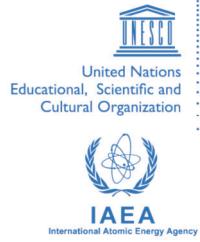




**The Abdus Salam
International Centre for Theoretical Physics**



2354-11

Summer School on Cosmology

16 - 27 July 2012

Introduction to Cosmology

S. Dodelson
FNAL & U. Chicago

Introduction to Cosmology: Smooth Universe

I. EVOLUTION OF THE SCALE FACTOR

- Einstein-Hilbert Action \leftrightarrow Einstein equations

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi GT_{\mu\nu}. \quad (1)$$

- Friedmann-Robertson-Walker metric

$$g_{\mu\nu} = \text{diag} \left[-1, a^2(t), a^2(t), a^2(t) \right] \quad (2)$$

- Friedmann Equation

$$H^2 = \frac{8\pi G\rho}{3} \quad (3)$$

- Acceleration

Space-space gives:

$$-2\frac{\ddot{a}}{a} = \frac{8\pi G}{3}\rho + 8\pi GP \quad (4)$$

Subtract twice this from Friedmann to get

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3} [\rho + 3P]. \quad (5)$$

- $f(R)$ Modification

$$S \rightarrow \frac{1}{16\pi G} \int d^4x \sqrt{-g} [R + f(R)] \quad (6)$$

results in acceleration equation

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3} [\rho + 3P] + f_R H^2 - \frac{f}{6} - \frac{\ddot{f}_R}{2}. \quad (7)$$

II. COSMOLOGICAL MODEL/INVENTORY

Express contribution from each species in units of the critical density $\rho_c \equiv 3H_0^2/8\pi G$, a constant.

- Photons

$$\Omega_\gamma = 2.47 \times 10^{-5} h^{-2}. \quad (8)$$

- Baryons

$$\Omega_b = 0.0226 h^{-2}. \quad (9)$$

- Cold Dark Matter

$$\Omega_b = 0.11 h^{-2}. \quad (10)$$

- Neutrinos

Massless:

$$\Omega_\nu = 0.68 \Omega_\gamma \quad (11)$$

Massive:

$$\Omega_\nu = 0.01 \left(\frac{\sum m_\nu}{1 \text{ eV}} \right). \quad (12)$$

- Dark Energy

$$\Omega_{de} = 0.73. \quad (13)$$

Equation of state $w \simeq -1$.

III. ENERGY-MOMENTUM CONSERVATION

-

$$T^\mu{}_{\nu;\mu} = \frac{\partial T^\mu{}_\nu}{\partial x^\mu} + \Gamma^\mu{}_{\alpha\mu} T^\alpha{}_\nu - \Gamma^\alpha{}_{\nu\mu} T^\mu{}_\alpha = 0 \quad (14)$$

- Diagonal stress-energy tensor

$$\frac{d \ln(\rho)}{dt} = -3H(1+w) \quad (15)$$

where $w \equiv P/\rho$.

IV. DISTANCES

A. Comoving Distance

$$\chi(z) = \int_0^z \frac{dz'}{H(z')} \quad (16)$$

B. Comoving Horizon

$$\eta(a) = \int_0^a \frac{da'}{a'} \frac{1}{a' H(a')} \quad (17)$$

C. Angular Diameter Distance

$$d_A = a\chi \quad (18)$$

D. Luminosity Distance

$$d_L = \frac{\chi}{a} \quad (19)$$

E. Example

Two SN 1992p ($m_1 = 16.08$; $z_1 = 0.026$) and 1997ap ($m_2 = 24.32$; $z_2 = 0.83$):

$$d_L(z_2) = 3470 h^{-1} \text{ Mpc} \quad (20)$$

as compared with the value for $\Omega_m = 1$: $d_L(z_2) = 2850 h^{-1}$ Mpc.

V. THERMAL HISTORY

Basic equation for $1 + 2 \leftrightarrow 3 + 4$:

$$\frac{1}{n_1^{(0)} a^3} \frac{d(n_1 a^3)}{d \ln(a)} = \frac{n_2^{(0)} \langle \sigma v \rangle}{H} \left\{ \frac{n_3 n_4}{n_3^{(0)} n_4^{(0)}} - \frac{n_1 n_2}{n_1^{(0)} n_2^{(0)}} \right\} \quad (21)$$

with

$$n_i^{(0)} \equiv g_i \int \frac{d^3 p}{(2\pi)^3} e^{-E_i/T} \rightarrow \begin{cases} g_i \left(\frac{m_i T}{2\pi} \right)^{3/2} e^{-m_i/T} & T \ll m_i \\ g_i \frac{T^3}{\pi^2} & T \gg m \end{cases} \quad (22)$$

A. Small Reaction Rate

In the limit $\langle\sigma v\rangle \ll H$, $n_1 \propto a^{-3}$.

- Neutrinos after $T \sim 5$ MeV
- Dark Matter after $T \sim m/20$
- Neutrons and protons after $T \sim 0.7$ MeV
- Free electrons after recombination

B. Large Reaction Rate

In the limit $\langle\sigma v\rangle \gg H$, two terms on the right hand side must vanish so densities stay in equilibrium

- Neutrinos before $T \sim 5$ MeV have $n_\nu \sim T^3$
- Dark Matter before $T \sim m/20$ falls exponentially as $n \propto (mT)^{3/2} e^{-m/T}$
- Neutrons and protons before $T \sim 0.7$ MeV : $n/p \propto e^{-\Delta m/T}$
- Free electrons before/during recombination (Saha):

$$\frac{x_e^2}{1 - x_e} = \frac{1}{n_b} \left(\frac{m_e T}{2\pi} \right)^{3/2} e^{-\epsilon_0/T} \quad (23)$$