2354-17

Summer School on Cosmology

16 - 27 July 2012

Statistics and Data Analysis - Lecture 2

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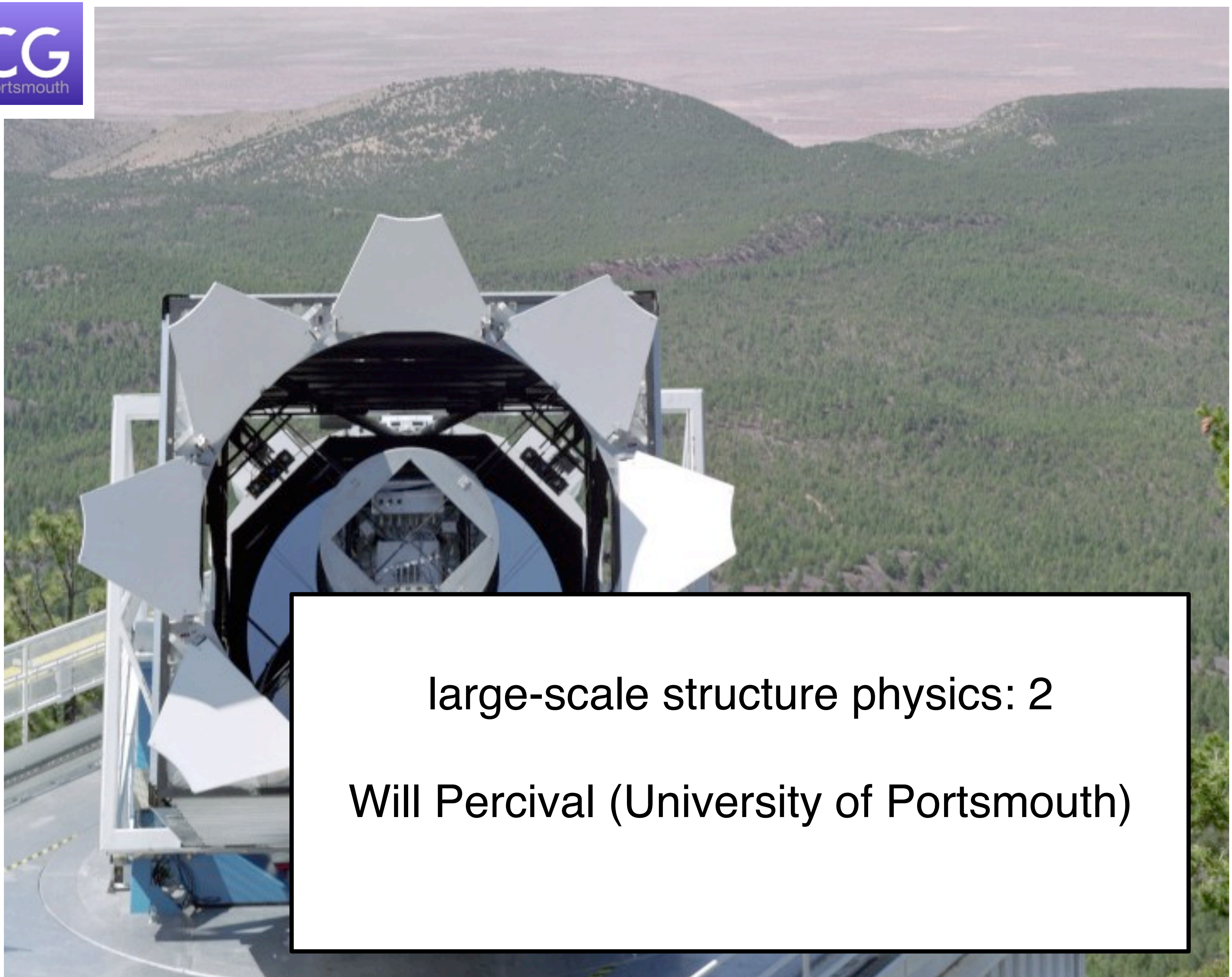
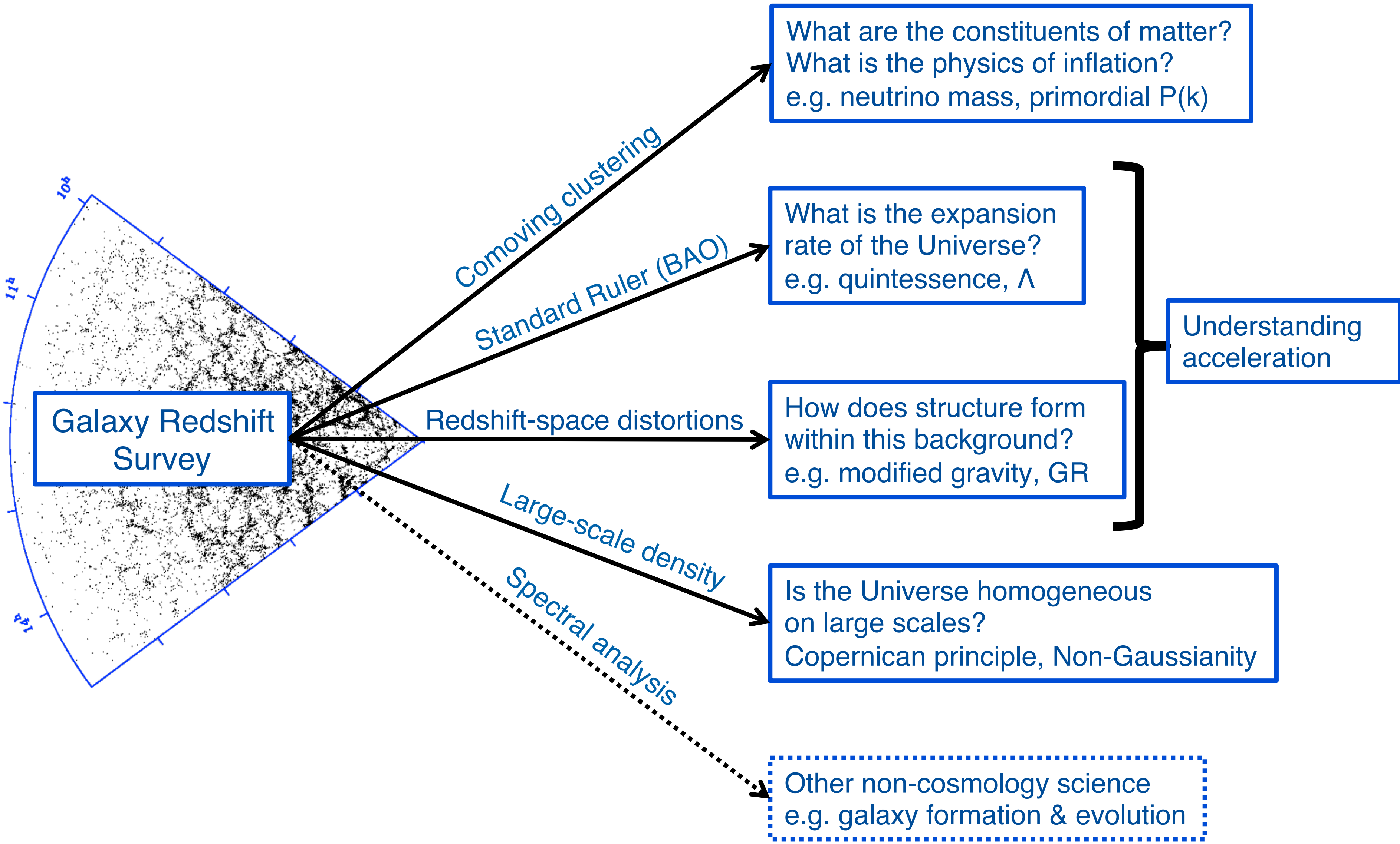


large-scale structure physics: 2

Will Percival (University of Portsmouth)

Cosmology from Spectroscopic Galaxy Surveys



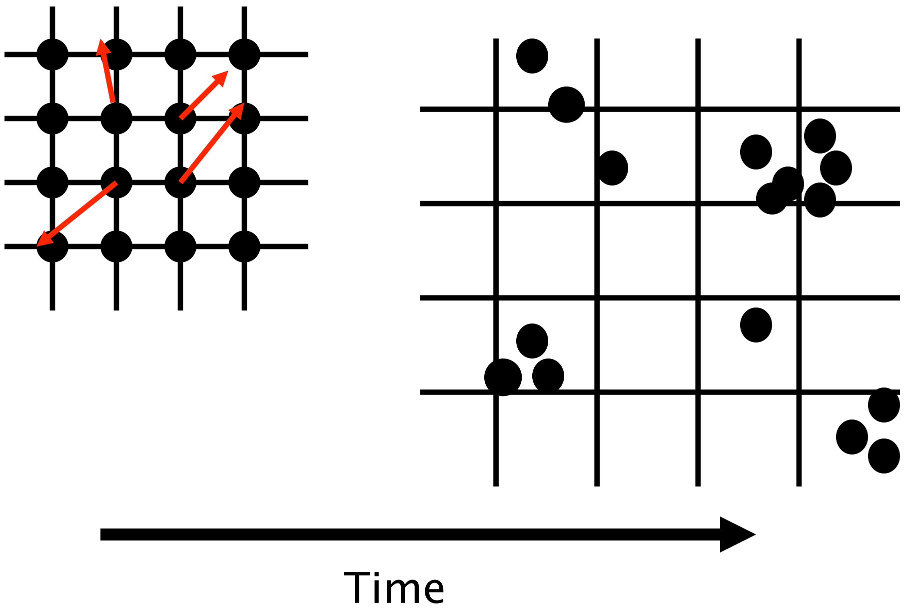
Peculiar velocities

Peculiar velocities

All of structure growth happens because of peculiar velocities

Initially distribution of matter is approximately homogeneous (δ is small)

Final distribution is clustered



Linear peculiar velocities

Work in comoving units and conformal time τ=t/a

For ease, define an acceleration vector $$\mathbf{g} = -\frac{\nabla\theta}{a}$$

The continuity and Poisson equations are

$$\frac{d\delta}{d\tau} + \nabla\cdot[(1+\delta)\mathbf{u}] \simeq \dot{\delta} + \nabla\cdot\mathbf{u} = 0 \qquad \nabla\cdot\mathbf{g} = -\frac{3}{2}\Omega_m H^2 a\delta$$

linear theory

Equations have solution $\mathbf{u} = -\frac{2f}{3H\Omega_m}\mathbf{g}$ (exercise!), where $f = \frac{\dot{\delta}}{aH\delta} = \frac{d\ln D}{d\ln a}$

This gives that

$$\delta = -\frac{\nabla\cdot\mathbf{u}}{aHf} \qquad P_u(k) = (aHf)^2 P(k) k^{-2}$$

Change from τ to t : remove factors of *a* in last 2 equations

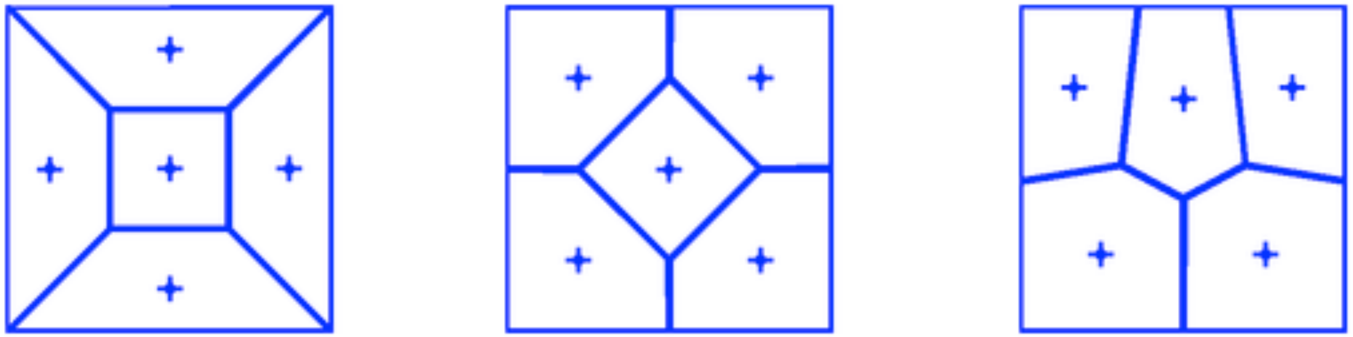
Measuring the velocity P(k) from simulations

Is surprisingly difficult:

for an overdensity field, no galaxies means overdensity = -1

for a velocity field, no objects means we do not know the velocity

Voroni tesselation: The partitioning of a plane/volume with n points into convex polygons such that each polygon contains exactly one generating point and every point in a given polygon is closer to its generating point than to any other.



Can then interpolate the velocities onto a grid and use standard FFT techniques to measure the velocity power spectrum

Measuring velocities of galaxies

1 Radial measurement of individual object velocities, using redshift and independent distance estimate (for example):
- – SN1a distances
- – Fundamental Plane of early-type galaxies
- – Tully-Fisher relation
- – Brightest cluster galaxies

2 Kinetic SZ observations of clusters
- – really only has power as a statistical measurement

3 Redshift-space distortions: statistical measure of properties of distribution

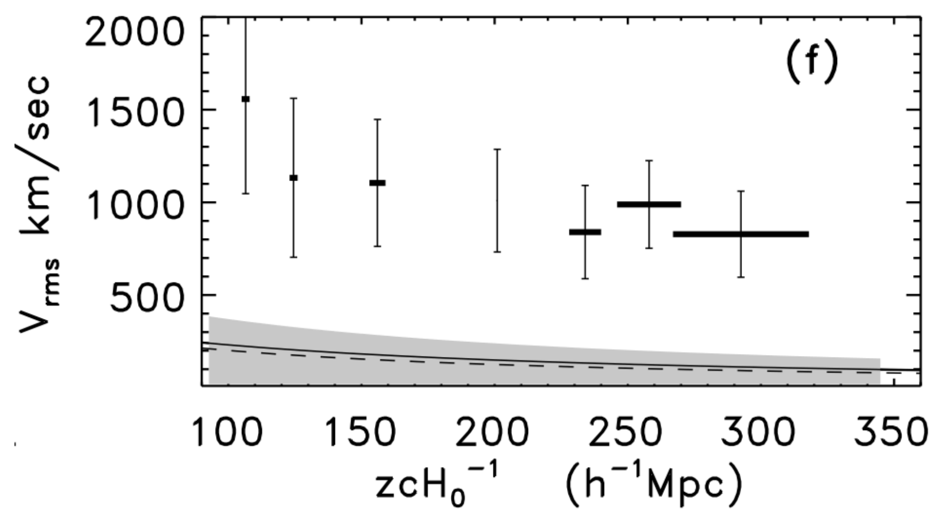
Fundamental plane: is a correlation between the effective radius, average surface brightness and central velocity dispersion of normal elliptical (or early-type) galaxies. Together the measurements fall on a plane within the more general 3D space.

Tully-Fisher relation: an empirical relationship between the intrinsic luminosity (proportional to the stellar mass) of a spiral galaxy and its velocity dispersion. (Tully & Fisher 1977)

Brightest cluster galaxies: assumes that the brightest elliptical galaxy in clusters have the same luminosity. Does it work? …. No.

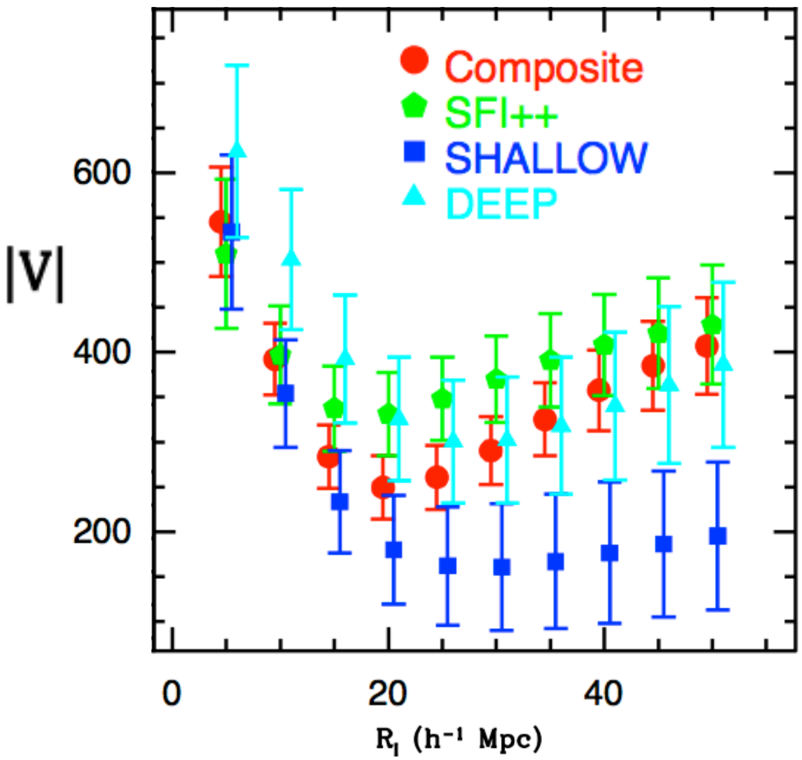
Kashlinsky et al., arXiv: 0809.3734

- Take WMAP 5 year CMB map
- Wiener filter to "optimally" remove CMB power spectrum
- Use known cluster positions from Reflex, BCS, CIZA, and calculate angular power spectrum of CMB at these locations
- Measure the dipole component
- Relate this to the dipole expected from clusters moving towards each other



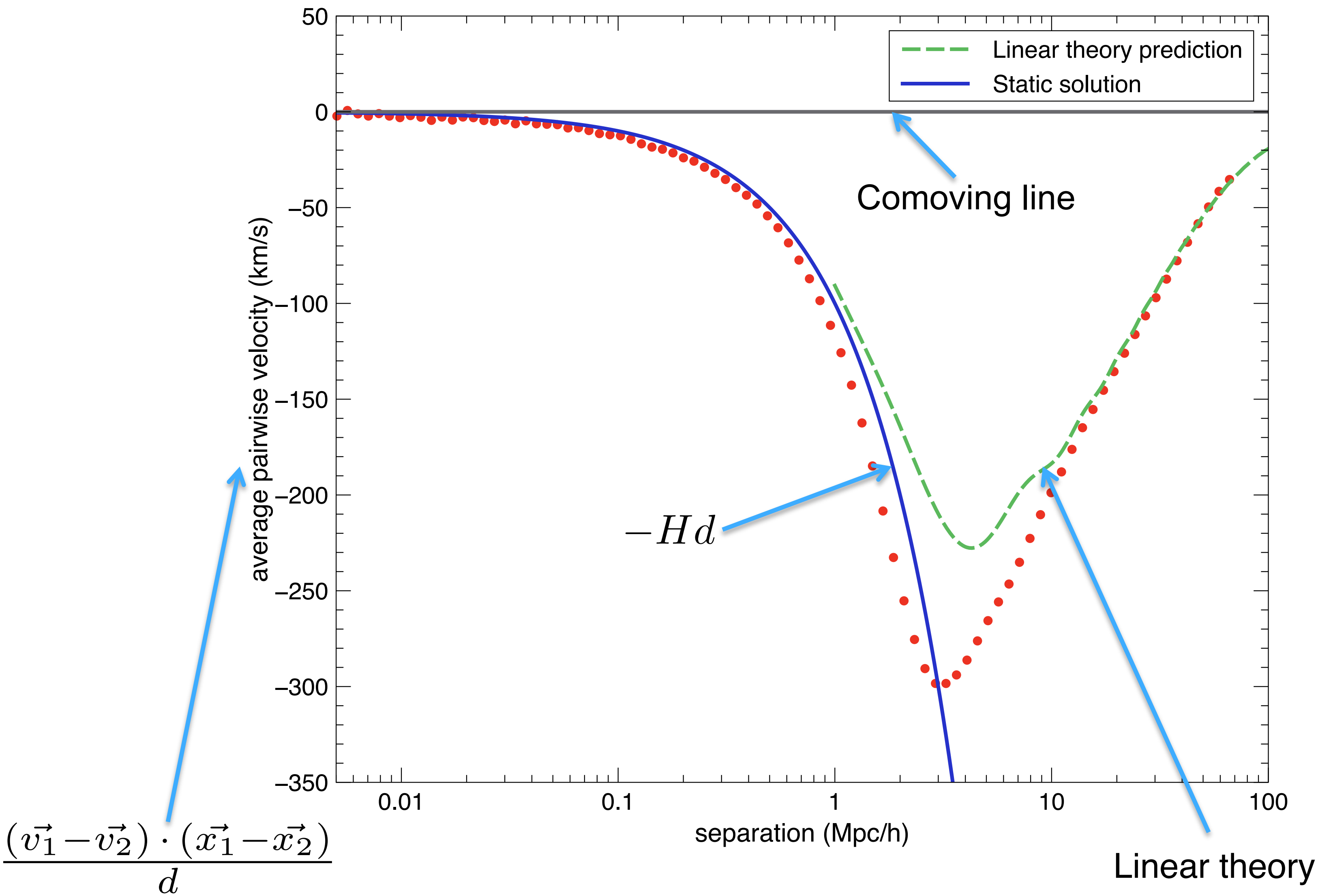
Watkins et al, arXiv: 0809.4041

- Use 9 data sets with peculiar velocities
- peculiar velocities measured using lots of different techniques
- Weighting scheme (minimum variance) to account for different windows for different surveys



	ML		BC	
	$\Omega_m = 0.258$	$\sigma_8 = 0.796$	$\Omega_m = 0.262$	$\sigma_8 = 0.863$
Expected 1D r.m.s.	112 km/s		121 km/s	
Survey	$\chi 2$	$P(>\chi 2)$	$\chi 2$	$P(>\chi 2)$
SHALLOW	1.54	0.6731	1.37	0.7126
DEEP	7.54	0.0565	6.76	0.0800
SFI++	11.23	0.0105	9.92	0.0193
COMPOSITE	11.52	0.0092	10.15	0.0173

Average galaxy pairwise velocity in the Millenium simulation



From Belloso et al. 2012: arXiv:1204.5761

Redshift-space distortions

Redshift-space distortions

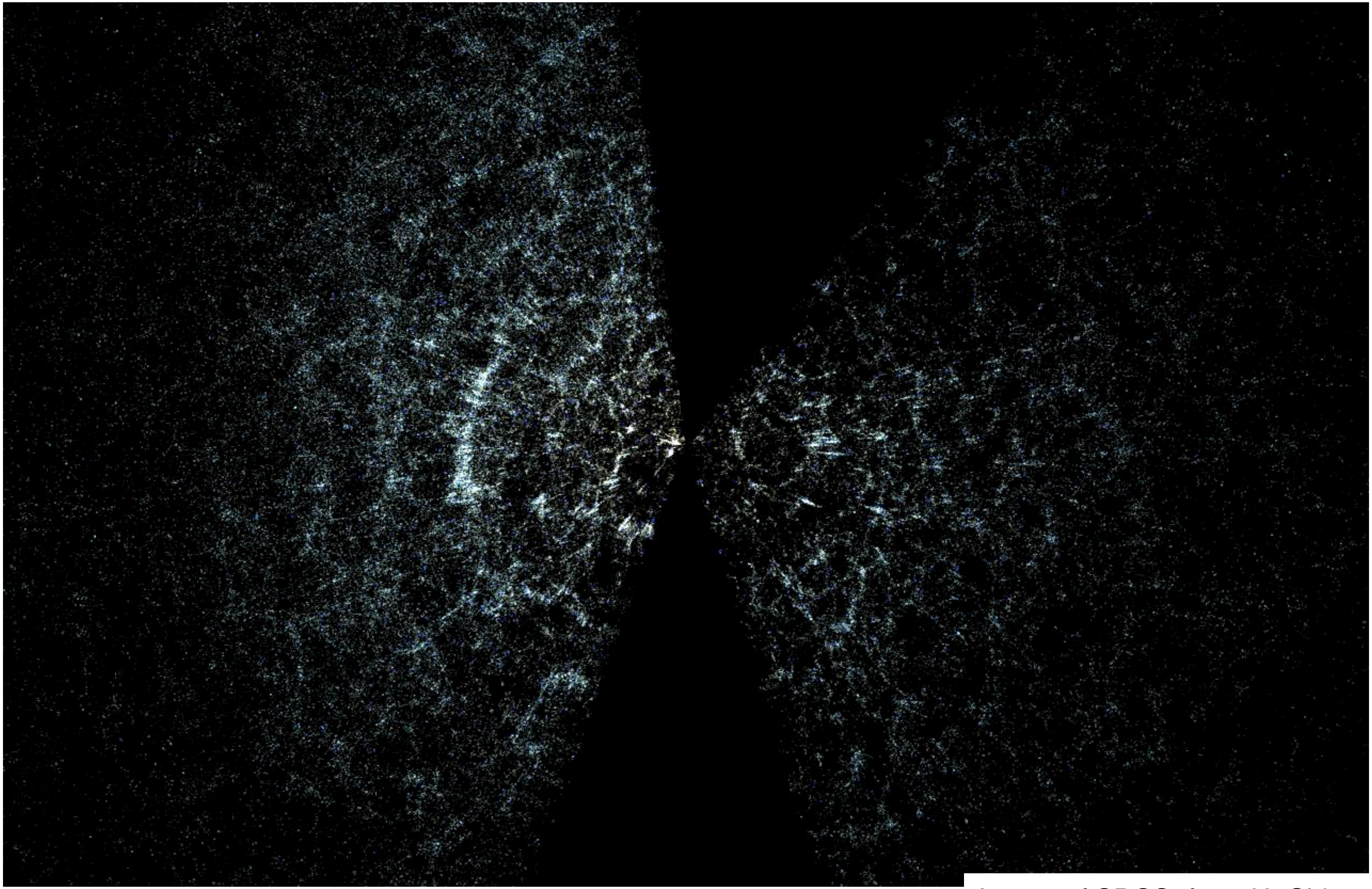


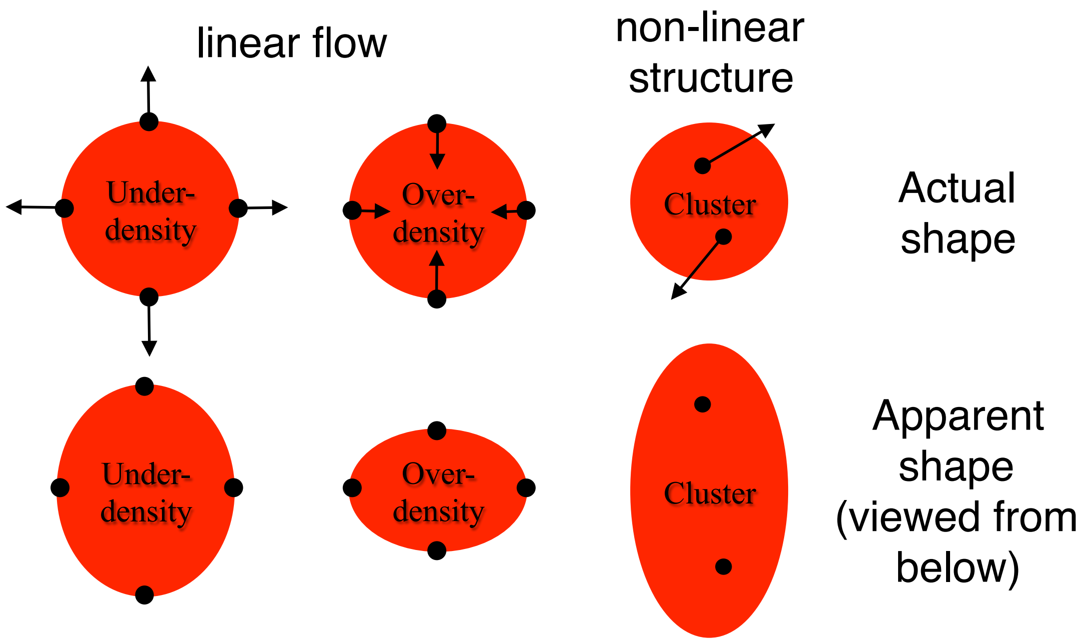
Image of SDSS, from U. Chicago

Redshift-space distortions

- Statistically compare "apparent" structure across and along the line-of-sight

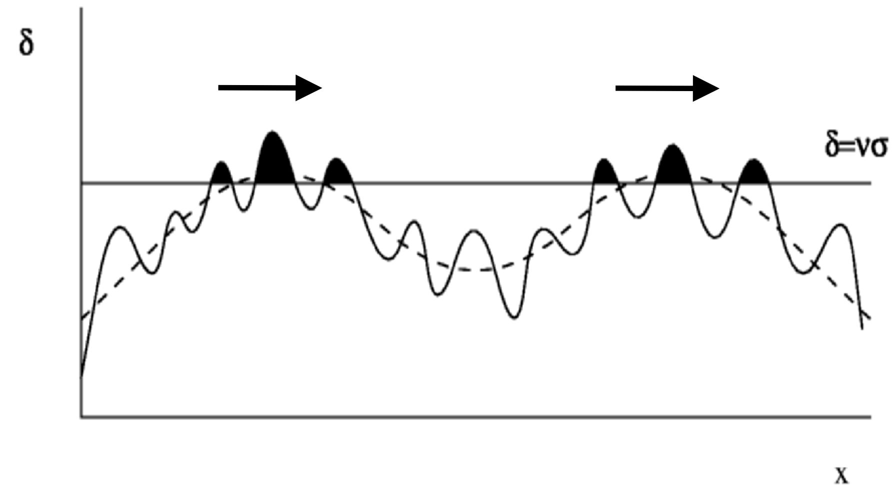
- Linear growth of structure enhances clustering signal, but only along line-of-sight

- Measurable effect!



Galaxies act as test particles in cosmological velocity field

- Locally, galaxies act as test particles in the flow of matter
- On large-scales, the distribution of galaxy velocities is unbiased if galaxies fully sample the velocity field
- expect a small peak velocity-bias due to motion of peaks in Gaussian random fields differing from that of the mass

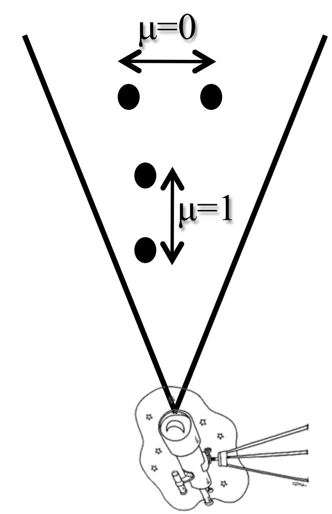


Redshift-space distortion theory

Transition from real to redshift space, with peculiar velocity v **in units of the Hubble flow**

$$\mathbf{s} = \mathbf{r} + v_{\rm los}\hat{\mathbf{r}}_{\rm los}$$

$$\mu = cos(\alpha)$$
$$\theta = \nabla \cdot \mathbf{u}$$



Jacobian for transformation

$$\frac{d^3 s}{d^3 r} = \left(1 + \frac{v_{\rm los}}{r_{\rm los}}\right)^2 \left(1 + \frac{dv_{\rm los}}{dr_{\rm los}}\right)$$

Conservation of galaxy number

$$n^r(\mathbf{r})d^3 r = n^s(\mathbf{s})d^3 s \qquad 1 + \delta_g^s = (1 + \delta_g^r)\frac{d^3 r}{d^3 s}\frac{\bar{n}^r(\mathbf{r})}{\bar{n}^s(\mathbf{s})}$$

Trick to understand velocity field derivative

$$\frac{\partial v_{\rm los}}{\partial r_{\rm los}} = \left(\frac{\partial}{\partial r_{\rm los}}\right)^2 \nabla^{-2}\theta = \left(\frac{k_{\rm los}}{k}\right)^2 \theta = \mu^2\theta, \quad \theta = \nabla \cdot \mathbf{v}$$

Gives to first order

$$\delta_g^s = \delta_g^r - \mu^2\theta$$

Kaiser 1987, MNRAS, 227, 1

what do linear z-space distortions measure?

linear scales,

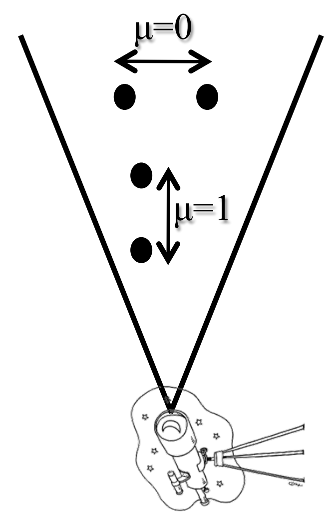
$$\delta_g^s(\mu) = \delta_g + \mu^2\theta$$
$$P_g^s(\mu) = \langle|\delta_g + \mu^2\theta|^2\rangle$$
$$= P_{gg} + 2\mu^2 P_{g\theta} + \mu^4 P_{\theta\theta}$$

Galaxy-galaxy power

Velocity-velocity power

Galaxy-velocity divergence cross power

$$\mu = cos(\alpha)$$
$$\theta = \nabla\cdot\mathbf{u}$$



In linear regime (remember from slide on linear velocities)

$$\delta_g = b\delta(\mathrm{mass}),\quad \theta = -f\delta(\mathrm{mass}),\quad f \equiv \frac{d\ln G}{d\ln a}$$

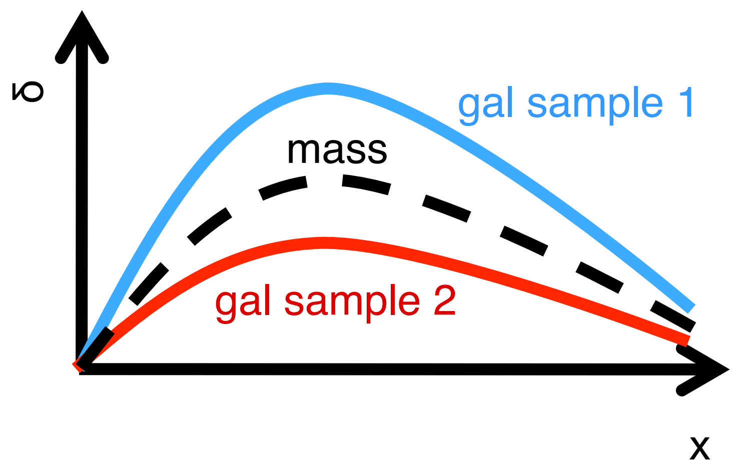
Linear growth rate

So, the simplest model for the power spectrum is

$$P_g^s(k,\mu) = \left[b + \mu^2 f\right]^2 P_{\mathrm{mass}}(k)$$

Kaiser 1987, MNRAS, 227, 1

Beating the cosmic variance limit



If we have 2 samples of galaxies (in real space) with different deterministic biases,

$$\delta_1 = b_1 \delta_{\rm mass}, \ \delta_2 = b_2 \delta_{\rm mass}$$

accuracy of b_1/b_2 measurement only depends on shot noise

In redshift-space this result generalizes to

$$\delta_1 = (b_1 + f\mu^2)\delta_{\rm mass}, \ \delta_2 = (b_2 + f\mu^2)\delta_{\rm mass}$$

giving b_1/b_2, f/b_1, and f/b_2 limited by only shot noise

Allows us to use non-radial modes to "extract" information about $f^2P(k)_{\rm mass}$

Does not affect errors on the power spectrum shape.

McDonald & Seljak, astro-ph/0808.1101

The break-down of the simple model

- Real-Redshift space mapping
  - Kaiser formula first order in δ and θ
  - on small scales, we need 2nd and 3rd order (δ, θ cross) terms
  - assumes irrotational velocity field

- Non-linear density field evolution
  - P_{gg} breaks from linear behaviour (small scale, late time)

- Non-linear velocity field evolution
  - $P_{\theta\theta}$ breaks from linear behaviour (small scale, late time)
  - Fingers-of-God

- Plane-parallel approximation breaks down for galaxy pairs with wide angular separation

- Assumes local, deterministic density bias

- RSD still provide the best method for determining ***f***

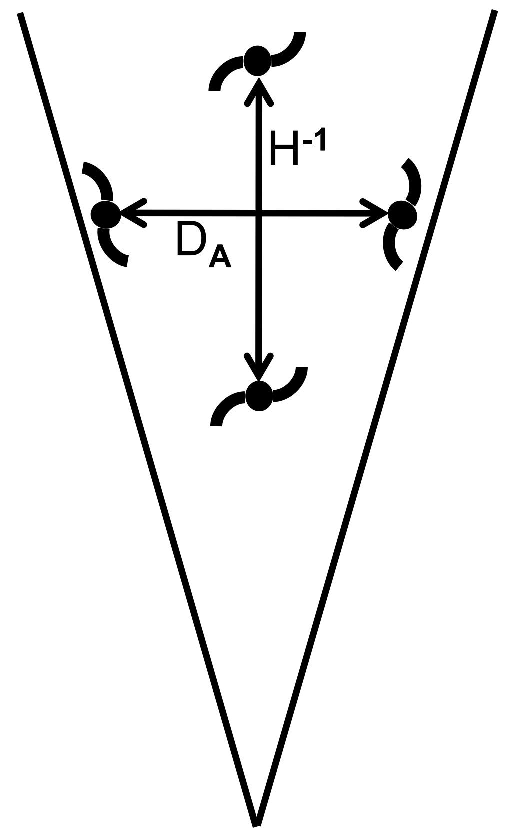
The Alcock-Paczynski Effect

The Alcock-Paczynski Effect

- If the Universe is isotropic, clustering is same radial & tangential
- Stretching at a single redshift slice (for galaxies expanding with Universe) depends on

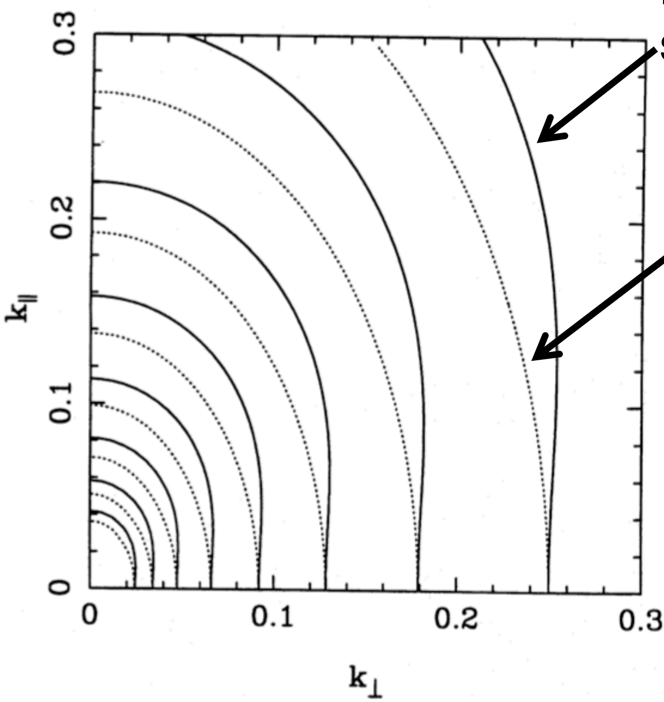
  $H^{-1}(z)$ (radial)

  $D_A(z)$ (angular)
- Analyze with wrong model -> see anisotropy
- AP effect measures $D_A(z)H(z)$
- RSD limits test to scales where can be modeled



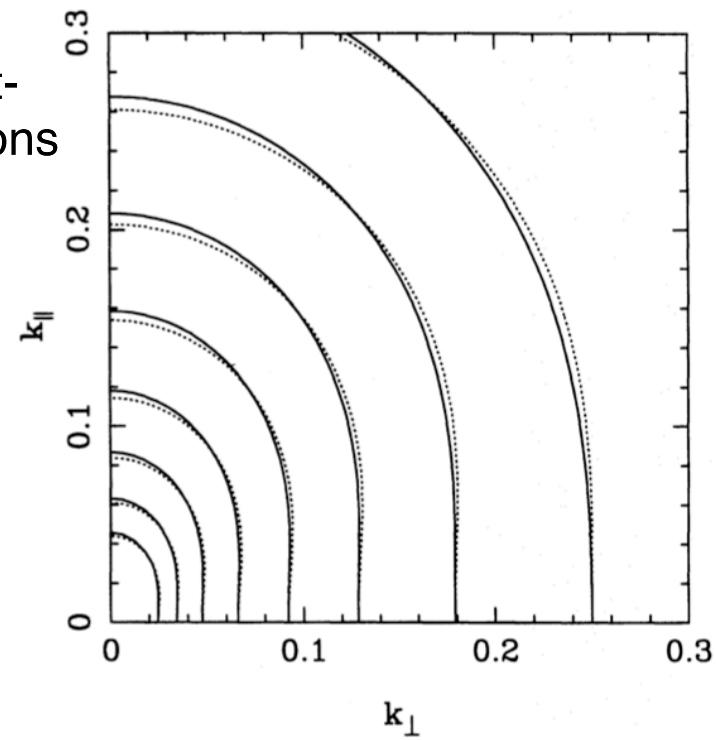
Linking z-space distortions and BAO

We should allow for the coupling between the redshift-space distortions and the geometrical squashing caused by getting the geometry wrong. Effects are not perfectly degenerate



Linear redshift-space distortions

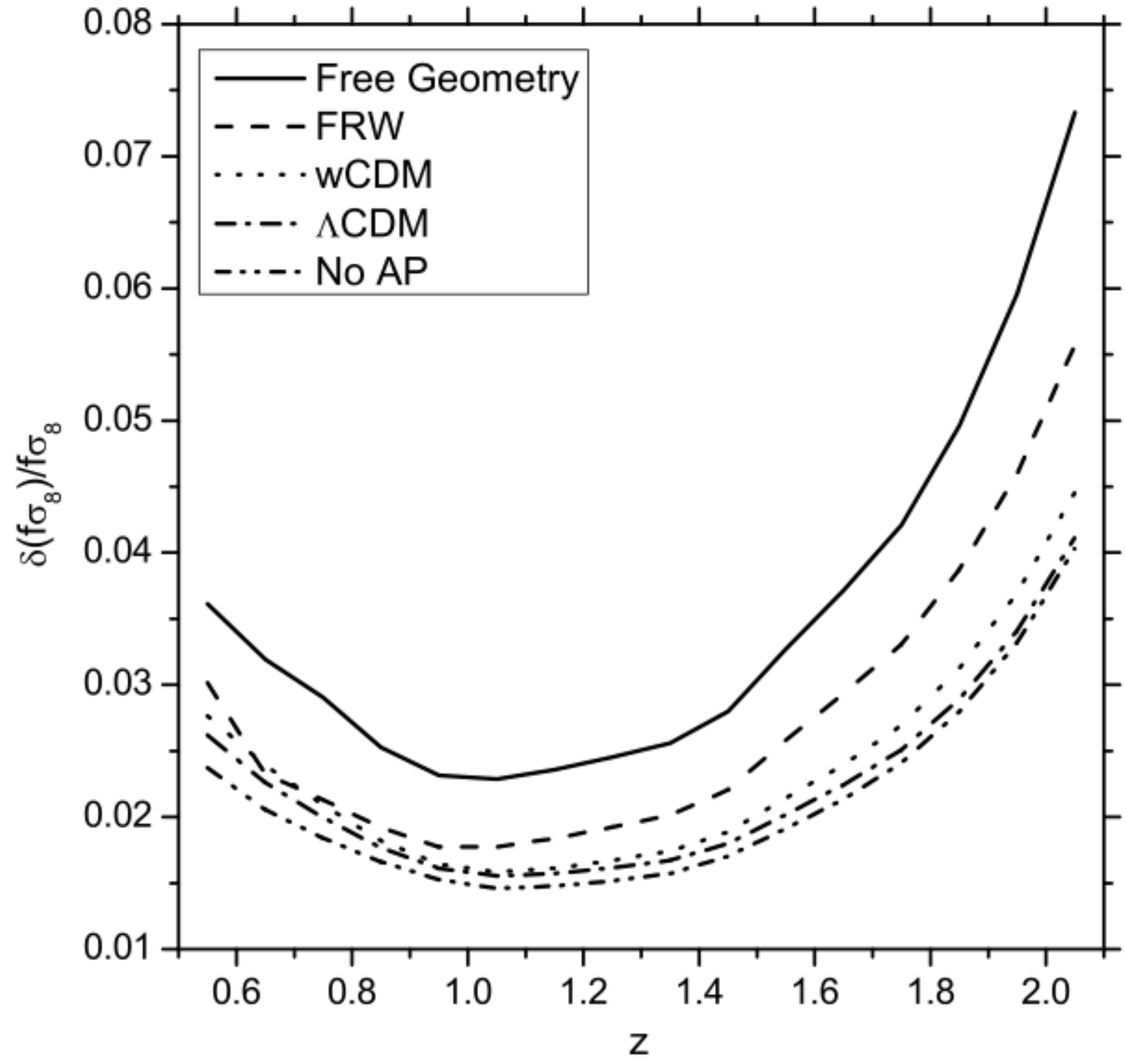
Geometric squashing



Fit to redshift-space distortions cannot mimic geometric squashing

Ballinger, Peacock & Heavens 1999, MNRAS, 282, 877

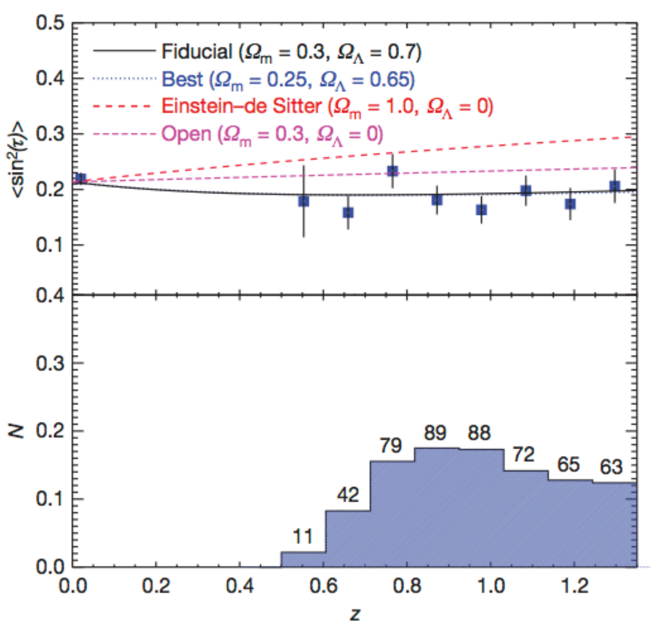
Degradation of RSD measurements by AP effect



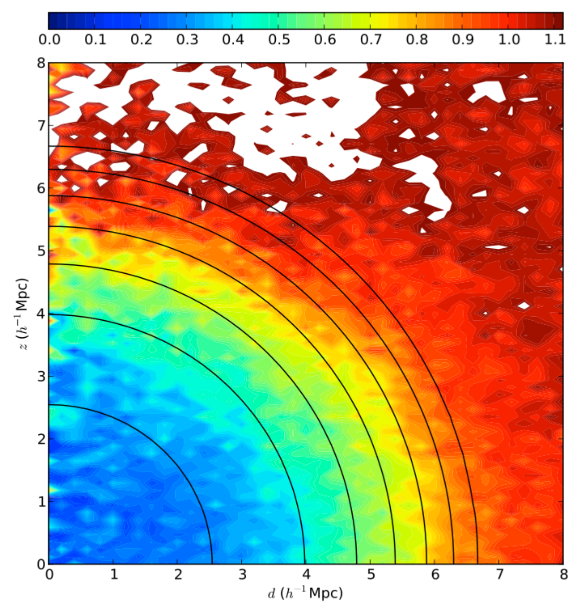
Samushia et al 2011, 410, 1993

Does needing to know RSD signal limit to large-scales?

- use isolated galaxy pairs
- Marinoni & Buzzi 2011
  - Nature 468, 539
- Jennings et al. 2012
  - MNRAS 420, 1079



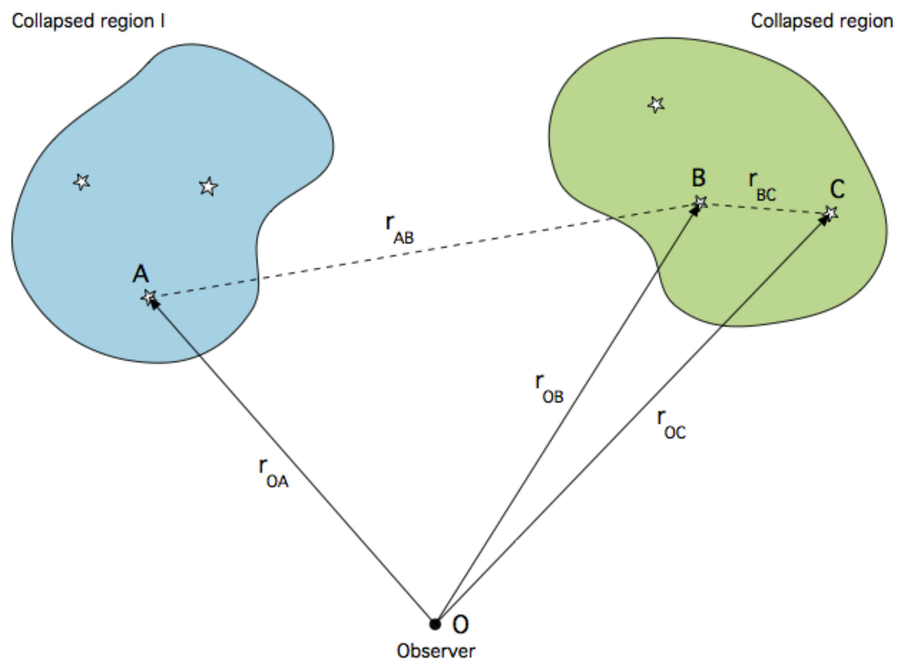
- use voids
- Lavaux & Wandelt 2011
  - arXiv:1110.0345



Both try to isolate objects where the RSD signal is known or weak

Collapsed structures

- Live in static region of space-time
- Velocity from growth exactly cancels Hubble expansion
- Two static galaxies in same structure have same observed redshift irrespective of distance from us
- Redshift difference only tells us properties of system
- Two collapsed similar regions observed in different background cosmologies give same Δz
- No cosmological information from Δz
- Cannot be used for AP tests



Belloso et al. 2012: arXiv:1204.5761

Primordial non-Gaussianity

Measuring primordial non-Gaussianity: f_{NL} g_{NL}

δ is sourced from a potential field **Φ**, whose form might not be Gaussian

$$\nabla^2\Phi(\mathbf{x}) = 4\pi G\delta(\mathbf{x})$$

$$\Phi(\mathbf{x}) \sim \phi(\mathbf{x}) + f_{NL}\phi^2(\mathbf{x}) + \ldots$$

skewness ~ f_{NL}
kurtosis ~ f_{NL}^2
...

ϕ is a Gaussian field. the non-linear terms in **Φ** make **Φ** non-Gaussian. This map completely specifies **Φ** statistics.

Salopek and Bond 1990;
Gangui, Lucchin, Matarrese, Mollerach 1994;
Komatsu and Spergel 2001

$$\Phi(\mathbf{x}) \sim \phi(\mathbf{x}) + g_{NL}\phi^3(\mathbf{x}) + \ldots$$

skewness ~ 0
kurtosis ~ g_{NL}
…

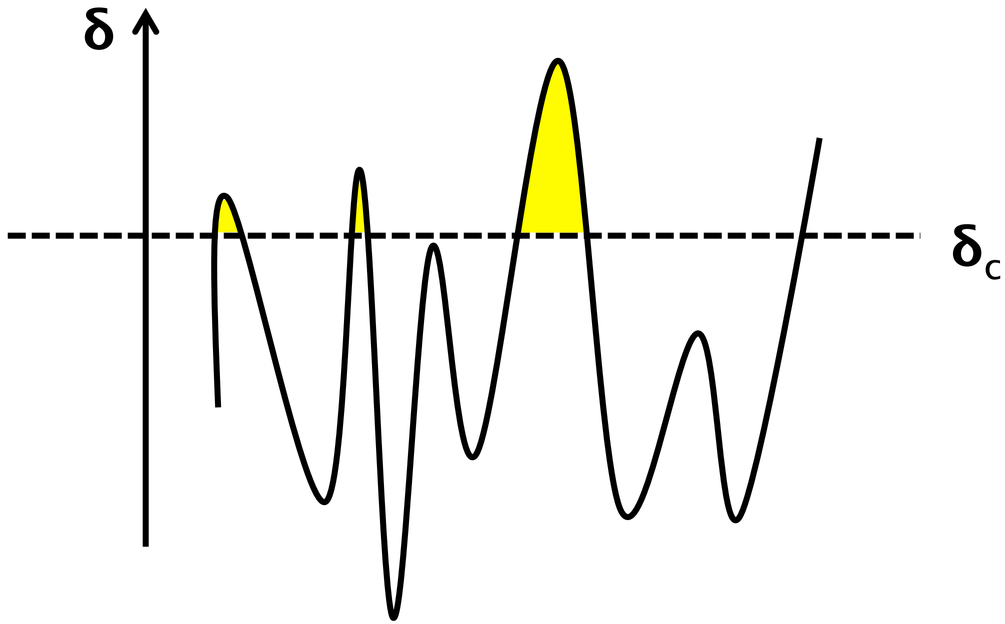
f_{NL} is not the only option for local potential fluctuations … you can go even further down this route …

Okamoto and Hu 2002;
Enqvist and Nurmi 2005

Non-local models introduce non-trivial higher order correlations in **Φ**

Measuring primordial non-Gaussianity: halo abundance

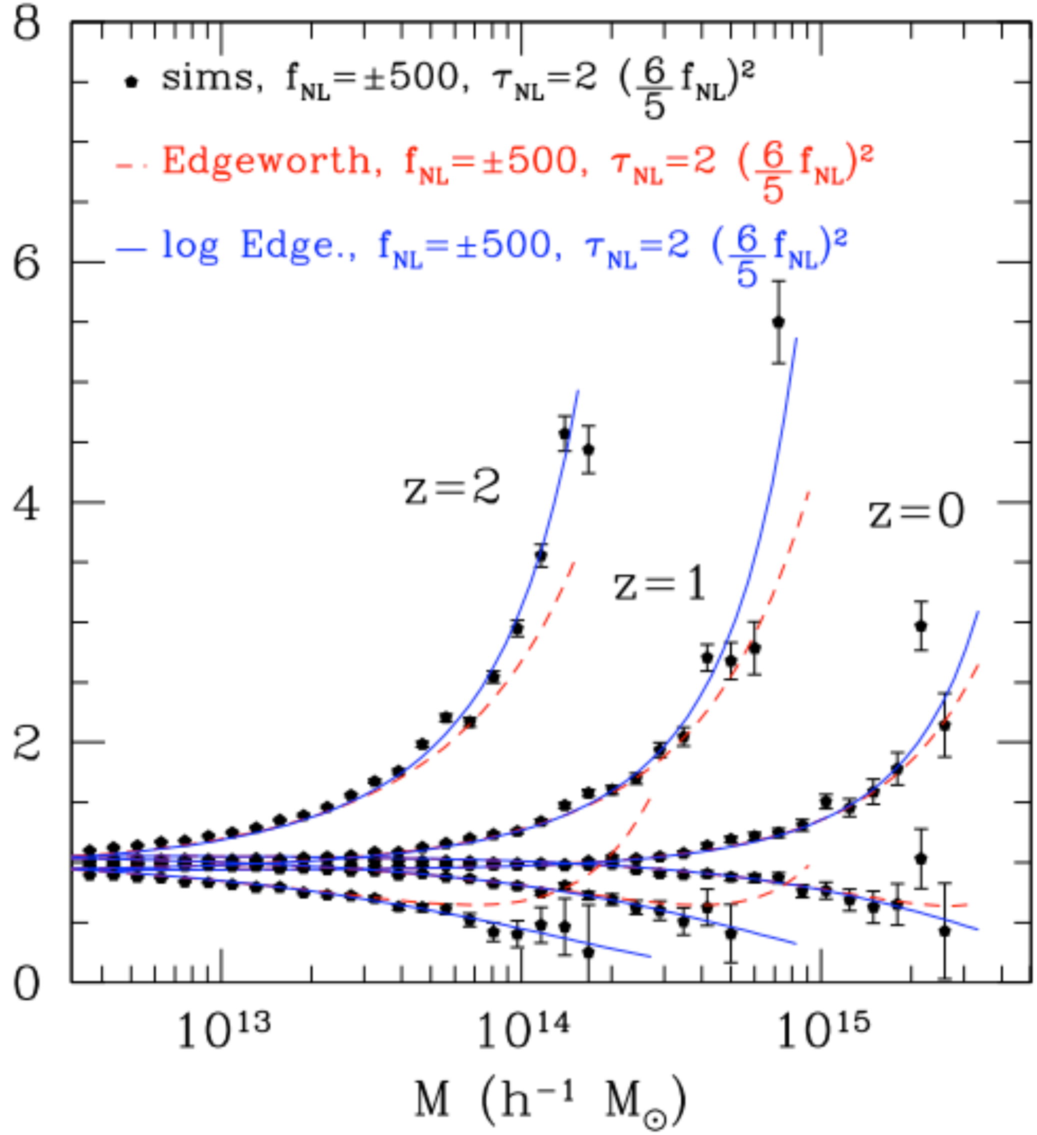
Dark matter halos form in the peaks of the density field



Non-Gaussianity changes the number density of the peaks

This in turn affects the halo mass function

Measuring primordial non-Gaussianity: halo abundance



Largest effect is seen at highest masses

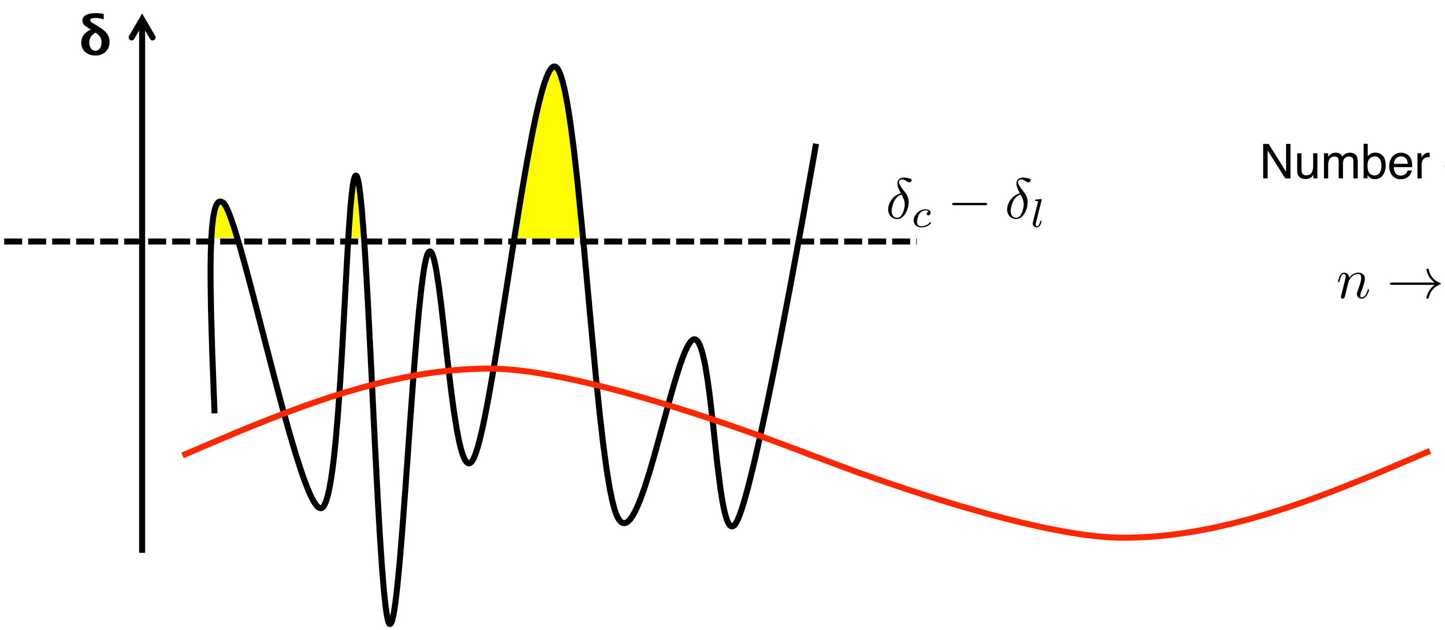
Insensitive to shape of bispectrum

But difficult to observe – relies on cluster masses being precisely known

LoVerde & Smith 2010

Peak-background split galaxy bias model

Halo formation much easier with additional long-wavelength fluctuation



Number density of halos

$$n \to n - \frac{dn}{d\delta_c}\delta_l$$

Directly from the large-scale mode

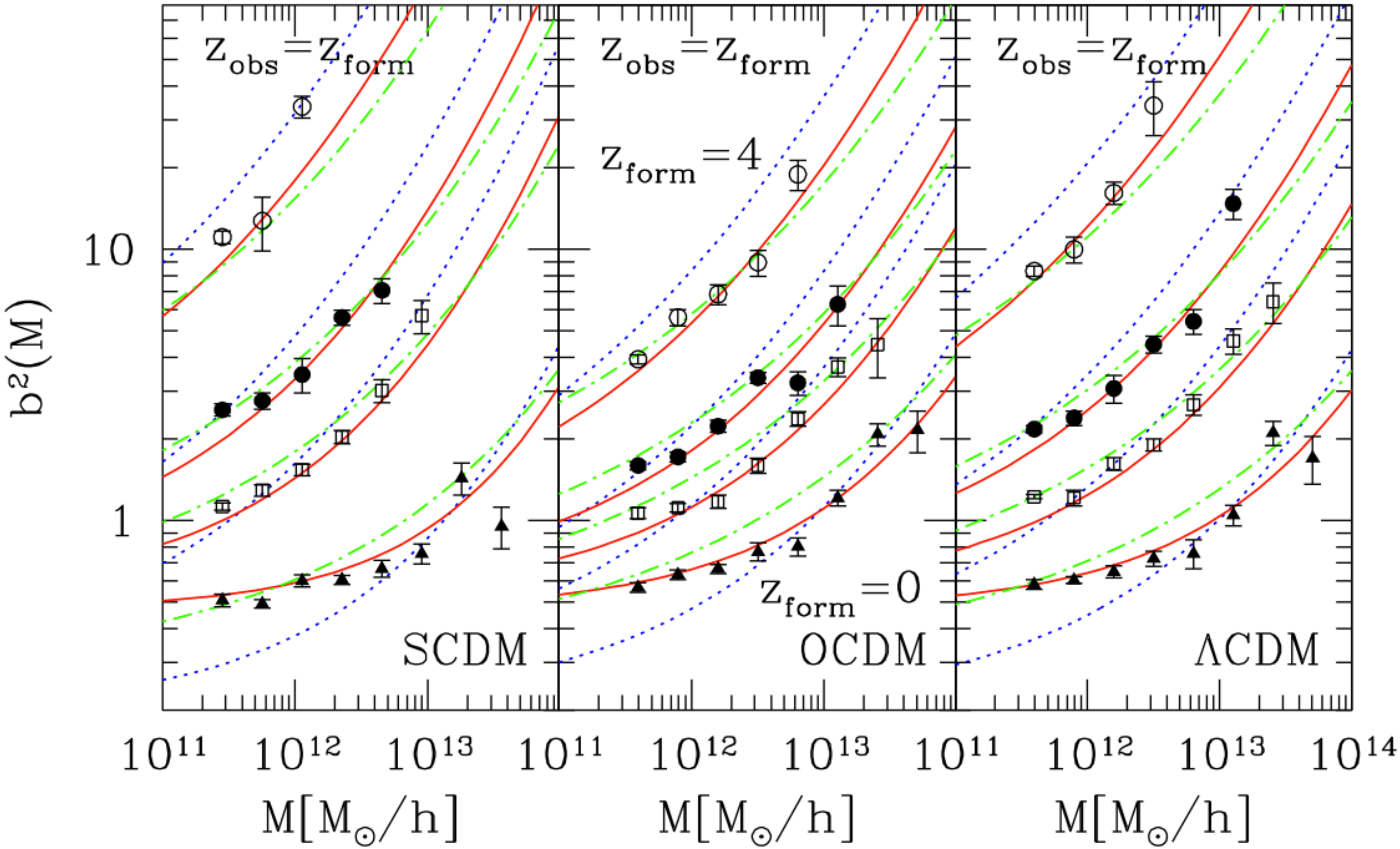
Leads to a revised density $\rho_{\rm new} = \left(1 + \frac{\Delta n}{n}\right)(1 + \delta_l)$

From the change in Number of haloes

To first order, this leads to a bias

$$\delta_{\rm new} = \left(1 + \frac{\Delta n}{n}\right)\delta_l \qquad b = 1 - \frac{d\ln n}{d\delta_c}$$

Peak-background split galaxy bias model



Sheth & Tormen 1999

This is altered by f_{NL} signal

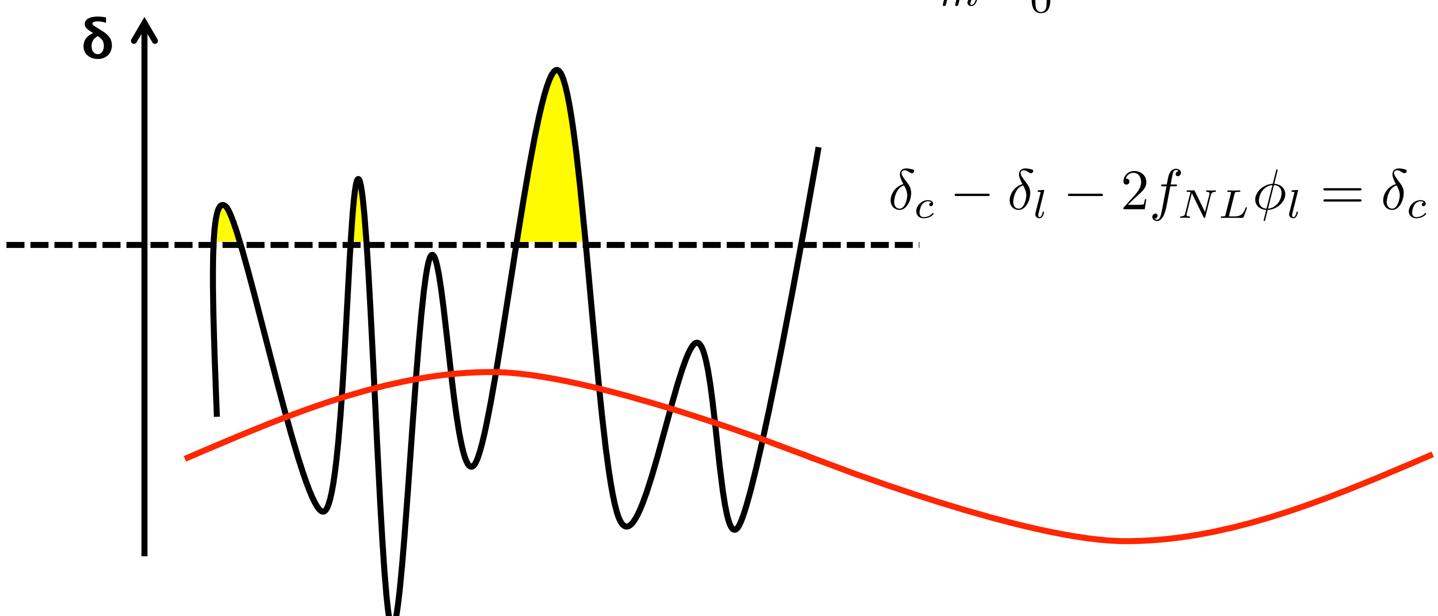
Now split non-Gaussian potential into long and short wavelength components

$$\Phi(\mathbf{x}) = \phi_l + \underbrace{f_{NL}\phi_l^2}_{\text{small}} + (1 + \underbrace{2f_{NL}\phi_l})\phi_s + f_{NL}\phi_s^2 + \text{cnst}$$

Link between potential and overdensity field shows how changing long wavelength potential component changes "critical density"

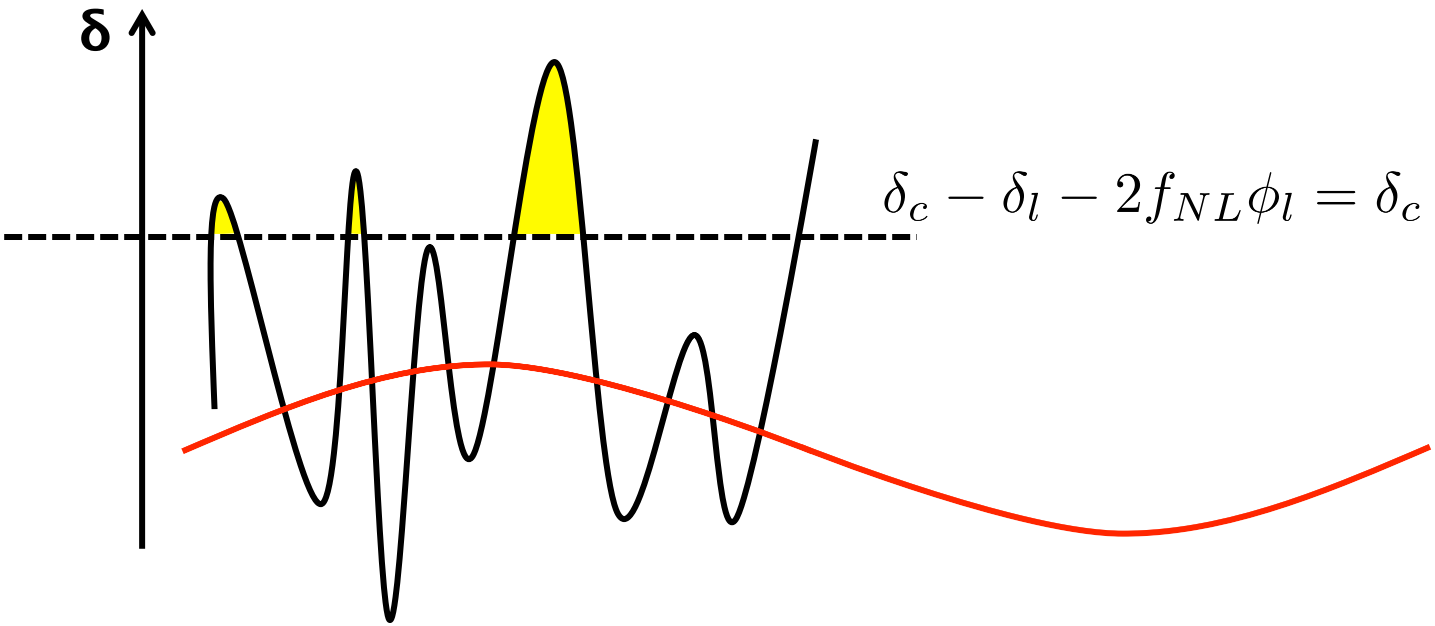
$$\delta_l(k) = \alpha(k)\Phi(k) \quad \alpha(k) = \frac{2c^2k^2T(k)D(z)}{3\Omega_m H_0^2}$$

$$\delta_c - \delta_l - 2f_{NL}\phi_l = \delta_c - \delta_l\left(1 + \frac{2f_{NL}}{\alpha(k)}\right)$$



Peak-background split for non-Gaussian primordial fluctuations

Halo formation much easier with additional long-wavelength fluctuation



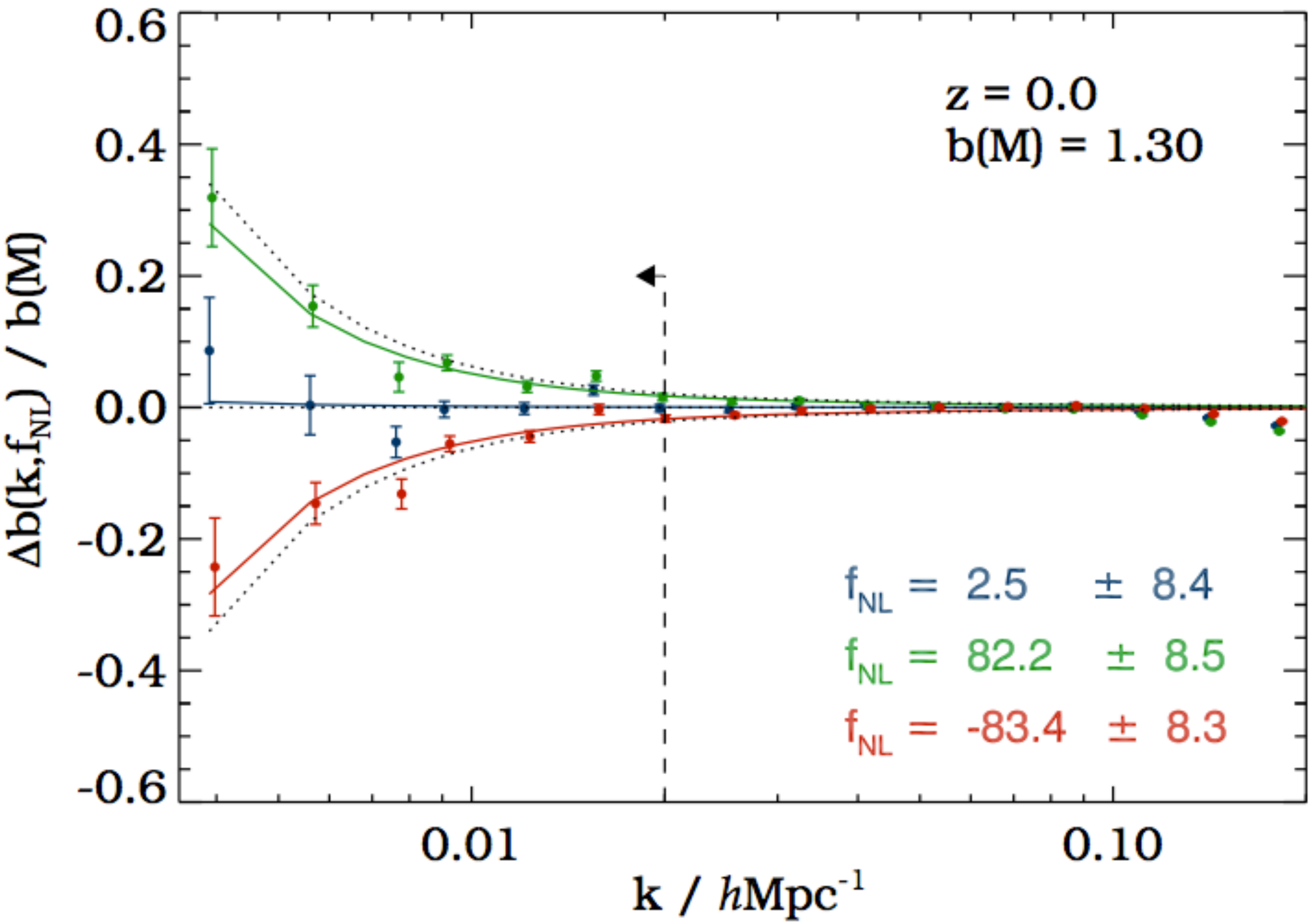
$$\delta_c - \delta_l - 2f_{NL}\phi_l = \delta_c - \delta_l \left(1 + \frac{2f_{NL}}{\alpha(k)}\right)$$

Number density of halos $$n \rightarrow n - \left(1 + \frac{2f_{NL}}{\alpha(k)}\right)\frac{dn}{d\delta_c}\delta_l$$

Leads to a revised bias

$$b = 1 - \left(1 + \frac{2f_{NL}}{\alpha(k)}\right)\frac{d\ln n}{d\delta_c}$$

K² dependence in simulations



Dalal, Doré, Huterer, Shirokov 2007 ; Smith, LoVerde 2010; Smith, Ferraro, LoVerde 2011; Pillepich, Porciani, Hahn 2008; Desjacques, Seljak, Iliev 2008; Grossi et al 2009; Shandera, Dalal, Huterer 2010; Hamaus et al. 2011

Pulling it all together

Physics from the linear galaxy power spectrum

Information from power spectrum shape
- Matter density
- Baryon Acoustic Oscillations
- Neutrino mass
- Inflation fluctuation spectrum
- f_{NL}

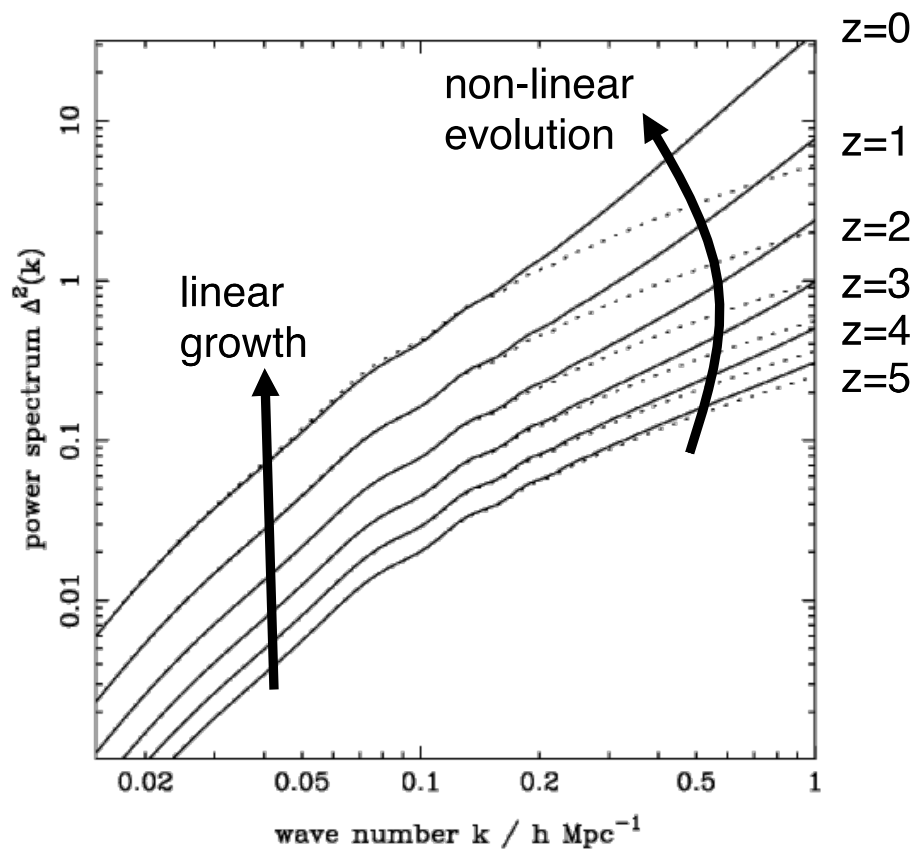
Information from geometry
- Galaxy clustering as a standard ruler
- BAO or full power spectrum
- Alcock-Paczynski effect

$$P_{\rm gal}(k,\mu,a) = k^n T^2(k) D^2(a)[b(a) + f(a)\mu^2]^2$$

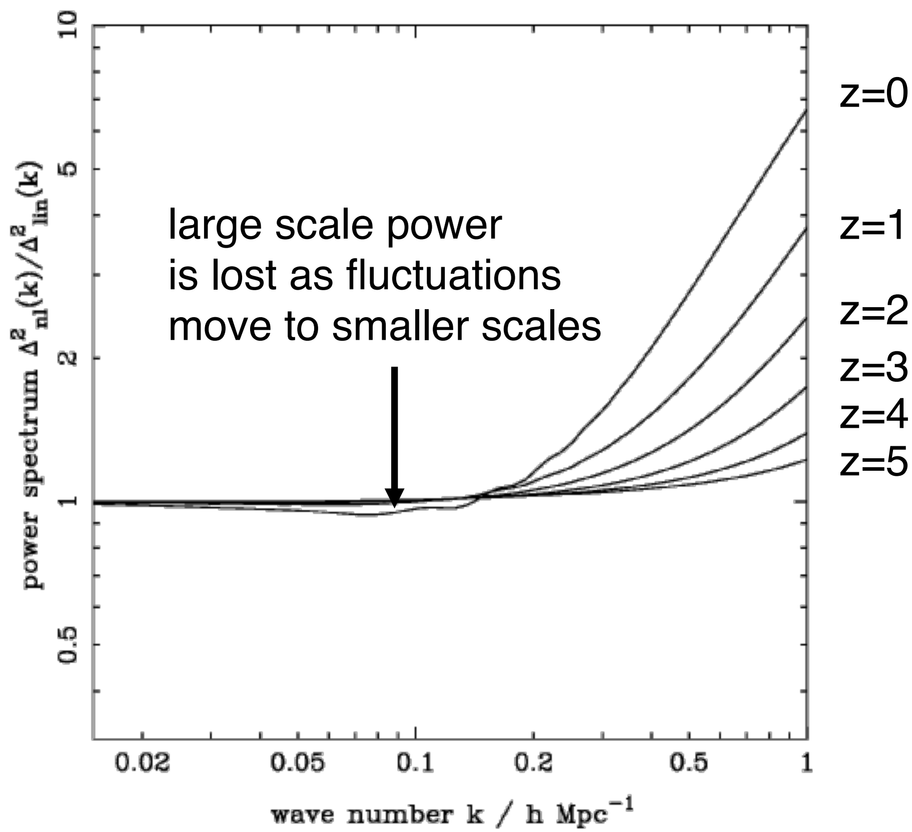
k = comoving wavenumber
μ = cos(angle to line-of-sight)
a = cosmological scale factor
b = galaxy bias factor
D = linear growth rate
f = dlnD/dlna

Information from structure growth
- amplitude of power spectrum
- redshift-space distortions

Linear vs Non-linear behaviour



Cannot easily measure growth directly from galaxy surveys as degenerate with galaxy bias



P(k) calculated from Smith et al. 2003, MNRAS, 341,1311 fitting formulae

Model parameters (describing LSS & CMB)

content of the Universe

total energy density
Ω_{tot} (=1?)
matter density
Ω_m
baryon density
Ω_b
neutrino density
Ω_n (=0?)
Neutrino species
f_n
dark energy eq^n of state
$w(a)$ (=-1?)
or **w_0, w_1**

perturbations after inflation

scalar spectral index
n_s (=1?)
normalisation
σ_8
running
$a = dn_s/dk$ (=0?)
tensor spectral index
n_t (=0?)
tensor/scalar ratio
r (=0?)

evolution to present day

Hubble parameter
h
Optical depth to CMB
τ

parameters usually marginalised and ignored

galaxy bias model
$b(k)$ (=cst?)
or **b, Q**
CMB beam error
B
CMB calibration error
C

Assume Gaussian, adiabatic fluctuations

Multi-parameter fits to multiple data sets

- Given CMB data, other data are used to help break degeneracies (although CMB is now doing a pretty good job by itself) and understand dark energy

- Main problem is keeping a handle on what is being constrained and why
  – difficult to allow for systematics
  – you have to believe all of the data!

- Have two sets of parameters
  – those you fix (part of the prior)
  – those you vary

- Need to define a prior
  – what set of models
  – what prior assumptions to make on them (usual to use uniform priors on physically motivated variables)

- Need a sampling method for exploring multi-dimensional parameter space

Markov-Chain Monte-Carlo method

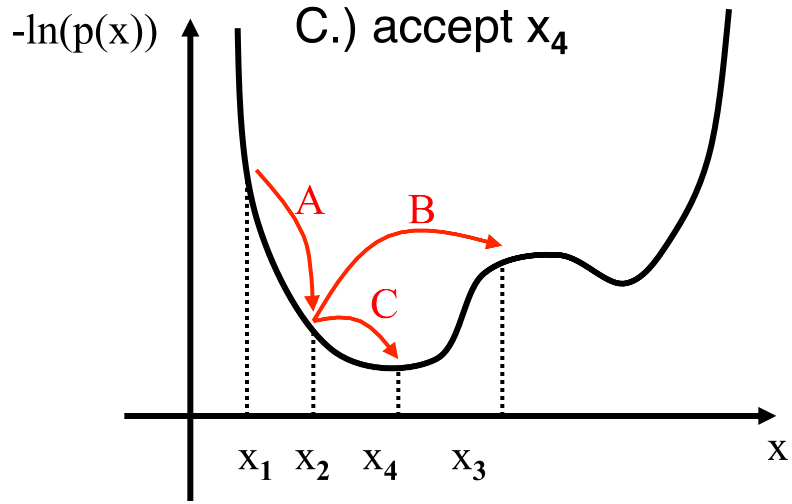
MCMC method maps the likelihood surface by building a chain of parameter values whose density at any location is proportional to the likelihood at that location p(x)

given a chain at parameter x, and a candidate for the next step x', then x' is accepted with probability

$$\begin{cases} 1 & p(x') > p(x) \\ p(x')/p(x) & \text{otherwise} \end{cases}$$

for any symmetric proposal distribution q(x|x') = q(x'|x), then an infinite number of steps leads to a chain in which the density of samples is proportional to p(x).

an example chain starting at x_1
A.) accept x_2
B.) reject x_3
C.) accept x_4

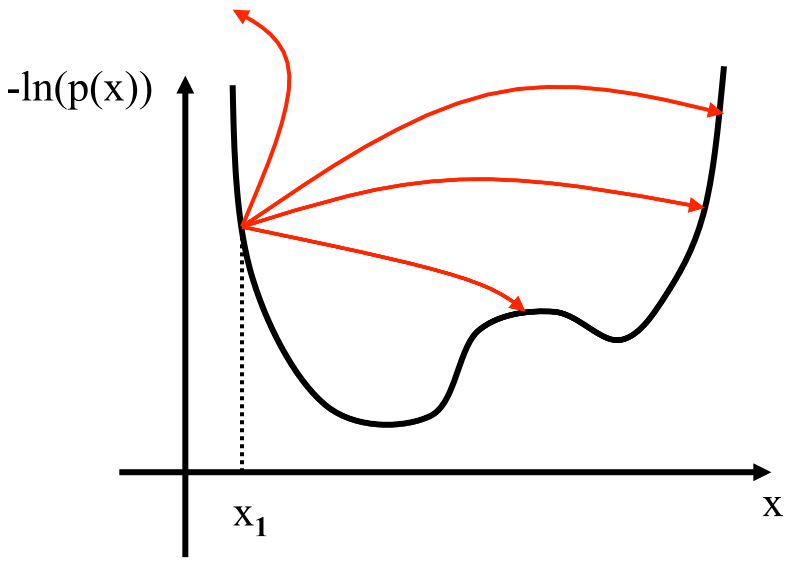


CHAIN: x_1, x_2, x_2, x_4, ...

MCMC problems: jump sizes

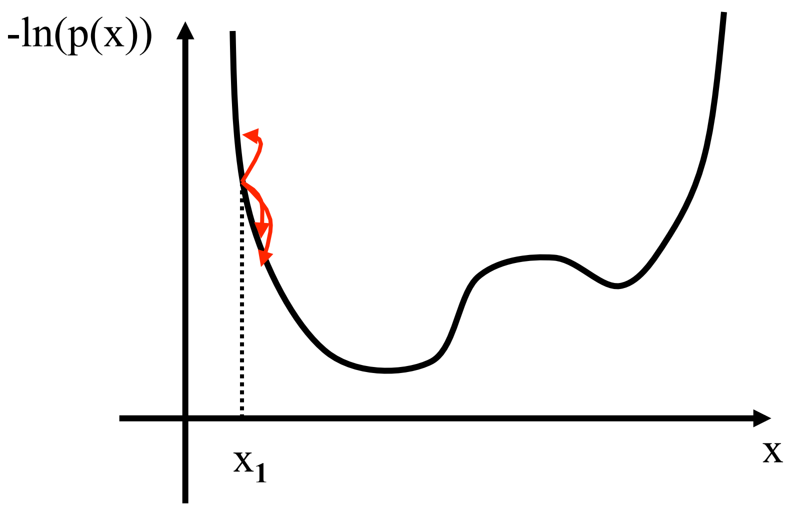
q(x) too broad

chain lacks mobility
as all candidates are
unlikely



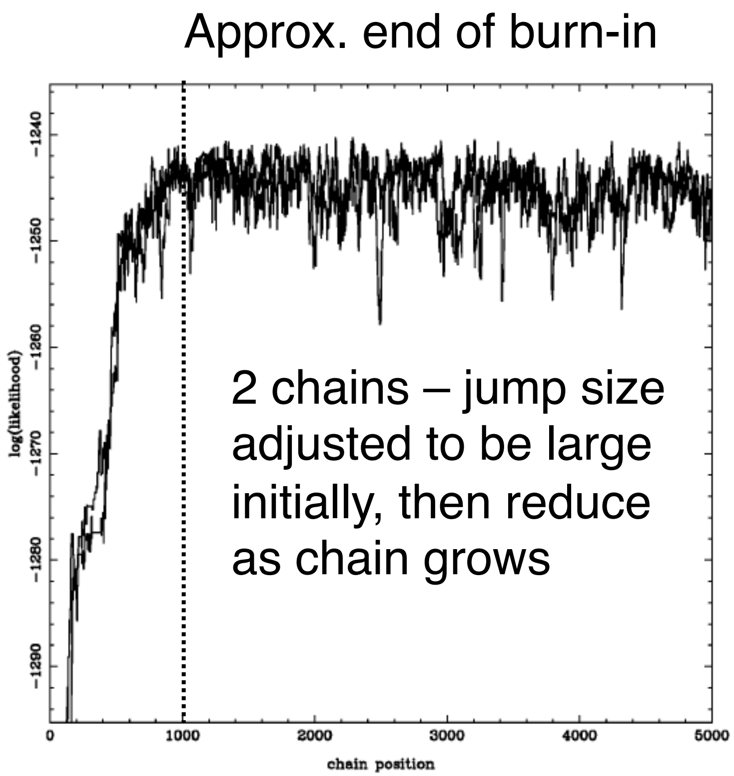
q(x) too narrow

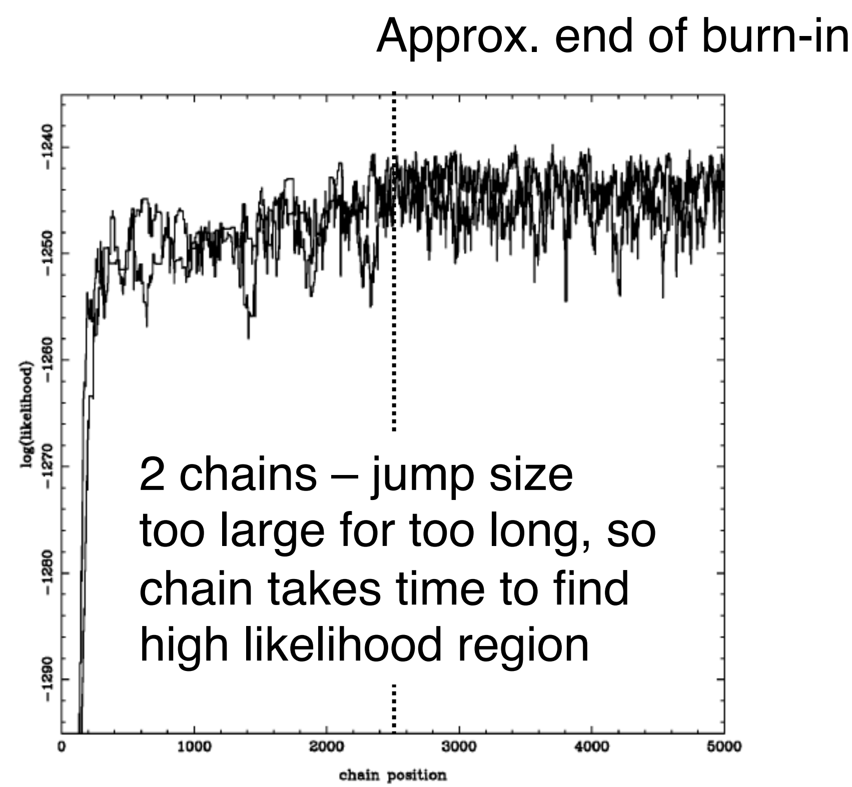
chain only moves
slowly to sample all
of parameter space



MCMC problems: burn in

Chain takes some time to reach a point where the initial position chosen has no influence on the statistics of the chain (dependent on the proposal distribution q(x))





MCMC problems: convergence

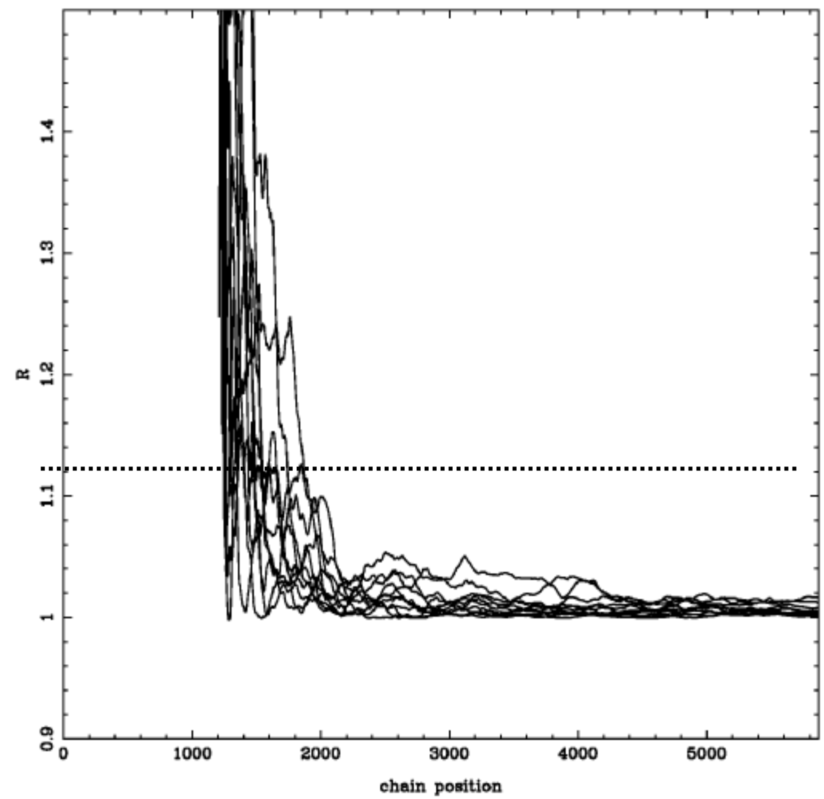
How do we know when the chain has sampled the likelihood surface sufficiently well, that the mean & std deviation for each parameter are well constrained?

Gelman & Rubin (1992) convergence test:

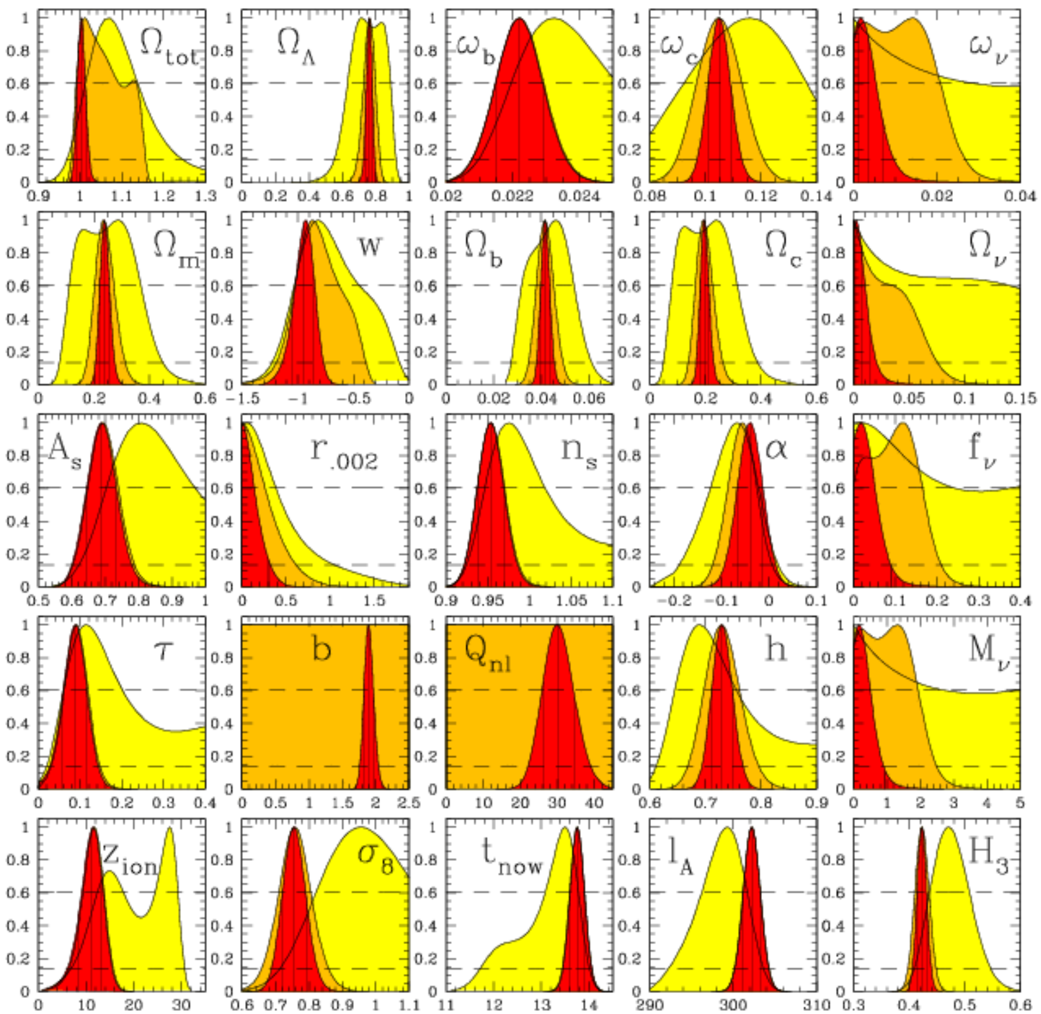
Given M chains (or sections of chain) of length N, Let W be the average variance calculated from individual chains, and B be the variance in the mean recovered from the M chains. Define

$$R = \frac{N-1}{N} + \frac{1}{W}\left(1 + \frac{B}{N}\right)$$

Then R is the ratio of two estimates of the variance. The numerator is unbiased if the chains fully sample the target, otherwise it is an overestimate. The denominator is an underestimate if the chains have not converged. Test: set a limit R<1.1



Resulting constraints



Tegmark et al, 2006, arXiv:0608632

Further reading

- Coles & Lucchin, “Cosmology: the origin and evolution of cosmic structure”, Wiley
  - Good peculiar velocities section
- RSD
  - review by Hamilton (1997), astro-ph/9708102
- Alcock-Paczynski
  - Alcock & Paczynski (1979), Nature 281, 358
- f_{NL}
  - review by Desjques & Seljak (2010), arXiv:1006.4763
- Combined constraints
  - CosmoMC, Lewis & Bridle (2002), astro-ph/0205436
  - WMAP papers
  - Sanchez et al. (2005), astro-ph/0507538
  - Tegmark et al. (2006), astro-ph/0608632
  - Spergel et al. (2007), ApJSS, 170, 3777