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Quantum Measurements in Continuous Time and Non-Markovian Evolutions

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Abstract:

The (linear and nonlinear) stochastic Schrödinger equation (SSE) originated from three different lines of research, all related to quantum open system theory: dynamical reduction theories (which aim to give a dynamical model of the reduction of the wave function in quantum measurement theory), measurements in continuous time (quantum detection theory), unravelling of master equations (in order to simplify the task of simulating quantum master equations). In quantum optics the whole theory is often referred as “quantum trajectory theory”; at present, the biggest part of the developments of this theory are in the Markov case.

To attach the non-Markov case, it is possible to modify the usual master equation and to introduce memory terms, but the problem is to guarantee the complete positivity of the resulting dynamical map; only restricted classes of non-Markovian master equations are known. On the other side, to modify the SSE is easiest, as the dynamical map arising from the mean of the projection on the stochastic wave function is automatically completely positive. Eventually this mean does not satisfy a closed equation, but this is not a conceptual problem. A more serious problem is the possibility of a measurement interpretation. The compatibility with the axiomatic structure of quantum mechanics and the possibility of interpreting the noises in the SSE as outputs of a measuerement in continuous time impose restrictions on the possible “non-Markovian modifications” of the SSE.

In this talk I want to present some proposals and results on the non-Markov SSE, which are compatible with quantum detaction theory. Our SSEs are related to two classes of non-Markovian dynamics, the so called Lindblad rate equation and a dynamics representing a quantum system interacting with a coloured bath and possibly controlled by a measurement based feedback. Let me stress that memory effects can arise because of the interaction with the external environment in cases in which the usual Markov approximations are not good, but also because we take under observation the system and we control it in some way by taking into account the results of the measurement. As soon as this feedback is not instantaneous, some memory effect is introduced.

The continuous observation allows also to see the effect of the memory terms. Typically a non-Markovian thermal-like bath modifies the the response of the system to a stimulating laser and the properties of the emitted fluorescence light; then, the effect of the non-Markovian dynamics can be seen in the spectrum of the output of the continuous measurement.

1. A. Barchielli, C. Pellegrini, *Jump-diffusion unravelling of a non Markovian generalized Lindblad master equation*, J. Math. Phys. **51** (2010) 112104; arXiv:1006.4527v1 [quant-ph].
2. A. Barchielli, P. Di Tella, C. Pellegrini, F. Petruccione, *Stochastic Schrödinger equations and memory*, to appear in *QP-PQ: Quantum Probability and White Noise Analysis* Vol. 27; arXiv:1006.3647v1 [quant-ph].
3. A. Barchielli, C. Pellegrini, F. Petruccione, *Stochastic Schrödinger equations with coloured noise*, EPL **91** (2010) 24001; arXiv:0911.2554v1 [quant-ph].
4. A. Barchielli, A.S. Holevo, *Constructing quantum measurement processes via classical stochastic calculus*, Stoch. Proc. Appl. **58** (1995) 293-317.