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School and Training Course on Dense Magnetized Plasma as a Source of Ionizing Radiations, their Diagnostics and Applications

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Energetic ions in Dense Magnetized Plasmaand the Fundamental Premise of Thermonuclear Fusion

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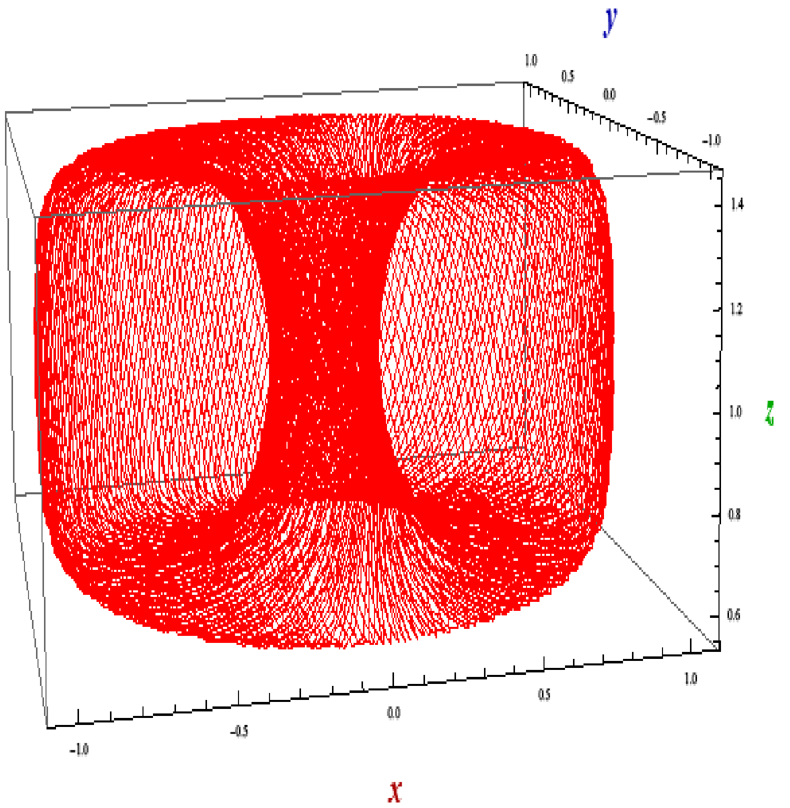
Directionality of energetic ions

- Energetic ions move along +z direction and also in –z direction
- They move in loops in r-z plane
- They have an azimuthal motion
- A toroidal trajectory agrees with this description



•Part-III: Recent developments

- Parts of the puzzle are missing (where to look for them?)

- Progress in theoretical status

What is the right question to ask?

- Beware the “Streetlight Effect“
  - Why Scientific Studies Are So Often Wrong: The Streetlight Effect: by David H. Freedman
    http://discovermagazine.com/2010/jul-aug/29-why-scientific-studies-often-wrong-streetlight-effect
  - Researchers tend to look for answers where *the looking is good*, rather than where the answers are likely to be hiding
  - Late at night, a police officer finds a drunk man crawling around on his hands and knees under a streetlight. The drunk man tells the officer he’s looking for his wallet. When the officer asks if he’s sure this is where he dropped the wallet, the man replies that he thinks he more likely dropped it across the street. Then why are you looking over here? the befuddled officer asks. Because the light’s better here, explains the drunk man.
- This joke contains an important truth: unless you ask the right question, you cannot get a right answer.

Which DMP data is most puzzling?

- Rotational motion of energetic deuterons
  - Why should rotation exist in azimuthally symmetric plasmas?
  - How do the deuterons decide whether to rotate in $+\theta$ or $-\theta$ direction, which should be completely equivalent?
  - Which other plasmas, if any, exhibit rotation?

Explore what is known about this

- Rotation is found in many plasmas

  ....and not understood so far!
  - Tokamaks (Rice et al Nucl. Fusion 47 (2007) 1618–1624)
  - Theta pinches (L.C. Steinhauer Phys. Fluids 24, 328 (1981);
  - Astrophysical plasmas(Sivaram and Arun, The Open Astron. Journal, 2012, 5, p7)
- Are there *device independent* theories which predict rotation?
  - Relaxation theories in Hall MHD plasmas
  - Sudan PRL, **42**, 1277 (1979); Turner, IEEETPS 14, 849 (1986).

Overview of Hall-MHD relaxation

- If dissipative phenomena are neglected, the two-fluid MHD equations conserve total energy, magnetic helicity and hybrid helicity.
- In the presence of dissipative phenomena, the total energy varies on a faster time scale than the two helicities. (L.C. Steinhauer and A. Ishida, PRL, 79 3423 (1997).
- Hence, on an intermediate time scale, the plasma should be described by a state of minimum energy with constant helicities.
- Velocity and other fields of such minimum energy state are a combination of curl eigenfunctions

Brief tutorial on Curl Eigenfunctions

- Relaxation theories predict a state composed of one or more eigenfunctions of curl in cylindrical geometry
  - Also known as Chandrasekhar-Kendall functions

Definition $\vec{\nabla}\times\vec{\chi} = sk\vec{\chi} \quad s=\pm 1,\ k\geq 0$

Substitute $\vec{\chi} = sk\vec{\nabla}\times(\psi\hat{z}) + \vec{\nabla}\times\vec{\nabla}\times(\psi\hat{z})$

The result is $\vec{\nabla}\times\vec{\chi} - sk\vec{\chi} = -\vec{\nabla}\times\left(\hat{z}\left[\nabla^2\psi + k^2\psi\right]\right)$

If $\nabla^2\psi + k^2\psi = 0$, $\vec{\chi}$ is an eigenfunction of curl

Curl eigenfunctions in cylindrical geometry

$\kappa \to$ axial mode number;

$\gamma \to$ radial mode number

$m \to$ azimuthal mode number

$$k^2 = \kappa^2 + \gamma^2$$

$$\psi_{m\kappa\gamma}(r,\theta,z) = J_m(\gamma r)\exp(im\theta - i\kappa z)$$

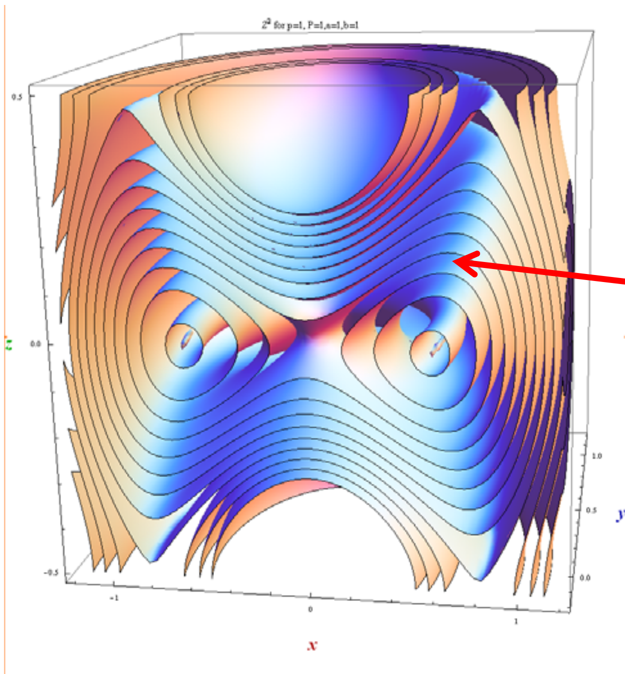
$$\vec{\chi}_{m\kappa\gamma s} = k^{-2}\exp(im\theta - i\kappa z)$$

$$\times\left(\begin{array}{l} \hat{r}\dfrac{i}{2}\gamma\big((sk+\kappa)J_{m+1}(\gamma r)+(sk-\kappa)J_{m-1}(\gamma r)\big) \\ +\hat{\theta}\dfrac{1}{2}\gamma\big((sk+\kappa)J_{m+1}(\gamma r)-(sk-\kappa)J_{m-1}(\gamma r)\big) \\ +\hat{z}\gamma^2 J_m(\gamma r) \end{array}\right)$$

Azimuthally Symmetric (m=0) Curl eigenfunctions

$$\vec{\chi}_{0\kappa\gamma s} = k^{-2}\exp(-i\kappa z)$$

$$\times\left(i\hat{r}\gamma\kappa J_1(\gamma r) + \hat{\theta}\gamma s k J_1(\gamma r) + \hat{z}\gamma^2 J_0(\gamma r)\right)$$

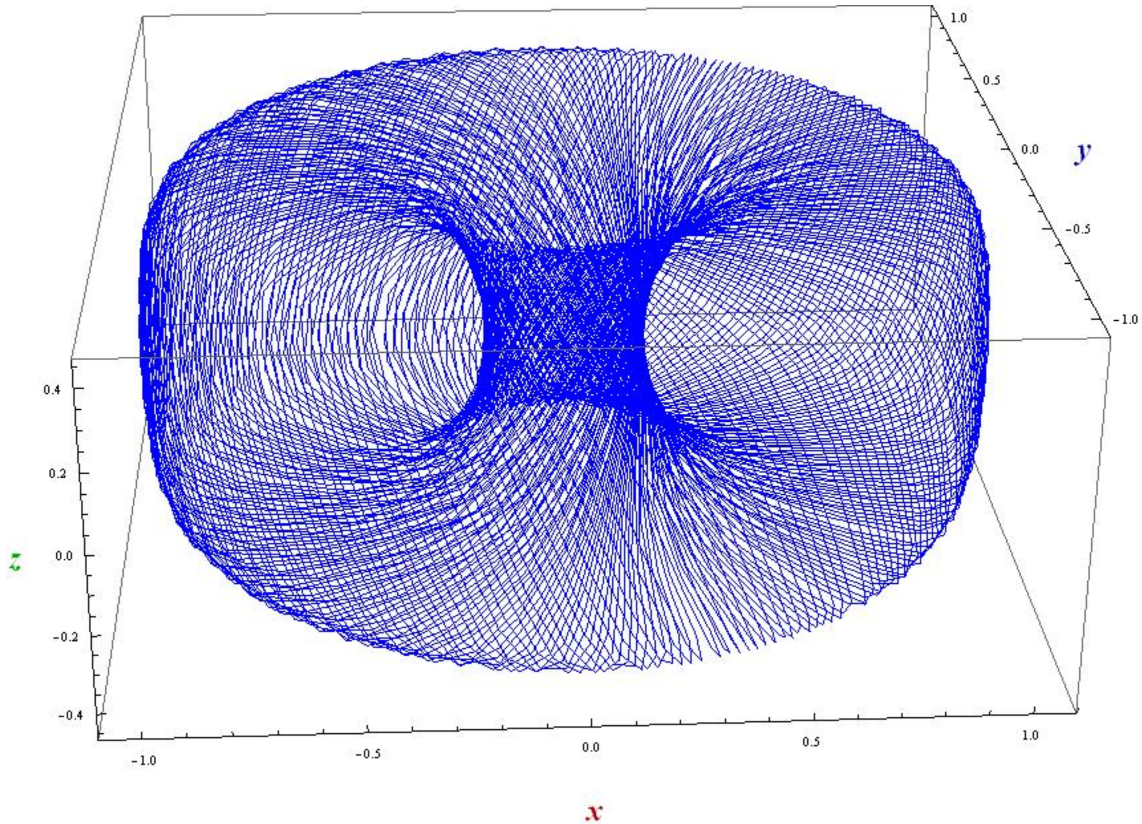


3-D contour plot of $\left|\vec{\chi}\right|^2$

Nested toroidal surfaces

Flow lines of Curl Eigenfunction

3-D parametric plot of the line $$\frac{dr}{\chi_r} = \frac{rd\theta}{\chi_\theta} = \frac{dz}{\chi_z} = \frac{ds}{\left|\vec{\chi}\left(r_0, \theta_0, z_0\right)\right|}$$



Endless line covering a toroidal surface densely

Relaxation theories yield toroidal flow patterns

Application to z-pinches

PHYSICS OF PLASMAS 18, 032508 (2011)

Evaluation of Turner relaxed state as a model of long-lived ion-trapping structures in plasma focus and Z-pinches

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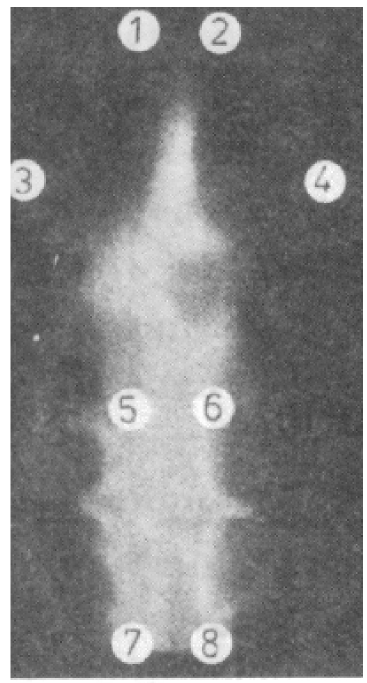
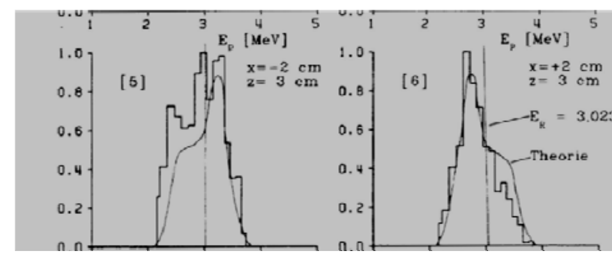
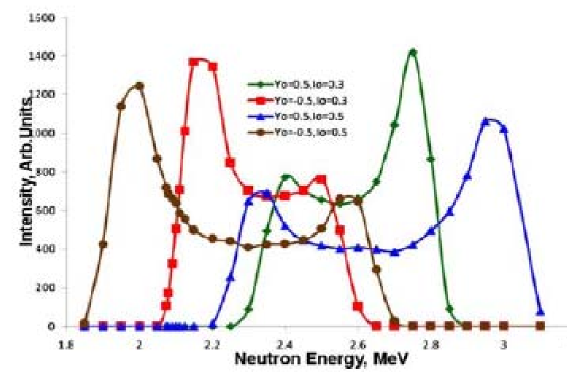
- Turner's relaxation theory adapted to Z-pinch problem
- Asymmetry of neutron spectra from laterally mirror symmetric points demonstrated







First clue

- The directionality of energetic ions in DPF can be linked with toroidal ion motion described by azimuthally symmetric (m=0) CK function

Is plasma relaxation the answer?

- Relaxation theories rely on having a conducting boundary close to the plasma, on which radial components of *all* the curl-eigenfunctions vanish
  - Not applicable in the case of DPF
- Directionality of energetic ions agrees with the topology of an m=0 CK function for ion motion.
  - Minimum energy is not the only criterion which yields this state: other extremum principles do the same.

Important Question

- How does a purely irrotational flow like DPF end up with a solenoidal component?
  - Equations show that growth of solenoidal fields requires initially non-zero solenoidal force fields
  - If $v_\theta, J_\theta, E_\theta, B_r$ and B_z are all zero initially, they remain zero forever

Just look at the equations

Fields marked in red are initially zero. They remain zero!

$$\nu \to e\,(\text{electrons}) \text{ or } i\,(\text{ions})$$

$$m_\nu n\left(\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + v_{z,\nu}\frac{\partial}{\partial z}v_r - \frac{v_{\theta,\nu}^2}{r}\right) = q_\nu n\left(E_r + v_{\theta,\nu}B_z - v_{z,\nu}B_\theta\right) - \frac{\partial p_\nu}{\partial r}$$

$$m_\nu n\left(\frac{\partial v_{\theta,\nu}}{\partial t} + v_r\frac{\partial v_{\theta,\nu}}{\partial r} + v_{z,\nu}\frac{\partial}{\partial z}v_{\theta,\nu} + \frac{v_r v_{\theta,\nu}}{r}\right) = q_\nu n\left(E_\theta + v_{z,\nu}B_r - v_r B_z\right)$$

$$m_\nu n\left(\frac{\partial v_{z,\nu}}{\partial t} + v_r\frac{\partial v_{z,\nu}}{\partial r} + v_{z,\nu}\frac{\partial}{\partial z}v_{z,\nu}\right) = q_\nu n\left(E_z + v_r B_\theta - v_{\theta,\nu}B_r\right) - \frac{\partial p_\nu}{\partial z}$$

$$\frac{\partial B_r}{\partial r} = -\frac{\partial B_z}{\partial z}$$

$$-\frac{\partial B_\theta}{\partial z} = \mu_0 J_r;\ \frac{\partial B_r}{\partial z} - \frac{\partial B_z}{\partial r} = \mu_0 J_\theta + \frac{1}{c^2}\frac{\partial E_\theta}{\partial t};\ \frac{1}{r}\frac{\partial}{\partial r}\left(rB_\theta\right) = \mu_0 J_z + \frac{1}{c^2}\frac{\partial E_z}{\partial t}$$

$$-\frac{\partial E_\theta}{\partial z} = -\frac{\partial B_r}{\partial t};\ \frac{\partial E_r}{\partial z} - \frac{\partial E_z}{\partial r} = -\frac{\partial B_\theta}{\partial t};\ \frac{1}{r}\frac{\partial}{\partial r}\left(rE_\theta\right) = -\frac{\partial B_z}{\partial t}$$

Observation: How to have a nonzero source term for rotation

- Assume that all fields have random fluctuations, $\vec{v} = \underline{\vec{v}} + \underset{\sim}{\vec{v}}$ which are correlated through dynamical equations

$$\frac{\partial \underline{v}_{\theta,\nu}}{\partial t} = -\underline{v}_r \frac{\partial \underline{v}_{\theta,\nu}}{\partial r} - \underline{v}_{z,\nu} \frac{\partial}{\partial z} \underline{v}_{\theta,\nu} - \frac{\underline{v}_r \underline{v}_{\theta,\nu}}{r}$$

$$+ \frac{q_\nu n}{m_\nu}\left(\underline{E}_\theta + \underline{v}_{z,\nu}\underline{B}_r - \underline{v}_r \underline{B}_z\right) + \frac{q_\nu n}{m_\nu}\left(\left\langle \underset{\sim}{v}_{z,\nu} \underset{\sim}{B}_r \right\rangle - \left\langle \underset{\sim}{v}_r \underset{\sim}{B}_z \right\rangle\right)$$

- Now the rotation has a source term proportional to average of product of two randomly fluctuating fields

Second clue

- Random fluctuations in plasma parameters may provide a small “seed” solenoidal flow.
- Interaction between irrotational and solenoidal plasma flows may lead to energy transfer leading to amplification of this “seed” solenoidal flow.

Task: Construction of a theory for interaction between solenoidal and irrotational flows incorporating random flucutaions

- Work in progress:
  - Auluck S K H, 2012 Coherent effects in the stochastic electrodynamics of two-fluid plasma, arXiv:1208.1573v1

Overview of the approach

- Express two-fluid equations in cylindrical Fourier space using CK functions as basis for solenoidal fields, Helmholtz potential for scalar fields and its gradient for irrotational fields.
- Construct a stochastic model of random electromagnetic fields (could be due to thermal radiation or any other origin)
- Separate time evolution equations for random and non-random parts of spectral amplitudes
- Show that solenoidal flow spectrum can spontaneously arise even when it is zero initially.

Orthogonal basis for solenoidal, scalar and irrotational fields

$$\int d^3\vec{R}\ \vec{\chi}^*_{m\kappa\gamma s}\cdot\vec{\chi}_{m'\kappa'\gamma's'} = 8\pi^2 k^{-2}\gamma\delta_{mm'}\delta_{ss'}\delta(\kappa'-\kappa)\delta(\gamma'-\gamma)$$

$$\int d^3\vec{R}\psi^*_{m\kappa\gamma}\psi_{m'\kappa'\gamma'} = 4\pi^2\gamma^{-1}\delta_{mm'}\delta(\kappa'-\kappa)\delta(\gamma'-\gamma)$$

$$\int d^3\vec{R}\vec{\nabla}\psi^*_{m\kappa\gamma}\cdot\vec{\nabla}\psi_{m'\kappa'\gamma'} = 4\pi^2 k^2\gamma^{-1}\delta_{m,m'}\delta(\kappa-\kappa')\delta(\gamma-\gamma')$$

$$\int d^3\vec{R}\vec{\nabla}\psi^*_{m\kappa\gamma}\cdot\vec{\chi}_{m'\kappa'\gamma's'} = 0$$

$$\int d^3\vec{R}\vec{\nabla}\psi_{m\kappa\gamma}\cdot\vec{\chi}_{m'\kappa'\gamma's'} = 0$$

Transformation between coordinate and mode number spaces

- For scalar fields S(r,θ,z,t)

$$S(r,\theta,z,t) = \sum_{m'=-\infty}^{\infty} \int_{-\infty}^{\infty} d\kappa' \int_{0}^{\infty} d\gamma' S_{m'}(\kappa',\gamma',t)\, \psi_{m'\kappa'\gamma'}(r,\theta,z)$$

$$S_{m}(\kappa,\gamma,t) = \frac{\gamma}{4\pi^2} \int_{0}^{2\pi} d\theta \int_{-\infty}^{\infty} dz \int_{0}^{\infty} r dr \psi^{*}_{m\kappa\gamma}(r,\theta,z) S(r,\theta,z,t)$$

Physically significant information is contained in the spectral coefficient $S_{m'}(\kappa',\gamma',t)$

Transformation between coordinate and mode number spaces

- For irrotational fields $\vec{F}_{IR}(r,\theta,z,t)$

$$\vec{F}_{IR}(r,\theta,z,t) = \sum_{m'=-\infty}^{\infty} \int_{-\infty}^{\infty} d\kappa' \int_{0}^{\infty} d\gamma' \mathcal{F}_{m'}^{IR}(\kappa',\gamma',t)\, k'^{-1} \vec{\nabla} \psi_{m'\kappa'\gamma'}(r,\theta,z)$$

$$\mathcal{F}_{m}^{IR}(\kappa,\gamma,t) = \frac{\gamma}{4\pi^2 k} \int_{0}^{2\pi} d\theta \int_{-\infty}^{\infty} dz \int_{0}^{\infty} r dr \vec{\nabla} \psi_{m\kappa\gamma}^{*}(r,\theta,z) \cdot \vec{F}_{IR}(r,\theta,z,t)$$

Transformation between coordinate and mode number spaces

- For solenoidal fields $\vec{F}_{SOL}\left(r,\theta,z,t\right)$

$$\vec{F}_{SOL}\left(r,\theta,z,t\right)=\sum_{s'}\sum_{m'=-\infty}^{\infty}\int_{-\infty}^{\infty}d\kappa'\int_{0}^{\infty}d\gamma'\mathcal{F}_{m's'}^{SOL}\left(\kappa',\gamma',t\right)\vec{\chi}_{m'\kappa'\gamma's'}\left(r,\theta,z\right)$$

$$\mathcal{F}_{ms}^{SOL}\left(\kappa,\gamma,t\right)=\frac{k^2}{8\pi^2\gamma}\int_{0}^{2\pi}d\theta\int_{-\infty}^{\infty}dz\int_{0}^{\infty}rdr\vec{\chi}_{m\kappa\gamma s}^{*}\left(r,\theta,z\right)\cdot\vec{F}_{SOL}\left(r,\theta,z,t\right)$$

Transformation of two-fluid equations into mode number space

- Perform all vector and differential operations in coordinate space on the basis functions
- Time evolution equations are obtained in mode number space (cylindrical Fourier space)
- A small (big??) problem: this procedure leads to expressions involving integrals of the form

$$\mathfrak{D}^{\gamma\ \gamma'\ \gamma''}_{m m' m''} \equiv \int_0^\infty r dr J_m(\gamma r) J_{m'}(\gamma' r) J_{m''}(\gamma'' r)$$

for which, unfortunately, there is no theory!!

Semi-empirical theory for triple-Bessel integrals

- Using approximate expression for Bessel function for large arguments and a large but finite upper limit of integration, an approximate relation has been obtained and numerically verified.
- An approximate "model" for this integral has been proposed in terms of Dirac delta functions
  - S K H Auluck (accepted for publication in The Mathematica Journal)

Stochastic model of random electromagnetic field

$$\vec{E}_R\left(r,\theta,z,t,\vec{L}\right)=\sum_{s'}\sum_{m'=-\infty}^{\infty}\int_{-\infty}^{\infty}d\kappa'\int_0^{\infty}d\gamma' e_{m's'}\left(\vec{L},\kappa',\gamma'\right)\exp\left(ick't\right)\vec{\chi}_{m'\kappa'\gamma's'}\left(r,\theta,z\right)$$

$$\vec{B}_R\left(r,\theta,z,t,\vec{L}\right)=\sum_{s'}\sum_{m'=-\infty}^{\infty}\int_{-\infty}^{\infty}d\kappa'\int_0^{\infty}d\gamma' c^{-1}is' e_{m's'}\left(\vec{L},\kappa',\gamma'\right)\exp\left(ick't\right)\vec{\chi}_{m'\kappa'\gamma's'}\left(r,\theta,z\right)$$

Random amplitude $\quad e_{ms}\left(\vec{L},\kappa,\gamma\right)\equiv\mathbb{F}\left(\kappa,\gamma\right)\mathbb{R}_{ms}\left(\vec{L},\kappa,\gamma\right)$

$\mathbb{R}_{ms}\left(\vec{L},\kappa,\gamma\right)$ is a dimensionless function of 3 random variables

$\vec{L}=\left(L_x,L_y,L_z\right)$ in the interval $\{-\infty,\infty\}$ having dimensions of length with the property

$$\left\langle\mathbb{R}^*_{ms}\left(\vec{L},\kappa,\gamma\right)\mathbb{R}_{m's'}\left(\vec{L},\kappa',\gamma'\right)\right\rangle=\delta_{mm'}\delta_{ss'}k^2\delta\left(\kappa-\kappa'\right)\delta\left(\gamma-\gamma'\right)$$

Stochastic model of random electromagnetic field

- The random field amplitude $\mathbb{F}(\mathrm{k})$ can be related to the energy density of random electromagnetic radiation
- As an example, for Planck radiation

$$\mathbb{F}(\mathrm{k}) = \mathcal{E}_0 \sqrt{\left\{1 + \frac{2}{\exp\left(\hbar c k / k_B T_r\right) - 1}\right\}}$$

$$\mathcal{E}_0 \equiv \sqrt{\frac{\hbar c}{4\pi^3 \varepsilon_0}} = 5.3655 \times 10^{-9} \text{ Volt} - \text{m}$$

The main idea

- Randomness in all fields is correlated through dynamical equations
- Hence, spectral coefficient of every field should be a sum of a random part and a non-random part, all random parts being proportional to the same function of random variables $\mathbb{R}_{\mathrm{ms}}\left(\vec{\mathrm{L}}, \kappa, \gamma\right)$
- This enables separation of equations for random and non-random parts of spectral coefficients

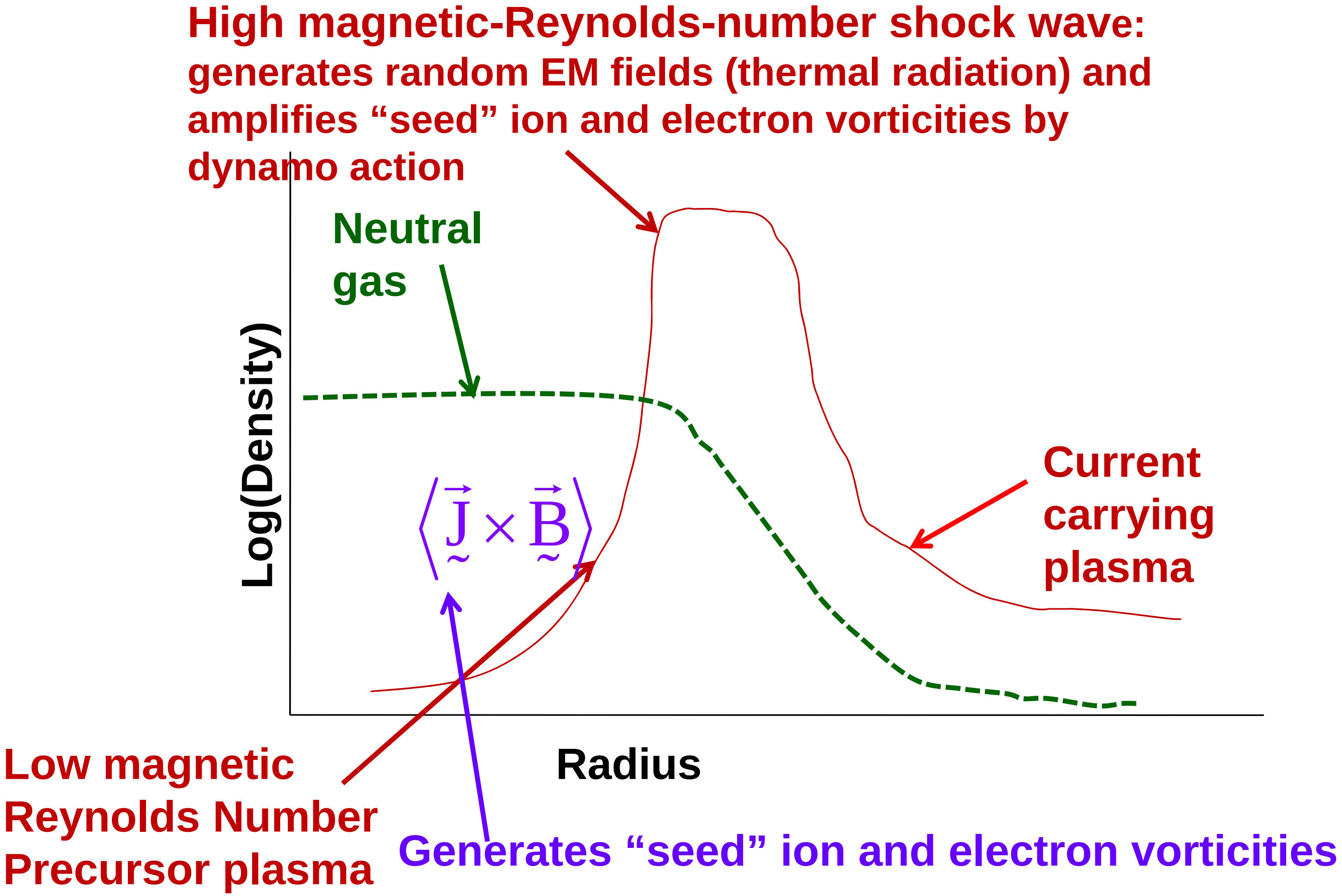
Outcome

- New source terms proportional to square of random EM field appear for non-random fields.
- They have azimuthal symmetry.
- The random fields are governed by a system of linear differential equations, in which m=0 non-random fields appear as adiabatic coefficients.
- Spontaneous generation of non-random azimuthally symmetric fields becomes possible.

Glimpse of solution to DPF puzzle

- Neutral gas ahead of plasma sheath is a partially ionized plasma.
- Random EM fields produced by the radiation due to plasma sheath drive fluctuating currents leading to 3 nonzero components of $\left\langle \underset{\sim}{\vec{\mathrm{J}}} \times \underset{\sim}{\vec{\mathrm{B}}} \right\rangle$ and hence 3 components of velocity.
- Subsequent compressive flow amplifies the 3 components of velocity generating both solenoidal and irrotational flows.

DPF sheath structure



Conclusion

- This discussion shows that the two-orders-of – magnitude-higher-than-thermonuclear fusion reaction rate in DPF, related to toroidal ion trajectories, *may be* an expected, predictable and controllable feature of plasma dynamics.
- Elaborating this requires a major departure from the traditional approach to plasma dynamics
  - Remember the "streetlight effect"!!

Fundamental premise of thermonuclear fusion may not be valid

- Lawson plasmas obeying Maxwellian distribution may not be *the only* viable approach to fusion energy.
- DPF or a related device may have a future as energy producing device

Thank you