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# Clusters of Galaxies - Lecture 1

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# Halo Abundance as a Cosmological Tool

The evolution of collapsed structures provides a range of cosmological constraints, including on the expansion history, the growth rate of structure and the nature of the underlying density field. Recent results suggest that systematic uncertainties can be controlled, but there remain challenges to scaling these studies up to large numbers of clusters.

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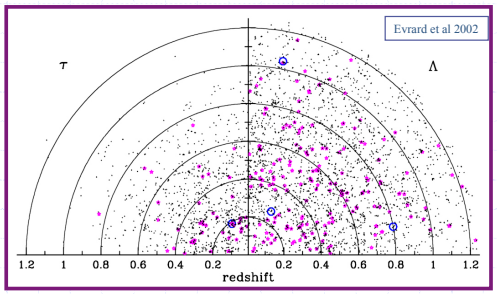
## Galaxy Cluster Topics

- Halo abundance as cosmological constraints
- Galaxy clusters and galaxy cluster surveys
- Cosmological constraints from cluster surveys
- Future Prospects



## Halos (Galaxy Clusters) Trace the Large Scale Structure

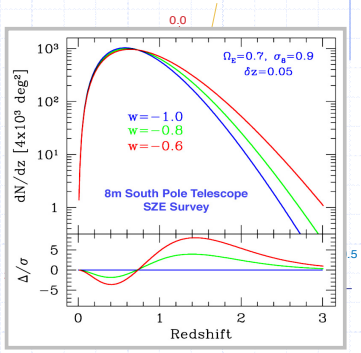
- Galaxy clusters are excellent tracers of structure formation
- A galaxy cluster survey is a powerful probe of the cosmic acceleration
  - As we probe to higher redshift we see clusters disappear, and the exact rate at which they disappear is sensitive to the growth rate of density perturbations and to the volume we are probing (i.e. the distance-redshift relation)






## Cluster Studies of Cosmology

- Clusters are easily observed, rare, high mass astrophysical objects.
- They have figured prominently in cosmological studies over the past two decades
  - Evolution of the cluster abundance
  - Baryon fractions
  - Cluster structure- concentration
  - Local mass function
  - Cluster power spectrum
  - Distances using SZE+Xray, baryon fractions, and isophotal sizes




Bahcall et al 1999

Redshift distribution is sensitive to distance-redshift relation and rate of structure growth

## Cluster Studies of Dark Energy

- Currently there are four leading methods for studying the dark energy:
  (see e.g. Albrecht et al 2006 – DETF report astro-ph/0609591)
  - Supernovae
  - Galaxy Clusters
  - Cosmic Shear
  - Galaxy Clustering
- Clusters offer a different handle on the combination of the growth rate of structure and the distance-redshift relation



## Overview: Halo Abundance as a Cosmological Tool

- Linear growth of density perturbations
- Non-linear dynamics of a perturbation
- Press Schechter model for abundance of collapsed halos
- Discussion of applications



## Linear and Nonlinear processes

- Most of the directly observable objects in the Universe have already transitioned well beyond the linear regime, and this presents a challenge
- Equations of motion can be integrated in N-body simulations, and extensive work has been done on developing analytical models to describe nonlinear evolution
- But the power spectrum of fluctuations in the microwave background, the clustering of objects on sufficiently large scale, probes of clustering that extend to the high redshift universe and the number density of collapsed massive objects like clusters all are examples of observations whose interpretation relies primarily on linear evolution of density perturbations



## Dynamics of linear perturbations

- One can study the evolution of density perturbations in the linear regime within a Newtonian framework
  - Euler $\frac{D\vec{v}}{dt} = -\frac{\nabla p}{\rho} - \nabla\Phi$
  - Energy $\frac{D\rho}{dt} = -\rho\nabla\cdot\vec{v}$
  - Poisson $\nabla^2\Phi = 4\pi G\rho$
  - Convective derivate $\frac{D}{dt} = \frac{\partial}{\partial t} + \vec{v}\cdot\nabla$
- Then introduce perturbations as $\rho=\rho_o+\delta\rho$ and $v=v_o+\delta v$ and $\Phi=\Phi_o+\delta\Phi$ and collect terms that are first order in the perturbed quantities. Introduce

$$\delta \equiv \frac{\delta\rho}{\rho_o}$$

Discussion follows Peacock

## First order perturbation evolution

- To first order in the perturbed quantities, the governing equations become:

$$\frac{d}{dt}\delta\vec{v} = -\frac{\nabla\delta p}{\rho_o} - \nabla\delta\Phi - (\delta\vec{v}\cdot\nabla)\vec{v}_o$$

$$\frac{d}{dt}\delta = -\nabla\cdot\delta\vec{v}$$

$$\nabla^2\delta\Phi = 4\pi G\rho_o\delta$$

where d/dt is the time derivative of an observer comoving with the unperturbed expansion of the universe

$$\frac{d}{dt} = \frac{\partial}{\partial t} + \vec{v}_o\cdot\nabla$$



## Comoving coordinates

- Through a transformation to comoving coordinates it is possible to describe the evolution of the perturbed quantities with respect to overall uniform expansion. We introduce

$$\vec{x}(t) = a(t)\vec{r}(t)$$
$$\delta\vec{v}(t) = a(t)\vec{u}(t)$$

and use $\nabla_x = \frac{1}{a}\nabla_r \qquad \vec{g} = \frac{\nabla\delta\Phi}{a}$

- The dynamical equations become

$$\dot{\vec{u}} + 2\frac{\dot{a}}{a}\vec{u} = \frac{\vec{g}}{a} - \frac{\nabla\delta p}{\rho_o}$$

$$\dot{\delta} = -\nabla\cdot\vec{u}$$

$$\nabla^2\delta\Phi = 4\pi G\rho_o\delta$$



## Differential equation for perturbation

- Using an equation of state definition of the sound speed

$$c_s^2 \equiv \frac{\partial p}{\partial\rho}$$

- And considering a plane wave disturbance

$$\delta \propto e^{-i\vec{k}\cdot\vec{r}}$$

where **k** is the comoving wavevector

we obtain:

$$\ddot{\delta} + 2\frac{\dot{a}}{a}\dot{\delta} = \delta\left(4\pi G\rho_o - \frac{c_s^2k^2}{a^2}\right)$$

In absence of pressure forces:

$$\boxed{\ddot{\delta} + 2\frac{\dot{a}}{a}\dot{\delta} = 4\pi G\rho_o\delta}$$



## Growth of Density Perturbations in the Linear Regime

- If we try a power law solution in t we obtain

$$\delta(t) \propto t^{2/3} \quad or \quad t^{-1} \quad \text{matter dominated}$$
$$\delta(t) \propto t^{1} \quad or \quad t^{-1} \quad \text{radiation dominated}$$

- Remember that for Ω=1 the scale factor grows as

$$a(t) \propto t^{2/3} \quad \text{matter dominated}$$
$$a(t) \propto t^{1/2} \quad \text{radiation dominated}$$

Giving us simple solutions for the growth of density perturbations for these two cases (early and intermediate times)

$$\delta \propto a \quad \text{matter dominated}$$
$$\delta \propto a^2 \quad \text{radiation dominated}$$

As dark energy comes to dominate at late times the solution deviates from these simple cases

## The Quasi-Linear Regime

- Density perturbations evolve linearly while $\delta$<<1 and then transition to the nonlinear regime and speed up
- An analytical connection between linear and non-linear collapse exists in the so-called spherical collapse model
- Consider a spherical tophat overdensity in an expanding universe with radius R and initial overdensity $\delta$
- Solutions parallel those for the evolution of the scale factor and time in a closed, matter dominated homogeneous and isotropic universe

$$\frac{a(t)}{a(t_o)} = (1-\cos\theta)\frac{\Omega_o}{2(\Omega_o-1)} \qquad H_o t = (\theta - \sin\theta)\frac{\Omega_o}{2(\Omega_o-1)^{3/2}}$$

Discussion follows Liddle and Lyth



## Linearized Scale Factor

- By examining those expressions at the maximum expansion $a_{max}$ and time of maximum expansion $t_{max}$ we can write

$$\frac{a(t)}{a_{\max}} = \frac{1}{2}(1-\cos\theta)$$
$$\frac{t}{t_{\max}} = \frac{1}{\pi}(\theta - \sin\theta)$$

- To study the linear regime of these solutions we can use the small parameter expansions of both expressions

$$\frac{a(t)}{a_{\max}} \cong \frac{\theta^2}{4} - \frac{\theta^4}{48} \quad \text{and} \quad \frac{t}{t_{\max}} \cong \frac{1}{\pi}\left(\frac{\theta^3}{6} - \frac{\theta^5}{120}\right)$$

- Combining these one can solve for a linearized scale factor $a_{lin}$

$$\frac{a_{lin}(t)}{a_{\max}} \cong \frac{1}{4}\left(6\pi\frac{t}{t_{\max}}\right)^{2/3}\left[1 - \frac{1}{20}\left(6\pi\frac{t}{t_{\max}}\right)^{2/3}\right]$$



## Linearized Evolution Cont

- Ignoring the bracketed expression we see a~$t^{2/3}$, which we recognize as the expression for the background evolution

$$\frac{a_{lin}(t)}{a_{\max}} \cong \frac{1}{4}\left(6\pi\frac{t}{t_{\max}}\right)^{2/3}\left[1 - \frac{1}{20}\left(6\pi\frac{t}{t_{\max}}\right)^{2/3}\right]$$

- The full expression is for the evolving perturbation, which expands more slowly than the background



## Linearized Evolution Cont

- Consider turnaround (a=$a_{max}$) and the perturbation overdensity

$$1 + \delta_{lin} = \frac{a_{back}^3}{a_{lin}^3}$$

- Substituting in the preceeding expressions gives

$$\delta_{lin} = \frac{3}{20}\left(6\pi\frac{t}{t_{\max}}\right)^{2/3}$$

- So at turnaround, t=$t_{max}$, the linear density contrast is

$$\delta_{lin}^{turn} = \frac{3}{20}(6\pi)^{2/3} \approx 1.06$$

## Collapse or Critical Overdensity

- After turnaround the collapse proceeds symmetrically to the expansion phase, and so at $t=2*t_{max}$ the perturbation has collapsed

$$\delta_{lin}^{coll} = \frac{3}{20}(12\pi)^{2/3} \approx 1.686$$

- So the linear density contrast of $\delta$~1.7 corresponds to a threshold density at which the underlying density perturbation would have collapsed and formed a halo
- Counting collapsed halos has become an issue of counting the fraction of halos of a given mass with linearly extrapolated overdensities exceeding 1.7!



## True Virial Overdensity

- The actual non-linear density contrast at turnaround is

$$1+\delta_{nonlin}^{turn} = \frac{a_{back}^3}{a_{\max}^3} = \frac{(6\pi)^2}{4^3} \approx 5.55$$

- If we assume that the collapsing object virializes at half the radius, its density will have gone up by a factor of 8. Relative to the background density the nonlinear overdensity of the collapsed halo is

$$1+\delta_{nonlin}^{vir} \approx 178$$

- N-body simulations confirm that the region of the halo with an overdensity of ~200 corresponds to the virialized portion of the halo



## Filtering the Cosmic Density Field on Mass Scale M

- How would we go about calculating the distribution of overdensities for perturbations with a mass scale M?
- Mass scale M can be connected to physical scale R using $M=V*\rho$ where $V=4/3\pi R^3$ and $\rho$ is the mean matter density

  Imagine we smooth the density field with a spherical tophat of radius R and examine the distribution of overdensities
- The variance of the density field smoothed on a scale M can be written

$$\sigma^2(M) = \frac{1}{2\pi^2}\int k^2\, dk\, P(k)\left|W(k,M)^2\right|$$

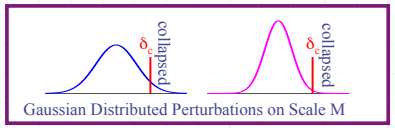
where $W(k,M)$ is the Fourier transform of the spherical tophat

and P(k) is the power spectrum of density perturbations



## Connecting linear growth to halo abundance evolution: The Press-Schechter Mass Function

- Consider the cosmic density field filtered on a mass scale M
  - Gaussian distribution of over/under density with width $\sigma$
  - Over cosmic time perturbations grow and $\sigma$ increases




- Consider the number density of collapsed (highly nonlinear) objects on a mass scale M
  - Note that density perturbations have collapsed by the time their linearly evolved overdensity exceeds some critical value $\delta_c$
  - Abundance (number density) of collapsed objects with mass M is then proportional to an integral over the tail of a Gaussian distribution

$$n(M,z) = \frac{\rho_b}{M}\frac{1}{\sqrt{2\pi}\sigma(M,z)}\int_{\delta_c}^{\infty} d\delta\, \exp\left\{\frac{-\delta^2}{2\sigma^2(M,z)}\right\} \sim n_{char} * f_{col}$$

## Halo Abundance

- Cluster mass function dn(M,z)/dM depends on mean matter density and amplitude of density fluctuations

$$\sigma^2(M) = \frac{1}{2\pi^2}\int k^2\, dk\, P(k)\left|W(k,M)^2\right|$$

where $W(k,M)$ is the Fourier transform of the spherical tophat

- Vintage Press-Schechter formalism

$$\frac{dn}{dM}(M,z) = -\sqrt{\frac{2}{\pi}}\frac{\rho_b}{M}\frac{d\sigma(M,z)}{dM}\frac{\delta_c}{\sigma^2(M,z)}\exp\left\{\frac{-\delta_c^2}{2\sigma^2(M,z)}\right\}$$

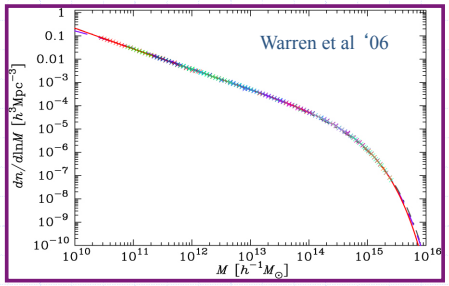
- Modern numerical simulations: Jenkins et al 2001

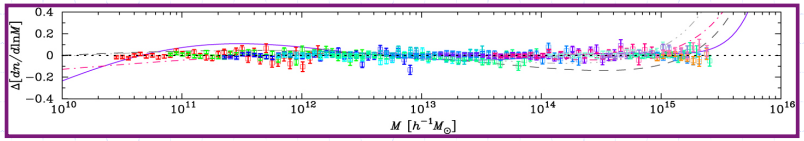
$$\frac{dn}{dM}(M,z) = -0.315\frac{\rho_b}{M}\frac{d\sigma(M,z)}{dM}\frac{1}{\sigma}\exp\left\{-\left|0.61-\log(D_z\sigma_M)\right|^{3.8}\right\}$$



## Mass Function Now Studied with Numerical Simulations of Structure Formation

- Extracted from sixteen billion particle dark matter simulation.
- Halos are defines using a friends of friends algorithm
- Halo masses are assigned using mass within spherical overdensity






## Cosmological Dependence

- Direct tests using N-body simulations in good agreement with theoretical models (Sheth and Tormen 1999) over 5 orders of magnitude in mass
- Halo abundance is sensitive to cosmology through
  - Mean background density
  - The power spectrum of density fluctuations
  - Linear growth rate of density perturbations
- The fitting functions have been testing over a wide range of ΛCDM cosmologies and have been shown to be accurate at <10% level
- Direct simulation required if better precision needed

Growth function sensitive to expansion history of Universe.

$$\ddot{\delta} + 2\frac{\dot{a}}{a}\dot{\delta} = 4\pi G\rho_o\delta$$

where $\delta = \frac{\delta\rho}{\rho_o}$ and $H = \frac{\dot{a}}{a}$



## Importance of Mass Definition

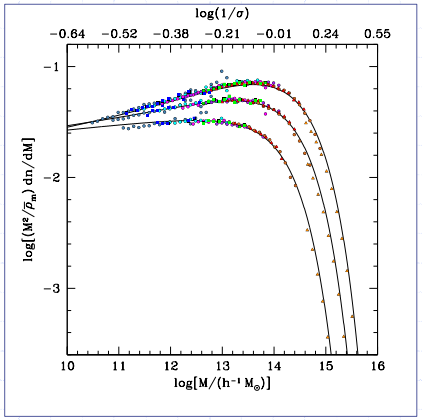
- Tinker et al have explored the dependence of the mass function on the spherical overdensity mass with particular overdensity Δ
  - Figure shows Δ=200, 800, 3200

$$\frac{dn}{dM} = \frac{\bar{\rho}_m}{M}f(\sigma)\frac{d\ln\sigma^{-1}}{dM}$$

  - Fitting function f(σ) has 4 free parameters

$$f(\sigma) = A\left[\left(\frac{\sigma}{b}\right)^{-a}+1\right]e^{-c/\sigma^2}$$

- Shape varies, but good fit possible for arbitrary overdensity Δ

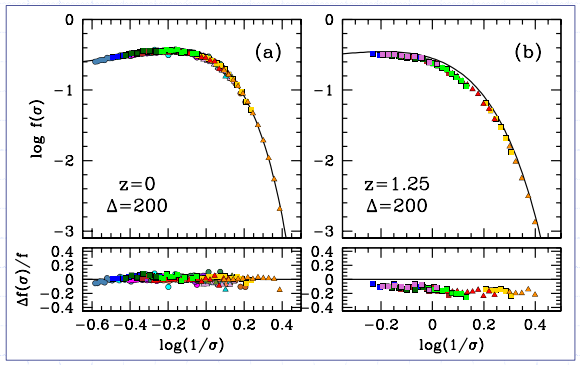



Tinker et al 2008

## Mass Function Shape Not Universal

- Evolution of mass function not completely described by change in σ(M) and <$\rho_m$>
  - 20% offset in figure below

$$\frac{dn}{dM} = \frac{\bar{\rho}_m}{M} f(\sigma) \frac{d\ln\sigma^{-1}}{dM} \qquad f(\sigma) = A\left[\left(\frac{\sigma}{b}\right)^{-a} + 1\right] e^{-c/\sigma^2}$$

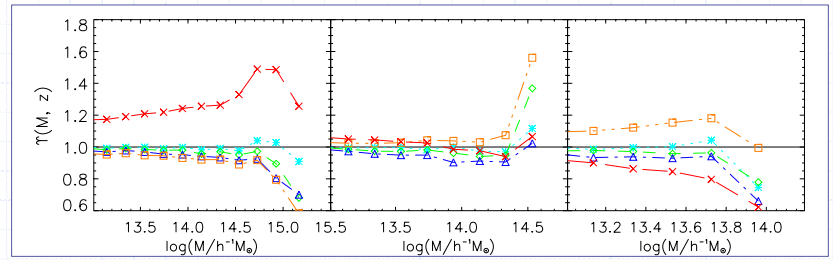



Tinker et al 2008



## Tests Underway in Interacting Dark Energy and Dark Matter Models

- Using public CoDECS simulations (Baldi 2011), Cui et al compared the mass functions to previously published fitting functions (Δ=200 below)
  - z=0, z=1, z=2 (left to right)
  - Reasonable agreement for most models, agreement worse at high mass
  - Halo abundance can be used to test these models

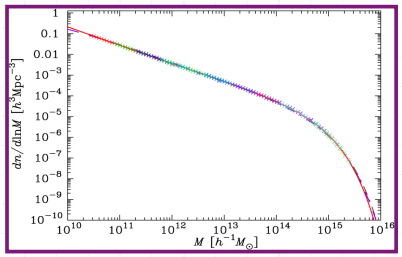



Cui, Baldi and Borgani 2012



## Consider the Application of this Method

- What is needed for this test to provide cosmological constraints?
  - Accurate theory- mass function must be well known in target model
  - Accurate mass measurements
  - Clean selection through some property related to mass




- What classes of objects qualify?
  - Galaxy Clusters (?)
  - Galaxies (?)



## What do we potentially learn?

- A study of halo abundances and its evolution provides direct information on matter density, linear growth rate of density perturbations, anything that impacts the transfer function, and the volume we are surveying as we probe to high redshift (i.e. the distance-redshift relation, as we will see)

$$\frac{dn}{dM} = \frac{\bar{\rho}_m}{M} f(\sigma) \frac{d\ln\sigma^{-1}}{dM}$$

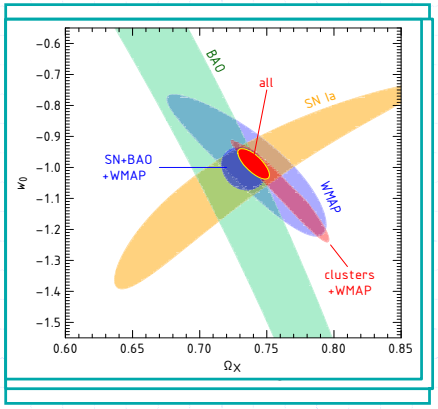
$$\sigma^2(M) = \frac{1}{2\pi^2}\int k^2\, dk\, P(k) \left|W(k,M)^2\right|$$

where $W(k,M)$ is the Fourier transform of the spherical tophat

- Halo abundance experiment probes growth rate of density perturbation and distance-redshift relation… possible to test consistency of GR internally

## Recent Results: 400 deg$^2$ ROSAT Archival Sample
### Vikhlinin et al 2009

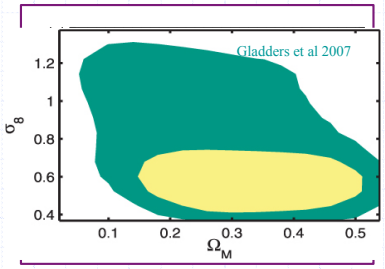
- Analysis:
  - 49 "local" + 37 z>0.35 clusters
  - Mass functions
  - 12 clusters at z>0.55 require Lambda
  - Independent constraints in good agreement with WMAP+ cosmology
  - w constrained to 0.2(clus)/0.05(all)






## Recent Results: Optical

- Recent analyses of X-ray cluster samples using existing datasets have generally been quite successful, but some problems have emerged with the optical samples
- SDSS and RCS were able to obtain interesting constraints on $\Omega_m$ and $\sigma_8$.
  - RCS- 956 clusters over 72 deg$^2$
  - SDSS- $10^4$ clusters over 7,400 deg$^2$






## Summary

- Halo abundance evolution is potentially a very powerful cosmological tool: distance-redshift, growth rate of structures, P(k) effects
- Theoretical predictions possible through direct simulation, and results for ΛCDM models consistent with simple theoretical picture of linear and quasi-linear density evolution
- Requires that halo masses be measured and that halos can be cleanly selected by observable property (closely) related to the mass
- Recent results quite promising…



## References


- *Articles from the current literature*
- *Cosmological Inflation and Large-Scale Structure,*
  Andrew Liddle & David Lyth, Cambridge University Press, 1999
- *Cosmological Physics,*
  John Peacock, Cambridge University Press, 2000