Permutationally Invariant Quantum Tomography

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We propose permutationally invariant (PI) tomography in many-qubit quantum experiments. Concretely, instead of the density matrix ϱ, we propose to determine the permutationally invariant part of the density matrix defined as

$$\varrho_{\rm PI} = \frac{1}{N!}\sum_k \Pi_k \varrho \Pi_k, \tag{1}$$

where Π_k are all the permutations of the qubits.

We develop a provably optimal scheme, which is feasible for large multi-qubit systems: For our method, the number of local measurement settings needed increases *quadratically* with the size of the system. For a setting, the same operator A_j has to be measured on all the qubits. Our approach is further motivated by the fact that almost all multipartite experiments are done with permutationally invariant quantum states. We demonstrate that these techniques are viable in practice by applying them to a photonic experiment observing a four-qubit symmetric Dicke state. The results are shown in Fig. 1 and Fig 2.

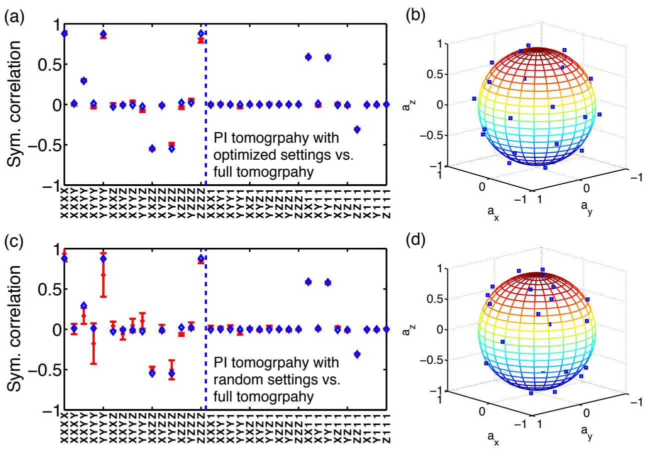

FIG. 1: (a) The 34 symmetrized correlations coming from (crosses with error bars) 15 permutationally invariant measurement settings with optimized A_j matrices for $N = 4$ qubits and (diamonds) from full tomography requiring 81 local settings. The average uncertainty of all symmetrized correlations obtained from full tomography is ± 0.022, and is not shown on the figure. The labels refer to symmetrized correlations. The results corresponding to the 15 full four-qubit correlations are left from the vertical dashed line. All correlations are between -1 and $+1$. (b) Measurement directions. A point at (a_x, a_y, a_z) corresponds to measuring operator $a_x X + a_y Y + a_z Z$. (c) Results for randomly chosen A_j matrices and (d) corresponding measurement directions.

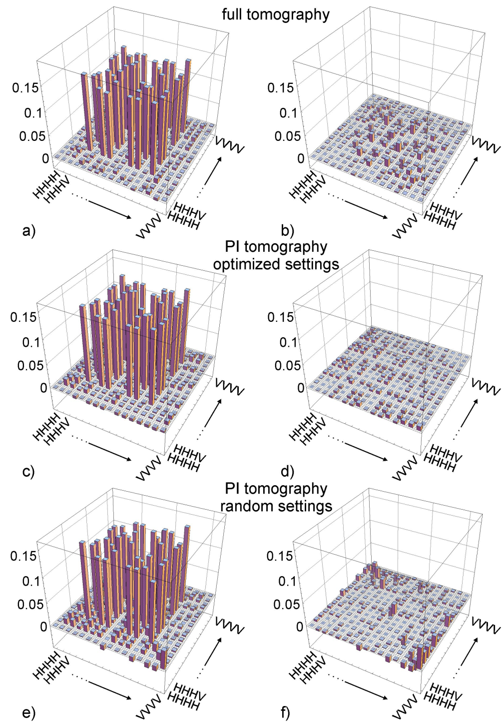

FIG. 2: (a) The real and (b) imaginary parts of the density matrix coming from full tomography. (c-d) The same for permutationally invariant tomography with optimized and (e-f) random measurement directions, respectively.

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[1] G. Tóth, W. Wieczorek, D. Gross, R. Krischek, C. Schwemmer, and H. Weinfurter, Phys. Rev. Lett. **105**, 250403 (2010).