

# Radiative Inverse Seesaw and Dark Matter

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1. The Inverse Seesaw Mechanism
2. Two Loop Radiative Inverse Seesaw
3. Dark Matter in Radiative Inverse Seesaw
4. Discussions and Conclusions

Collaborators: Gang Guo and Guan-Nan Li, arXiv:1207.6308 (to appear in JHEP)

Seesaw mechanism is one of the popular mechanism which can explain why neutrino masses are so much lighter than their charged lepton partners.

Where is the scale of seesaw, can it be close to the scale where LHC can say something? Usually larger than a few TeV

Inverse seesaw can further bring down the scale. If radiatively realized, the scale can be even lower. LHC may be able to say something about it.

Is neutrino mass generation mechanism related another fundamental problem of modern physics, dark matter? May be!

I will try convince you that indeed it is possibility to realize the above.

# 1. The Inverse Seesaw Mechanism

## The Seesaw Mechanism

The Seesaw mechanism refers to the neutrino mass matrix of the form

$$L_m = -\frac{1}{2}(\nu_L^c, \nu_R) \begin{pmatrix} 0 & m_D \\ m_D^T & M_R \end{pmatrix} \begin{pmatrix} \nu_L \\ \nu_R^c \end{pmatrix}.$$

For one generation, if  $M_R \gg m_D$ , the eigenmasses are

$$m_\nu \approx -m_D M_R^{-1} m_D^T, \quad m_N \approx M_R$$

A very nice way to explain why light neutrino masses are so much lighter than their charged lepton partners.

$$\begin{aligned} m_{\nu_1} &= m_e^2/M_R \rightarrow M_R = m_e^2/m_{\nu_1}. \text{ For } m_{\nu_1} = 0.1\text{eV}, M_R = 2.5\text{TeV} \\ m_{\nu_2} &= m_\mu^2/M_R \rightarrow M_R = m_\mu^2/m_{\nu_1}. \text{ For } m_{\nu_2} = 0.1\text{eV}, M_R = 10^8\text{GeV} \\ m_{\nu_3} &= m_\tau^2/M_R \rightarrow M_R = m_\tau^2/m_{\nu_1}. \text{ For } m_{\nu_3} = 0.1\text{eV}, M_R = 3 \times 10^{10}\text{GeV}. \end{aligned}$$

# Inverse Seesaw

The inverse seesaw neutrino mass matrix  $M_\nu$  is the mass matrix resulted from the effective Lagrangian

$$L_m = -\bar{\nu}_L m_D N_R - \bar{N}_L M N_R - \frac{1}{2} \bar{N}_R^c \mu_R N_R - \frac{1}{2} \bar{N}_L \mu_L N_L^c + h.c.$$

where  $\nu_L$  is the light active neutrino,  $N_{L,R}$  are heavy neutrinos.

In the bases  $(\nu_L^c, N_R, N_L^c)^T$ ,  $M_\nu$  is given by

$$M_\nu = \begin{pmatrix} 0 & m_D & 0 \\ m_D^T & \mu_R & M^T \\ 0 & M & \mu_L \end{pmatrix}$$

With the hierarchy  $\mu_L \sim \mu_R \ll m_D \ll M$ , the light neutrino mass matrix  $m_\nu$ , defined by  $L_{mass} = -(1/2)\nu_L m_\nu \nu_L^c$ , to order  $(m_D/M)^2$  is given by

$$m_\nu = m_D M^{-1} \mu_L (M^{-1})^T m_D^T.$$

**M can be much lower than that in usual Seesaw.**

# An example of Inverse Seesaw model

$$M_\nu = \begin{pmatrix} 0 & m_D & 0 \\ m_D^T & \mu_R & M^T \\ 0 & M & \mu_L \end{pmatrix}$$

introduce a leptonic doublet  $D_{L,R} : (2, -1/2)$

to realize the inverse seesaw.

along with a singlet  $S$  and a triplet  $\Delta : (3, -1)$

a global  $U(1)_D$  symmetry to distinguish  $D_L$  and  $L_L$ .

Under this symmetry  $D_{L,R} \rightarrow \exp[i\alpha_D]D_{L,R}$ ,  $S \rightarrow \exp[-i\alpha_D]S$ ,  $\Delta \rightarrow \exp[2i\alpha_D]\Delta$ ,

and other fields do not transform.

$$L_D = -\bar{L}_L Y_D D_R S - \bar{D}_L M D_R - \frac{1}{2} \bar{D}_L Y_L D_L^c \Delta - \frac{1}{2} \bar{D}_R^c Y_L D_R \Delta^\dagger + h.c.$$

If both  $S$  and  $\Delta$  both develop non-zero vev's, the inverse seesaw mechanism is realized.

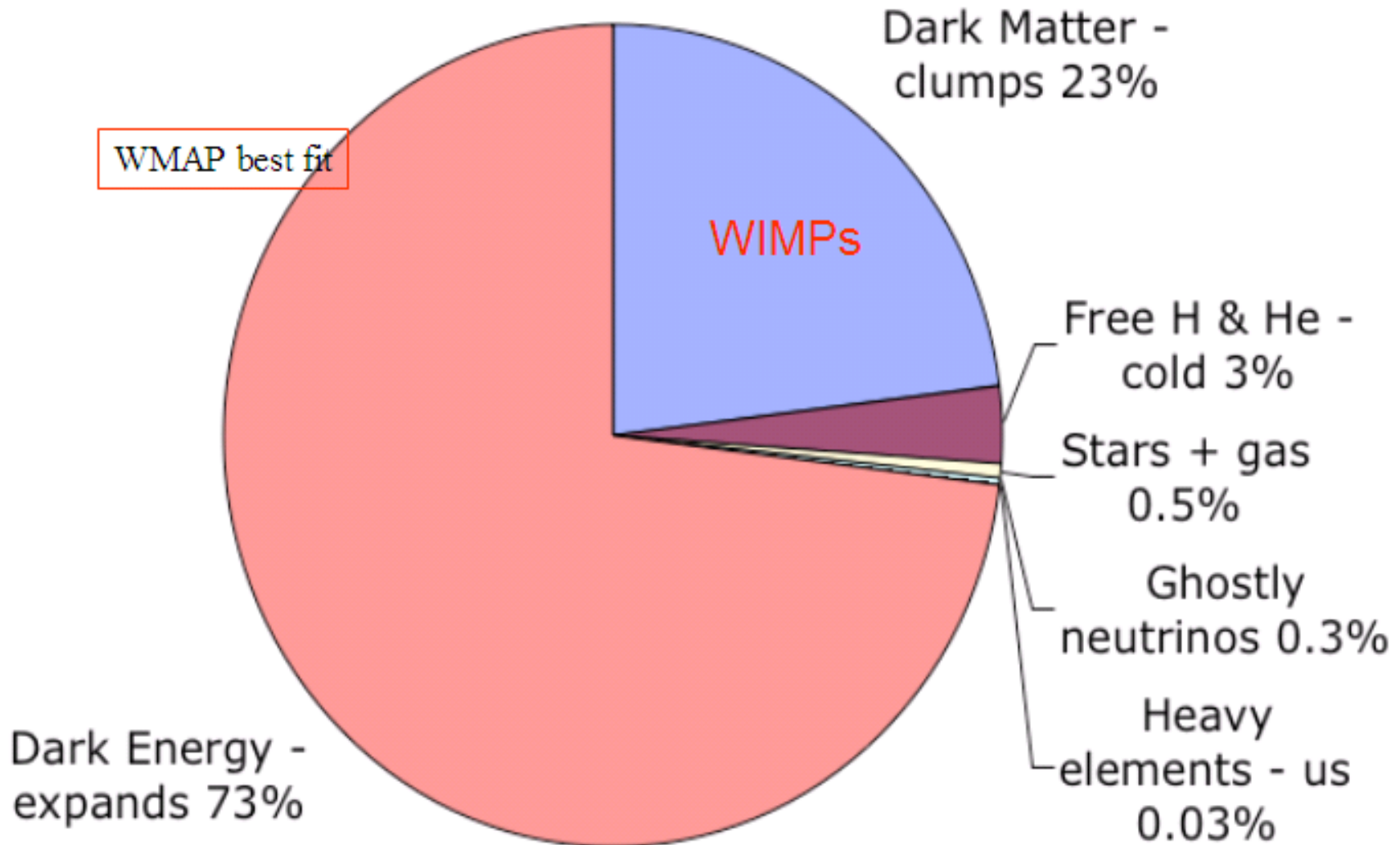
This model, however, will have a Goldstone boson due to breaking of the global  $U(1)_D$  symmetry which may be problematic.

To avoid the existence of a Goldstone boson in the theory, extension is needed.

Introduce more fields, or softly break  $U(1)_D$  by adding a term of the form  $H\Delta H$

# Link Neutrino Physics to Dark Matter physics

## Composition of the Cosmos



Usual seesaw,  $N$  (heavy neutrino) mixes with light  $\nu_L$ , although the mixing is small of order  $\sqrt{m_\nu/m_N}$ , but it is still too large to keep  $N$  long lived to be candidate for dark matter (smaller mixing may do the job with small  $N$  mass),  $N$  cannot be WIMP dark matter candidate.

Need additional symmetry to guarantee the stability of  $N$ .

# Radiative seesaw and dark matter

E. Ma, PRD73:077301,2006 introduce right handed neutrinos  $N_R$  and a new Higgs doublet  $\eta$

$$SU(2)_L \times U(1)_Y \times \bar{X} Z_2$$

$$(\nu_i, l_i) \sim (2, -1/2; +), \quad l_i^c \sim (1, 1; +), \quad N_i \sim (1, 0; -),$$

$$(\phi^+, \phi^0) \sim (2, 1/2; +), \quad (\eta^+, \eta^0) \sim (2, 1/2; -).$$

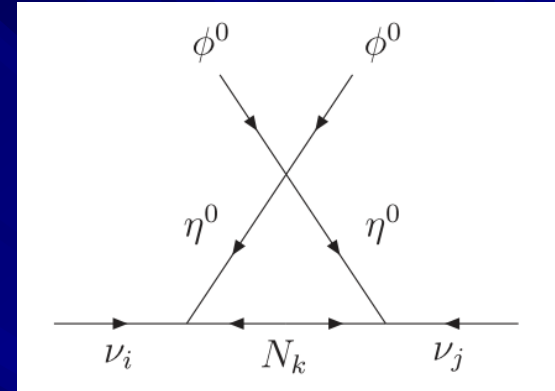
$$\mathcal{L}_Y = f_{ij}(\phi^- \nu_i + \bar{\phi}^0 l_i^c) + h_{ij}(\nu_i \eta^0 - l_j \eta^+) N_j + \text{H.c.}$$

$$\frac{1}{2} M_i N_i N_i + \text{H.c.} \quad \frac{1}{2} \lambda_5 (\Phi^\dagger \eta)^2 + \text{H.c.}$$

under  $Z_2$

$$N_R \rightarrow -N_R$$

$$\eta \rightarrow -\eta$$



$$(\mathcal{M}_\nu)_{ij} = \sum_k \frac{h_{ik} h_{jk} M_k}{16\pi^2} \left[ \frac{m_R^2}{m_R^2 - M_k^2} \ln \frac{m_R^2}{M_k^2} - \frac{m_I^2}{m_I^2 - M_k^2} \ln \frac{m_I^2}{M_k^2} \right],$$

$$\text{If } m_0^2 \simeq M_k^2, \text{ then } (\mathcal{M}_\nu)_{ij} \simeq \frac{\lambda_5 v^2}{16\pi^2} \sum_k \frac{h_{ik} h_{jk}}{M_k}.$$

$\eta$  no vev,  $Z_2$  not broken. No usual seesaw neutrino masses. One loop will generate them. If neutral  $\eta$  is the lightest particle, it can be DM. If one of the  $N$  is the lightest, it is the DM.

DM mass related to radiative seesaw neutrino mass scale.

Radiative Seesaw can further lower the seesaw scale and having dark matter candidate as a bonus!

What about Radiative inverse seesaw with dark matter?



## 2. Two Loop Radiative Inverse Seesaw

Besides the problem of Goldstone boson, the example model given earlier does not have dark matter candidates.

To avoid the existence of Goldstone bosons, do not let relevant scalars to have non-zero vevs (without adding soft breaking terms). It implies that one cannot have vev for  $S$  and  $\Delta$ . The global  $U(1)$  symmetry is not broken.

Bonus: Unbroken symmetry  $\rightarrow$  guarantee the existence of stable particles  $\rightarrow$  dark matter candidate.

Problem: light neutrinos remains massless.

Solution: Radiatively generate neutrino masses.

# The $U(1)_D$ model

introduce a leptonic doublet  $D_{L,R} : (2, -1/2)$  to realize the inverse seesaw.

along with a singlet  $S$  and a triplet  $\Delta : (3, -1)$

a global  $U(1)_D$  symmetry to distinguish  $D_L$  and  $L_L$ .

Under this symmetry  $D_{L,R} \rightarrow \exp[i\alpha_D]D_{L,R}$ ,  $S \rightarrow \exp[-i\alpha_D]S$ ,  $\Delta \rightarrow \exp[2i\alpha_D]\Delta$ ,

and other fields do not transform.

$$L_D = -\bar{L}_L Y_D D_R S - \bar{D}_L M D_R - \frac{1}{2} \bar{D}_L Y_L D_L^c \Delta - \frac{1}{2} \bar{D}_R^c Y_L D_R \Delta^\dagger + h.c.$$

If both  $S$  and  $\Delta$  both develop non-zero vev's, the inverse seesaw mechanism is realized.

This model, however, will have a Goldstone boson due to breaking of the global  $U(1)_D$  symmetry which may be problematic.

To avoid the existence of a Goldstone boson in the theory, extension is needed.

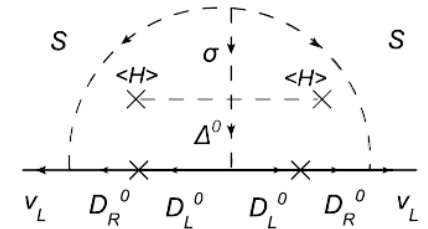
# Cure:

introduce another singlet  $\sigma$  which transforms under the  $U(1)_D$  as  $\sigma \rightarrow \exp[2i\alpha_D]\sigma$ .

The allowed renormalizable terms in the potential  $V_D$  are given by

$$V_D = -\mu_H^2 H^\dagger H + \lambda_H (H^\dagger H)^2 + \mu_S^2 S^\dagger S + \lambda_S (S^\dagger S)^2 + \mu_\sigma^2 \sigma^\dagger \sigma + \lambda_\sigma (\sigma^\dagger \sigma)^2 \\ + \mu_\Delta^2 \Delta^\dagger \Delta + \lambda_\Delta^\alpha (\Delta^\dagger \Delta \Delta^\dagger \Delta)_\alpha + \sum_{ii} \lambda_{ij} i^\dagger i j^\dagger j + (\mu_{S\sigma} S^2 \sigma + \lambda_{\Delta\sigma H} H \Delta \sigma^\dagger H + h.c.)$$

where the sum  $\sum_{ij}$  is over all possible  $i$  and  $j$  to be one of the  $H$ ,  $S$ ,  $\sigma$  and  $\Delta$ .



## The two loop diagram break lepton number, but conserve $U(1)_D$ number

In the above  $\mu_i^2$  are all larger than zero. The potential only allows  $H$  to have a non-zero vev  $v_H$ . The theory after spontaneous symmetry breaking from  $SU(2)_L \times U(1)_Y$  to  $U(1)_{em}$ , the  $U(1)_D$  global symmetry is unbroken.

At the tree level, light neutrinos are massless.

Non-zero neutrino masses can only be generated at two loop level

This is similar to the Babu-Zee model two loop neutrino mass generation

but with the light charged leptons in the loop replaced by new heavy particles.

in the bases where  $M$  is diagonalized  $m_\nu^{ij} = \frac{v_H Y_D^{ik} (\lambda_{\Delta\sigma H} \mu_{S\sigma} Y_L^{kl}) Y_D^{jl} v_H}{M_{kk}^2} \kappa_{kl}$ ,

$\kappa_{kl}$  is defined as:  $\kappa_{kl} = \delta_{kl} \frac{1}{2(4\pi)^4} \frac{1}{(1 - m_S^2/M_{kk}^2)^2} [g(m_{\phi_1}, m_S, m_S) - g(m_{\phi_1}, M_{kk}, m_S) - g(m_{\phi_1}, m_S, M_{kk}) + g(m_{\phi_1}, M_{kk}, M_{kk})]$

$$g(m_1, m_2, m_3) = \int_0^1 dx [1 + Sp(1 - \mu^2) - \frac{\mu^2}{1 - \mu^2} \log \mu^2] \quad Sp(z) = - \int^z \frac{\ln(1-t)}{t} dt$$

$$\text{with } \mu^2 = \frac{ax+b(1-x)}{x(1-x)}, a = \frac{m_2^2}{m_1^2}, b = \frac{m_3^2}{m_1^2}.$$

$$M_\nu = \begin{pmatrix} 0 & m_D & 0 \\ m_D^T & \mu_R & M^T \\ 0 & M & \mu_L \end{pmatrix}$$

If one identifies, effectively,

$$m_D = Y_D v_H, M = \text{diag}(M_{ii}) \text{ and } \mu_L = (\mu_L^{ij}) \quad \mu_L^{ij} = (\lambda_{\Delta\sigma H} \mu_{s\sigma}) Y_L^{ij} \kappa_{ij},$$

the light neutrino mass matrix is effectively an inverse seesaw mass form.

We therefore refer this as radiative inverse seesaw mechanism.

# Fitting to data

## Neutrino mixing and mass differences from Global data fitting

Fogli et al, arXiv: 1205:5204

Results of the global  $3\nu$  oscillation analysis, in terms of best-fit values and allowed  $1$ ,  $2$  and  $3\sigma$  ranges for the  $3\nu$  mass-mixing parameters. We remind that  $\Delta m^2$  is defined herein as  $m_3^2 - (m_1^2 + m_2^2)/2$ , with  $+\Delta m^2$  for NH and  $-\Delta m^2$  for IH.

Parameter	Best fit	$1\sigma$ range	$2\sigma$ range	$3\sigma$ range
$\delta m^2/10^{-5} \text{ eV}^2$ (NH or IH)	7.54	7.32 – 7.80	7.15 – 8.00	6.99 – 8.18
$\sin^2 \theta_{12}/10^{-1}$ (NH or IH)	3.07	2.91 – 3.25	2.75 – 3.42	2.59 – 3.59
$\Delta m^2/10^{-3} \text{ eV}^2$ (NH)	2.43	2.33 – 2.49	2.27 – 2.55	2.19 – 2.62
$\Delta m^2/10^{-3} \text{ eV}^2$ (IH)	2.42	2.31 – 2.49	2.26 – 2.53	2.17 – 2.61
$\sin^2 \theta_{13}/10^{-2}$ (NH)	2.41	2.16 – 2.66	1.93 – 2.90	1.69 – 3.13
$\sin^2 \theta_{13}/10^{-2}$ (IH)	2.44	2.19 – 2.67	1.94 – 2.91	1.71 – 3.15
$\sin^2 \theta_{23}/10^{-1}$ (NH)	3.86	3.65 – 4.10	3.48 – 4.48	3.31 – 6.37
$\sin^2 \theta_{23}/10^{-1}$ (IH)	3.92	3.70 – 4.31	$3.53 - 4.84 \oplus 5.43 - 6.41$	3.35 – 6.63
$\delta/\pi$ (NH)	1.08	0.77 – 1.36	—	—
$\delta/\pi$ (IH)	1.09	0.83 – 1.47	—	—

# Example for neutrino mss solutions

choose  $Y_L$  diagonal and  $Y_D = y_D U_{PMNS}$ ,

For the normal hierarchy, choose  $Y_L = \text{diag}(1, 1.05, 2.01) \times 10^{-2}$ ,  $y_D = 10^{-2}$ ,

$\lambda_{\Delta\sigma H} = 0.1$ ,  $\mu_{S\sigma} = 100\text{GeV}$ ,  $m_\phi = 300\text{GeV}$ ,  $m_S = 150\text{GeV}$ ,  $M_{ii} = 500\text{GeV}$ ,

the three neutrino mass  $2.804 \times 10^{-2}\text{eV}$ ,  $2.936 \times 10^{-2}\text{eV}$ ,  $5.636 \times 10^{-2}\text{eV}$ ,

For inverted hierarchy case,  $Y_L = \text{diag}(1.297, 1.317, 0.100) \times 10^{-2}$ ,

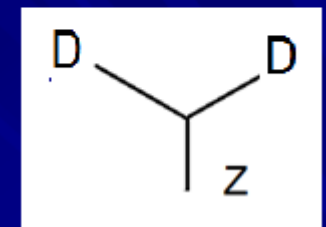
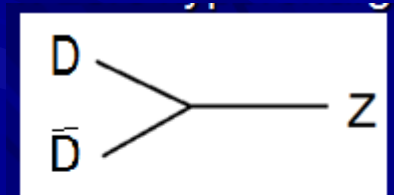
the neutrino masses will be  $4.90 \times 10^{-2}\text{eV}$ ,  $4.98 \times 10^{-2}\text{eV}$ ,  $3.78 \times 10^{-3}\text{eV}$ ,

### 3. Dark Matter in Radiative Inverse Seesaw

In the  $U(1)_D$  models, since the global symmetry  $U(1)_D$  is not broken, there is at least one stable new particle in the model. They are potential dark matter candidates.

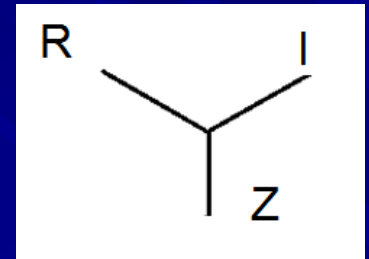
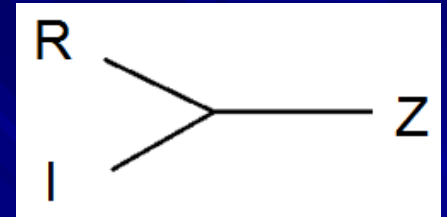
The  $U(1)_D$  model:  $S$ ,  $\sigma$ , and neutral component in  $D$  and  $\Delta$ .

The neutral component in  $D$  has hypercharge and has a non-zero coupling to  $Z$ . Can fit relic density through s-channel  $Z$  exchange, but leads to a too large direct detection cross section through t-channel  $Z$  exchange. Out!



Neutral components in  $\Delta$  in both models also have non-zero hypercharge and couple to  $Z$ - $\text{Re}(\Delta)$ - $\text{Im}(\Delta)$ .

If there is a mass splitting  $\delta = m_R - m_I$  larger than 100 KeV or so between  $\text{Re}(\Delta)$  and  $\text{Im}(\Delta)$ , It is possible to satisfy relic density and direct detection constraints, through inelastic dark matter idea.



But in both models, there is no mass splitting,  $\delta = 0$ , inelastic dark matter mechanism is ineffective. Out!

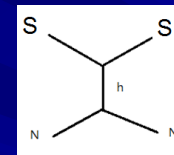
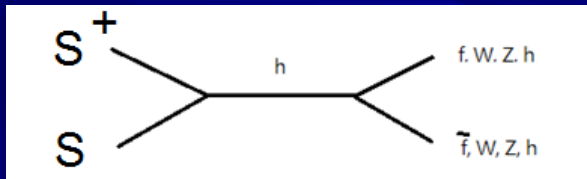
$\sigma$  mixes with neutral component in  $\Delta$ , if mixing is large, ruled out. Also, there is a  $Z_2$  unbroken symmetry for  $S \rightarrow -S$ ,  $D \rightarrow -D$  and others do not transform,  $S$  or neutral  $D$  is also stable. Two component dark matter,  $\sigma$  and  $S$ . ( $D$  is out due to non-zero hypercharge)



$S$  in  $U(1)_D$  model can be a natural dark matter candidate. It is a complex singlet dark matter model. It is a Higgs portal dark matter model. The relic density and direct detection is through s- and t-channel higgs  $h$  exchange, respectively, via,

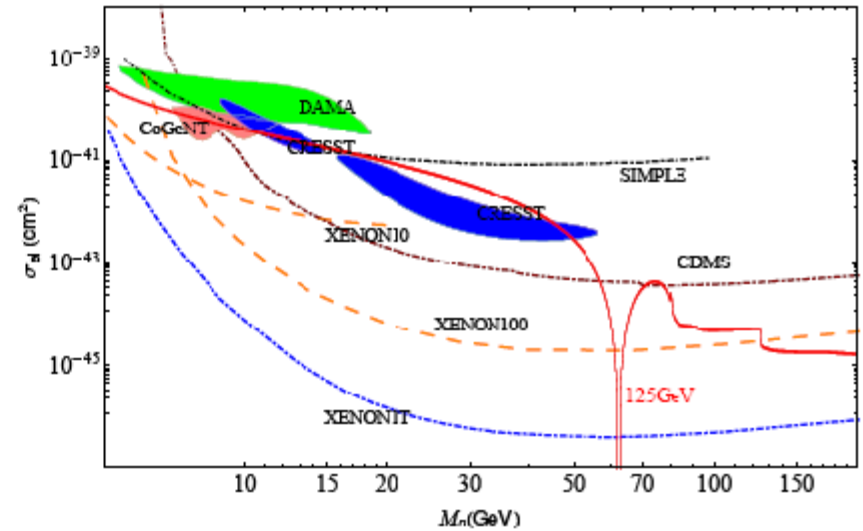
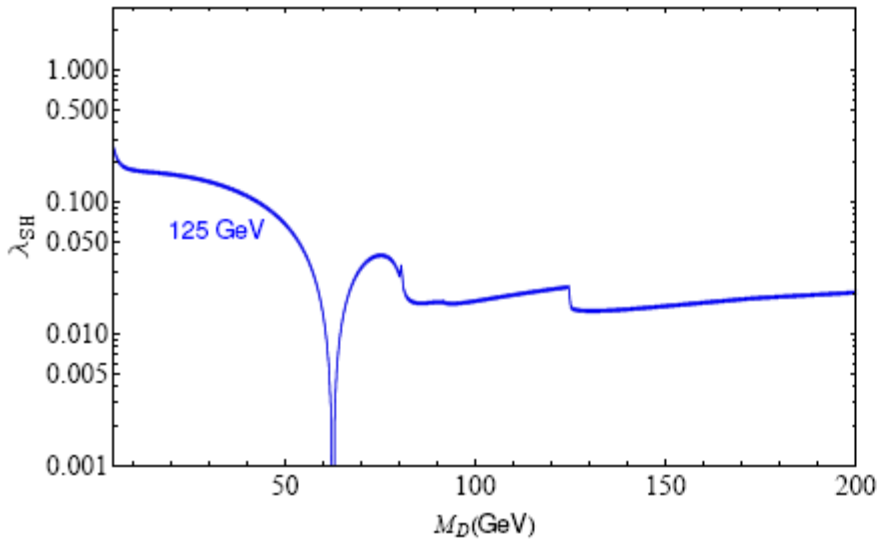
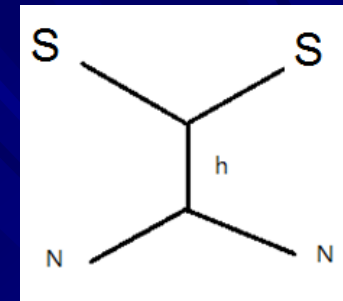
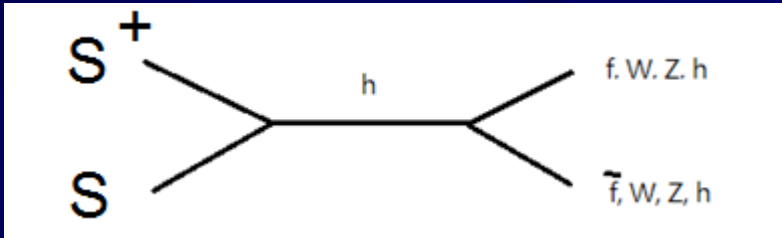
$$\lambda_{SH} S^\dagger S H^\dagger H = \frac{1}{2} \lambda_{SH} (v_H^2 + 2v_H h + hh) S S^\dagger .$$

The first term will modify the mass of  $S$  from  $\mu_S^2$  to  $M_D^2 = \mu_S^2 + \lambda_{SH} v_H^2 / 2$ .



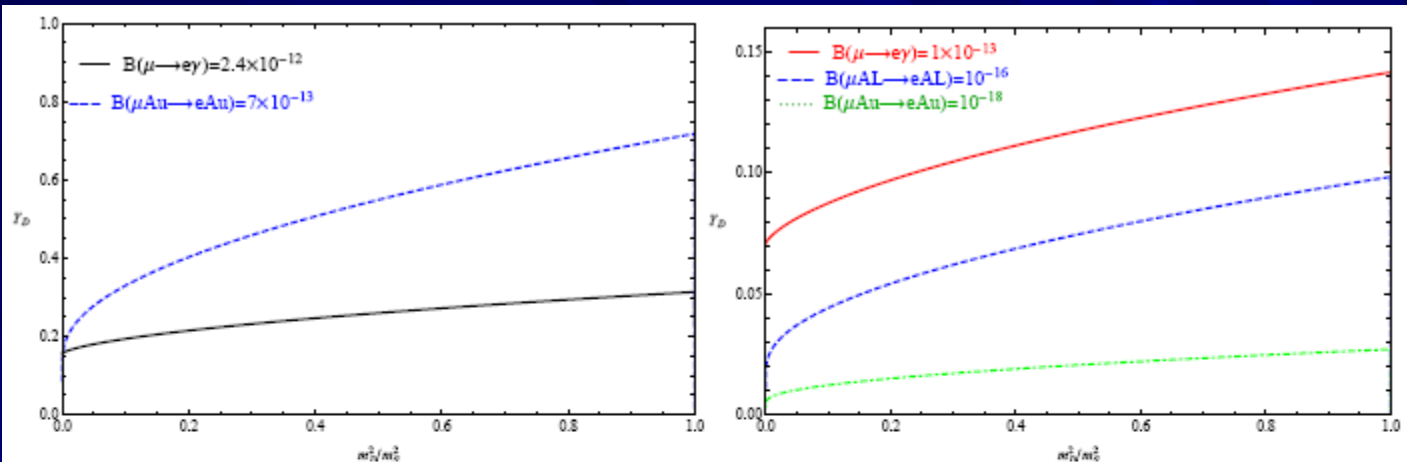
Real and imaginary parts of  $S$  have degenerate mass.

Higgs doublet tree level couplings to SM particles not modified.  
 $h$  is almost SM Higgs. Take  $m_h = 125$  GeV for analysis.



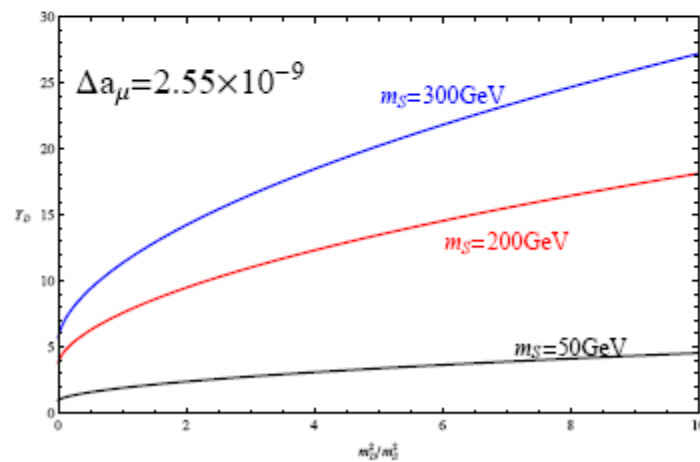
Consistent with all data, (left: relic density  $\Omega h^2 = 0.11$ , right: direct search) if dark matter is about half of Higgs mass or larger than 130 GeV

# Large Yukawa coupling YD, may induce large leptonic FCNC at one loop level.



Current experimental bounds

Future prospects



g-2 constants

# 4. Discussions

## Several other new signatures

May have enhanced:  $h \rightarrow \gamma\gamma$ , due to charged  $\Delta$  and  $\eta$  particle exchanges at one loop.

New low masses of charge components, can be produced and detected by their decays into a charged SM particles and dark matter carrying large missing transverse energy.

## Conclusions

Possible to construct models with two loop radiative inverse seesaw neutrino masses and at the same time with viable dark matter candidate.