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Lie Algebraic Methods for the Analysis and Control of Quantum Systems; Applications to Quantum Walks

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Abstract:

The theory of Lie algebras and Lie groups has proved to be an invaluable tool in the analysis and control of quantum mechanical systems. In this talk I will summarize three recent contributions in this area. First I will show how classical results on the decomposition of finite dimensional Lie algebras lead to a general description of the structure of quantum dynamics [3]. The dynamics may or may not be subject to a control action, for example an appropriately shaped electro-magnetic field. In the case a control action is present, the problem is to shape the control so as to have the desired dynamics. The second contribution discussed [2] gives a general method to control a finite dimensional quantum system to a desired target up to arbitrary accuracy. This consists of appropriately choosing a sequence of Hamiltonians from an allowed set and an algorithm to switch between different Hamiltonians. This contribution can be seen as a constructive proof of the Lie algebra rank condition [5] an important result in geometric control theory which can be considered the starting point of the Lie algebraic approach to quantum system. In the third contribution we will see how Lie algebraic ideas can be used to solve the problem of simulation of the dynamics of a continuous quantum walk on a graph by a number of steps of a discrete time quantum walk on the same graph. This problem, which was posed in [6] was solved with different techniques in [1] and [4] which generalized a technique previously given in [7]. We shall take the example of the quantum walk on the cycle and discuss the complexity of the simulation, that is, how the number of steps needed varies with the number of nodes of the graph.

References

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