Pairing in Atomic Fermi Mixtures

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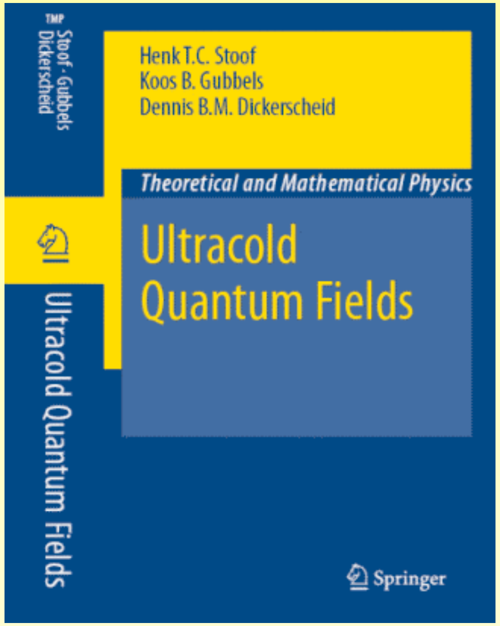
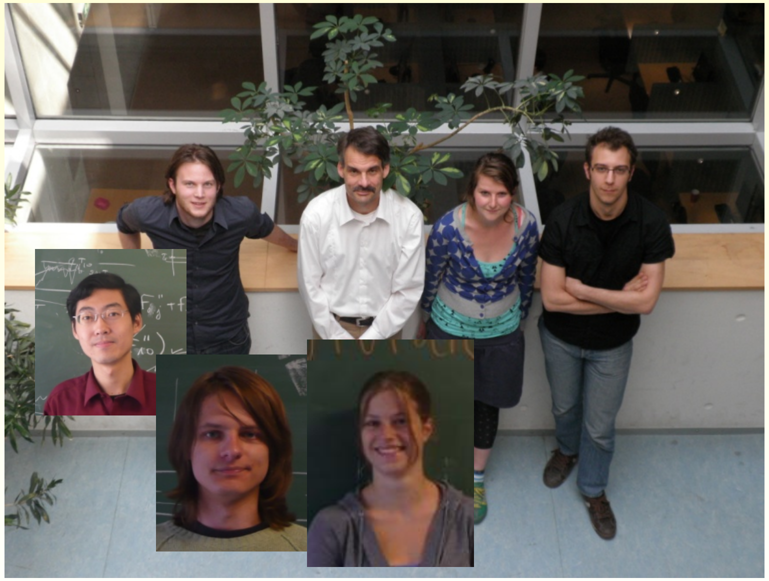
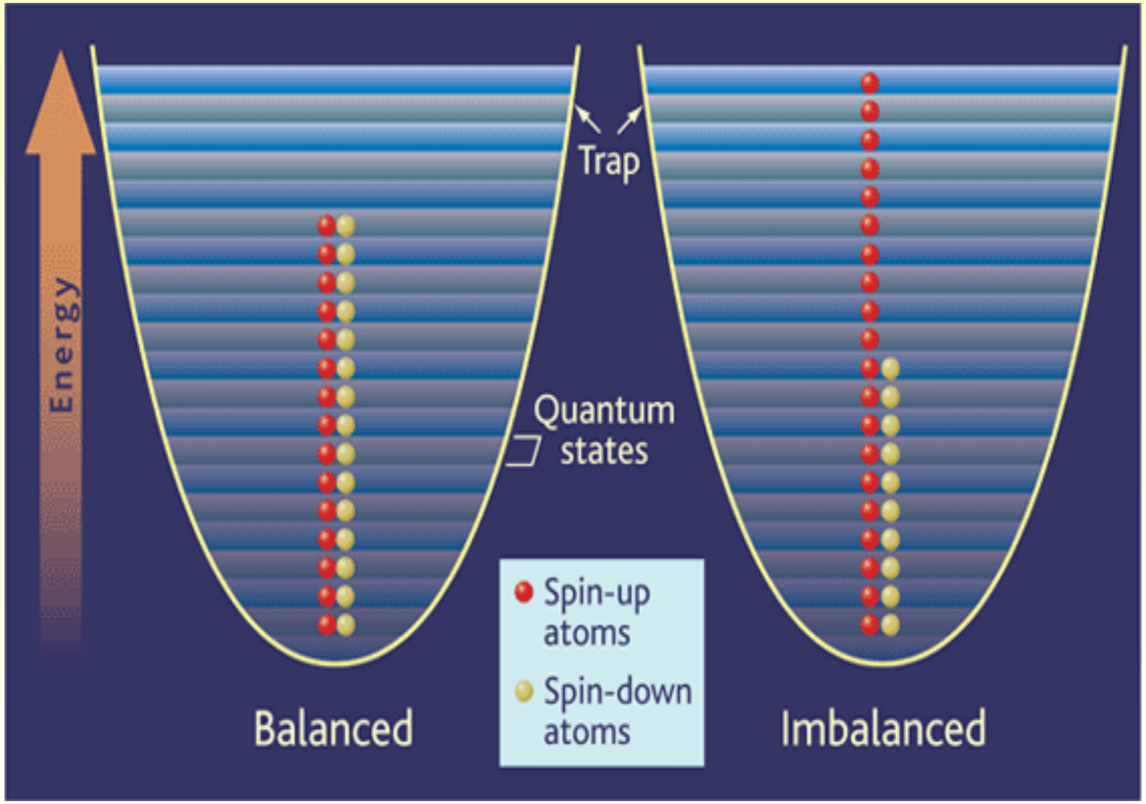


- Introduction
- BCS Theory
- The BEC-BCS Crossover and Imbalanced Fermi Mixtures
- Towards Gravity Dual (AdS/CFT)



Ideal Fermi Gases I

- Experiments are always in a trap:



Ideal Fermi Gases II

- Number of states below a certain energy (for one spin state) is:

$$\boxed{N(\varepsilon) = \frac{1}{(\mathrm{h}\omega)^3}\int_0^{\varepsilon} d\varepsilon_x \int_0^{\varepsilon-\varepsilon_x} d\varepsilon_y \int_0^{\varepsilon-\varepsilon_x-\varepsilon_y} d\varepsilon_z = \frac{\varepsilon^3}{6(\mathrm{h}\omega)^3}}$$

- This means that the Fermi energy is:

$$\boxed{\varepsilon_F = (6N)^{1/3}\,\mathrm{h}\omega}$$

Ideal Fermi Gases III

- Differently: For the homogeneous gas: $n = k_F^3 / 6\pi^2$. So in the trap

$$\boxed{n(\mathbf{x}) = \frac{1}{6\pi^2}\left\{\frac{2m}{\hbar^2}[\varepsilon_F - V(\mathbf{x})]\right\}^{3/2}}$$

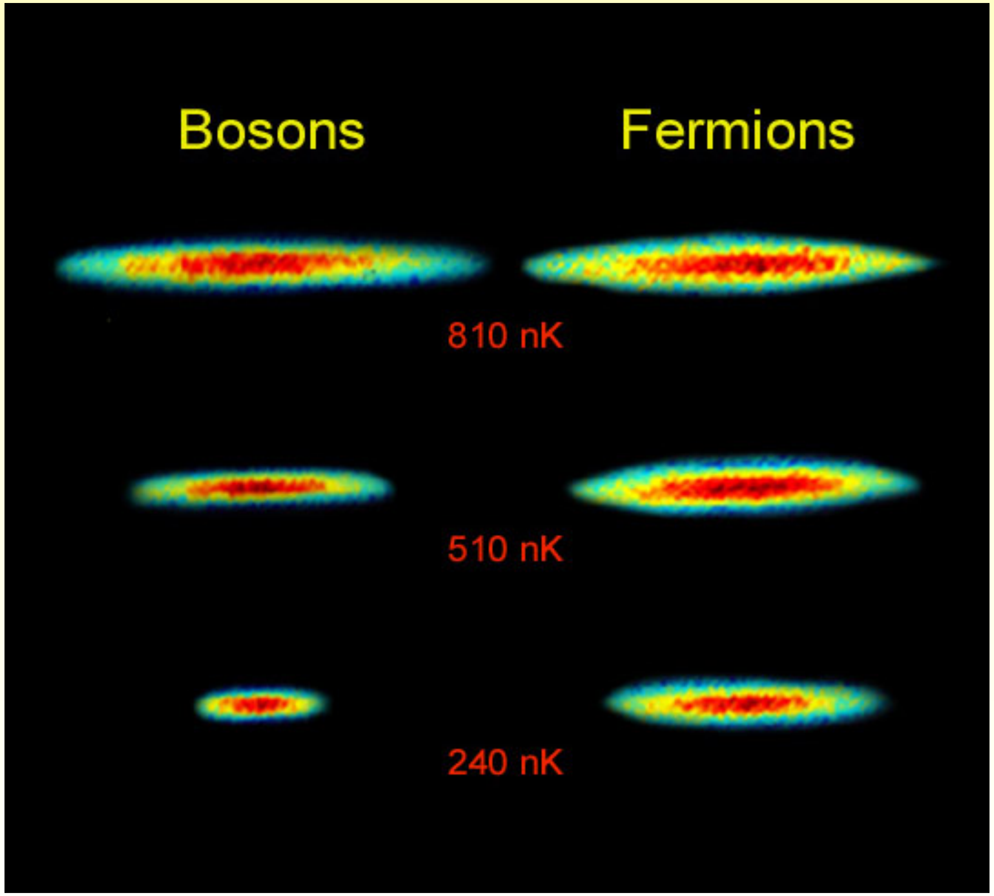
- By integrating over space we find again:

$$\boxed{\varepsilon_F = (6N)^{1/3}\hbar\omega}$$

- Note that the size of the cloud is: $R = \sqrt{\dfrac{2\varepsilon_F}{m\omega^2}}$

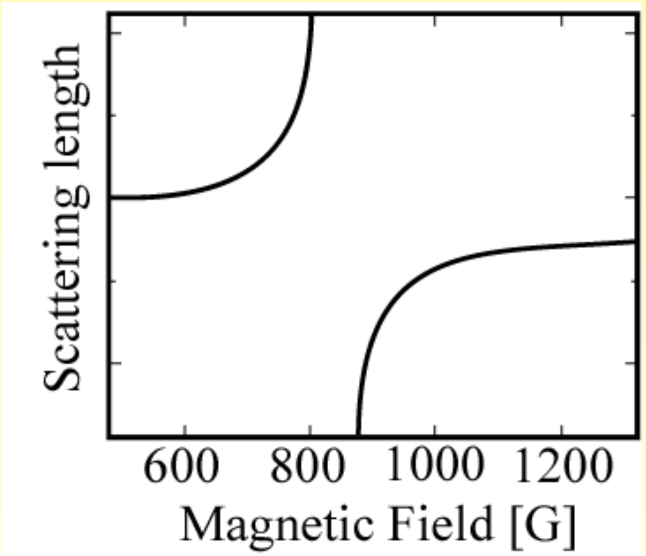
Ideal Fermi Gases IV

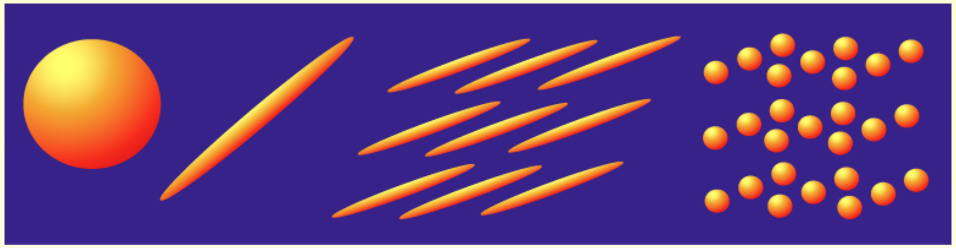
- Comparison between bosons and fermions:



Ultracold Fermi Mixtures I

- Experimental control over:
  - temperature and density
  - external potentials, disorder
  - number of particles, their quantum state
  - and even interactions!



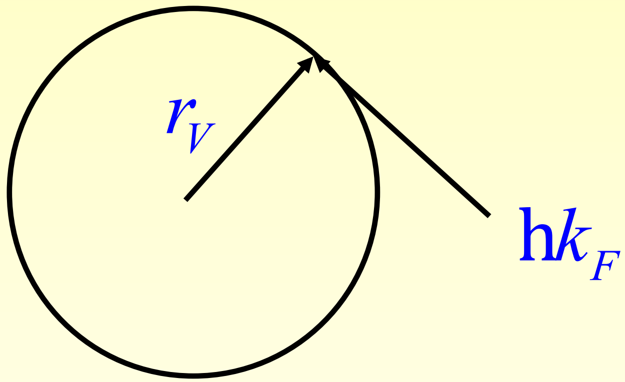


- Degenerate Fermi mixtures
  - Neutron stars ($T = 10^6$ K, $T_F = 10^{11}$ K, $T = 10^{-5}\, T_F$)
  - (High-Tc) superconductors ($T = 10^2$ K, $T_F = 10^5$ K, $T = 10^{-3}\, T_F$)
  - Ultracold atomic Fermi gases ($T = 10^2$ nK, $T_F = \mu$K, $T = 10^{-1}\, T_F$)

Ultracold Fermi Mixtures II

- Collisions are *s*-wave

$$\boxed{\mathrm{h}k_F r_V = \mathrm{h}}$$

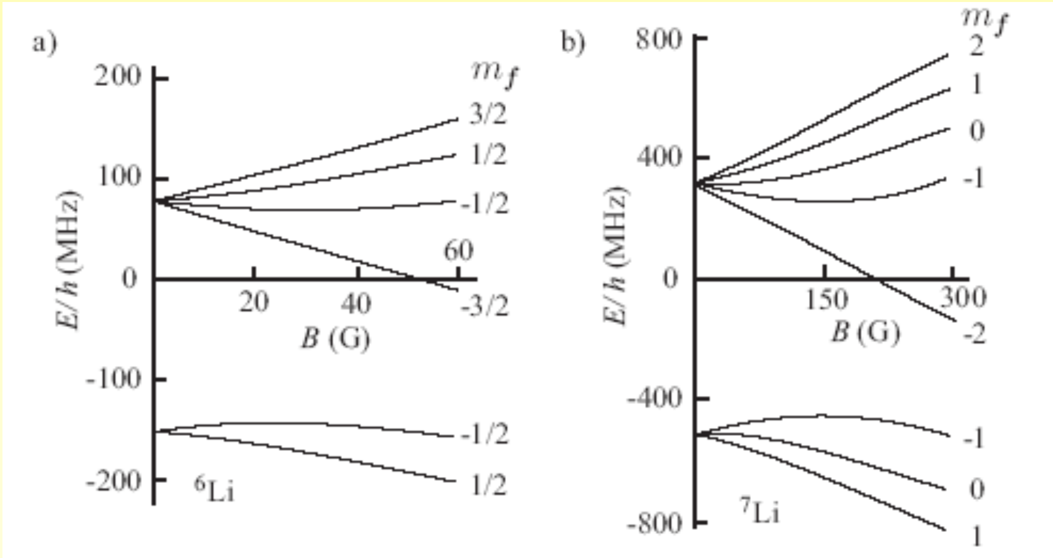


and we thus only have interactions between two different spin states.

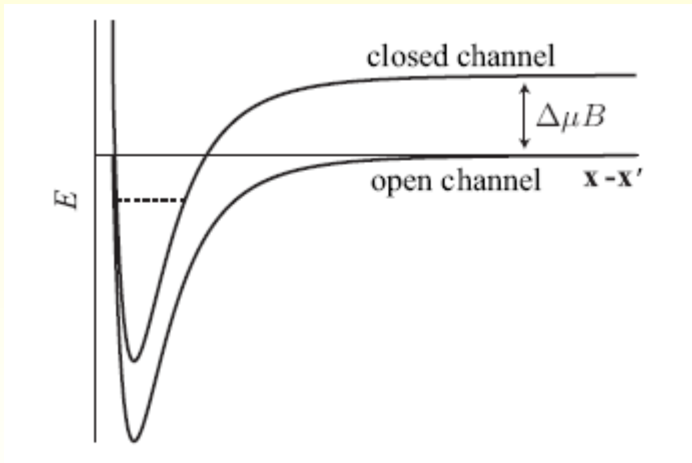
- This implies also: $n r_V^3 = 1$

Ultracold Fermi Mixtures III

- Hyperfine and Zeeman interactions:

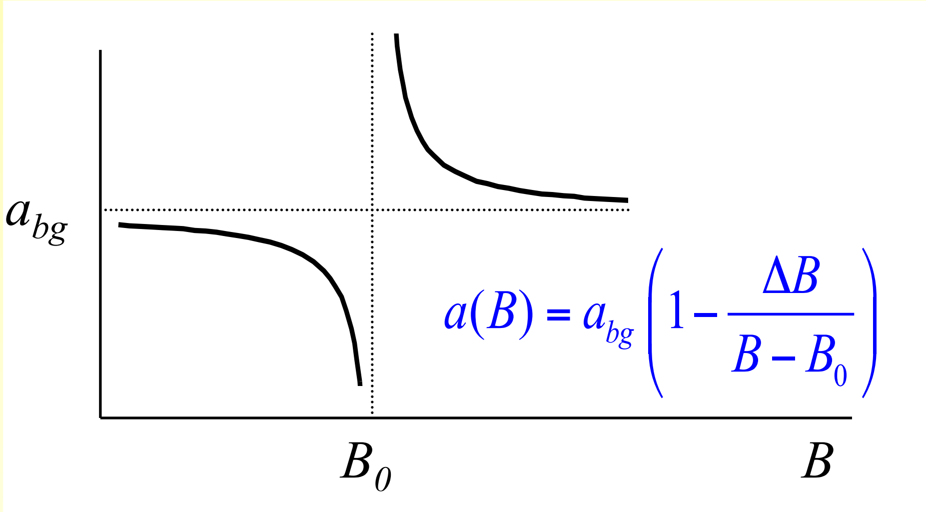


- Central or exchange interaction
- Together they lead to Feshbach resonances!

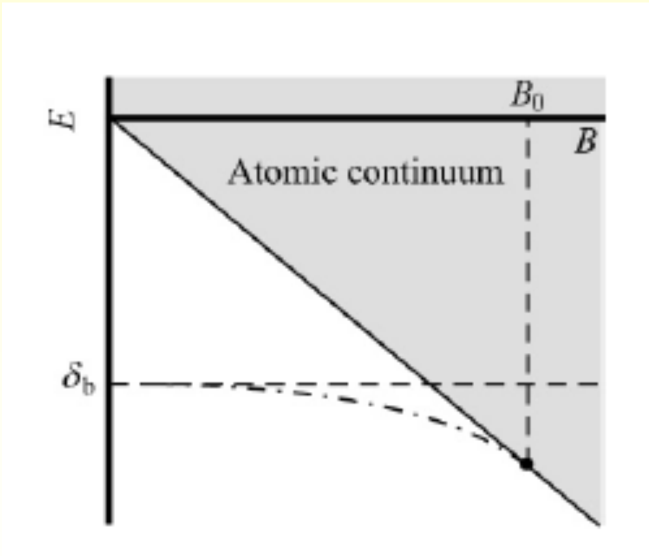


Ultracold Fermi Mixtures IV

- Interaction strength or scattering length:



$$a(B) = a_{bg}\left(1 - \frac{\Delta B}{B - B_0}\right)$$



- Binding energy:

$$\boxed{E_b(B) = -\frac{h^2}{m\left[a(B)\right]^2} \propto -\left(B - B_0\right)^2}$$

Superfluidity I

- Flow without friction. Described by a macroscopic wave function:

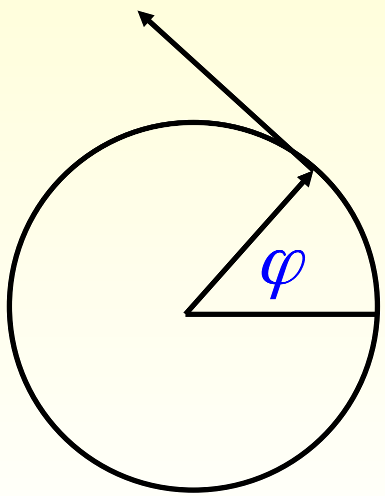
$$\boxed{\Psi(\mathbf{x}) = \sqrt{n_s}\, e^{i(m\mathbf{v}_s/\hbar)\cdot\mathbf{x}}}$$

  or more general $\mathbf{v}_s(\mathbf{x}) = \hbar\nabla\vartheta(\mathbf{x})/m$ and $n_s(\mathbf{x}) = |\Psi(\mathbf{x})|^2$.

- This implies the existence of quantized vortices with

$$\boxed{\vartheta(\mathbf{x}) = l\,\varphi}$$

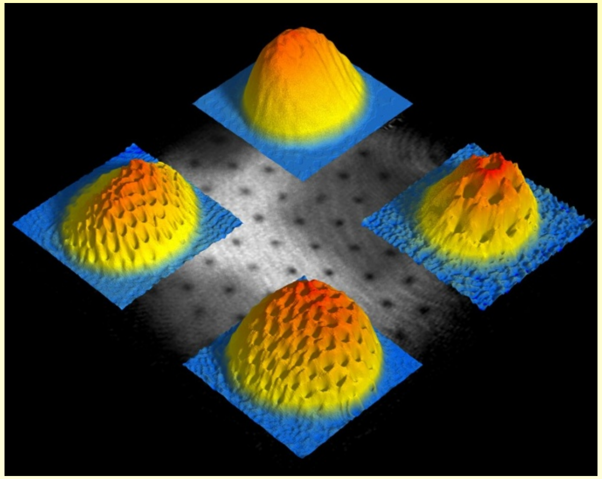
  which is really the trademark of superfluidity.



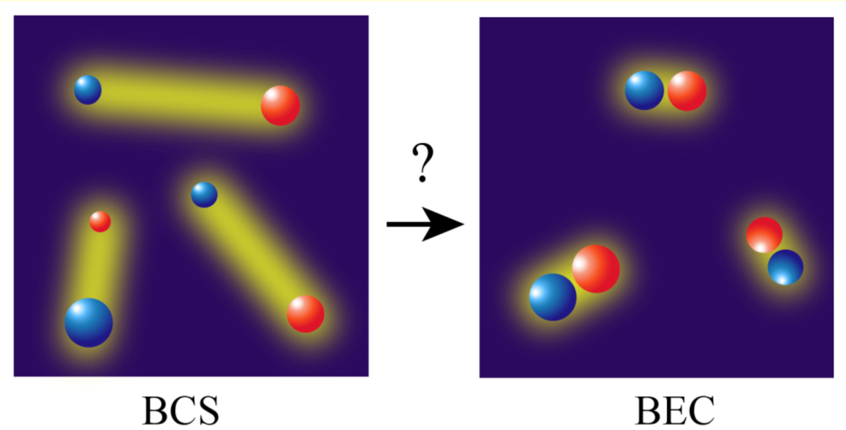
$$\mathbf{v}_s = \frac{l\,\hbar}{2\pi m r}\mathbf{e}_\varphi$$

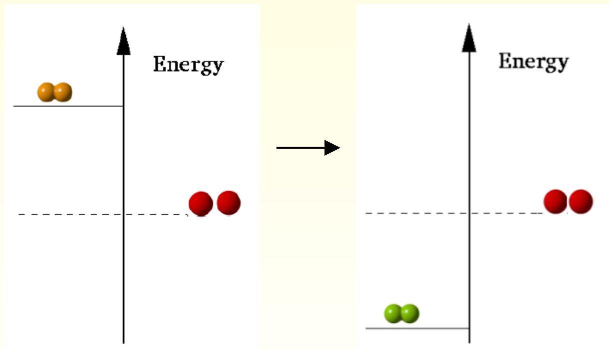
Superfluidity II

- Observed in a rotating Bose-Einstein condensate:



- What about a Fermi gas?

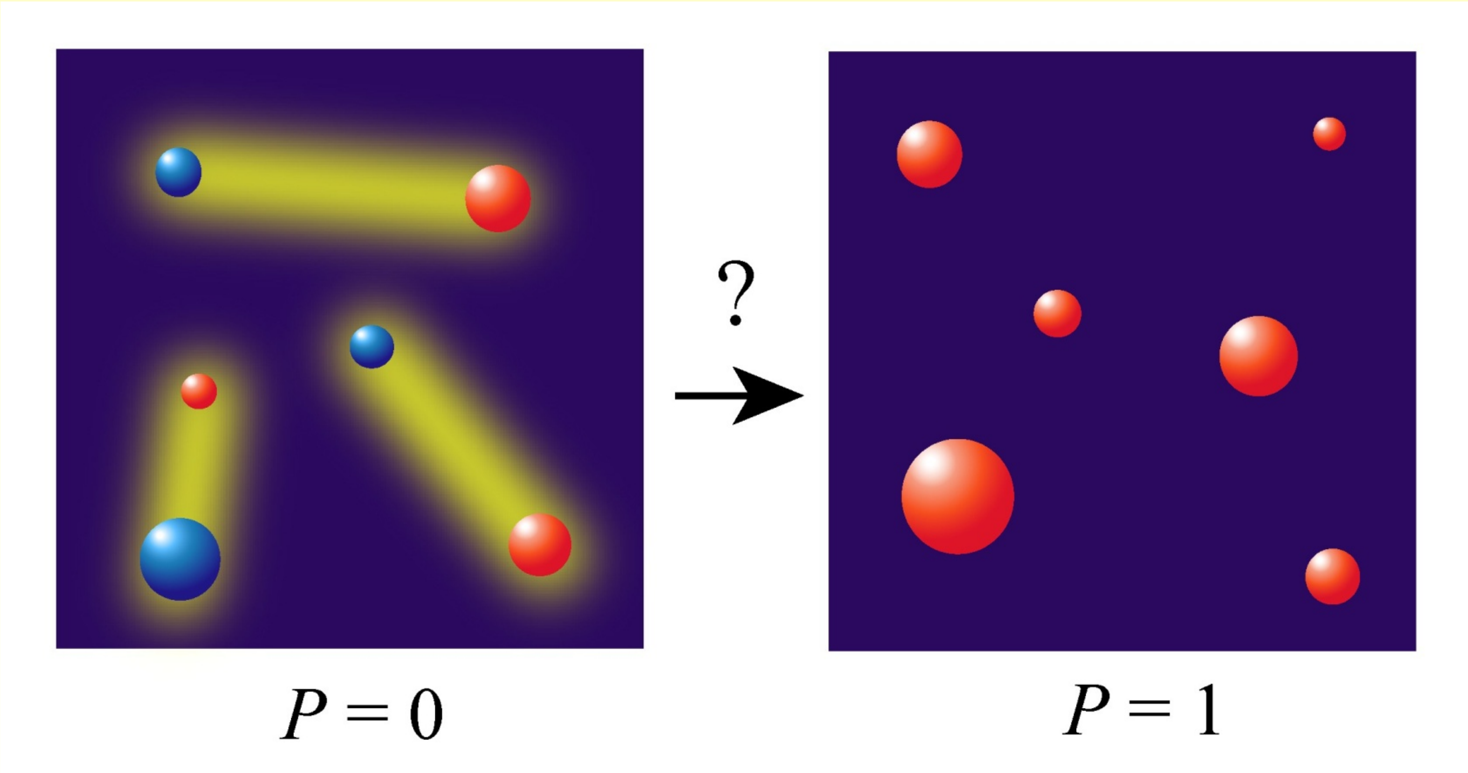




Superfluidity III

- Recently much debate over:

$$P = \frac{N_\uparrow - N_\downarrow}{N_\uparrow + N_\downarrow}$$



BEC I

- In second-quantization language the hamiltonian is

$$\boxed{\begin{aligned}\hat{H} &= \int d\mathbf{x}\,\hat{\psi}^{\dagger}(\mathbf{x})\left\{-\frac{\hbar^2\nabla^2}{2m}-\mu\right\}\hat{\psi}(\mathbf{x}) + \ldots \\ &\ldots + \frac{1}{2}V_0\int d\mathbf{x}\,\hat{\psi}^{\dagger}(\mathbf{x})\hat{\psi}^{\dagger}(\mathbf{x})\hat{\psi}(\mathbf{x})\hat{\psi}(\mathbf{x}),\end{aligned}}$$

- How do we treat Bose-Einstein condensation now?

BEC II

- Our most simple variational ground-state wave function for a Bose-Einstein condensed gas is now

$$\boxed{|\Psi\rangle \propto \left(\int d\mathbf{x}\,\phi(\mathbf{x})\hat{\psi}^{\dagger}(\mathbf{x})\right)^{N}|0\rangle\ .}$$

- However, for N ? 1 we expect that we are also allowed to use the more convenient wave function

$$\boxed{|\Psi\rangle \propto \exp\left(\int d\mathbf{x}\,\phi(\mathbf{x})\hat{\psi}^{\dagger}(\mathbf{x})\right)|0\rangle\ .}$$

BEC III

- The latter ground-state wave function has the property that

$$\boxed{\hat{\psi}(\mathbf{x})\left|\Psi\right\rangle = \phi(\mathbf{x})\left|\Psi\right\rangle \, .}$$

- This suggests that Bose-Einstein condensation is associated with spontaneous symmetry breaking, i.e.,

$$\boxed{\left\langle \hat{\psi}(\mathbf{x}) \right\rangle \neq 0.}$$

- This is the macroscopic wavefunction of superfluidity!

Symmetry Breaking I

- It is nice to understand spontaneous symmetry breaking a bit better. At a fixed number we have

$$\boxed{\left|N\right\rangle \propto \frac{1}{\sqrt{N!}}\left(\int dx\,\phi(x)\psi^{\dagger}(x)\right)^{N}\left|0\right\rangle .}$$

- At fixed phase we have, however,

$$\boxed{\left|\vartheta\right\rangle \propto \sum_{N}\frac{\exp\left(iN\vartheta\right)}{\sqrt{N!}}\left|N\right\rangle .}$$

Symmetry Breaking II

- This shows that the phase and the number of particles are conjugate variables, i.e.,

$$\boxed{[N,\vartheta]_- = -i\,.}$$

- Moreover, the energy obeys due to the definition of the chemical potential

$$\boxed{E \simeq E_0 + \mu\Delta N + \frac{1}{2}\frac{d\mu}{dN}\Delta N^2.}$$

Symmetry Breaking III

- The thermodynamic potential thus obeys

$$\boxed{\Omega \; ; \; \Omega_0 + \frac{1}{2}\frac{d\mu}{dN}\Delta N^2,}$$

- which leads to the Schrödinger equation

$$\boxed{i\mathrm{h}\frac{\partial}{\partial t}\Psi(\vartheta) = \frac{1}{2}\frac{d\mu}{dN}\left(\frac{1}{i}\frac{\partial}{\partial\vartheta} - N\right)^2 \Psi(\vartheta).}$$

Symmetry Breaking IV

- So the absolute ground state of the gas is the symmetry unbroken state

$$\boxed{\Psi(\vartheta) = \frac{1}{\sqrt{2\pi}} \exp(iN\vartheta).}$$

- However, if N ? 1 it takes a very long time for the gas to 'diffuse' to this state and we can safely assume that

$$\boxed{\Psi(\vartheta) = \delta(\vartheta).}$$

BCS I

- In second-quantization language the hamiltonian is

$$\boxed{\hat{H} = \sum_{\sigma} \int d\mathbf{x}\, \hat{\psi}^{\dagger}_{\sigma}(\mathbf{x}) \left\{ -\frac{\hbar^2 \nabla^2}{2m} - \mu_{\sigma} \right\} \hat{\psi}_{\sigma}(\mathbf{x}) + \ldots \\ \ldots + V_0 \int d\mathbf{x}\, \hat{\psi}^{\dagger}_{\uparrow}(\mathbf{x}) \hat{\psi}^{\dagger}_{\downarrow}(\mathbf{x}) \hat{\psi}_{\downarrow}(\mathbf{x}) \hat{\psi}_{\uparrow}(\mathbf{x}),}$$

- Now we have Bose-Einstein condensation of pairs so:

$$\boxed{\left\langle \hat{\psi}_{\downarrow}(\mathbf{x}) \hat{\psi}_{\uparrow}(\mathbf{x}) \right\rangle \neq 0.}$$

BCS II

- Introducing $\Delta = V_0 \left\langle \hat{\psi}_{\downarrow}(\mathbf{x}) \hat{\psi}_{\uparrow}(\mathbf{x}) \right\rangle$ the Hamiltonian can be approximated by

$$\hat{H} \simeq \sum_{\sigma} \int d\mathbf{x}\, \hat{\psi}^{\dagger}_{\sigma}(\mathbf{x}) \left\{ -\frac{\hbar^2 \nabla^2}{2m} - \mu_{\sigma} \right\} \hat{\psi}_{\sigma}(\mathbf{x}) + \ldots$$

$$\ldots + \int d\mathbf{x}\, \Delta \hat{\psi}^{\dagger}_{\uparrow}(\mathbf{x}) \hat{\psi}^{\dagger}_{\downarrow}(\mathbf{x}) + \int d\mathbf{x}\, \Delta^{*} \hat{\psi}_{\downarrow}(\mathbf{x}) \hat{\psi}_{\uparrow}(\mathbf{x}),$$

- This is thus a mean-field theory!

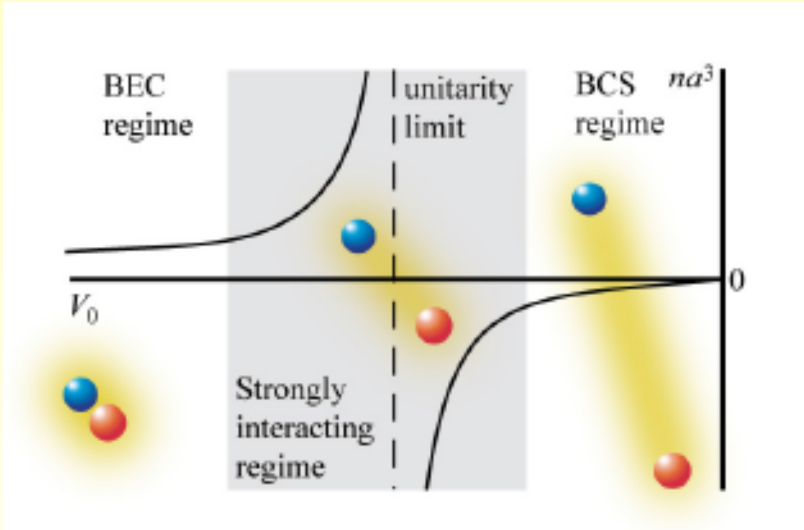
Zero Temperature I

- The microscopic Hamiltonian

$$\hat{H} = \sum_{\mathbf{k},\alpha} (\epsilon_{\mathbf{k}} - \mu \;\;) \hat{\psi}^\dagger_{\mathbf{k},\alpha} \hat{\psi}_{\mathbf{k},\alpha} + \frac{V_0(\Lambda)}{V} \sum_{\mathbf{K},\mathbf{k},\mathbf{k}'} \hat{\psi}^\dagger_{\mathbf{K}-\mathbf{k}',\uparrow} \hat{\psi}^\dagger_{\mathbf{k}',\downarrow} \hat{\psi}_{\mathbf{K}-\mathbf{k},\downarrow} \hat{\psi}_{\mathbf{k},\uparrow}$$

- Interaction vs. scattering length

$$V_0(\Lambda) = \frac{4\pi\hbar^2 a}{m} \frac{\pi}{\pi - 2a\Lambda}$$



- The BCS Ansatz

$$|\Psi_{\rm BCS}\rangle = \prod_{\mathbf{k}} \left(u_{\mathbf{k}} + v_{\mathbf{k}} \hat{\psi}^\dagger_{-\mathbf{k},\uparrow} \hat{\psi}^\dagger_{\mathbf{k},\downarrow} \right) |0\rangle$$

- Expectation values

$$\begin{aligned} \langle \Psi_{\rm BCS} | \hat{\psi}^\dagger_{\mathbf{k},\alpha} \hat{\psi}_{\mathbf{k},\alpha} | \Psi_{\rm BCS} \rangle &= v^2_{\mathbf{k}}, \\ \langle \Psi_{\rm BCS} | \hat{\psi}_{\mathbf{k},\downarrow} \hat{\psi}_{-\mathbf{k},\uparrow} | \Psi_{\rm BCS} \rangle &= u_{\mathbf{k}} v_{\mathbf{k}}. \end{aligned}$$

Zero Temperature II

- Normalization and minimization of $\langle \Psi_{\mathrm{BCS}} | \hat{H} | \Psi_{\mathrm{BCS}} \rangle$,

$$v_{\mathbf{k}}^2 = 1 - u_{\mathbf{k}}^2 = \frac{1}{2}\left(1 - \frac{\epsilon_{\mathbf{k}} - \mu}{\hbar\omega_{\mathbf{k}}}\right) \text{ with}$$

$$\hbar\omega_{\mathbf{k}} = \sqrt{(\epsilon_{\mathbf{k}} - \mu)^2 + \Delta^2} \quad .$$

- Gap and number equation:

$$\Delta \equiv -\frac{V_0(\Lambda)}{V} \sum_{\mathbf{k}} u_{\mathbf{k}} v_{\mathbf{k}}$$

$$n = \frac{2}{V} \sum_{\mathbf{k}} v_{\mathbf{k}}^2$$

can be easily solved numerically.

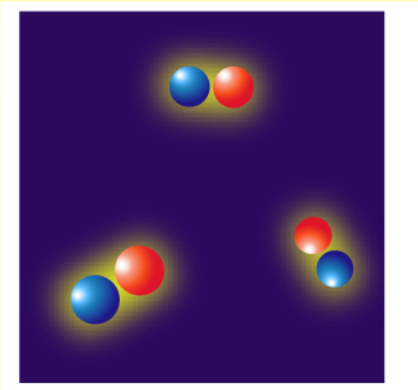
The BEC-BCS Crossover I

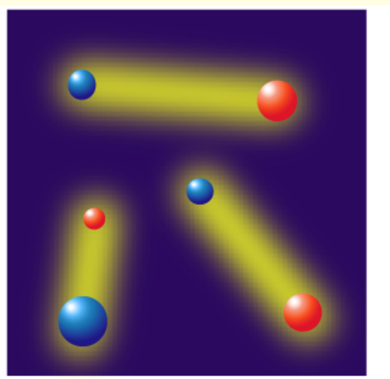
- Cooper condensate wavefunction:

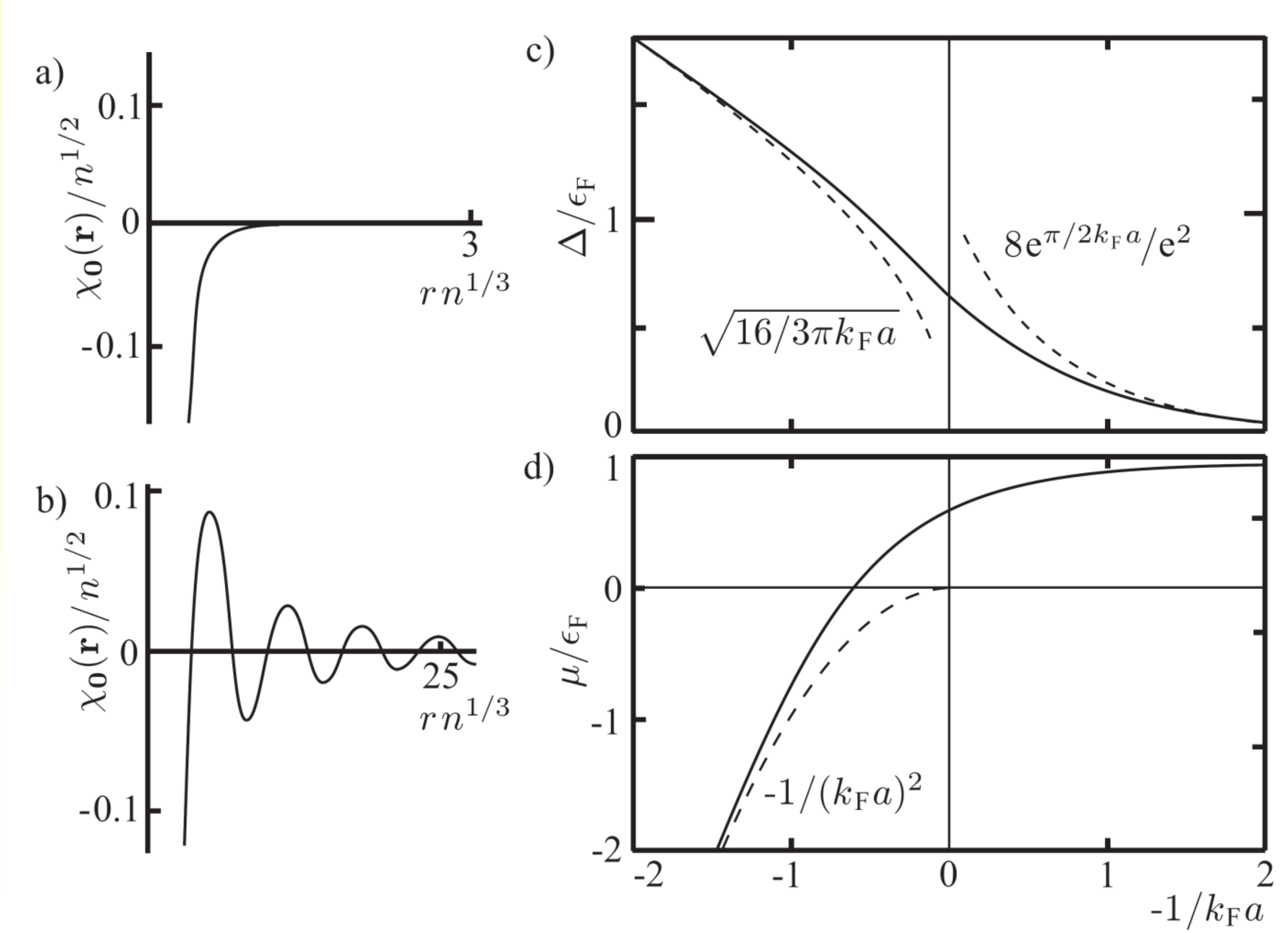
$$\phi_{\mathbf{0}}(\mathbf{r}) = -\frac{1}{V}\sum_{\mathbf{k}} u_{\mathbf{k}} v_{\mathbf{k}} e^{-i\mathbf{k}\mathbf{r}}$$

- Fermi energy

$$\epsilon_{\mathrm{F}} = \hbar^2 k_{\mathrm{F}}^2/2m = \hbar^2(3\pi^2 n)^{2/3}/2m$$

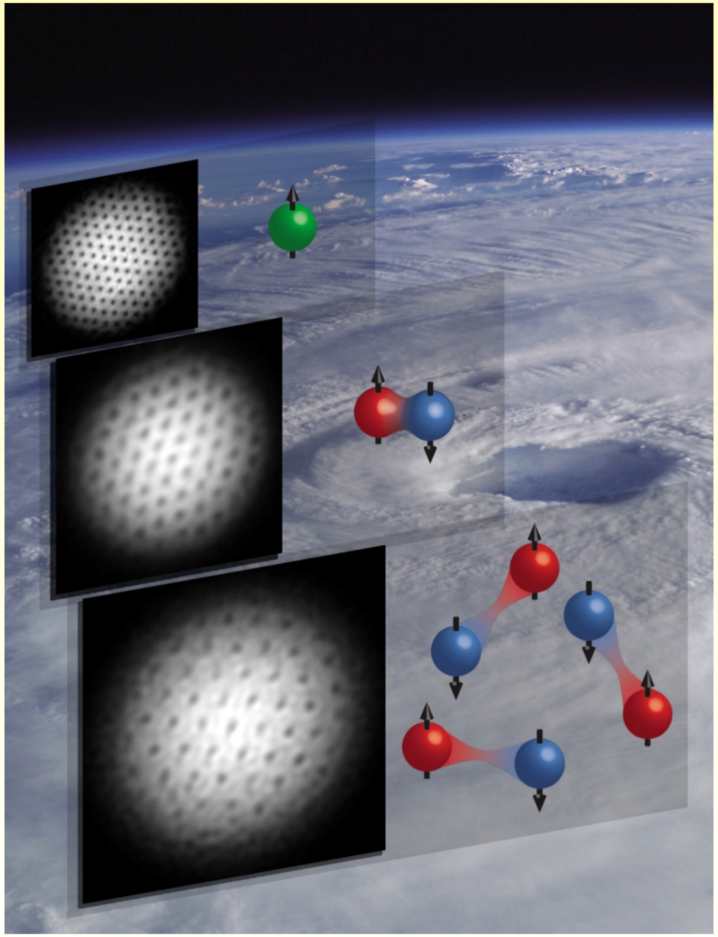




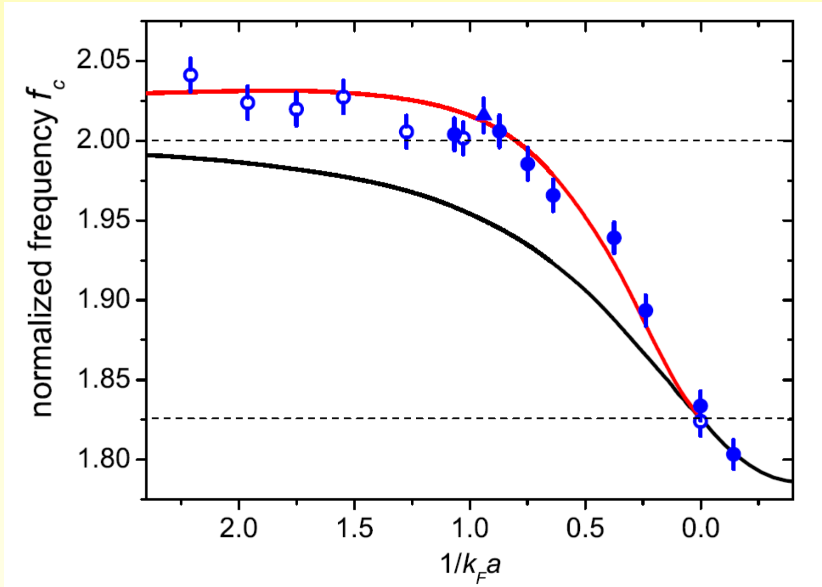


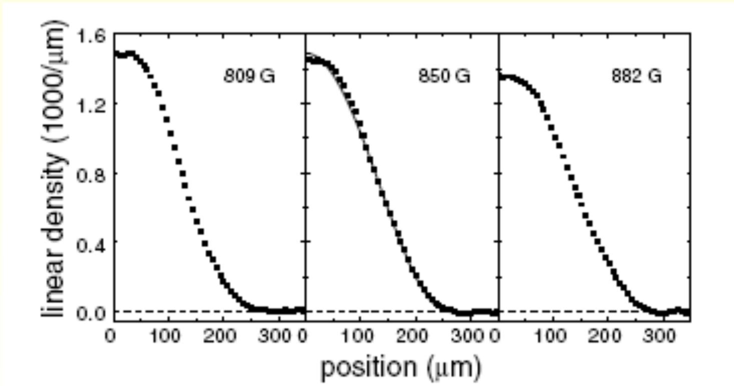
The BEC-BCS Crossover II

MIT: the study of vortices



Innsbruck: the study of collective modes





Summary

- BEC:

$$\boxed{\left|\Psi\right\rangle \propto \exp\left(\int d\mathbf{x}\,\phi(\mathbf{x})\hat{\psi}^{\dagger}(\mathbf{x})\right)\left|0\right\rangle .}$$

- BCS:

$$\boxed{\left|\Psi\right\rangle \propto \exp\left(\int d\mathbf{x}\,d\mathbf{x}'\,\phi(\mathbf{x},\mathbf{x}')\hat{\psi}^{\dagger}_{\downarrow}(\mathbf{x})\hat{\psi}^{\dagger}_{\uparrow}(\mathbf{x}')\right)\left|0\right\rangle}$$

leads to gap equation for $\boldsymbol{\Delta}(\mathbf{x}) = V_0\left\langle\hat{\psi}_{\downarrow}(\mathbf{x})\hat{\psi}_{\uparrow}(\mathbf{x})\right\rangle$.

Imbalanced Fermi Gas at Unitarity

- Mean-field substitution, where $V_0\langle\hat{\psi}_\downarrow(\mathbf{x})\hat{\psi}_\uparrow(\mathbf{x})\rangle \equiv \Delta$, such that

$$V_0\hat{\psi}_\uparrow^\dagger(\mathbf{x})\hat{\psi}_\downarrow^\dagger(\mathbf{x})\hat{\psi}_\downarrow(\mathbf{x})\hat{\psi}_\uparrow(\mathbf{x}) \to \Delta^*\hat{\psi}_\downarrow(\mathbf{x})\hat{\psi}_\uparrow(\mathbf{x}) + \Delta\hat{\psi}_\uparrow^\dagger(\mathbf{x})\hat{\psi}_\downarrow^\dagger(\mathbf{x}) - \Delta^2/V_0$$

- Mean-field Hamiltonian,

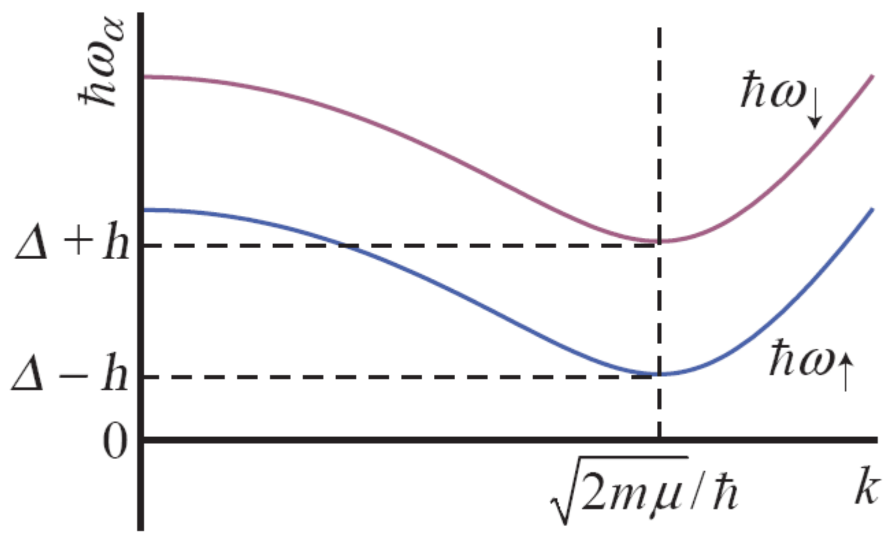
$$\frac{\hat{H}}{V} = \sum_{\mathbf{k}}\frac{\epsilon_{\mathbf{k}}-\mu_\downarrow}{V} + \frac{1}{V}\sum_{\mathbf{k}}\left[\hat{\psi}_{\mathbf{k},\uparrow}^\dagger, \hat{\psi}_{-\mathbf{k},\downarrow}\right]\begin{bmatrix}\epsilon_{\mathbf{k}}-\mu_\uparrow & \Delta \\ \Delta^* & \mu_\downarrow-\epsilon_{\mathbf{k}}\end{bmatrix}\begin{bmatrix}\hat{\psi}_{\mathbf{k},\uparrow} \\ \hat{\psi}_{-\mathbf{k},\downarrow}^\dagger\end{bmatrix} - \frac{|\Delta|^2}{V_0}$$

- The Bogoliubov quasi-particles,

$$\begin{aligned}\hat{\phi}_{\mathbf{k},\uparrow} &= u_{\mathbf{k}}\hat{\psi}_{\mathbf{k},\uparrow} - v_{\mathbf{k}}\hat{\psi}_{-\mathbf{k},\downarrow}^\dagger \\ \hat{\phi}_{-\mathbf{k},\downarrow}^\dagger &= v_{\mathbf{k}}\hat{\psi}_{\mathbf{k},\uparrow} + u_{\mathbf{k}}\hat{\psi}_{-\mathbf{k},\downarrow}^\dagger\end{aligned}$$

- The quasi-particle dispersions,

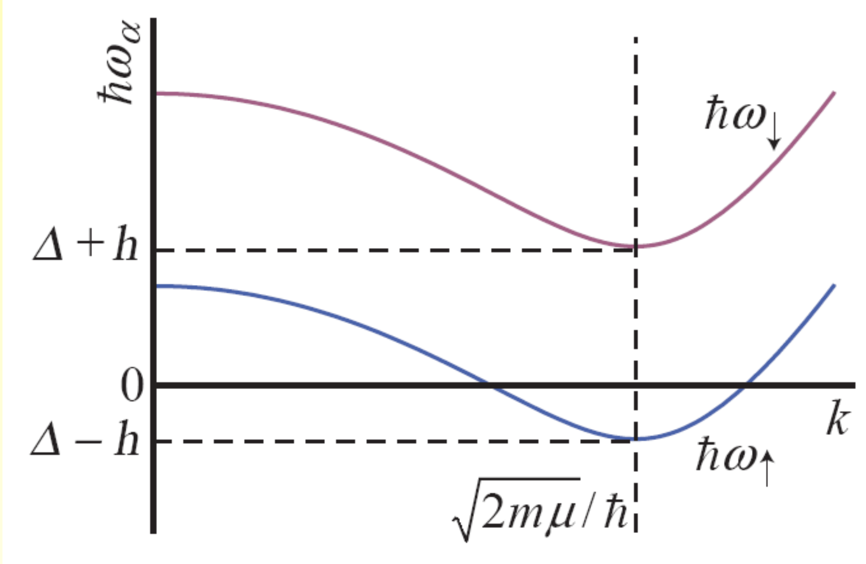
$$\hbar\omega_{\mathbf{k},\uparrow/\downarrow} = \mp h + \sqrt{(\epsilon_{\mathbf{k}}-\mu)^2+\Delta^2}$$



Sarma Phase

- BCS ground state energy and ideal gas of quasi-particles

$$\frac{\hat{H}}{V} = -\frac{\Delta^2}{V_0(\Lambda)} + \frac{1}{V}\sum_{\mathbf{k},\alpha}\left\{-\hbar\omega_{\mathbf{k},\alpha}v_{\mathbf{k}}^2 + \hbar\omega_{\mathbf{k},\alpha}\hat{\phi}^\dagger_{\mathbf{k},\alpha}\hat{\phi}_{\mathbf{k},\alpha}\right\}$$

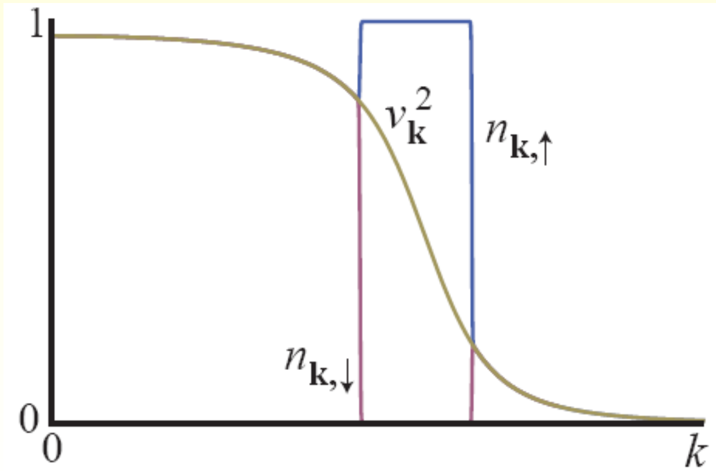


- In principle, majority becomes gapless, when $\Delta = h$.

- Then, ground state becomes gapless polarized superfluid.

- Occupation numbers:

$$\langle\hat{\psi}^\dagger_{\mathbf{k},\alpha}\hat{\psi}_{\mathbf{k},\alpha}\rangle \equiv n_{\mathbf{k},\alpha}$$



- Typically, Sarma phase is unstable at zero temperature.

Thermodynamics

- Thermodynamic potential density

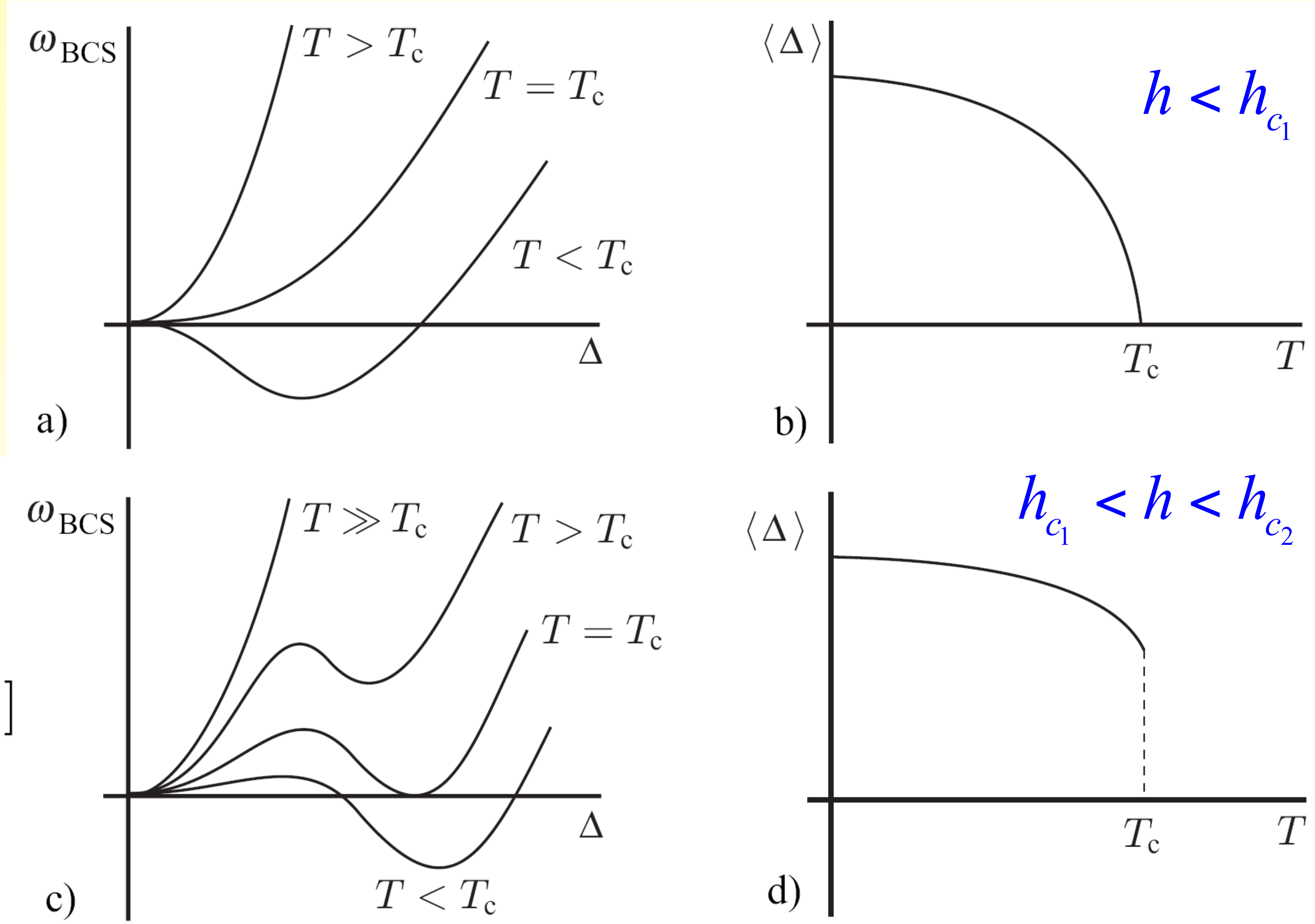
$$\omega_{\mathrm{BCS}} = -\frac{\Delta^2}{V_0(\Lambda)} + \frac{1}{V}\sum_{\mathbf{k}}\{\hbar\omega_{\mathbf{k}} - \epsilon_{\mathbf{k}} + \mu\} - \frac{1}{\beta V}\sum_{\mathbf{k},\alpha}\log\left(1 + e^{-\beta\hbar\omega_{\mathbf{k},\alpha}}\right).$$

-Second-order transition:

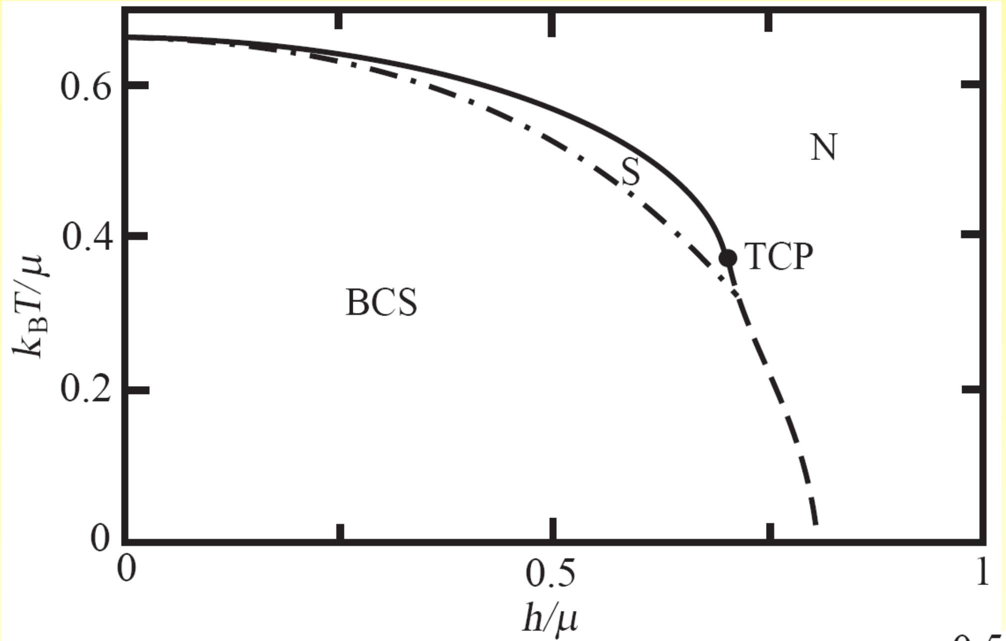
$$\left.\frac{\partial\omega_{\mathrm{BCS}}[\Delta]}{\partial\Delta^2}\right|_{\Delta=0} = 0$$

-First-order transition:

$$\omega_{\mathrm{BCS}}[0] = \omega_{\mathrm{BCS}}[\langle\Delta\rangle]$$



Homogeneous Phase Diagram



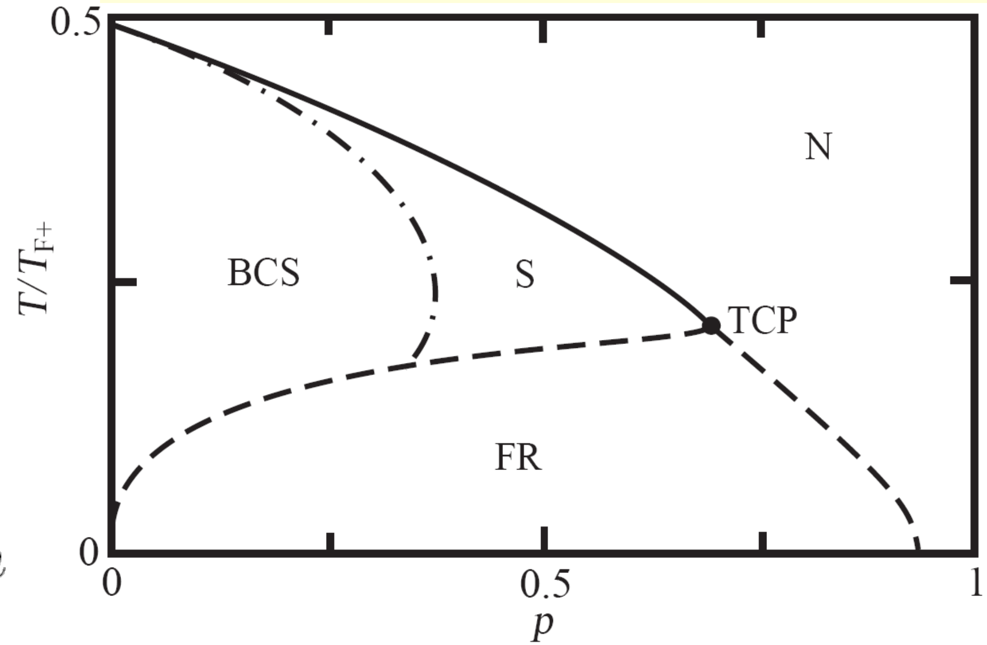
- Crossover from fully gapped BCS superfluid to gapless Sarma (S) superfluid, when $\Delta = h$

- Forbidden region (FR) gives rise to phase separation.

- (Local) polarization: $p = \dfrac{n_\uparrow - n_\downarrow}{n_\uparrow + n_\downarrow}$

- Fermi temperature:

$$k_\mathrm{B} T_{\mathrm{F},\alpha} \equiv \epsilon_{F,\alpha} = \hbar^2 (6\pi^2 n_\alpha)^{2/3} / 2m$$

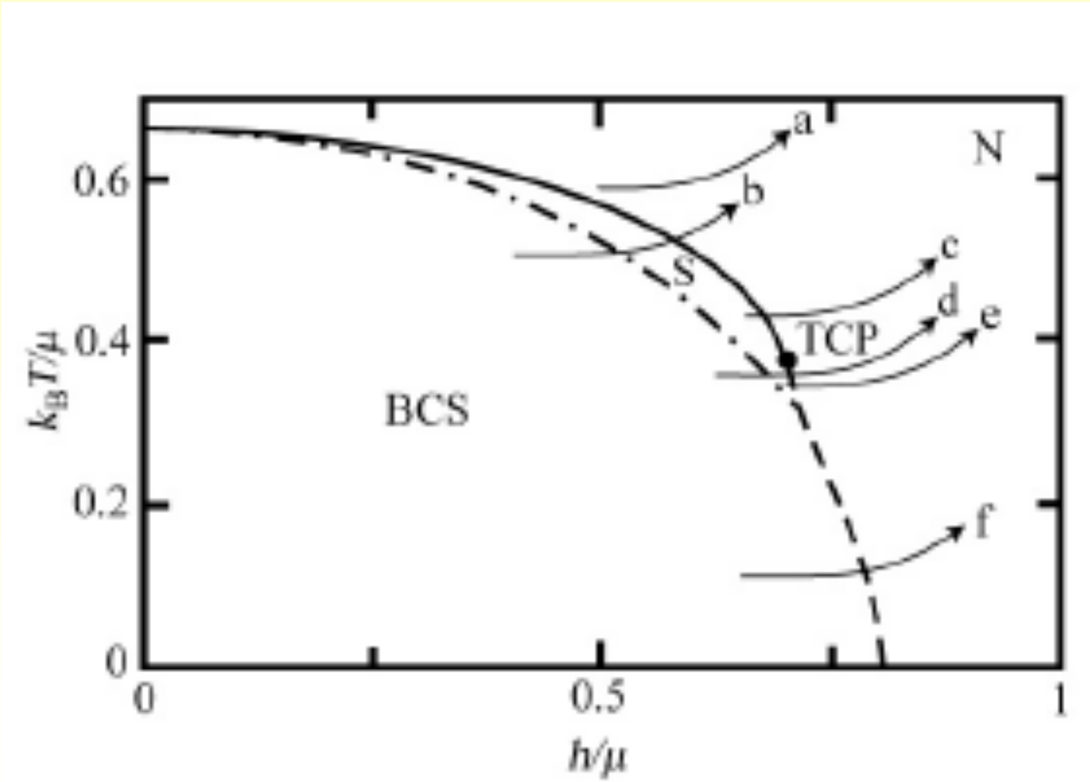


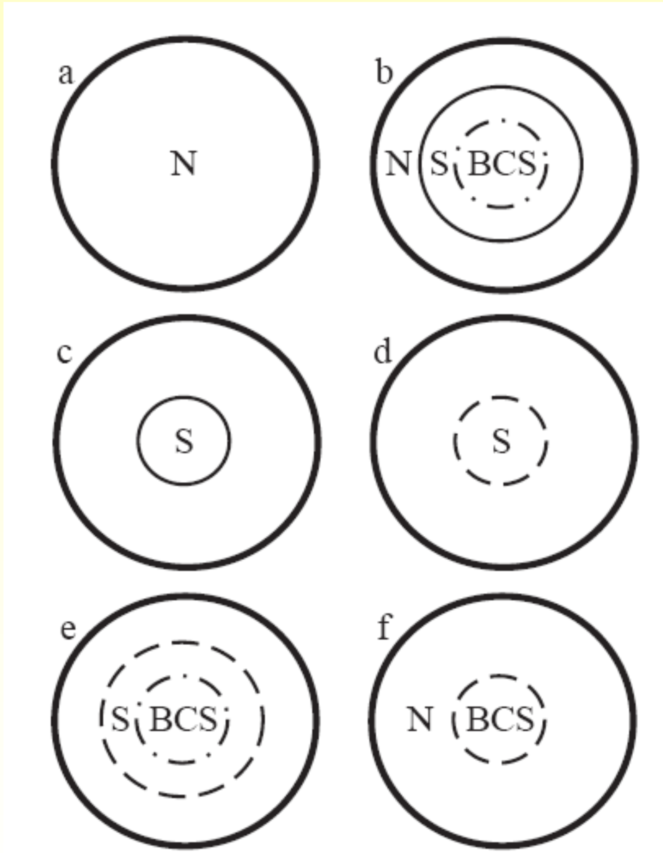
The Local-Density Approximation (LDA)

-Trapping potential: $V_{\text{trap}}(\mathbf{r}) = \frac{1}{2} m \omega_{\text{trap}}^2 r^2$

- LDA : $\mu_\alpha(\mathbf{r}) \equiv \mu_\alpha - V_{\text{trap}}(\mathbf{r})$

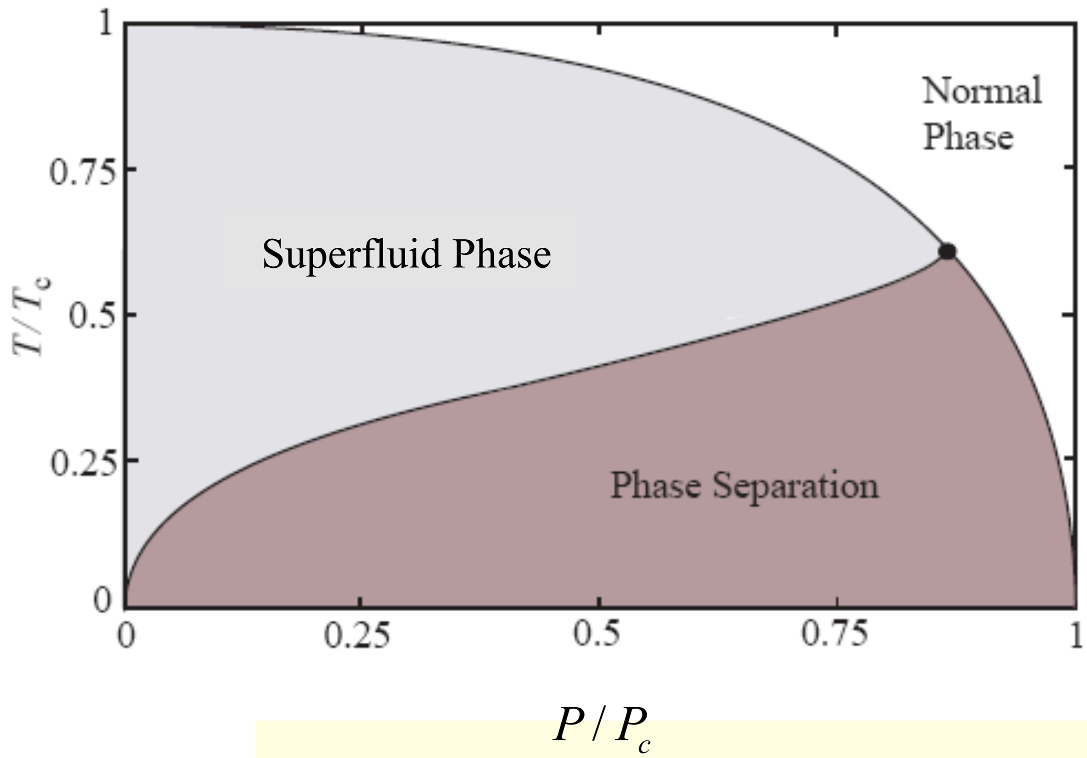
- In the trap: decreasing $\mu(\mathbf{r})$, constant *h*.

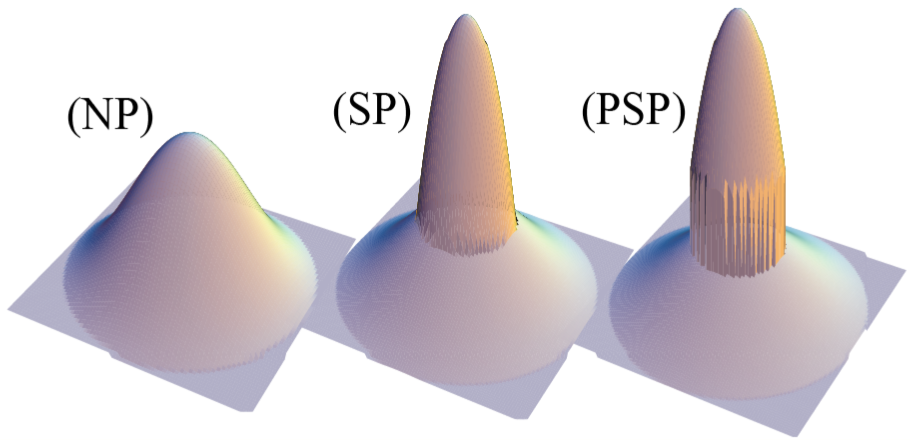




Phase Diagram in a Trap

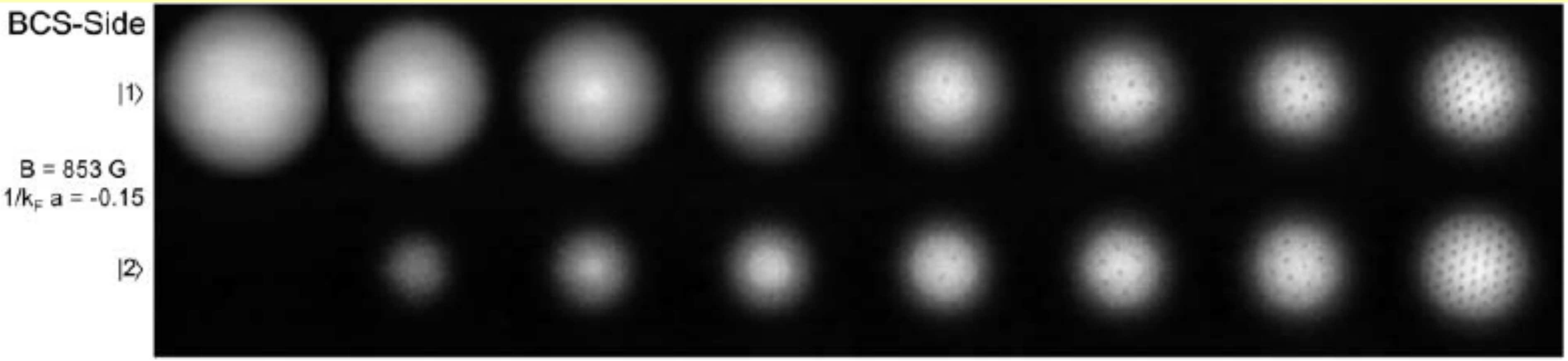
- Superfluid phase: 2nd order transition in the trap.
- Phase separation: 1st order transition in the trap.
- Normal phase: normal throughout trap.

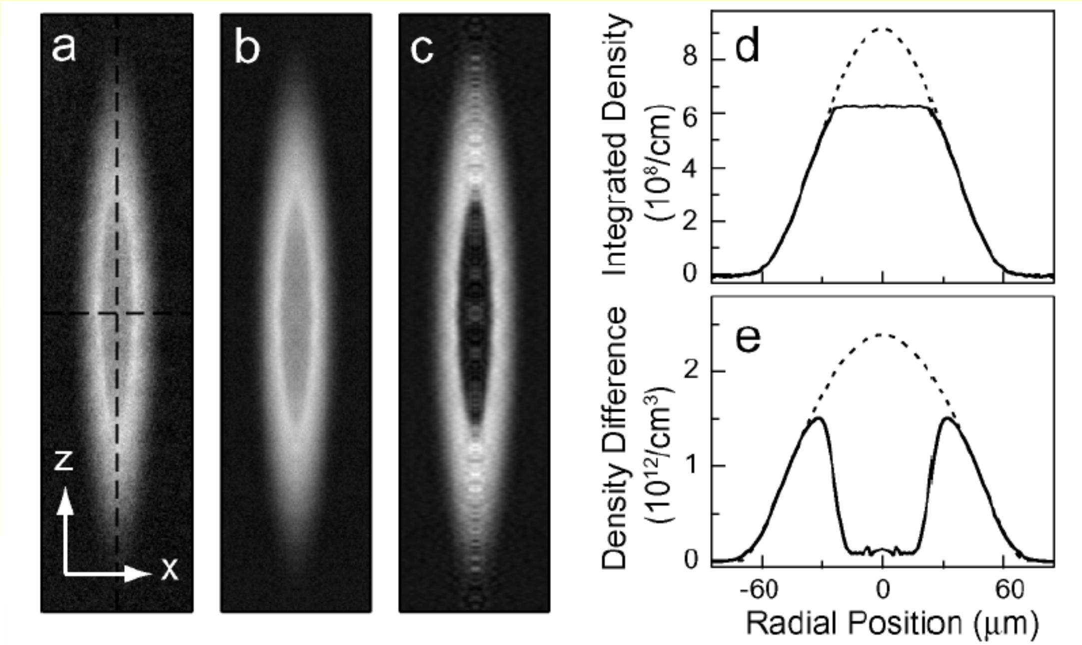




(Global) polarization: $P = \frac{N_\uparrow - N_\downarrow}{N_\uparrow + N_\downarrow}$

‘Old’ MIT Experiments



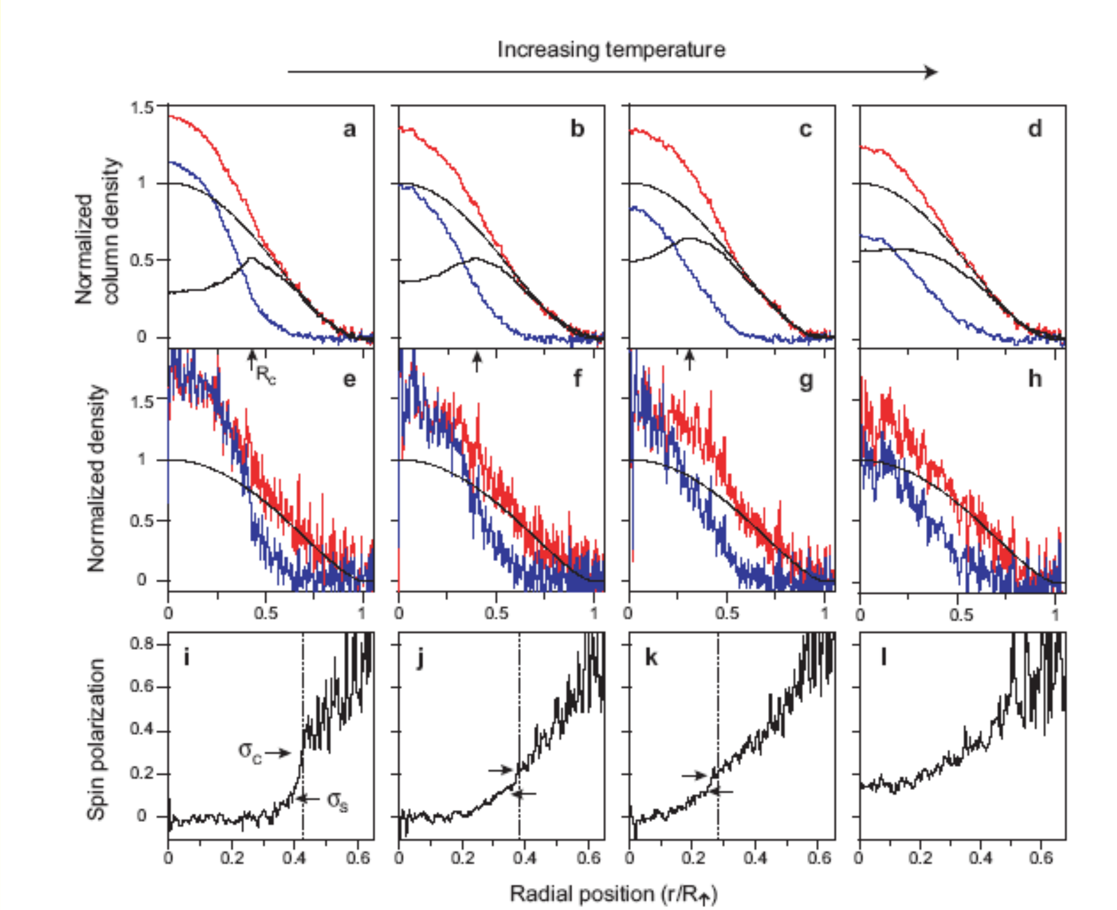


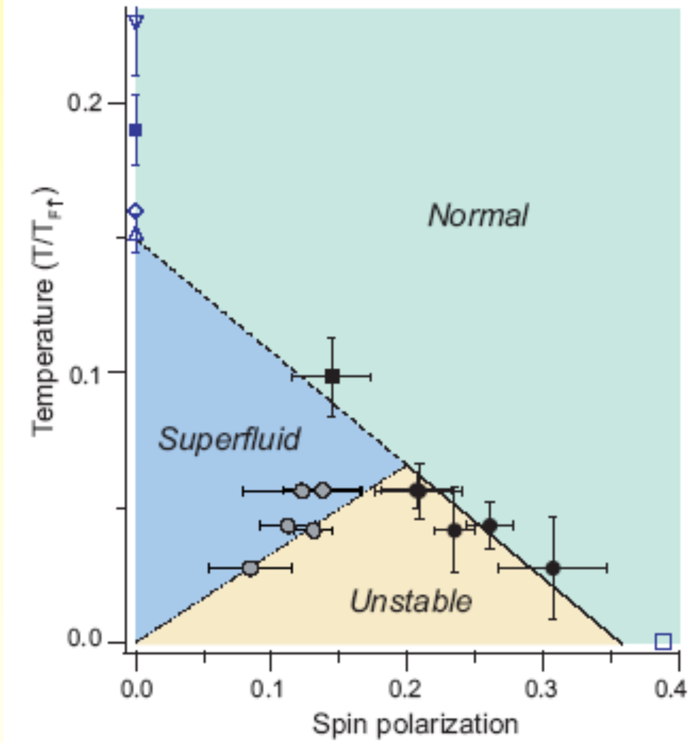
Shell structure with fully paired core and normal outer region.

1^{st} or 2^{nd} order?

‘New’ MIT Experiment: Homogeneous Phase Diagram

- Measurements locally in the trap. Now MIT also experimentally shows phase separation.

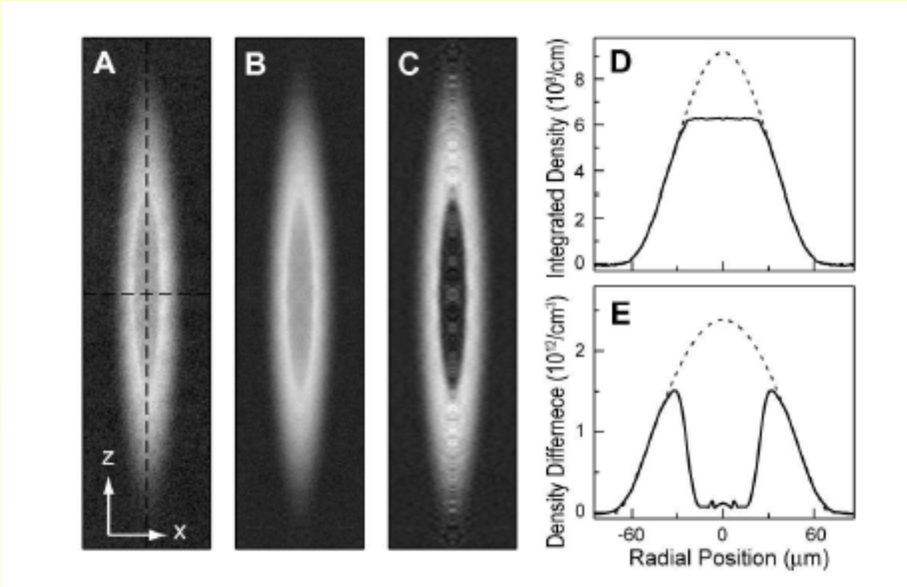




Shin *et al.*, Nature **451**, 689 (2008)

The Rice Experiments

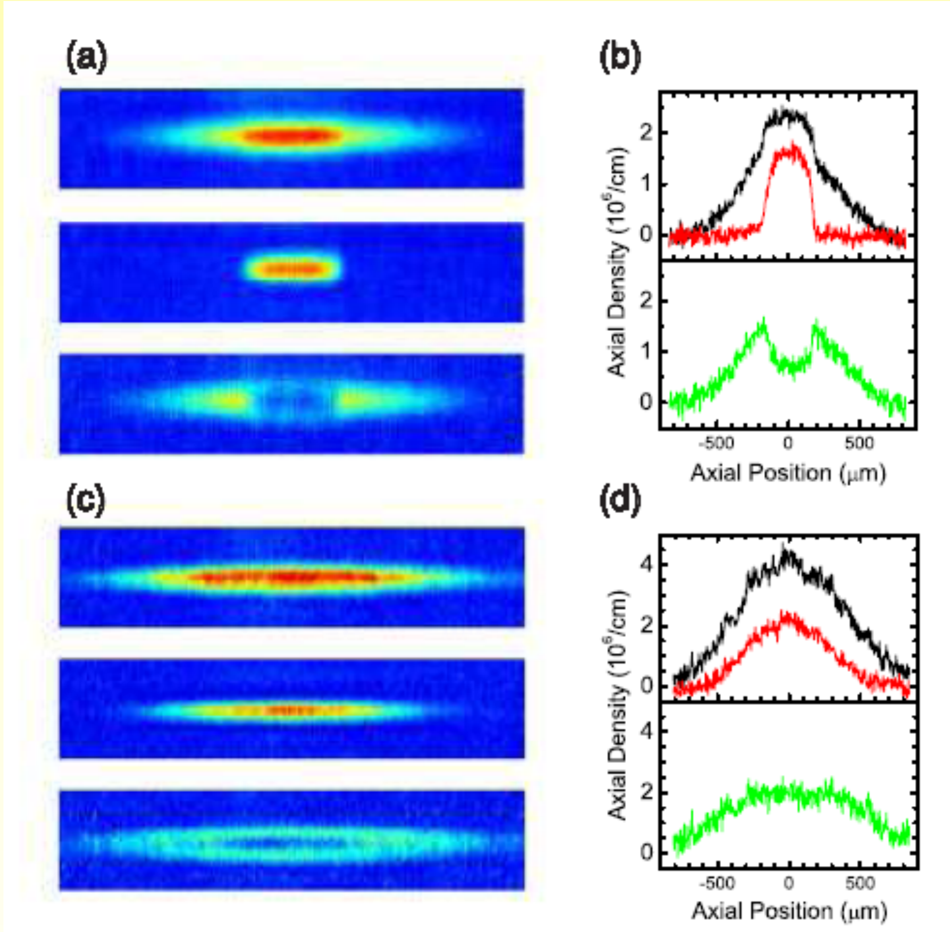
• Typical gas clouds with super-fluid core at MIT.



Shin *et al.* , PRL **97**, 030401 (2006)

• No deformation

• Gas clouds at lowest and higher temperature of Rice

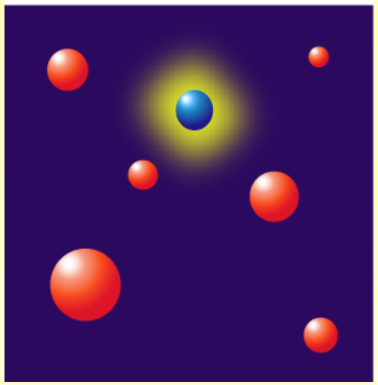


Partridge *et al.*, PRL **97**, 190407 (2006)

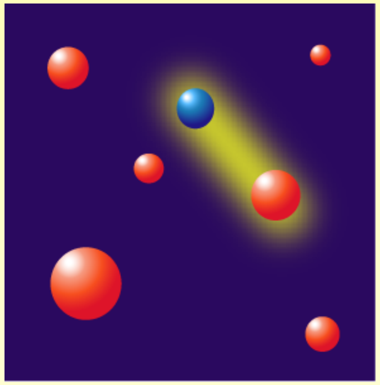
a) Deformation c) No deformation

Zero *T*, Unitary, Normal Phase: MC Equation of State

- Spin-down particle in sea of spin-up particles: fermion or Cooper pair?

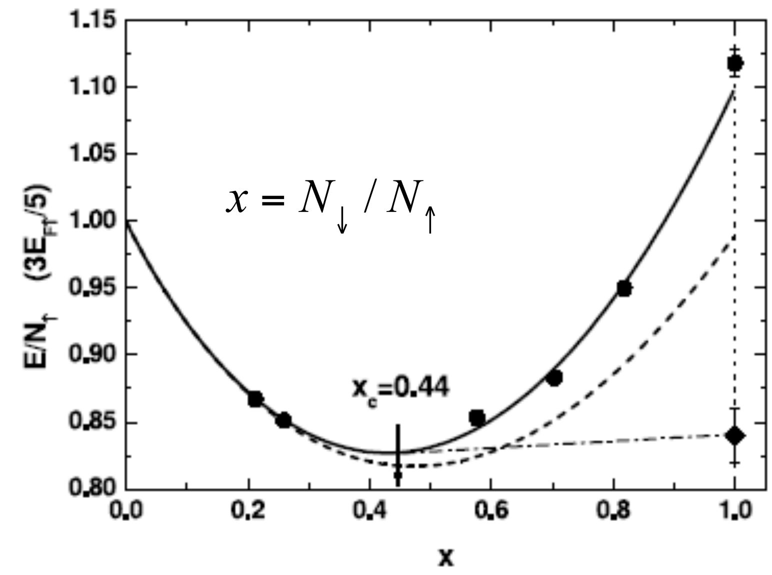


or



?

Quantum Monte Carlo equation of state



Lobo *et al.* , PRL **97**, 200403 (2006)

- Dashed line

$$E(N_\uparrow, N_\downarrow) = \frac{3}{5} E_{F,\uparrow} N_\uparrow + \frac{3}{5} E_{F,\downarrow} N_\downarrow - 0.6 E_{F,\uparrow} N_\downarrow$$

Remember! $h\Sigma_\downarrow = -0.6 E_{F,\uparrow}$

- Quantum phase transition at
  $P = 0.39$ (homogeneous) and
  $P = 0.78$ (trap with LDA)!

Renormalization Group Approach

- Integrate out modes in high-momentum shell Λ of width dΛ. Absorb result in couplings. Integrate out new shell, etc.

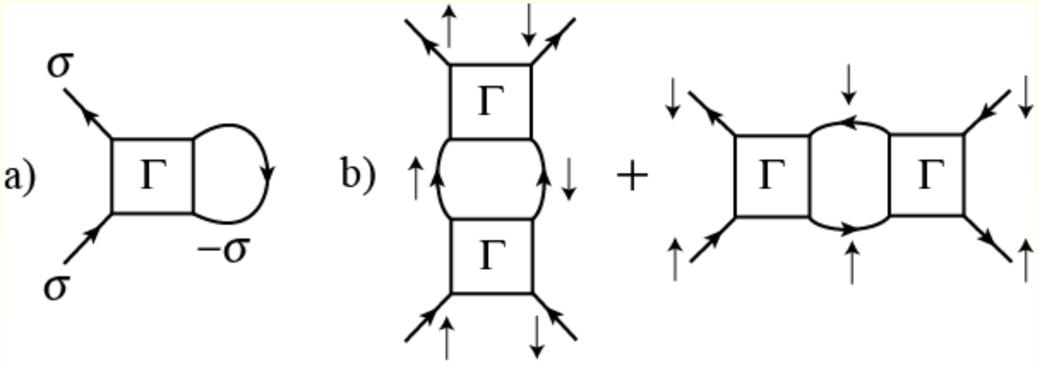
- Use RG as (non-perturbative) method to solve iteratively many-body problem.

- Starting point. The microscopic action

$$S = \sum_{\mathbf{k},n} \phi^{*}_{\sigma,\mathbf{k},n}(-i\hbar\omega_n - \varepsilon_{\mathbf{k}} - \mu_{\sigma})\phi_{\sigma,\mathbf{k},n} + \sum_{\substack{\mathbf{q},\mathbf{k},\mathbf{k}',\\ m,n,n'}} \Gamma_{\mathbf{q},m}\phi^{*}_{\uparrow,\mathbf{q}\text{-}\mathbf{k},m-n}\phi^{*}_{\downarrow,\mathbf{k},n}\phi_{\downarrow,\mathbf{k}',n'}\phi_{\uparrow,\mathbf{q}\text{-}\mathbf{k}',m-n'}$$

- Technically, we have to calculate one-loop diagrams.

- Infinitesimal width makes higher-loop diagrams vanish.

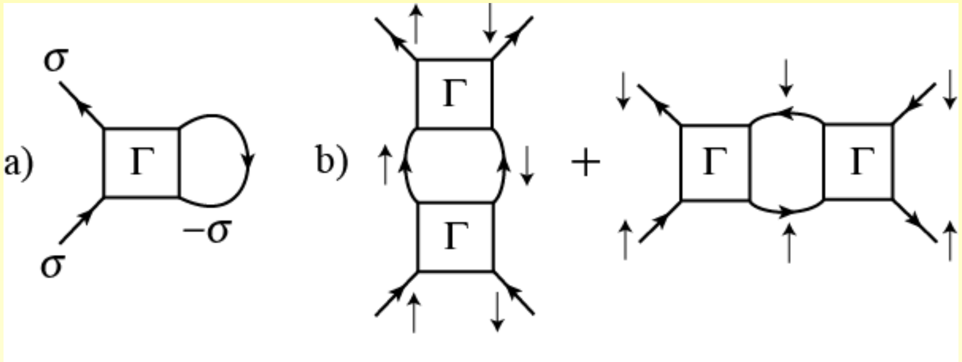


RG Theory for Imbalanced Fermi Gas

- Integrate out momenta in a shell Λ of infinitesimal width dΛ.
  Renormalization of chemical potentials determines fermionic self-energy.

$$d\mu_\sigma = -\frac{\Lambda^2}{2\pi^2}\Gamma_{\mathbf{0},0}N_{-\sigma}d\Lambda$$

Due to infinitesimal width higher loop diagrams vanish!



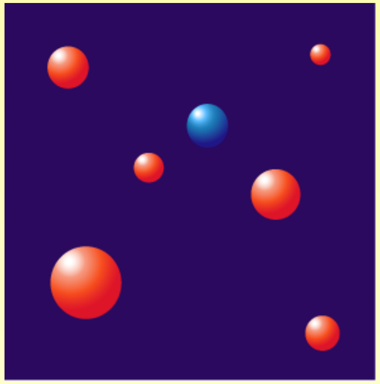
- Interaction: 'ladder diagram' (scattering of particles), 'bubble diagram' (screening by particle-hole excitations). Coupled diff. equations!

$$d\Gamma^{-1}_{\mathbf{0},0} = \frac{\Lambda^2}{2\pi^2}\left(\frac{1-N_\uparrow-N_\downarrow}{2(\varepsilon_\Lambda-\mu)}-\frac{N_\uparrow-N_\downarrow}{2h}\right)d\Lambda \ , \quad N_\sigma = \frac{1}{e^{\beta(\varepsilon_\Lambda-\mu_\sigma)}+1} \quad , \quad \varepsilon_\Lambda = \frac{\hbar^2\Lambda^2}{2m}$$

- Phase transition: $\Gamma_{\mathbf{0},0}(\infty)$ diverges (Thouless criterion). Self-energies diverge. Unphysical! CM-momentum/frequency dependence important!

$$\Gamma^{-1}_{\mathbf{q},m} = \Gamma^{-1}_{\mathbf{0},0} - Z_q^{-1}q^2 + Z_\omega^{-1}i\hbar\omega_m$$

RG Theory for *T*=0, Unitary, Extremely Imbalanced Gas



- One spin-down particle in a sea of spin up particles. Density of spin-down particles is zero. Self energy due to strong interactions (unitarity limit).

$$d\Gamma_{\mathbf{0},0}^{-1} = \frac{\Lambda^2}{2\pi^2}\left(\frac{1-N_\uparrow}{2(\varepsilon_\Lambda - \mu_\downarrow)} - \frac{N_\uparrow}{2h}\right) d\Lambda$$

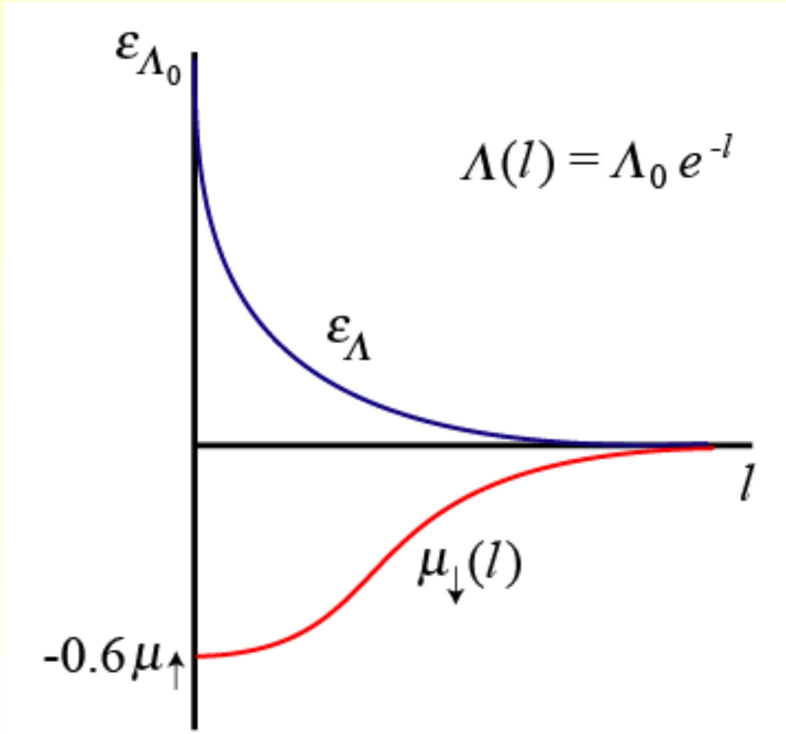
$$d\mu_\downarrow = -\frac{\Lambda^2}{2\pi^2(\Gamma_{\mathbf{0},0}^{-1} - Z_q q^2)} N_\uparrow d\Lambda$$

- QPT from zero to nonzero down-density at

$$E_{F,\downarrow} = \mu_\downarrow(\infty) = 0 = \mu_\downarrow(0) - \mathrm{h}\Sigma_\downarrow$$

$$\mathrm{h}\Sigma_\downarrow = -0.6\mu_\uparrow = -0.6E_{F,\uparrow}$$

- Crucial to let chemical potential flow!



RG Theory: Weakly Interacting, Balanced Fermi Gas

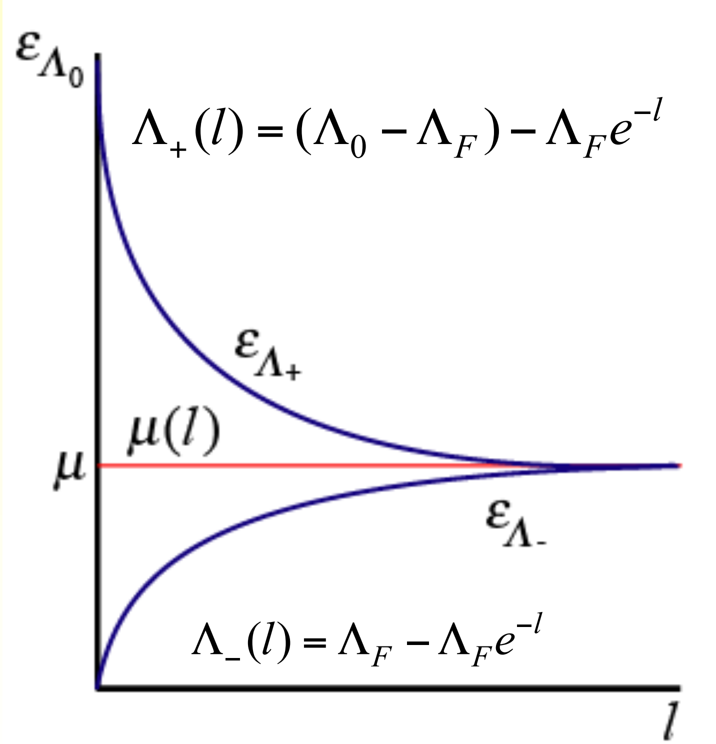
- In the extremely weakly interacting limit the chemical potentials don't renormalize anymore, i.e. the selfenergies go to zero

$$d\Gamma^{-1}_{\mathbf{0},0} = \frac{\Lambda^2}{2\pi^2}\left(\frac{1-2N}{2(\varepsilon_\Lambda-\mu)} - \beta N(1-N)\right)d\Lambda$$

- Differential form of gap equation with Gorkov's correction.

- Flow to (stationary) Fermi surface. Natural endpoint because here excitations of lowest energy.

- Exactly solvable! Leads to the BCS transition temperature reduced by a factor of e (with relative momentum it is 2.2)

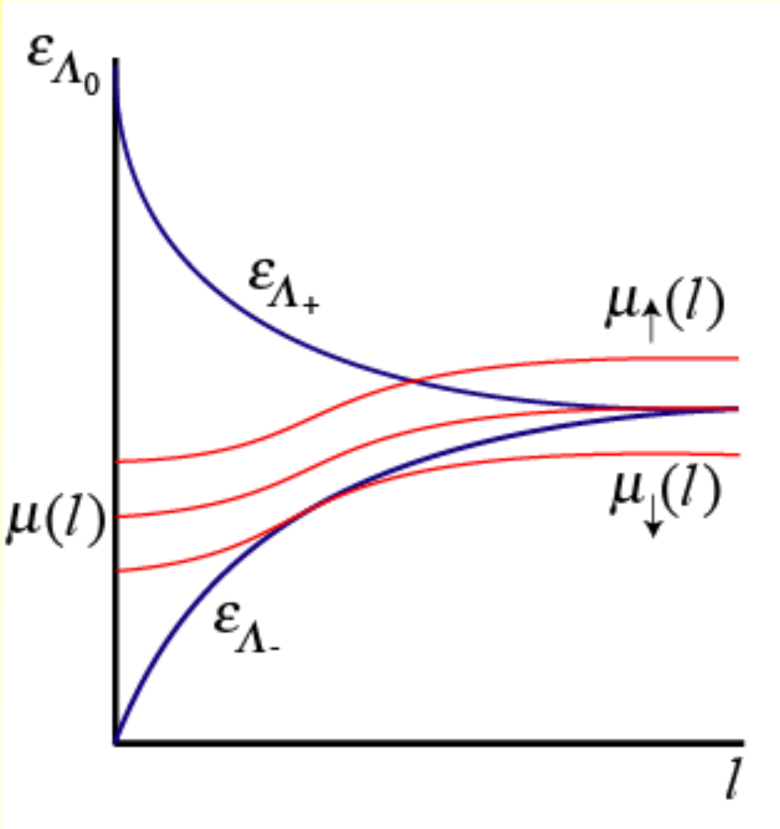


RG Theory: Unitarity Limit, Imbalanced Case

- Three Fermi levels in the system! But only one pole in RG equations

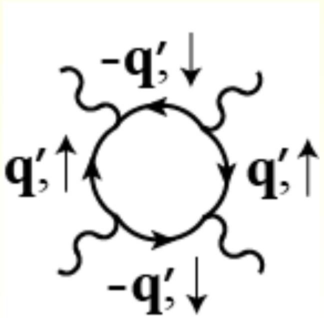
$$d\Gamma^{-1}_{\mathbf{0},0} = \frac{\Lambda^2}{2\pi^2}\left(\frac{1-N_\uparrow - N_\downarrow}{2(\varepsilon_\Lambda - \mu)} - \frac{N_\uparrow - N_\downarrow}{2h}\right)d\Lambda$$

$$d\mu_\sigma = -\frac{\Lambda^2}{2\pi^2(\Gamma^{-1}_{\mathbf{0},0} - Z_q q^2)}N_{-\sigma}d\Lambda$$



- Flow automatically to average Fermi level

- Tricritical point determined by following class of Feynman diagram



$$F_L = a_0 + a_2\Delta^2 + a_4\Delta^4 + \ldots$$

$$a_2 = a_4 = 0$$

- Flowing of $\mu(l)$ crucial, since $\mu(0)/\mu(\infty) = 1 + \beta$

Results

RG equations:

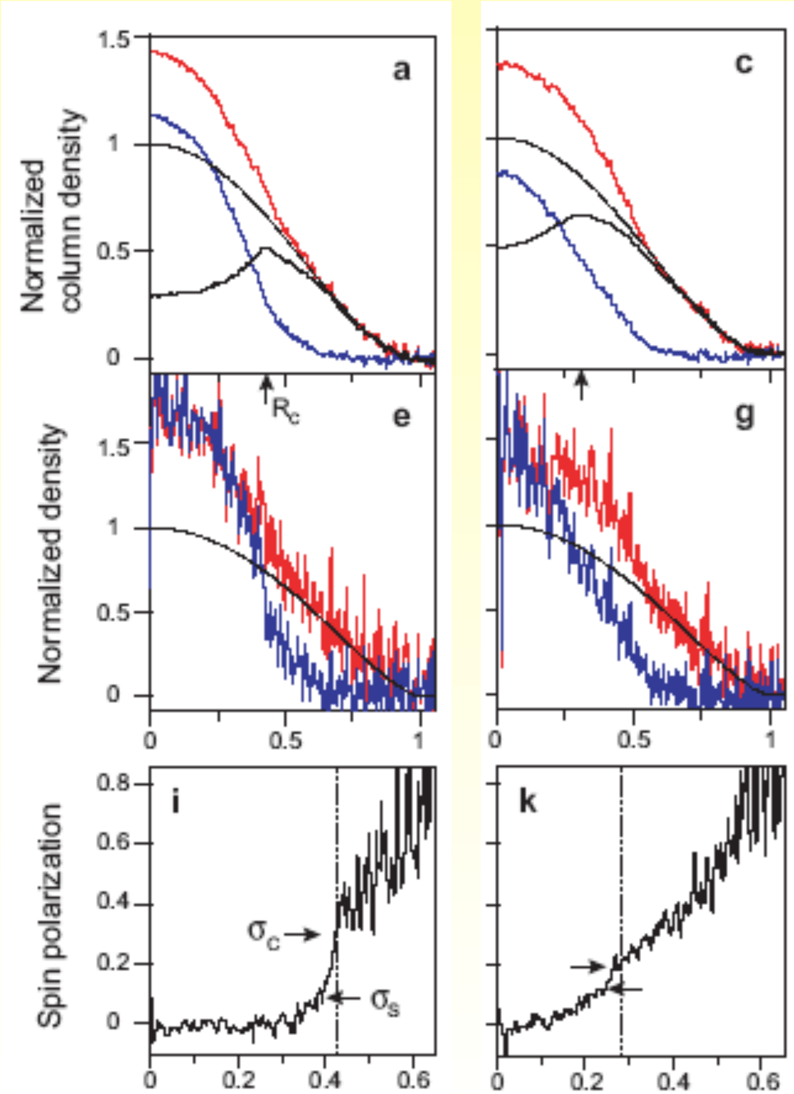
$$d\Gamma^{-1}_{\mathbf{0},0} = \frac{\Lambda^2}{2\pi^2}\left(\frac{1-f_\uparrow - f_\downarrow}{2(\varepsilon_\Lambda - \mu)} - \frac{f_\uparrow - f_\downarrow}{2h}\right) d\Lambda,$$

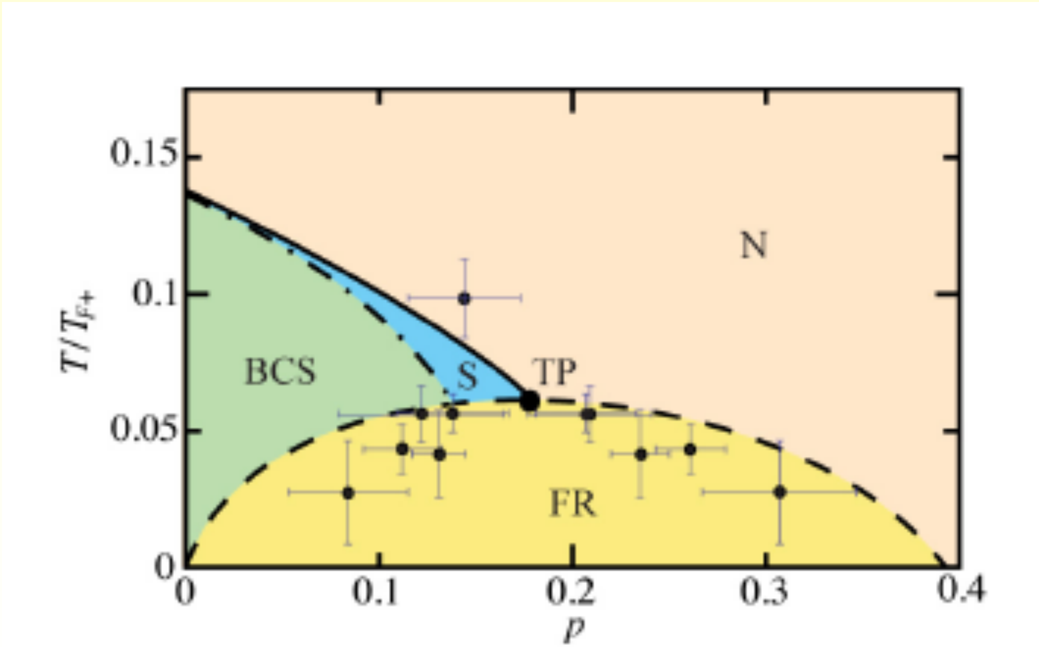
$$d\mu_\sigma = -\frac{\Lambda^2}{2\pi^2(\Gamma^{-1}_{\mathbf{0},0} - Z_q q^2)} f_{-\sigma} d\Lambda,$$

with $\Gamma^{-1}_{\mathbf{q},m} = \Gamma^{-1}_{\mathbf{0},0} - Z_q^{-1} q^2$,

and $f_\sigma = \dfrac{1}{e^{\beta(\varepsilon_\Lambda - \mu_\sigma)} + 1}$.

Experiments:





Conclusion and Outlook I

- Ultracold quantum gases are ideal quantum simulators. The field is able to address fundamental questions on many-body quantum systems in great detail.

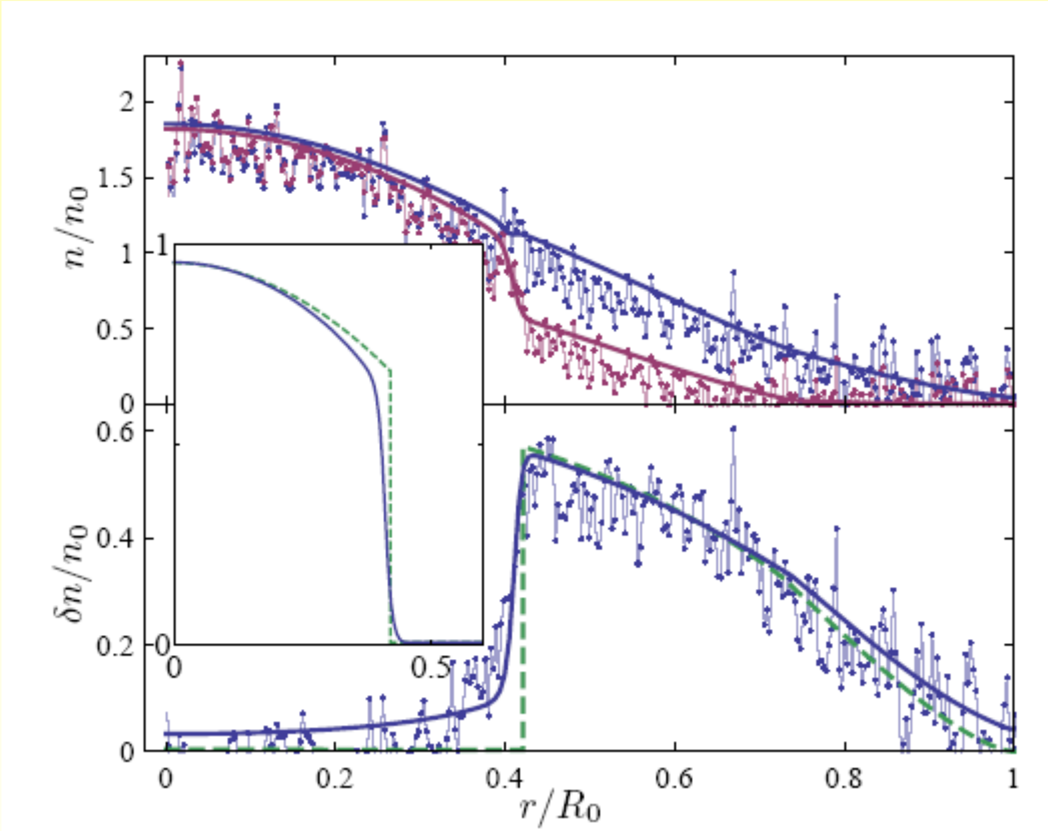
- A first good example is the detailed study of the BEC-BCS crossover, which gives a unified view on BEC- and BCS-like superfluidity.

- A second example is the study of the strongly interacting Fermi mixture with a population imbalance, whose phase diagram is important to condensed matter, nuclear matter and astrophysics.

- Many more examples are under way. Fermi mixtures with a mass imbalance, with long-ranged anisotropic dipole interactions, the doped fermionic Hubbard model, etc. We recently worked on:

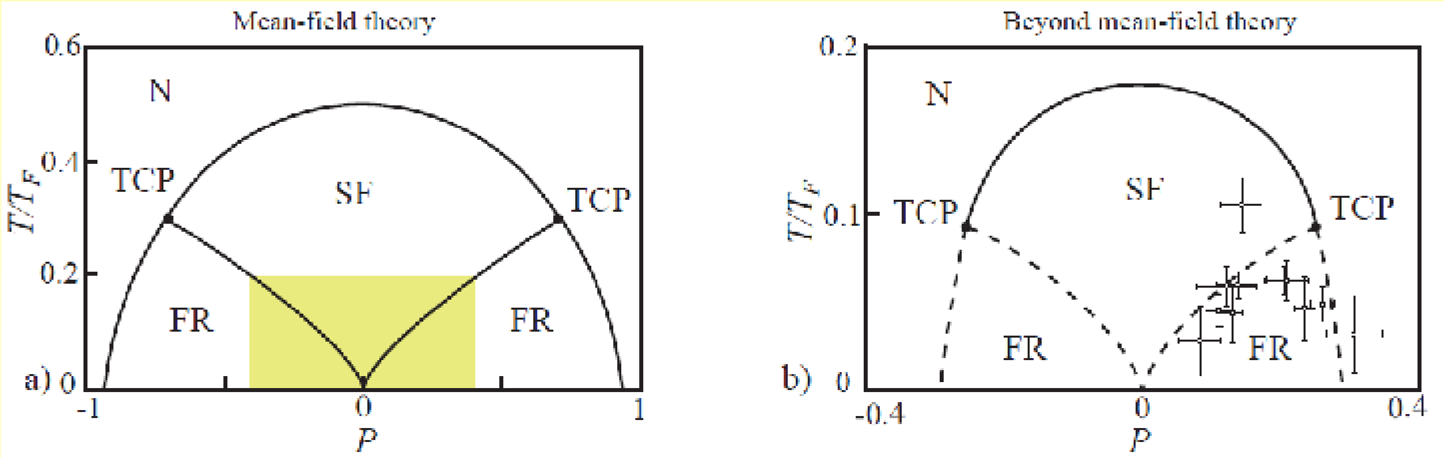
Conclusion and Outlook II

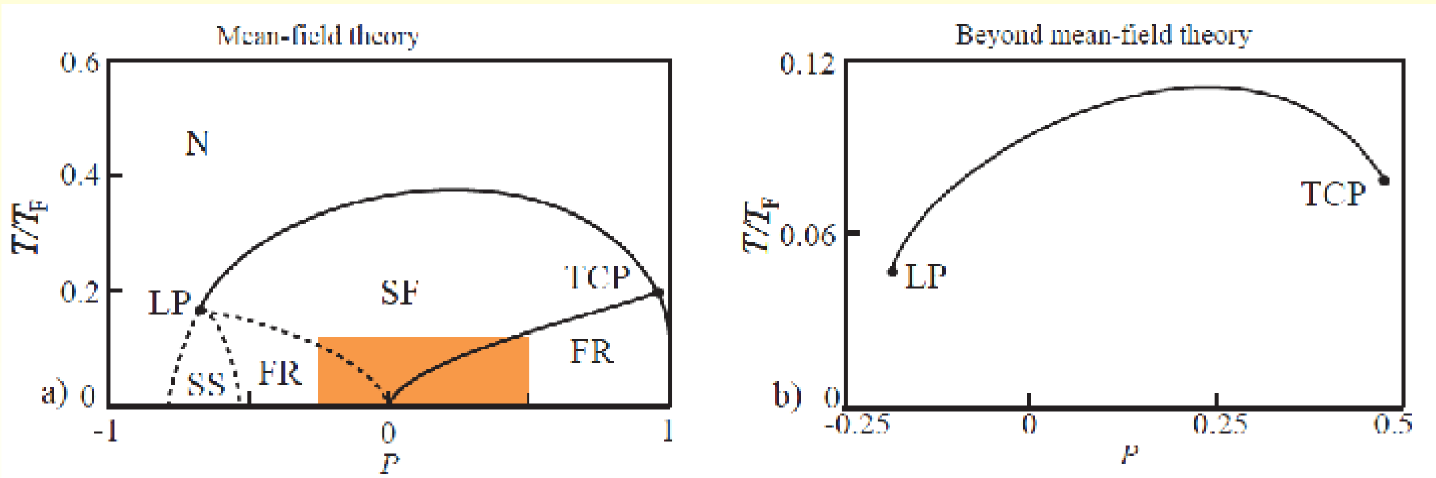
- Superfluid-normal interface and surface tension:



Conclusion and Outlook III

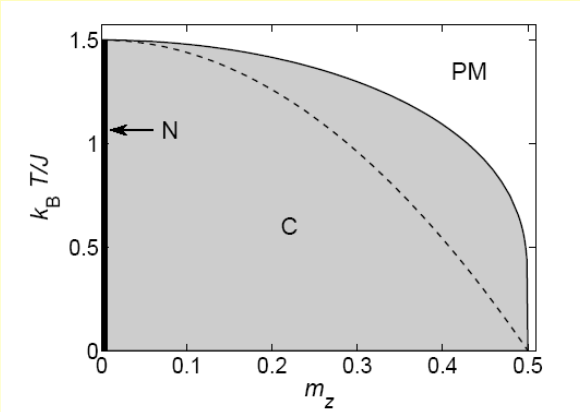
• Mass imbalance (^{6}Li-^{40}K) at unitarity:

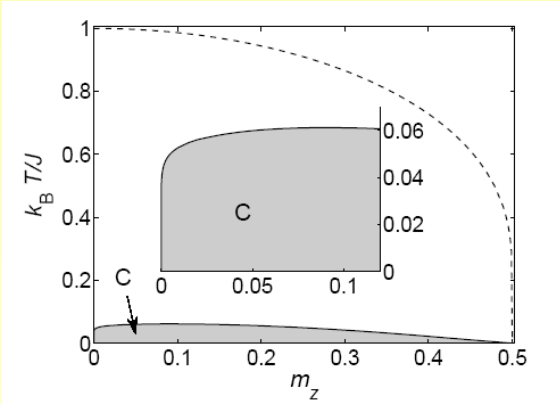


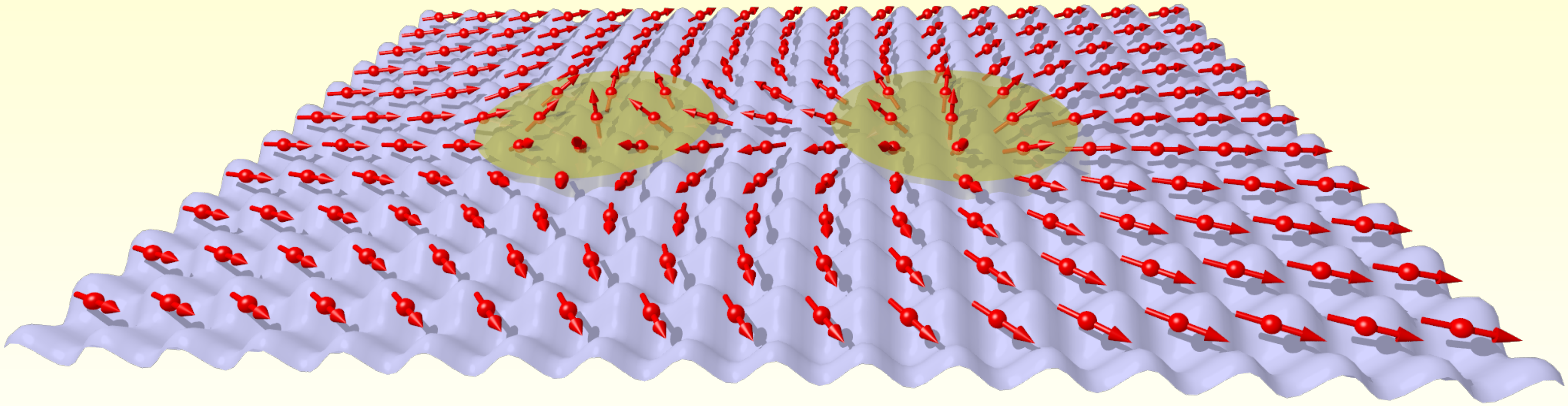


Conclusion and Outlook IV

• Imbalanced antiferromagnet:

3D: 

2D: 



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