Flavor Symmetry relating neutrino mixing angles to neutrino masses

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Plan of my talk

1 Introduction

Breaking with Tri-bimaximal mixing Paradigm

2 A_4 model to realize largeθ_{13}

3 Large θ_{13} and the neutrino masses

4 Towards minimal model

Texture with Split Seesaw

5 Summary

1 Introduction

Reactor and LBL Experiments suggest us the breaking with tri-bimaximal paradigm !

$$\sin^2\theta_{12} = 1/3,\ \sin^2\theta_{23} = 1/2,\ \sin^2\theta_{13} = 0,$$

$$U_{\text{tri-bimaximal}} = \begin{pmatrix} \sqrt{2/3} & \sqrt{1/3} & 0 \\ -\sqrt{1/6} & \sqrt{1/3} & -\sqrt{1/2} \\ -\sqrt{1/6} & \sqrt{1/3} & \sqrt{1/2} \end{pmatrix}$$

Harrison, Perkins, Scott (2002)

Simple modification of the tri-bimaximal mixing gives us the relation among masses and mixing angles.

Tri-bimaximal Mixing realized in

$$M_{\rm TBM} = \frac{m_1+m_3}{2}\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix} + \frac{m_2-m_1}{3}\begin{pmatrix}1&1&1\\1&1&1\\1&1&1\end{pmatrix} + \frac{m_1-m_3}{2}\begin{pmatrix}1&0&0\\0&0&1\\0&1&0\end{pmatrix}$$

independent of neutrino masses.

Additional Matrices break Tri-bimaximal mixing

$$\begin{pmatrix}0&0&1\\0&1&0\\1&0&0\end{pmatrix}, \quad \begin{pmatrix}0&1&0\\1&0&0\\0&0&1\end{pmatrix}$$

which appear in A_4, S_4, $\Delta(27)$ flavor symmetries.

Mixing angles are no more independent of mass eigenvalues.

Let us see this situation in A_4 model,

E. Ma and G. Rajasekaran, PRD64(2001)113012

2 A_4 Model to realize largeθ_{13}

Modify G. Altarelli, F. Feruglio, Nucl.Phys. B720 (2005) 64

	(l_e, l_μ, l_τ)	e^c	μ^c	τ^c	$h_{u,d}$	ϕ_l	ϕ_ν	ξ	ξ'
$SU(2)$	2	1	1	1	2	1	1	1	1
A_4	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}''$	$\mathbf{1}'$	$\mathbf{1}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}'$
Z_3	ω	ω^2	ω^2	ω^2	1	1	ω	ω	ω

$$\mathbf{3} \times \mathbf{3} = \mathbf{3} + \mathbf{3} + \mathbf{1} + \mathbf{1}' + \mathbf{1}''$$

$$\mathbf{3} \times \mathbf{3} \Rightarrow \mathbf{1} \quad = a_1 * b_1 + a_2 * b_3 + a_3 * b_2$$

$$\mathbf{3} \times \mathbf{3} \Rightarrow \mathbf{1}' \quad = a_1 * b_2 + a_2 * b_1 + a_3 * b_3$$

$$\mathbf{3} \times \mathbf{3} \Rightarrow \mathbf{1}'' \quad = a_1 * b_3 + a_2 * b_2 + a_3 * b_1$$

ξ $\qquad$ ξ' $\qquad$ remark ξ''

$$\mathbf{1} \times \mathbf{1} \Rightarrow \mathbf{1} \quad , \quad \mathbf{1}'' \times \mathbf{1}' \Rightarrow \mathbf{1} \quad , \quad 1' \times 1'' \to 1$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \qquad \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \qquad \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Y. Simizu, M. Tanimoto, A. Watanabe, PTP 126, 81(2011)

Additional Matrix

$$M_\nu = a\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix} + b\begin{pmatrix}1&1&1\\1&1&1\\1&1&1\end{pmatrix} + c\begin{pmatrix}1&0&0\\0&0&1\\0&1&0\end{pmatrix} + d\begin{pmatrix}0&0&1\\0&1&0\\1&0&0\end{pmatrix}$$

$$a = \frac{y^\nu_{\phi_\nu}\alpha_\nu v_u^2}{\Lambda}, \qquad b = -\frac{y^\nu_{\phi_\nu}\alpha_\nu v_u^2}{3\Lambda}, \qquad c = \frac{y^\nu_{\xi}\alpha_\xi v_u^2}{\Lambda}, \qquad d = \frac{y^\nu_{\xi'}\alpha_{\xi'} v_u^2}{\Lambda}$$

There is one relation $a = -3b$

$$m_1 = a + \sqrt{c^2 + d^2 - cd}$$
$$m_2 = c + d$$
$$m_3 = -a + \sqrt{c^2 + d^2 - cd}$$

Neglecting phases,
3 parameters

For normal hierarchical limit m1<< m2<< m3, we have

$$m_1 \simeq 0 \longrightarrow \Delta m^2_{\rm atm} \simeq 4(c^2 + d^2 - cd), \quad \Delta m^2_{\rm sol} \simeq (c+d)^2$$

After rotating by $V_{\text{tri-bi}}$, we get

$$M_\nu = V_{\text{tri-bi}} \begin{pmatrix} a+c-\frac{d}{2} & 0 & \frac{\sqrt{3}}{2}d \\ 0 & a+3b+c+d & 0 \\ \frac{\sqrt{3}}{2}d & 0 & a-c+\frac{d}{2} \end{pmatrix} V_{\text{tri-bi}}^T$$

$$a = -3b$$

$$U_{\text{MNS}} = V_{\text{tri-bi}} \begin{pmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{pmatrix}$$

$$\tan 2\theta = \frac{\sqrt{3}d}{-2c+d}$$

Mixing matrix elements are given

$$|U_{e2}| = \frac{1}{\sqrt{3}}\ , \quad |U_{e3}| = \frac{2}{\sqrt{6}}|\sin\theta|\ , \quad |U_{\mu 3}| = \left|-\frac{1}{\sqrt{6}}\sin\theta - \frac{1}{\sqrt{2}}\cos\theta\right|$$

Then, we obtain

$$\sin\theta_{13} = \frac{2}{\sqrt{6}}|\sin\theta| \qquad \sin\theta_{23} = \frac{1}{\cos\theta_{13}}\left(\frac{1}{\sqrt{2}}\cos\theta + \frac{1}{\sqrt{6}}\sin\theta\right)$$

where sinθ is fixed by neutrino masses

$$\tan 2\theta = \frac{\sqrt{3}d}{-2c+d} \qquad \frac{\Delta m_{31}^2}{\Delta m_{21}^2} = \frac{4(c^2+d^2-cd)}{(c+d)^2}$$

for normal hierarchy limit m1<< m2<< m3

Taking account exp. with 3σ $\frac{\Delta m^2_{31}}{\Delta m^2_{21}} = 26 \sim 38$

we obtain $\sin\theta = 0.16 \sim 0.18$ for normal hierarchy.

Then, we predict

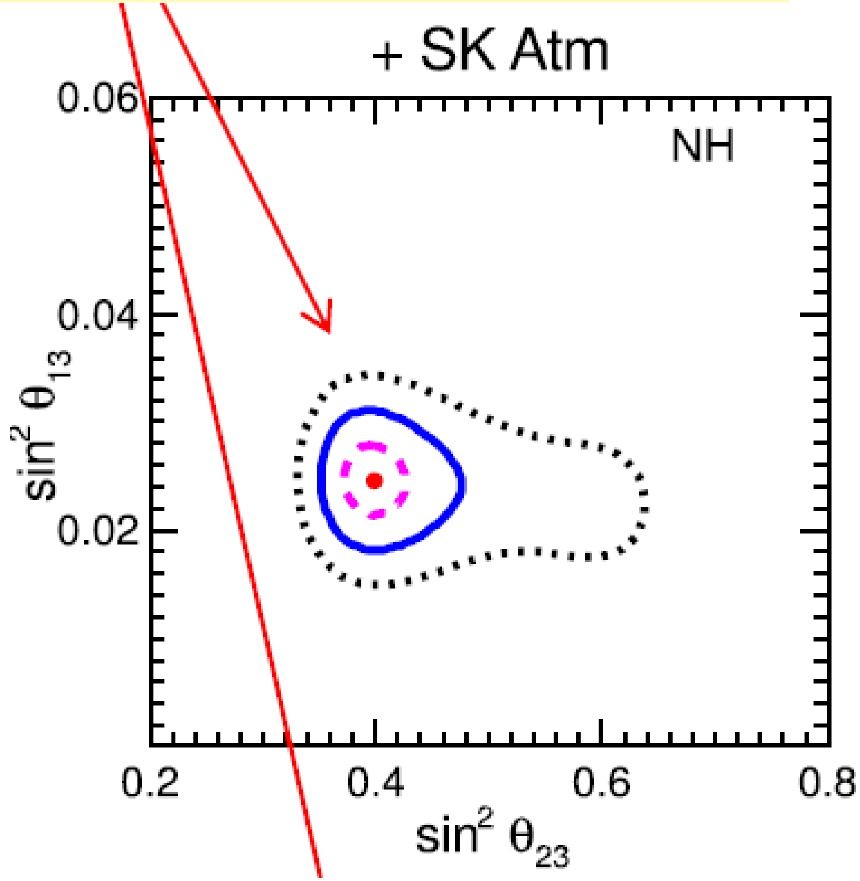
$$\sin\theta_{13} = 0.133 \sim 0.146$$

Exp. (Daya Bay 3σ) $\sin\theta_{13} = 0.12 \sim 0.18$

$$\sin\theta_{23} = 0.64 - 0.65, \quad 0.77 \sim 0.78$$

$$\sin\theta_{23}^2 = 0.40 - 0.42, \quad 0.59 \sim 0.61$$

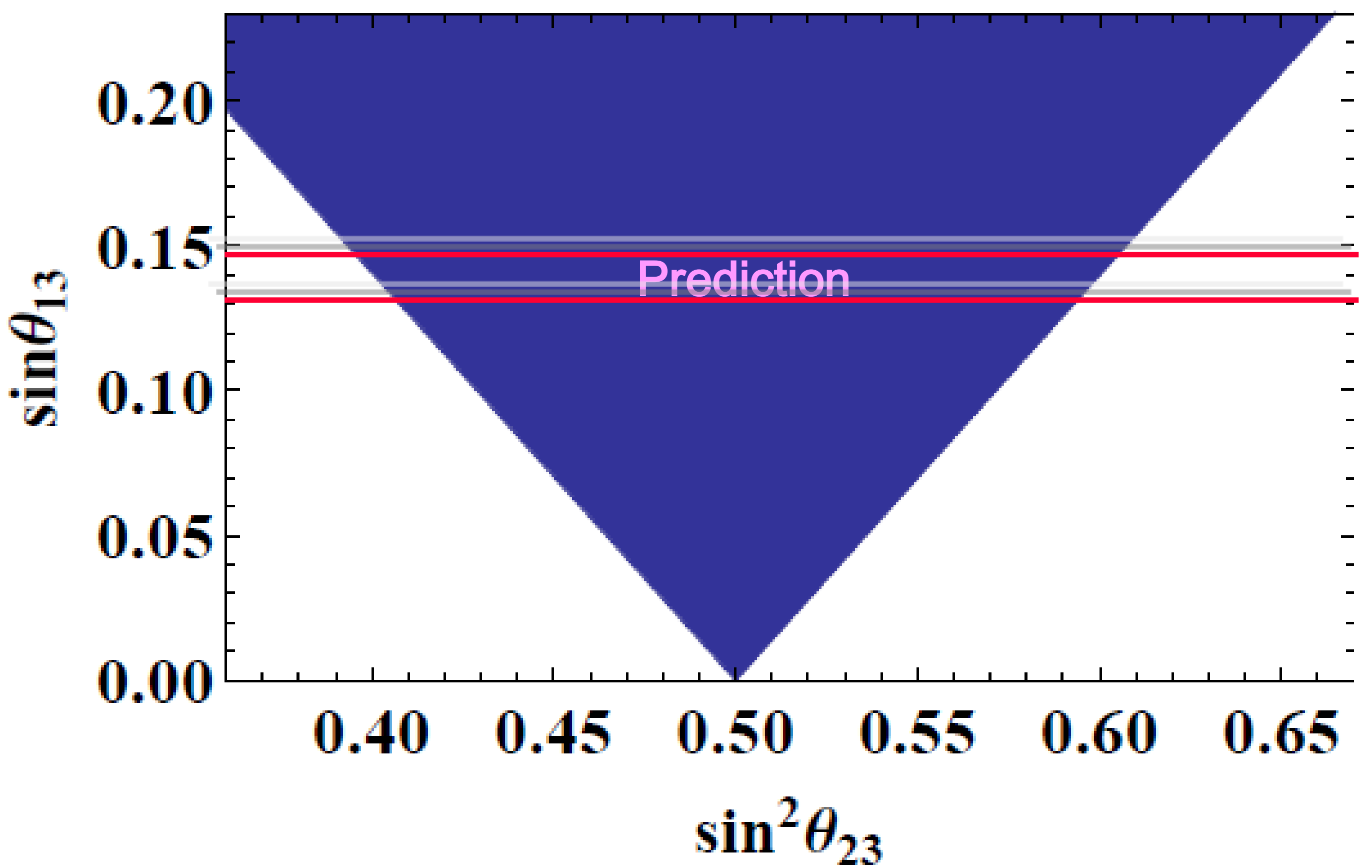
the preference for θ_{23} in the 1st octant is corroborated



Normal Hierarchy

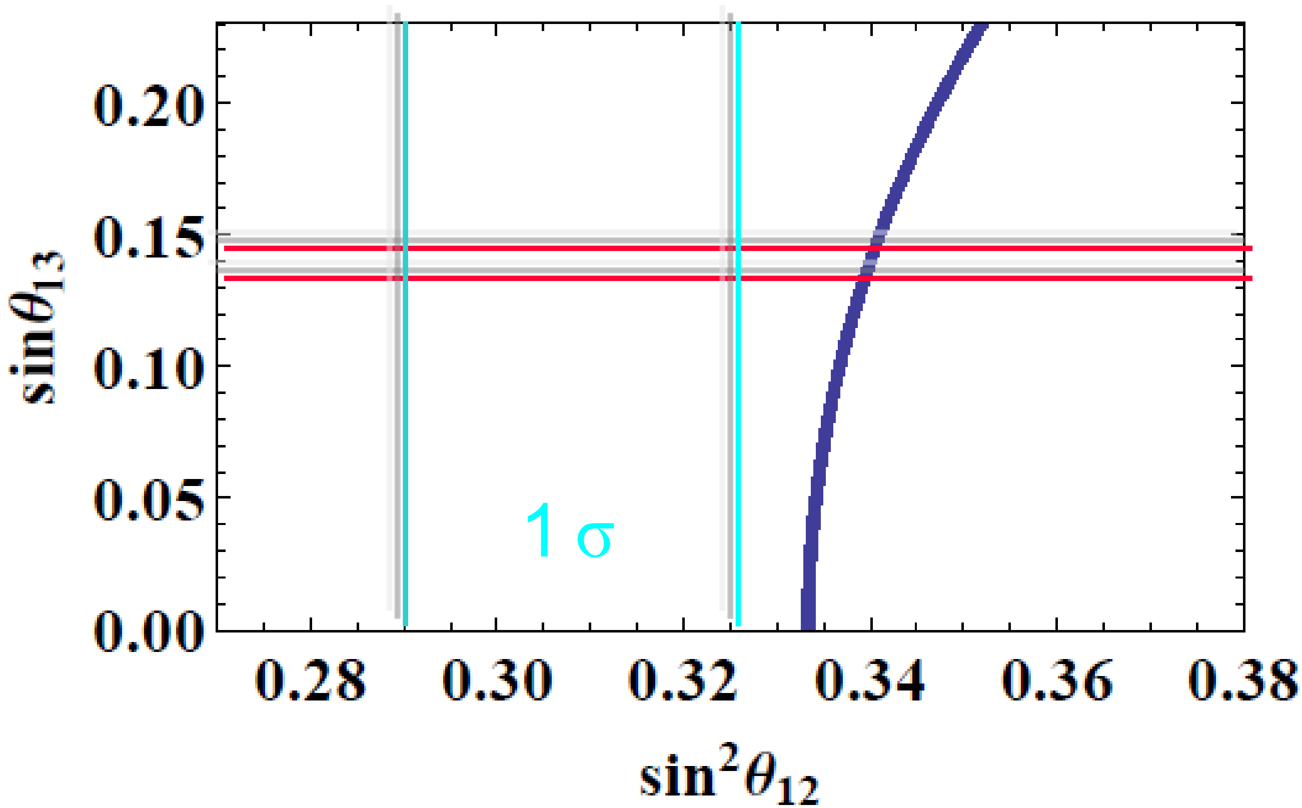
Taking account phase, we obtain allowed region

$$\sin\theta_{13} = \frac{2}{\sqrt{6}} |\sin\theta e^{i\phi}| \qquad \sin\theta_{23} = \frac{1}{\cos\theta_{13}} \left(\frac{1}{\sqrt{2}} \cos\theta + \frac{1}{\sqrt{6}} \sin\theta e^{i\phi} \right)$$

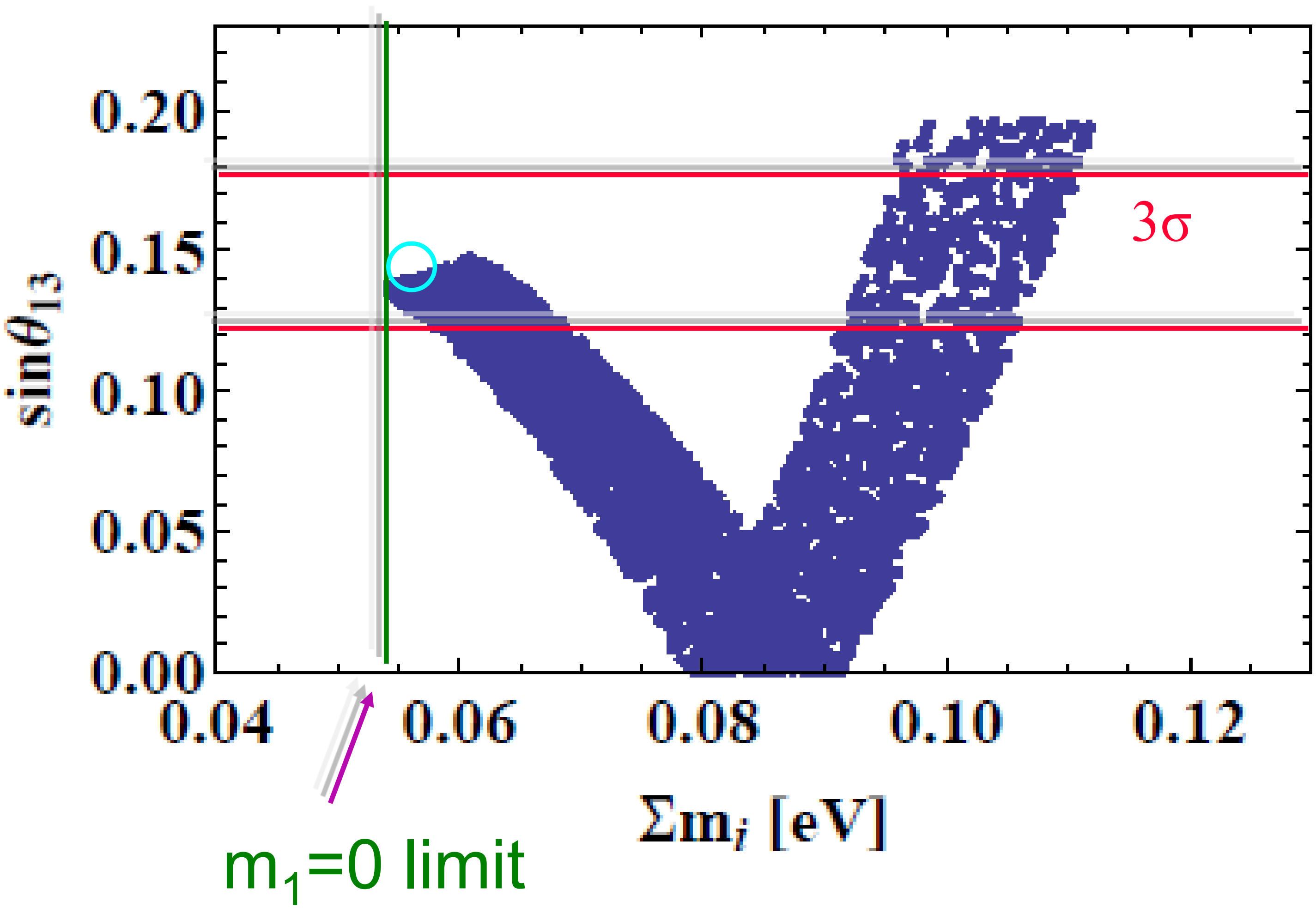


Since $|U_{e2}| = \frac{1}{\sqrt{3}}$, $\sin^2\theta_{12}$ is larger than 1/3 !

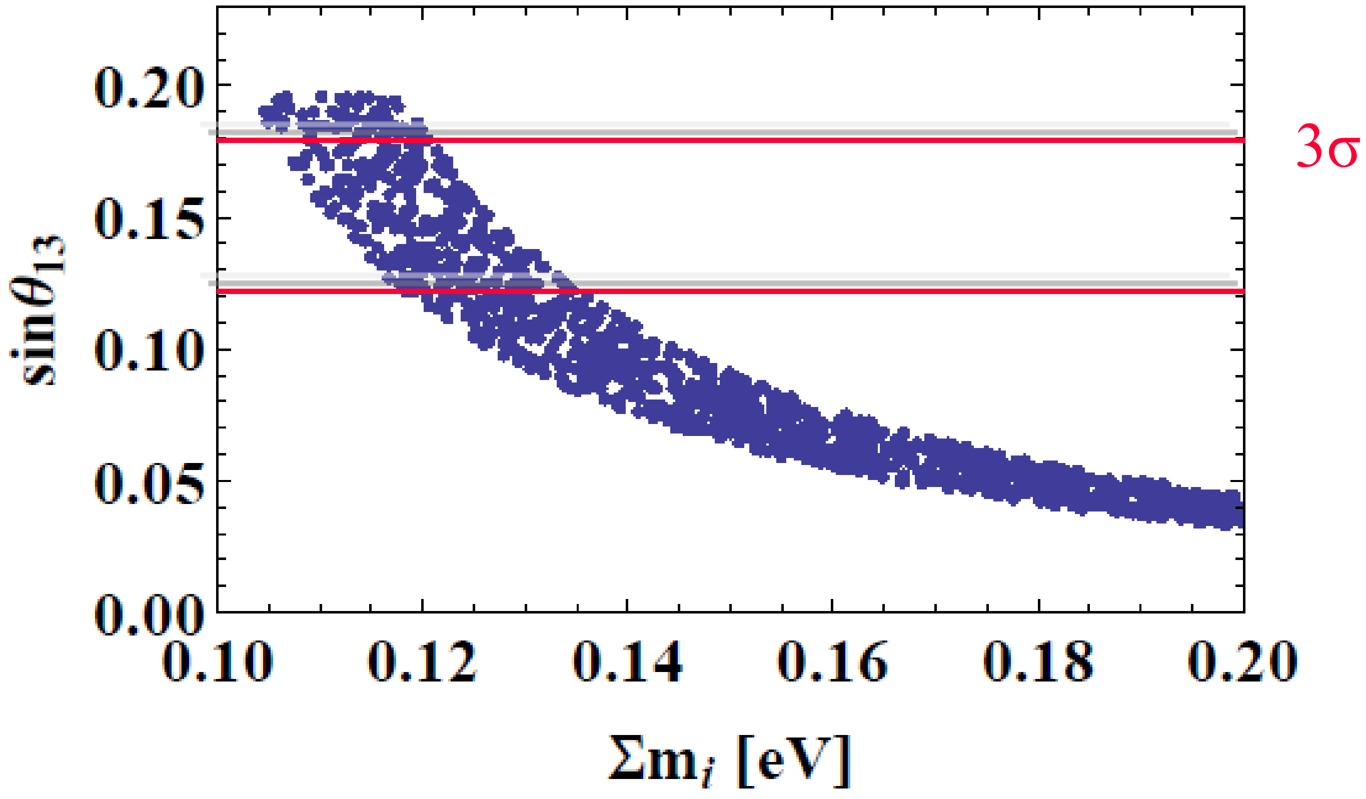
$$\sin\theta_{12} = \frac{U_{e2}}{\cos\theta_{13}} = \frac{1}{\sqrt{3(1-\sin^2\theta_{13})}}$$



Normal Hierarchy of Neutrino Masses



Inverted Hierarchy of Neutrino Masses



Remark: Three combinations give same result.

Present analyses

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \text{ and } \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

ξ and ξ'

1 and 1'

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \text{ and } \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

ξ and ξ''

1 and 1''

$$\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \text{ and } \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

ξ' and ξ''

1' and 1''

3 Large θ_{13} and the neutrino masses

Daya Bay results

$$\sin\theta_{13} = 0.15 \pm 0.01$$ Order of Cabbibo angle !?

What does this value of θ_{13} indicate ?

$$\text{Recall} \quad \sin\theta_C \simeq \sqrt{\frac{m_d}{m_s}} \simeq 0.2$$

Θ_{ij} could be related to neutrino masses explicitly.

Ratio of Neutrino Mass Squared differences

$$\frac{\Delta m^2_{\rm sol}}{\Delta m^2_{\rm atm}} = 0.026 \sim 0.038 = \mathcal{O}(\lambda^2)$$

$$\sqrt{\frac{\Delta m^2_{\rm sol}}{\Delta m^2_{\rm atm}}} = 0.160 \sim 0.196 = \mathcal{O}(\lambda)$$ θ_{13} ?

$$\sqrt[4]{\frac{\Delta m^2_{\rm sol}}{\Delta m^2_{\rm atm}}} = 0.40 \sim 0.44 = \mathcal{O}(\sqrt{\lambda})$$

$$\sqrt[8]{\frac{\Delta m^2_{\rm sol}}{\Delta m^2_{\rm atm}}} = 0.63 \sim 0.66 = \mathcal{O}(\sqrt[4]{\lambda})$$ θ_{23} ?

λ=0.2

Texture zeros give Large θ_{13}

in terms of neutrino masses

Fukugita, Tanimoto, Yanagida,
PLB 562(2003) 273 [arXiv:/0303177].

$$m_E = \begin{pmatrix} 0 & A_\ell & 0 \\ A_\ell & 0 & B_\ell \\ 0 & B_\ell & C_\ell \end{pmatrix}, \qquad m_{\nu D} = \begin{pmatrix} 0 & A_\nu & 0 \\ A_\nu & 0 & B_\nu \\ 0 & B_\nu & C_\nu \end{pmatrix}$$

$$M_R = M_0 \mathbf{I}$$

$$m_i = \left(m_{\nu D}^T M_R^{-1} m_{\nu D} \right)_i$$

$$U = U_\ell^\dagger \, Q \, U_\nu \qquad Q = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\sigma} & 0 \\ 0 & 0 & e^{i\tau} \end{pmatrix}$$

Fritzsch Texture : free parameters are m_1, σ, τ

Lepton Mixing Matrix

Fukugita, Shimizu, Tanimoto, Yanagida (PLB 2012), arXiv:1204.2389

$m_i = m_{Di}{}^2 / M_R$

$$U_{e2} \simeq -\left(\frac{m_1}{m_2}\right)^{1/4} + \left(\frac{m_e}{m_\mu}\right)^{1/2} e^{i\sigma}$$

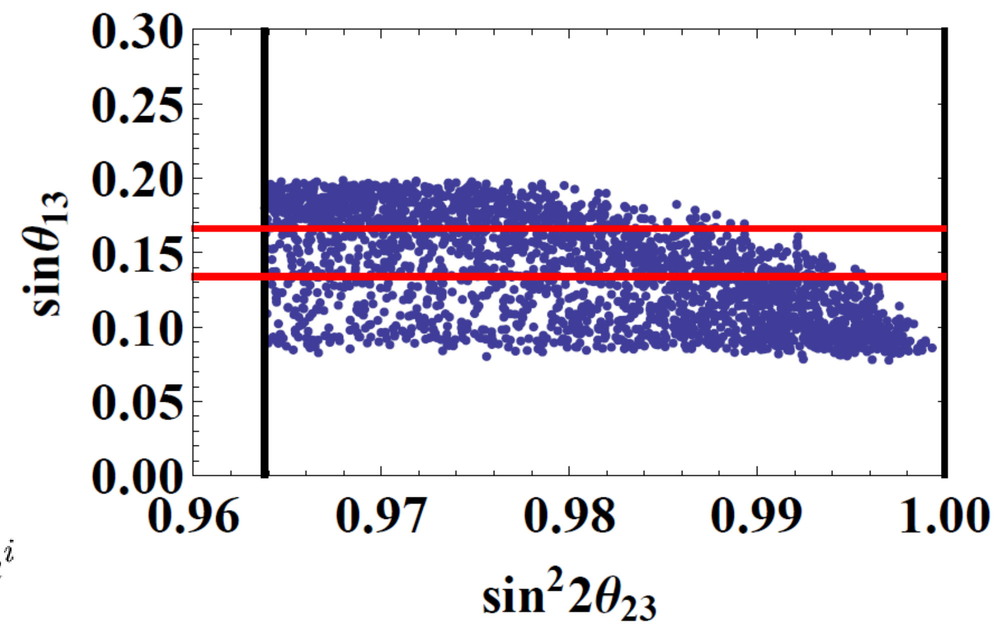
$$U_{\mu 1} \simeq \left(\frac{m_1}{m_2}\right)^{1/4} e^{i\sigma} - \left(\frac{m_e}{m_\mu}\right)^{1/2}$$

$$U_{\mu 3} \simeq \left(\frac{m_2}{m_3}\right)^{1/4} e^{i\sigma} - \left(\frac{m_\mu}{m_\tau}\right)^{1/2} e^{i\tau}$$

$$U_{\tau 2} \simeq -\left(\frac{m_2}{m_3}\right)^{1/4} e^{i\tau} + \left(\frac{m_\mu}{m_\tau}\right)^{1/2} e^{i}$$

$$U_{e3} \simeq \left(\frac{m_e}{m_\mu}\right)^{1/2} U_{\mu 3} + \left(\frac{m_2}{m_3}\right)^{1/2} \left(\frac{m_1}{m_3}\right)^{1/4}$$

$$U_{\tau 1} \simeq \left(\frac{m_1}{m_2}\right)^{1/4} U_{\tau 2}$$



Normal Mass Hierarchy

$$\sin\theta_{23} \simeq \sqrt[8]{\frac{\Delta m^2_{\rm sol}}{\Delta m^2_{\rm atm}}} = 0.63 \sim 0.66 = \mathcal{O}(\sqrt[4]{\lambda})$$

$$\boxed{\sin\theta_{13} \simeq (\sin\theta_{23})^3 \sin\theta_{12} \simeq 0.16}$$

4 Towards minimal model

Consider Texture with Split Seesaw

A.Kusenko, F.Takahashi, T.Yanagida, Phys.Lett.B 693 (2010) 144

$$M_{R1} \sim \mathcal{O}(\mathrm{keV}) \ll M_{R2},\ M_{R3} \sim \mathcal{O}(10^{12}\mathrm{GeV})$$

$$Y_{1i} \ll Y_{2i},\ Y_{3i}$$

M_{R1} is the sterile neutrino: Dark Matter Candidate

Realized in 5D theory compactified on S^1/Z_2

$$M_R^{3\times 3} = \begin{pmatrix} M_{R1}^{1\times 1} & 0 \\ 0 & M_R^{2\times 2} \end{pmatrix}$$

$$m_D = \begin{pmatrix} Y_{3\times 1} & Y_{3\times 2} \end{pmatrix} v_u$$

After Seesaw

$$M_\nu = Y_{3\times 2}(M_R^{2\times 2})^{-1} Y_{2\times 3}^T \; v_u^2 + \sum_i Y_{1i} M_{R1}^{-1} Y_{1i}^T v_u^2$$

$\downarrow$ **Flavor Mixing**

$\downarrow$ No effect on flavor mixing
Lightest neutrino mass
Rank 1 matrix

We need 3×2 Dirac mass matrix and
2×2 Right-handed Majorana mass matrix,
which give observed flavor mixing angles.

One Example

$$M_R^{2\times 2} = \begin{pmatrix} M_2 & 0 \\ 0 & M_3 \end{pmatrix}, \quad m_D = \begin{pmatrix} a_2 & a_3\lambda \\ a_2 & a_3 \\ -a_2 & a_3 \end{pmatrix}$$

S.F. King and C. Luhn, arXiv:1112.1959, JHEP, 1203(2012) 036

which are realized in A_4 model

$$\mathbf{M}_\nu = \frac{m_2}{3}\begin{pmatrix} 1 & 1 & -1 \\ 1 & 1 & -1 \\ -1 & -1 & 1 \end{pmatrix} + \frac{m_3}{2}\begin{pmatrix} \lambda^2 & \lambda & \lambda \\ \lambda & 1 & 1 \\ \lambda & 1 & 1 \end{pmatrix}$$

$$\sin\theta_{13} \simeq \frac{\lambda}{\sqrt{2}}$$

λ is Cabibbo angle

We present another texture, which predicts clear correlations among mixing angles.

Y.Shimizu, R.Takahashi, M.T

$$M_R^{2\times 2} = M_R \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad m_D = \begin{pmatrix} 0 & \frac{X-Y}{2} \\ \frac{1}{\sqrt{2}} & X \\ \frac{1}{\sqrt{2}} & Y \end{pmatrix}$$

These could be realized in S4 model.

$$M_\nu = \begin{pmatrix} \frac{1}{4}(X-Y)^2 & \frac{1}{2}X(X-Y) & \frac{1}{2}(X-Y)Y \\ \frac{1}{2}X(X-Y) & \frac{1}{2}+X^2 & \frac{1}{2}+XY \\ \frac{1}{2}(X-Y)Y & \frac{1}{2}+XY & \frac{1}{2}+Y^2 \end{pmatrix}$$

In tri-bimaximal mixing Basis

$$M_\nu = U_{\rm TBM}^T \begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{3}{4}(X-Y)^2 & \frac{1}{2}\sqrt{\frac{3}{2}}(X-Y)(X+Y) \\ 0 & \frac{1}{2}\sqrt{\frac{3}{2}}(X-Y)(X+Y) & 1+\frac{1}{2}(X+Y)^2 \end{pmatrix} U_{\rm TBM}$$

$$U_{\rm TBM} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

$$U_{\text{lepton}} = U_{\text{TBM}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{pmatrix}$$

$$\tan 2\theta \simeq \sqrt{\frac{3}{2}}(X-Y)(X+Y) = \sqrt{6}\lambda$$

$$\frac{m_2}{m_3} \simeq \frac{3}{4}(X-Y)^2 = r$$

Reparametrization including phase s

$$X = \frac{2re^{2i\alpha} + 3\lambda e^{i\delta}}{2\sqrt{3re^{2i\alpha}}}, \quad Y = \frac{-2re^{2i\alpha} + 3\lambda e^{i\delta}}{2\sqrt{3re^{2i\alpha}}}$$

$$r \simeq \lambda$$

$$|U_{e2}| = \left|\frac{\cos\theta}{\sqrt{3}}\right| \simeq \sqrt{\frac{1}{3} - \frac{\lambda^2}{2}}, \quad |U_{e3}| = \left|\frac{\sin\theta}{\sqrt{3}}\right| \simeq \frac{\lambda}{\sqrt{2}},$$

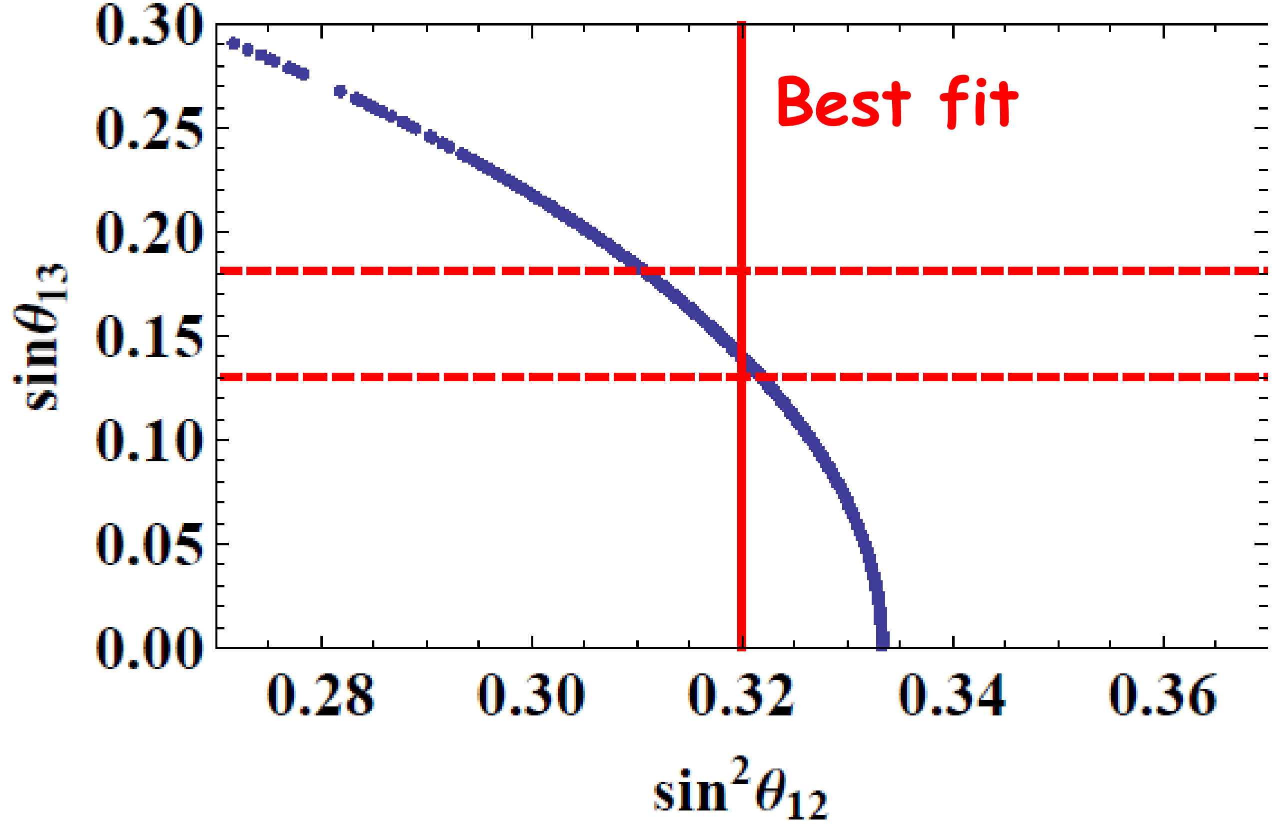
$$|U_{\mu 3}| = \left|\frac{\cos\theta}{\sqrt{2}} + \frac{\sin\theta}{\sqrt{3}}\right| \simeq \left|\sqrt{\frac{1}{2} - \frac{3\lambda^2}{4}} + \frac{\sqrt{6}}{2}\lambda\right|.$$

$$\sin^2\theta_{12} \simeq \tfrac{1}{3}(1 - \lambda^2)$$

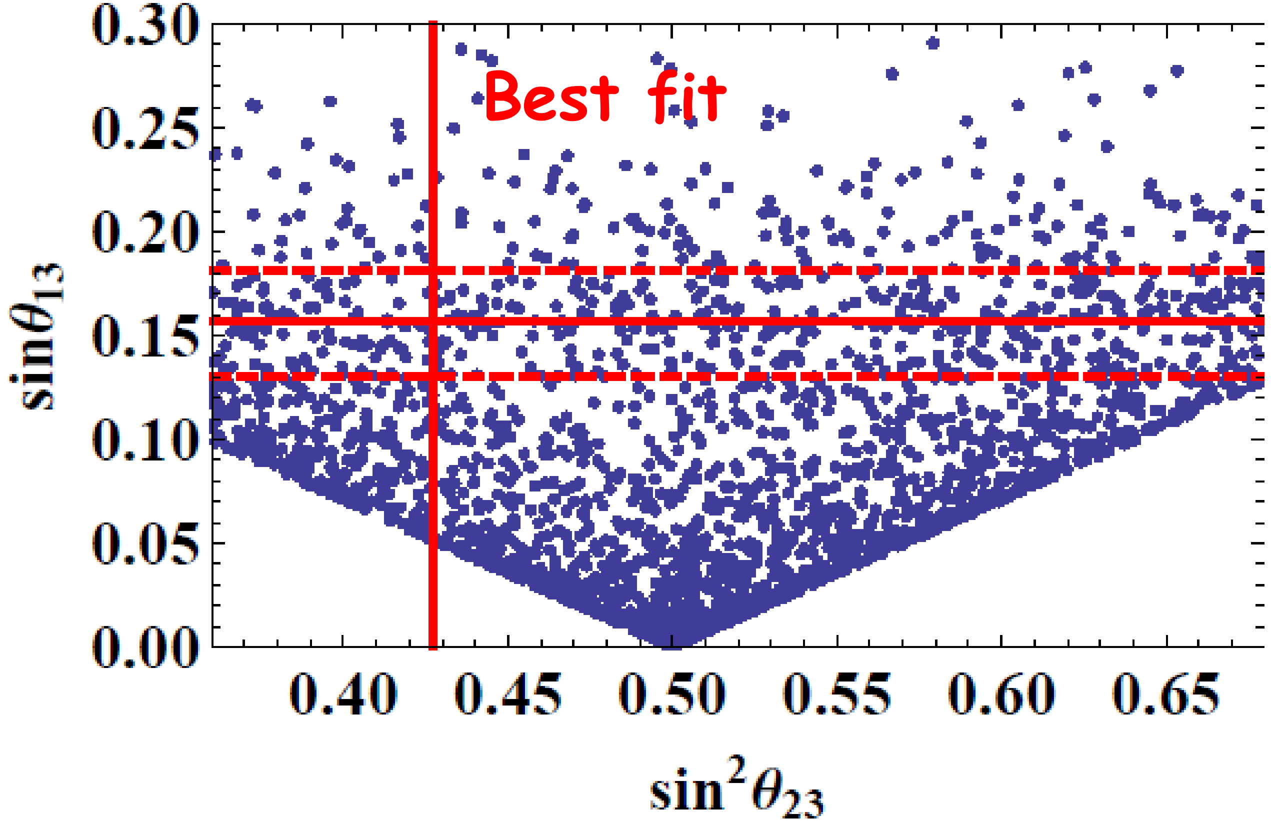
$$\sin\theta_{13} \simeq \frac{\lambda}{\sqrt{2}}$$

$\sin\theta_{23}$ **depends on phases !**

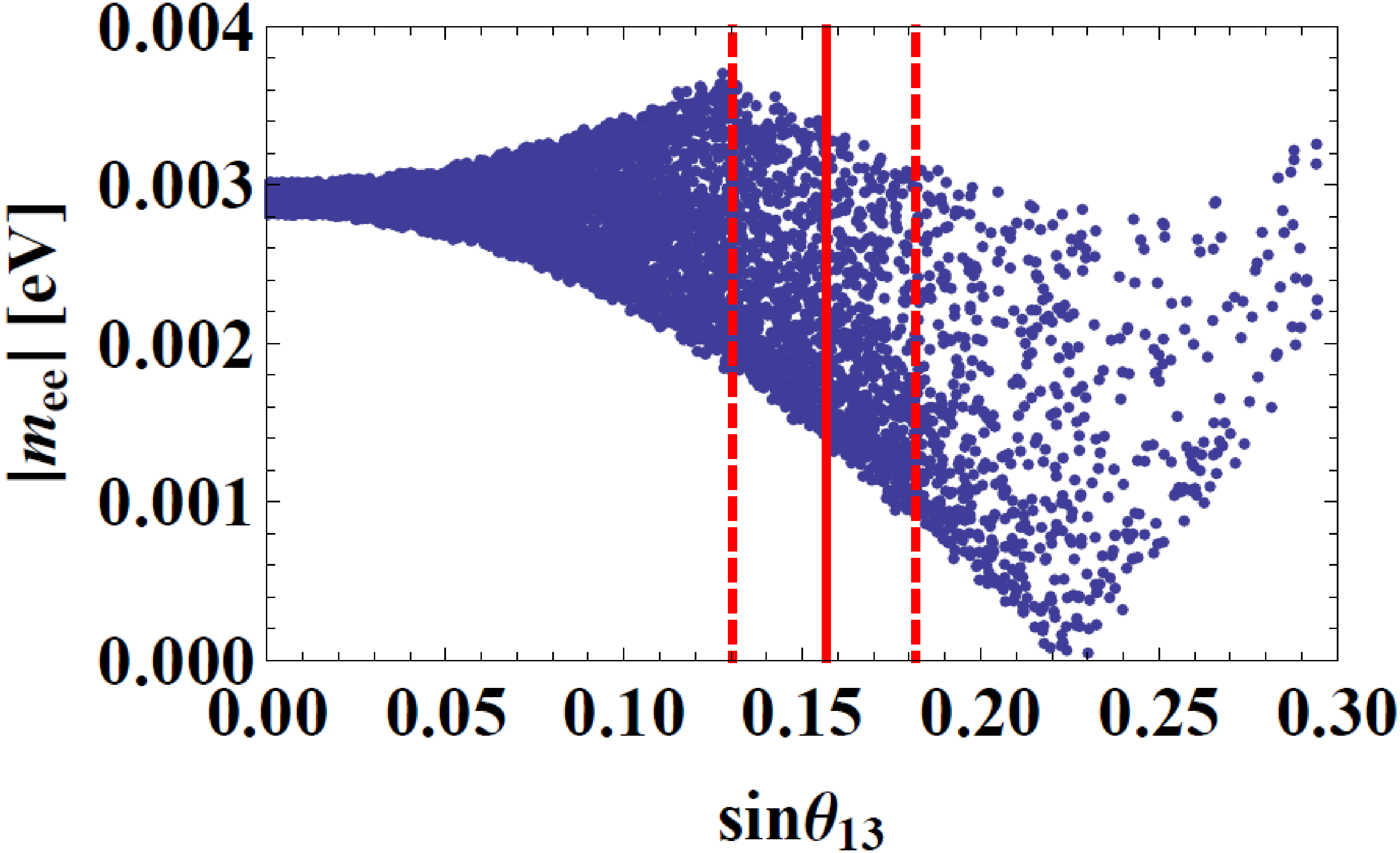
$$\sin\theta_{23} = \frac{1}{\sqrt{2}} + \frac{\sqrt{6}}{2}\lambda e^{i\phi}$$



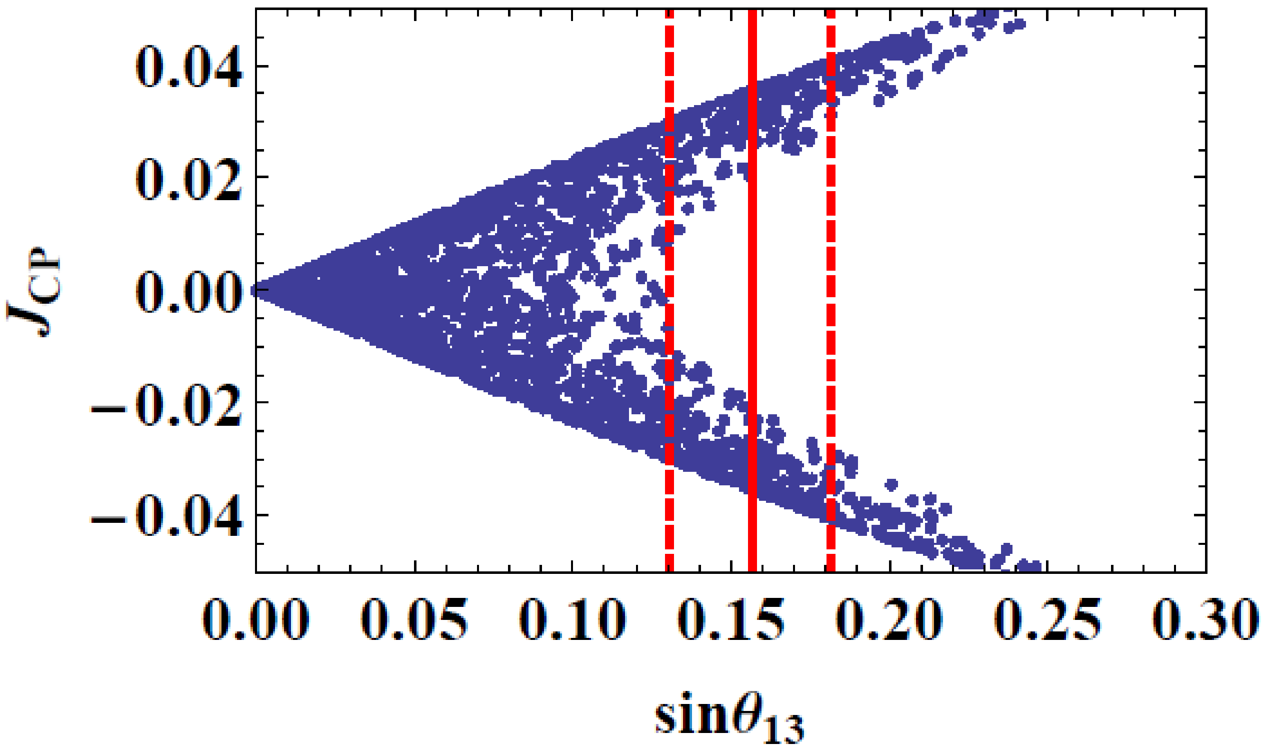
Best fit
0.30
0.25
0.20
0.15
0.10
0.05
0.00
0.28
0.30
0.32
0.34
0.36
$\sin\theta_{13}$
$\sin^2\theta_{12}$



0.30
0.25
0.20
0.15
0.10
0.05
0.00
0.40
0.45
0.50
0.55
0.60
0.65
$\sin\theta_{13}$
$\sin^2\theta_{23}$
Best fit



0.004
0.003
0.002
0.001
0.000
$|m_{ee}|$ [eV]
0.00
0.05
0.10
0.15
0.20
0.25
0.30
$\sin\theta_{13}$



0.04
0.02
0.00
−0.02
−0.04
J_{CP}
0.00
0.05
0.10
0.15
0.20
0.25
0.30
$\sin\theta_{13}$

5 Summary

Impact of large $\theta_{13} \fallingdotseq 0.15$

☆A4 flavor model is easily modified with large θ_{13}.

☆There is relations among mixing angles for the specific textures which is testable in the future.

☆Mixing angle depends on the neutrino masses.

We should consider the origin of neutrino mass spectrum !

Why $\sqrt{\frac{\Delta m^2_{\rm sol}}{\Delta m^2_{\rm atm}}} = 0.160 \sim 0.196 = \mathcal{O}(\lambda)$?