2354-26

Summer School on Cosmology

16 - 27 July 2012

Lensing - Lecture 1

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Gravitational lensing

Henk Hoekstra

What is it good for?

Gravitational lensing has developed into a key tool to measure masses of objects, irrespective of their dynamical state.

Another important application is the study of the statistical properties of the matter distribution to constrain cosmological parameters.

Other useful applications are the detailed study of distant magnified objects and the clumpiness of halos through microlensing. The latter can also be used for population studies of planets.

Some history

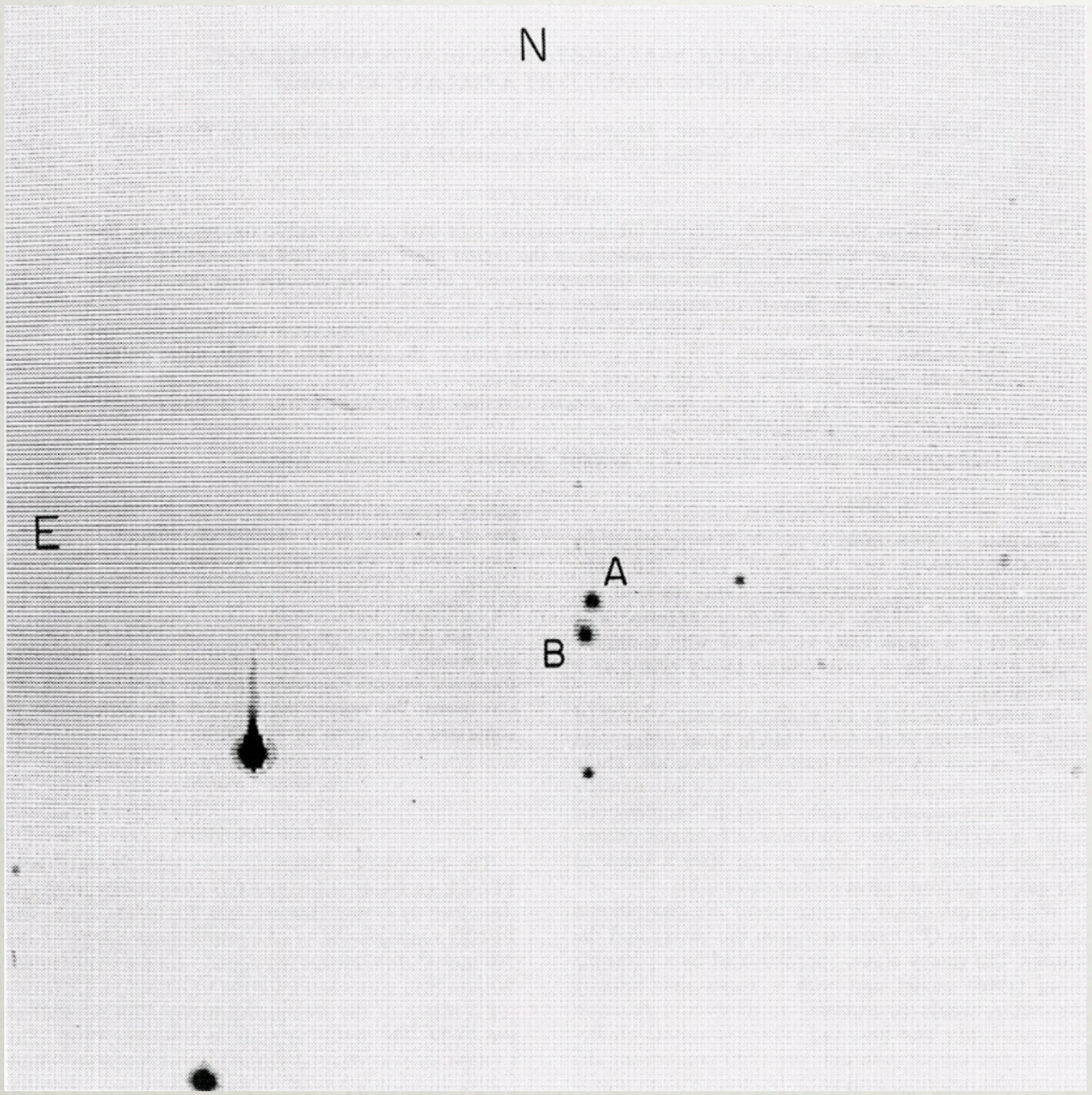
That gravitational lensing occurs was long known: it was used by Eddington to test General Relativity.

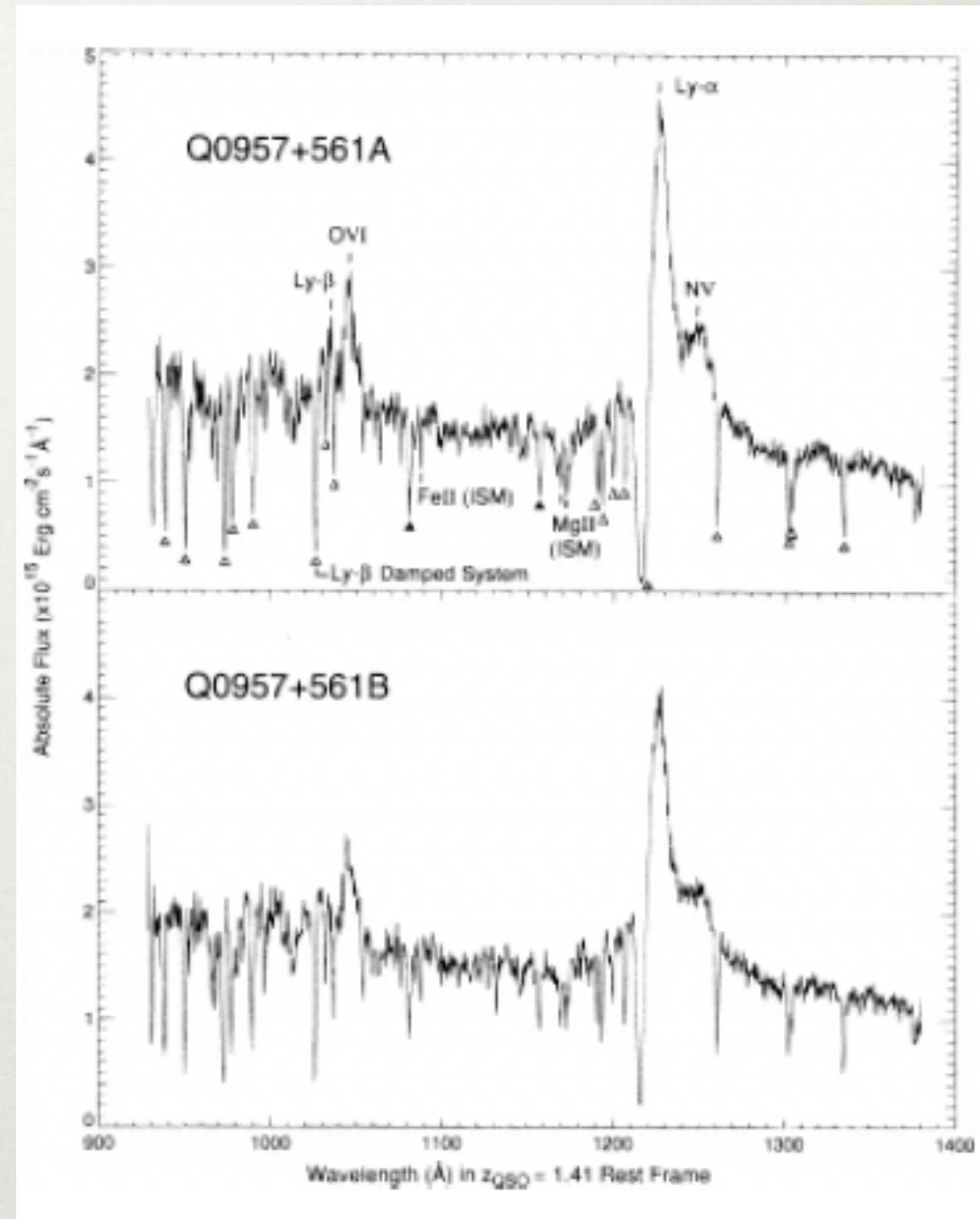
Its use for the study of galaxies/clusters was proposed in the '30s by Zwicky.

Some theoretical work was done in the '60s, but it was considered mostly a waste of time...

A major milestone was the discovery of the first lensed quasar in 1979.

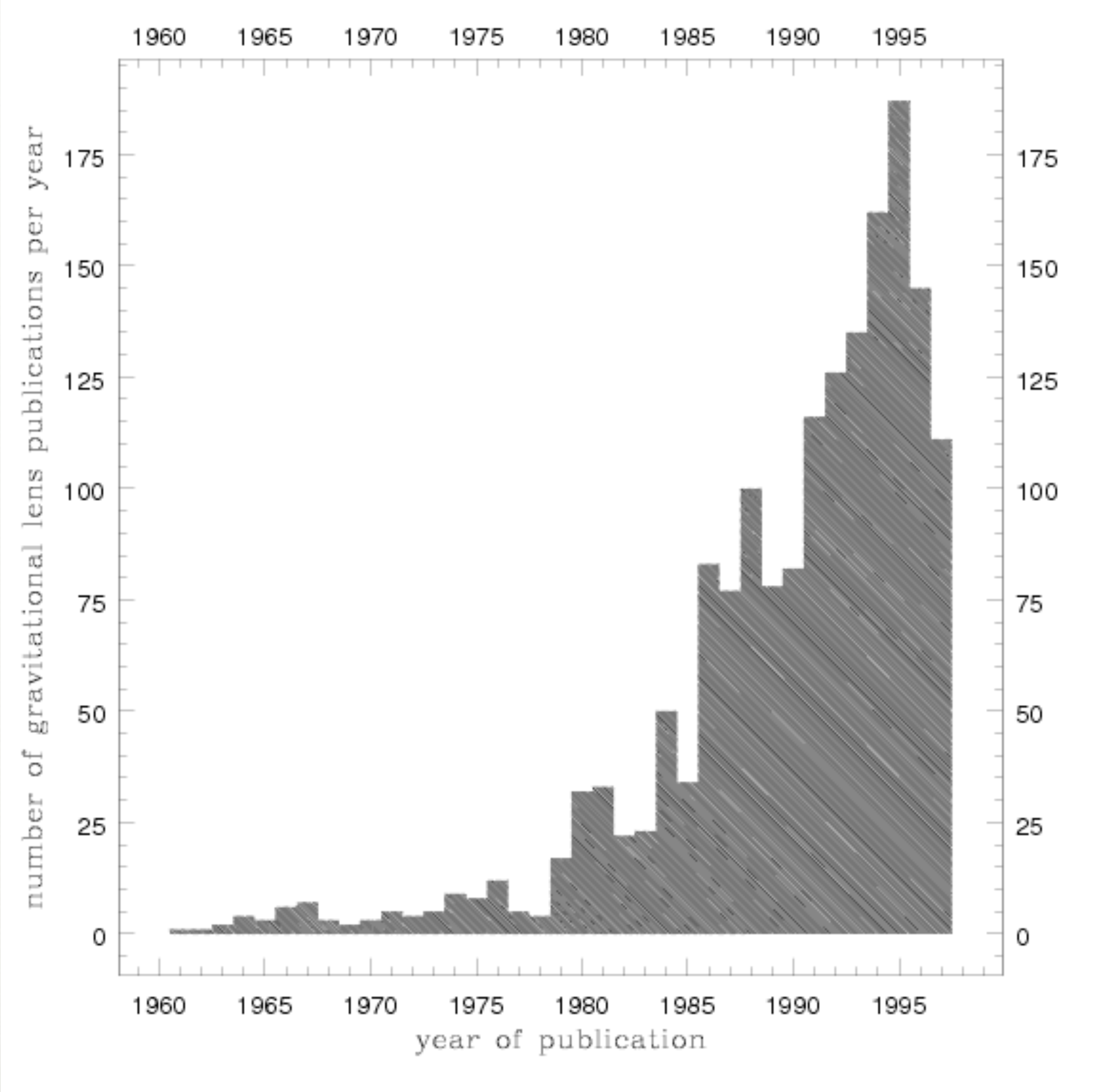
First Lens





First lens QSO0957+561 was discovered in 1979 (Walsh et al.)

RAPIDLY CHANGING...



Rapidly changing...

Number of refereed papers with "gravitational lensing" mentioned in the abstract in ADS.

1980	27
1985	35
1990	116
1995	220
2000	301
2005	315
2010	351
2011	333

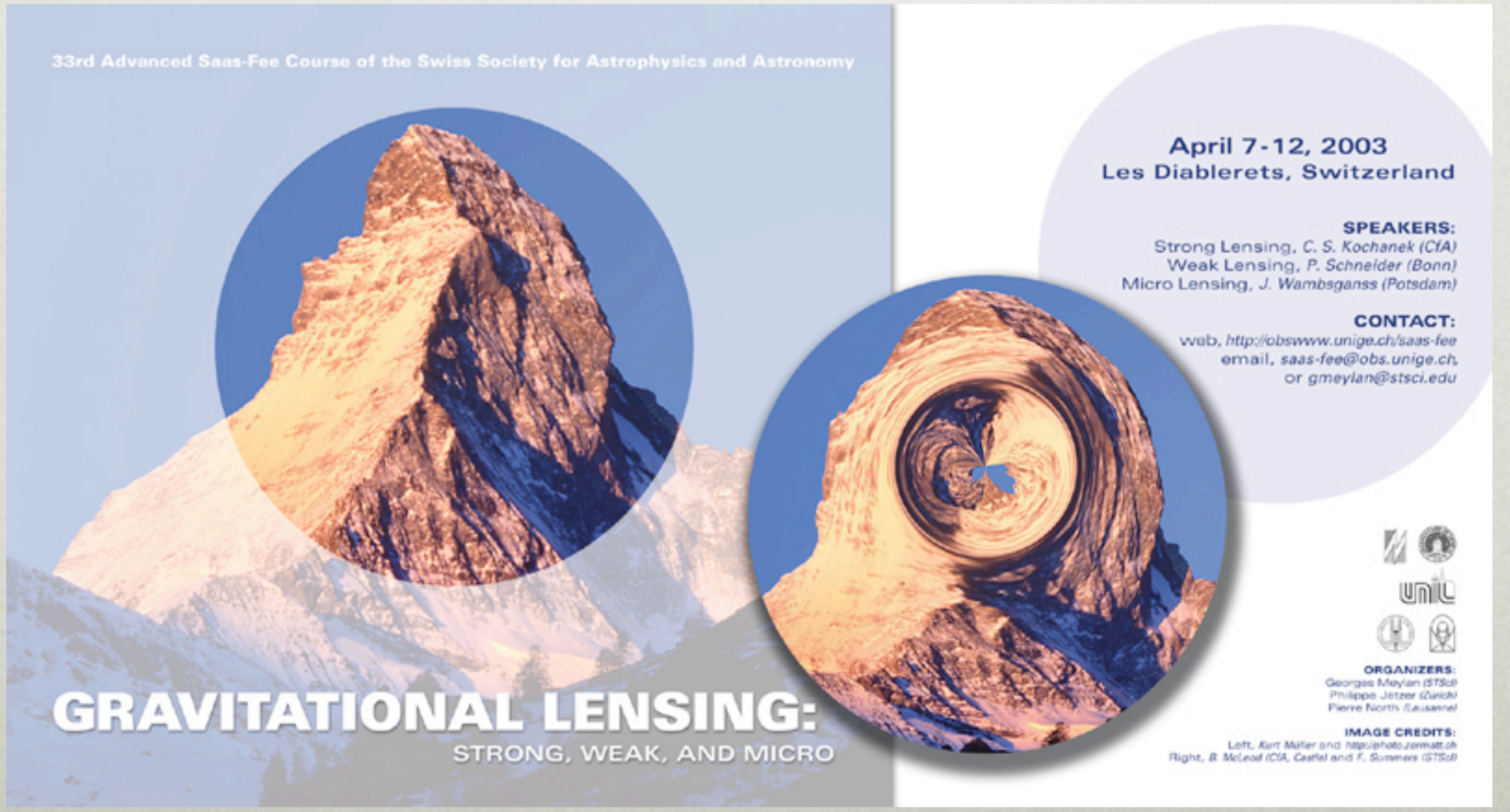
Four lectures

- Introduction
- Cosmic shear/Interpretation
- Measurements/Tests
- Applications/Results

Useful resources

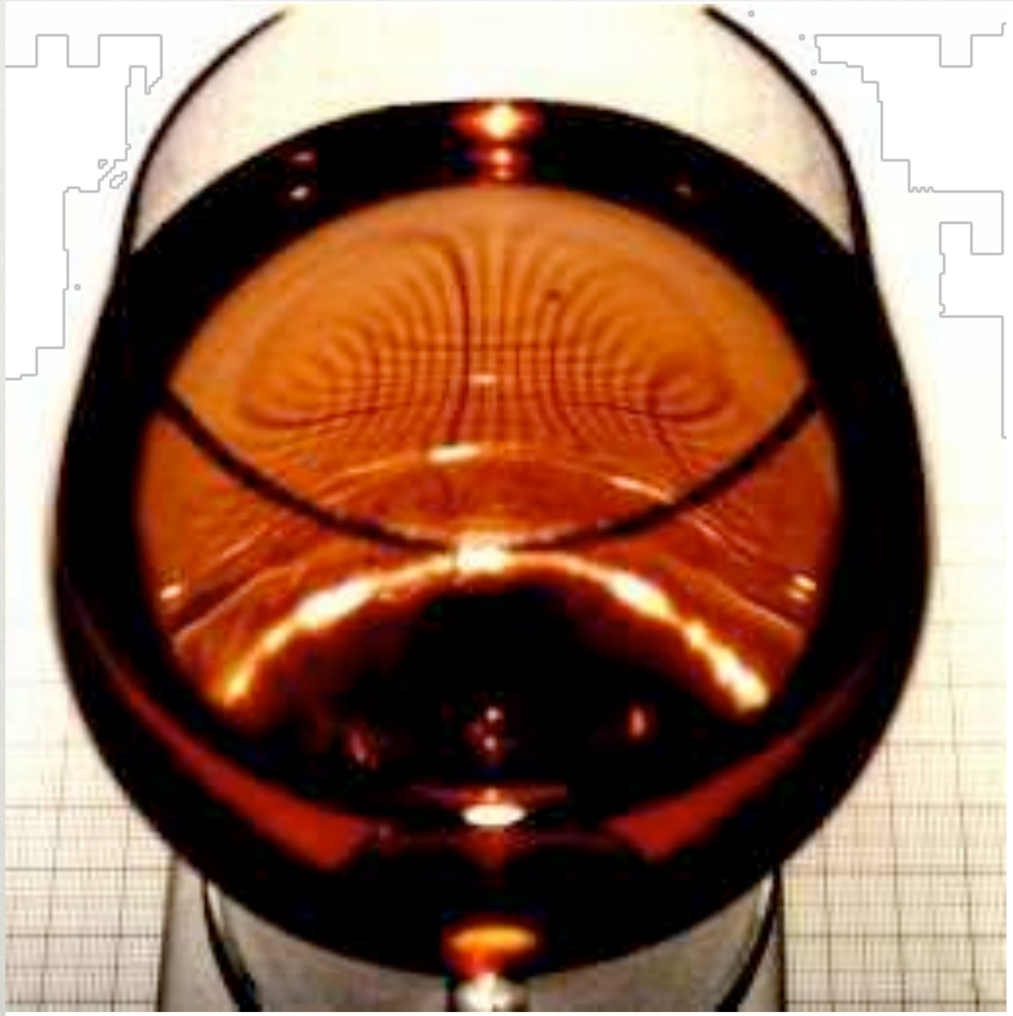
Lecture notes by P. Schneider for the 33rd SAAS-FEE Advanced Course:

http://www.astro.uni-bonn.de/~peter/SaasFee.html



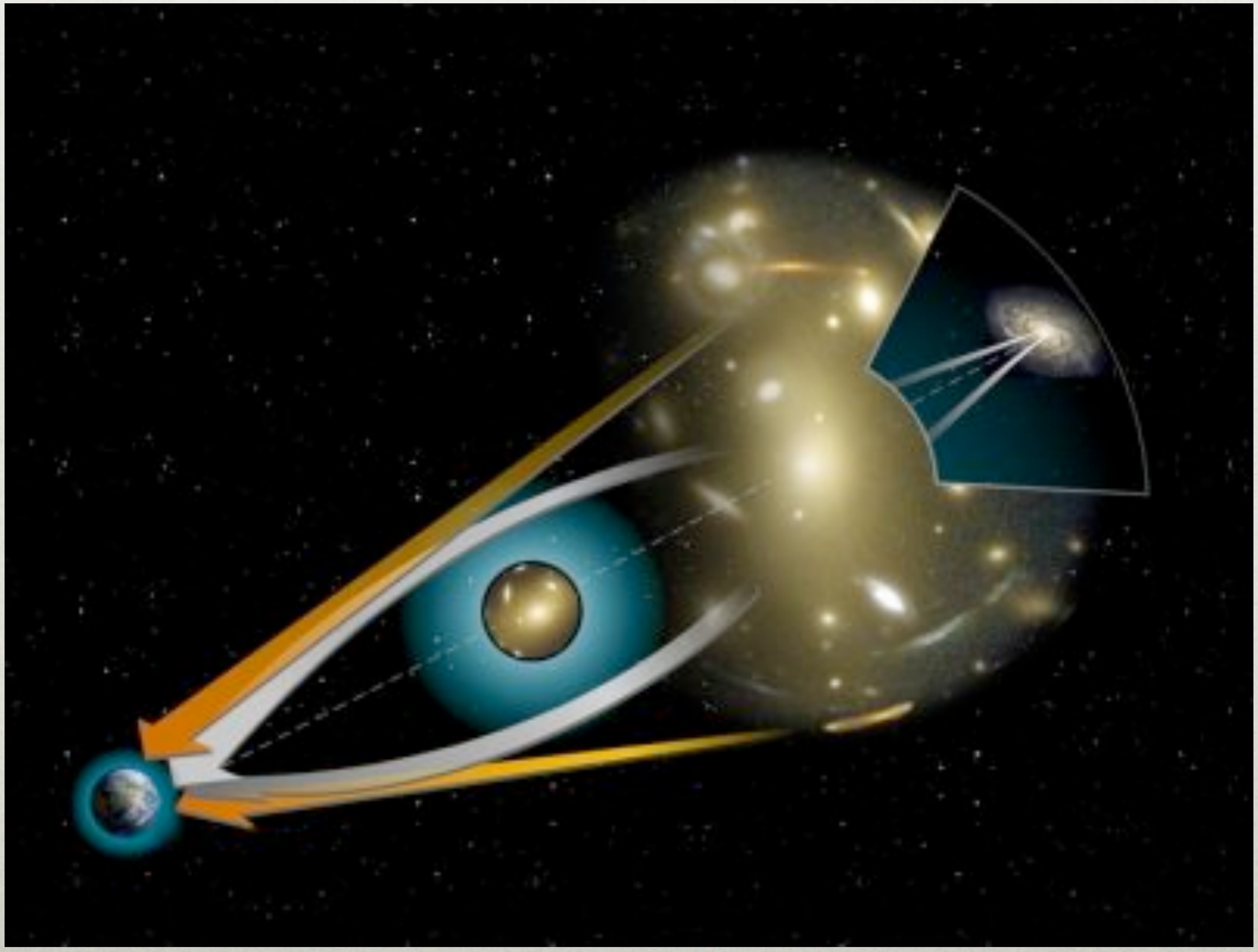
Introduction

Light rays are deflected when they propagate through an inhomogenuous *medium* (following Fermat's principle)

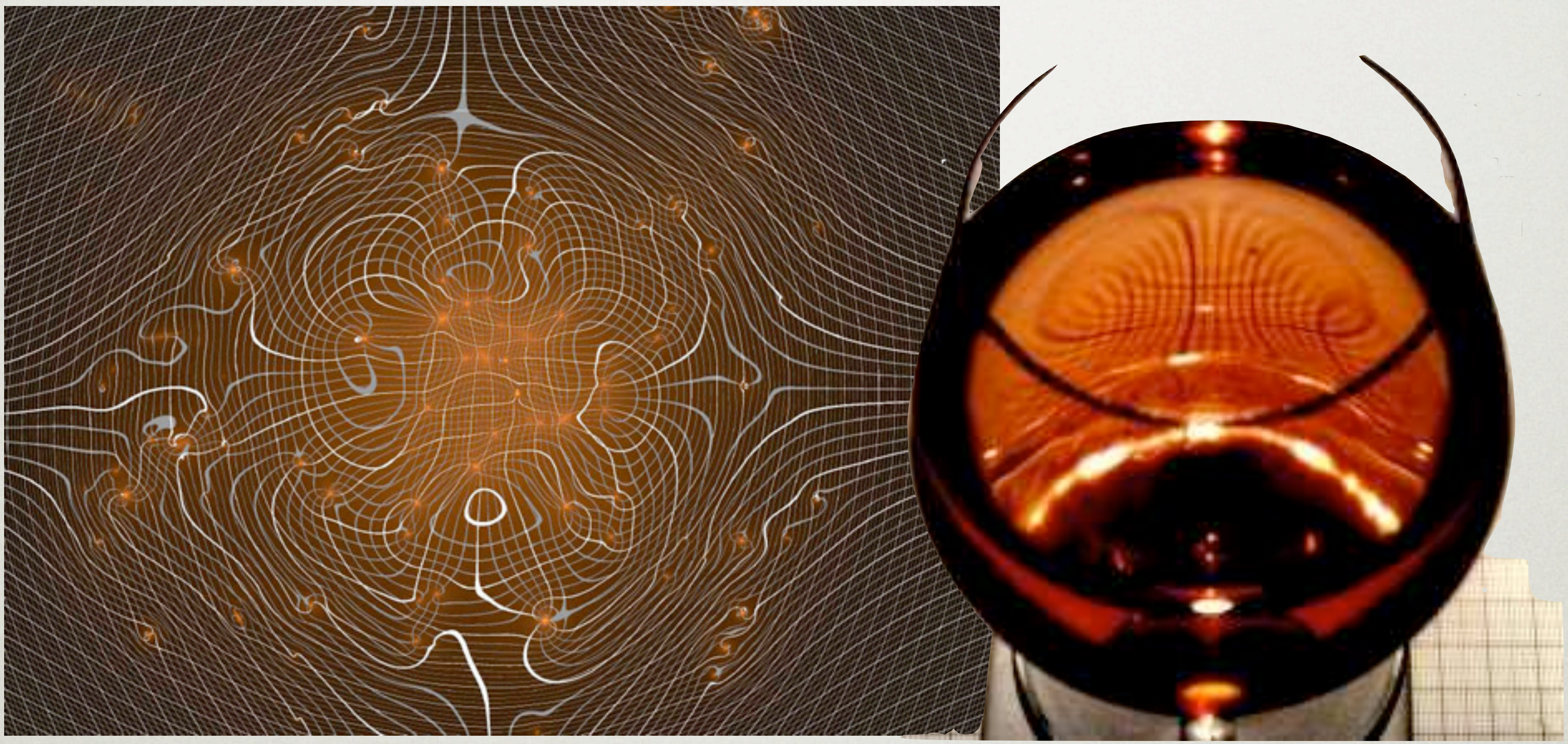


Introduction

Light rays are deflected when they propagate through an inhomogenuous *gravitational field*.

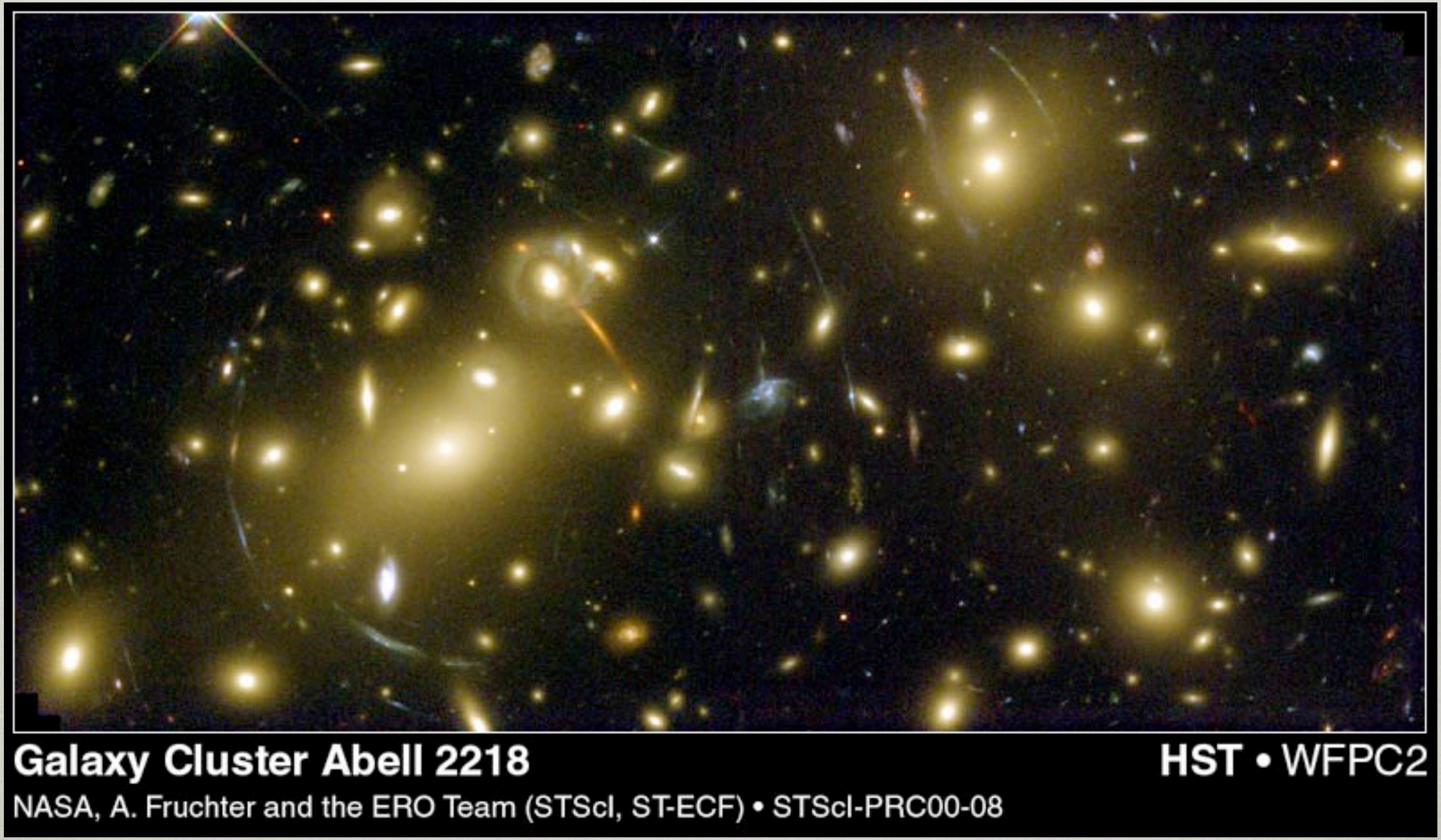


Introduction



The cluster mass distribution causes a distortion in the shapes of background galaxies. The leads to spectacular lensing examples.

Introduction



Lensing=Optics

Let us assume:

- gravitational field is weak
- deflection angles are small
- deflection occurs on scales << scale of the Universe

We can define a coordinate system in which the GR line element is

$$d\tau^2 = (c^2 + 2\Phi)dt^2 - (1 - 2\Phi / c^2)ds^2$$

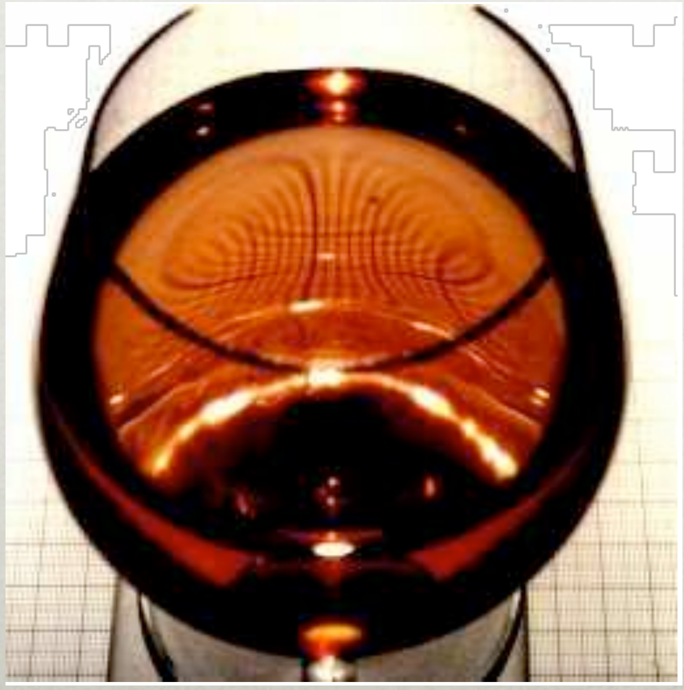
and Φ is the Newtonian potential

Lensing=Optics

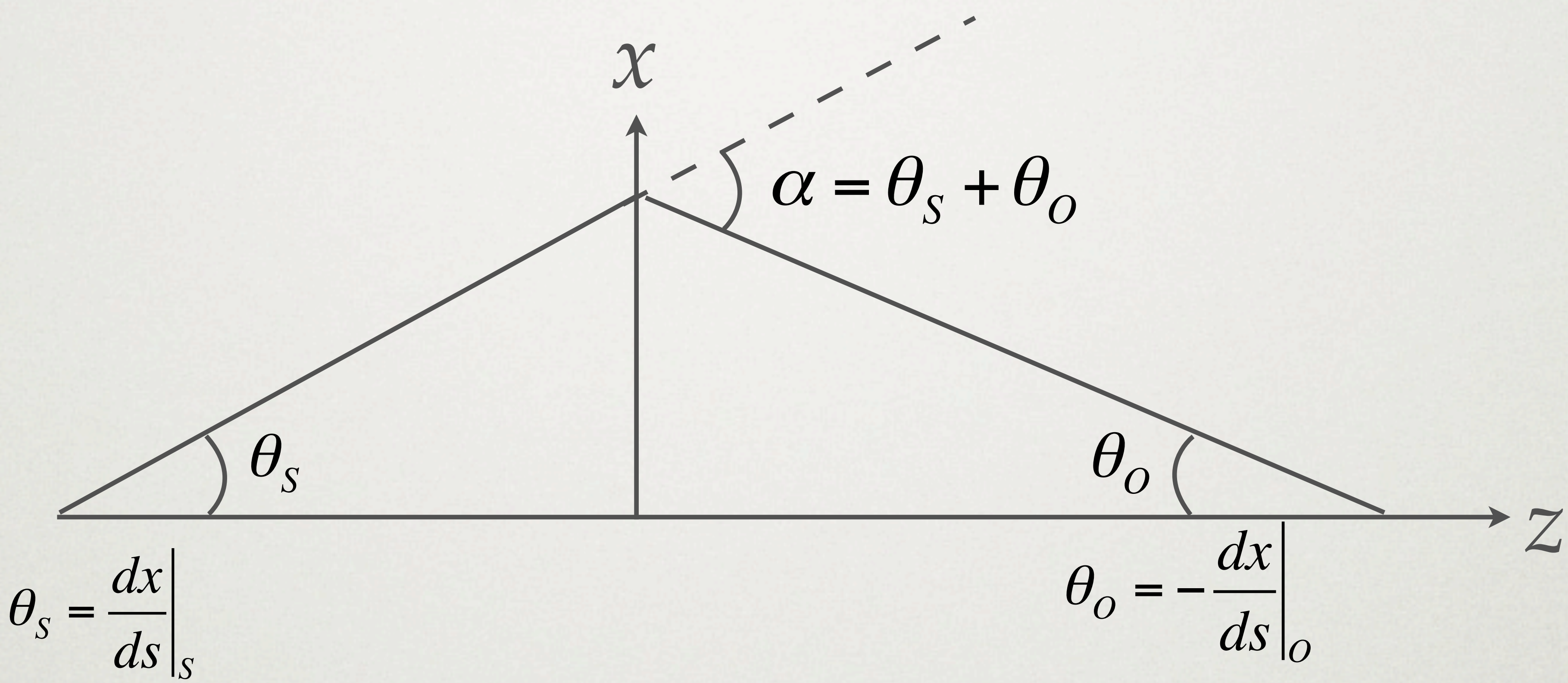
We can now use Fermat's principle: $d\tau$ has to vanish

$$dt = \sqrt{\frac{1-2\Phi/c^2}{c^2+2\Phi}}ds \approx \frac{1}{c}\left(1-2\frac{\Phi}{c^2}\right)ds = \frac{n}{c}ds$$

Newtonian potential plays role of "index of refraction"



Lensing=Optics



Photons follow the path for which the light travel time

$$t = \frac{1}{c}\int n \cdot ds$$

is stationary to small changes in the path.

Lensing=Optics

Through variational analysis one can show that

$$\vec{\alpha} = \int_S^O ds \vec{\nabla}_\perp n = -\frac{2}{c^2} \int_S^O ds \vec{\nabla}_\perp \Phi$$

Using $\Phi(\vec{x}) = -G \int d^3 x' \frac{\rho(\vec{x}')}{\left|\vec{x} - \vec{x}'\right|}$

We find that:

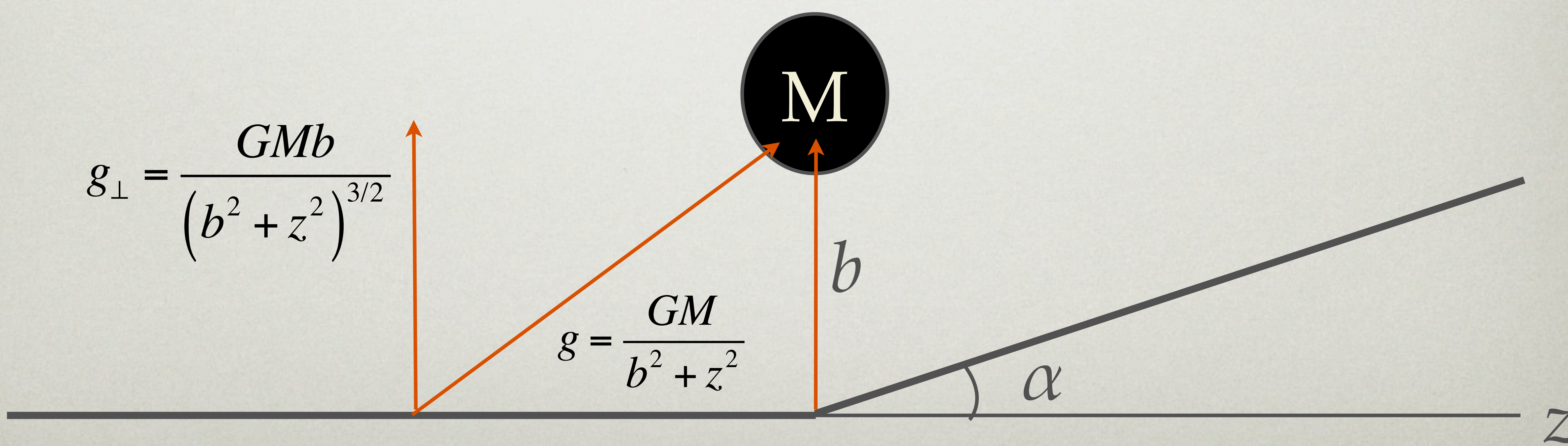
$$\alpha(\vec{x}) = -\frac{4G}{c^2} \vec{\nabla} \int d^2 \vec{x}' \Sigma(\vec{x}') \ln\left|\vec{x} - \vec{x}'\right|$$

Deflection by a point mass

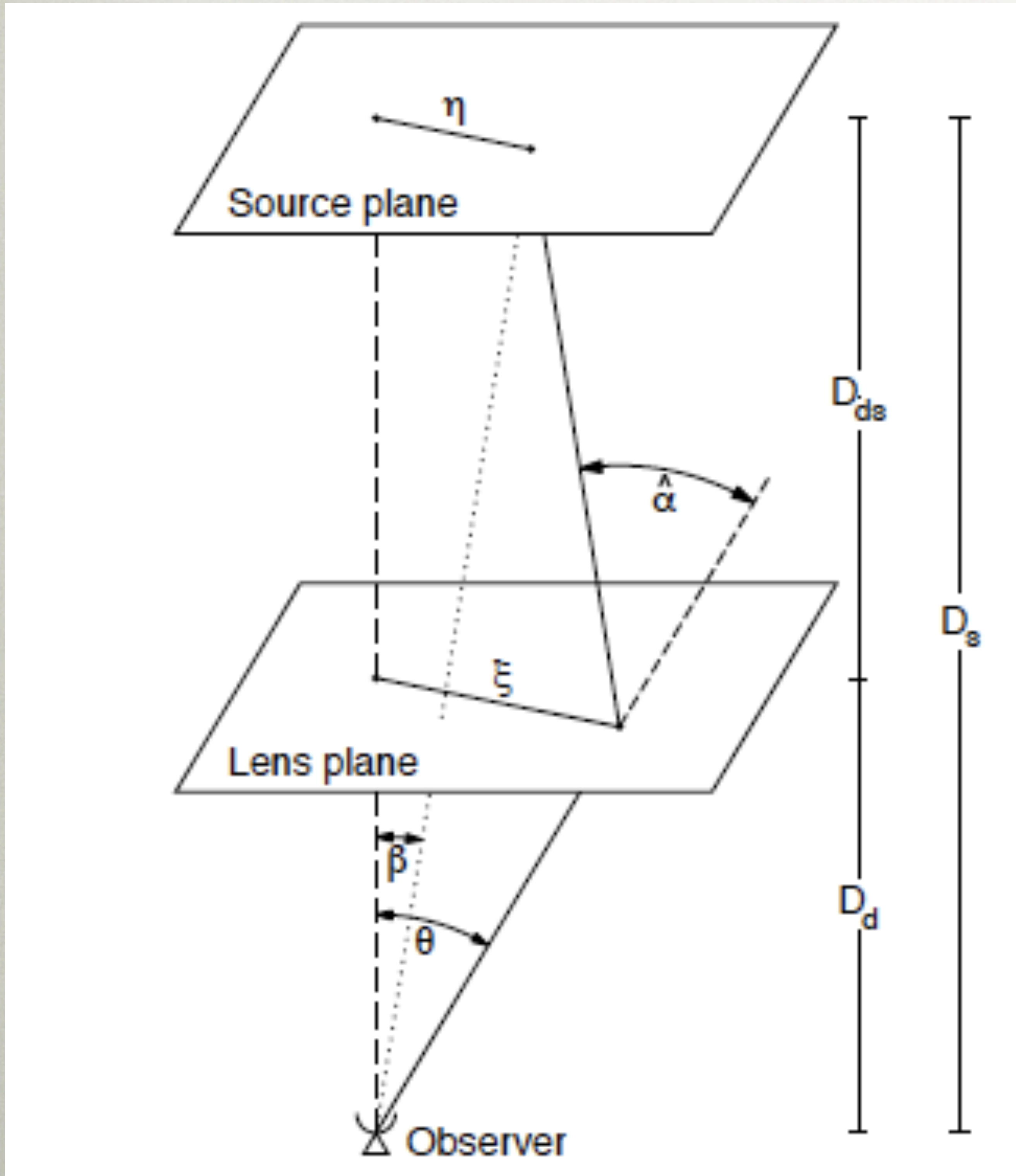
GR weak field deflection is 2x Newtonian:

$$\alpha = 2 \times \frac{1}{c} \int \frac{dz}{c} g_{\perp} = \frac{4GM}{bc^2}$$

Any mass distribution is just the sum of point masses, which leads to the previous result in terms of surface mass density.



Lens equation



$$\boldsymbol{\eta} = \frac{D_\mathrm{s}}{D_\mathrm{d}}\,\boldsymbol{\xi} - D_\mathrm{ds}\hat{\boldsymbol{\alpha}}(\boldsymbol{\xi})$$

If we introduce angular coordinates:

$$\boldsymbol{\eta} = D_\mathrm{s}\boldsymbol{\beta} \ \text{ and } \ \boldsymbol{\xi} = D_\mathrm{d}\boldsymbol{\theta}$$

and define the scaled deflection angle, then we obtain the lens equation:

$$\boldsymbol{\beta} = \boldsymbol{\theta} - \frac{D_\mathrm{ds}}{D_\mathrm{s}}\,\hat{\boldsymbol{\alpha}}(D_\mathrm{d}\boldsymbol{\theta}) \equiv \boldsymbol{\theta} - \boldsymbol{\alpha}(\boldsymbol{\theta})$$

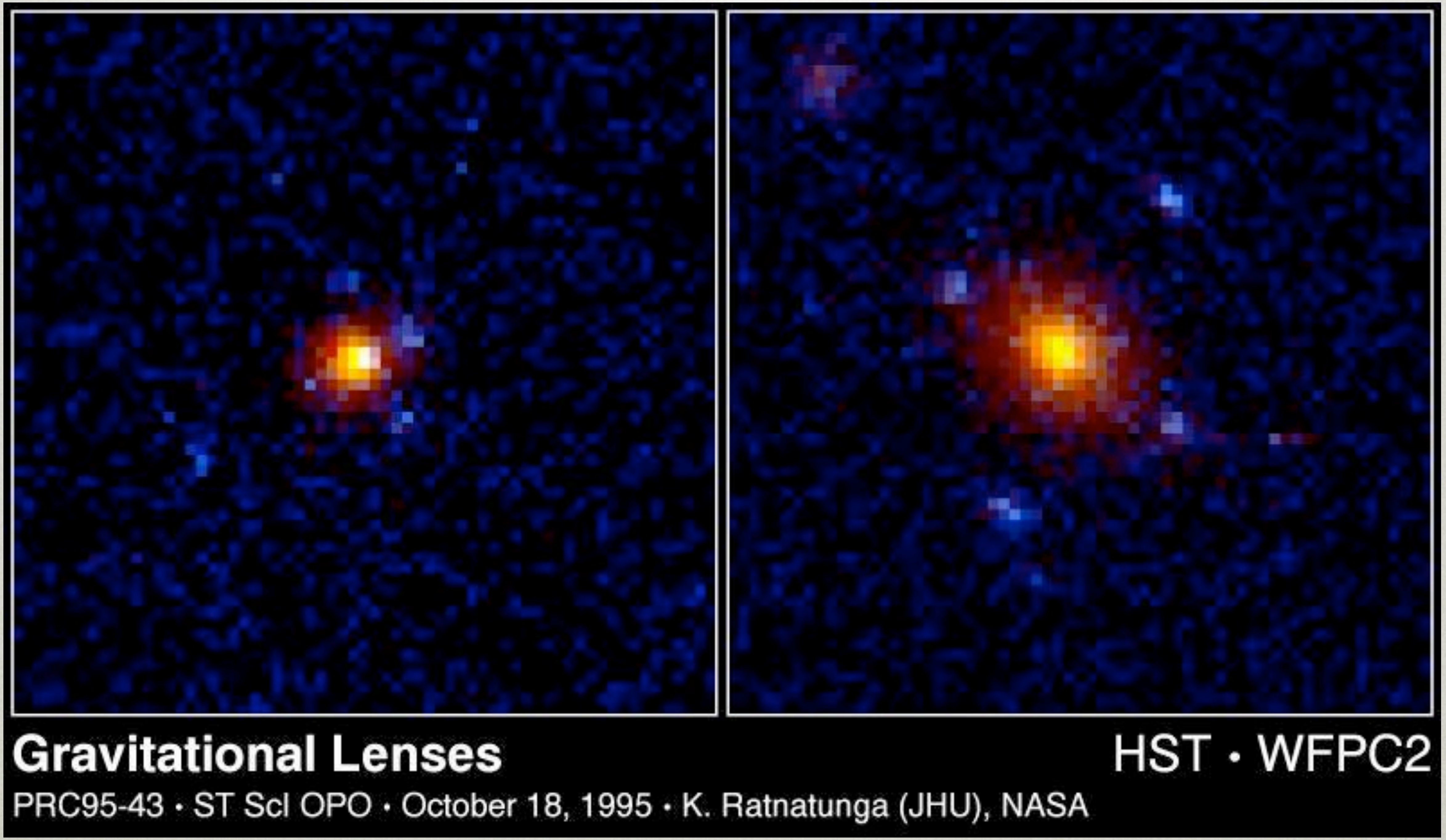
Lens equation

$$\beta = \boldsymbol{\theta} - \boldsymbol{\alpha}(\boldsymbol{\theta})$$

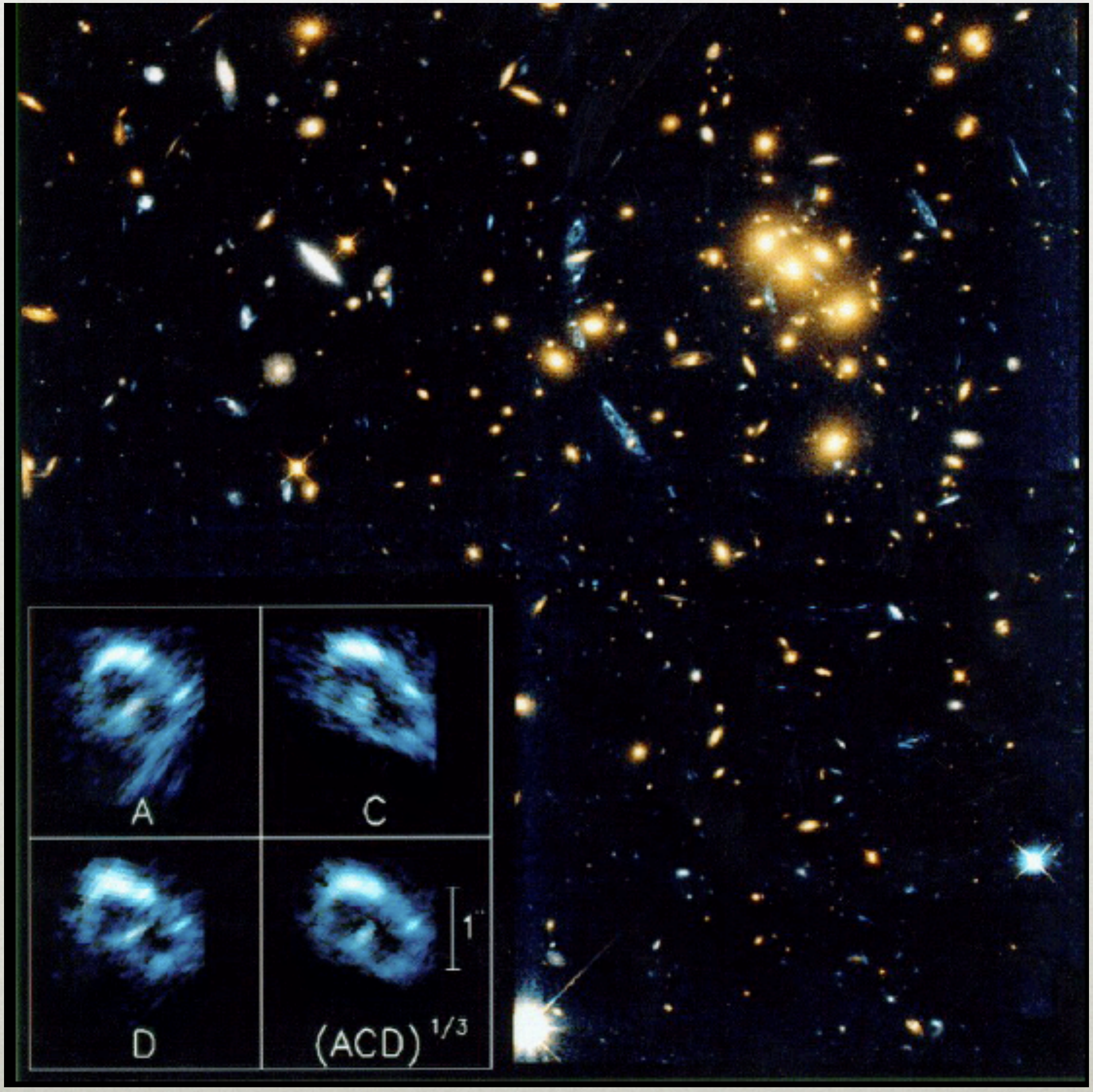
The mapping from image to source plane is easy. This is not the case for the mapping from source to image plane:

A source with true position β will be observed at all positions θ that satisfy the lens equation. Multiple solutions are possible: a single source can be observed at several positions on the sky!

Strong lensing



Strong lensing



Convenient notation

It is convenient to rescale the surface density using

$$\kappa(\theta) = \frac{\Sigma}{\Sigma_{crit}}, \qquad \Sigma_{crit} \equiv \frac{c^2}{4\pi G}\frac{D_s}{D_{ls}D_l}$$

If the convergence $\kappa \geq 1$ then the lens equation is guaranteed to have multiple solutions (sufficient but not necessary condition)

Source Redshifts

$$\Sigma_{crit} \equiv \frac{c^2}{4\pi G} \frac{D_s}{D_{ls} D_l}$$

To relate the lensing signal to physical quantities we need to know the redshifts of the sources.

Spectroscopy is expensive, but lensing kernel is broad and photometric redshift are sufficient.

Convenient notation

The deflection angle is given by

$$\alpha(\theta) = \frac{1}{\pi}\int d^2\vartheta \cdot \kappa(\vartheta)\frac{\theta - \vartheta}{\left|\theta - \vartheta\right|^2} \equiv \vec{\nabla}\Psi(\theta)$$

where the deflection potential is defined as

$$\Psi(\theta) = \frac{1}{\pi}\int d^2\vartheta \cdot \kappa(\vartheta)\ln\left|\theta - \vartheta\right|$$

Note the "Poisson equation": $\nabla^2\Psi(\theta) = 2\kappa(\theta)$

Effects of lensing

The 3 "D"s of gravitational lensing

1. Delay
2. Deflection
3. Distortion

There is a 4th one called "flexion"

Delay

Delay: 0^{th} derivative

longer path

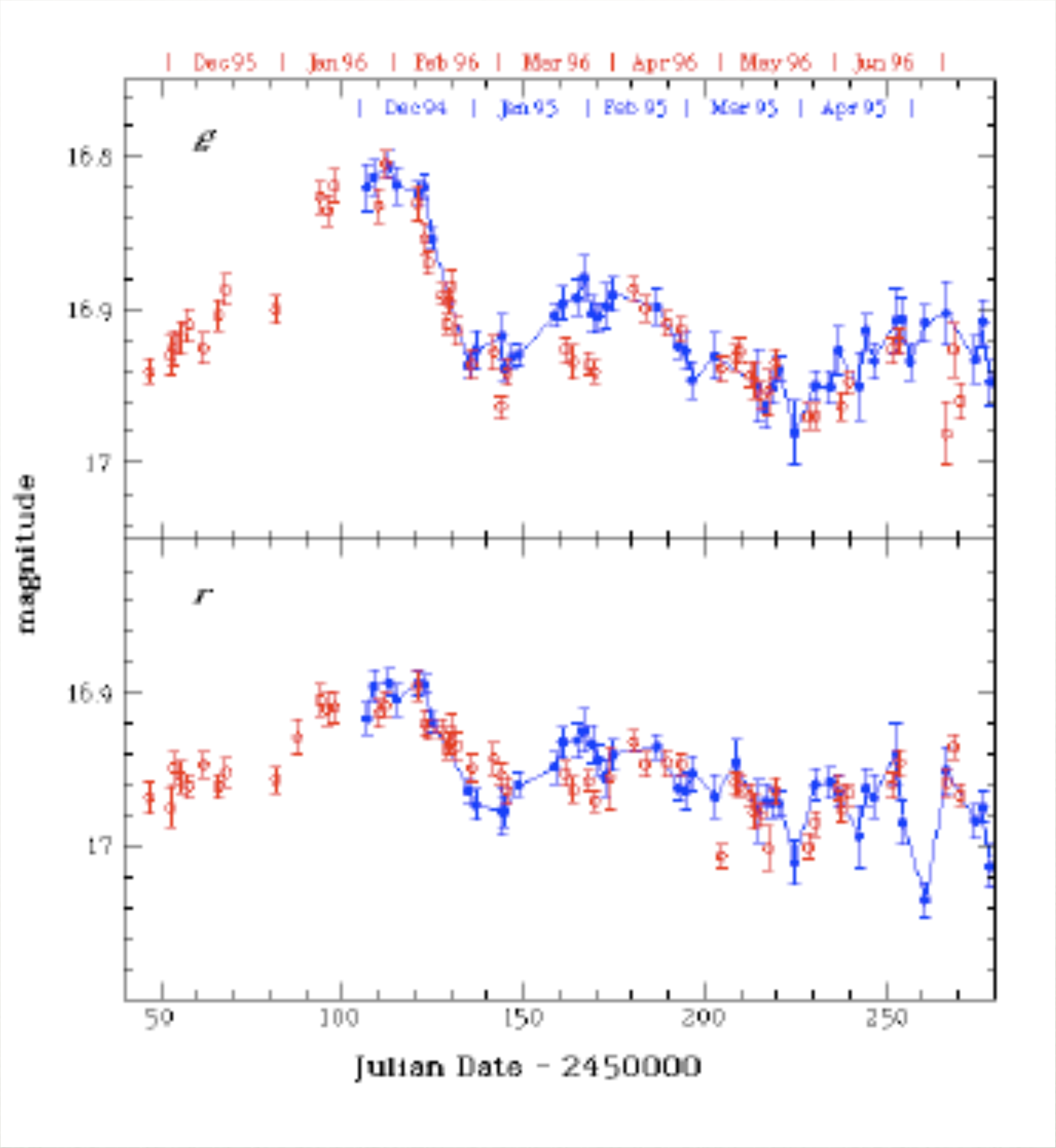
$$\tau = \frac{1+z_L}{H_0}\frac{d_L d_S}{d_{LS}}\left[\frac{1}{2}(\vec{\theta}-\vec{\beta})^2 - \psi_{2D}(\vec{\theta})\right]$$

"slowing down"

$$\vec{\theta} = \vec{\theta}_A, \vec{\theta}_B, \vec{\theta}_C, \vec{\theta}_D, ...$$

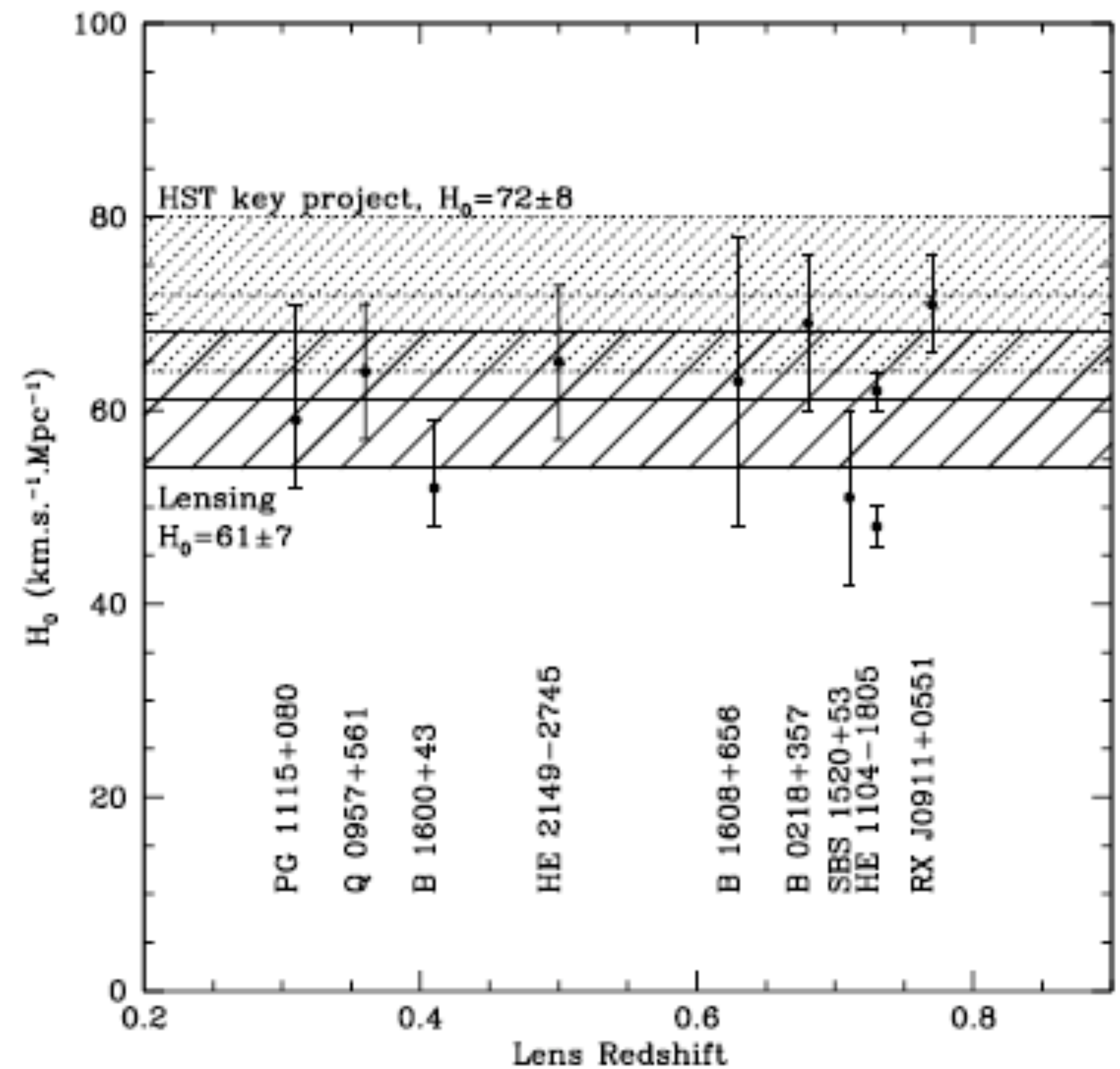
Units: $(\text{angle})^2/H_0$

Time Delay



time delay of 417 days

Time Delay



[Courbin (2003)]

Deflection

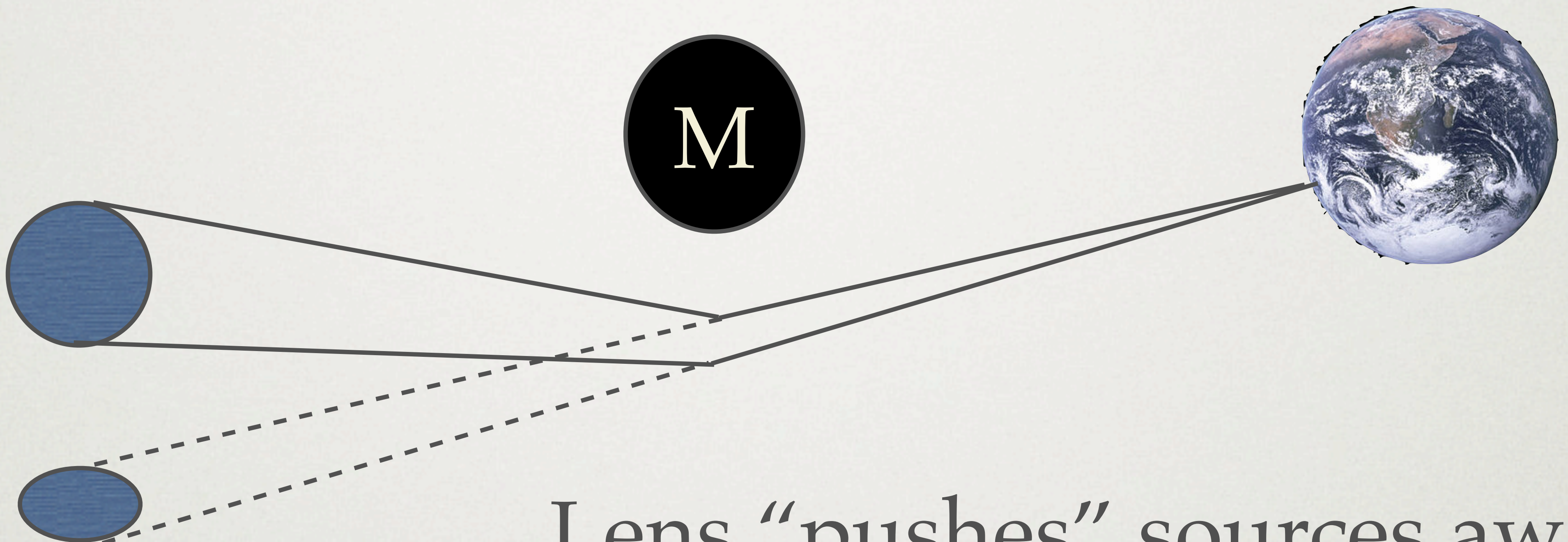
Deflection: 1st derivative

$$\vec{\nabla}\tau = 0$$

$$\vec{\theta} - \vec{\beta} - \vec{\nabla}\psi_{2D} = 0$$

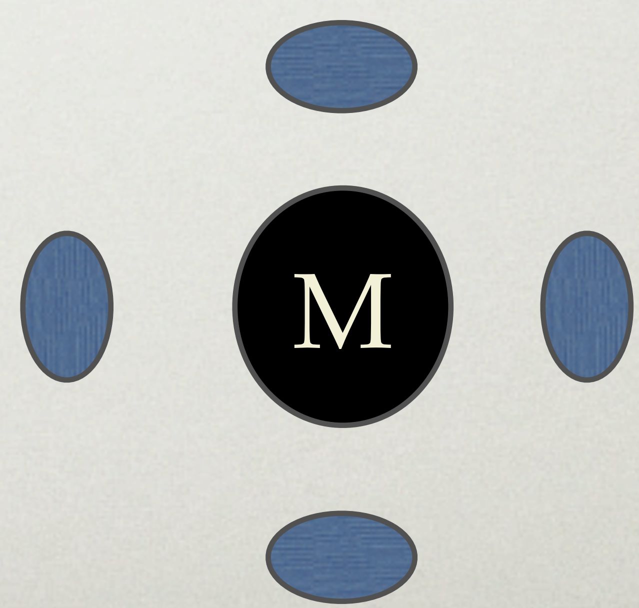
Units: (angle)^1

Differential deflection



Lens “pushes” sources away

radial squeezing:



Distortion

Distortion: 2nd derivative

$$\frac{\partial \vec{\beta}}{\partial \vec{\theta}} = \begin{pmatrix} 1 - \frac{\partial^2 \psi}{\partial \theta_x^2} & -\frac{\partial^2 \psi}{\partial \theta_x \partial \theta_y} \\ -\frac{\partial^2 \psi}{\partial \theta_x \partial \theta_y} & 1 - \frac{\partial^2 \psi}{\partial \theta_y^2} \end{pmatrix}$$

$$\left| \frac{\partial \vec{\beta}}{\partial \vec{\theta}} \right| = 1/\mu$$

Units: $(\text{angle})^0$

Weak lensing

The effect of lensing is to remap the images of extended sources, while conserving surface brightness

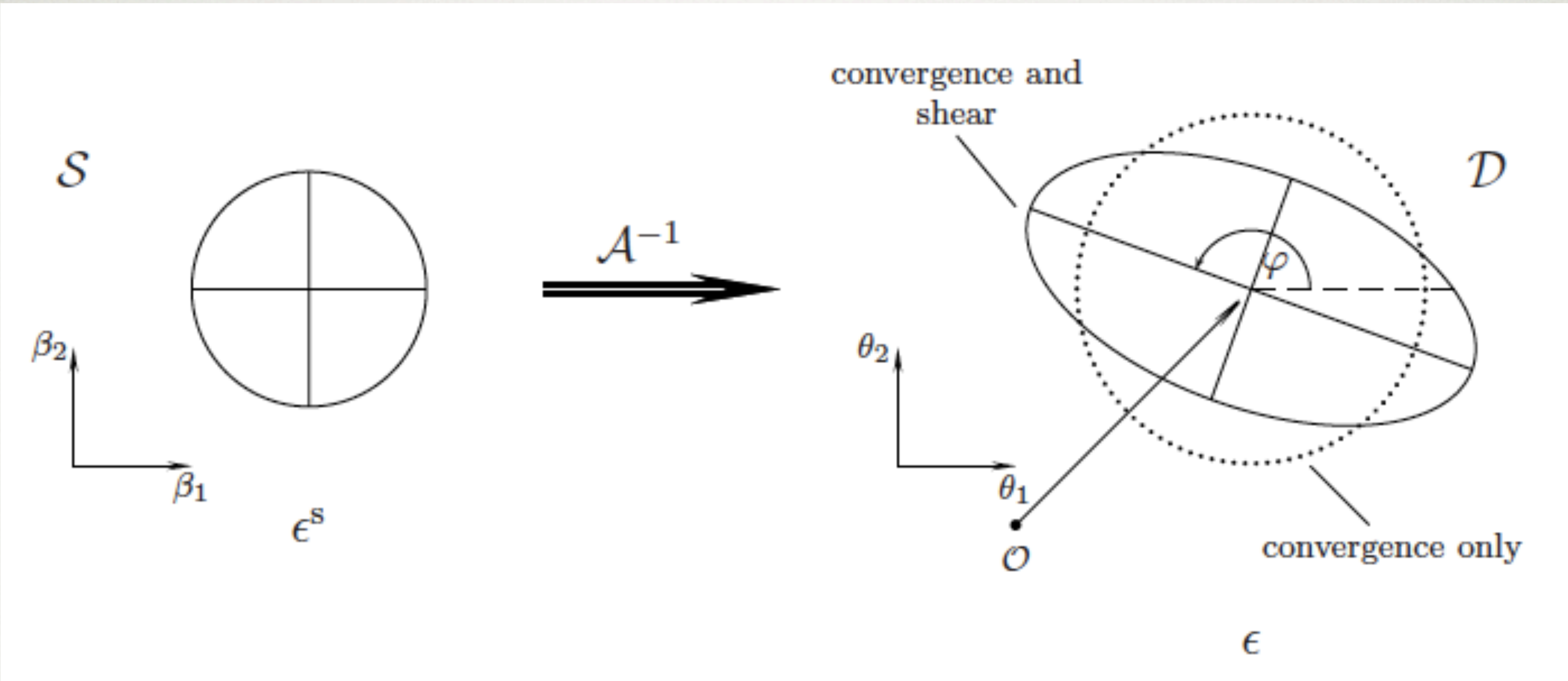
$$I(\boldsymbol{\theta}) = I^{(\mathrm{s})}[\boldsymbol{\beta}(\boldsymbol{\theta})]$$

If the source is small compared to the scale on which the deflection angle changes:

$$I(\boldsymbol{\theta}) = I^{(\mathrm{s})}[\boldsymbol{\beta}_0 + \mathcal{A}(\boldsymbol{\theta}_0) \cdot (\boldsymbol{\theta} - \boldsymbol{\theta}_0)]$$

$$\mathcal{A}(\boldsymbol{\theta}) = (1-\kappa) \begin{pmatrix} 1 - g_1 & -g_2 \\ -g_2 & 1 + g_1 \end{pmatrix}, \text{ where } g(\boldsymbol{\theta}) = \frac{\gamma(\boldsymbol{\theta})}{[1 - \kappa(\boldsymbol{\theta})]}$$

Weak lensing



Two effects: *shearing* and *magnification*

Observables

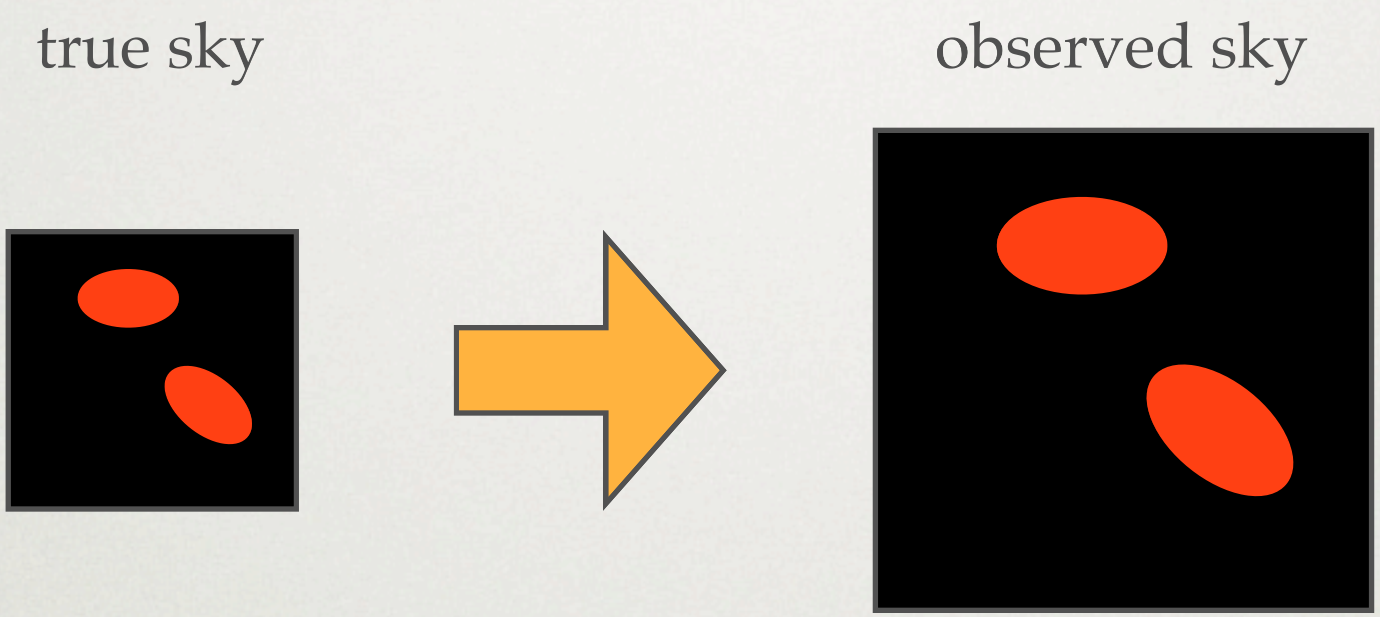
reduced shear: $$g_i = \frac{\gamma_i}{(1-\kappa)}$$

$$|g| = \frac{1-b/a}{1+b/a} \quad \Leftrightarrow \quad \frac{b}{a} = \frac{1-|g|}{1+|g|}$$

magnification: $$\mu = \frac{1}{\det \mathcal{A}} = \frac{1}{(1-\kappa)^2 - |\gamma|^2} = \frac{1}{(1-\kappa)^2\ (1-|g|^2)}$$

Magnification

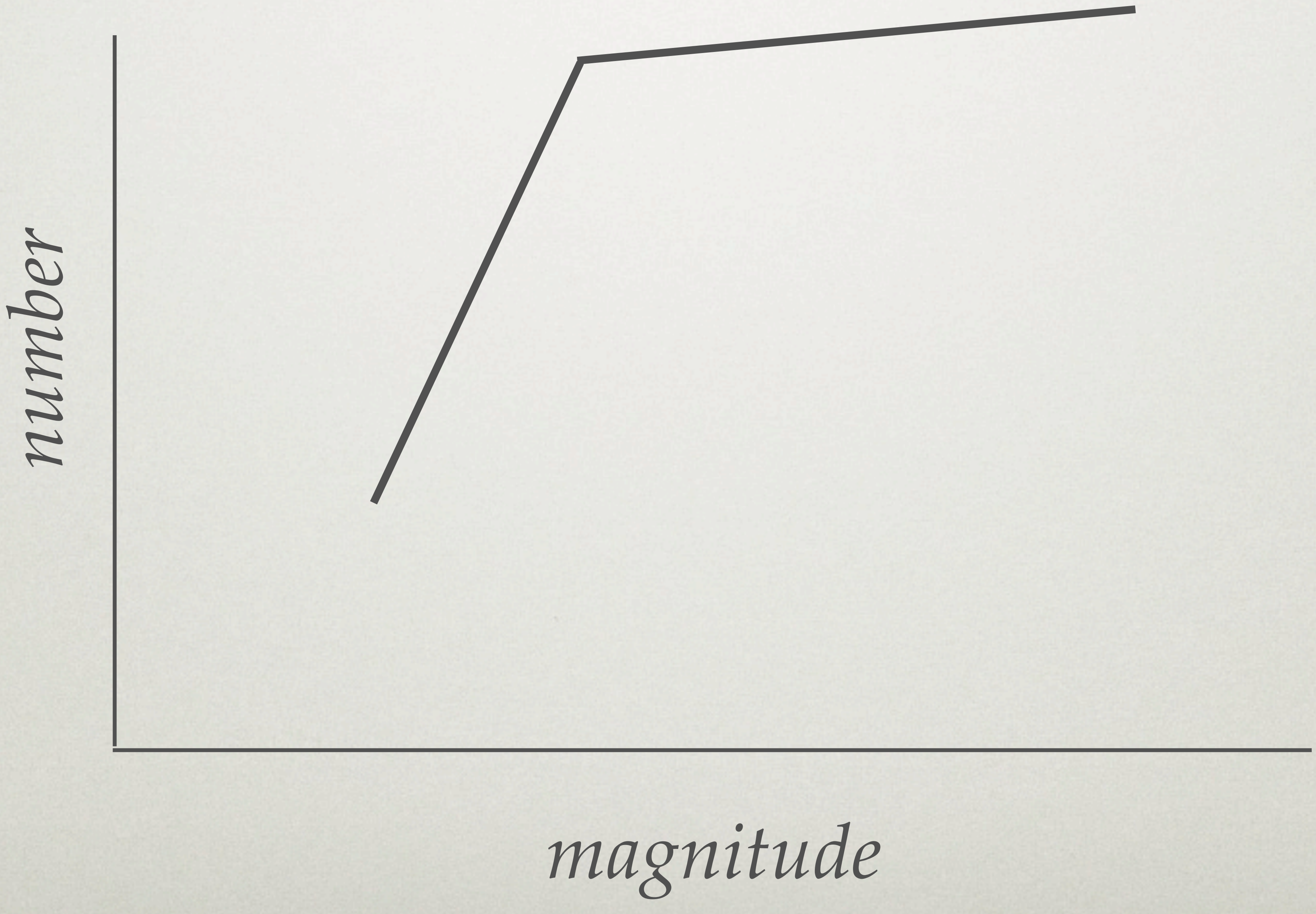
Magnification has two effects:



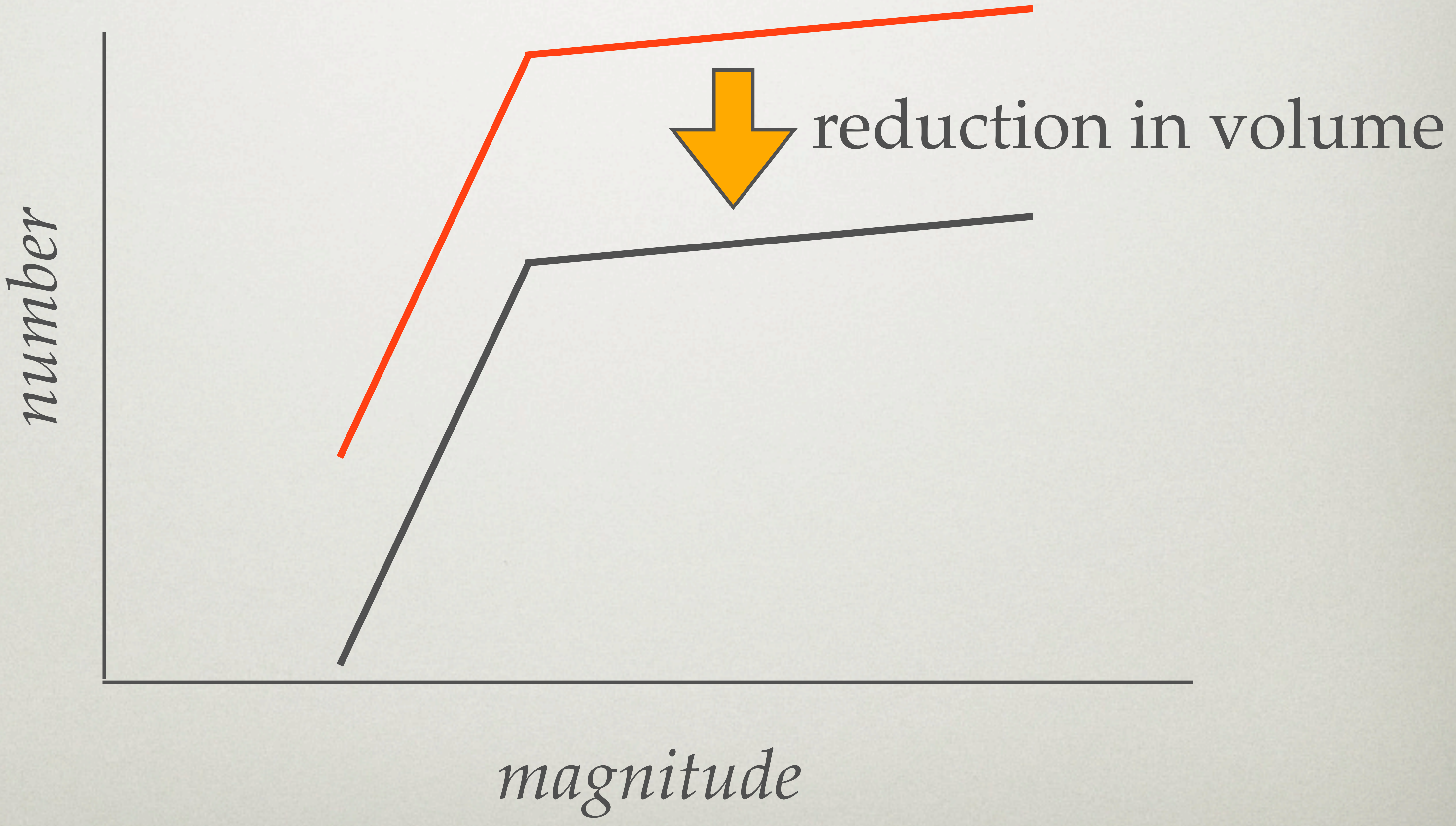
- true survey area is $1/\mu$ times larger
- objects are μ times larger/brighter

$$n(>S,z) = \frac{1}{\mu(\boldsymbol{\theta},z)}\, n_0\left(>\frac{S}{\mu(\boldsymbol{\theta},z)}, z\right)$$

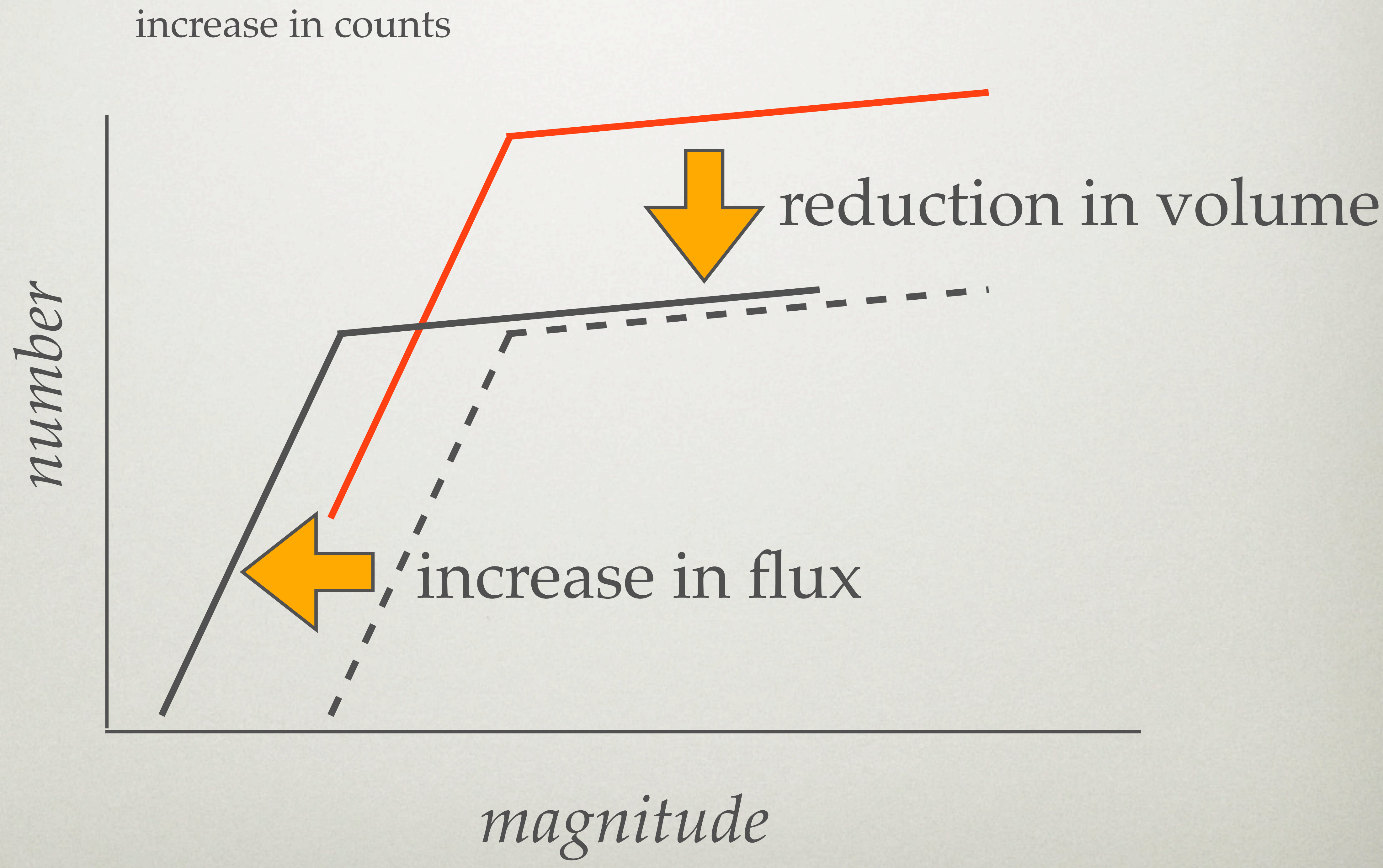
Magnification



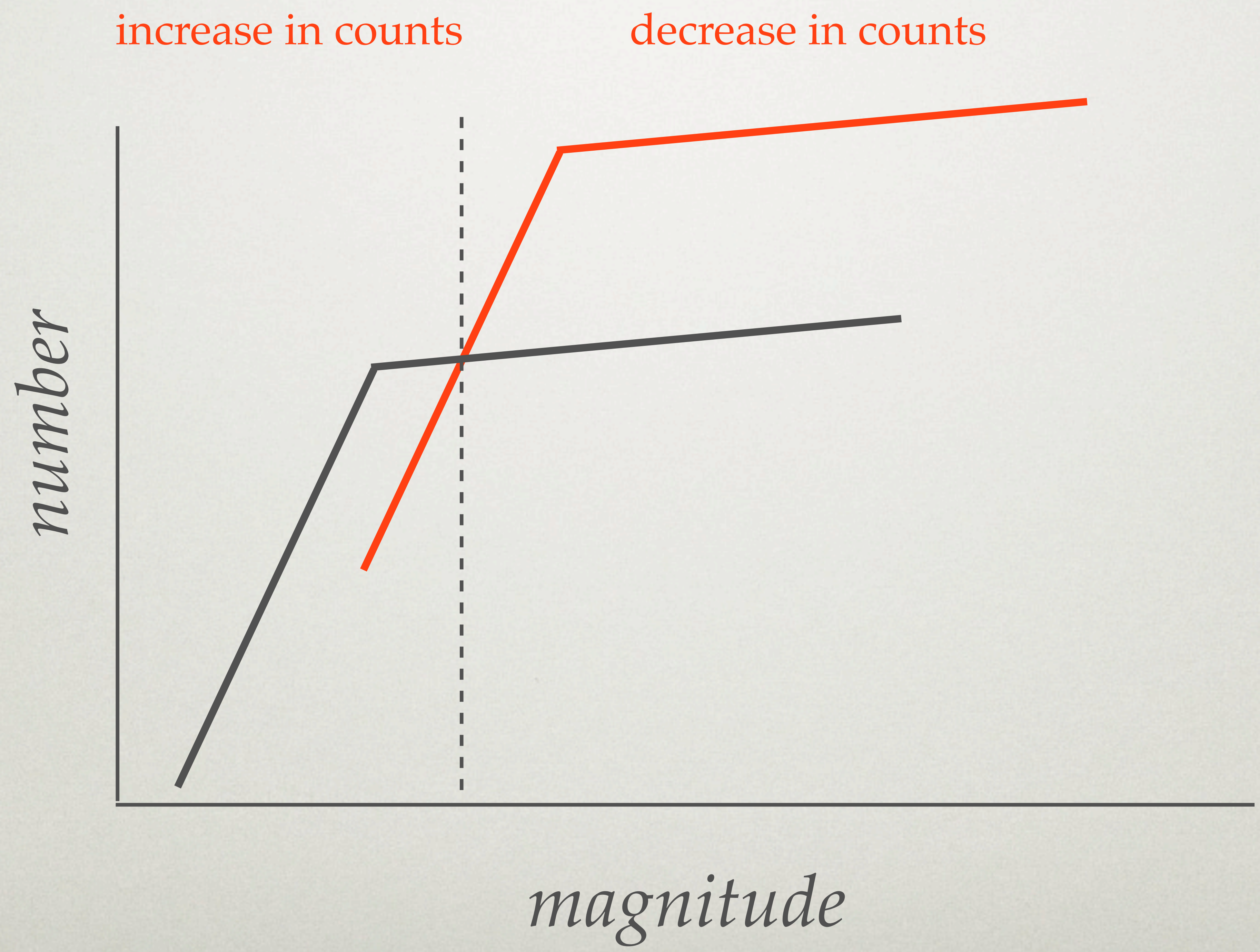
Magnification



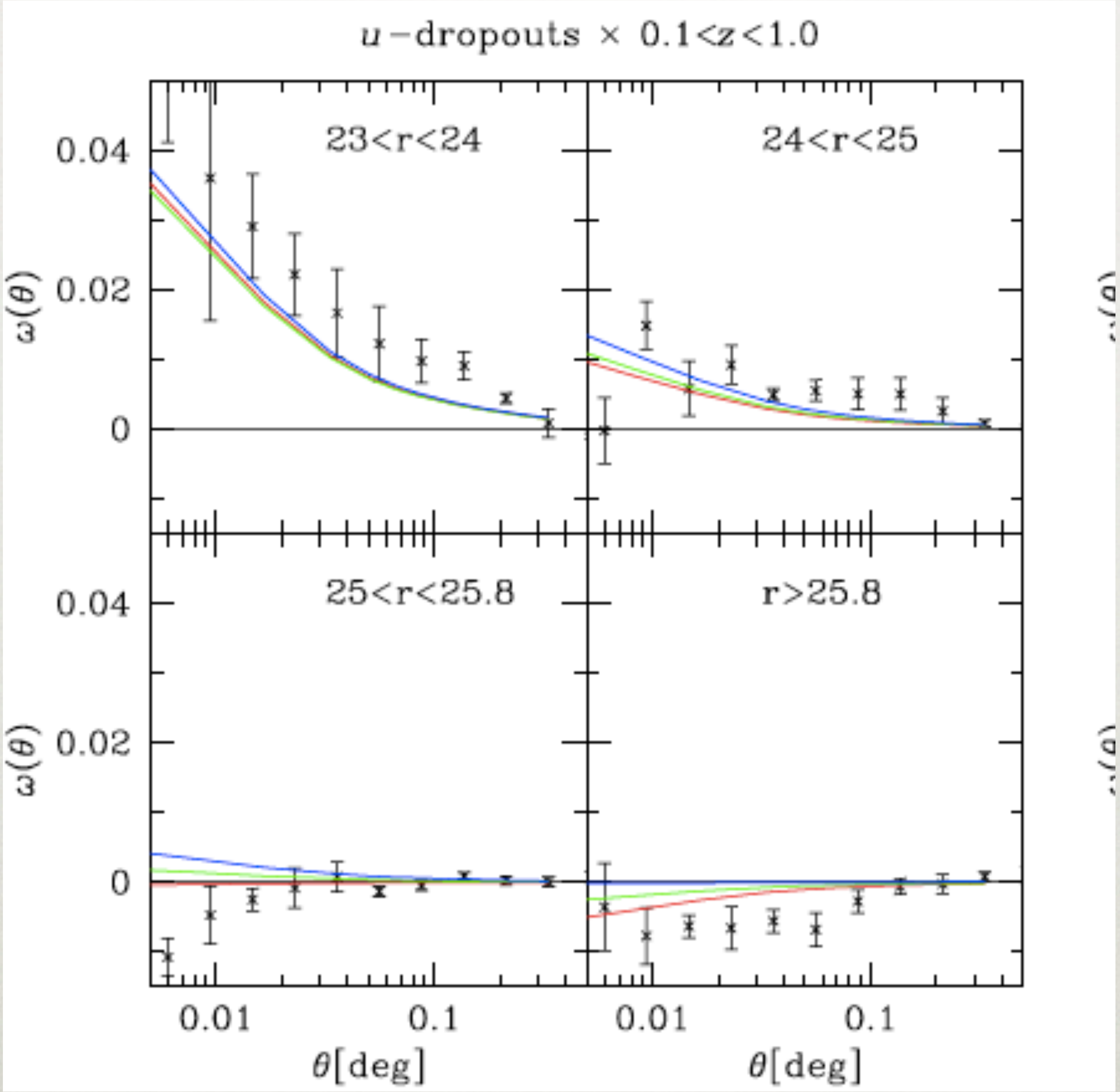
Magnification



Magnification

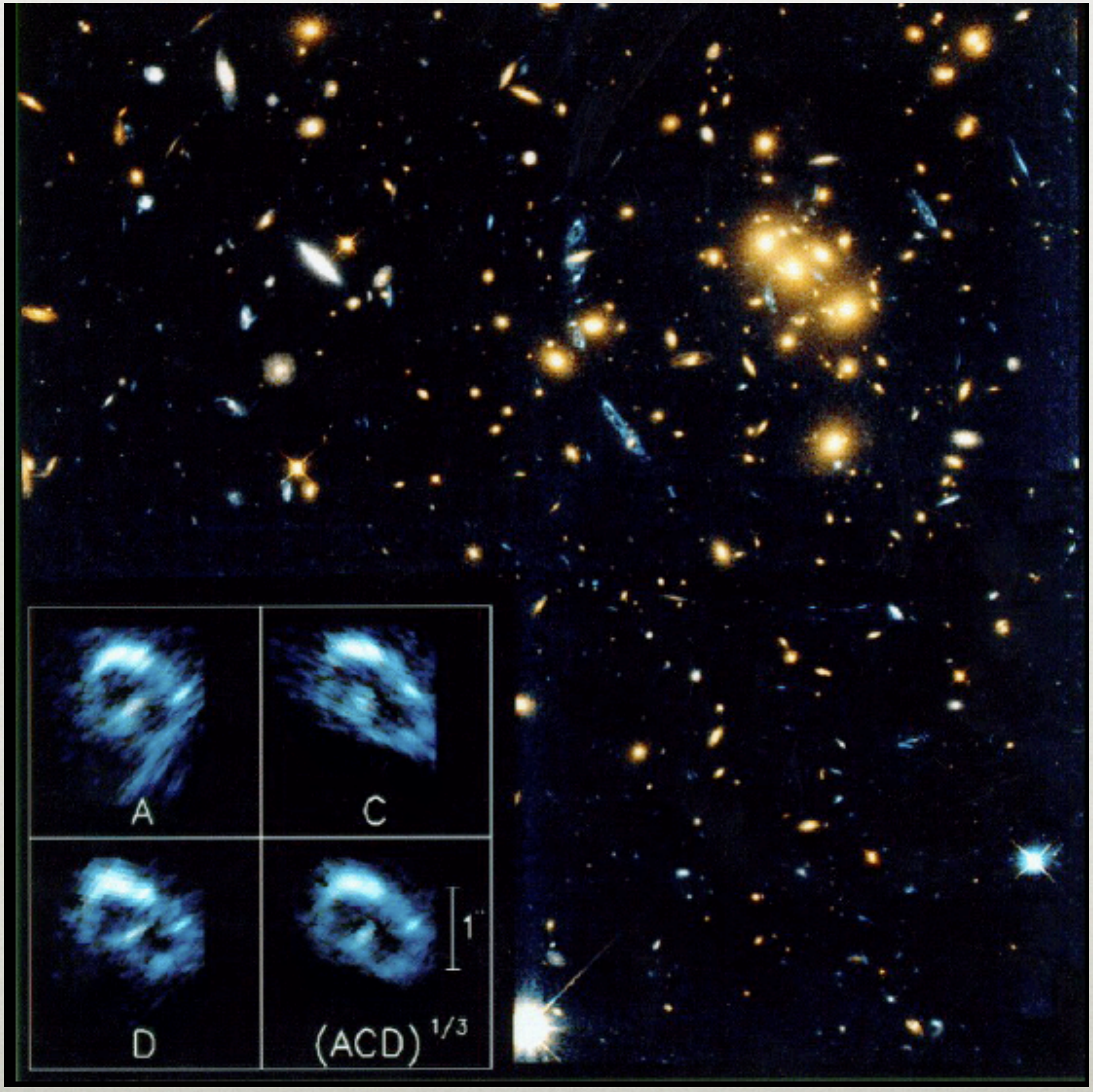


Magnification around galaxies

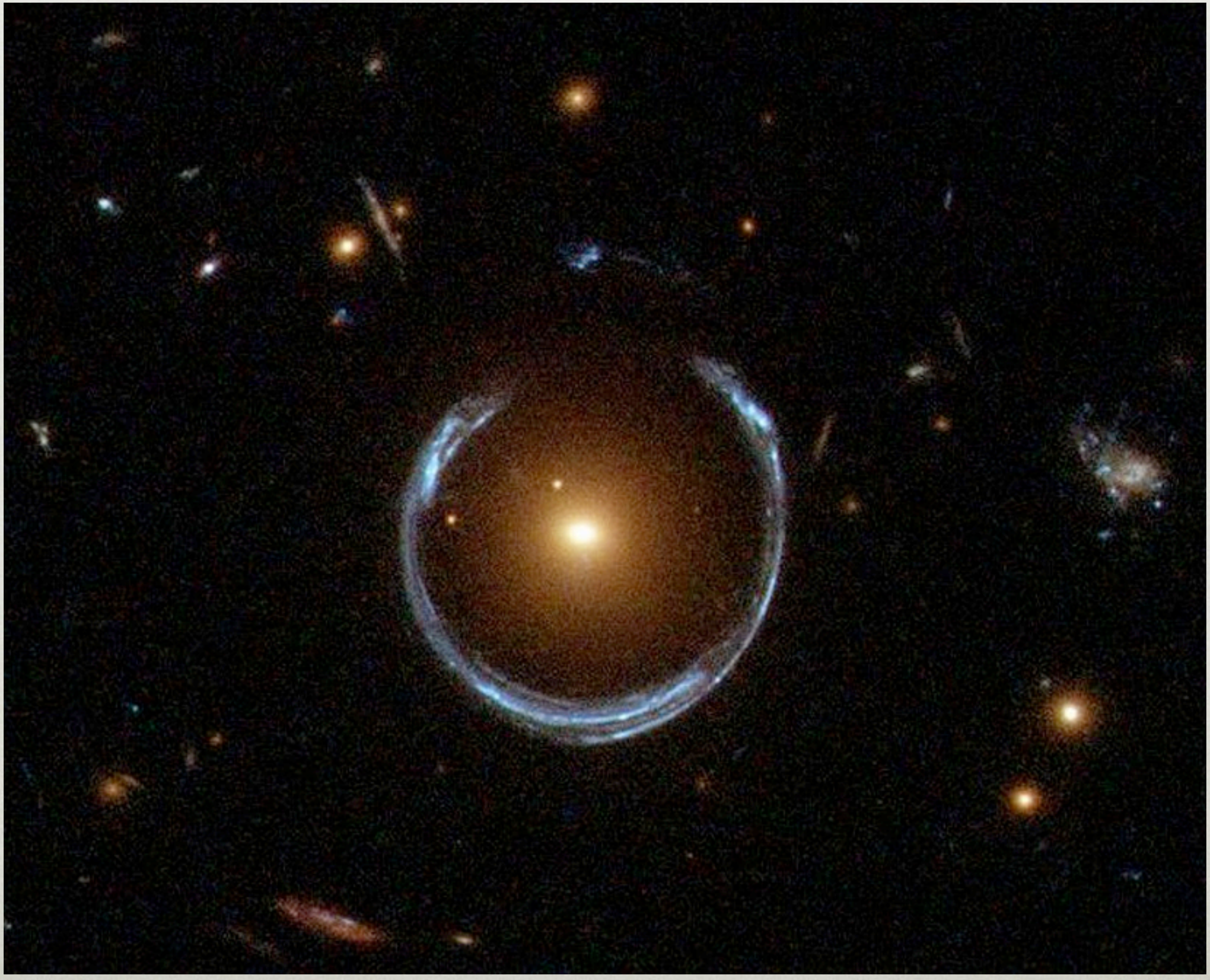


Hildebrandt et al. (2009)

Gravitational telescope

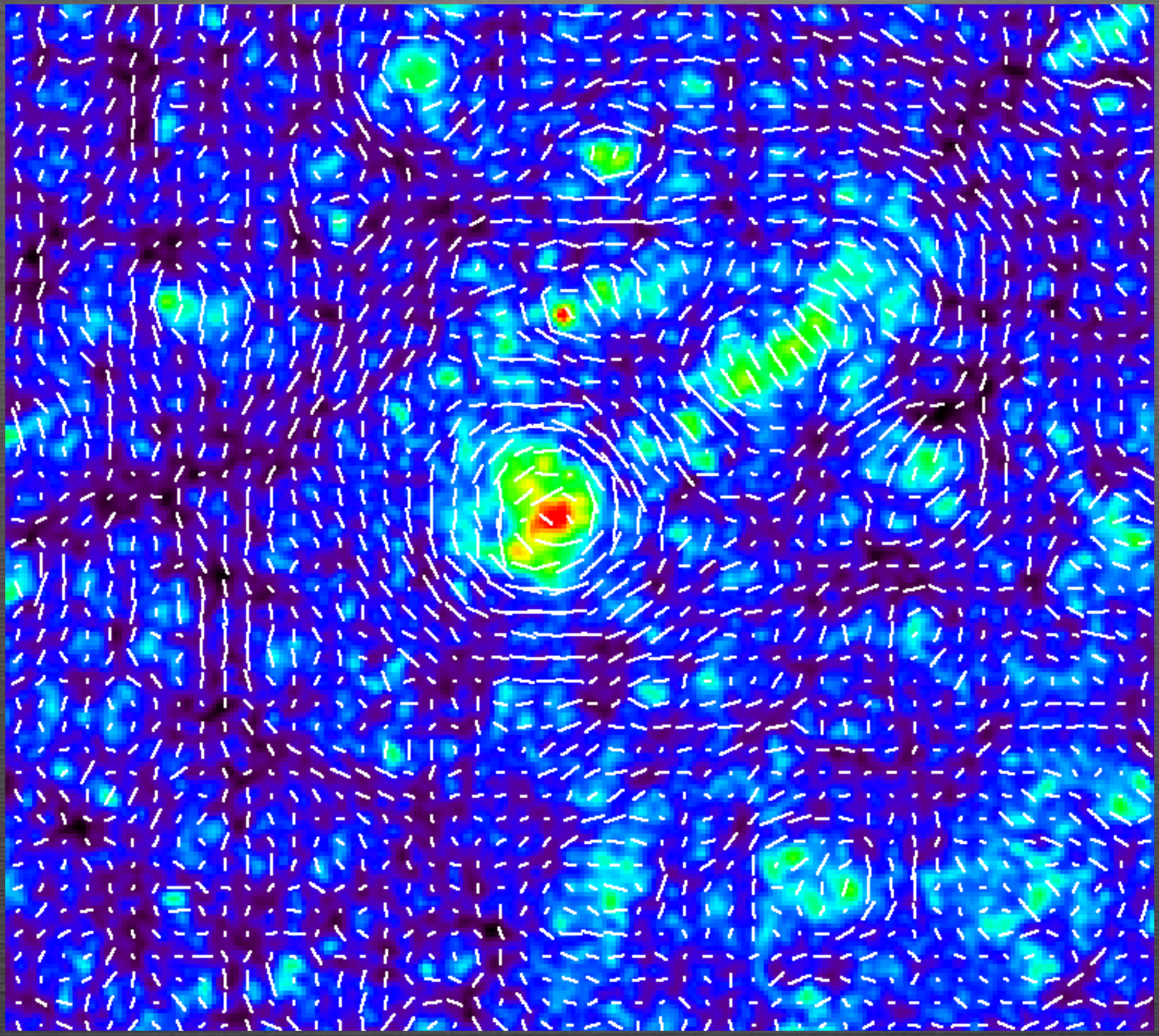


Gravitational telescope



We can 'see' dark matter

Courtesy B. Jain



In the absence of noise we would be able to map the matter distribution in the universe (even "dark" clusters).

Convenient notation

Recall that: $$\Psi(\theta) = \frac{1}{\pi}\int d^2\vartheta \cdot \kappa(\vartheta)\ln\left|\theta - \vartheta\right|$$

$$\vec{\alpha}(\theta) = \vec{\nabla}\Psi(\theta)$$

$$\nabla^2\Psi(\theta) = 2\kappa(\theta)$$

but also that:

$$\gamma_1 = \frac{1}{2}\left(\frac{\partial^2\Psi}{\partial^2 x_1} - \frac{\partial^2\Psi}{\partial^2 x_2}\right) \quad \textit{and} \quad \gamma_2 = \frac{\partial^2\Psi}{\partial x_1 \partial x_2}$$

The shear and the convergence are related

Mass reconstruction

$$\gamma(\boldsymbol{\theta}) = \frac{1}{\pi}\int_{\mathbb{R}^2} \mathrm{d}^2\theta' \, \mathcal{D}(\boldsymbol{\theta}-\boldsymbol{\theta}') \, \kappa(\boldsymbol{\theta}') \, , \quad \text{with kernel}$$

$$\mathcal{D}(\boldsymbol{\theta}) \equiv \frac{\theta_2^2-\theta_1^2-2\mathrm{i}\theta_1\theta_2}{|\boldsymbol{\theta}|^4} = \frac{-1}{(\theta_1-\mathrm{i}\theta_2)^2} \, .$$

Hence γ is a convolution of κ with a kernel *D*.
In Fourier space this becomes:

$$\hat{\gamma}(\boldsymbol{\ell}) = \pi^{-1}\hat{\mathcal{D}}(\boldsymbol{\ell}) \, \hat{\kappa}(\boldsymbol{\ell}) \quad \text{for} \quad \boldsymbol{\ell} \neq \mathbf{0} \, .$$

Mass reconstruction

With inversion: $\hat{\kappa}(\boldsymbol{\ell}) = \pi^{-1}\hat{\gamma}(\boldsymbol{\ell})\,\hat{\mathcal{D}}^*(\boldsymbol{\ell}) \quad \text{for} \quad \boldsymbol{\ell} \neq \mathbf{0}$

where

$$\hat{\mathcal{D}}(\boldsymbol{\ell}) = \pi \frac{\left(\ell_1^2 - \ell_2^2 + 2\mathrm{i}\ell_1\ell_2\right)}{|\boldsymbol{\ell}|^2}$$

was used (this implies $\mathcal{D}\mathcal{D}^* = \pi^2$).

Fourier back-transformation then yields

$$\kappa(\boldsymbol{\theta}) - \kappa_0 = \frac{1}{\pi}\int_{\mathbb{R}^2} \mathrm{d}^2\theta'\,\mathcal{D}^*(\boldsymbol{\theta}-\boldsymbol{\theta}')\,\gamma(\boldsymbol{\theta}')$$

Kaiser & Squires (1993)

The surface density can be recovered up to a constant: *mass-sheet degeneracy*

Mass reconstruction

The surface density is a real quantity and the imaginary part of the integral should vanish.

Both shear components are not independent: there is redundancy in the measurements, which can be used to check for residual systematics (see tomorrow).

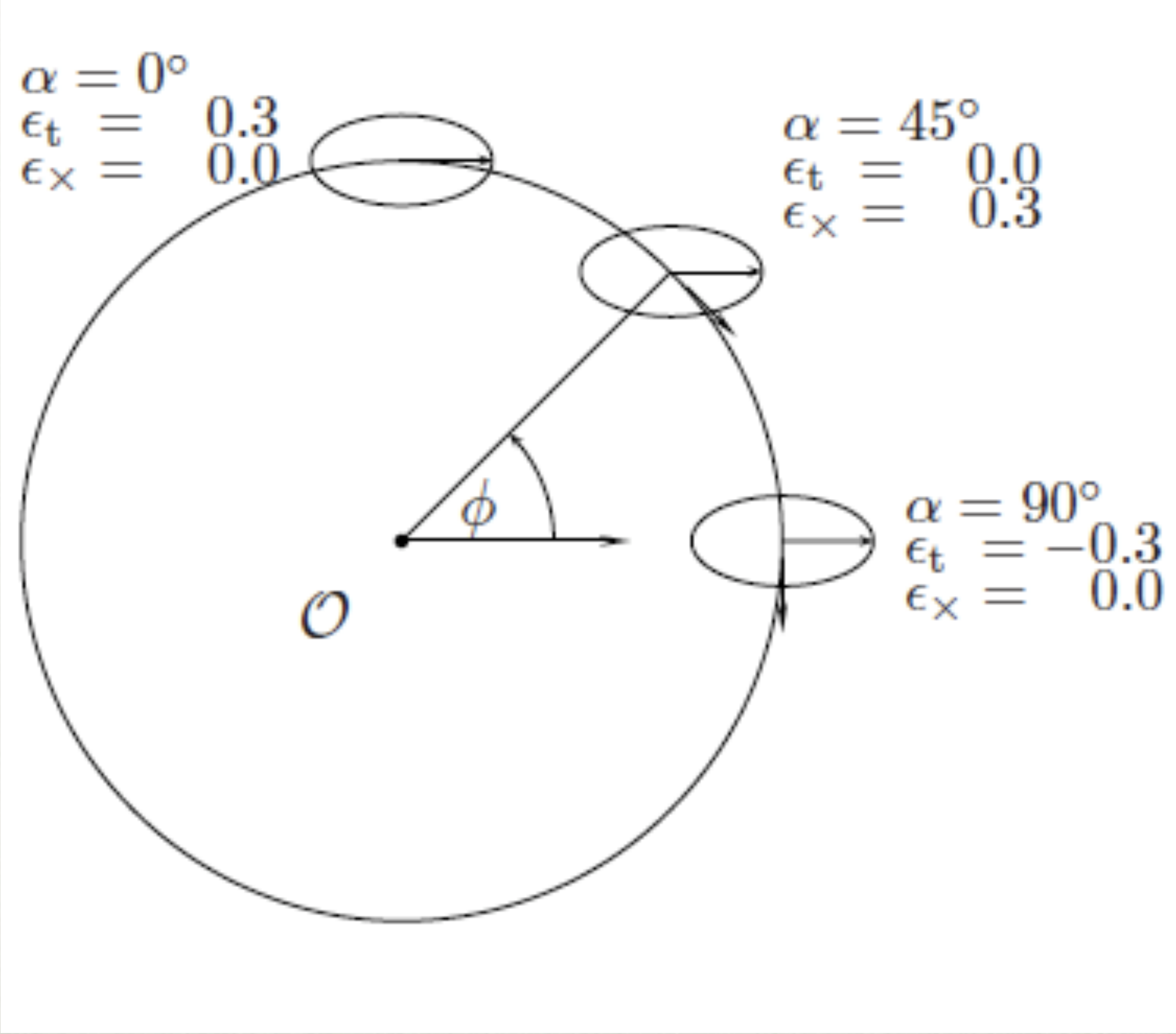
The integral extends to infinity, but data only for finite field. This is particularly important when the field-of-view is small and finite field methods have been developed to solve this problem.

Mass reconstruction

We can “see” dark matter!

Tangential shear

$$\gamma_t = -\Re[\gamma e^{-2i\phi}] \quad and \quad \gamma_\times = -\Im[\gamma e^{-2i\phi}]$$



Tangential shear

Gauss' theorem: $\int_0^{\theta} d^2\vartheta \cdot \nabla \bullet \nabla\Psi = \theta \oint d\varphi \nabla\Psi \bullet \vec{n}$

$$m(\theta) = \frac{1}{\pi}\int_0^{\theta} d^2\vartheta \cdot \kappa(\vartheta) = \frac{\theta}{2\pi}\oint d\varphi \frac{d\Psi}{d\theta}$$

$$\frac{dm}{d\theta} = \frac{m}{\theta} + \frac{\theta}{2\pi}\oint d\varphi \frac{d^2\Psi}{d^2\theta} = \frac{m}{\theta} + \theta\left[\langle\kappa(\theta)\rangle - \langle\gamma_t(\theta)\rangle\right]$$

where we used that $\frac{d^2\Psi}{d^2\theta} = \kappa - \gamma_T$

Tangential shear

$$m(\theta) = \theta^2 \bar{\kappa}(\theta) \Rightarrow \frac{dm}{d\theta} = 2\theta\bar{\kappa}(\theta)$$

$$\langle \gamma_t \rangle(\theta) = \bar{\kappa}(\theta) - \langle \kappa \rangle(\theta)$$

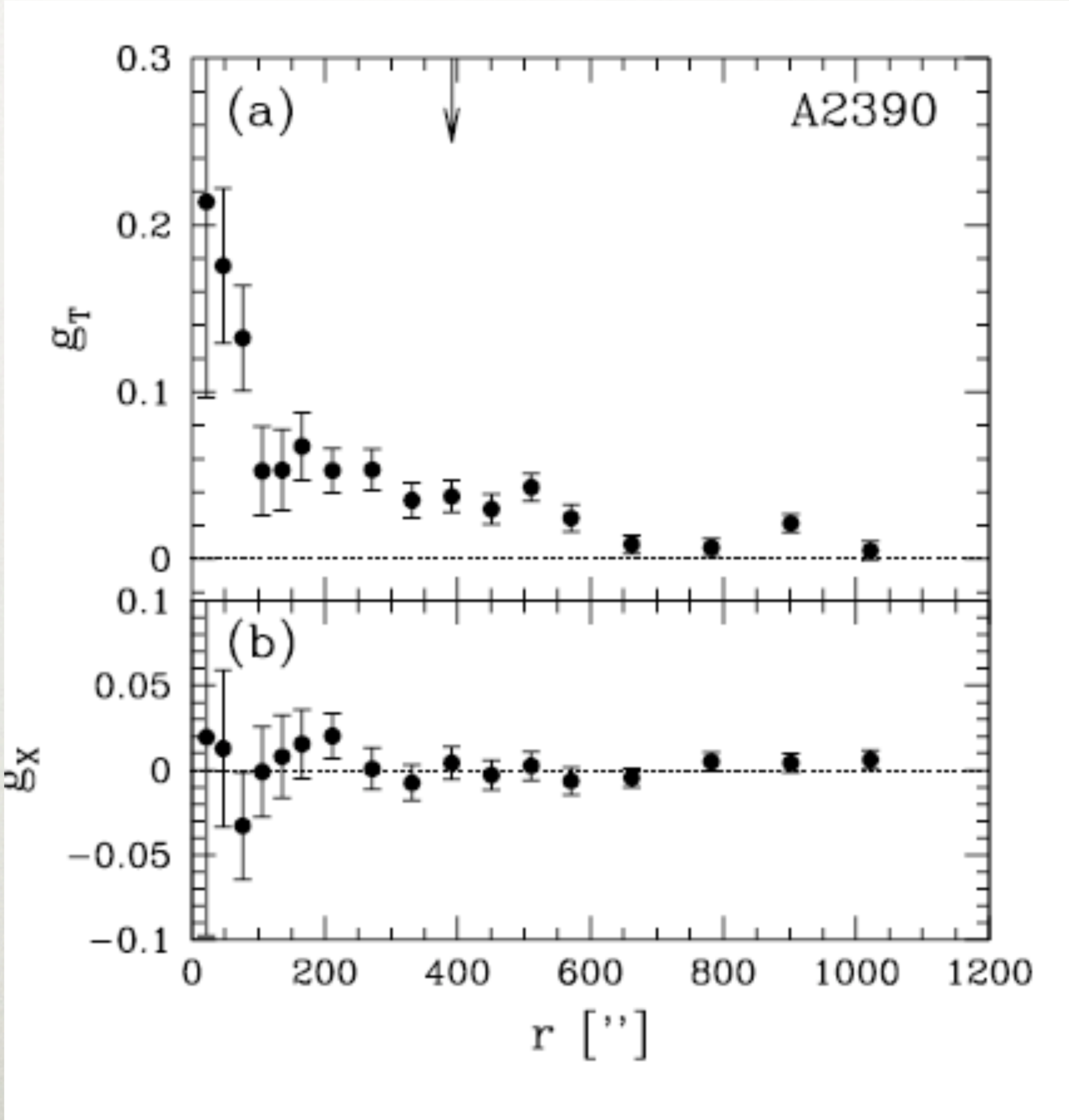
The tangential shear provides a direct measure of the mass contrast. This can be used to estimate projected masses within a radius with minimal assumptions about the radial matter distribution.

Tangential shear

“mass contrast”

$$g_T(r) \propto \langle \Sigma(<r) \rangle - \langle \Sigma(r) \rangle$$

Aperture masses



Weak lensing S/N

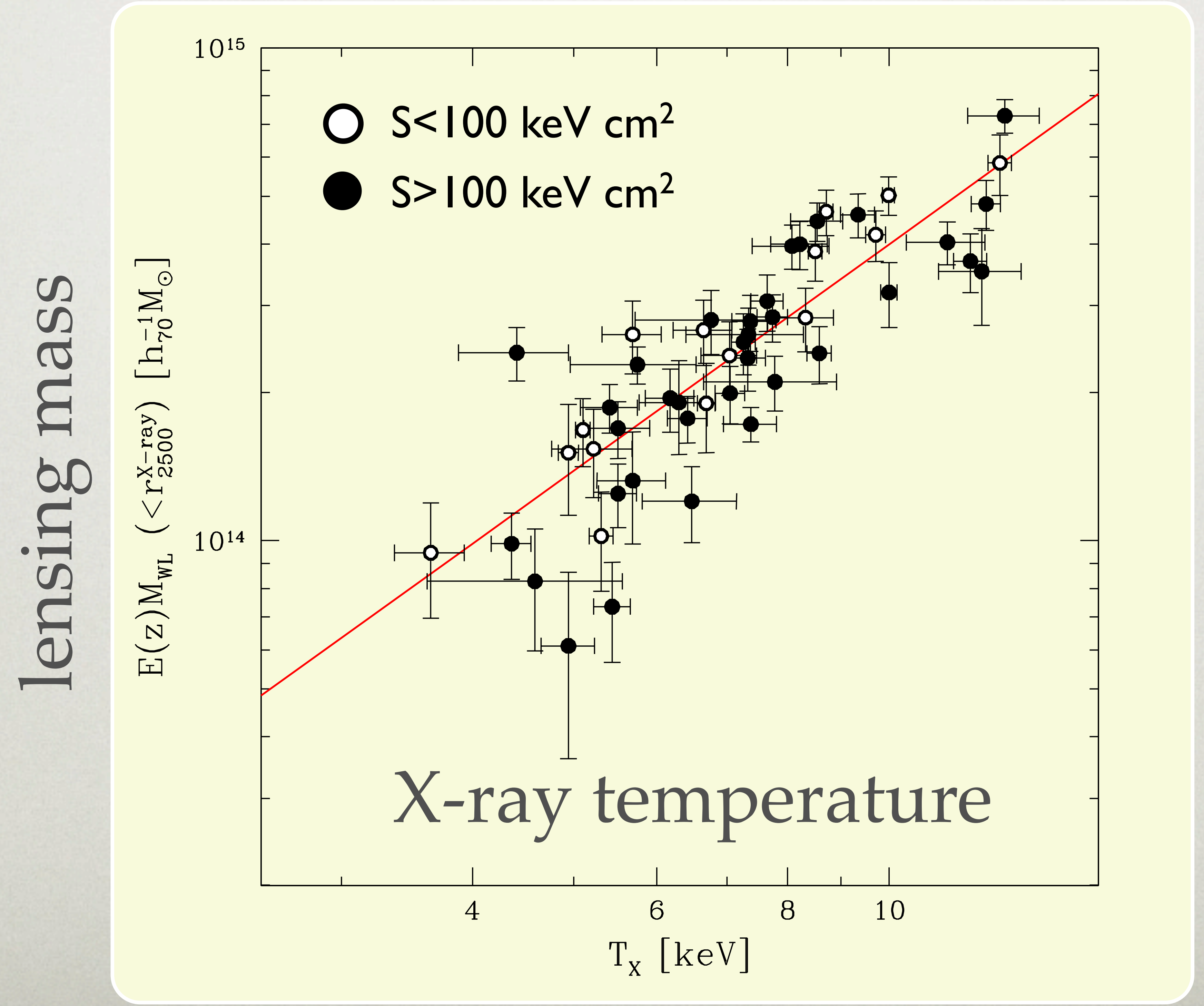
The signal-to-noise for an individual lens (described by isothermal sphere with l.o.s. velocity dispersion σ_v) is

$$\frac{\mathrm{S}}{\mathrm{N}} \approx 4 \left(\frac{n}{20\,\mathrm{arc\ min}^{-2}} \right)^{1/2} \left(\frac{\sigma_\epsilon}{0.3} \right)^{-1} \left(\frac{\sigma_v}{600\,\mathrm{km\,s}^{-1}} \right)^2 \frac{\langle D_{\mathrm{ds}}/D_{\mathrm{s}} \rangle}{0.6} \, .$$

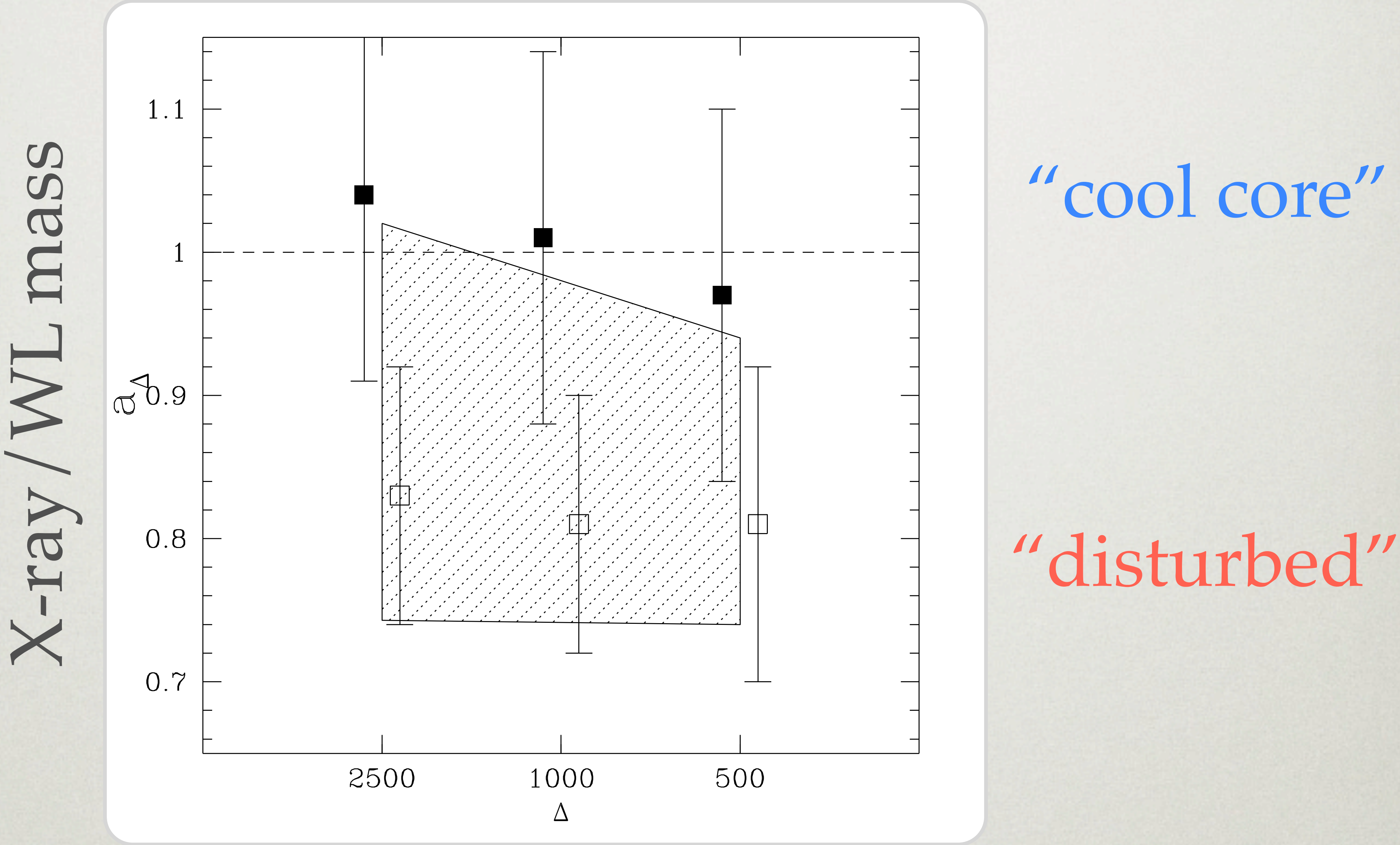
- Clusters of galaxies ($\sigma_v \gtrsim 600\,\mathrm{km/s}$) can be detected with $\mathrm{S/N} \gtrsim 4$ from weak lensing;
- individual galaxies ($\sigma_v \sim 200\,\mathrm{km/s}$) are too weak as lenses to be detected individually

Nature's weighing scales

Mass-X-ray properties



Testing X-ray masses



Mahdavi et al. (in prep):

gas is not always in hydrostatic equilibrium.

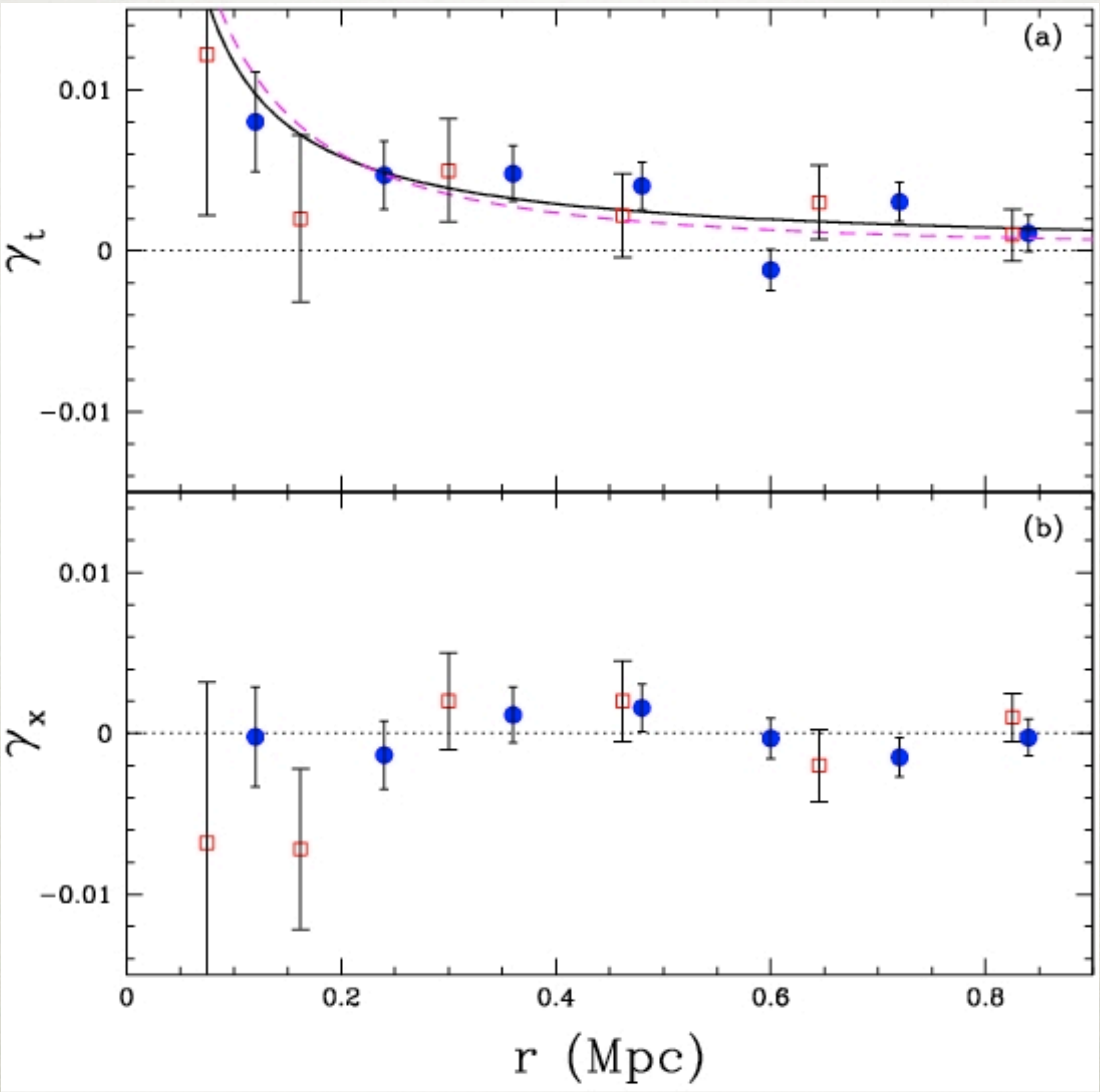
Stacking signals

If the masses are too low, one can still learn about the cluster properties by stacking the signal of many systems. This is for instance done for galaxy groups (Hoekstra et al. 2001; Parker et al. 2006).

Similarly, although SDSS imaging is not deep enough to study the masses of individual clusters, the signals of similar systems can be combined.

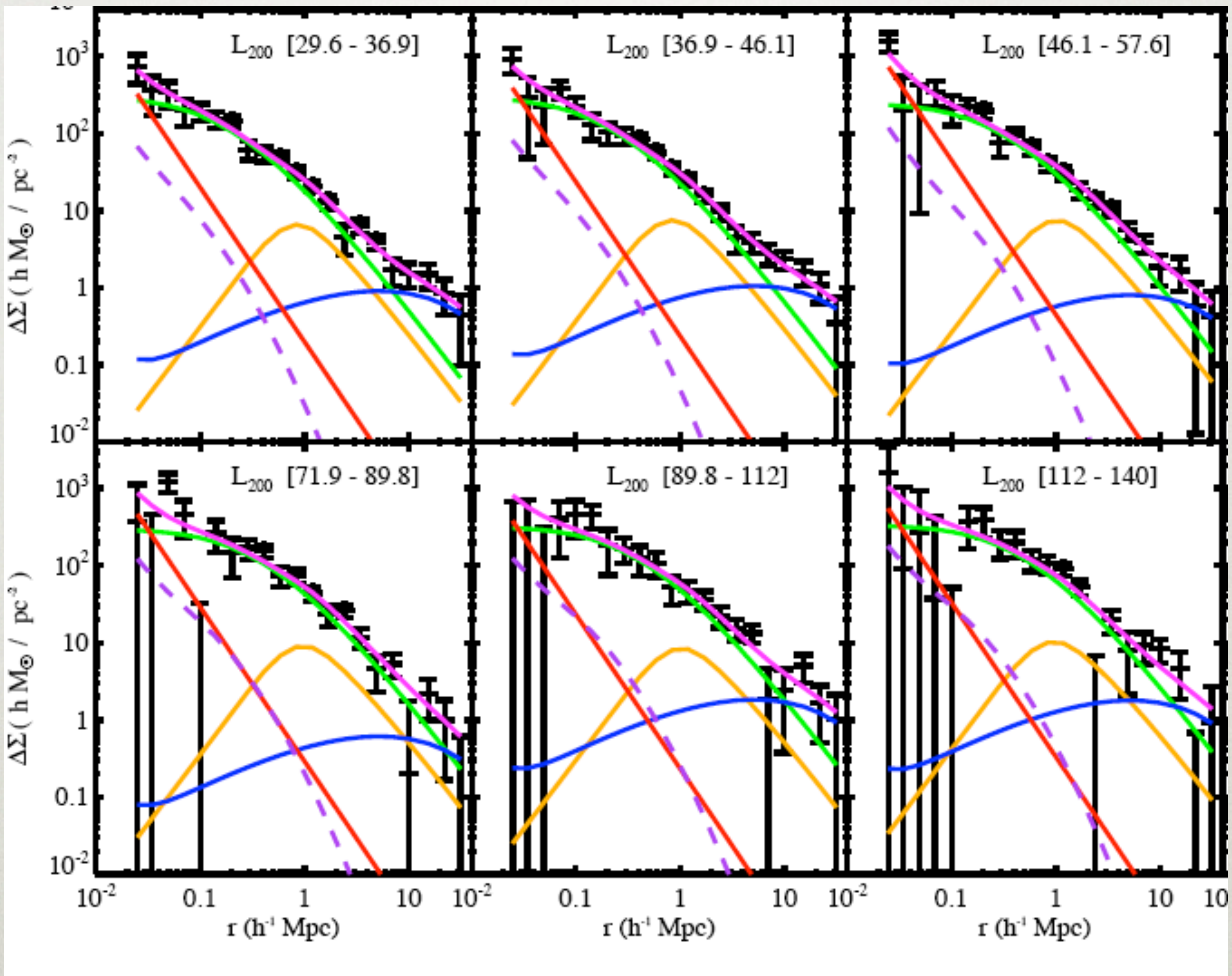
For instance this allows studies of the cluster mass profile out to large radii

Lensing by groups



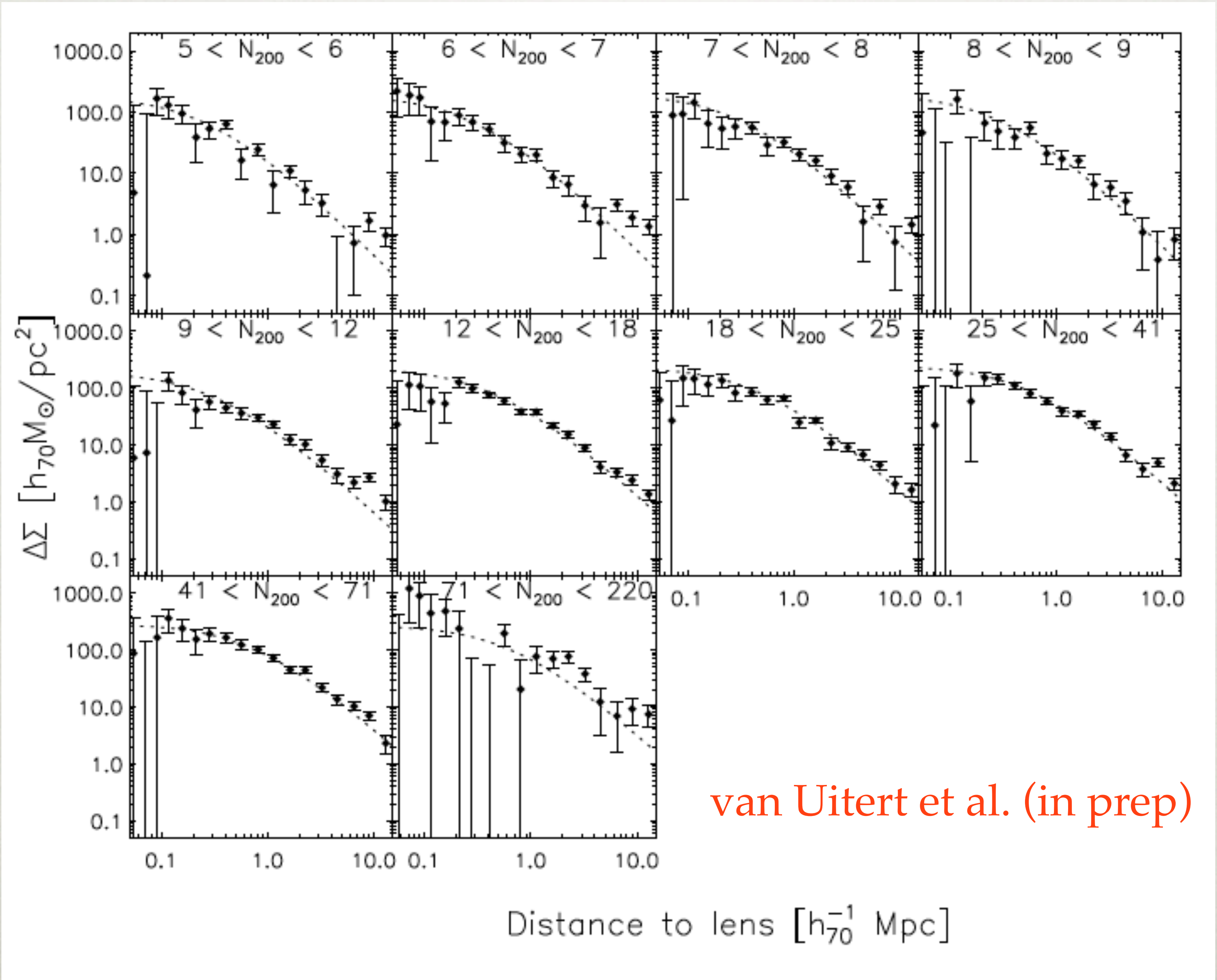
Parker et al. (2005)

Cluster density profiles



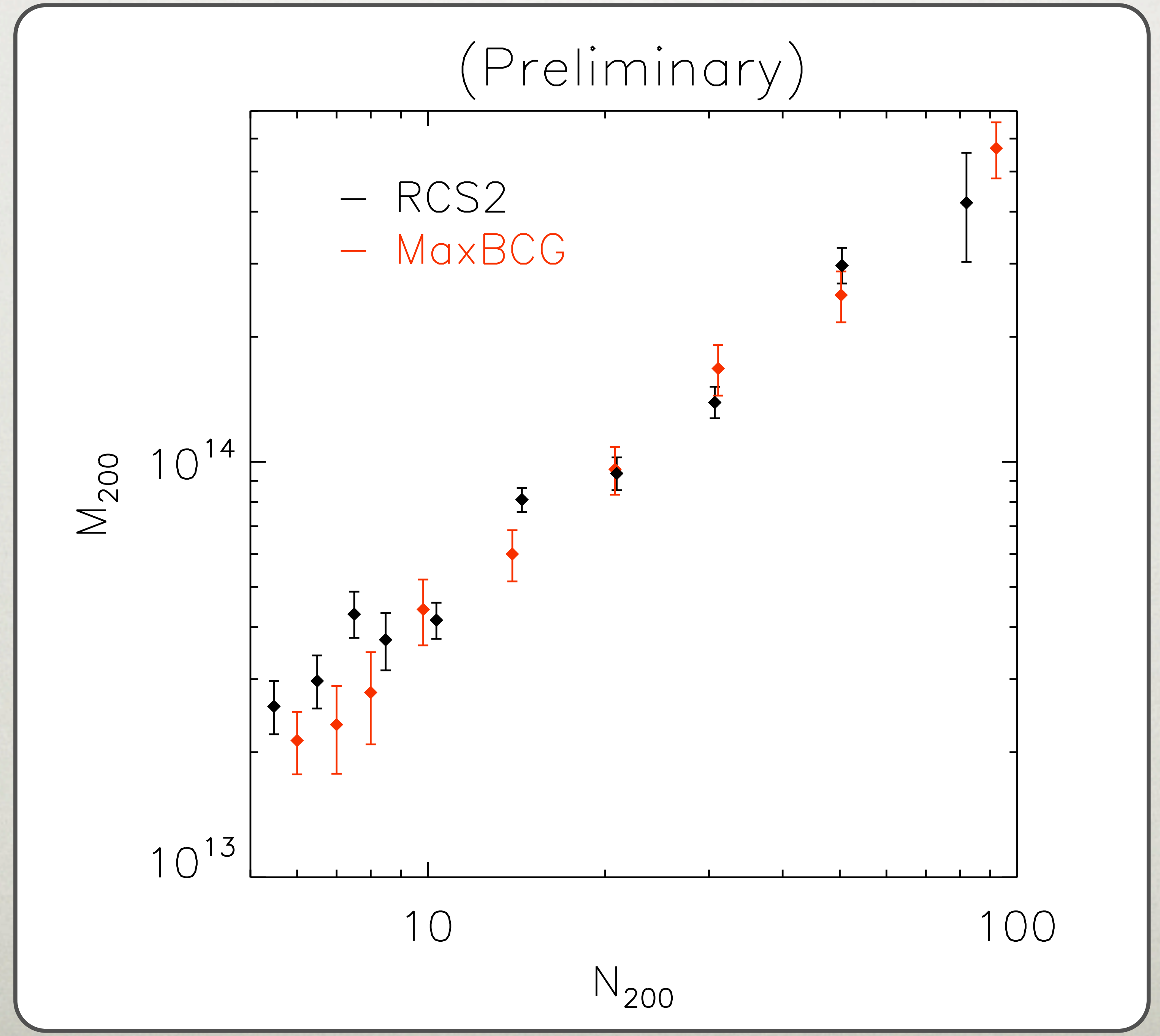
Johnston et al. (2007)

RCS2 - 28,000 clusters

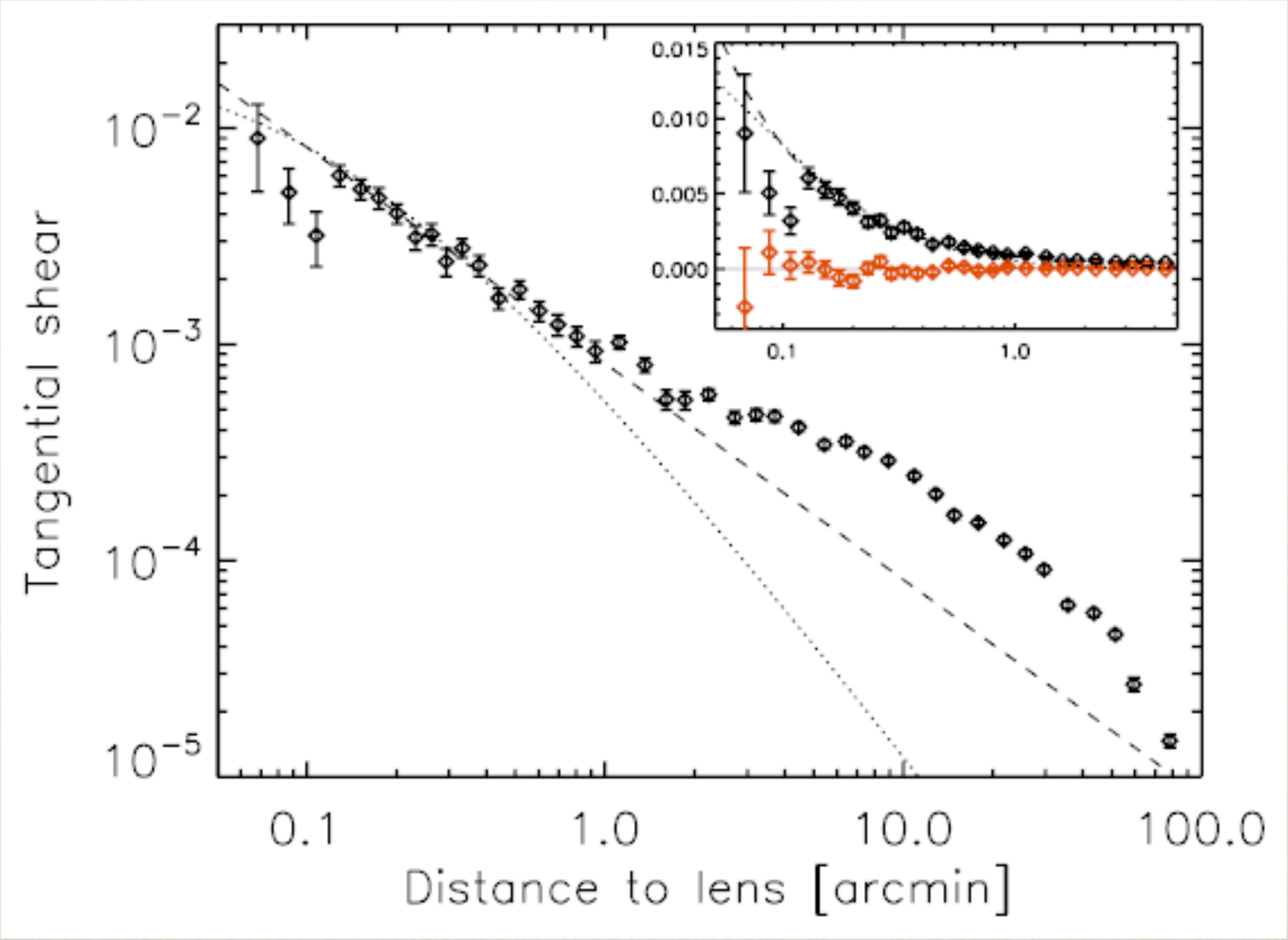


RCS2 - 28,000 clusters

van Uitert et al. (in prep)



Around galaxies



RCS2: 800 square degrees (van Uitert, in prep.)

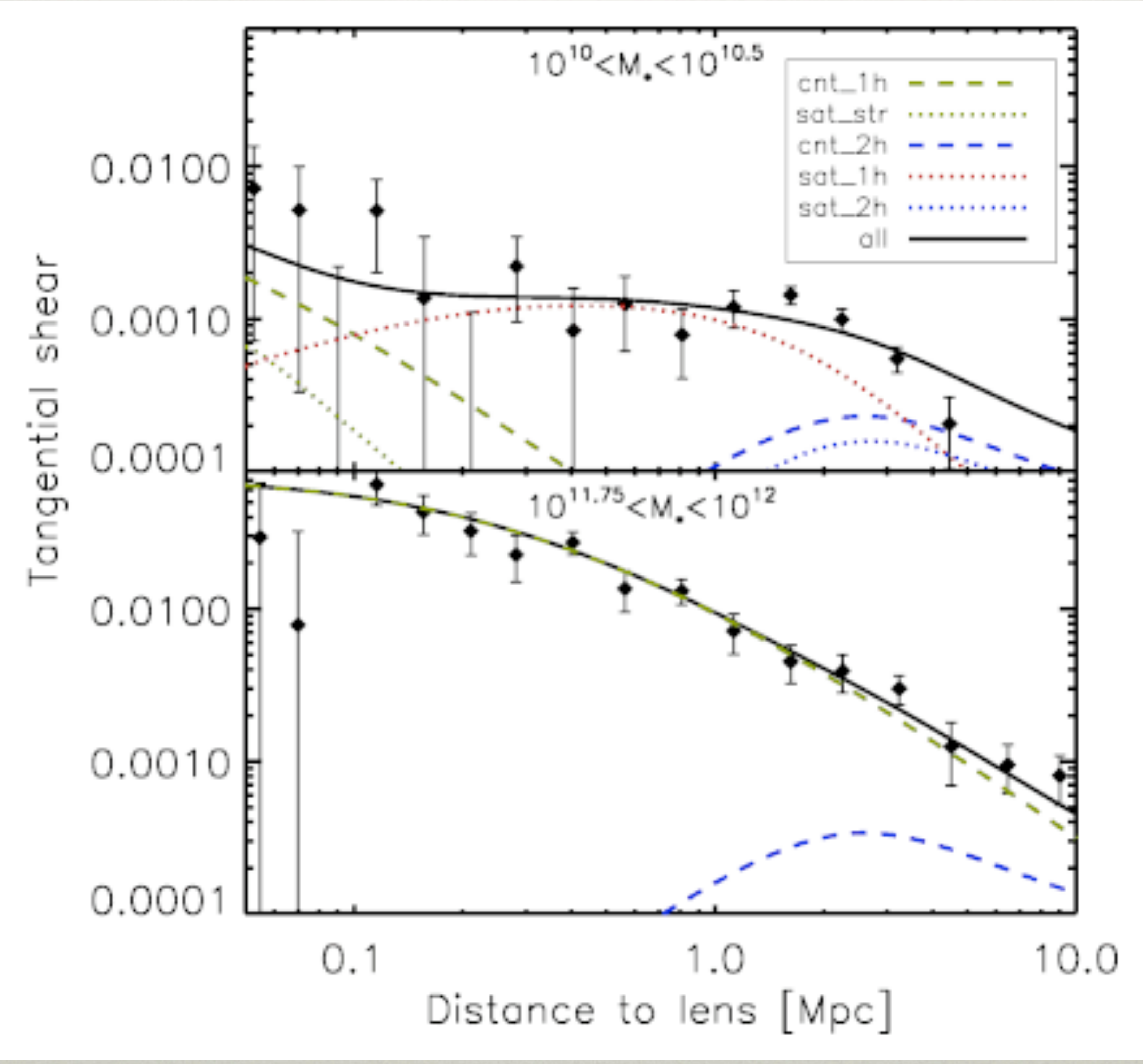
How to interpret the signal?

The signal *(the galaxy-mass cross-correlation function)* is the convolution of the dark matter distribution around galaxies and the clustering properties of the lenses.

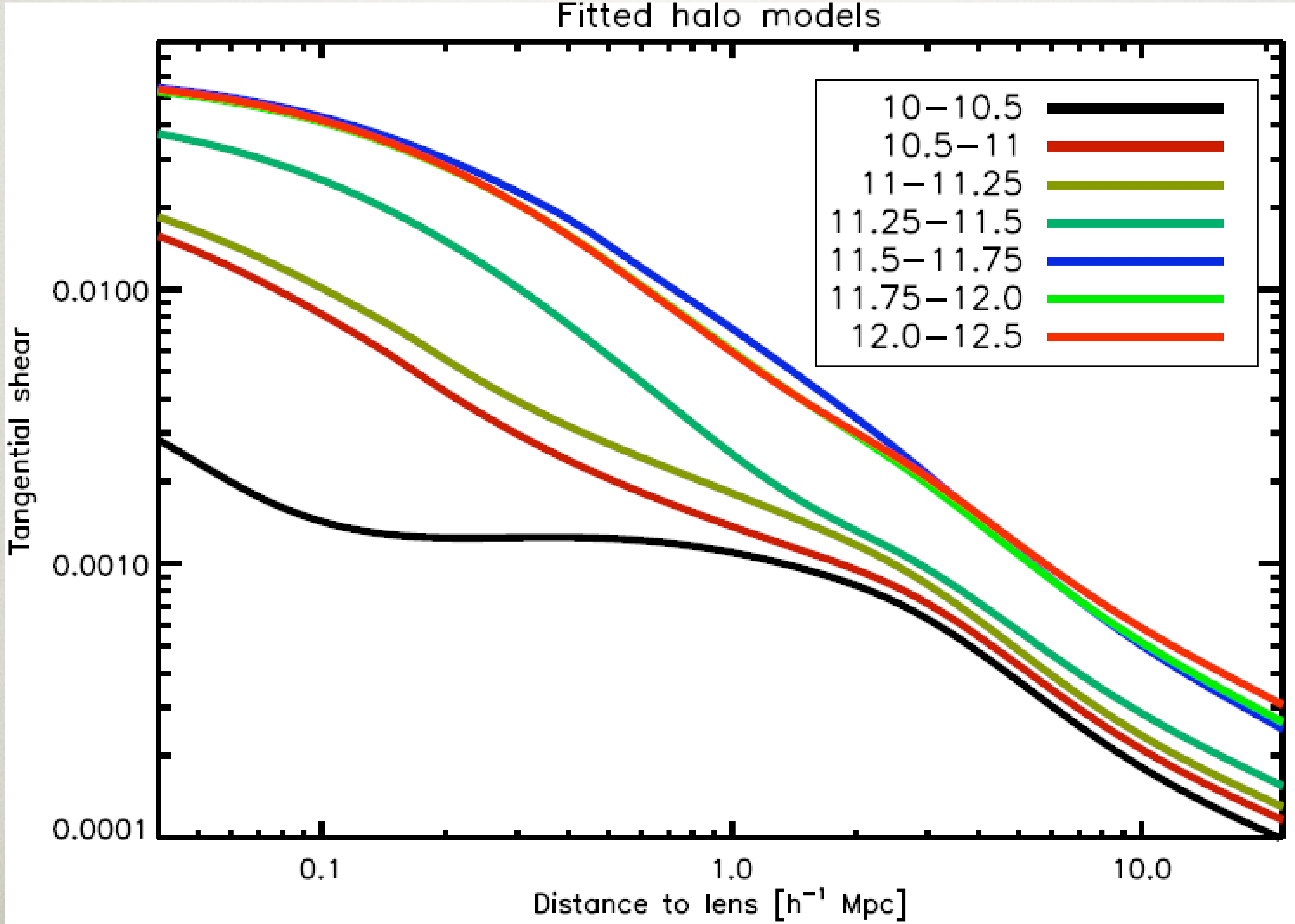
We have some options to infer information about the properties of the dark matter halos around galaxies:

- interpret the data in the context of a model (simulations/analytical)
- deconvolve the correlation function
- look at isolated halos

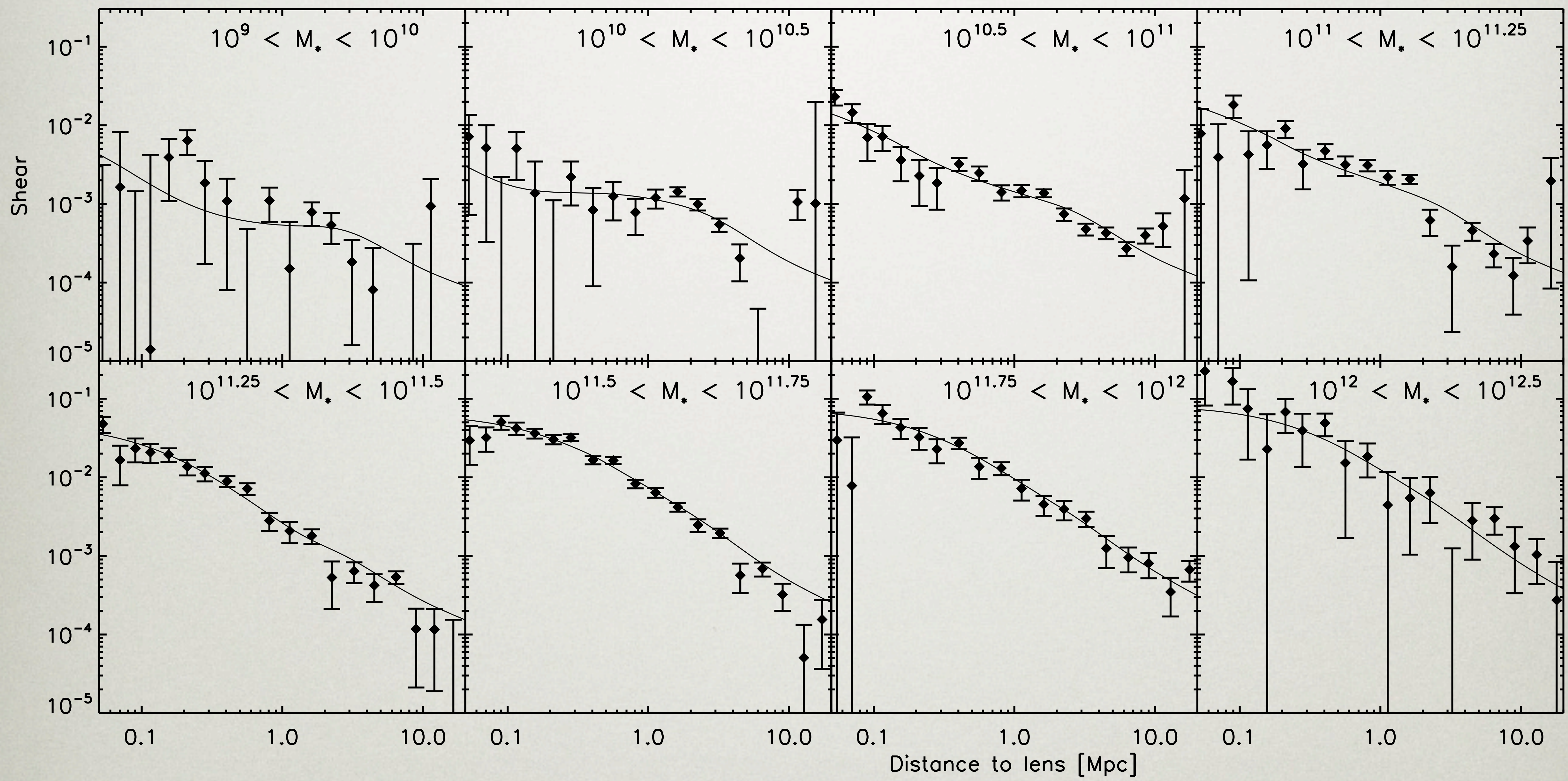
Halo-model interpretation



Halo-model interpretation

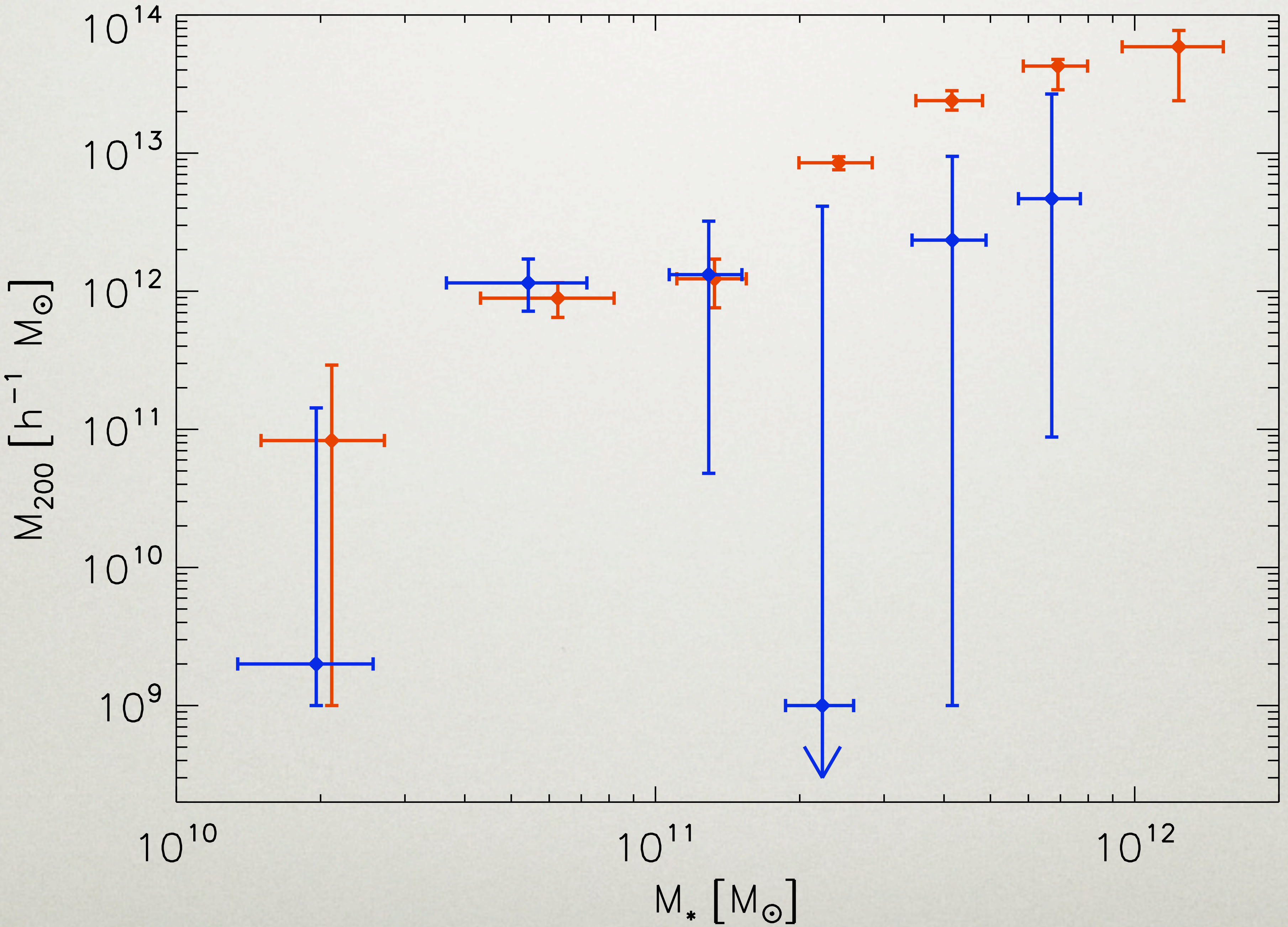


Signal vs stellar mass



overlap of RCS2 with SDSS

M_{200}-M_{STAR} RELATION



Conclusions

Weak lensing studies of clusters, groups and galaxies provide important information to link observations to simulations, which in turn leads to a better understanding of baryon physics.

Sample sizes are increasing rapidly (KiDS, DES, Euclid). Therefore it is important that the analyses become more sophisticated.