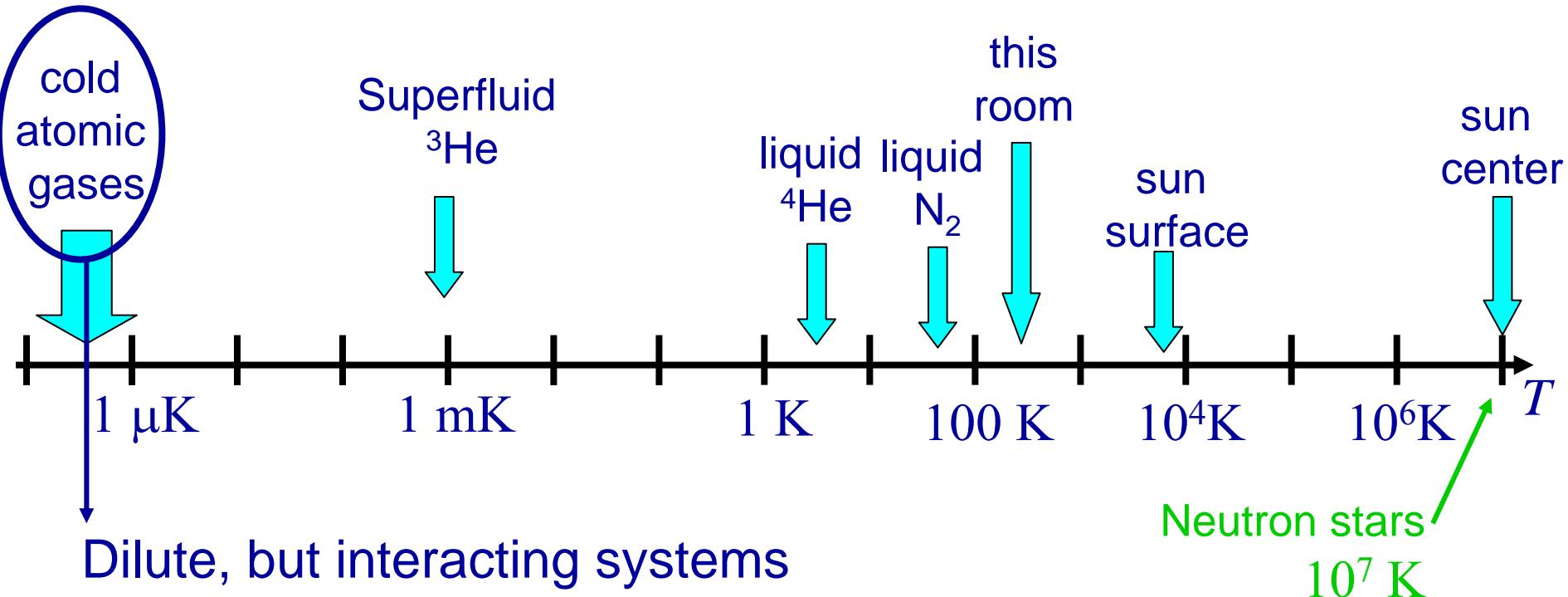
Ecole Normale Supérieure, Paris



# Summary of lectures

- Introduction to quantum gases
  - Bose-Einstein condensation
  - Experimental techniques
  - Tuning interaction between atoms
- 
- Ultracold fermions: the crossover between Bose and Fermi superfluid
  - Thermodynamics of quantum gases
  - 2D gases
  - Perspectives

# Temperature scale of cold gases



Typical density:  $\rho = 10^{13}$  to  $10^{15}$  atoms/cm<sup>3</sup>

Interatomic distance  $0.1$  to  $0.5 \mu\text{m}$   $\gg$  range of interatomic potentials

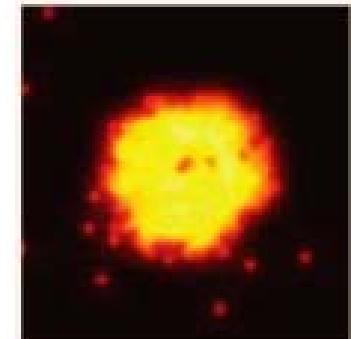
$E_{\text{int}} \gg \hbar\omega$  quantum of the motion in the trap

$E_{\text{int}} \gg k_B T$  thermal energy

Equilibrium properties and dynamics are governed by interactions

# Many-Body Physics with Cold Gases

Diluteness: atom-atom interactions described by 2-body (and 3 body) physics. At low energy: a single parameter, the scattering length  $a$



Control of the sign and magnitude of interaction

Control of trapping parameters:

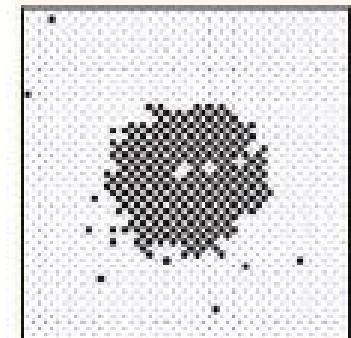
access to time dependent phenomena,  
out of equilibrium situations, 1D, 2D, 3D

Simplicity of detection

Comparison with quantum Many-Body theories:

Gross-Pitaevskii, Bose and Fermi Hubbard models,  
search for exotic phases, dipolar gases  
disorder effects, Anderson localization, ...

Sherson et al., MPQ 2010



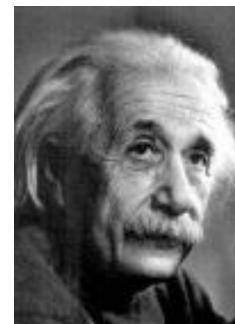
Link with condensed matter (high T<sub>c</sub> superconductors,  
magnetism in lattices), astrophysics (neutron stars)  
Nuclear physics, high energy physics (quark-gluon plasma),



Quantum simulation with cold atoms « a la Feynman »

# Bose-Einstein Condensation

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1924

**Prix Nobel de physique 1997**

**S. Chu, C. Cohen Tannoudji, W. Phillips**  
**Manipulation d'atomes par laser**



**Prix Nobel de physique 2001**

**E. Cornell, W. Ketterle, C. Wieman**  
**Condensation de Bose-Einstein**



# Fermions and Bosons

## Fermions

Particles with half- integer spin

Examples : electron, proton, neutron

2 identical fermions cannot occupy the same quantum state  
(Pauli exclusion principle)

## Bosons

Particles with integer spin

Examples : photon, atoms, molecules,...

Bose statistics: tendency to occupy the same quantum state

## Composite systems: atoms

An atom is a boson if it contains an even number of fermions  
(Ex : H, He<sup>4</sup>, Li<sup>7</sup>, Na<sup>23</sup>, Rb<sup>87</sup>), or a fermion if it contains  
an odd number of fermions (Ex : D, He<sup>3</sup>, Li<sup>6</sup>, Sr<sup>88</sup>)

# Bose Condensation of an ideal gas

N identical particles without interaction in a box or a trap

Hamiltonian  $\hat{H} = \hat{h}_1 + \hat{h}_2 + \hat{h}_3 + \dots \hat{h}_N$

Basis of eigenvectors of one body- hamiltonian  $\{|\lambda\rangle\}$

$$\hat{h} |\lambda\rangle = \varepsilon_\lambda |\lambda\rangle$$

$$\hat{H} = \sum_{\lambda} \varepsilon_{\lambda} a_{\lambda}^{\dagger} a_{\lambda} \quad \text{et} \quad \hat{N} = \sum_{\lambda} a_{\lambda}^{\dagger} a_{\lambda}$$

Basis of eigen states in Fock space:  $|N_{\lambda}, N_{\lambda'}, N_{\lambda''}, \dots\rangle$   
where  $N_{\lambda}$  are occupation numbers of an individual quantum state  
0 or 1 for fermions,  
positive integer for bosons

Total number of particles  $N = \sum_{\lambda} N_{\lambda}$

# Quantum statistics

**Fermions:**

$$N_\lambda = \frac{1}{e^{\beta(\varepsilon_\lambda - \mu)} + 1}$$

$\mu$ : chemical potential:

Energy to add a particle to the system

$$\beta = 1/k_B T$$

$\mu \rightarrow -\infty$  Boltmann gas

$\mu$  positive and large compared to  $kT$ : degenerate Fermi gas

**Bosons:**

$$N_\lambda = \frac{1}{e^{\beta(\varepsilon_\lambda - \mu)} - 1}$$

$\mu$  can take all values from  $-\infty$  to  $\varepsilon_{\min}$

When  $\mu$  tends toward  $\varepsilon_{\min}$ ,  $N_0$  tends to infinity:  $N_0 \simeq \frac{k_B T}{\varepsilon_{\min} - \mu}$   
Saturation of excited states

# Bose-Einstein condensation of an ideal Bose gas

N identical bosons in a trap, at thermal equilibrium at temperature T

$$\lambda_{DB} = \frac{\hbar}{\sqrt{2\pi m k_B T}}$$

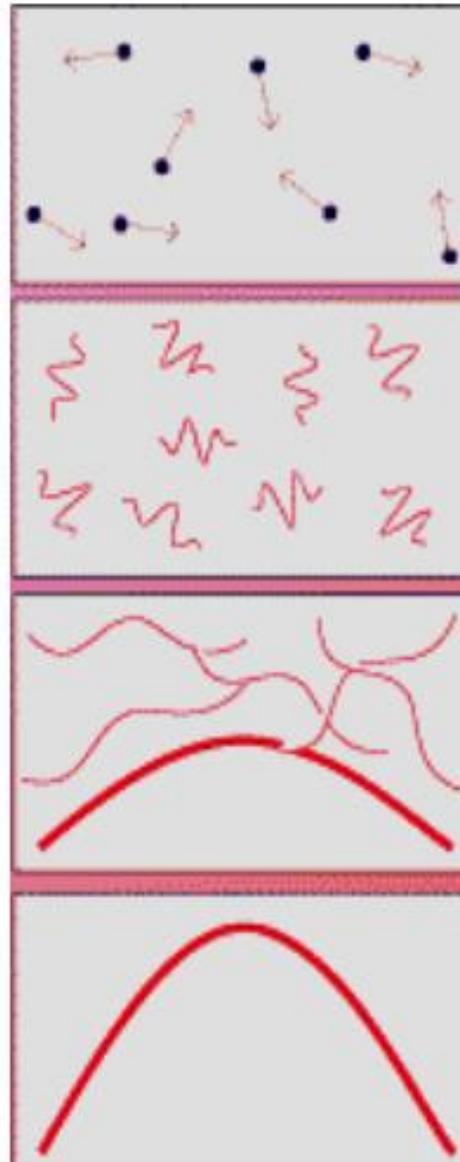
When the temperature T is lowered, the de Broglie wavelength increases. When  $T < T_c$ , a macroscopic number of bosons  $N_0$  condenses in the trap ground state.

The critical temperature  $T_c$  corresponds to a situation where the de Broglie wavelength becomes on the order of the average distance between particles.

The waves associated to different atoms overlap and interfere.

$$\lambda_{DB} = \frac{\hbar}{\sqrt{2\pi m k_B T}}$$

Pictorial image  
when temperature  
is lowered



$T \gg T_C$

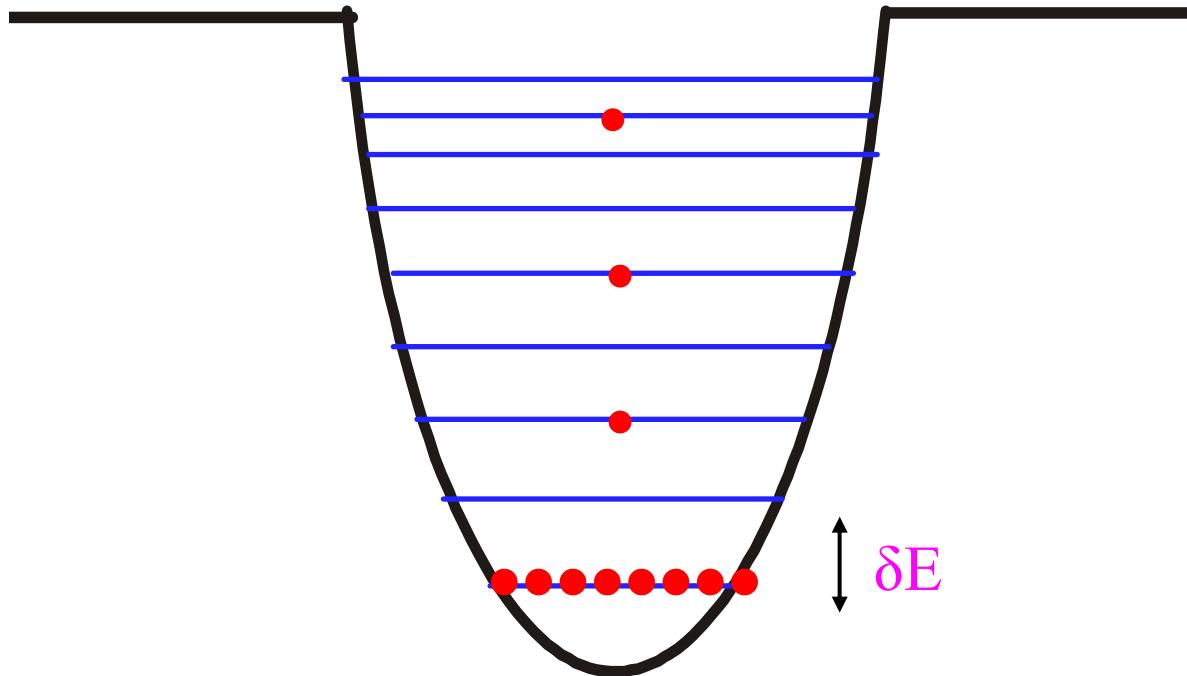
$T > T_C$

$T \simeq T_C$

$T \ll T_C$

# Boson accumulation in the trap ground state

$$T \ll T_c$$



BEC is not a trivial thermal effect that occurs when thermal excitation energy  $k_B T_c$  is smaller than the trap energy level spacing  $\delta E$  between levels

# Bose-Einstein condensation: order of magnitude



Dilute gas of atoms at temperature  $T$  confined in harmonic potential :

$$V(\vec{r}) = \frac{1}{2}m\omega^2 r^2$$

Condensation threshold:

$$N = 1.202 \left( \frac{k_B T}{\hbar \omega} \right)^3 \quad \xleftrightarrow{k_B T \gg \hbar \omega}$$

$$n_0 \lambda^3 = 2.612$$

$$\begin{cases} n_0 : \text{central density} \\ \lambda = \frac{\hbar}{\sqrt{2\pi m k_B T}} \end{cases}$$

Liquid helium :

$10^{27}$  atoms/m<sup>3</sup>

$$n_0^{-1/3} = 10 \text{ \AA} \quad T \sim 1 \text{ K}$$

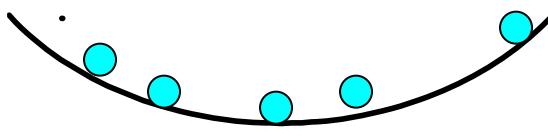
Gaseous condensate

$10^{19}$  at/m<sup>3</sup>

$$n_0^{-1/3} = 0.5 \text{ \mu m} \quad T \sim 1 \text{ \mu K}$$

# Magnetic trap

$F=1, m=1$



$F=1, m=0$

$F=1, m=-1$

$z$

$$E = -\vec{\mu} \cdot \vec{B} = +|\vec{\mu}| |\vec{B}|$$

Local minimum of  $|B|$   
+ spin polarisation

$$V = |\mu| / |B|$$

Maxwell's equations:  
No max of  $|B|$  in vacuum.

Atoms cannot be magnetically trapped in the lower energy state.

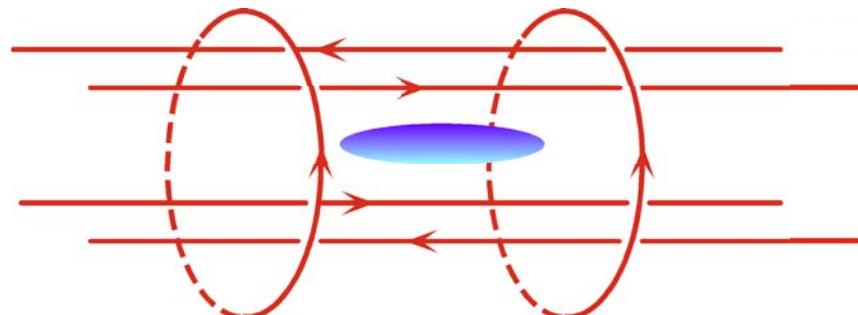
## ▲ Two-body inelastic collisions

Example: Ioffe-Pritchard trap

Trap depth 1 mK

Loaded with laser cooled atoms

Or cryo-cooled atoms (Harvard)



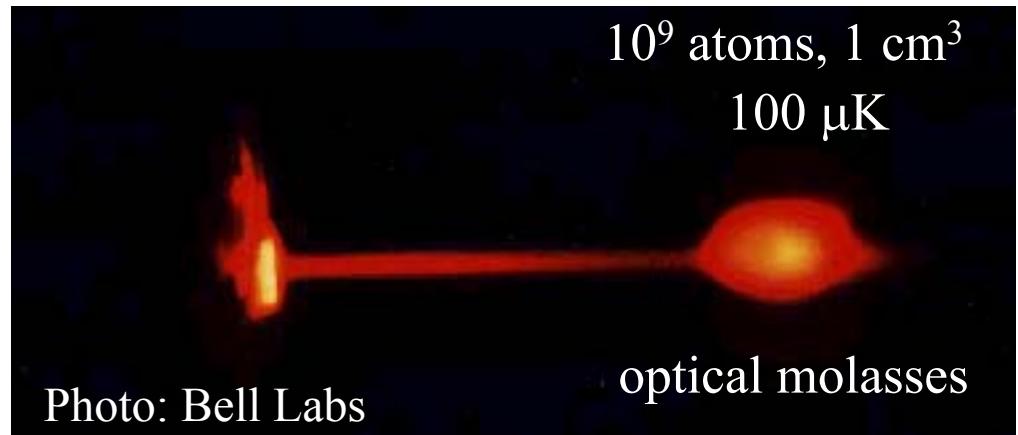
# Preparation of a quantum gas

- Create an atomic beam of atoms or a vapor
- Laser cooling to  $\sim 100 \text{ } \mu\text{K}$        $n\lambda^3 \approx 10^{-6}$
- Magnetic trapping or optical trapping
- Evaporative cooling to  $\sim 1 \text{ } \mu\text{K}$      $n\lambda^3 = 2.612$

# Loading a magnetic or optical trap

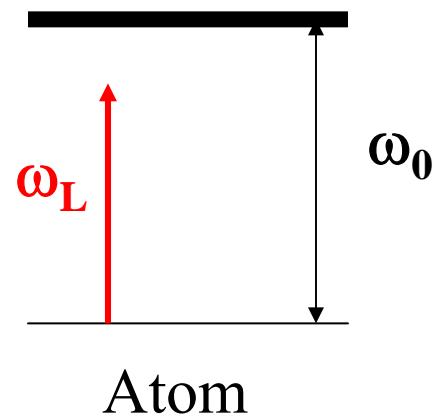
## Laser slowing and laser cooling

$$n\lambda^3 \approx 10^{-6}$$



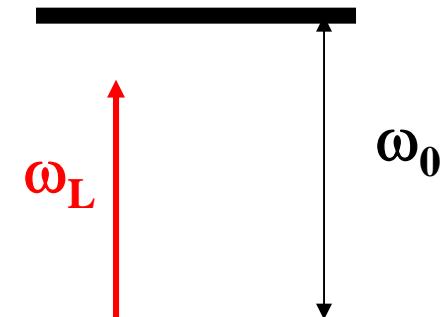
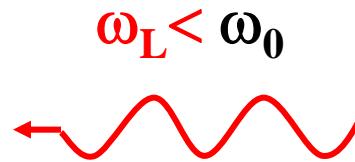
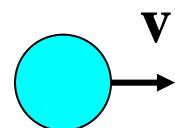
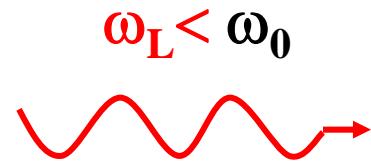
## Radiation pressure of the laser

$$F(z, v) = \frac{\hbar k \Gamma}{2} \frac{s_0}{1 + s_0 + 4[\delta + kv - \mu' B(z)/\hbar]^2/\Gamma^2}.$$

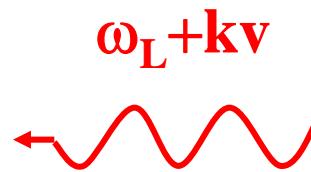
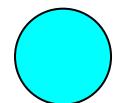
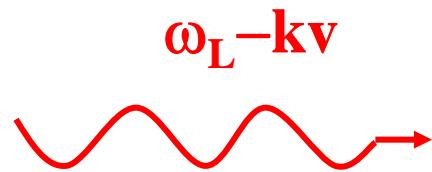


# Doppler Cooling

## Doppler effect



Laboratory frame

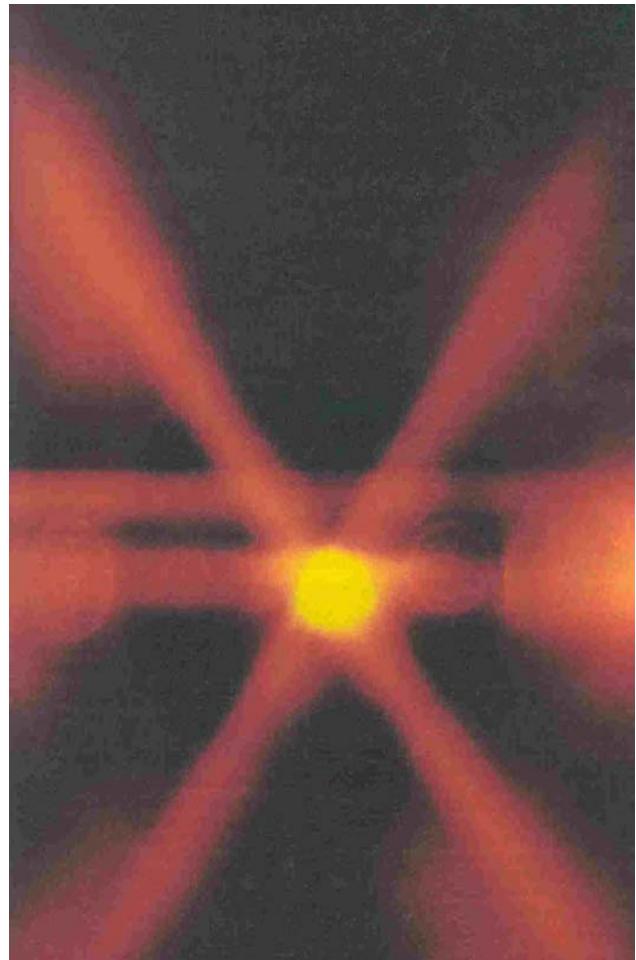


Atom frame

Absorption of the photon  $\omega_L + kv$ , followed by a spontaneous emission equiprobable in all directions of the space.

Act as:  $F = -\alpha v$  (friction force) =  $mdv/dt$

# Optical Molasses

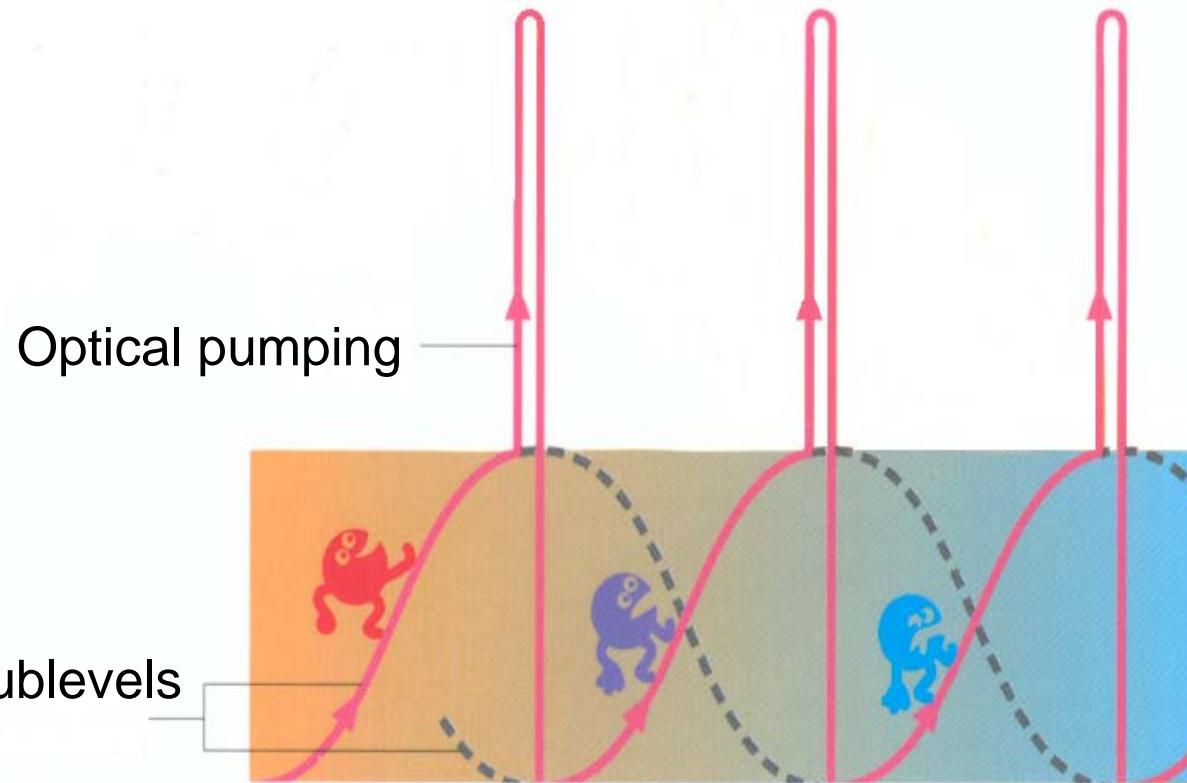


S. Chu, Scientific American, 174, 1992

Na molasses

# Sisyphus cooling

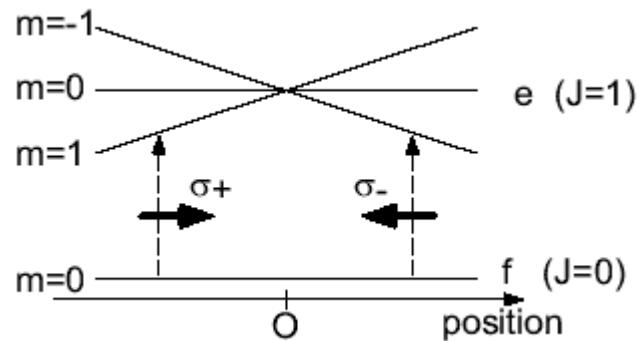
J. Dalibard, C. Cohen-Tannoudji



$$k_B T = U_0 / 4$$

**Limit Temperature: about 10 times recoil energy**

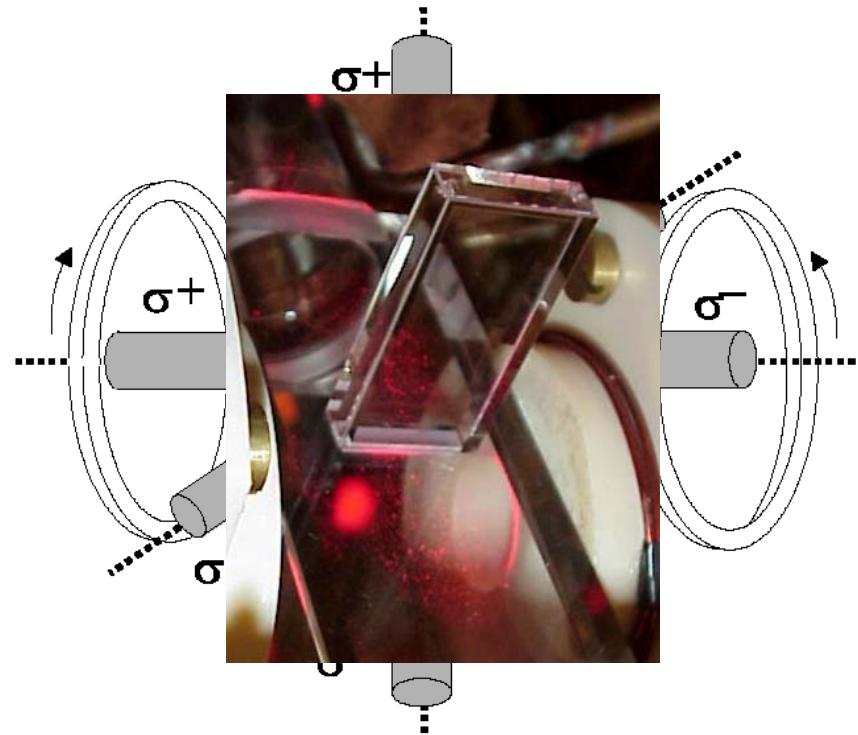
# Magneto-optical trap



$$\mathbf{F} = -\alpha \mathbf{v} - k \mathbf{r}$$

3D Molasses  
Doppler effect

Trapping  
Zeeman effect

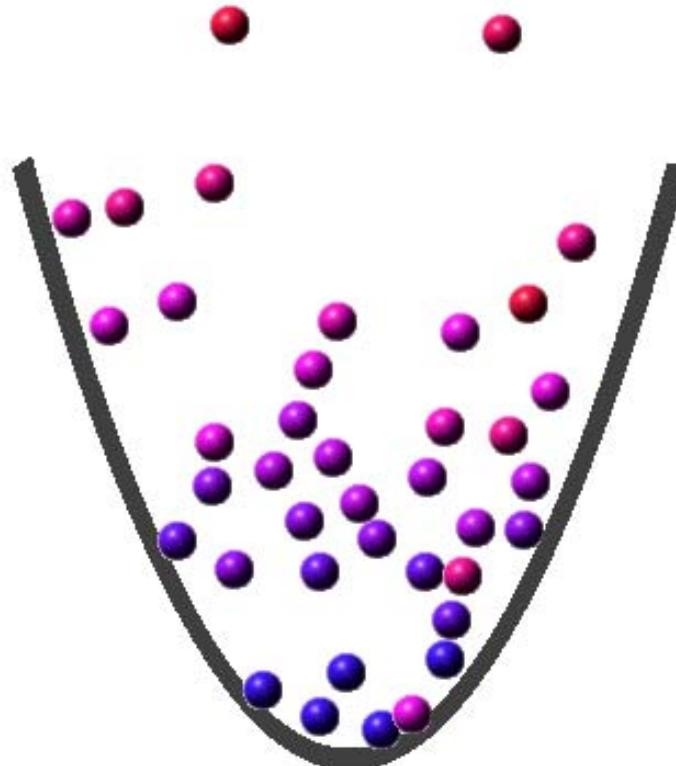


$b' = 10 \text{ Gauss / cm}$   
 $I = \text{a few mW per beam}$

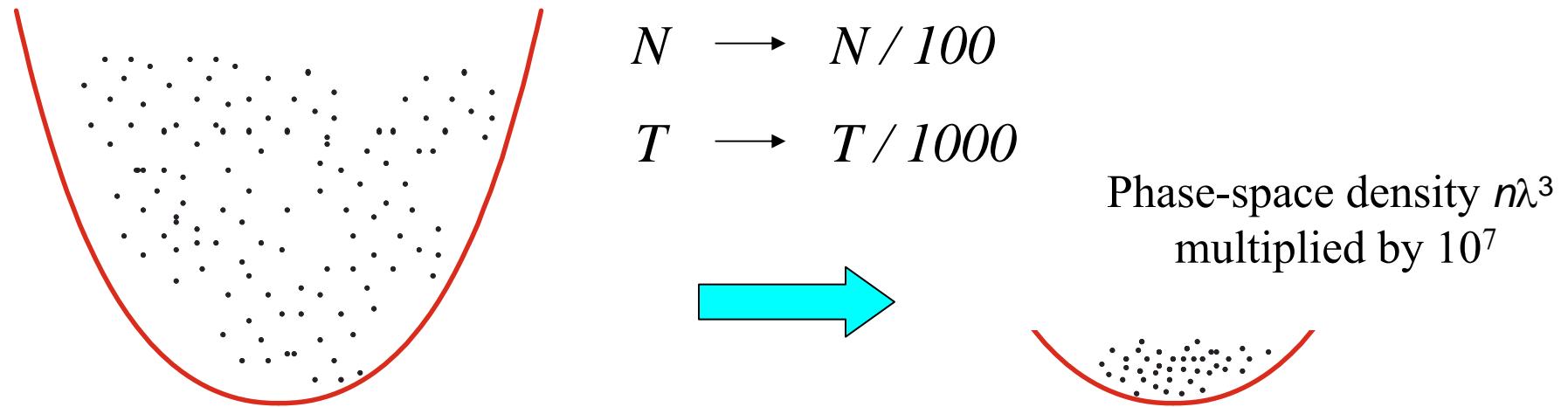
# Evaporative cooling

- The only known method to reach quantum degeneracy
  - Remove hot atoms
  - Elastic collisions ensure re-thermalisation

$$\gamma_{elastic} / \gamma_{inelastic} \geq 150$$

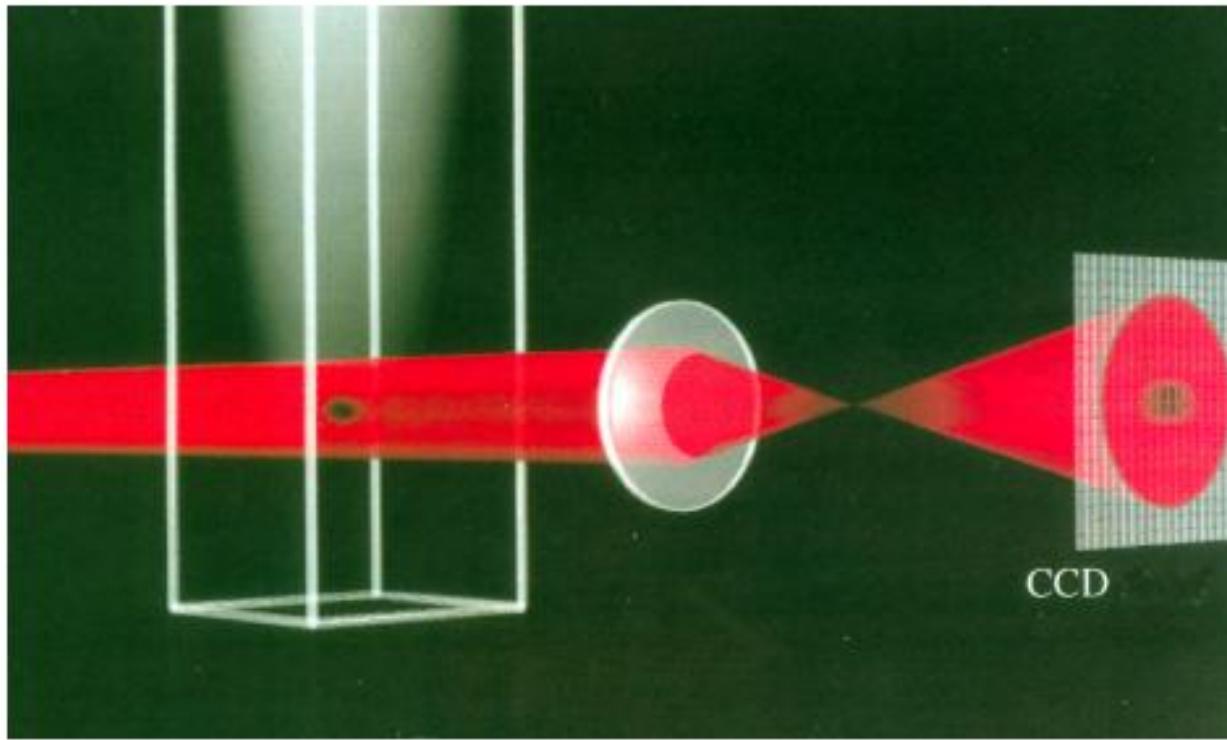


## Evaporative cooling (2)



Duration : 2 to 30 seconds,  $N_f=10^5$  to  $10^7$  atoms,  $T_f=0.2$  to  $2 \mu\text{K}$

# Imaging cold atomic clouds and condensates

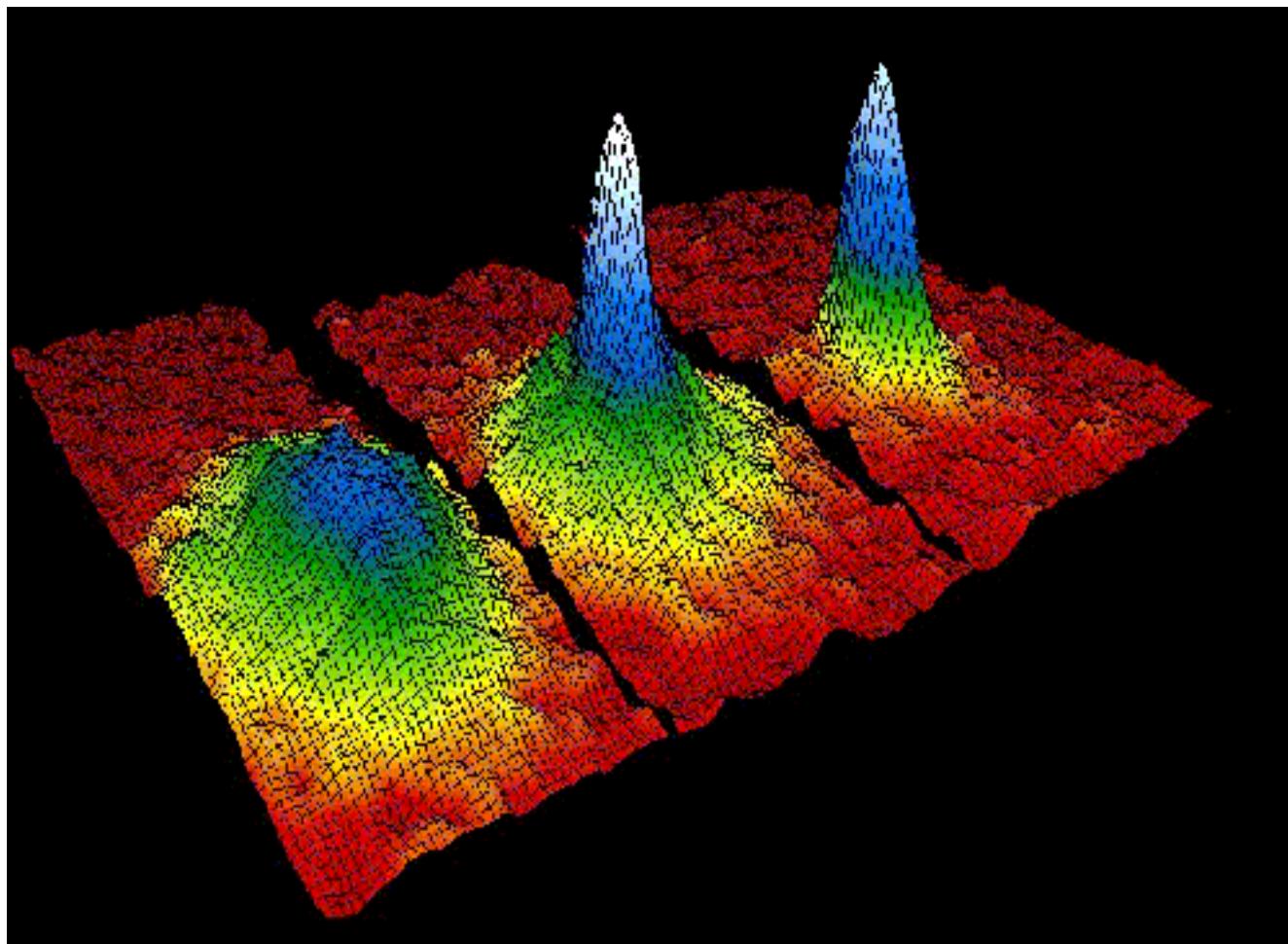


## Absorption imaging

- 1) In situ measurement: spatial distribution in the trap
- 2) After time of flight expansion: velocity distribution

# Bose-Einstein Condensation in Rubidium 87

## JILA - Boulder



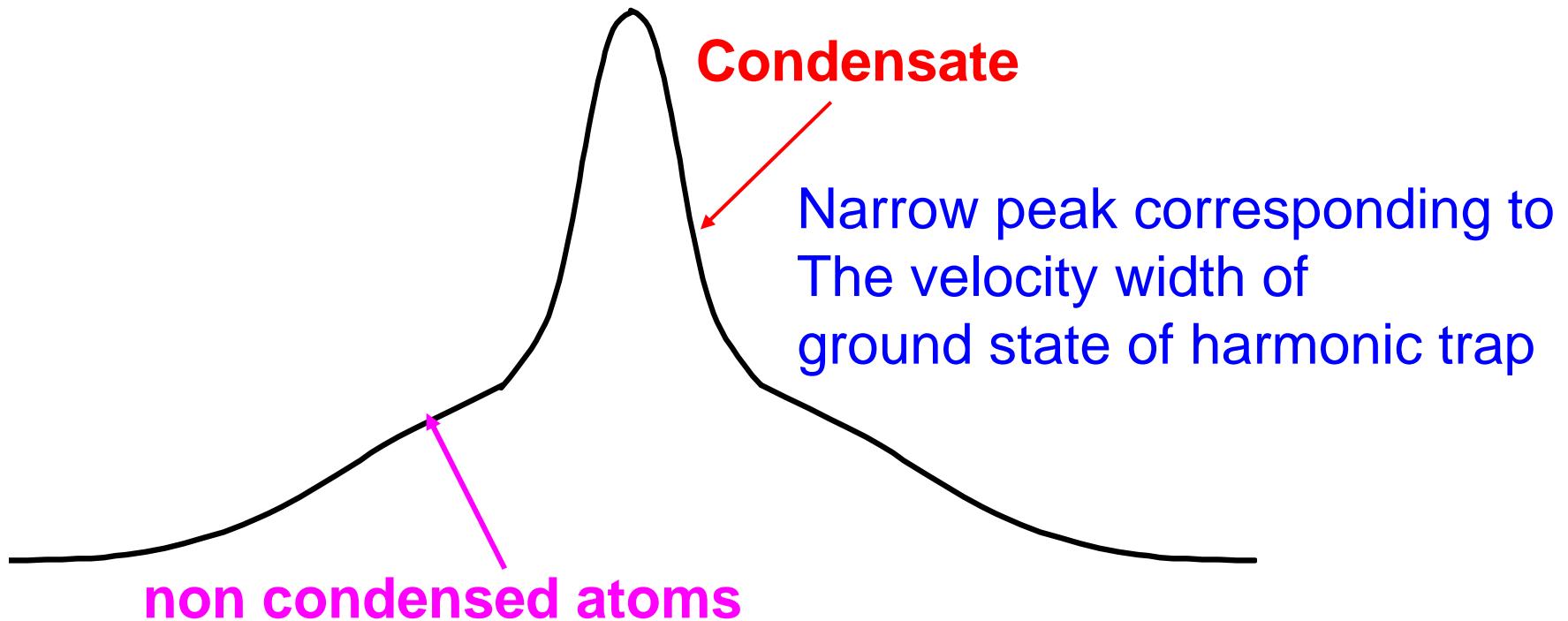
1000 atoms in ground state of magnetic trap.

Remark:  
Metastable systems  
The true ground state of Rb at 1  $\mu$ K is a piece of solid

Science, 269, 198 (1995)  
M. Anderson, E. Cornell and C. Wieman

+ Sodium, Lithium, Hydrogen, Potassium  
Helium (2s state), Cesium, Ytterbium,  
Calcium, Strontium

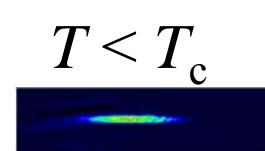
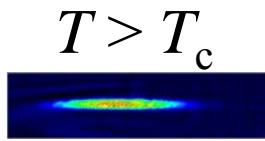
# Bimodal distributions



Thermal atoms in excited states: broader distribution

# Condensate signature

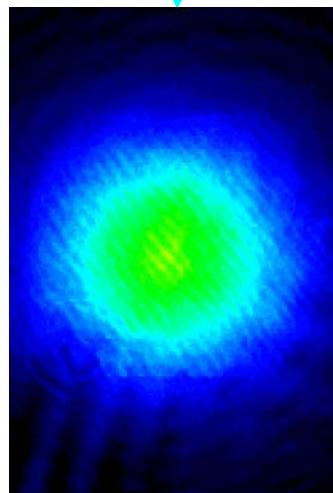
A few millions atoms in anisotropic magnetic trap



0,5 to 1  $\mu\text{K}$

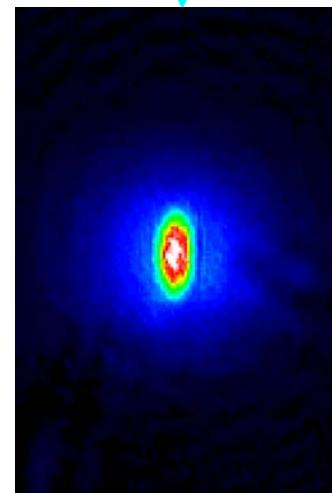
Time of flight

Boltzmann  
Gas



$$\frac{1}{2}mv_i^2 = \frac{1}{2}kT$$

isotropic



condensate

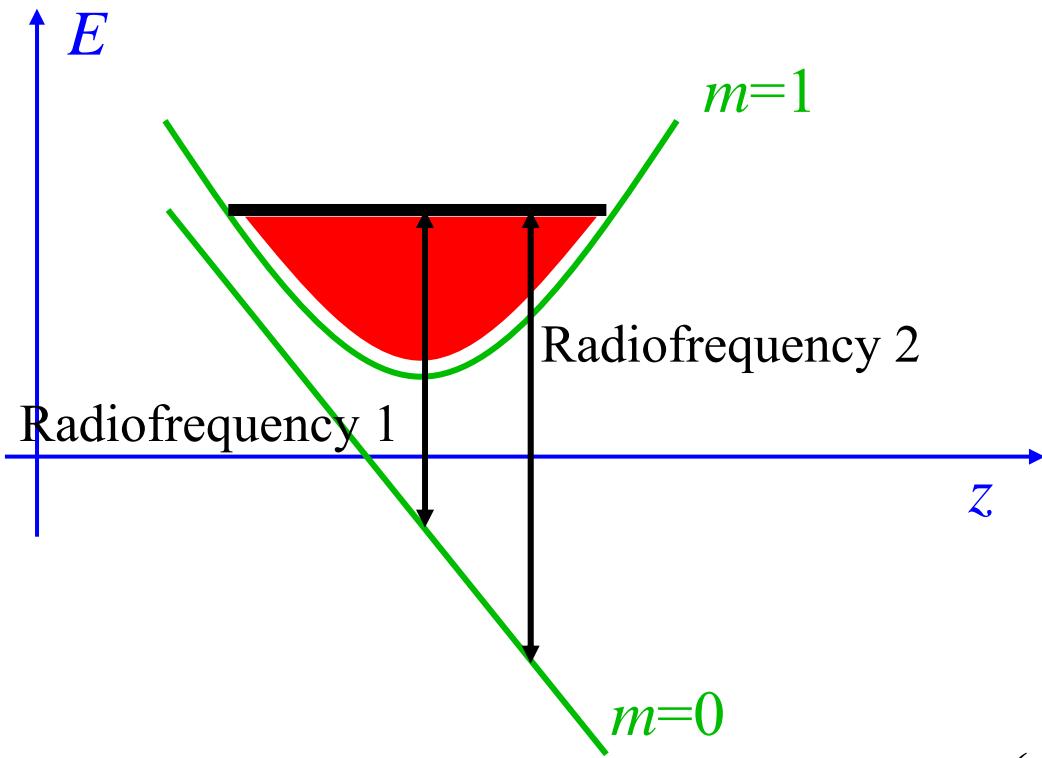
$$\frac{1}{2}mv_i^2 = \frac{1}{4}\hbar\omega_i$$

without  
interactions

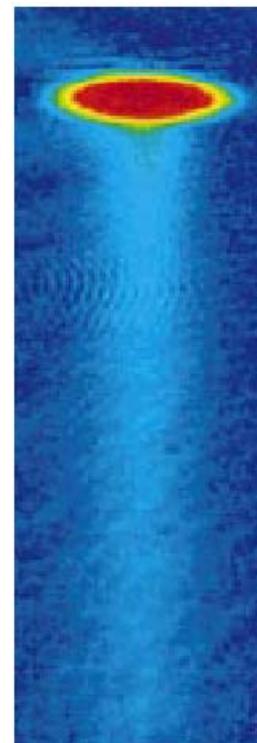
anisotropic

# Coherence of Bose-Einstein condensates

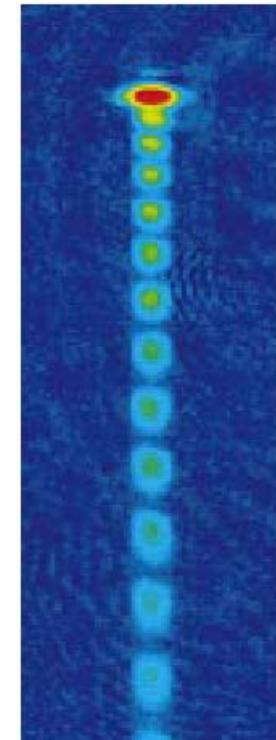
Young slit experiment, Munich, 2002



$$T > T_c$$



$$T < T_c$$



High contrast reveals  
macroscopic occupation  
of single quantum state

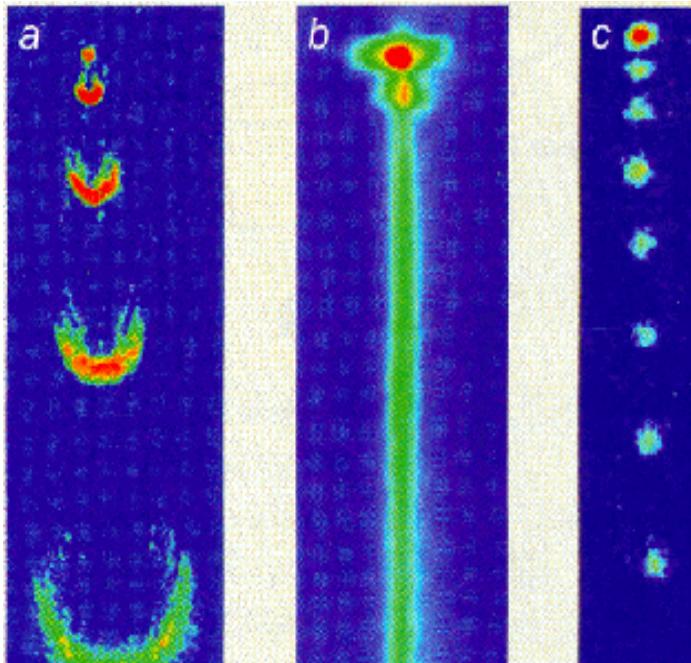
$$n_{out}(z) = |\psi_{\text{out}}(z - z_1) + \psi_{\text{out}}(z - z_2)|^2$$

$$\sim \frac{1}{\sqrt{z}} \left\{ 2 + 2 \cos \left( q\sqrt{z} + (\omega_1 - \omega_2)t \right) \right\}$$

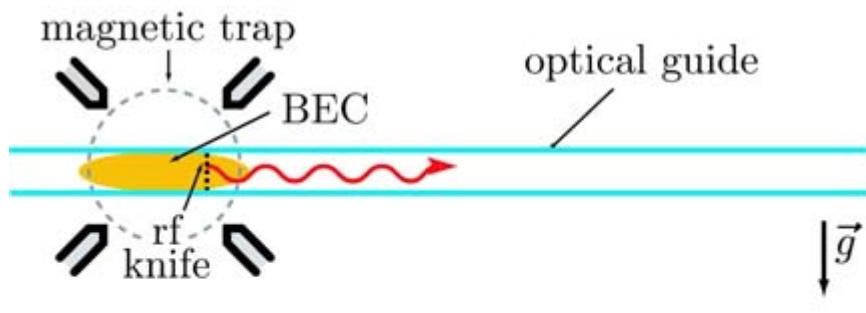
$$q = m(z_2 - z_1)\sqrt{2g}/\hbar$$

# Examples of Atom lasers

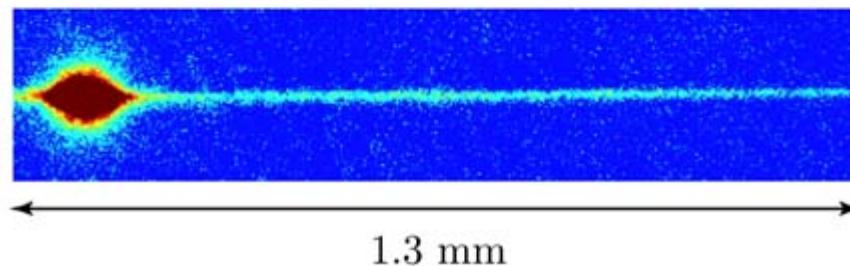
MIT



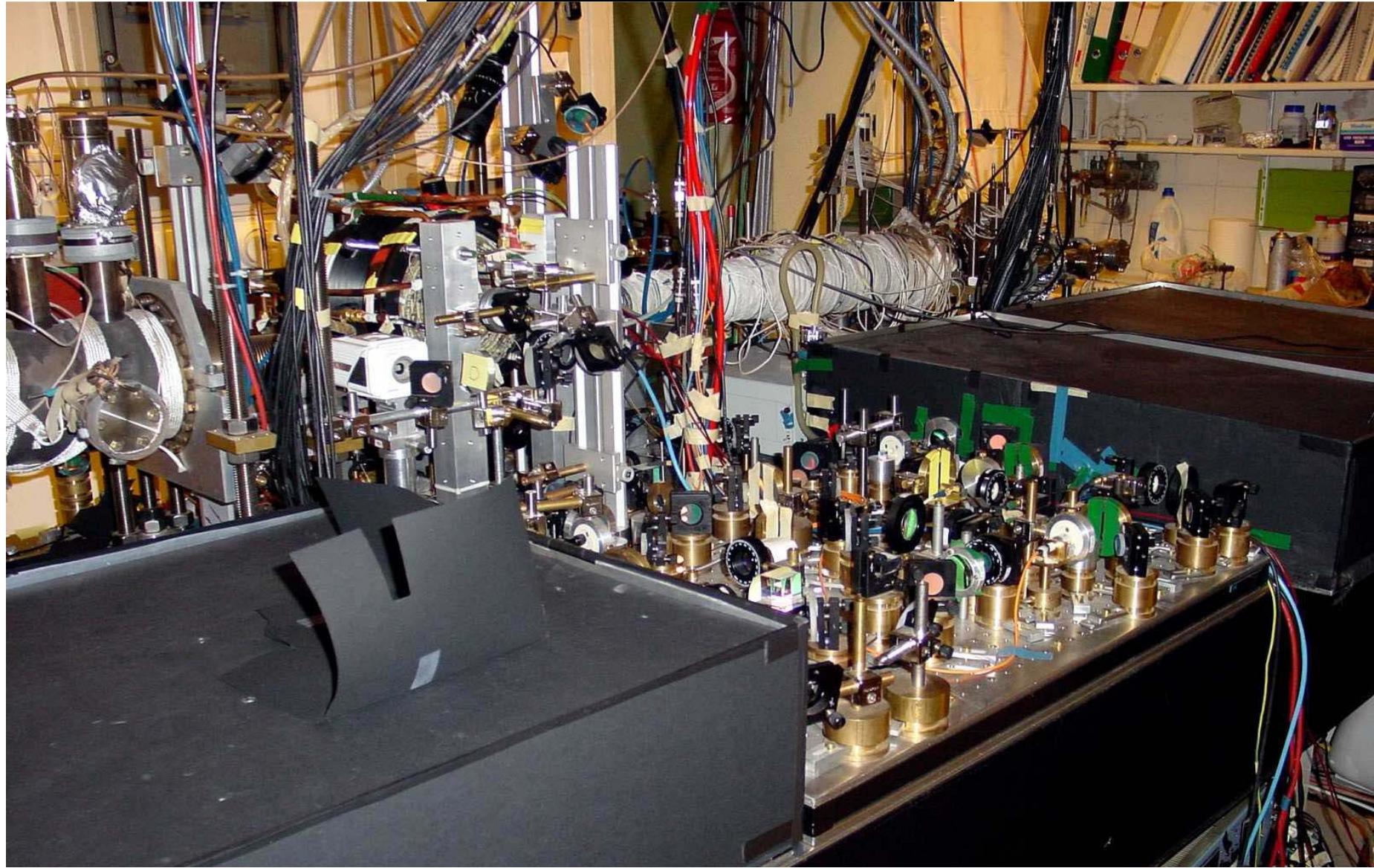
YALE



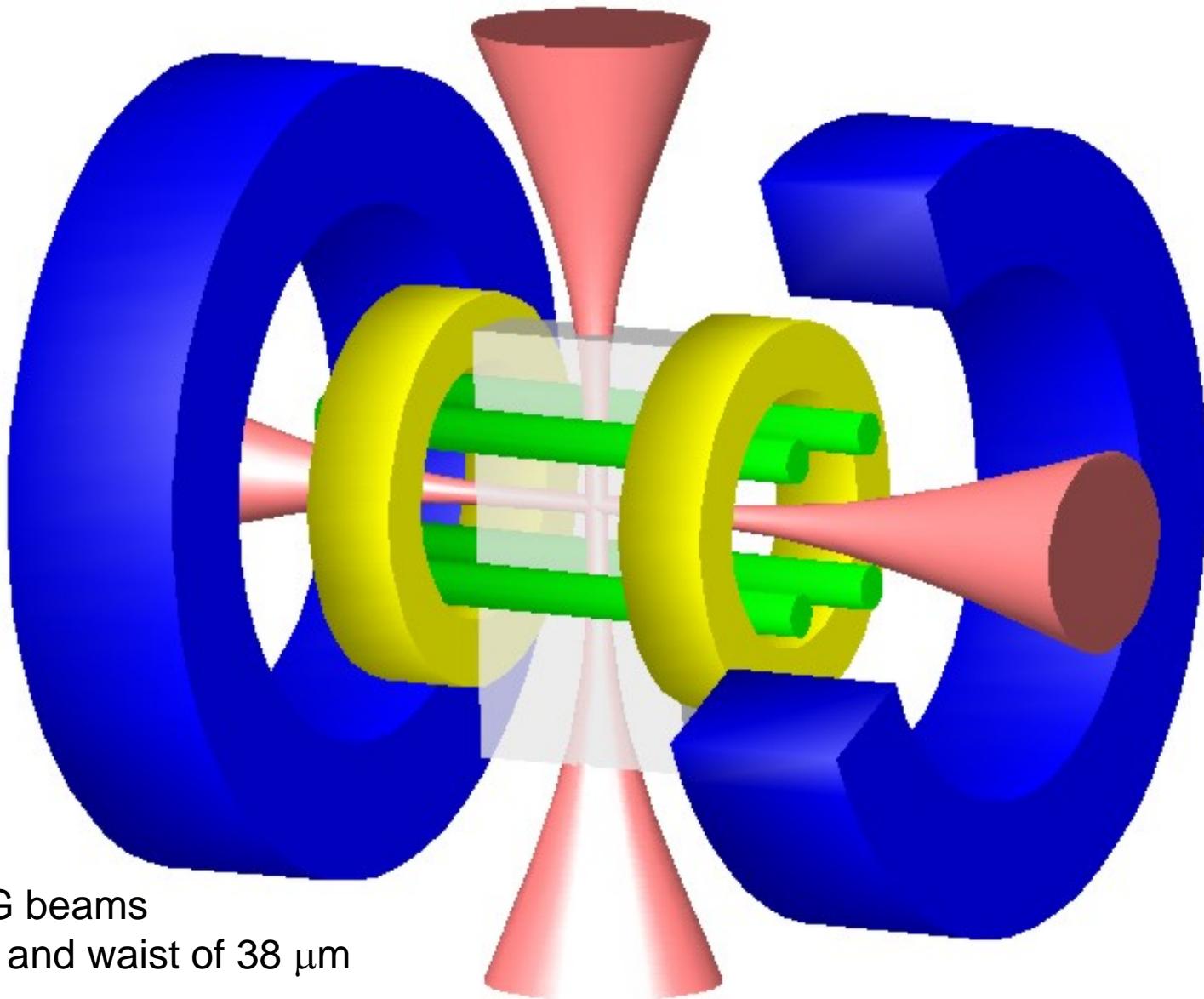
ORSAY



# A typical experiment



# The core of the experiment



# Optical Trapping

Laser field

$$\mathbf{E}(\mathbf{r},t) = \tilde{E}(\mathbf{r})\mathbf{e} e^{-i\omega t} + c.c$$

Induced dipole

$$\mathbf{p}(\mathbf{r},t) = \tilde{p}(\mathbf{r})\mathbf{e} e^{-i\omega t} + c.c$$

$$\tilde{p} = \alpha \tilde{E}$$

polarizability

$$\alpha = \frac{e^2}{m_e} \frac{1}{\omega_0^2 - \omega^2 - i\omega\Gamma}$$

Interaction energy

$$U_{dip} = -\frac{1}{2} \langle \mathbf{p} \mathbf{E} \rangle$$

Dipole potential

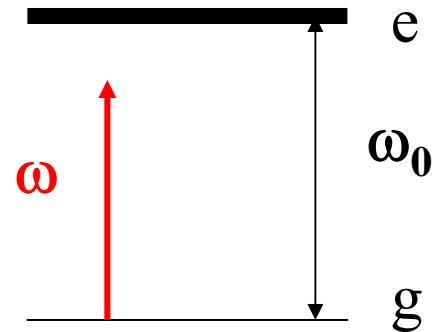
$$U_{dip} = -\frac{1}{2\epsilon_0 c} Re(\alpha) I$$

*I*: laser intensity

Potential depth  $\sim 1$  mK

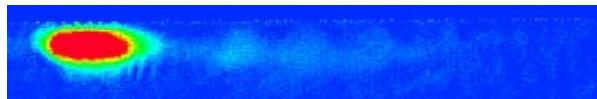
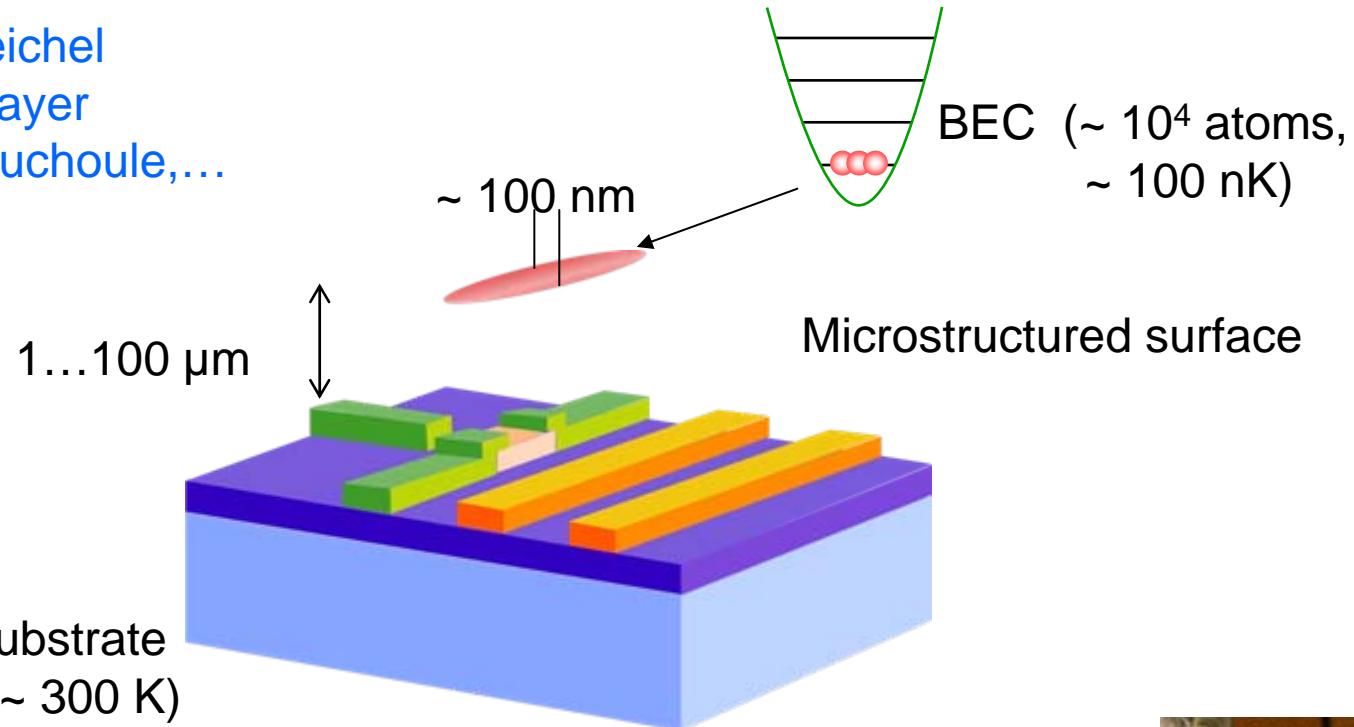
Dipole force

$$F_{dip} = -\nabla U_{dip} = \frac{1}{2\epsilon_0 c} Re(\alpha) \nabla I$$



# Atom chips

ENS: J. Reichel  
Schmiedmayer  
Vuletic, Bouchoule,...



*Complex manipulation*



*miniaturizing*

**Next lecture:**

**Tuning Atom-atom interactions  
and applications**