Stitching the Yukawa quilt in the light of θ_{13}

G. Ross, BENE 2012

$$\sin^2\theta_{13} = 0.022 \pm 0.004 \ (0.023 \pm 0.004)$$

8.5^0

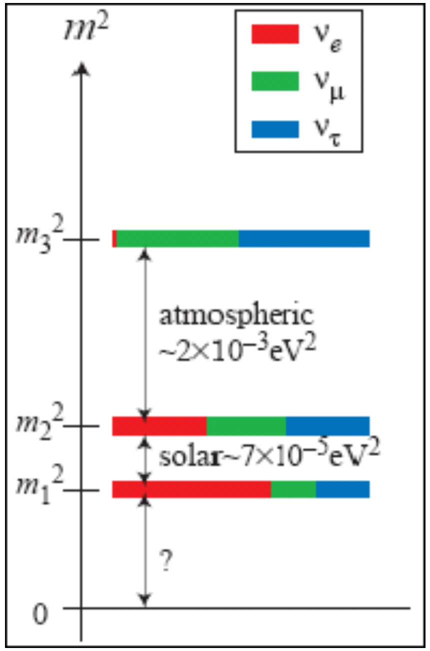
Chooz, Daya Bay, RENO, T2K, MINOS

Neutrino mixing

…near tri-bi-maximal

$$\sin^2\theta_{12} = 0.320^{+0.015}_{-0.017}$$
$$\sin^2\theta_{23} = 0.49^{+0.08}_{-0.05} \qquad \pm 1\sigma$$
$$\sin^2\theta_{13} = 0.026^{+0.003}_{-0.004}$$

Forero, Tortola, Valle
Fogli, Lisi, Marrone, Montanino, Palazzo, Rotuno



Cabibbo haze:

$$\theta_{13}^{\nu} = 0, \quad \theta_{13} \approx \theta_{12}^{l} \sin\theta_{23}^{\nu} \approx \frac{\theta_{12}^{l}}{\sqrt{2}}$$

$\left\{ \right.$ $M^{q,l}$: small mixing …dominated by θ_C

M^{ν} : tri-bi-maximal

$$L_m^{\nu} = m_{@}\left[\tfrac{1}{\sqrt{2}}(\nu_\mu + \nu_\tau)\right]^2 + m_{\odot}\left[\tfrac{1}{\sqrt{3}}(\nu_e + \nu_\mu - \nu_\tau)\right]^2$$

Datta, Everett, Ramond
Marzocca, Petkov, Romanino, Spinrath
Antusch et al

If $\theta_{12}^{l} = \theta_C$ (GUT?), $\theta_{13} = 9^0$!

…but $\theta_{12}^{l} = \theta_C$ inconsistent with other plausible mass & GUT relations

Mass relations:

$$\theta_C = \left| \sqrt{\frac{m_d}{m_s}} - e^{i\delta} \sqrt{\frac{m_u}{m_c}} \right|$$

$$m_\tau(M_{GUT}) = m_b(M_{GUT})$$

$$m_\mu(M_{GUT}) = 3 m_s(M_{GUT})$$

$$m_e(M_{GUT}) = \frac{1}{3} m_s(M_{GUT})$$

$$(TZ \Rightarrow Det(M_d) = Det(M_l))$$

$$M^d = m_b \begin{pmatrix} <\varepsilon^4 & \varepsilon^3 & . \\ \varepsilon^3 & \varepsilon^2 & \varepsilon^2 \\ . & . & 1 \end{pmatrix}$$

Gatto et al, Weinberg, Fritzsch

$$M^l = m_b \begin{pmatrix} <\varepsilon^4 & 1\varepsilon^3 & . \\ 1\varepsilon^3 & 3\varepsilon^2 & \varepsilon^2 \\ . & . & 1 \end{pmatrix}$$

Georgi-Jarlskog

Parameters	Input SUSY Parameters					
$\tan\beta$	1.3	10	38	50	38	38
γ_b	0	0	0	0	-0.22	$+0.22$
γ_d	0	0	0	0	-0.21	$+0.21$
γ_t	0	0	0	0	0	-0.44
Parameters	**Comparison with GUT Mass Ratios**					
$(m_b/m_\tau)(M_X)$	$1.00^{+0.04}_{-0.4}$	0.73(3)	0.73(3)	0.73(4)	1.00(4)	1.00(4)
$(3m_s/m_\mu)(M_X)$	$0.70^{+0.8}_{-0.05}$	0.69(8)	0.69(8)	0.69(8)	0.9(1)	0.6(1)
$(m_d/3\,m_e)(M_X)$	0.82(7)	0.83(7)	0.83(7)	0.83(7)	1.05(8)	0.68(6)
$(\frac{\det Y^d}{\det Y^e})(M_X)$	$0.57^{+0.08}_{-0.26}$	0.42(7)	0.42(7)	0.42(7)	0.92(14)	0.39(7)

GGR, Serna

Mass relations: $(\sim 10\%)$

$$\theta_C = \left| \sqrt{\frac{m_d}{m_s}} - e^{i\delta} \sqrt{\frac{m_u}{m_c}} \right|$$

$$M^d = m_b \begin{pmatrix} < \varepsilon^4 & \varepsilon^3 & \varepsilon^4 \\ \varepsilon^3 & \varepsilon^2 & \varepsilon^2 \\ . & . & 1 \end{pmatrix}$$

$$m_\tau(M_{GUT}) = m_b(M_{GUT})$$

$$m_\mu(M_{GUT}) = 3 m_s(M_{GUT})$$

$$m_e(M_{GUT}) = \frac{1}{3} m_s(M_{GUT})$$

$$\left| \frac{V_{td}}{V_{ts}} \right| = \sqrt{\frac{m_d}{m_s}}$$

Hall, Rasin

…but $\theta_{12}^l = \theta_C$ inconsistent with other plausible GUT relations

$$\theta_{12}^l = \theta_C$$

$$\theta_{13} = \frac{\theta_{12}^l}{\sqrt{2}} = 9^0$$

$$\Rightarrow \quad M^l = m_b \begin{pmatrix} \varepsilon^4 & \varepsilon^3 & . \\ \varepsilon^3 & 3\varepsilon^2 & \varepsilon^2 \\ . & . & 1 \end{pmatrix}$$

c.f.

$$\theta_C = \left| \sqrt{\frac{m_d}{m_s}} - e^{i\delta} \sqrt{\frac{m_u}{m_c}} \right|$$

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$$m_\mu(M_{GUT}) = 3m_s(M_{GUT})$$

$$m_e(M_{GUT}) = \frac{1}{3} m_s(M_{GUT})$$

$$M^l = m_b \begin{pmatrix} < \varepsilon^4 & 1\varepsilon^3 & . \\ 1\varepsilon^3 & 3\varepsilon^2 & \varepsilon^2 \\ . & . & 1 \end{pmatrix}$$

$$\theta_{12}^l = \sqrt{\frac{m_e}{m_\mu}} = \frac{\theta_C}{3}$$

$$\theta_{13} = \frac{\theta_{12}^l}{\sqrt{2}} = 3^0$$

…but $\theta_{12}^{l} = \theta_C$ inconsistent with other plausible GUT relations

$$\theta_{12}^{l} = \theta_C$$

$$\theta_{13} = \frac{\theta_{12}^{l}}{\sqrt{2}} = 9^0$$

$$\Rightarrow \quad M^l = m_b \begin{pmatrix} \varepsilon^4 & \varepsilon^3 & . \\ \varepsilon^3 & 3\varepsilon^2 & \varepsilon^2 \\ . & . & 1 \end{pmatrix}$$

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$$\theta_{12}^{l} = \sqrt{\frac{m_e}{m_\mu}} = \frac{\theta_C}{3}$$

$$\theta_{13} = \frac{\theta_{12}^{l}}{\sqrt{2}} = 3^0$$

To keep these relations need departure from tri-bi-maximal mixing in ν sector…

Combination of charged lepton and neutrino contributions in symmetry basis

$U_{PMNS} = U_{23}U_{13}U_{12}$

In small s_{13}^{ν} and charged lepton angles approximation

$$
\begin{aligned}
s_{13}e^{-i\delta_{13}} &= s_{13}^{\nu}e^{-i\delta_{13}^{\nu}} - s_{12}^{l}s_{23}^{\nu}e^{-i(\delta_{23}^{\nu}+\delta_{12}^{e})} - s_{13}^{l}c_{23}^{\nu}e^{-i\delta_{13}^{l}} \\
s_{12}e^{-i\delta_{12}} &= s_{12}^{\nu}e^{-i\delta_{12}^{\nu}} - s_{12}^{l}c_{12}^{\nu}c_{23}^{\nu}e^{-i\delta_{12}^{l}} + s_{13}^{l}c_{12}^{\nu}s_{23}^{\nu}e^{i(\delta_{23}^{\nu}-\delta_{13}^{e})} \\
s_{23}e^{-i\delta_{23}} &= s_{23}^{\nu}e^{-i\delta_{23}^{\nu}} - s_{23}^{l}c_{23}^{\nu}e^{-i\delta_{23}^{l}}
\end{aligned}
$$

Antusch King

Charged lepton contribution:

$\theta_{12}^{l} = \theta_{C}$ GUT + dimension 5 operator

Antusch, Maurer, Gross, Sluka

$\theta_{12}^{l} = \theta_{C}/3$ GUT + dimension 4 operator

$\theta_{12}^{l} = 0$

Neutrino contribution :

Perturb around TBM mixing

Neutrino contribution - perturb around TBM mixing:

Hall, GGR

$$\nu_{@} \approx \nu_a = \frac{1}{\sqrt{2}}\left(\nu_\mu + \nu_\tau\right)$$
$$\nu_{\odot} \approx \nu_b = \frac{1}{\sqrt{3}}\left(\nu_e + \nu_\mu - \nu_\tau\right)$$
$$\nu_{\perp} \approx \nu_c = \frac{1}{\sqrt{6}}\left(2\nu_e - \nu_\mu + \nu_\tau\right)$$

- ν_a, ν_b mixing

$$\varepsilon = s_{13}^{\nu} e^{i\Delta}$$

$$L_m^{\nu} = m_{@}(\nu_a + \sqrt{3}\,\varepsilon\,\nu_b)^2 + m_{\odot}(\nu_b - \sqrt{3}\,\varepsilon\,\nu_a)^2 + m_{\perp}(\nu_c)^2$$

$$V^{\nu_L} = U_{23}^{\nu} U_{13}^{\nu} U_{12}^{\nu} P^{\nu} \qquad U_{ij}^{e,\nu} = \begin{pmatrix} c_{ij}^{e,\nu} & s_{ij}^{e,\nu} e^{-i\delta_{ij}^{e,\nu}} \\ -s_{ij}^{e,\nu} e^{i\delta_{ij}^{e,\nu}} & c_{ij}^{e,\nu} \end{pmatrix}$$

Δ constrained

Dirac ~~CP~~ phase

$U_{PMNS} = R_{23} U_{13} R_{12} P$

$$s_{13}^{\nu} = |\varepsilon|, \qquad s_{23}^{\nu} = \left|\frac{1}{\sqrt{2}} + e^{-i\Delta} s_{13}^{\nu}\right|, \qquad s_{12}^{\nu} = \frac{1}{\sqrt{3}}$$

$$\delta^{\nu} = \delta_{13}^{\nu} - \delta_{23}^{\nu} - \delta_{12}^{\nu} = \Delta - 3\sqrt{2}\, s_{13}^{\nu} \sin\Delta$$

Neutrino contribution - perturb around TBM mixing:

Hall, GGR

$$\nu_{@} \approx \nu_a = \frac{1}{\sqrt{2}}\left(\nu_\mu - \nu_\tau\right)$$

$$\nu_{\odot} \approx \nu_b = \frac{1}{\sqrt{3}}\left(\nu_e + \nu_\mu + \nu_\tau\right)$$

$$\nu_{\perp} \approx \nu_c = \frac{1}{\sqrt{6}}\left(2\nu_e - \nu_\mu - \nu_\tau\right)$$

- ν_a, ν_b mixing

$$s_{13}^{\nu} = |\varepsilon|, \qquad s_{23}^{\nu} = \left|\frac{1}{\sqrt{2}} + e^{-i\Delta} s_{13}^{\nu}\right|, \qquad s_{12}^{\nu} = \frac{1}{\sqrt{3}}$$

- ν_a, ν_c mixing

$$s_{13}^{\nu} = |\varepsilon|, \qquad s_{23}^{\nu} = \left|\frac{1}{\sqrt{2}} - \frac{1}{2} e^{-i\Delta} s_{13}^{\nu}\right|, \qquad s_{12}^{\nu} = \frac{1}{\sqrt{3}}$$

- ν_b, ν_c mixing

$$s_{13}^{\nu} = 0$$

More general mixing:

e.g.

Correlation? …. can be made natural (alignment field $\langle\phi_1\rangle = (1,0,0)$)

$$L_M \approx M_{@}(\nu_a + \frac{\epsilon}{\sqrt{3}}(\nu_b + \sqrt{2}\nu_c))^2 + M_{\odot}(\nu_b - \frac{\epsilon}{\sqrt{3}}\nu_a)^2 + M_{\perp}(\nu_c - \sqrt{\frac{2}{3}}\epsilon\nu_a)^2$$

$$s_{13}^{\nu} = |\varepsilon|, \qquad s_{23}^{\nu} = \frac{1}{\sqrt{2}}, \qquad s_{12}^{\nu} = \frac{1}{\sqrt{3}}$$

King

Δ unconstrained…but spoils mass relations

Implications of θ_{13} measurement

Illustrate using 1σ range of θ_{23}, θ_{13} to predict Dirac phase †

Allow for unknown lepton phases

	$\lvert\delta\rvert/\pi$ (1σ)	$\lvert\delta\rvert/\pi$ (3σ)
$s^l_{12}=0$	0.46–0.8	0.19–1
$s^l_{12}=\sqrt{\frac{m_e}{m_\mu}}$	0.47–0.73	0.07– 1
$s^\nu_{13}=0$	0.48 – 0.57	0.41 – 0.70

$\nu_a\nu_b$ mixing

	$\lvert\delta\rvert/\pi$ (1σ)	$\lvert\delta\rvert/\pi$ (3σ)
$s^l_{12}=0$	0–0.39	0–1
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$s^\nu_{13}=0$	0.48 – 0.57	0.41 – 0.70

$\nu_a\nu_c$ mixing

†

Parameter	Best fit	1σ range	3σ range	Best Fit	1σ range	3σ range
$\sin^2\theta_{12}/10^{-1}$ (NH or IH)	3.07	2.91 – 3.25	2.59 – 3.59	3.2	3.03-3.36	2.7-3.7
$\sin^2\theta_{13}/10^{-2}$ (NH)	2.41	2.16 – 2.66	1.69 – 3.13	2.46	2.18-2.75	1.7-3.3
$\sin^2\theta_{13}/10^{-2}$ (IH)	2.44	2.19 – 2.67	1.71 – 3.15	2.23	2.23-2.76	1.7-3.3
$\sin^2\theta_{23}/10^{-1}$ (NH)	3.86	3.65 – 4.10	3.31 – 6.37	6.13	4.61-6.35	3.6-6.8
$\sin^2\theta_{23}/10^{-1}$ (IH)	3.92	3.70 – 4.31	3.35 – 6.63	6.0	5.69-6.26	3.7-6.7
δ/π (NH)	1.08	0.77 – 1.36	—	0.80	—	—
δ/π (IH)	1.09	0.83 – 1.47	—	-0.03	—	—

Fogli, Lisi, Marrone, Montanino, Palazzo, Rotuno

Forero, Tortola, Valle

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$\sin^2\theta_{23}/10^{-1}$ (IH)	3.92	3.70 – 4.31	3.35 – 6.63	6.0	5.69-6.26	3.7-6.7
δ/π (NH)	1.08	0.77 – 1.36	—	0.80	—	—
δ/π (IH)	1.09	0.83 – 1.47	—	-0.03	—	—

Fogli, Lisi, Marrone, Montanino, Palazzo, Rotuno

Forero, Tortola, Valle

	s^2_{12}
$s^l_{12}=0$	0.33
$s^l_{12}=\sqrt{\frac{m_e}{m_\mu}}$	0.29–0.38
$s^\nu_{13}=0$	0.2 – 0.5

Origin of approximate TBM : an example

$$L_m^{\nu} = m_{@}\left[\tfrac{1}{\sqrt{2}}(\nu_\mu - \nu_\tau)\right]^2 + m_{\odot}\left[\tfrac{1}{\sqrt{3}}(\nu_e + \nu_\mu + \nu_\tau)\right]^2$$

$$\equiv m_{@}\left(\tfrac{1}{\sqrt{2}}\theta_{23}^i \nu_i\right)^2 + m_{\odot}\left(\tfrac{1}{\sqrt{3}}\theta_{123}^i \nu_i\right)^2$$

$$\theta_{23} = (0,1,-1), \quad \theta_{123} = (1,1,1)$$

Relate θ_i to dynamical variables, familons ϕ_i $\qquad \theta_i \propto <\phi_i>$

ϕ_i triplets of non Abelian discrete family symmetry, $G \subset SU(3)_f$

Vacuum alignment $\qquad <\phi_{23}> = \varepsilon(0,1,-1), \quad <\phi_{123}> = \varepsilon^2(1,1,1)$

Vacuum alignment

$e.g.$ $Z_3 \ltimes Z_n$

ϕ_i	$Z_3\phi_i$	$Z_n'\phi_i$
ϕ_1	$\to \phi_2$	$\to \phi_1$
ϕ_2	$\to \phi_3$	$\to \alpha\phi_2$
ϕ_3	$\to \phi_1$	$\to \alpha^2\phi_3$

$\alpha^3 = 1$

Choice of discrete symmetry

Vacuum structure : $Z_3 \ltimes Z_n \to \begin{cases} Z_3, & \langle\phi\rangle = (1,1,1) \quad \lambda > 0 \\ Z_n, & \langle\phi\rangle = (0,0,1) \quad \lambda < 0 \end{cases}$

$$V(\phi) = -m^2\, \phi^{\dagger i}\phi_i + \ldots + \lambda\, m^2 \phi^{\dagger i}\phi_i \phi^{\dagger i}\phi_i$$

Vacuum alignment

e.g. $Z_3 \ltimes Z_3$

ϕ_i	$Z_3\phi_i$	$Z'_n\phi_i$
ϕ_1	$\to \phi_2$	$\to \phi_1$
ϕ_2	$\to \phi_3$	$\to \alpha\phi_2$
ϕ_3	$\to \phi_1$	$\to \alpha^2\phi_3$

$\alpha^3 = 1$

Choice of discrete symmetry

$$\text{Vacuum structure}: \ Z_3 \ltimes Z_3 \ \to \begin{cases} Z_3, \quad \langle\phi\rangle = (1,1,1) \quad \lambda > 0 \\ Z_3, \quad \langle\phi\rangle = (0,0,1) \quad \lambda < 0 \end{cases}$$

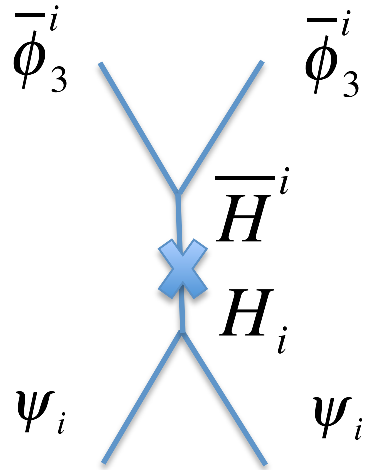
$$V(\phi) = -m^2\, \phi^{\dagger i}\phi_i + \ldots + \lambda\, m^2 \phi^{\dagger i}\phi_i \phi^{\dagger i}\phi_i$$

$$+ |\phi_{123}\phi_{23}|^2 \quad \Rightarrow \quad \phi_{123} \propto (0,1,-1)$$

$$e.g. \quad G = \Delta(6n^2) \sim (Z_n \ltimes Z_n) \times S_3$$ non Abelian discrete group (n>3)

Escobar, Luhn

Field	$SU(4)$	$SU(2)_L$	$SU(2)_R$	$\Delta(6n^2)$	Z_5	Z_4^R
Ψ	4	2	1	3_{1_l}	0	1
Ψ^c	$\bar{4}$	1	2	3_{1_l}	0	1
θ	8	1	2	1	1	0
ϕ_1	1	1	1	3_{1_l}	2	0
$\bar{\phi}_3$	1	1	1	$3_{1_{-l}}$	2	0
$\bar{\phi}_{23}$	1	1	1	$3_{1_{-l}}$	3	0
$\bar{\phi}_{123}$	1	1	1	$3_{2_{-l}}$	1	0



Structure through Higgs mediators

$H_{\text{light}} \simeq X_3$

GGR, Varzielas

Dirac mass structure

m_t comes from D=4 term

$$L_{Dirac}^{q,l,\nu} / H = \alpha\, \psi_i \bar{\phi}_3^i \psi_i^c \bar{\phi}_3^i + \beta \left(\psi_i \bar{\phi}_{123}^i \psi_j^c \bar{\phi}_{23}^j + \psi_i \bar{\phi}_{23}^i \psi_j^c \bar{\phi}_{123}^j \right) + \gamma\, \psi_i \bar{\phi}_{23}^i \psi_j^c \bar{\phi}_{23}^j \Sigma_{45}$$

$\alpha, \beta, \gamma = O(1)$

$$M^d = m_b \begin{pmatrix} <\varepsilon^4 & \varepsilon^3 & \varepsilon^3 \\ \varepsilon^3 & \varepsilon^2 & \varepsilon^2 \\ \varepsilon^2 & \varepsilon^2 & 1 \end{pmatrix} \qquad M^l = m_b \begin{pmatrix} <\varepsilon^4 & 1\varepsilon^3 & 1\varepsilon^3 \\ 1\varepsilon^3 & 3\varepsilon^2 & 3\varepsilon^2 \\ 1\varepsilon^3 & 3\varepsilon^2 & 1 \end{pmatrix}$$

$$e.g.\quad G=\Delta(6n^2)\sim\left(Z_n\ltimes Z_n\right)\times S_3$$ non Abelian discrete group

Field	$SU(4)$	$SU(2)_L$	$SU(2)_R$	$\Delta(6n^2)$	Z_5	Z_4^R
Ψ	4	2	1	3_{1_l}	0	1
Ψ^c	$\bar{4}$	1	2	3_{1_l}	0	1
θ	8	1	2	1	1	0
ϕ_1	1	1	1	3_{1_l}	2	0
$\bar{\phi}_3$	1	1	1	$3_{1_{-l}}$	2	0
$\bar{\phi}_{23}$	1	1	1	$3_{1_{-l}}$	3	0
$\bar{\phi}_{123}$	1	1	1	$3_{2_{-l}}$	1	0

Majorana mass structure

$$L_{Dirac}^{q,l,\nu}/H=\alpha\,\psi_i\bar{\phi}_3^i\psi_i^c\bar{\phi}_3^i+\beta\left(\psi_i\bar{\phi}_{123}^i\psi_j^c\bar{\phi}_{23}^j+\psi_i\bar{\phi}_{23}^i\psi_j^c\bar{\phi}_{123}^j\right)+\gamma\,\psi_i\bar{\phi}_{23}^i\psi_j^c\bar{\phi}_{23}^j\Sigma_{45}$$

$$L_M=\theta\psi_i^c\psi_j^c\left(a\bar{\phi}_3^i\bar{\phi}_3^i+b\varepsilon^3\bar{\phi}_{23}^i\bar{\phi}_{23}^i+c\varepsilon^2\bar{\phi}_{123}^i\bar{\phi}_{123}^i+d\varepsilon^3\bar{\phi}_{23}^i\bar{\phi}_{123}^i\varepsilon\right)$$

$a,b,c,d=O(1)$

$e.g.\quad G=\Delta(6n^2)\sim(Z_n\ltimes Z_n)\times S_3$ non Abelian discrete group

Field	$SU(4)$	$SU(2)_L$	$SU(2)_R$	$\Delta(6n^2)$	Z_5	Z_4^R
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$\bar{\phi}_{123}$	1	1	1	$3_{2_{-l}}$	1	0

$M_1 < M_2 << M_3$

Majorana mass structure

$$L^{q,l,\nu}_{Dirac}=\alpha\,\psi_i\bar{\phi}_3^i\psi^c_i\bar{\phi}_3^i+\beta\left(\psi_i\bar{\phi}_{123}^i\psi^c_j\bar{\phi}_{23}^j+\psi_i\bar{\phi}_{23}^i\psi^c_j\bar{\phi}_{123}^j\right)+\gamma\,\psi_i\bar{\phi}_{23}^i\psi^c_j\bar{\phi}_{23}^j\Sigma_{45}$$

small

$$L_M=\theta\psi^c_i\psi^c_j\left(a\bar{\phi}_3^i\bar{\phi}_3^i+b\varepsilon^3\bar{\phi}_{23}^i\bar{\phi}_{23}^i+c\varepsilon^2\bar{\phi}_{123}^i\bar{\phi}_{123}^i+d\varepsilon^3\bar{\phi}_{23}^i\bar{\phi}_{123}^i\varepsilon\right)$$

$$L^\nu_{eff}/H^2=\frac{\beta^2}{M_1}\psi_i\phi^i_{123}\psi_j\phi^j_{123}+\frac{\beta^2}{M_2}\psi_i\phi^i_{23}\psi_j\phi^j_{23}+\frac{(\alpha+\beta)^2}{M_3}\psi_i\phi^i_3\psi_j\phi^j_3$$

$<\phi_{23}>=\varepsilon(0,1,-1),$

$<\phi_{123}>=\varepsilon^2(1,1,1)$

$$\Rightarrow\frac{m_\odot}{m_@}=O(\varepsilon)$$

$e.g.\quad G=\Delta(6n^2)\sim(Z_n \ltimes Z_n)\times S_3$ non Abelian discrete group

Field	$SU(4)$	$SU(2)_L$	$SU(2)_R$	$\Delta(6n^2)$	Z_5	Z_4^R
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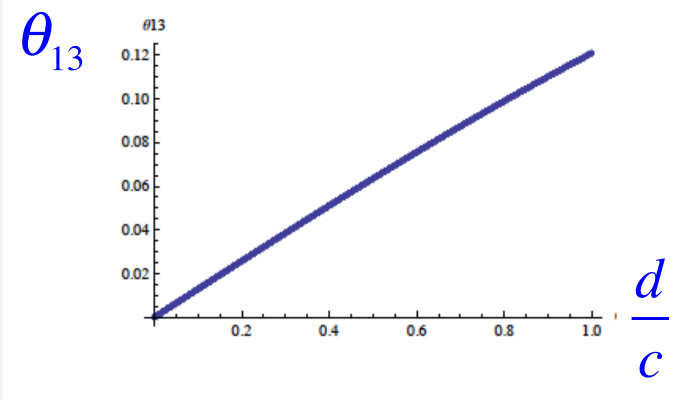
$\bar{\phi}_{23}$	1	1	1	$3_{1_{-l}}$	3	0
$\bar{\phi}_{123}$	1	1	1	$3_{2_{-l}}$	1	0



Majorana mass structure

breaks $\phi_{123}\to-\phi_{123}$ symmetry

$$L^{q,l,\nu}_{Dirac}=\alpha\,\psi_i\bar{\phi}^i_3\psi^c_i\bar{\phi}^i_3+\beta\left(\psi_i\bar{\phi}^i_{123}\psi^c_j\bar{\phi}^j_{23}+\psi_i\bar{\phi}^i_{23}\psi^c_j\bar{\phi}^j_{123}\right)+\gamma\,\psi_i\bar{\phi}^i_{23}\psi^c_j\bar{\phi}^j_{23}\Sigma_{45}$$

$$L_M=\theta\psi^c_i\psi^c_j\left(a\bar{\phi}^i_3\bar{\phi}^i_3+b\varepsilon^2\bar{\phi}^i_{23}\bar{\phi}^i_{23}+c\varepsilon\bar{\phi}^i_{123}\bar{\phi}^i_{123}+d\varepsilon^2\bar{\phi}^i_{23}\bar{\phi}^i_{123}\varepsilon\right)$$

$\nu_a\nu_b$ mixing

$$L^{\nu}_{eff}=\frac{\beta^2}{M_1}\psi_i\phi^i_{123}\psi_j\phi^j_{123}+\frac{\beta^2}{M_2}\psi_i\phi^i_{23}\psi_j\phi^j_{23}+\frac{\beta^2(d/c)\varepsilon}{M_2}\psi_i\phi^i_{23}\psi_j\phi^j_{123}$$

$\Delta(6n^2)$ model

- Higgs mediators – m_t unsuppressed
- $Z_{4R} - \mu$ term and nucleon decay under control
- Neutrino mass hierarchy limited $\frac{m_{\odot}}{m_{@}} = O(\varepsilon)$
- $\theta_{13} = O(\varepsilon)$
- Light familinos weakly coupled – LSP?- visible sector NLSP charged?

Summary

- $\theta_{13} \approx 9^0$ unexpected and requires modification of Yukawa quilt

- Can be accommodated by perturbation of TBM mixing and/or leptonic contribution

  Correlated changes in θ_{12}, θ_{23}

  +Large CP violation typical

- Discrete non Abelian family symmetry can generate approximate TBM mixing +

$$\theta_C = \left| \sqrt{\frac{m_d}{m_s}} - e^{i\delta}\sqrt{\frac{m_u}{m_c}} \right|$$

$$\left|\frac{V_{ub}}{V_{cb}}\right| \approx \sqrt{\frac{m_u}{m_c}}$$

$$\left|\frac{V_{td}}{V_{ts}}\right| \approx \sqrt{\frac{m_d}{m_s}}$$

$$m_\tau(M_{GUT}) = m_b(M_{GUT})$$

$$m_\mu(M_{GUT}) = 3m_s(M_{GUT})$$

$$m_e(M_{GUT}) = \frac{1}{3}m_s(M_{GUT})$$

…6+ parameter free postdictions

$$\Delta(6n^2) \sim (Z_n \times Z_n) \times S_3$$

$$\mathbf{3}_{1(l)} \quad : \quad \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \mapsto \begin{pmatrix} x_2 \\ x_3 \\ x_1 \end{pmatrix}_a , \quad \begin{pmatrix} x_3 \\ x_2 \\ x_1 \end{pmatrix}_b , \quad \begin{pmatrix} \eta^l x_1 \\ \eta^{-l} x_2 \\ x_3 \end{pmatrix}_c ,$$

$$\mathbf{3}_{2(l)} \quad : \quad \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \mapsto \begin{pmatrix} x_2 \\ x_3 \\ x_1 \end{pmatrix}_a , \quad \begin{pmatrix} -x_3 \\ -x_2 \\ -x_1 \end{pmatrix}_b , \quad \begin{pmatrix} \eta^l x_1 \\ \eta^{-l} x_2 \\ x_3 \end{pmatrix}_c$$

Escobar, Luhn

$$\Delta(6n^2) \sim (Z_n \times Z_n) \times S_3$$

- $\mathbf{3}_{1(l)} \otimes \mathbf{3}_{1(l')} = \mathbf{3}_{1(l+l')} + \mathbf{6}_{\widetilde{(l,-l')}}$. Vector x will transform as a $\mathbf{3}_{1(l)}$ and y as a $\mathbf{3}_{1(l')}$.

$$\mathbf{3}_{1(l+l')} \quad : \quad \begin{pmatrix} x_1y_1 \\ x_2y_2 \\ x_3y_3 \end{pmatrix}, \qquad \mathbf{6}_{(-l,l-l')} \quad : \quad \begin{pmatrix} x_1y_2 \\ x_2y_3 \\ x_3y_1 \\ x_3y_2 \\ x_2y_1 \\ x_1y_3 \end{pmatrix}, \tag{B.13}$$

- $\mathbf{3}_{1(l)} \otimes \mathbf{3}_{2(l')} = \mathbf{3}_{2(l+l')} + \mathbf{6}_{\widetilde{(l,-l')}}$. Vector x will transform as a $\mathbf{3}_{1(l)}$ and y as a $\mathbf{3}_{2(l')}$.

$$\mathbf{3}_{2(l+l')} \quad : \quad \begin{pmatrix} x_1y_1 \\ x_2y_2 \\ x_3y_3 \end{pmatrix}, \qquad \mathbf{6}_{(-l,l-l')} \quad : \quad \begin{pmatrix} x_1y_2 \\ x_2y_3 \\ x_3y_1 \\ -x_3y_2 \\ -x_2y_1 \\ -x_1y_3 \end{pmatrix}, \tag{B.14}$$

$$6_{l,-l} = 3_{1,-l} + 3_{2,-l}$$

$$-l \equiv (n-1)l \quad \Rightarrow \quad n > 3$$

Escobar, Luhn

Vacuum alignment

$$P \supset \; < P > \phi_{23}\phi_{123}^2 \rightarrow m_{3/2}\phi_{23}\phi_{123}^2$$

$$V_{tree} = m_{3/2}^2 |\phi_{123}|^4 + m_{3/2}^2 |\phi_{123}\phi_{23}|^2$$

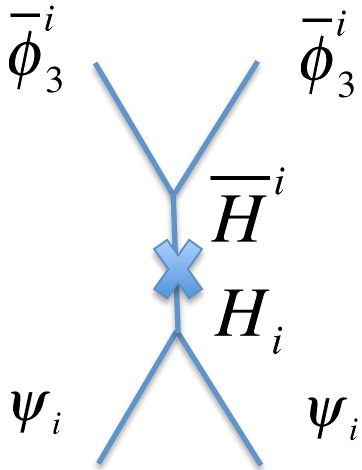
$$V_{rad} = \alpha m_{3/2}^2 |\phi_2|^4 + \beta m_{3/2}^2 |\phi_3|^4 + \gamma m_{3/2}^2 |\phi_2\phi_3|^4 + \delta m_{3/2}^2 |\phi_2\phi_{23}|^2 + \ldots$$

$$\phi_{123} \propto (1,1,1), \quad \phi_2 \propto (1,0,0), \quad \phi_3 \propto (0,0,1), \quad \phi_{23} \propto (0,1,-1)$$

$$\alpha, \beta < 0, \quad \gamma, \delta > 0$$

Field	$SU(4)$	$SU(2)_L$	$SU(2)_R$	$\Delta(6n^2)$	Z_5	Z_4^R
Ψ	4	2	1	3_{1_l}	0	1
Ψ^c	$\bar{4}$	1	2	3_{1_l}	0	1
θ	8	1	2	1	1	0
H	1	2	2	1	1	0
X	1	2	2	$3_{1_{-2l}}$	0	0
$\bar{X}$	1	2	2	$3_{1_{2l}}$	0	2
Y	1	2	2	$3_{1_{-2l}} + 3_{1_l}$	2	0
$\bar{Y}$	1	2	2	$3_{1_{2l}} + 3_{1_{-l}}$	3	2
Z	1	2	2	3_{2_l}	0	0
$\bar{Z}$	1	2	2	$3_{1_{-l}}$	0	2
Σ	15	1	3	1	3	0
ϕ	1	1	1	$1'$	0	0
ϕ_1	1	1	1	3_{1_l}	1	0
$\bar{\phi}_3$	1	1	1	$3_{1_{-l}}$	2	0
$\bar{\phi}_{23}$	1	1	1	$3_{1_{-l}}$	3	0
$\bar{\phi}_{123}$	1	1	1	$3_{2_{-l}}$	1	0

Table 1. Field and symmetry content of the model.



m_t comes from D=4 term

$$H_l \approx X_3 + \left(\frac{\bar{\phi}_{23}^3}{\bar{\phi}_3^3}\right)^2 \frac{M_X^a}{M_X^b}\left(a(Y_2^{(a)} + Y_3^{(a)}) - 2bY_1^{(b)}\right) + \left(\frac{\bar{\phi}_{23}^3 \bar{\phi}_{123}^3}{(\bar{\phi}_3^3)^2}\right)\frac{M_X^a}{M_X^c}(2Z_1 - Z_2 - Z_3) + H\frac{M\, M_X^a}{(\bar{\phi}_3^3)^2}$$

Majorana mass terms allowed by $Z_5 \times Z_{4R}$:

$$P_M = \theta \Psi_i^c \Psi_j^c \left(\frac{\bar{\phi}_3^i \bar{\phi}_3^j}{M_X^2} + \frac{\bar{\phi}_{23}^i \bar{\phi}_{23}^j \left[\bar{\phi}_3 \bar{\phi}_3 \bar{\phi}_{23} \bar{\phi}_{23} \bar{\phi}_{23} \right]}{M_X^7} \right)$$

$$+ \theta \Psi_i^c \Psi_j^c \left(\frac{\bar{\phi}_{123}^i \bar{\phi}_{123}^j \left[\bar{\phi}_3 \bar{\phi}_3 \bar{\phi}_3 \bar{\phi}_{23} \bar{\phi}_{23} \right]}{M_X^7} + \frac{\bar{\phi}_{23}^i \bar{\phi}_{123}^j \left[\bar{\phi}_3 \bar{\phi}_3 \bar{\phi}_3 \bar{\phi}_{23} \bar{\phi}_{123} \right]}{M_X^7} \right)$$