The Abdus Salam International Centre for Theoretical Physics

2354-27

Summer School on Cosmology

16 - 27 July 2012

Lensing - Lecture 2

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Cosmic shear

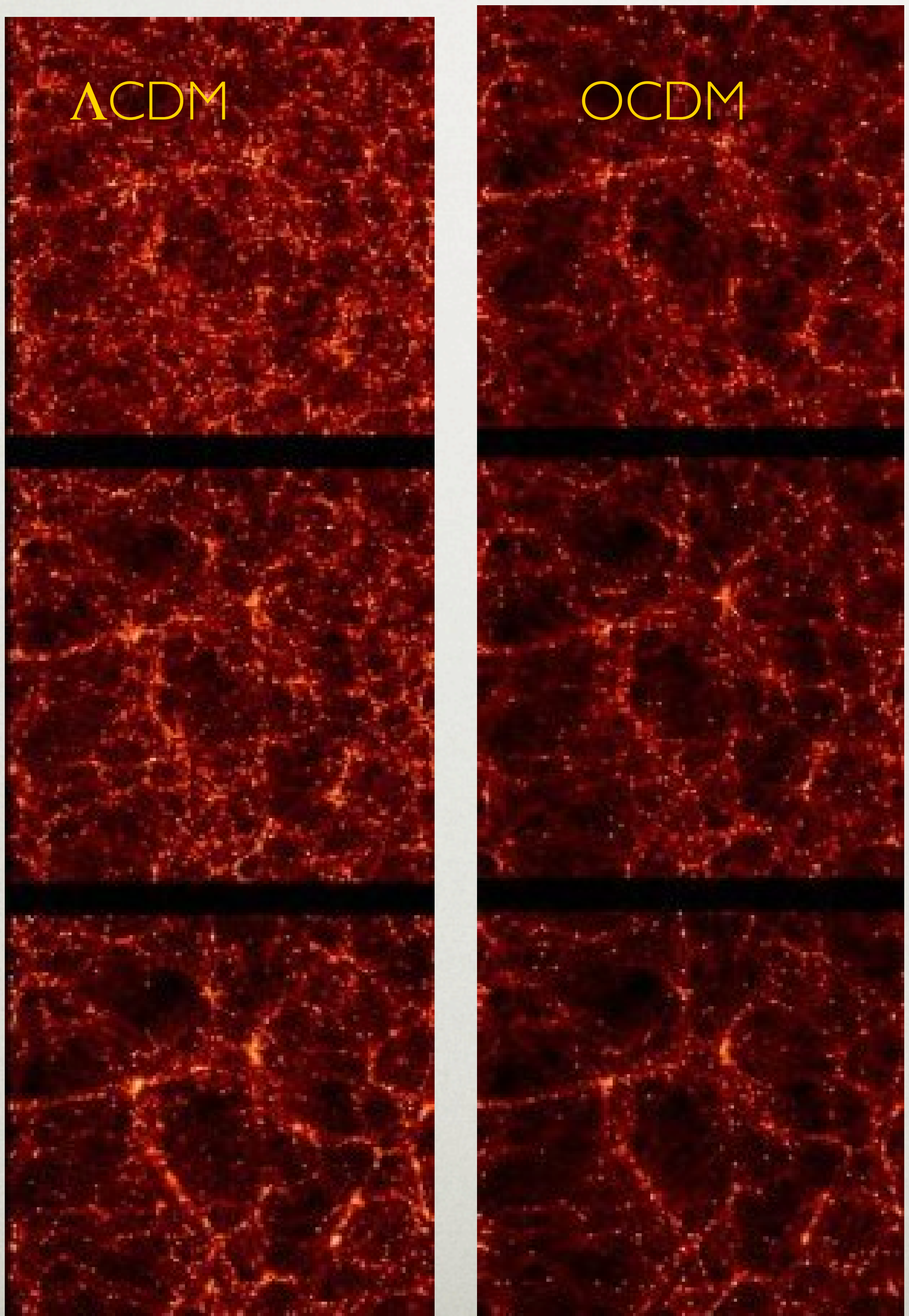
weak lensing by large scale structure

Spot the difference

z=3

z=1

z=0



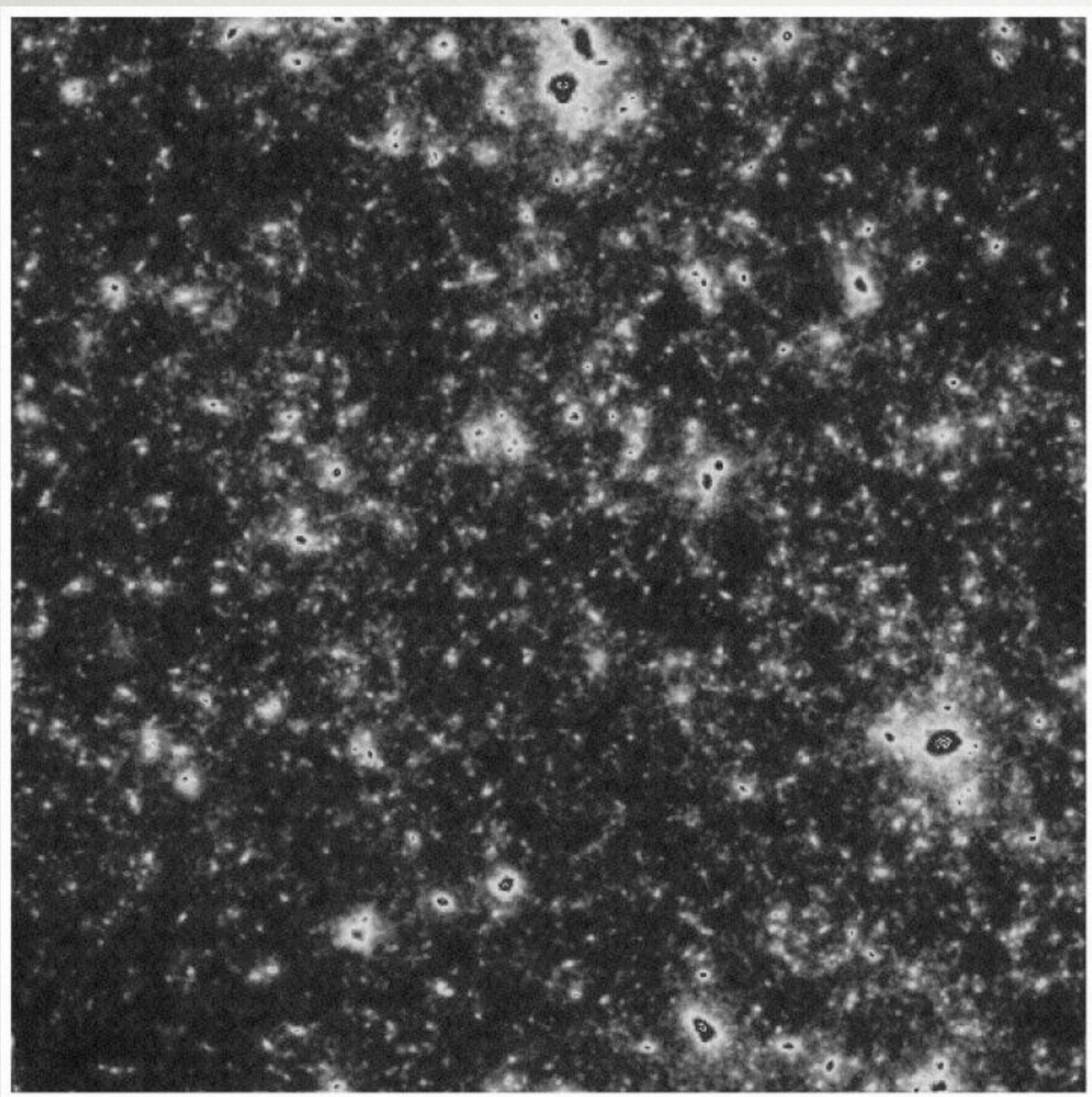
Kauffmann et al.

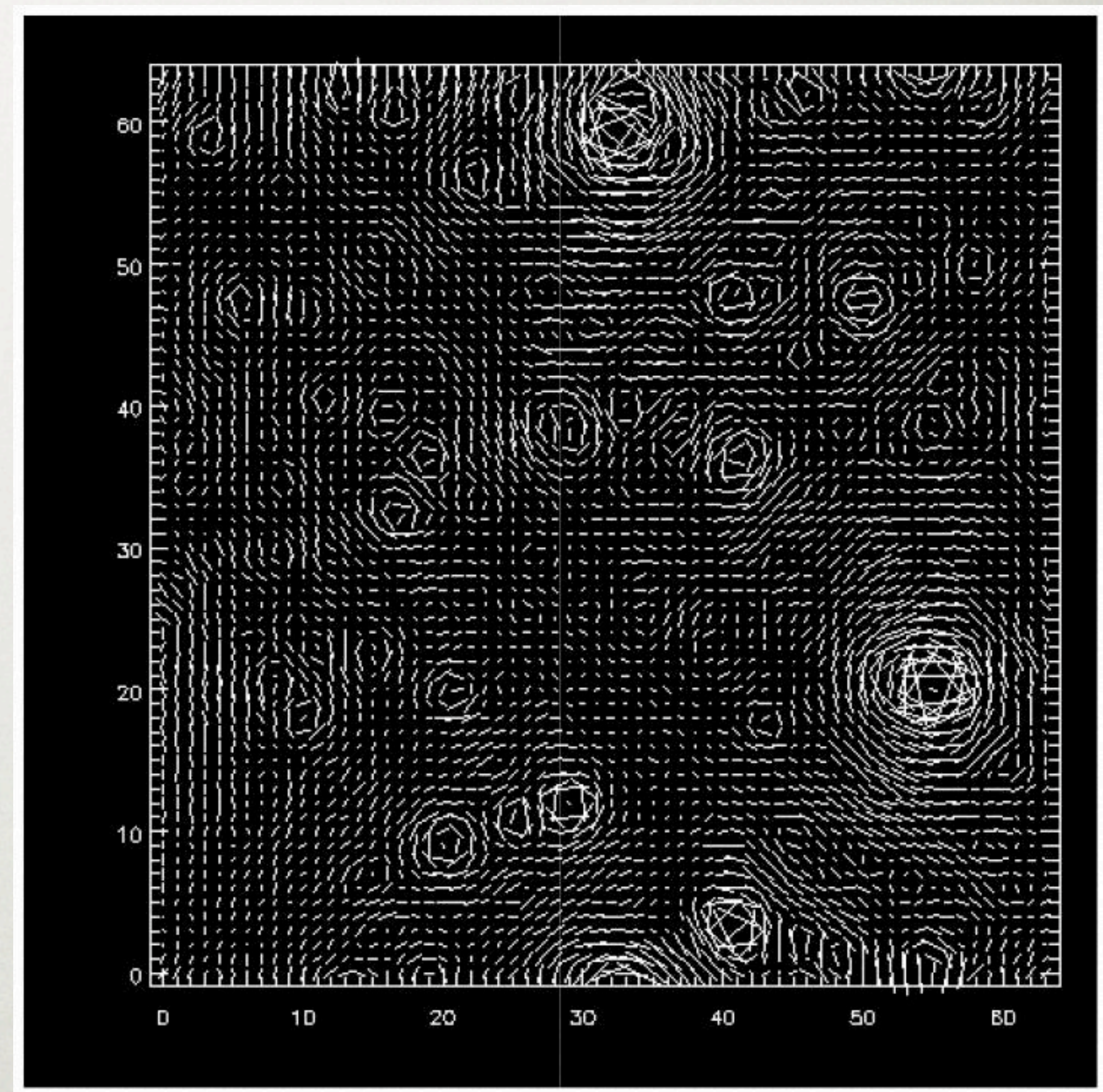
Different values for cosmological parameters lead to a different distribution of (dark) matter and a different evolution.

The clustering properties of matter as a function of scale and redshift can be used as a tool to measure the cosmology!

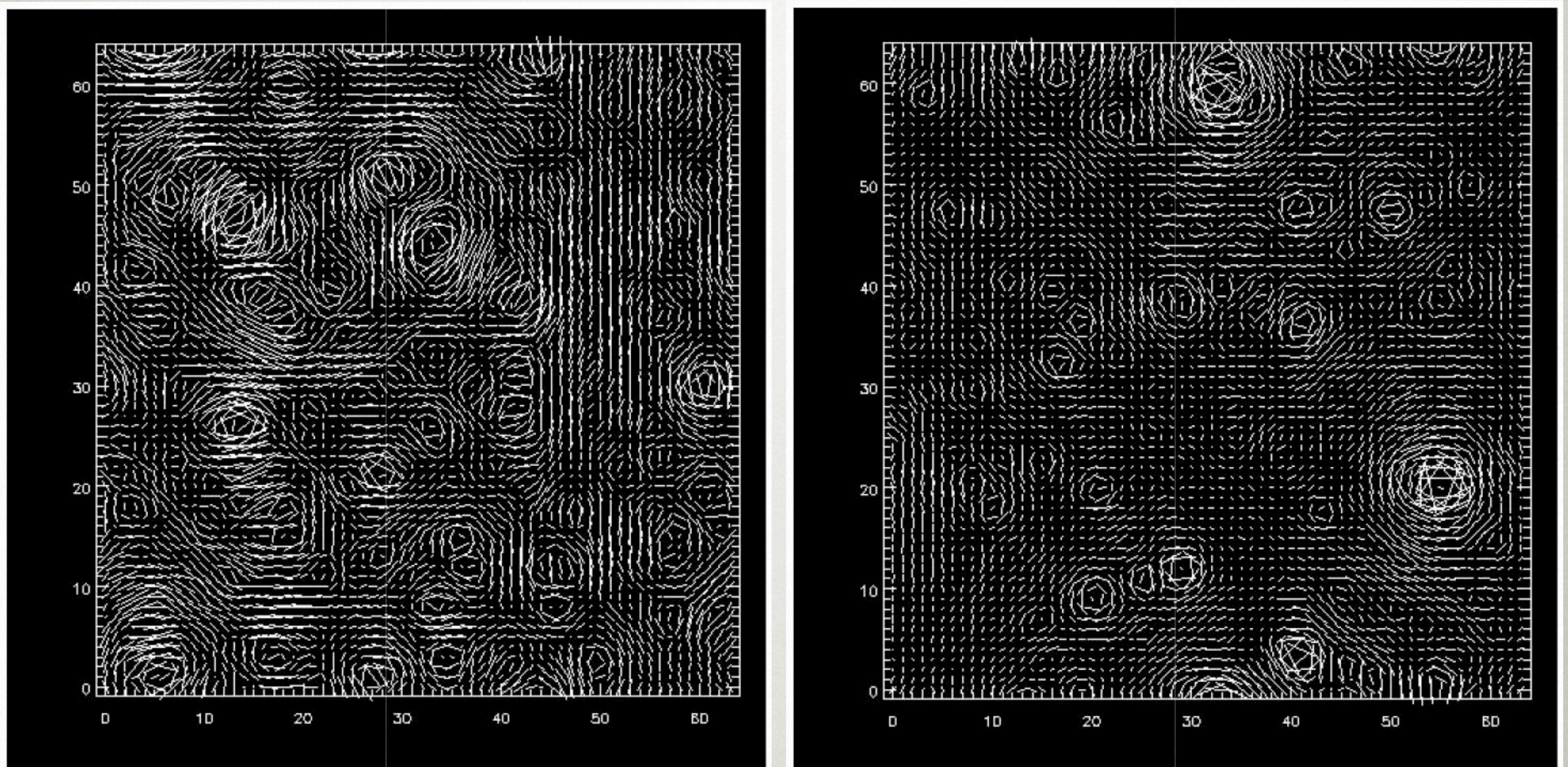
But... how to measure this?

CONVERGENCE VS SHEAR





SHEAR VARIANCE

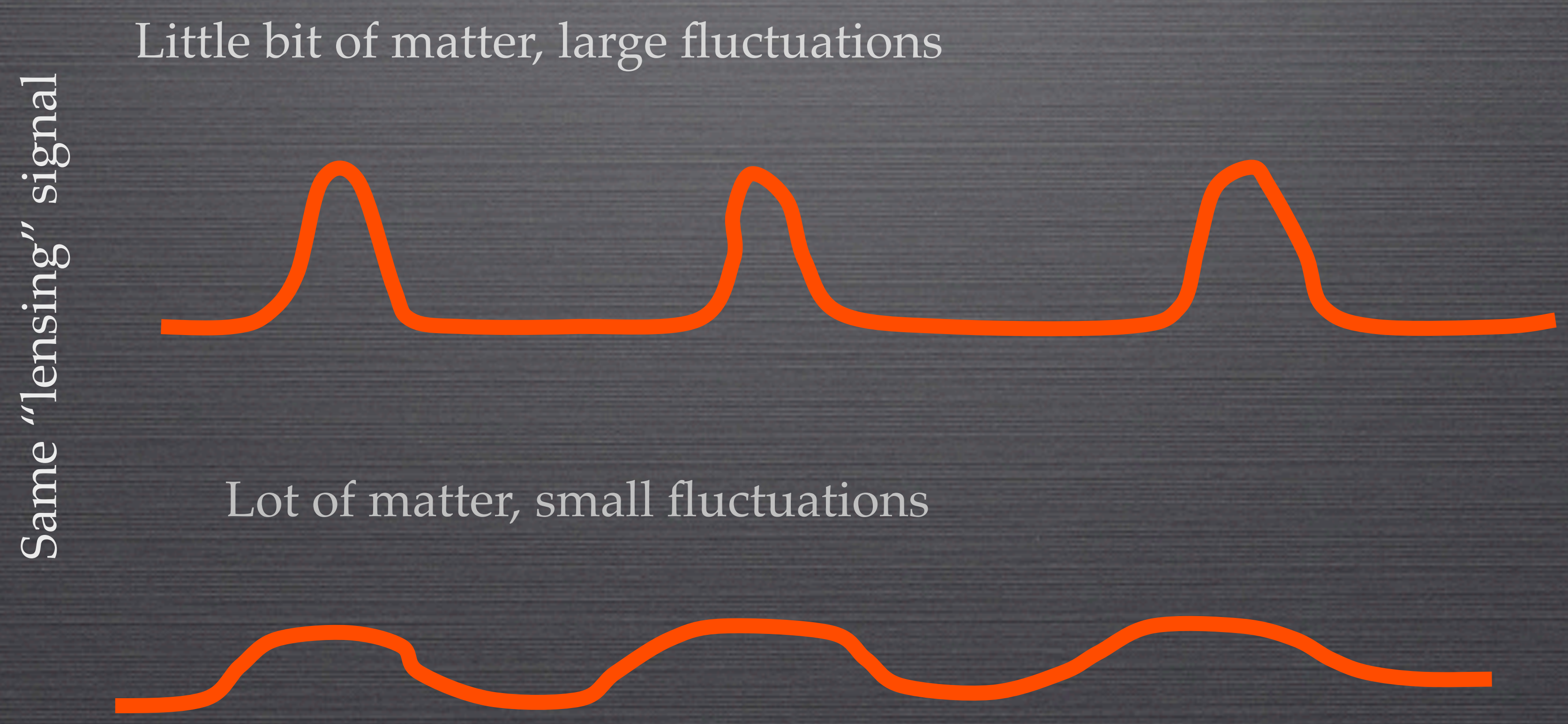


high-density Universe $\Omega_m = 1$

low-density Universe $\Omega_m = 0.3$

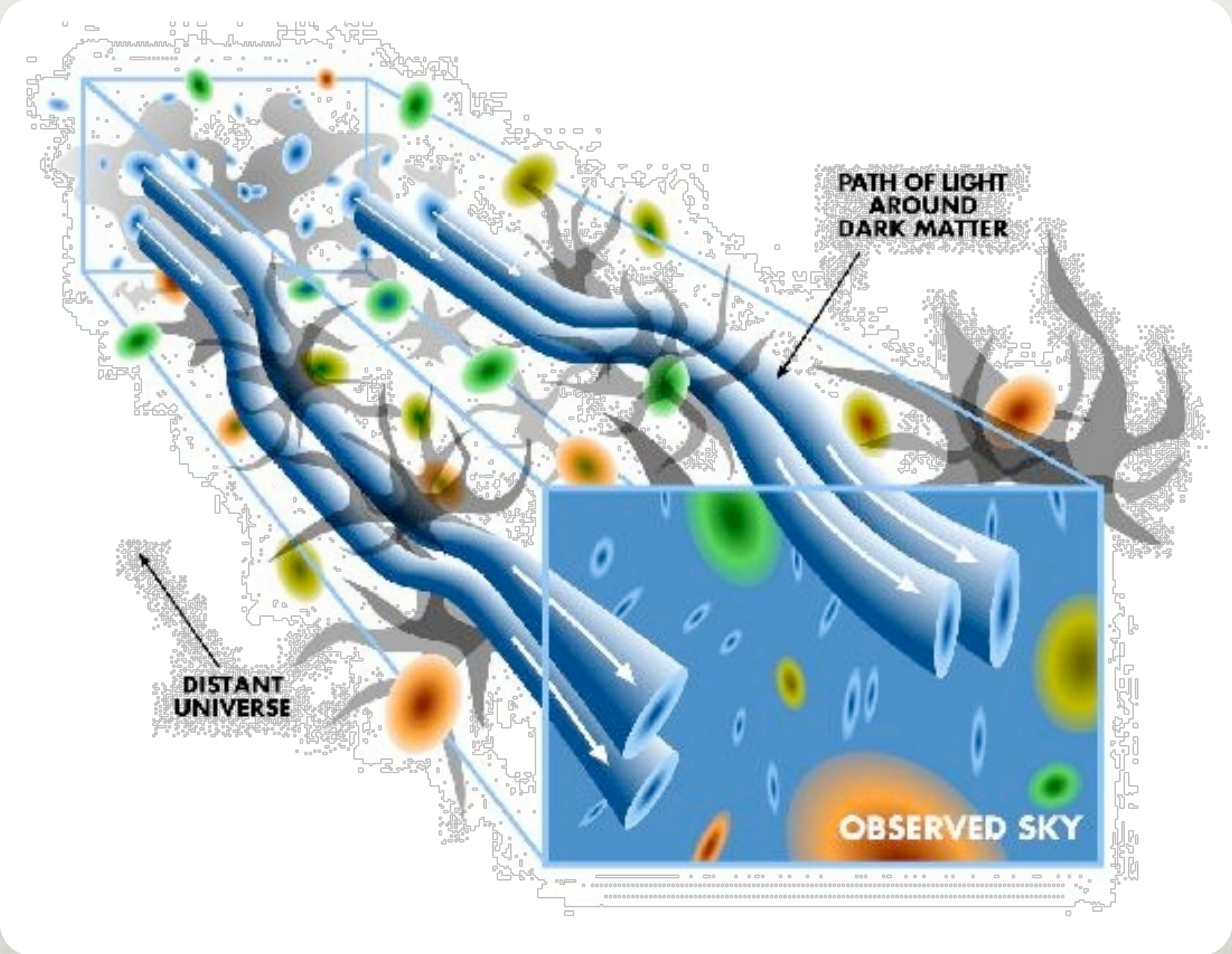
How to quantify?

The cosmic shear signal is mainly a measurement of the variance in the density fluctuations.



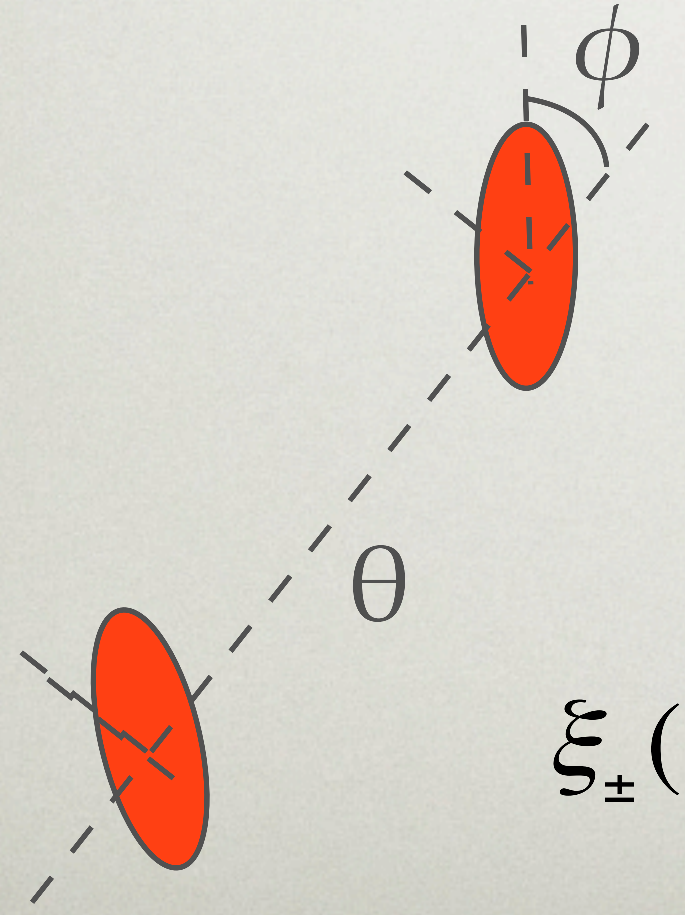
signal scales approximately $\propto (\Omega_{\mathrm{m}}^{0.6}\sigma_8)^2 z_s^{1.6}$

Ellipticity correlations

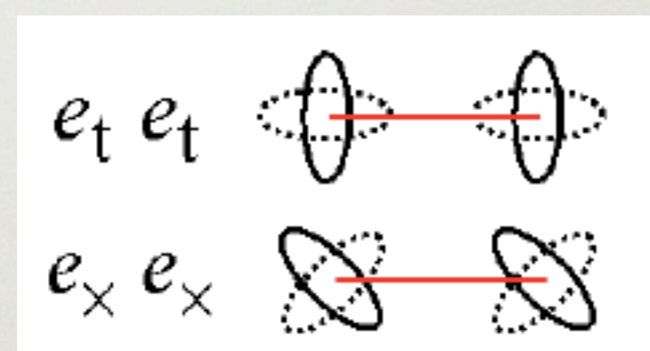


Ellipticity correlations

The shapes of galaxies become aligned as their light rays are deflected by common structures along the line-of-sight.

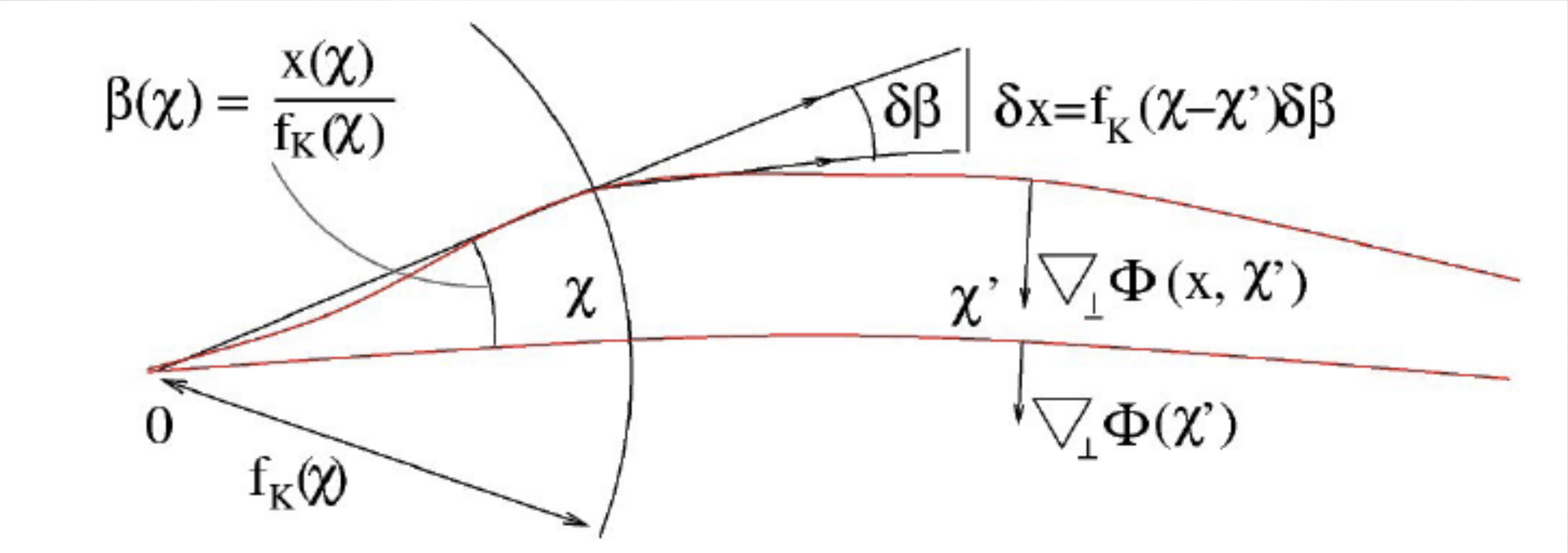


$$\gamma_t = -\Re[\gamma e^{-2i\phi}] \quad and \quad \gamma_\times = -\Im[\gamma e^{-2i\phi}]$$



$$\xi_\pm(\theta) = \left\langle \gamma_t(\vartheta)\gamma_t(\vartheta+\theta) \right\rangle \pm \left\langle \gamma_x(\vartheta)\gamma_x(\vartheta+\theta) \right\rangle$$

Lensing by LSS



$$\frac{d^2\mathbf{x}}{d\chi^2} + K\mathbf{x} = -\frac{2}{c^2}\left[\nabla_\perp \Phi(\mathbf{x}(\boldsymbol{\theta}, \chi), \chi) - \nabla_\perp \Phi^{(0)}(\chi)\right]$$

Φ : Newtonian gravitational potential, χ: comoving distance,
x: transverse comoving separation, K: curvature of the Universe
$\nabla_\perp = (\partial/\partial x_1, \partial/\partial x_2)$: Transverse comoving gradient operator.

Lensing by LSS

Solution:

$$\mathbf{x}(\boldsymbol{\theta},\chi) = f_K(\chi)\,\boldsymbol{\theta} - \frac{2}{c^2}\int_0^{\chi} \mathrm{d}\chi' f_K(\chi-\chi')\left[\nabla_\perp \Phi(\mathbf{x}(\boldsymbol{\theta},\chi'),\chi') - \nabla_\perp \Phi^{(0)}(\chi')\right]$$

$$f_K(\chi) = \begin{cases} \frac{1}{\sqrt{K}} \sin\left(\sqrt{K}\chi\right) & K > 0 \\ \chi & K = 0 \\ \frac{1}{\sqrt{-K}} \sinh\left(\sqrt{-K}\chi\right) & K < 0 \end{cases}$$

$$D^{\mathrm{ang}}_{z_1,z_2} = a(z_2) f_K[\chi(z_2) - \chi(z_1)]$$

Lensing by LSS

$\beta = \mathbf{x}/f_K$ hypothetical unlensed position for a source plane at χ.
As in 2D lensing define the **Jacobian matrix**

$$\begin{aligned}\mathcal{A}_{ij}(\boldsymbol{\theta},\chi) &= \frac{\partial \beta_i}{\partial \theta_j} = \frac{1}{f_K(\chi)}\frac{\partial x_i}{\partial \theta_j} \\ &= \delta_{ij} - \frac{2}{c^2}\int_0^\chi \mathrm{d}\chi' \frac{f_K(\chi-\chi')f_K(\chi')}{f_K(\chi)}\frac{\partial^2 \Phi(\mathbf{x}(\boldsymbol{\theta},\chi'),\chi')}{\partial x_i \partial x_k}\mathcal{A}_{kj}(\boldsymbol{\theta},\chi')\end{aligned}$$

Expand *A* in powers of potential and truncate the non-linear terms:

$$\mathcal{A}_{ij}(\boldsymbol{\theta},\chi) = \delta_{ij} - \frac{2}{c^2}\int_0^\chi \mathrm{d}\chi' \frac{f_K(\chi-\chi')f_K(\chi')}{f_K(\chi)}\frac{\partial^2 \Phi(\mathbf{x}=f_K(\chi')\boldsymbol{\theta},\chi')}{\partial x_i \partial x_j}$$

Lensing by LSS

Born approximation: to first order in the potential, the distortion can be approximated by the integral along the unperturbed light ray, with correction of order Φ^2 (precision ~few %).

Analogous to 2D lensing we define the deflection potential

$$\Psi(\boldsymbol{\theta}, \chi) \equiv \frac{2}{c^2} \int_0^{\chi} \mathrm{d}\chi' \frac{f_K(\chi - \chi')}{f_K(\chi) f_K(\chi')} \Phi(f_K(\chi')\boldsymbol{\theta}, \chi')$$

$$\kappa = \frac{1}{2}(\Psi_{,11} + \Psi_{,22}) \quad \gamma = \frac{1}{2}(\Psi_{,11} - \Psi_{,22}) + \mathrm{i}\Psi_{,12}$$

Lensing by LSS

The 3-d Poisson equation in comoving coordinates is:

$$\nabla^2 \Phi = \frac{3H_0^2 \Omega_m}{2a} \delta$$

which yields a convergence:

$$\boxed{\kappa(\boldsymbol{\theta}, \chi) \quad = \quad \frac{3H_0^2 \Omega_\mathrm{m}}{2c^2} \int_0^\chi \mathrm{d}\chi' \, \frac{f_K(\chi - \chi') f_K(\chi')}{f_K(\chi)} \frac{\delta(f_K(\chi')\boldsymbol{\theta}, \chi')}{a(\chi')}}$$

Lensing by LSS

For sources distributed in redshift: $p_z(z)\mathrm{d}z = p_\chi(\chi)\mathrm{d}\chi$

$$\kappa(\boldsymbol{\theta}) = \int \mathrm{d}\chi\, p_\chi(\chi)\kappa(\boldsymbol{\theta},\chi) = \frac{3H_0^2\Omega_\mathrm{m}}{2c^2}\int_0^{\chi_\mathrm{h}} \mathrm{d}\chi'\, g(\chi') f_K(\chi') \frac{\delta(f_K(\chi')\boldsymbol{\theta},\chi')}{a(\chi')}$$

Effective source-redshift weighted lens efficiency factor

$$g(\chi') = \int_{\chi'}^{\chi_\mathrm{h}} \mathrm{d}\chi\, p_\chi(\chi) \frac{f_K(\chi-\chi')}{f_K(\chi)}$$

How can we relate this to the measurements and the cosmological model?

Limber's equation

δ homogeneous and isotropic 3D random field $\Rightarrow$ 2D projection

$$c_i(\boldsymbol{\theta}) = \int \mathrm{d}\chi \, q_i(\chi)\delta(f_K(\chi)\boldsymbol{\theta}, \chi)$$

is also a homogeneous and isotropic field with correlation function

$$C_{12} = \langle c_1(\boldsymbol{\varphi_1}) c_2(\boldsymbol{\varphi_2}) \rangle = C_{12}(\varphi = |\boldsymbol{\varphi_1} - \boldsymbol{\varphi_2}|)$$

and power spectrum

$$P_{12}(\ell) = \int \mathrm{d}\varphi C_{12}(\varphi) \mathrm{e}^{\mathrm{i}\ell\varphi} = \int \mathrm{d}\chi \frac{q_1(\chi) q_2(\chi)}{f_K^2(\chi)} P_\delta\left(\frac{\ell}{f_K(\chi)}, \chi\right)$$

Convergence power spectrum

$\kappa(\boldsymbol{\theta})$ is such a projection with $q_1(\chi) = q_2(\chi) = \frac{3H_0^2\Omega_{\rm m}}{2c^2}\frac{g(\chi)f_K(\chi)}{a}$

$$P_\kappa(\ell) = \frac{9H_0^4\Omega_{\rm m}^2}{4c^4}\int_0^{\chi_{\rm h}} {\rm d}\chi \frac{g^2(\chi)}{a^2(\chi)} P_\delta\left(\frac{\ell}{f_K(\chi)}, \chi\right)$$

geometry cosmology

But we do not measure κ...

Convergence power spectrum

Recall: shear and convergence are related; this allowed us to make mass reconstructions.

Fourier transform and inverse:

$$\hat{\kappa}(\boldsymbol{\ell}) = \int \mathrm{d}^2\theta \mathrm{e}^{\mathrm{i}\boldsymbol{\ell}\cdot\boldsymbol{\theta}} \kappa(\boldsymbol{\theta}) \,, \quad \kappa(\boldsymbol{\theta}) = \frac{1}{(2\pi)^2} \int \mathrm{d}^2\ell \mathrm{e}^{-\mathrm{i}\boldsymbol{\ell}\cdot\boldsymbol{\theta}} \hat{\kappa}(\boldsymbol{\ell})$$

Real space: $\partial/\partial\theta_j \Leftrightarrow$ Fourier space: $\times - \mathrm{i}\ell_j$.

$$2\hat{\kappa}(\boldsymbol{\ell}) = -(|\boldsymbol{\ell}|^2)\hat{\Psi}(\ell) \,, \quad 2\hat{\gamma}(\boldsymbol{\ell}) = -(\ell_1^2 - \ell_2^2 + 2\mathrm{i}\ell_1\ell_2)\hat{\Psi}(\ell)$$

$$\hat{\gamma}(\boldsymbol{\ell}) = \left(\frac{\ell_1^2 - \ell_2^2 + 2\mathrm{i}\ell_1\ell_2}{|\boldsymbol{\ell}|^2}\right)\hat{\kappa}(\boldsymbol{\ell}) = \mathrm{e}^{2\mathrm{i}\beta}\hat{\kappa}(\boldsymbol{\ell})$$

Convergence power spectrum

The convergence and shear have the same power spectrum:

$$\langle \hat{\gamma}(\boldsymbol{\ell})\hat{\gamma}^*(\boldsymbol{\ell}') \rangle = \langle \hat{\kappa}(\boldsymbol{\ell})\hat{\kappa}^*(\boldsymbol{\ell}') \rangle = (2\pi)^2 \delta_{\mathrm{D}}(\boldsymbol{\ell} - \boldsymbol{\ell}') P_\kappa(\ell)$$

$\boxed{\text{We can use the observed shear statistics to directly constrain the convergence power spectrum!}}$

Measuring variance

Top-hat variance

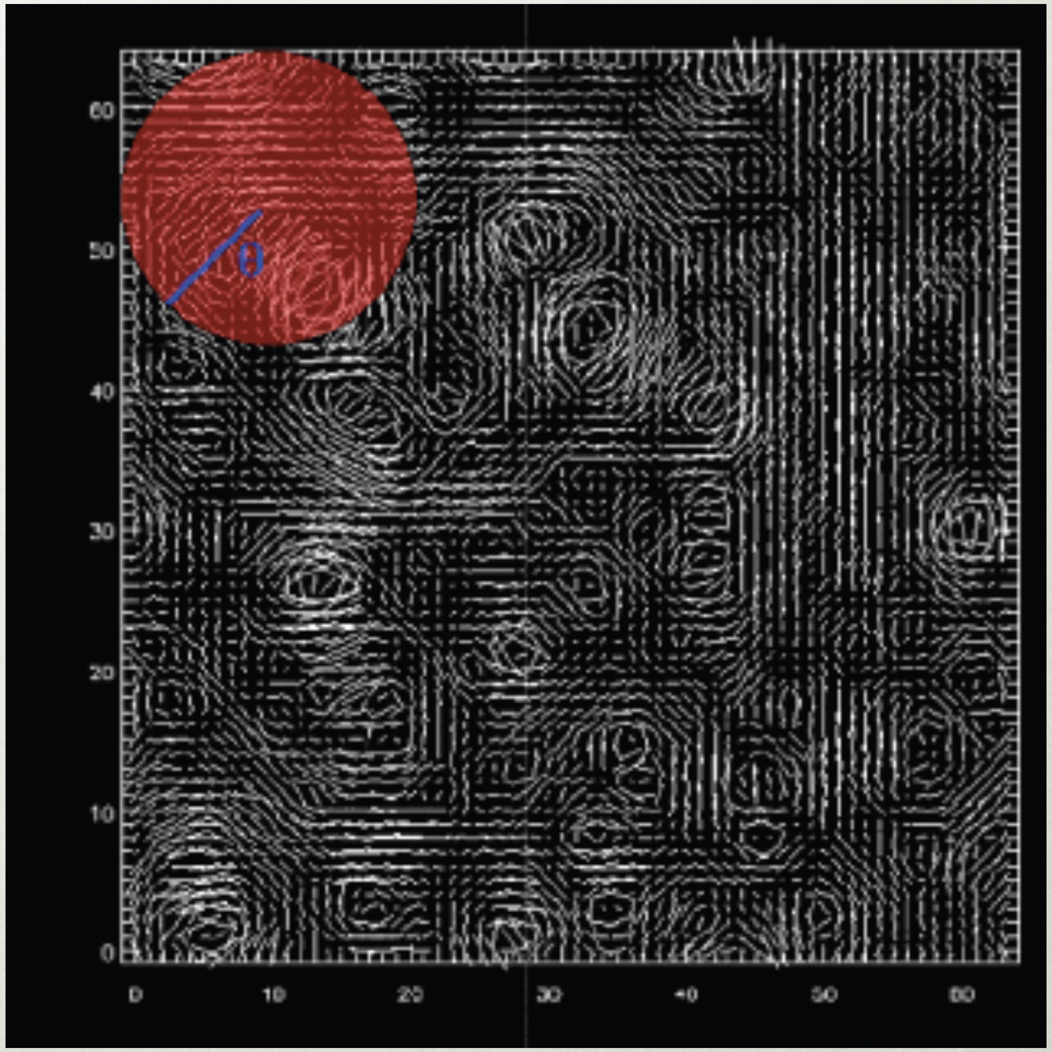
Mean shear in a circular aperture

$\bar{\gamma}(\theta) = \frac{1}{\pi\theta^2} \int d^2\theta' \, \gamma(\vec{\theta'})$

still has a phase $\Rightarrow \langle\bar{\gamma}\rangle(\theta) = 0$

Top-hat shear dispersion

$|\bar{\gamma}|^2(\theta) = \bar{\gamma}\bar{\gamma}^*(\theta)$ with unbiased estimator $\frac{1}{N(N-1)} \sum_{i \neq j} e_i e_j^*$



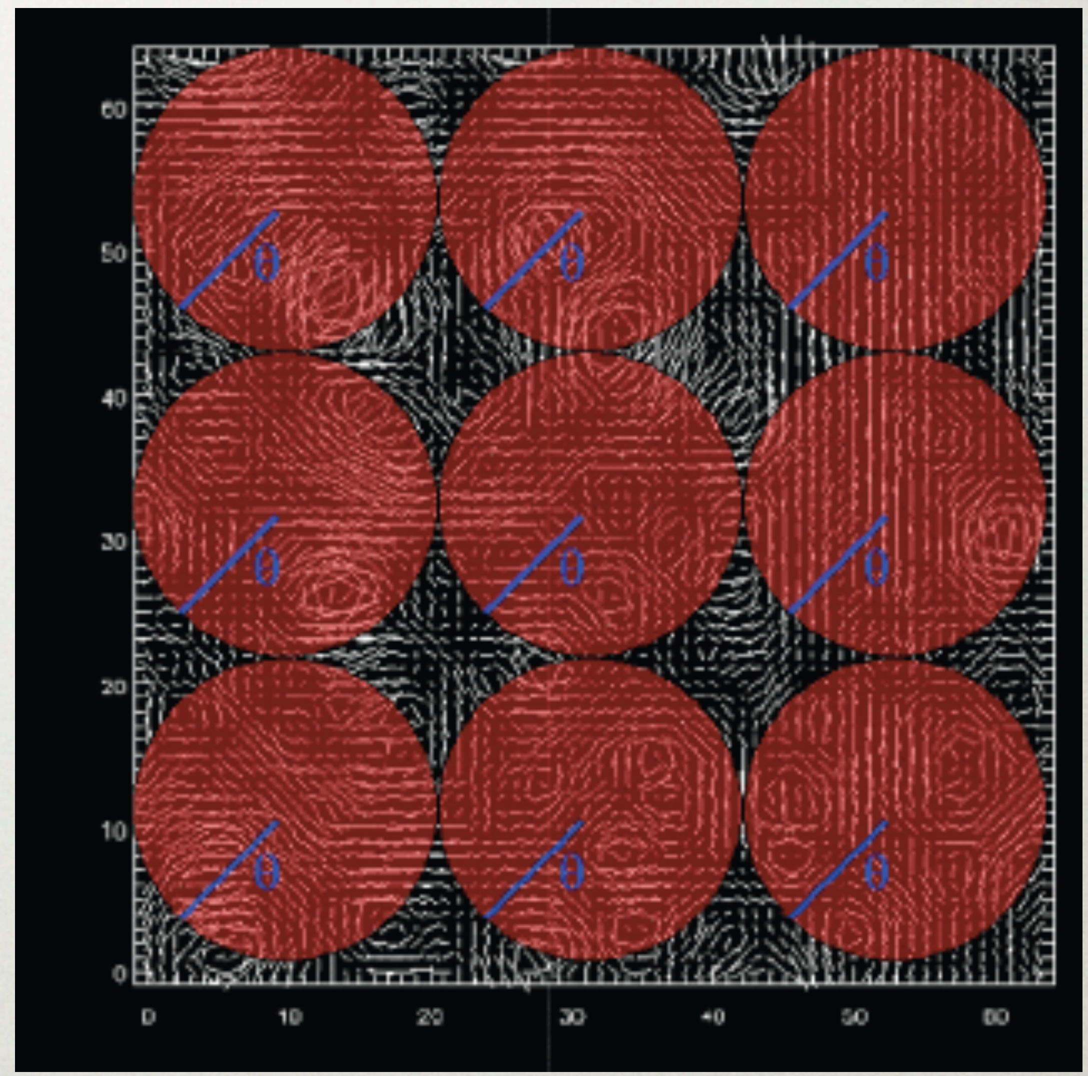
Measuring variance

Top-hat variance

Average over many independent apertures: $\langle|\bar{\gamma}|^2\rangle(\theta) = \langle\bar{\gamma}\bar{\gamma}^*\rangle(\theta)$

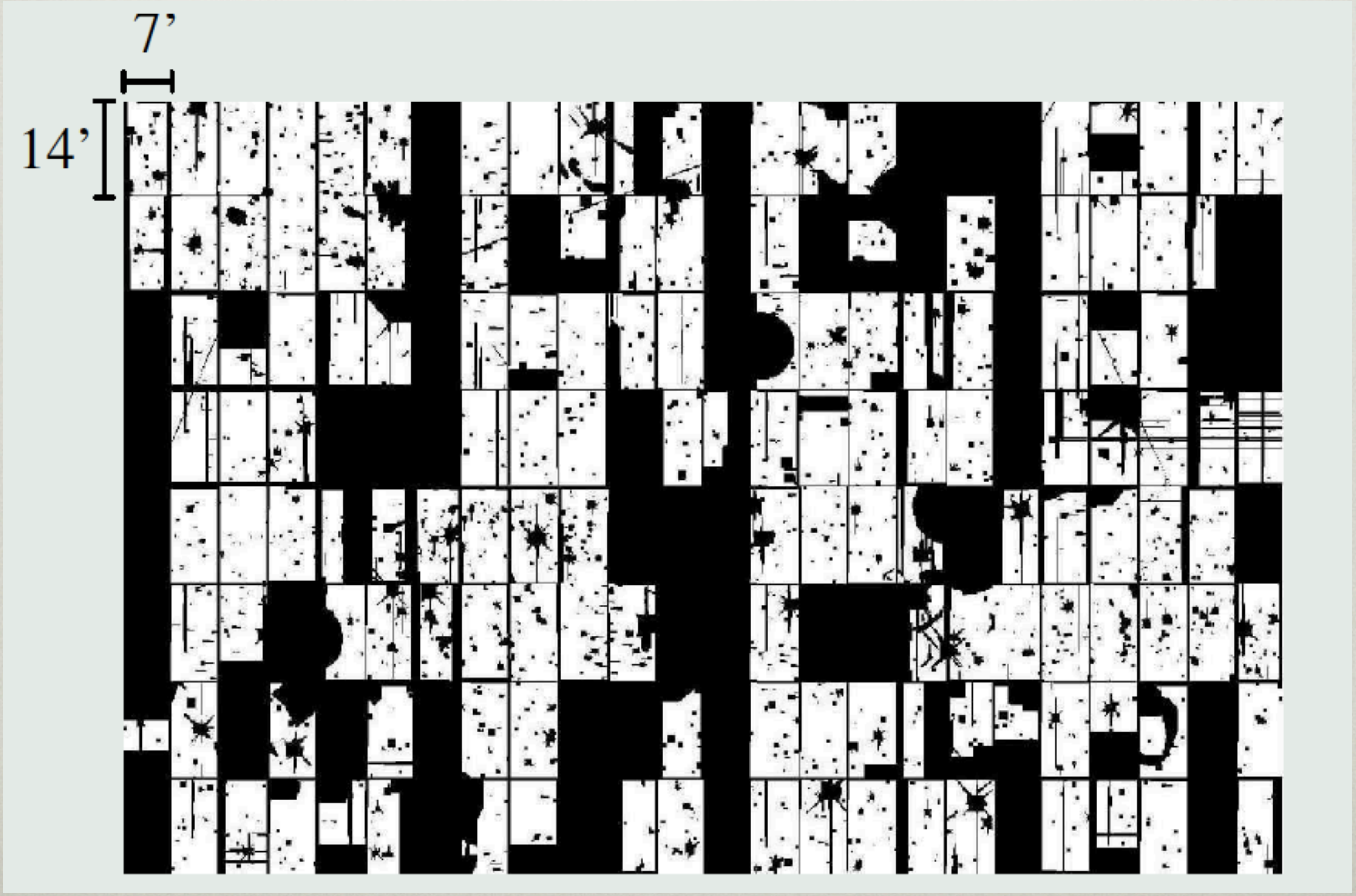
$$\langle|\bar{\gamma}|^2\rangle(\theta) = \frac{1}{2\pi}\int \mathrm{d}\ell\,\ell P_\kappa(\ell) W_{\mathrm{TH}}(\ell\theta)\,,$$

$W_{\mathrm{TH}}(\eta) = 4\mathrm{J}_1^2(\eta)/\eta^2$ top-hat filter, J_n: n^{th} Bessel function.



Problems: mixes power on different scales and cannot properly account for gaps in the data.

Measuring variance



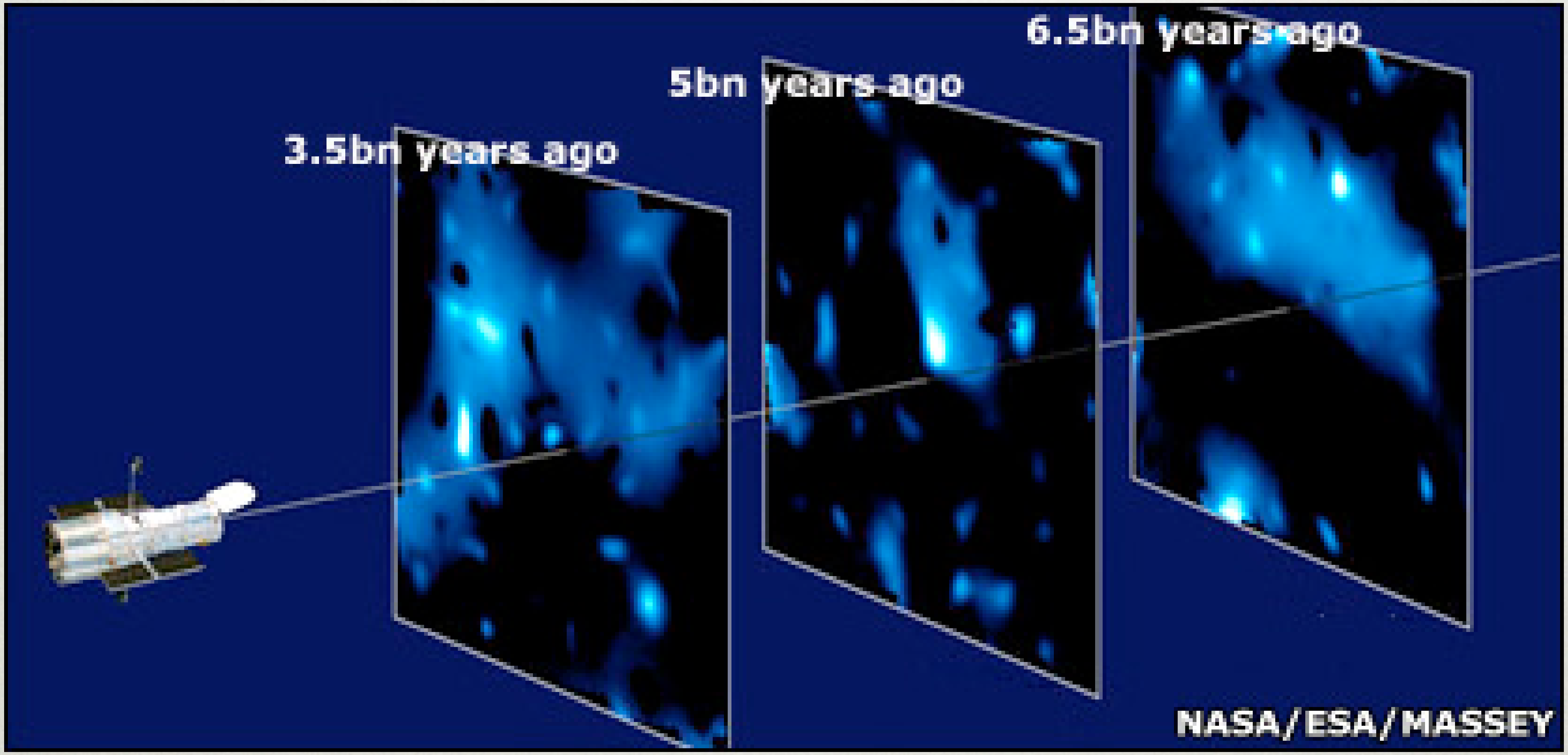
Ellipticity correlations

The ellipticity correlation functions are more convenient to use in practice:

$$\xi_+(\theta) = \frac{1}{2\pi} \int_0^\infty \mathrm{d}\ell\, \ell\, \mathrm{J}_0(\ell\theta) P_\kappa(\ell)$$

$$\xi_-(\theta) = \frac{1}{2\pi} \int_0^\infty \mathrm{d}\ell\, \ell\, \mathrm{J}_4(\ell\theta) P_\kappa(\ell)$$

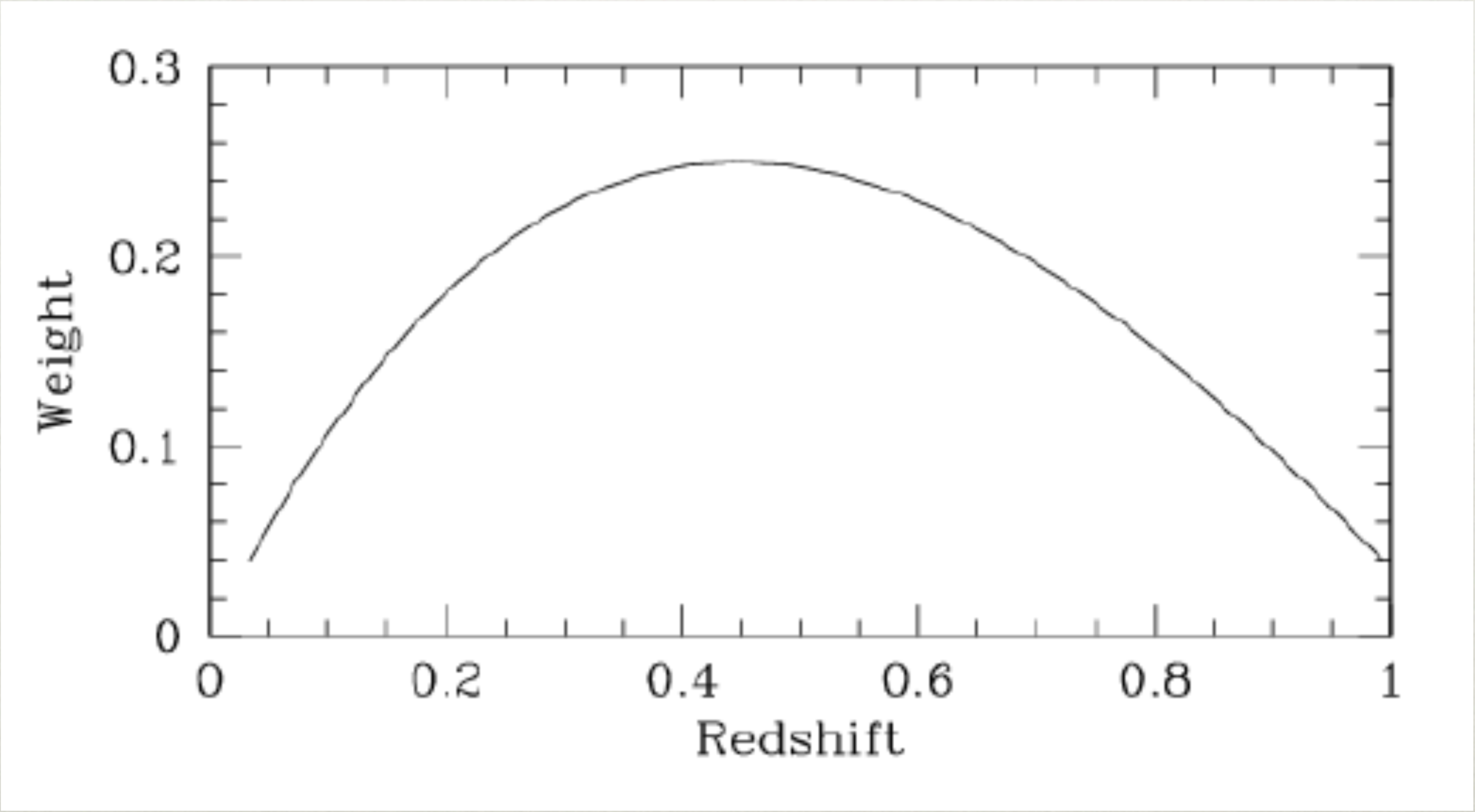
Slicing the universe



We need to measure the matter distribution as a function of redshift: in addition to the shapes, weak lensing tomography requires redshift information for the sources.

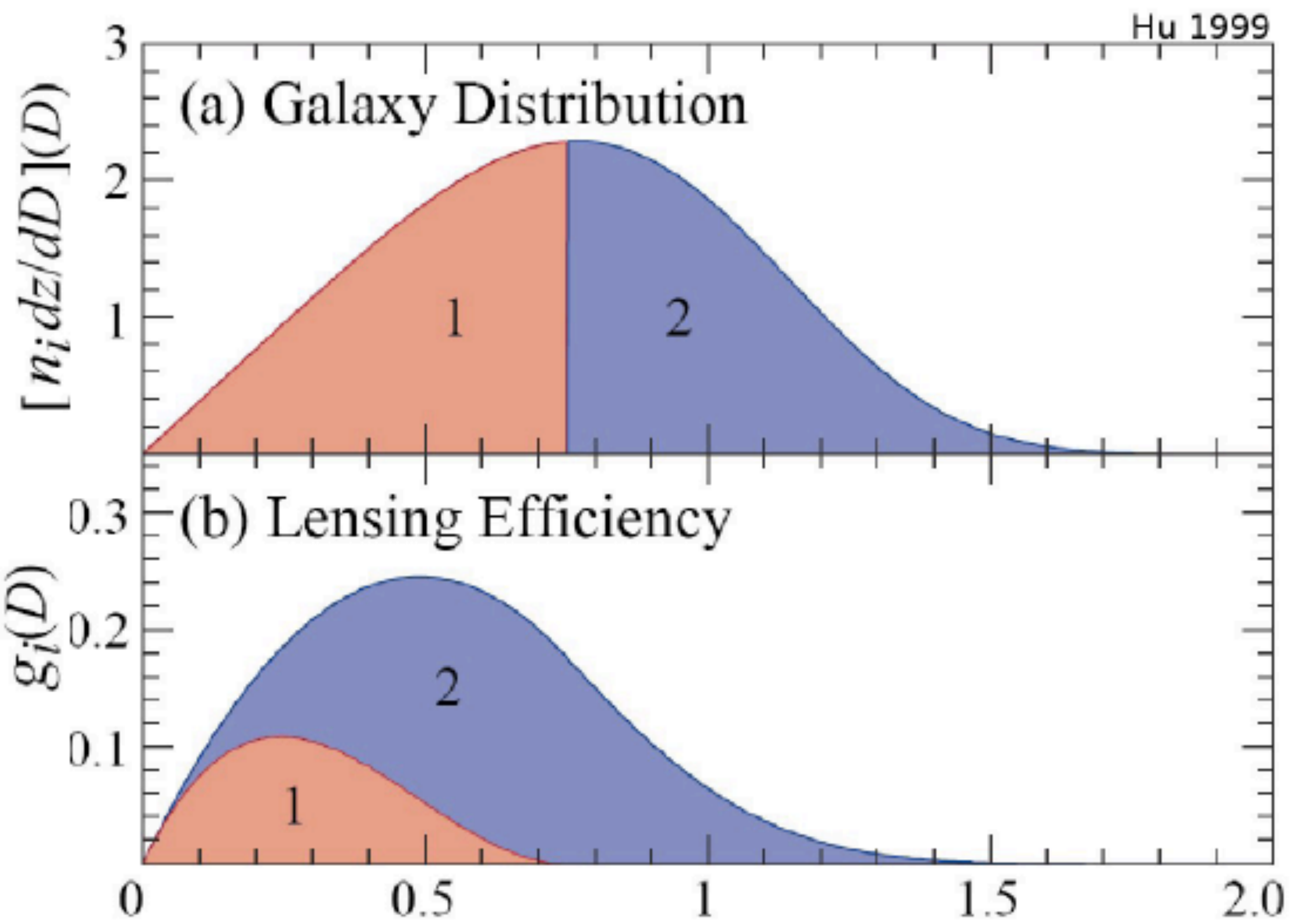
Slicing the universe

The lensing kernel is most sensitive to structure halfway between the observer and the source. But the kernel is broad: we do not need precise redshifts for the sources.



Slicing the universe

Cosmic shear is sensitive to everything along the line-of-sight...



Here: $g(\chi) = \mathbf{a}(\chi)\mathbf{f_K}(\chi) \int_\chi^{\chi_h} \mathrm{d}\chi' \, p_\chi(\chi') \frac{f_K(\chi'-\chi)}{f_K(\chi')}$.

Slicing the universe

$$P_\kappa(\ell) = \frac{9H_0^4\Omega_{\rm m}^2}{4c^4}\int_o^{\chi_{\rm h}} {\rm d}\chi \frac{g^2(\chi)}{a^2(\chi)} P_\delta\left(\frac{\ell}{f_K(\chi)}, \chi\right)$$

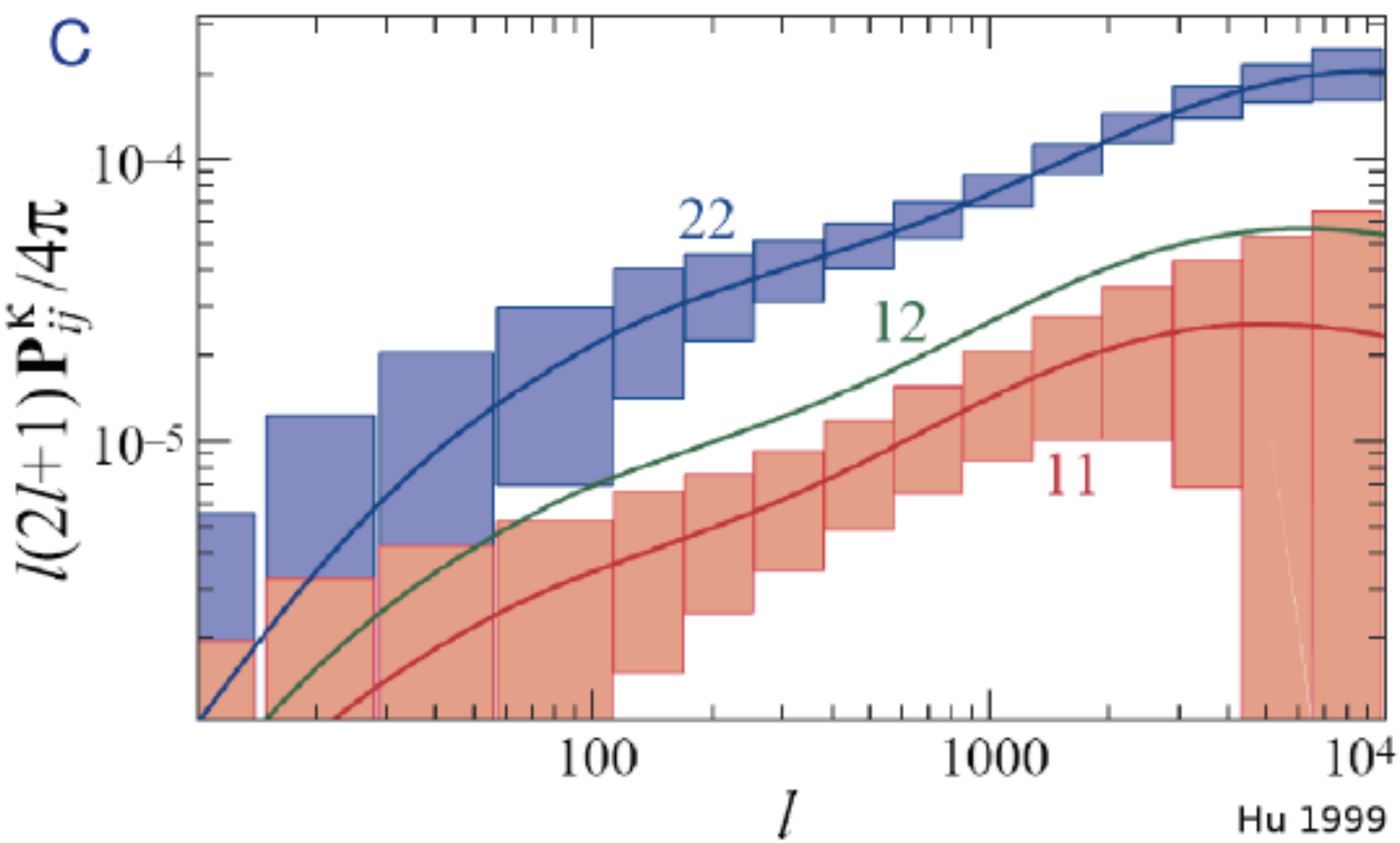
$$g(\chi) = \int_\chi^{\chi_{\rm h}} {\rm d}\chi' \, p_\chi(\chi') \frac{f_K(\chi' - \chi)}{f_K(\chi')}$$

For tomography:

$$P_\kappa(\ell) \Rightarrow P_\kappa^{ij}(\ell)$$

$$g^2(\chi) \Rightarrow g_i(\chi) g_j(\chi)$$

Slicing the universe



Why is P_κ^{22} so much larger than P_κ^{11}?

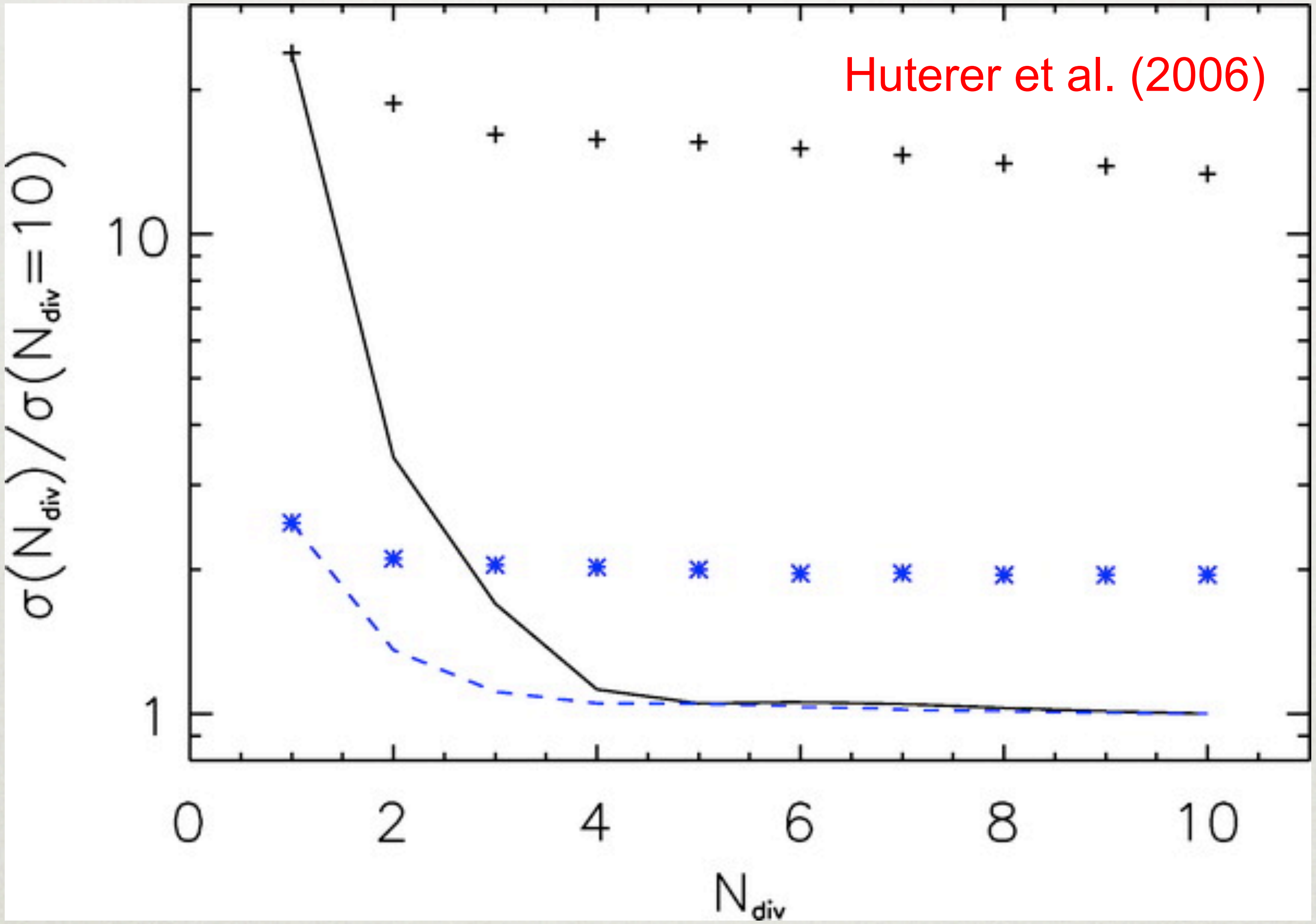
Slicing the universe

Tomography allows us to break the inherent degeneracy between normalization and matter density!

constrain the growth of structure

- test GR on cosmological scales
- constrain dark energy properties

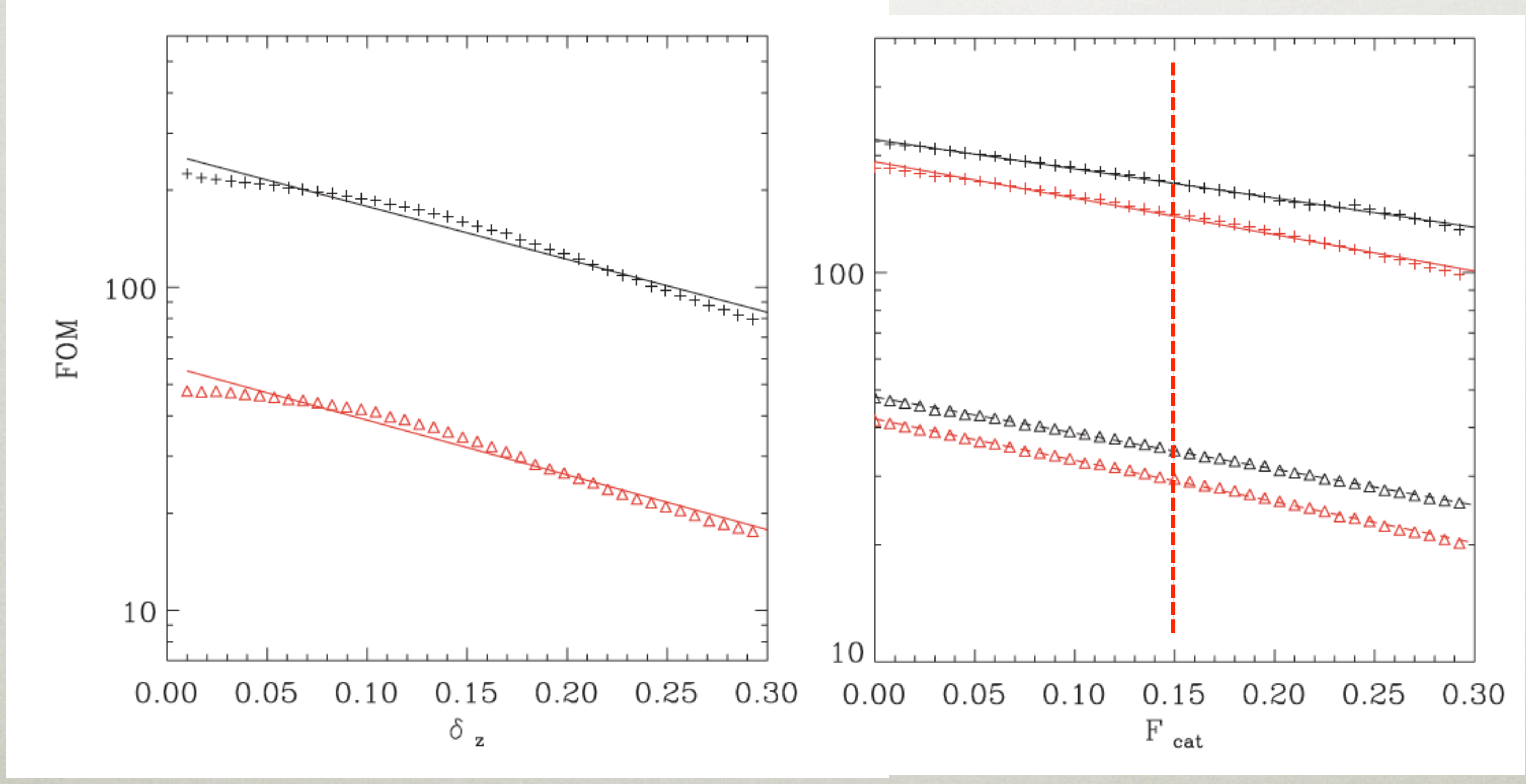
Slicing the universe



Because the kernel is broad the tomographic bins are very correlated. The gain saturates quickly with numer of bins.

Slicing the universe

Amara & Refregier (2007): impact on FoM by photometric redshift errors and outliers.

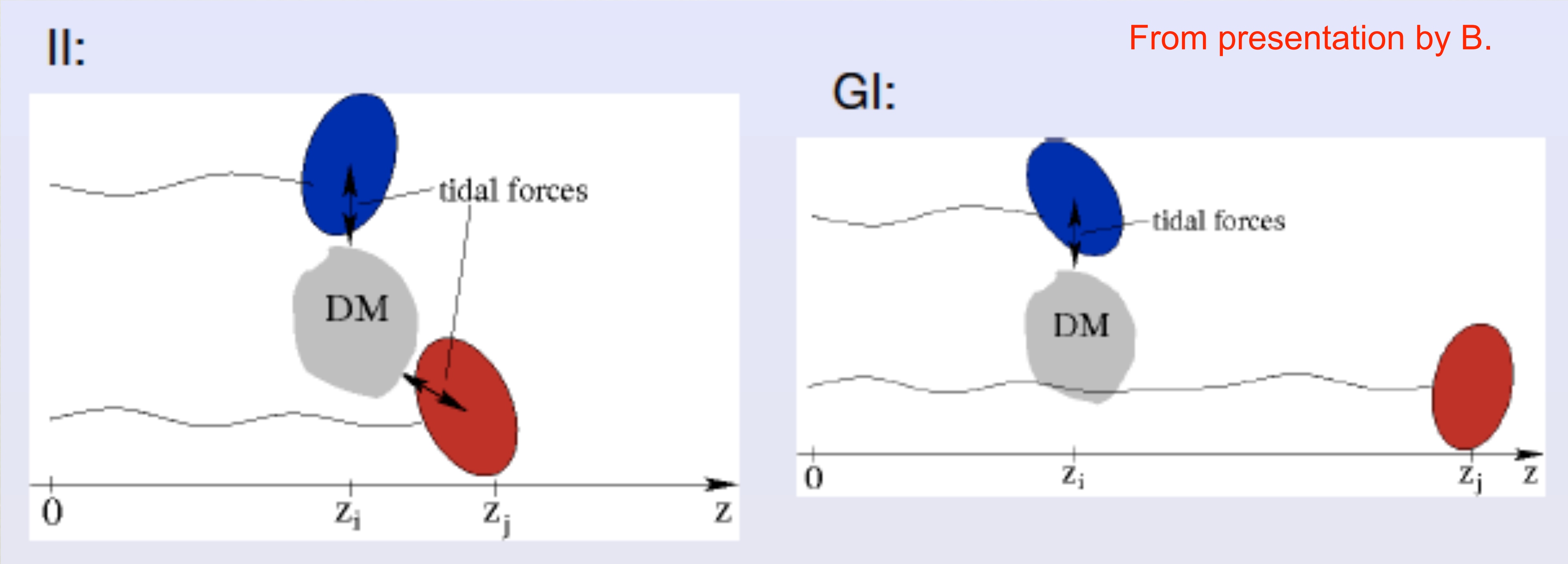


Intrinsic alignments

Gravitational lensing is not the only source of shape alignments.The local gravitational tidal field generates torques and shear forces.

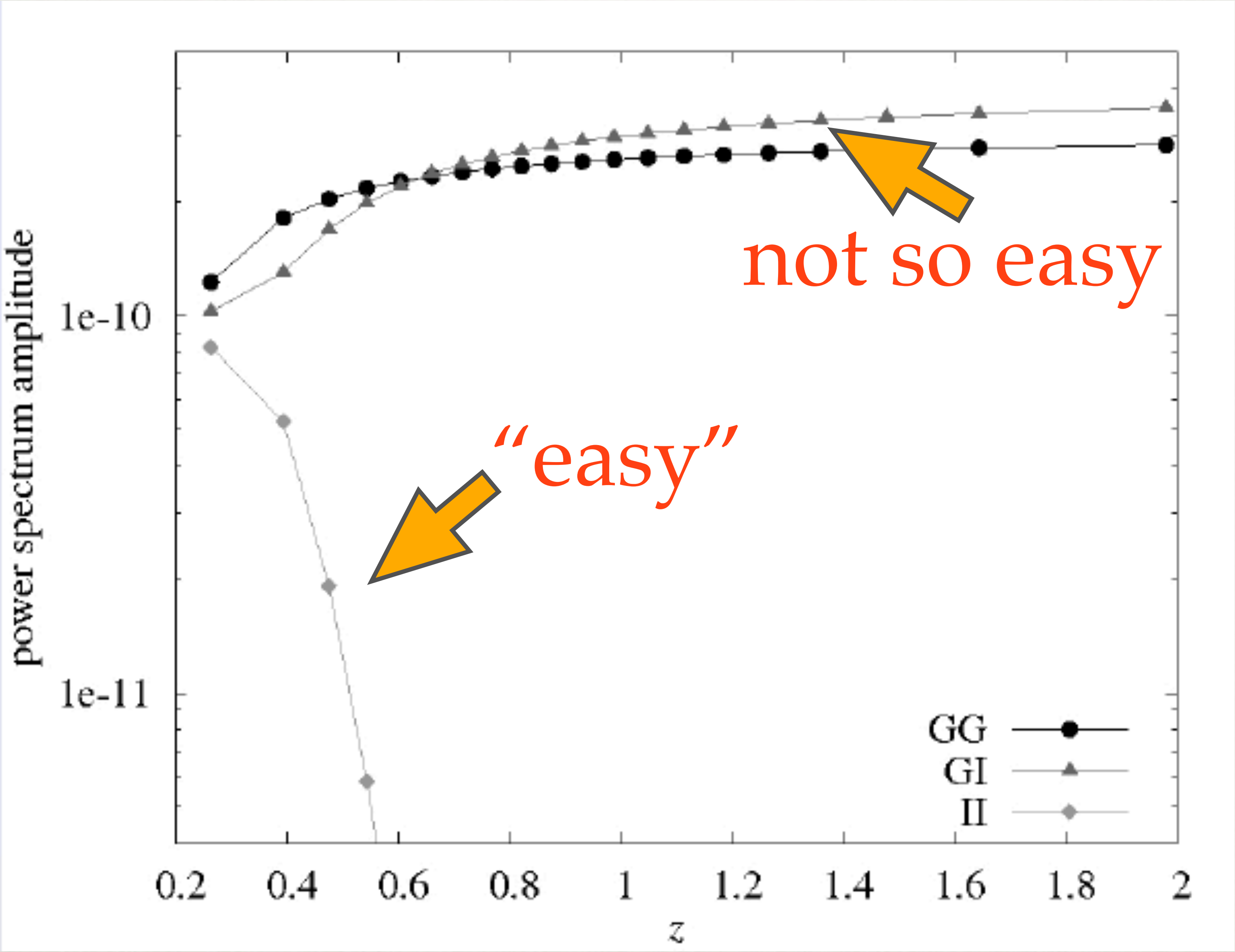
Intrinsic alignments

As a result shapes and angular momenta of galaxies are intrinsically aligned and lead to additional contributions to the ellipticity correlation function:



2pt correlation: $$\langle\epsilon_i\epsilon_j\rangle = \underbrace{\langle\gamma_i\gamma_j\rangle}_{\text{GG}} + \underbrace{\langle\epsilon_i^s\epsilon_j^s\rangle}_{\text{II}} + \underbrace{\langle\gamma_i\epsilon_j^s\rangle + \langle\epsilon_i^s\gamma_j\rangle}_{\text{GI}}$$

Intrinsic alignments



Can we use the different redshift dependence?

Intrinsic alignments

Find models for intrinsic alignment power spectra $P_{\delta\mathrm{I}}$ and P_{II}

Use well-known redshift dependence to separate lensing and IA signals in model-independent way

$$C_{\mathrm{GG}}^{(ij)}(\ell) = \int_0^{\chi_{\mathrm{hor}}} \mathrm{d}\chi \, \frac{q^{(i)}(\chi)\, q^{(j)}(\chi)}{\chi^2} \, P_\delta\left(\frac{\ell}{\chi}, \chi\right)$$

$$C_{\mathrm{GI}}^{(ij)}(\ell) \approx \int_0^{\chi_{\mathrm{hor}}} \mathrm{d}\chi \, \frac{p^{(i)}(\chi)\, q^{(j)}(\chi)}{\chi^2} \, P_{\delta\mathrm{I}}\left(\frac{\ell}{\chi}, \chi\right)$$

$$C_{\mathrm{II}}^{(ij)}(\ell) = \int_0^{\chi_{\mathrm{hor}}} \mathrm{d}\chi \, \frac{p^{(i)}(\chi)\, p^{(j)}(\chi)}{\chi^2} \, P_{\mathrm{II}}\left(\frac{\ell}{\chi}, \chi\right)$$

$q^{(i)}(\chi) \propto \left\langle \frac{D_{ds}}{D_s} \right\rangle_i$ lensing efficiency

$p^{(i)}(\chi)$ probability distribution of χ

Intrinsic alignments

Joachimi & Bridle (2010)

$$\boxed{C^{(ij)}_{\epsilon\epsilon}(\ell)} = C^{(ij)}_{\mathrm{GG}}(\ell) + \boxed{C^{(ij)}_{\mathrm{IG}}(\ell) + C^{(ji)}_{\mathrm{IG}}(\ell)} + C^{(ij)}_{\mathrm{II}}(\ell)$$

$$\boxed{C^{(ij)}_{nn}(\ell)} = \boxed{C^{(ij)}_{\mathrm{gg}}(\ell)} + C^{(ij)}_{\mathrm{gm}}(\ell) + C^{(ji)}_{\mathrm{gm}}(\ell) + C^{(ij)}_{\mathrm{mm}}(\ell)$$

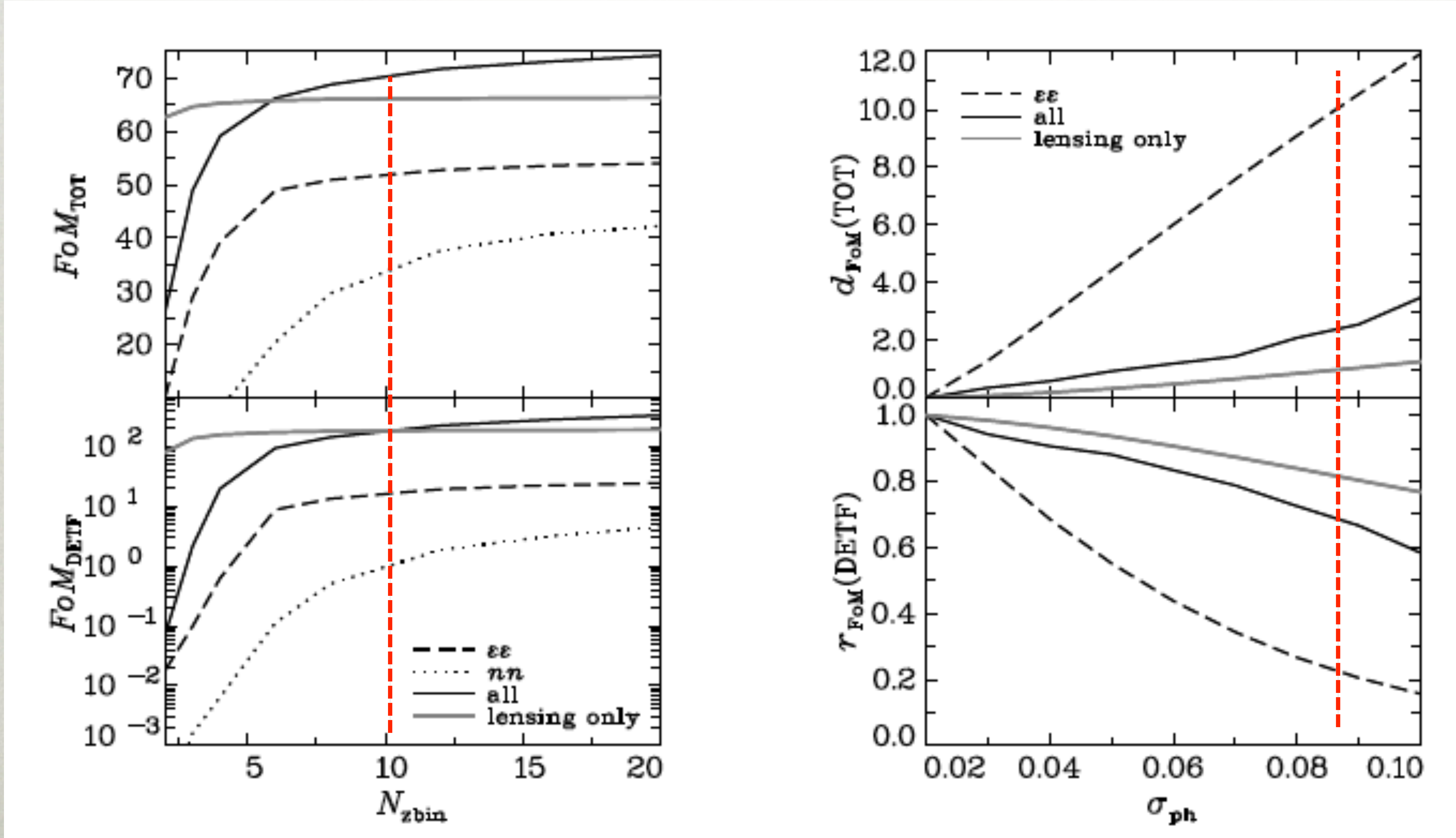
$$\boxed{C^{(ij)}_{n\epsilon}(\ell)} = C^{(ij)}_{\mathrm{gG}}(\ell) + \boxed{C^{(ij)}_{\mathrm{gI}}(\ell)} + C^{(ij)}_{\mathrm{mG}}(\ell) + C^{(ij)}_{\mathrm{mI}}(\ell)\,,$$

Observable

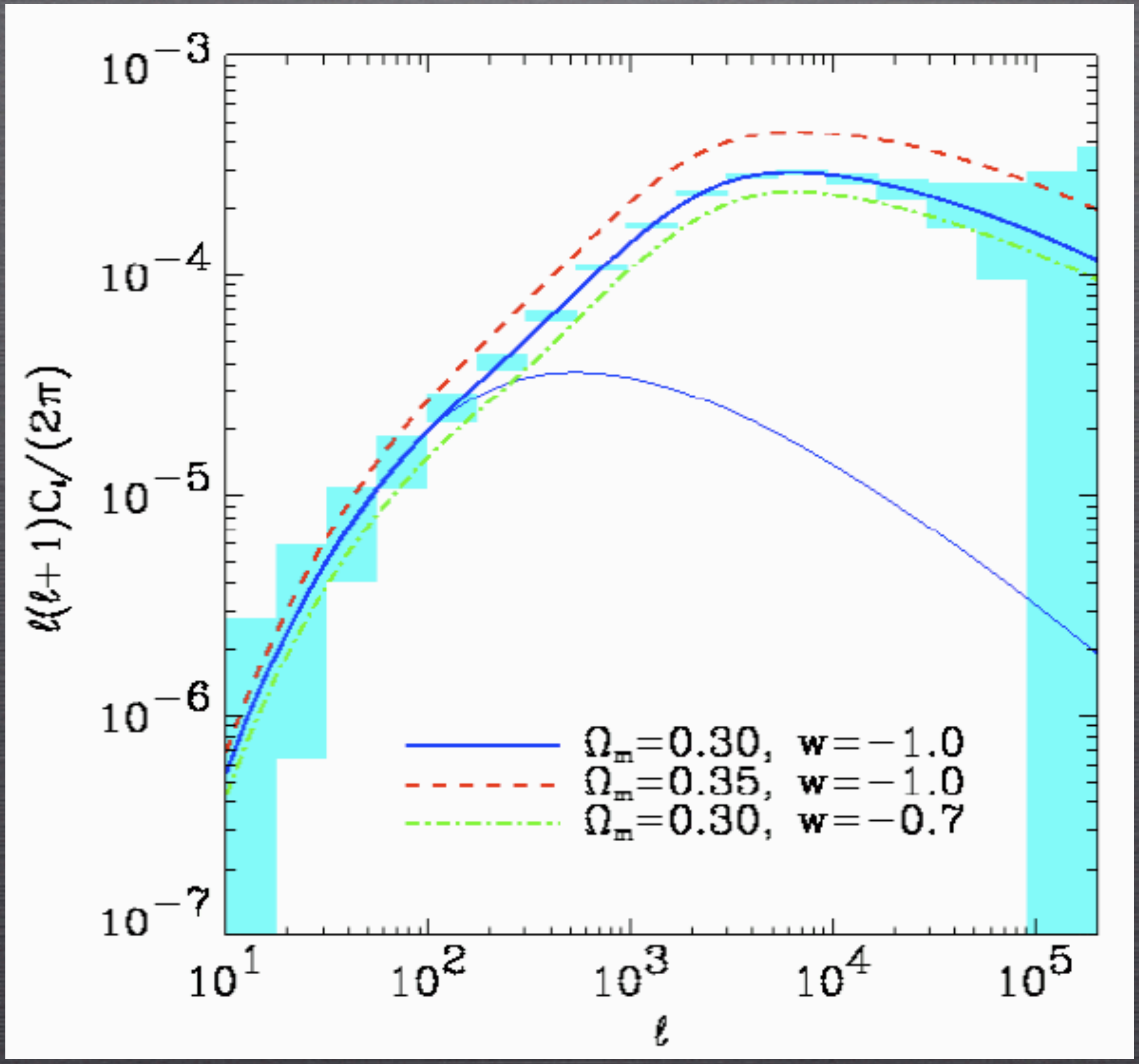
G:	gravitational shear
I:	intrinsic shear
g:	intrinsic number densities
m:	magnification effects

Intrinsic alignments

Joachimi & Bridle (2010)



Predicted power spectrum



MATTER POWER SPECTRUM

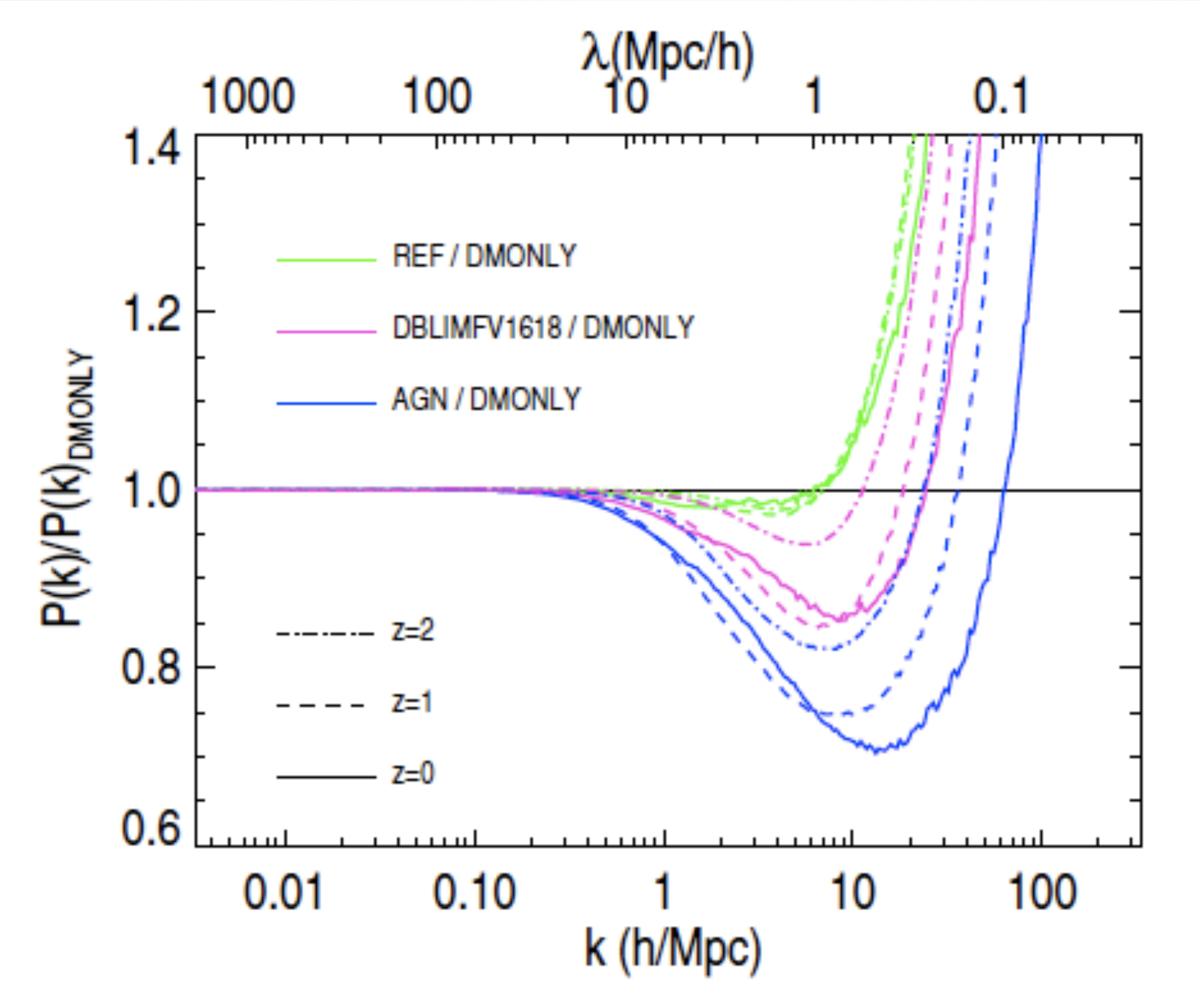
The largest contribution to the weak lensing power spectrum comes from scales that correspond to groups of galaxies, i.e. non-linear structures. To relate the observations to cosmological parameters we need very accurate predictions from numerical simulations

The lensing signal is sensitive to the total matter power spectrum, not just that of dark matter. If baryons trace the dark matter perfectly then "simple" n-body simulations might be sufficient, but recent work suggests that feedback processes can redistribute a large fraction of the baryons.

→ Hydro-simulations to infer "real" *C(l)*

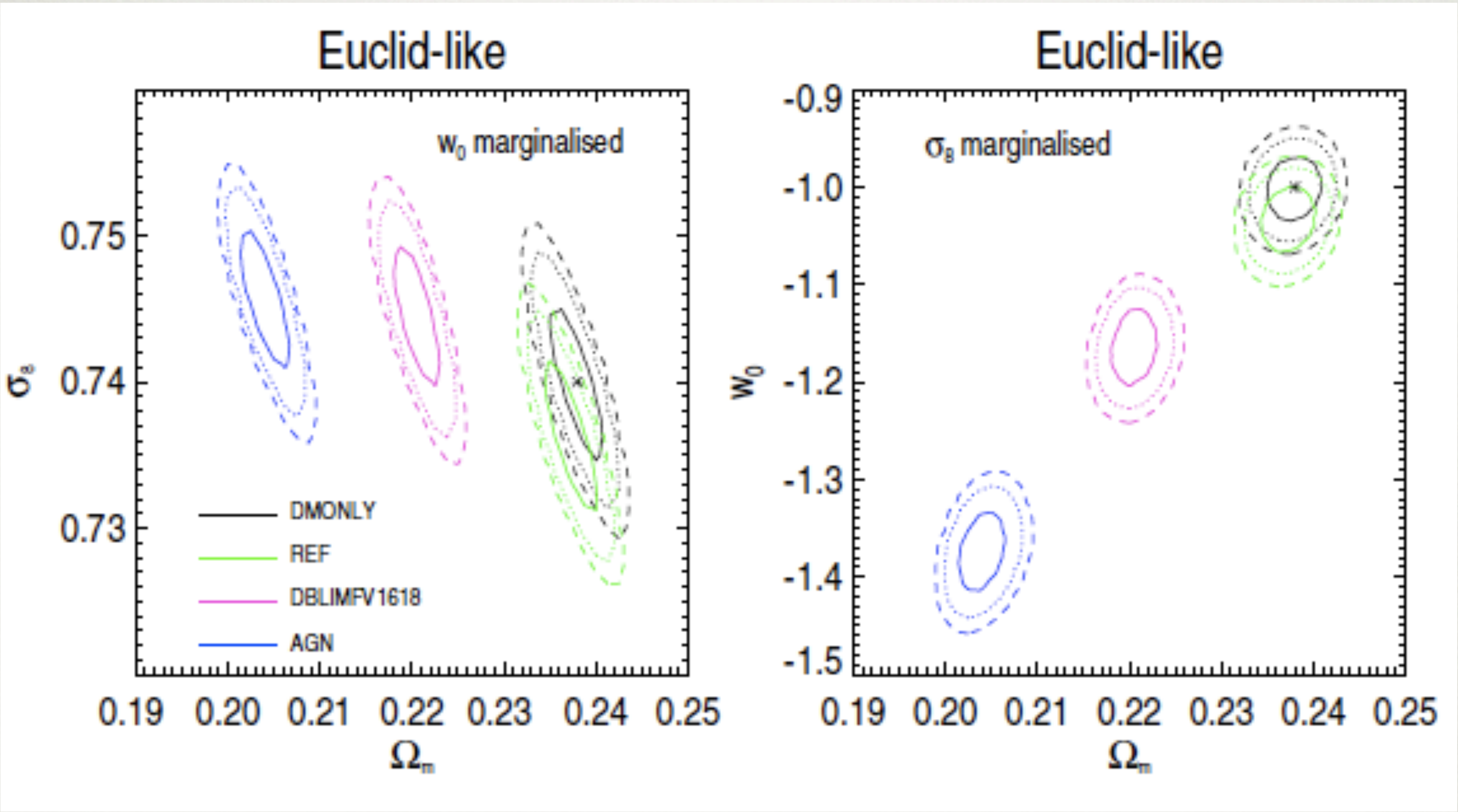
→ Recipe to convert n-body into "real" *C(l)* e.g. Semboloni et al. (2011)

MATTER POWER SPECTRUM



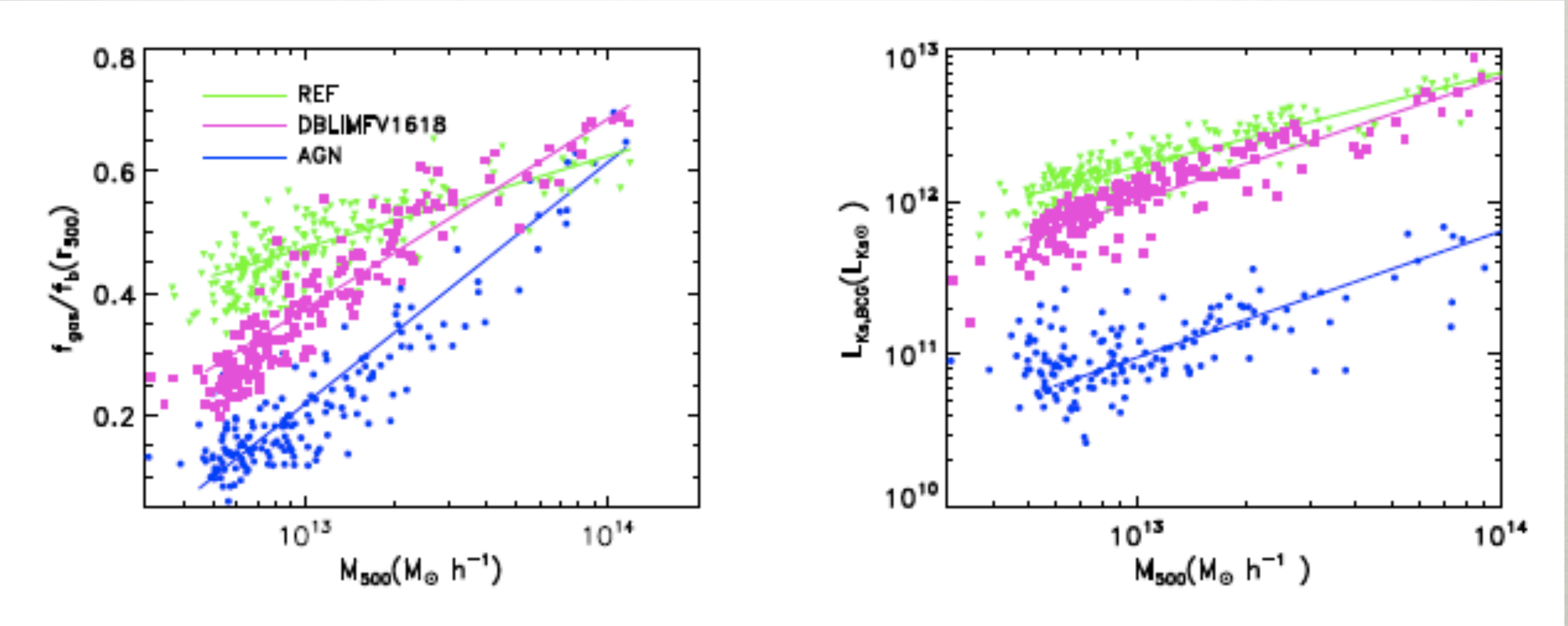
van Daalen et al. (2011): feedback processes can modify the matter power spectrum significantly on scales that are important for cosmic shear.

CANNOT IGNORE FEEDBACK



Semboloni et al. (2011): ignoring feedback may lead to large biases.
We cannot just use bigger dark matter-only simulations.

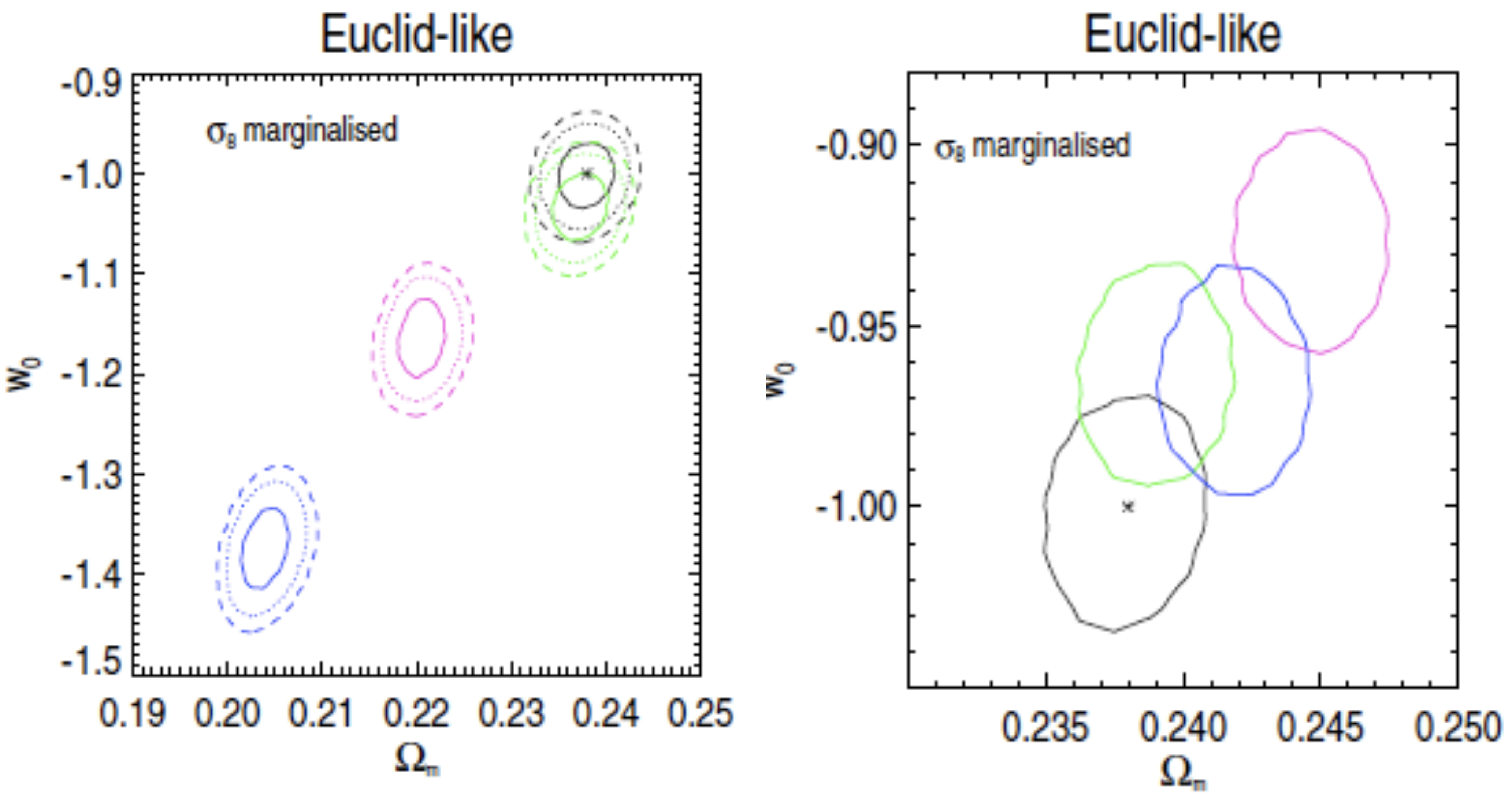
QUANTIFY FEEDBACK



Current simple halo model where:
- galaxies are point masses with a luminosity
- gas follows beta-model with some fraction removed

MODEL THE FEEDBACK

Semboloni et al. (2011): biases can be reduced

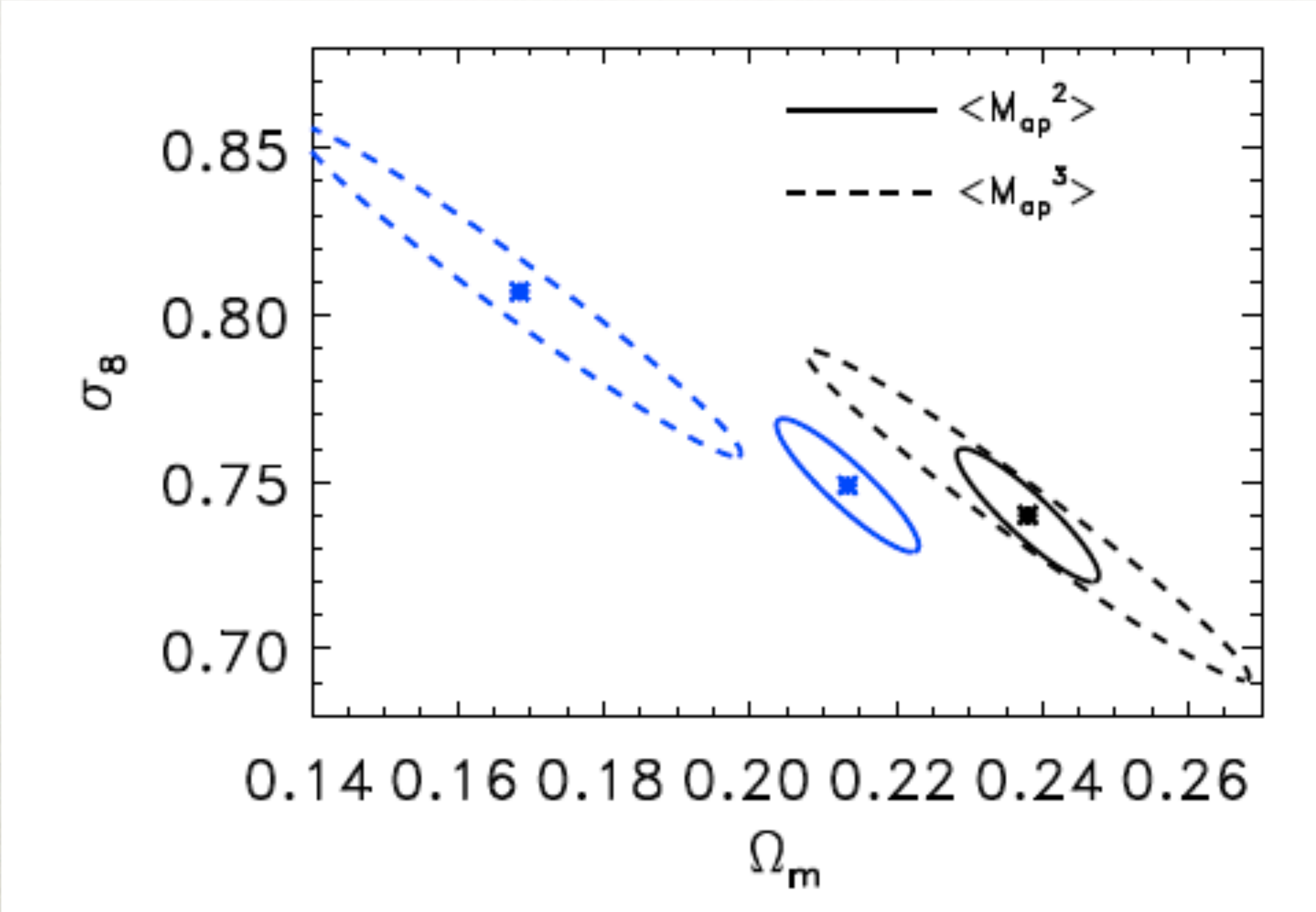


Ignoring feedback

Accounting for feedback

Higher order statistics

Semboloni et al. (in prep)



Comparison of 2- and 3-point statistics can be used to test the fidelity of the feedback model (or perhaps even help to calibrate)

Conclusions

Cosmic shear provides a direct way to study the statistical properties of the matter distribution in the Universe.

A correct cosmological interpretation of the signal requires accurate predictions for the non-linear power spectrum.

Two important astrophysical effects, intrinsic alignments and baryon physics can not be ignored in future projects.