Dynamical Yukawas

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Dynamical life...

1) μ-->e conversion: sensitivity to singlet fermions in nature

(Alonso, Dhen, Hambye, Gavela last week)

2) Dynamical Yukawas

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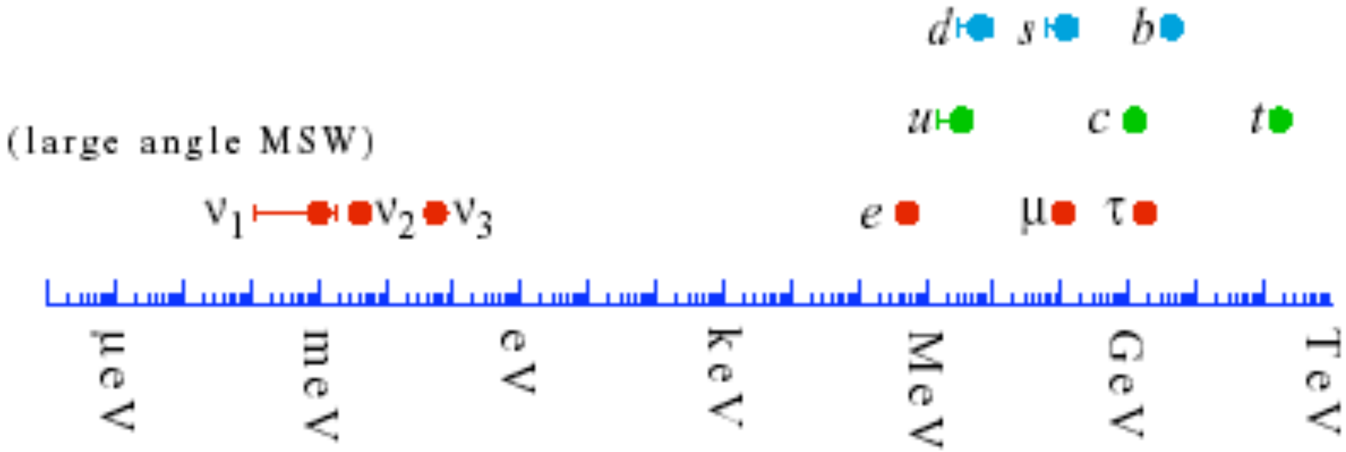
(Alonso, D.Hernandez, Gavela, Merlo 2012)

Neutrino light on flavour ?



Neutrinos lighter because Majorana?

Leptons

$$V_{PMNS} = \begin{pmatrix} 0.8 & 0.5 & \sim 9^o \\ -0.4 & 0.5 & -0.7 \\ -0.4 & 0.5 & +0.7 \end{pmatrix}$$

Quarks

$$V_{CKM} = \begin{pmatrix} \sim 1 & \lambda & \lambda^3 \\ \lambda & \sim 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & \sim 1 \end{pmatrix} \quad \lambda \sim 0.2$$

Why so different?

Neutrino are optimal windows into the exotic -dark- sectors

* Can mix with new neutral fermions, heavy or light

* Interactions not obscured by strong and e.m. ones

Dark portals

Only two singlet combinations in SM with d < 4:

$H^+ H$ Bosonic

$\overline{L}\ H$ Fermionic

Any hidden sector, singlet under SM, can couple to the dark portals

SM portals into Dark Matter

The only possible SM-**DM** renormalizable couplings (d<= 4) are:

* To scalar DM: **Higgs** portal to **DM** scalar singlets **S**

$$\mathcal{L}_{SM} \ldots\ldots + \lambda\,(|\mathbf{H}|^2 - v^2)^2 + \lambda' |\mathbf{H}|^2 \mathbf{S}^2 + \mu\,|\mathbf{H}|^2 \mathbf{S} \ldots$$

Text

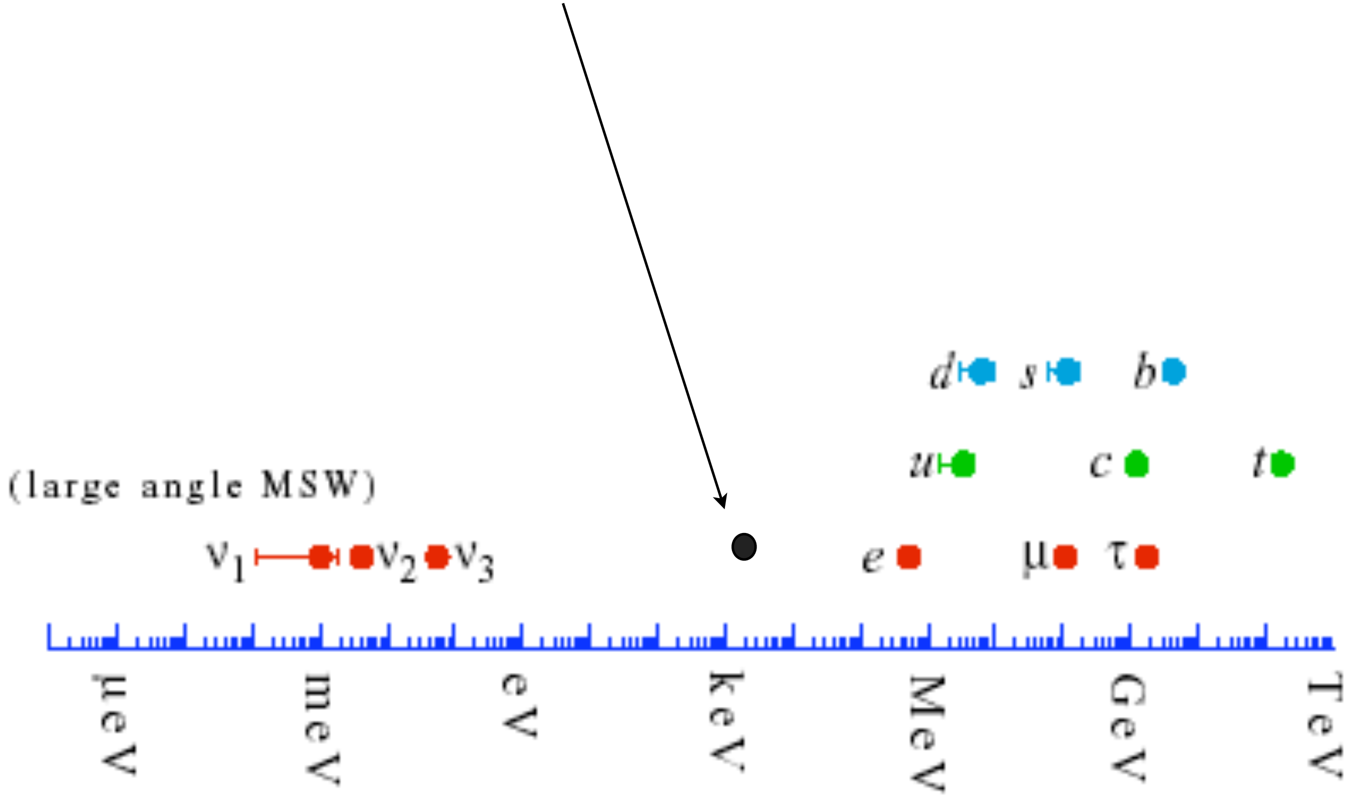
* To fermionic DM: **Lepton-Higgs** portal to **DM** fermion singlets $\boldsymbol{\Psi}$

Yukawa coupling

$$\mathcal{L}_{SM} \ldots\ldots + Y\,(\bar{\mathbf{L}}\,\mathbf{H}\boldsymbol{\Psi})$$

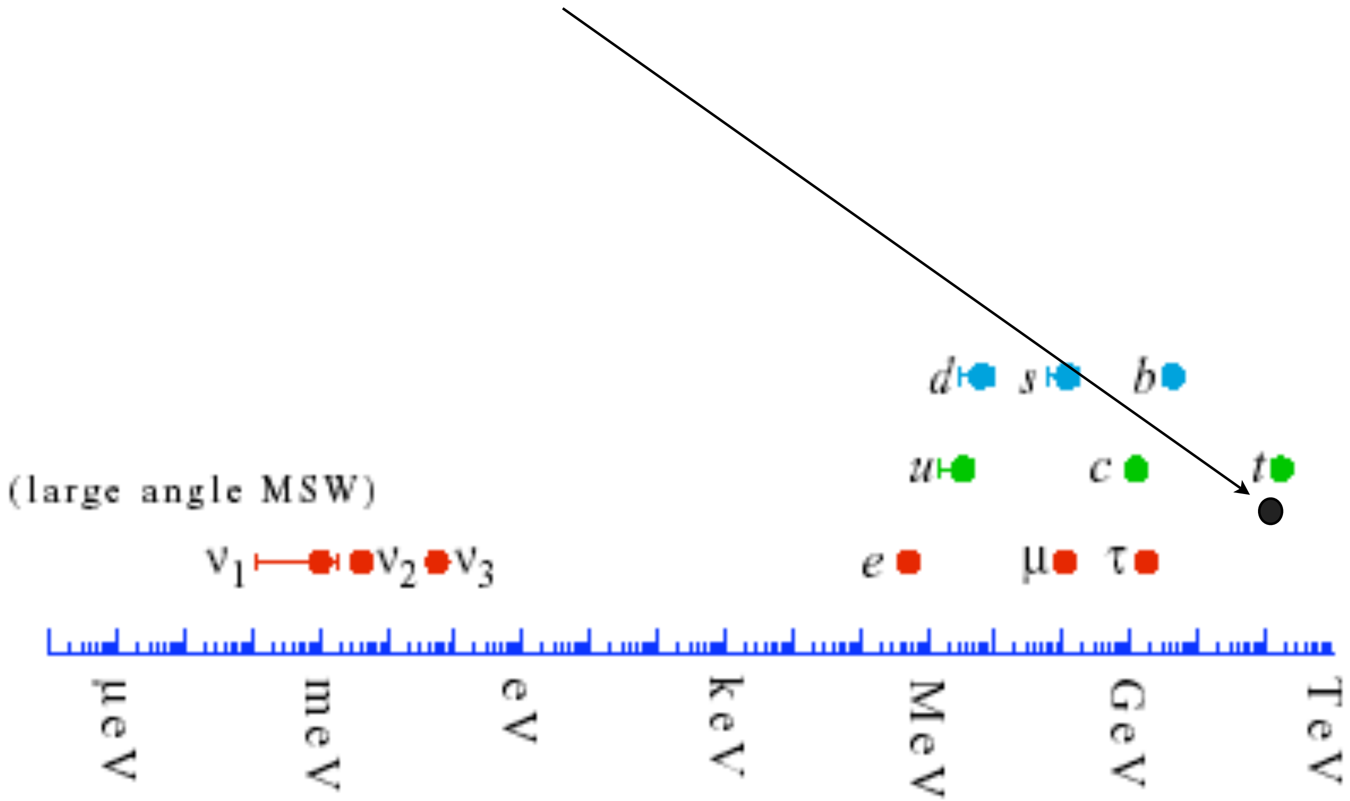
fermion singlets $\boldsymbol{\Psi}$ = "right-handed" neutrino

DARK FLAVOURS ?



.... they can be fermions

DARK FLAVOURS ?



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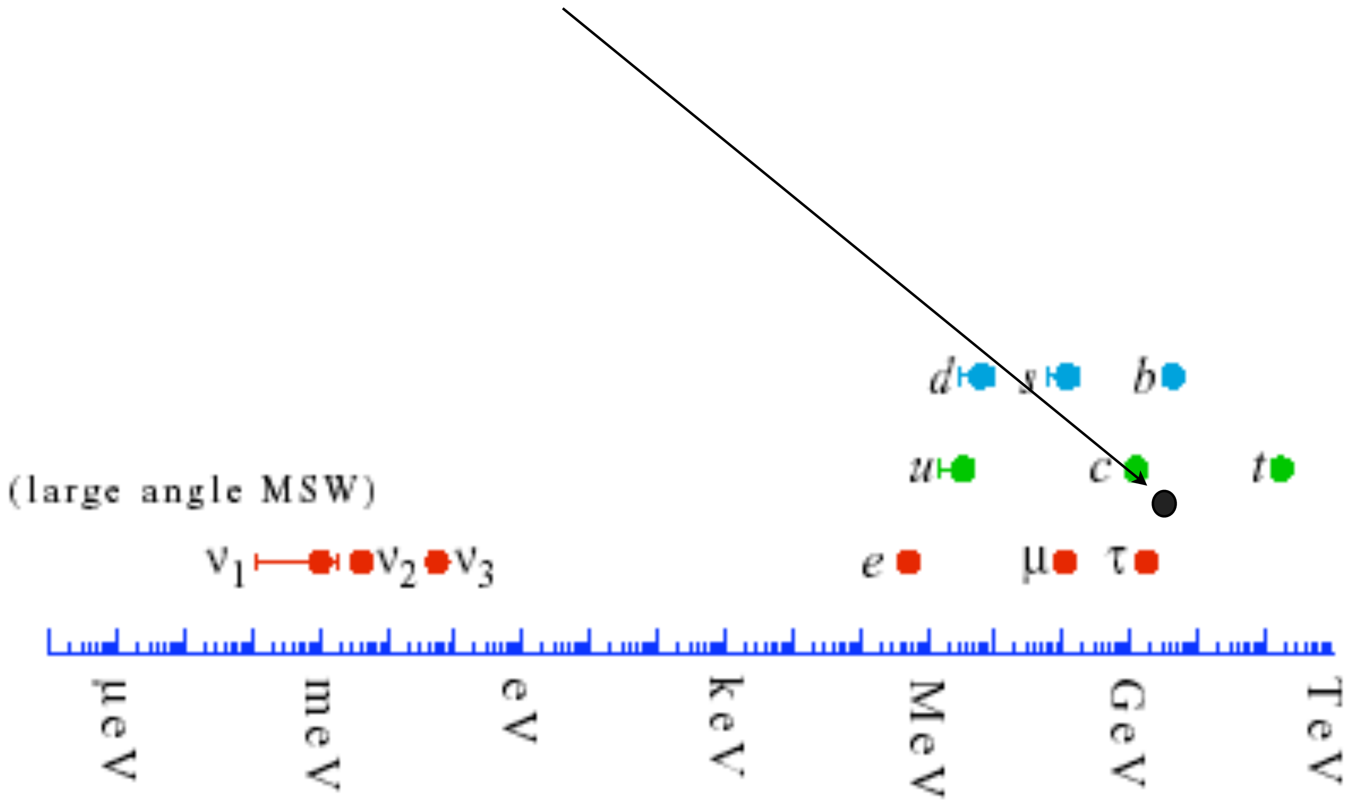

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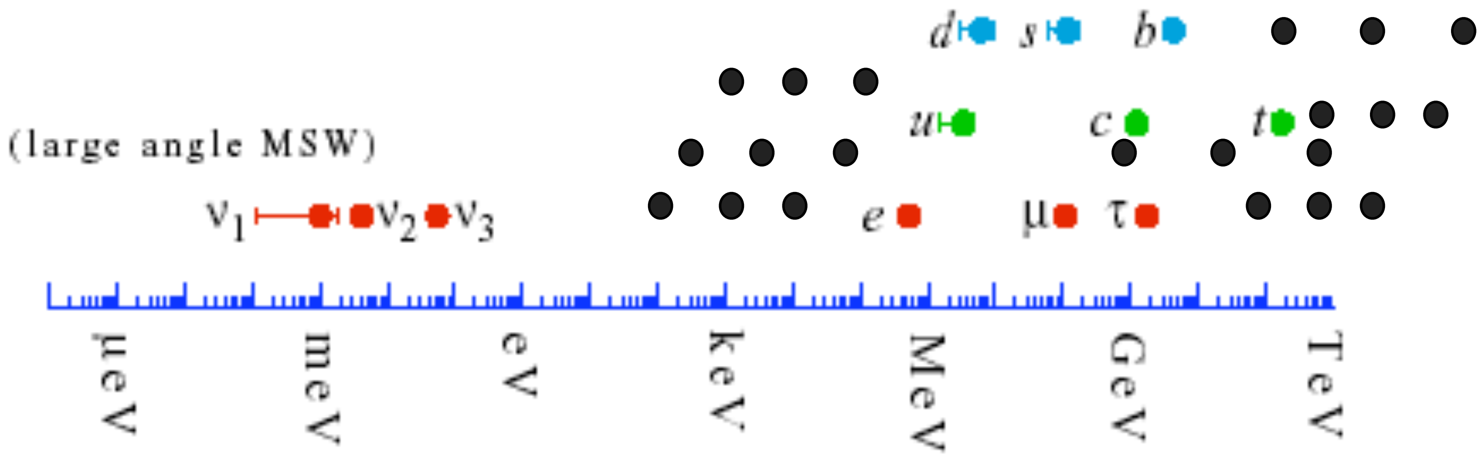

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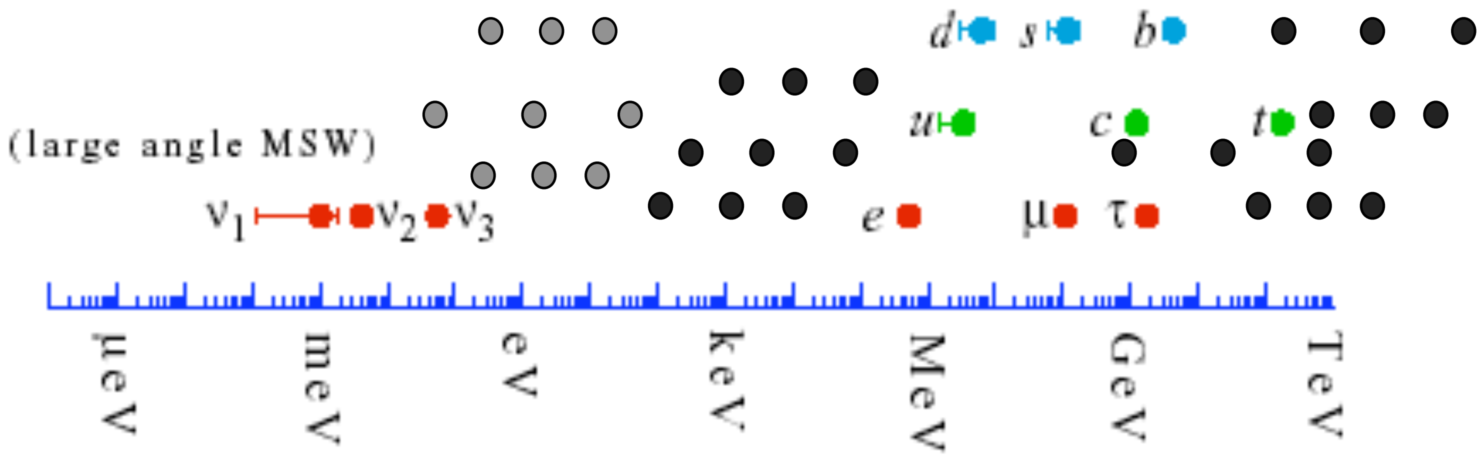
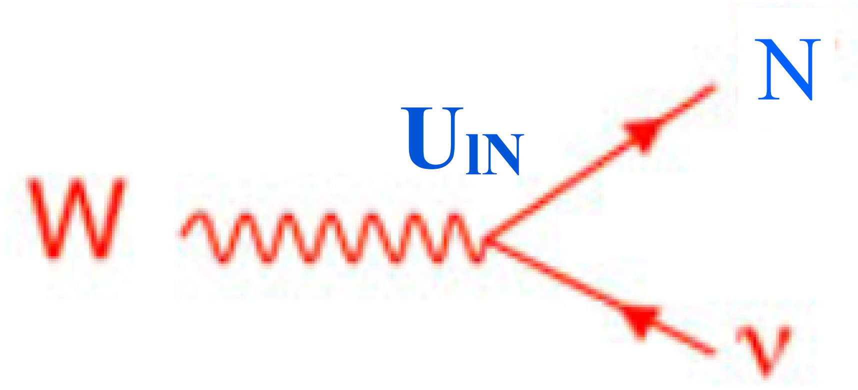


Assume that singlet fermion(s) N exists in nature



?

What are the limits on their mass m_N and mixings U_{lN} ?

Can we observe them?

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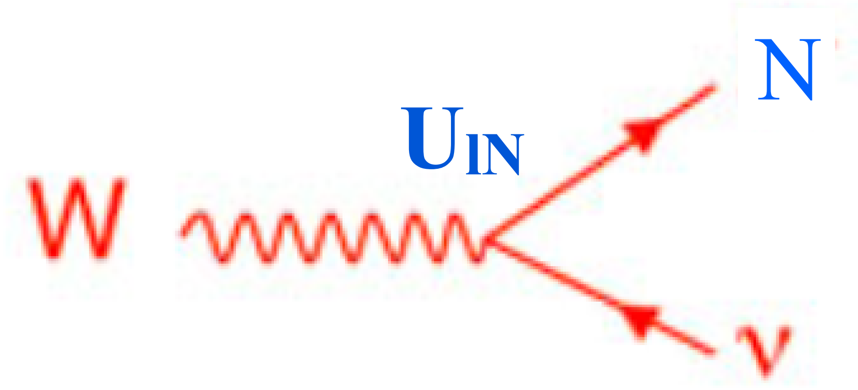

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Can we observe them?

The paradigm model: Seesaw type-I N_R

$$\mathcal{L} = \mathcal{L}_{SM} + i\overline{N_R}\partial\!\!\!/ N_R - \left[\overline{N_R} Y_N \tilde{\phi}^\dagger \ell_L + \frac{1}{2}\overline{N_R} M N_R{}^c + h.c.\right]$$

In type I seesaw $\mathcal{L} = \mathcal{L}_{SM} + i\overline{N_R}\partial\!\!\!/ N_R - \left[\overline{N_R}\,\mathbf{Y}\,\tilde{\phi}^\dagger \ell_L + \frac{1}{2}\overline{N_R}\,\mathbf{M}\,N_R{}^c + h.c.\right]$

$$U_{lN} \sim \mathbf{Y}\, v / \mathbf{M}$$

Observability requires: **$\mathbf{M} < 100$ TeV, $\mathbf{Y}$ "large"**

seesaws with approximate LN symmetry

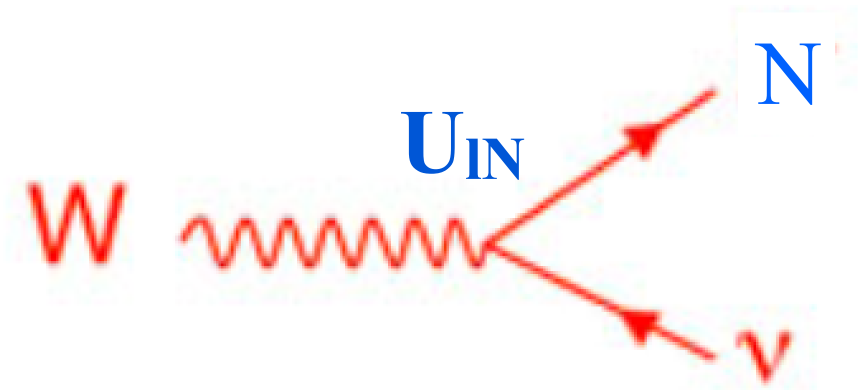
(e.g. inverse and direct seesaws)

(-> ~ degenerate heavy neutrinos)

Wyler+Wolfenstein 83, Mohapatra+Valle 86, Branco+Grimus+Lavoura 89, Gonzalez-Garcia+Valle 89, Ilakovac+Pilaftsis 95, Barbieri+Hambye+Romanino 03, Raidal+Strumia+Turzynski 05, Kersten+Smirnov 07, Abada+Biggio+Bonnet+Gavela+Hambye 07, Shaposhnikov 07, Asaka+Blanchet 08, Gavela+Hambye+D. Hernandez+ P. Hernandez 09

But let us remain model-independent

Assume that singlet fermion(s) N exists in nature



?

What are the limits on their mass m_N and mixings U_{lN} ?

Can we observe them?

Consider together

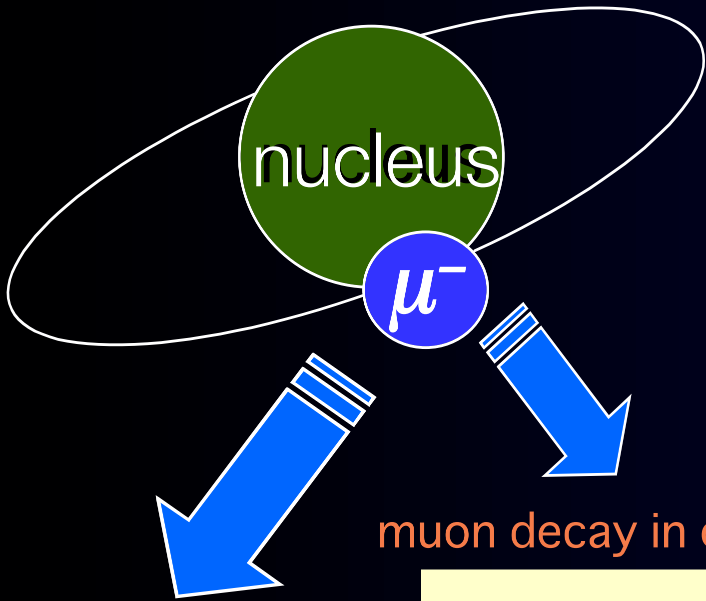
μ-->e conversion

μ-->e γ

μ-->e e e

What is Muon to Electron Conversion?

1s state in a muonic atom



muon decay in orbit

$$\mu^- \rightarrow e^- \nu \bar{\nu}$$

nuclear muon capture

$$\mu^- + (A,Z) \rightarrow \nu_\mu + (A, Z-1)$$

courtesy of Yoshi Kuno

Neutrino-less muon nuclear capture

$$\mu^- + (A,Z) \rightarrow e^- + (A,Z)$$

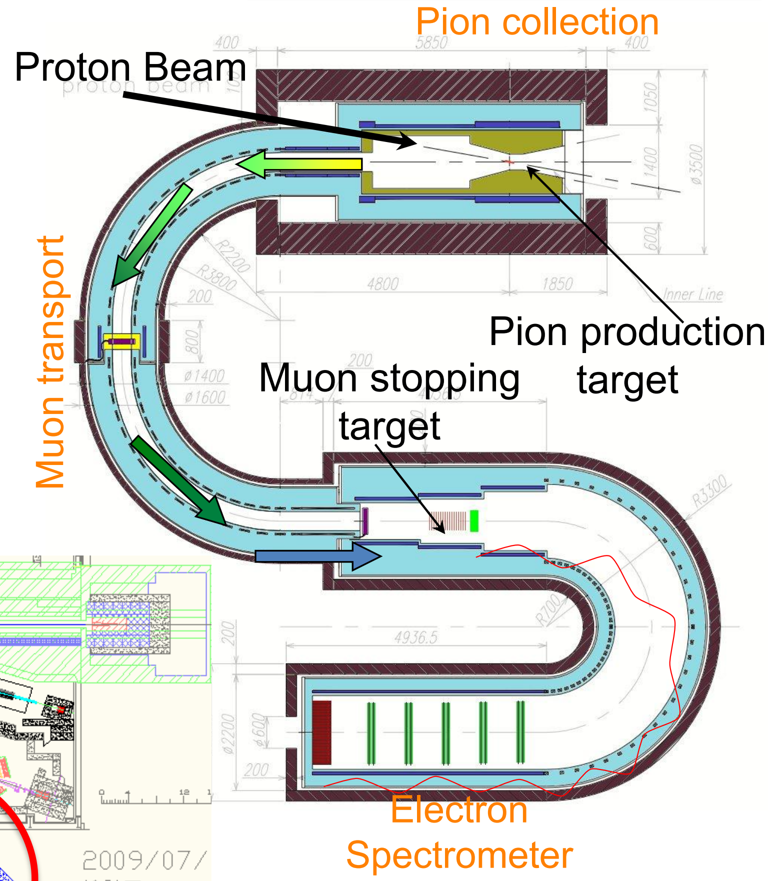
Event Signature :
a single mono-energetic electron of 100 MeV

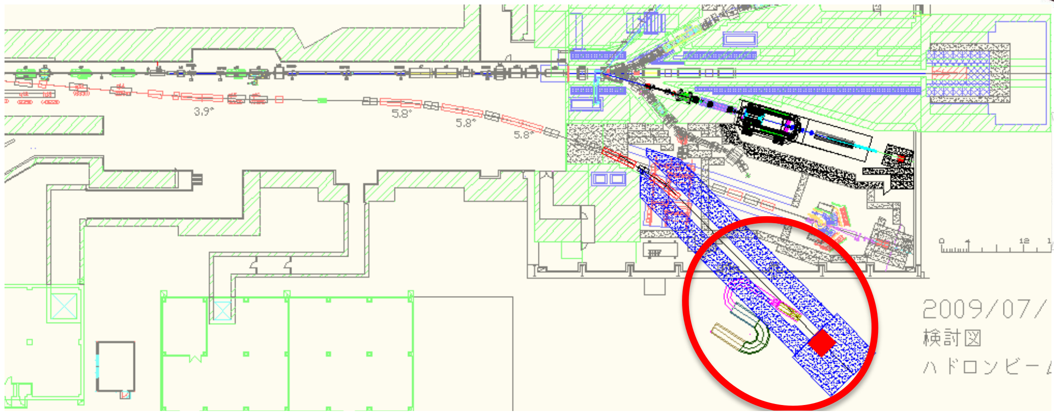
Backgrounds:
(1) physics backgrounds
ex. muon decay in orbit (DIO)
(2) beam-related backgrounds
ex. radiative pion capture, muon decay in flight,
(3) cosmic rays, false tracking

COMET μ-e conv. search

Phase-I phys run in 2017
Full COMET run in 2021-2022

- Search for cLFV mu-e conv.
  - **10^{-16}** sensitivity (Target S.E.S. 2.6×10^{-17})
  - Improve **O(10^4)** than present upper bound such as SINDRUM-II BR[$\mu^- + Au \rightarrow e^- + Au$] < 7×10^{-13}
- Signature: 105MeV monochromatic electron
- Beam requirement
  - 8GeV bunched slow extraction
  - 1.6×10^{21} pot needed to reach goal
  - 7 uA (56kW) x 4 SN year (4×10^7sec)
  - Extinction <10^{-9}





courtesy of Yoshi Kuno

now **expected**

μ-->e conversion

$R^{Ti}_{\mu\to e} < 4.3\times 10^{-12}$ ----------------> $\lesssim 10^{-18}$

$R^{Au}_{\mu\to e} < 7\times 10^{-13}$

$R^{Pb}_{\mu\to e} < 4.6\times 10^{-11}$

$R^{Al}_{\mu\to e} \lesssim 10^{-16}$

μ-->e γ

$Br(\mu \to e\gamma) < 2.4\cdot 10^{-12}$ ----------------> $< 10^{-14}$

μ-->e e e

$Br(\mu \to eee) < 10^{-12}$ ----------------> $< 10^{-16}$

μ-->e conversion

We performed an exact one-loop calculation.
Only approximations used are to neglect:

-- the electron mass compared to muon mass

-- the 3 light neutrino masses compared to heavy ones
(that is, assume $m_N > \text{eV}$)

-- higher orders in the external momentum versus M_W, as usual

μ-->e conversion

mass eigenstates $n_i = \nu_1, \nu_2, \nu_3, N_1 \ldots N_k$

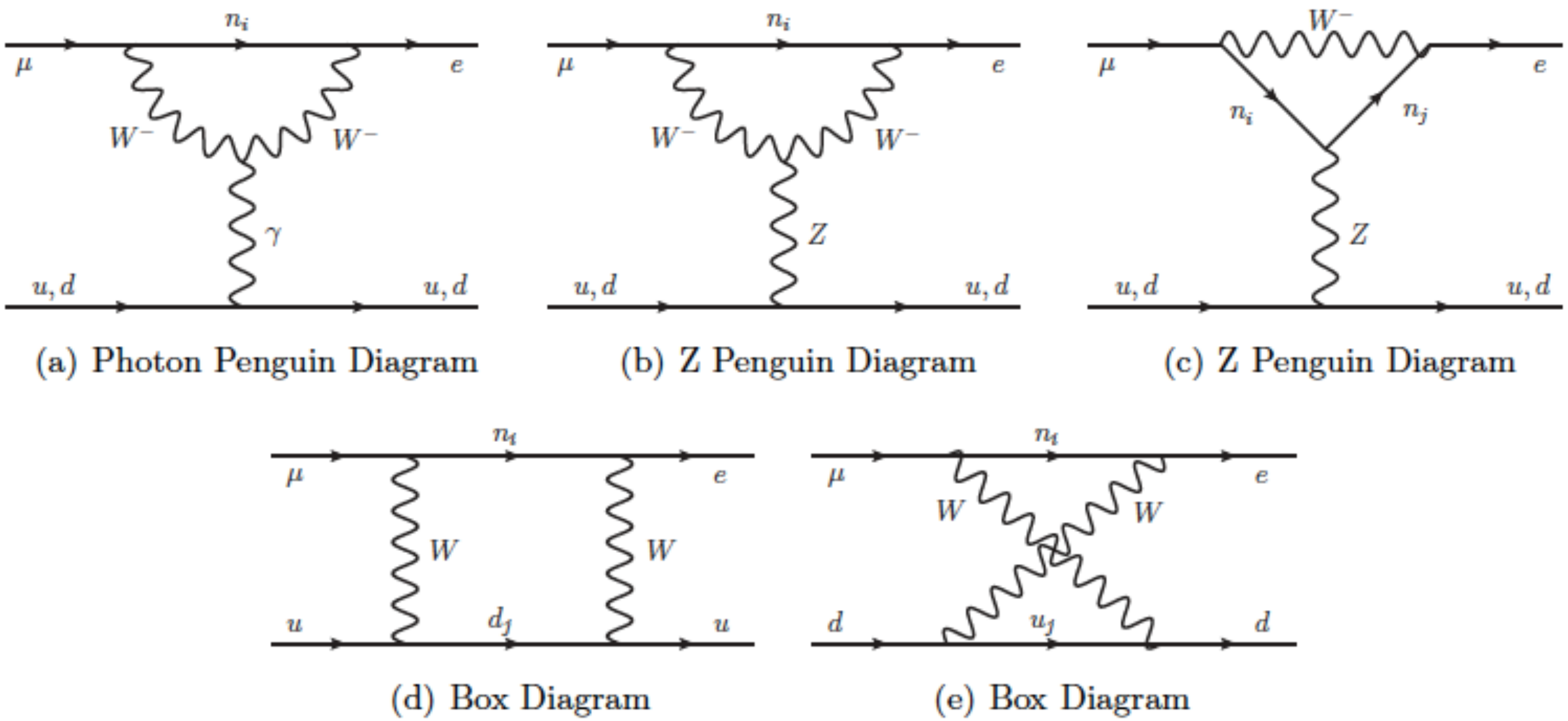


Figure 1: The five classes of diagrams contributing to μ to e conversion in the type-I seesaw model.

μ-->e conversion

μ-->e γ

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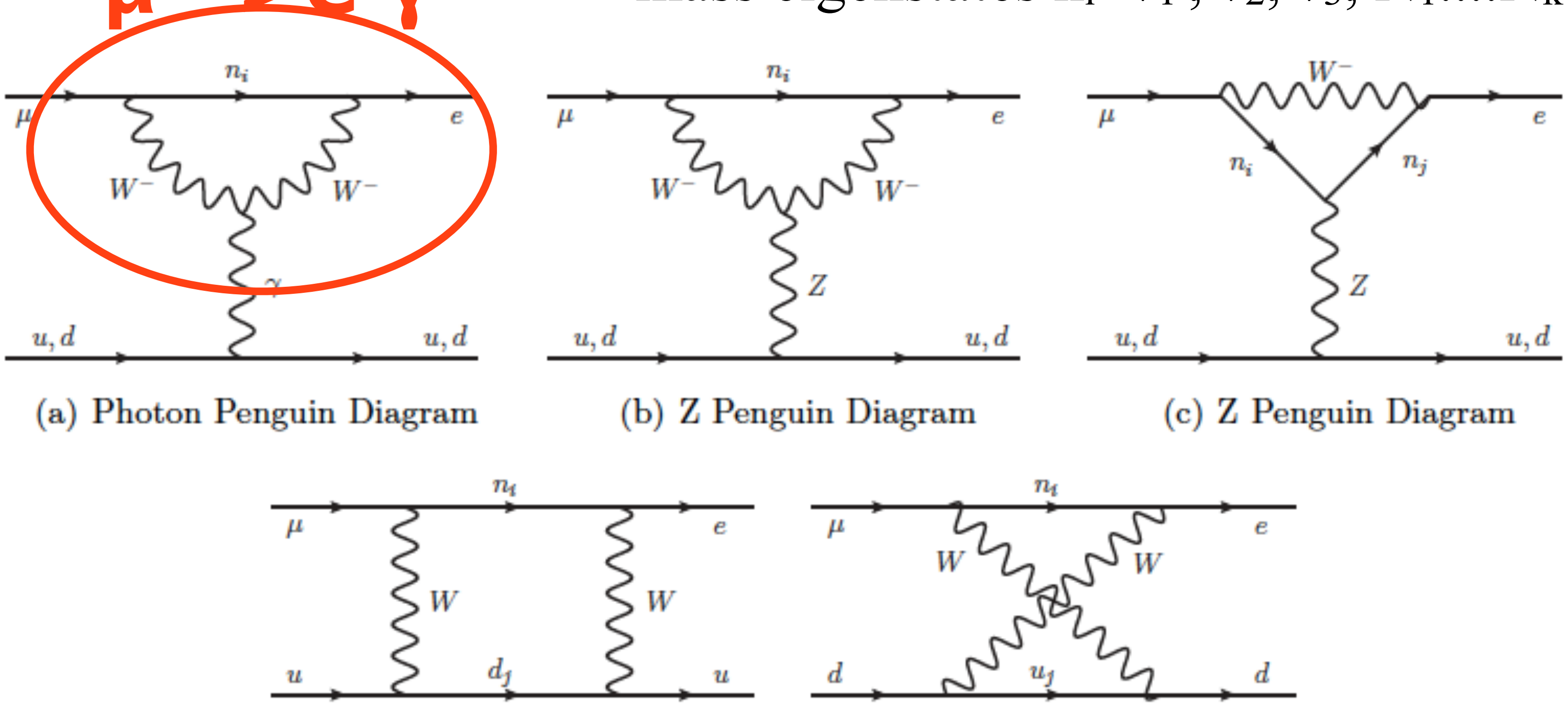

(a) Photon Penguin Diagram (b) Z Penguin Diagram (c) Z Penguin Diagram

(d) Box Diagram (e) Box Diagram

Figure 1: The five classes of diagrams contributing to μ to e conversion in the type-I seesaw model.

They share just one form factor ("dipole")

μ-->e conversion

μ-->e ee

mass eigenstates $n_i = \nu_1, \nu_2, \nu_3, N_1 \ldots N_k$

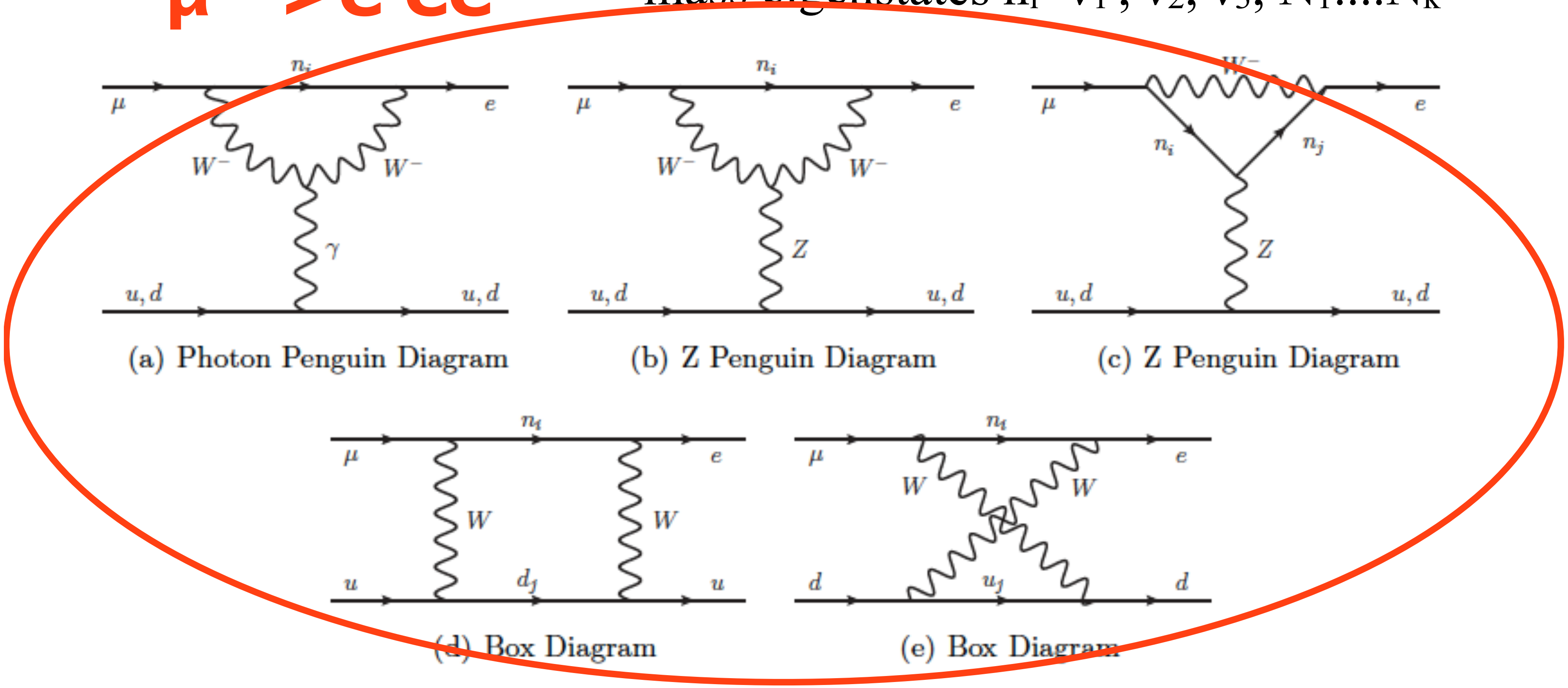

Figure 1: The five classes of diagrams contributing to μ to e conversion in the type-I seesaw model.

Share all form factors, in different combinations

μ-->e conversion

Many people before us computed it for singlet fermions:
Riazzudin+Marshak+Mohapatra 91,
Chang+Ng 94,
Ioannisian+Pilaftsis00,
Pilaftsis and Underwood05,
Deppish+Kosmas+Valle06,
Ilakovac+Pilaftsis09,
Deppish+Pilaftsis11,
Ding+Ibarra+Molinaro+Petcov12,
Aristizabal Sierra+Degee+Kamenik12

typical applications assumed masses over 100 GeV or TeV

Not two among those papers completely agree with each other, or they are not complete

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We agree for logarithmic dependence

typical applications assumed masses over 100 GeV or TeV

Not two among those papers completely agree with each other, or they are not complete

μ-->e conversion

Two types of form factors contribute:

- **with logarithmic dependence on m_N.**
- **without**

BOTH are important and must be taken into account

i.e.: to omit constant terms may change the rates by orders of magnitude for scales in the TeV range

We did many checks to our results, e.g.:

* For "dipole" for factors .... check with $b \dashrightarrow s\, l^{+}\, l^{-}$

* For the other form factors .... agreement with $\mu \dashrightarrow eee$ form factors

....

another check: **Decoupling limits**

* Large mass $m_N >> m_W$

In the seesaw, for $m_N \to \infty$ the remaining theory is renormalizable (SM) --> rate must vanish then.
Our results do decouple for $x_N = m_N^2/M_W^2 \gg 1$

$$\Gamma \sim (\log x_N)^2/x_N^2\,, \qquad \text{for } \mu \to \text{eee} \quad \text{and} \quad \mu \to \text{e conversion}\,,$$
$$\Gamma \sim 1/x_N^2\,, \qquad \text{for } \mu \to \text{e}\gamma\,.$$

* Low mass $m_N << m_W$

they also vanish for m_N --> 0

$x_N = m_N^2/M_W^2 \ll 1$

$$\Gamma \sim x_N^2(\log x_N)^2\,, \qquad \text{for } \mu \to \text{eee} \quad \text{and} \quad \mu \to \text{e conversion}\,;$$
$$\Gamma \sim x_N^2\,, \qquad \text{for } \mu \to \text{e}\gamma\,.$$

RESULTS

* Large mass $m_N >> m_W$

When one m_N scale dominates, as for degenerate heavy neutrinos or very hierarchical spectra, the ratio of any two μ-e transitions only depends on m_N (Chu, Dhen, Hambye 11)

Besides, μ-e conversion vanishes at some large m_N

(Dinh, Ibarra, Molinaro, Petcov 12)

For instance, we find that for light nuclei (αZ<<1), it vanishes as

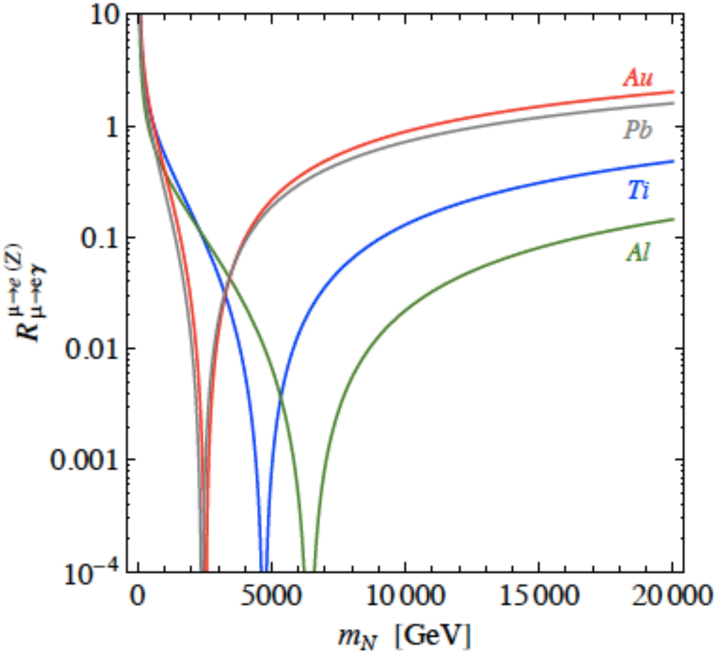
$$m_N^2\Big|_0 = M_W^2 \exp\left(\frac{\frac{9}{8}(A-Z)+\left(\frac{9}{8}+\frac{31 s_W^2}{12}\right)Z}{\frac{3}{8}(A-Z)+\left(\frac{4 s_W^2}{3}-\frac{3}{8}\right)Z}\right)$$

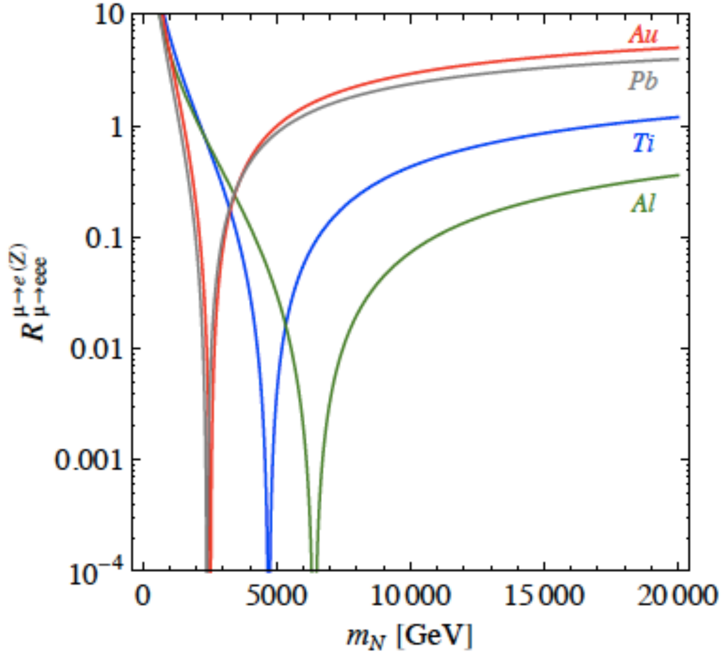
exponential sensitivity

The ratios of two e-μ transitions may vanish....

we obtain:

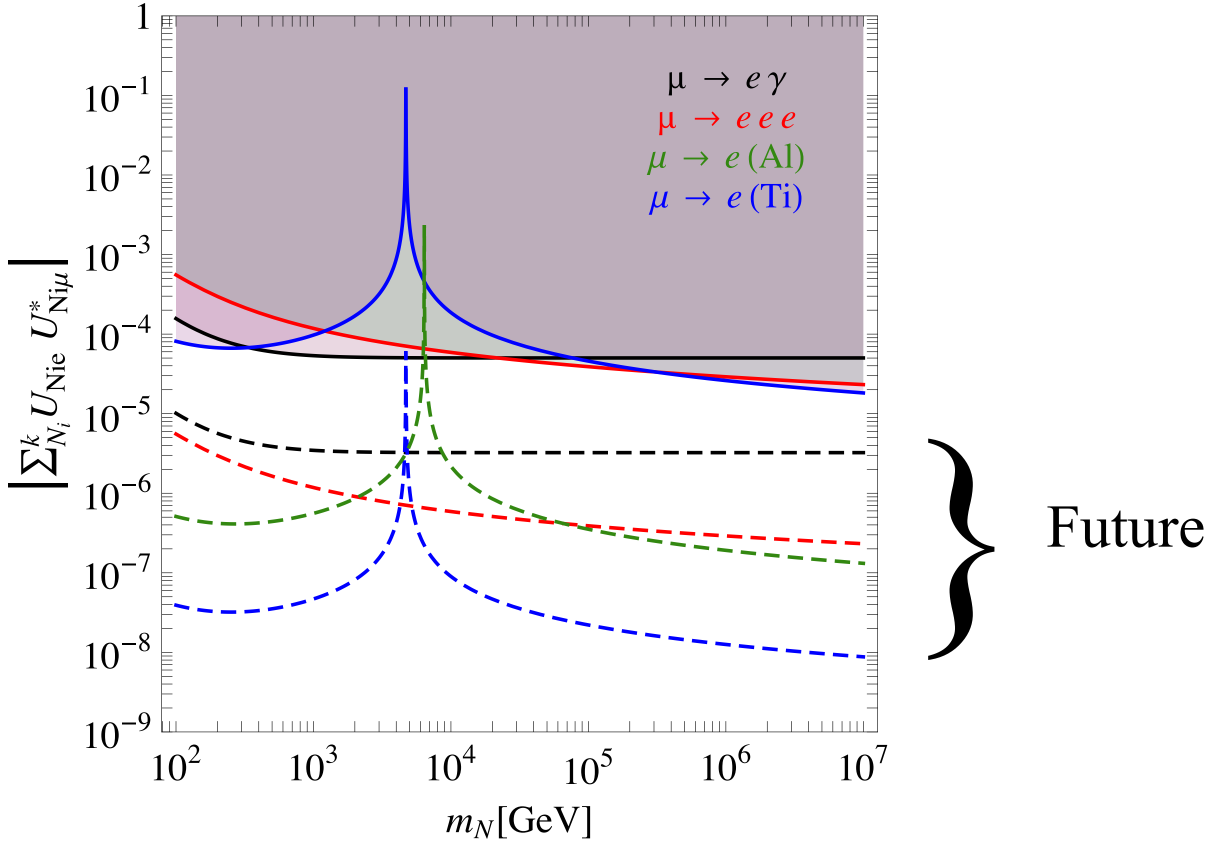
$$\frac{\mu\text{-e conversion}}{\mu \to e\,\gamma} \qquad\qquad \frac{\mu\text{-e conversion}}{\mu \to e\,e\,e}$$





...typically vanishes for m_N in 2- 7 TeV range

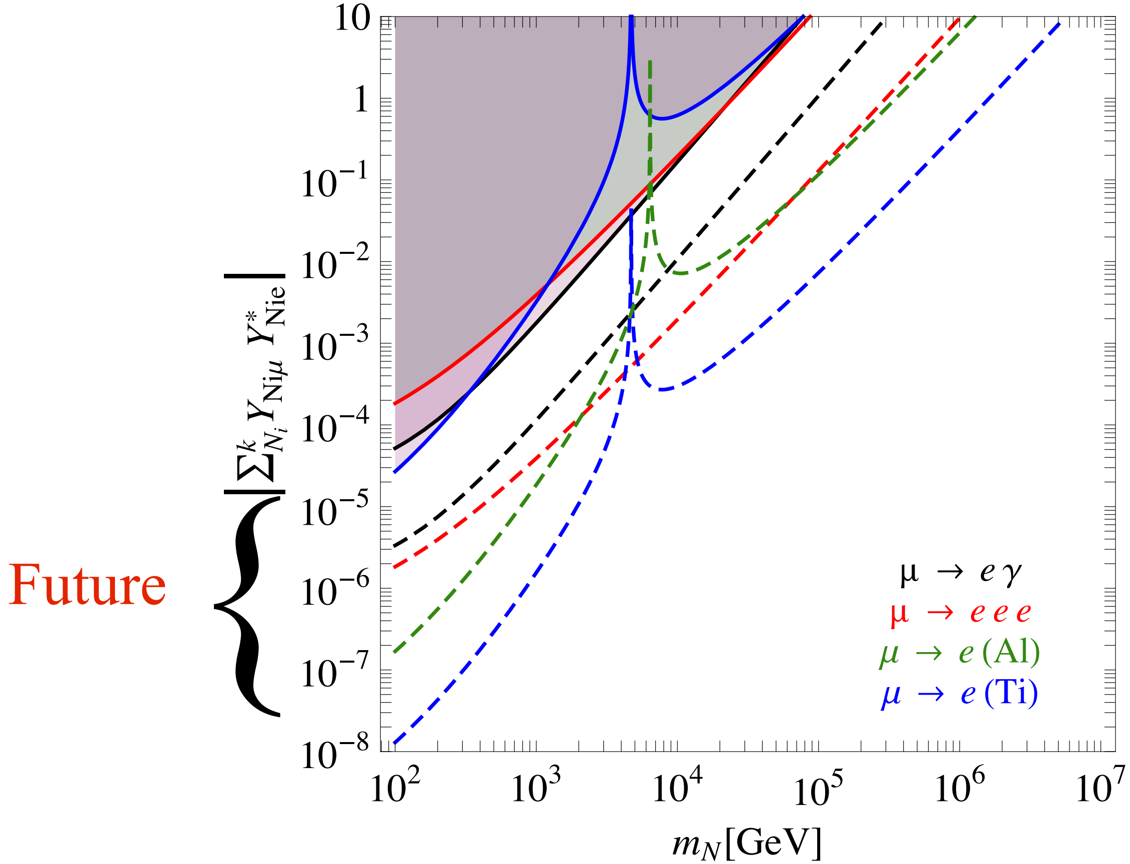
|$U_{\mu N}$ U_{eN}*| versus m_N



Sensitivity up to m_N ~ 6000 TeV for Ti

For the particular case of seesaw I : $U_{lN} \sim Y\, v / M$

$|Y_{\mu N}\, Y_{eN}^*|$ versus m_N



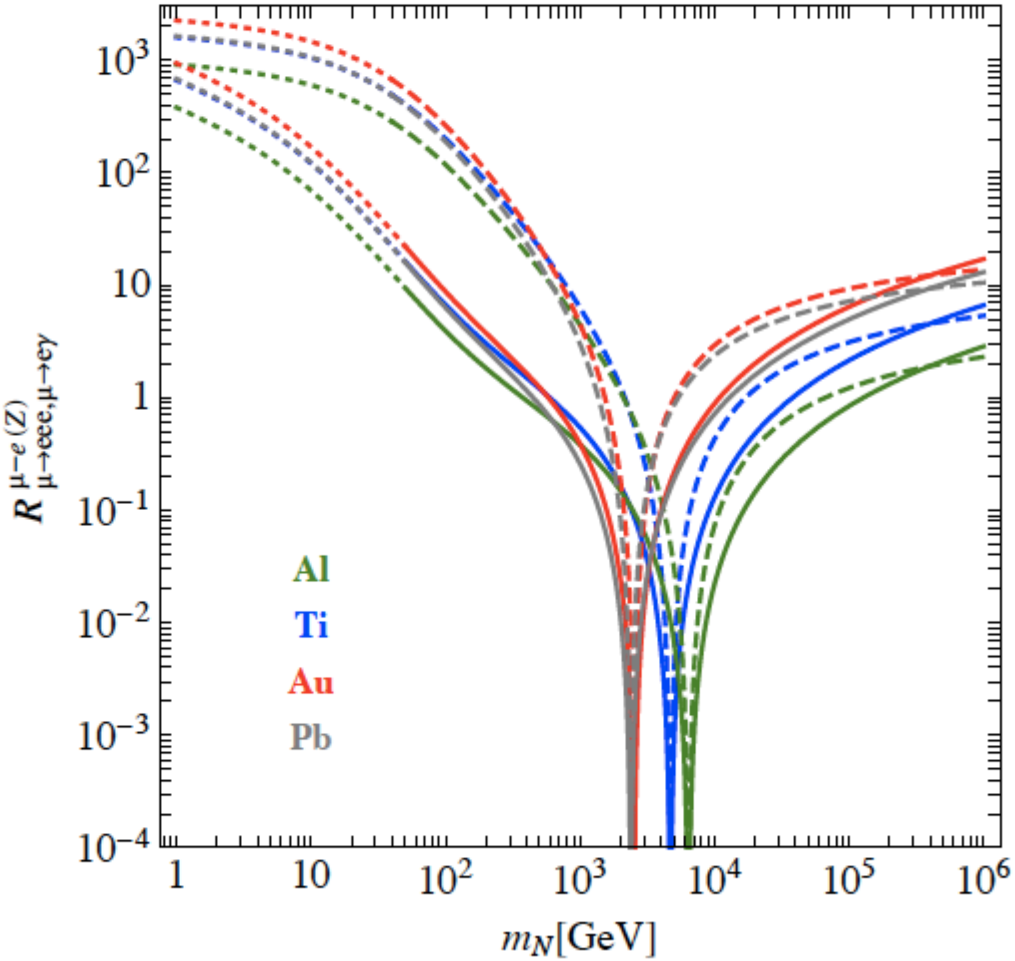
Sensitivity up to $m_N \sim 6000$ TeV for Ti

* Low mass regime eV << m_N << m_W

.... de Gouvea 05...

* Low mass regime eV << m_N << m_W

μ--> e conversion does not vanish for low mass



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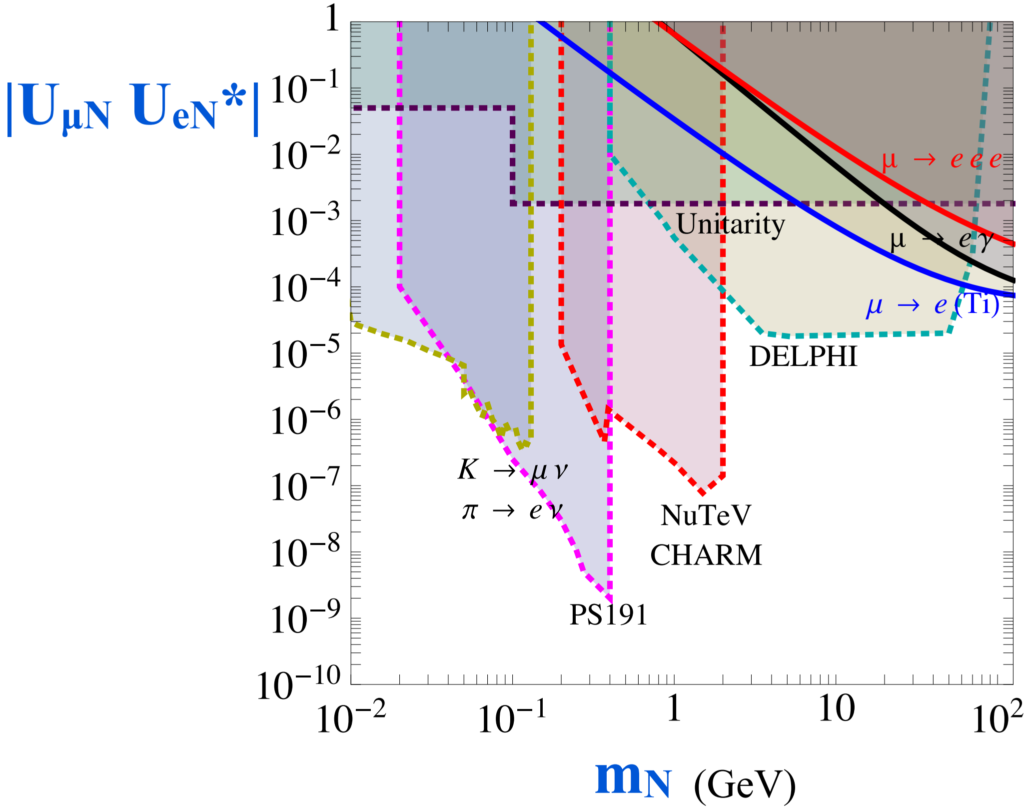

Peak decays+PS191+NuTev/CHARM+Delphi: Atre+Han+Pascoli+Zhang 09...... Richayskiy+Ivashko 12

Unitarity: Antusch+Biggio+Fdez-Martinez+Gavela+Lopez-Pavon 06; Antusch+Bauman+ Fedez-Martinez 09

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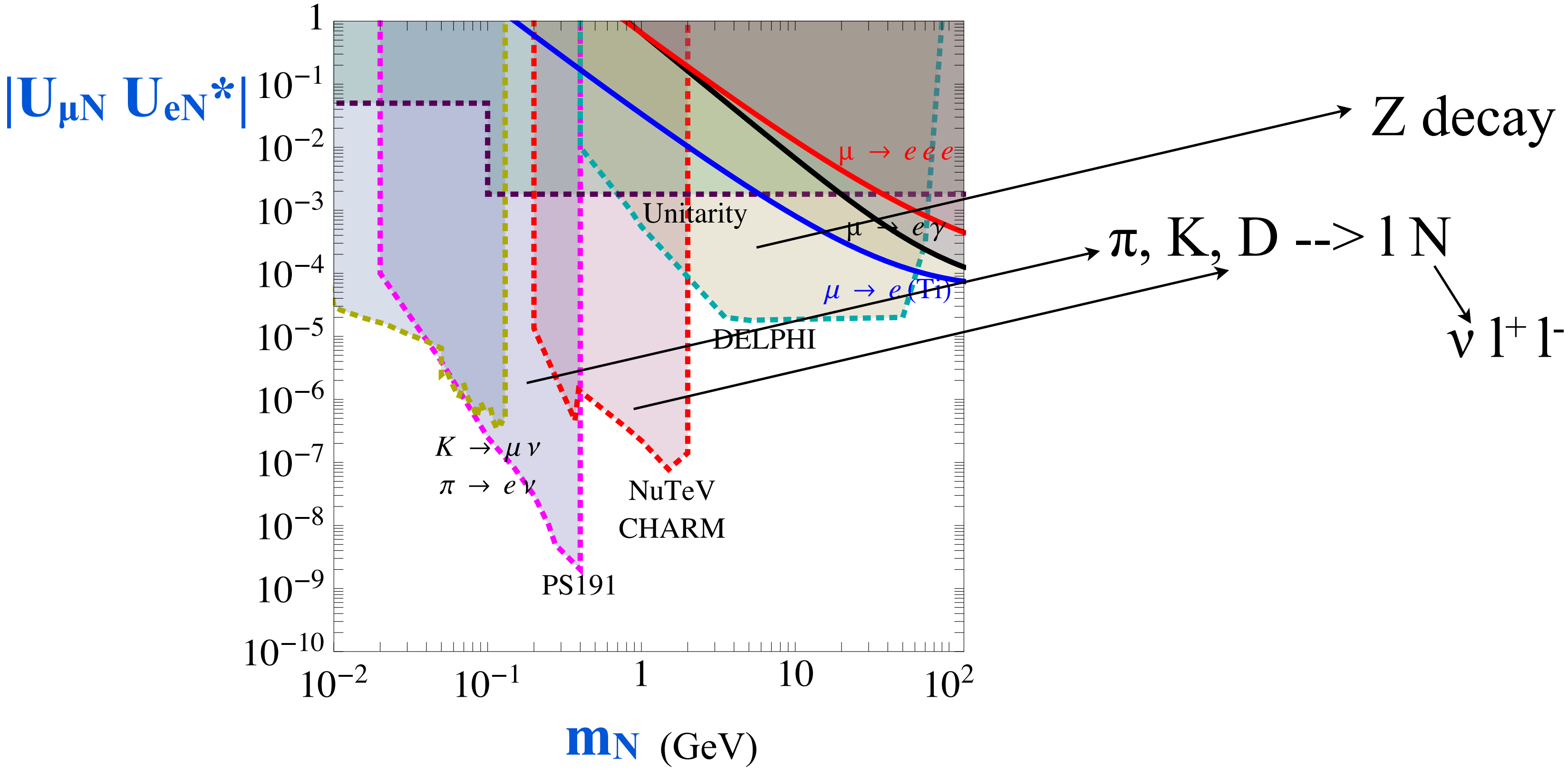

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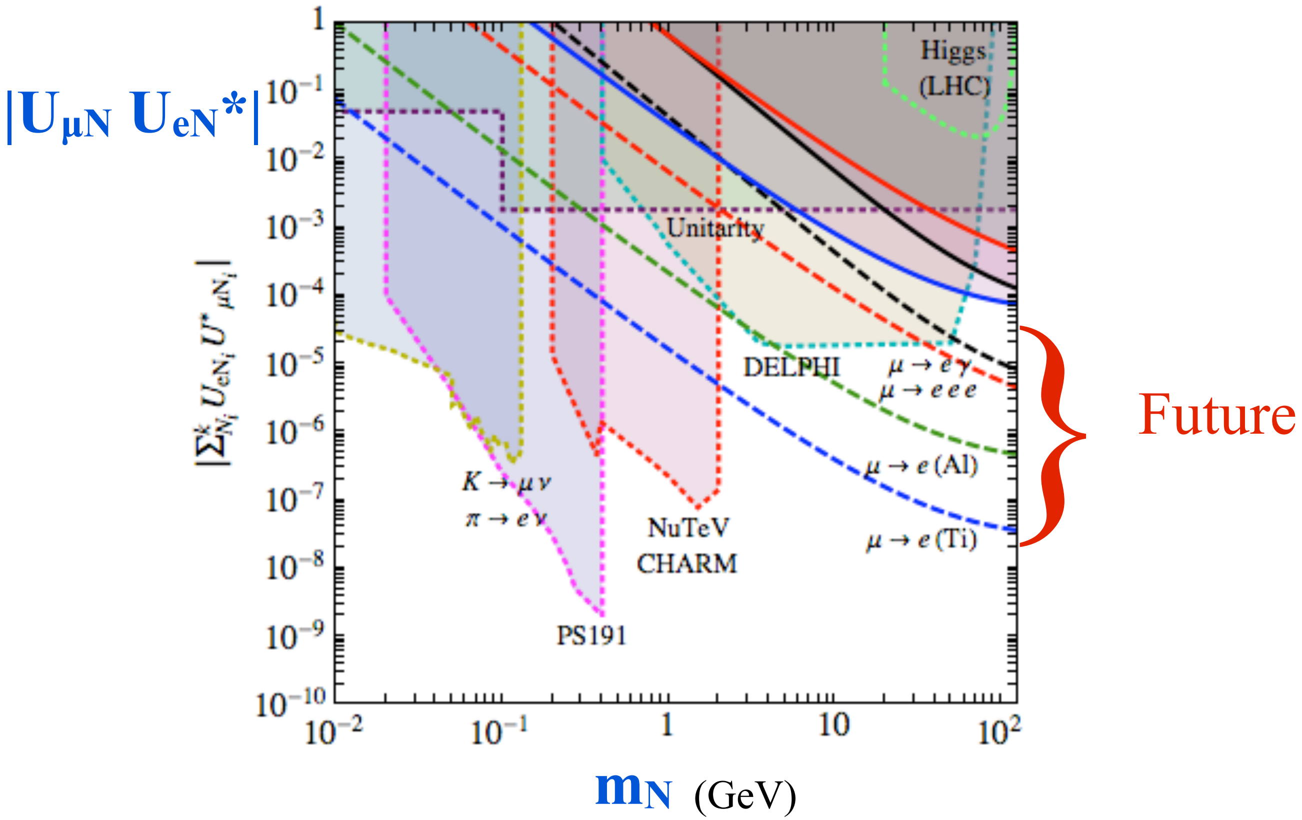

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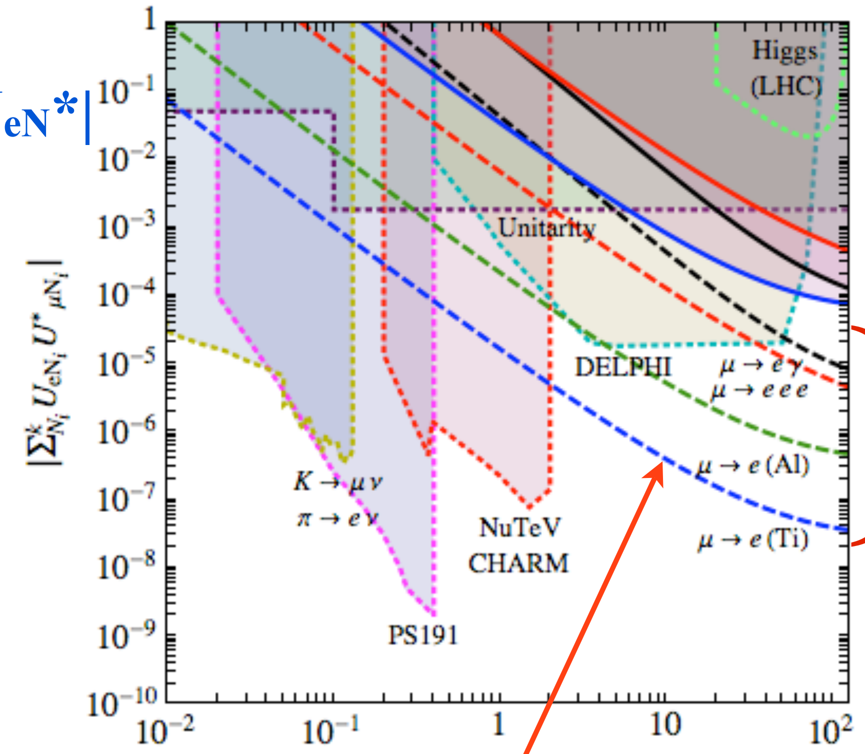
$|U_{\mu N} U_{eN}^*|$



Future

m_N (GeV)

This experiment (considered alone) will probe masses down to m_N=2Mev

Higgs decay (LHC)

e.g. **H --> ν N**

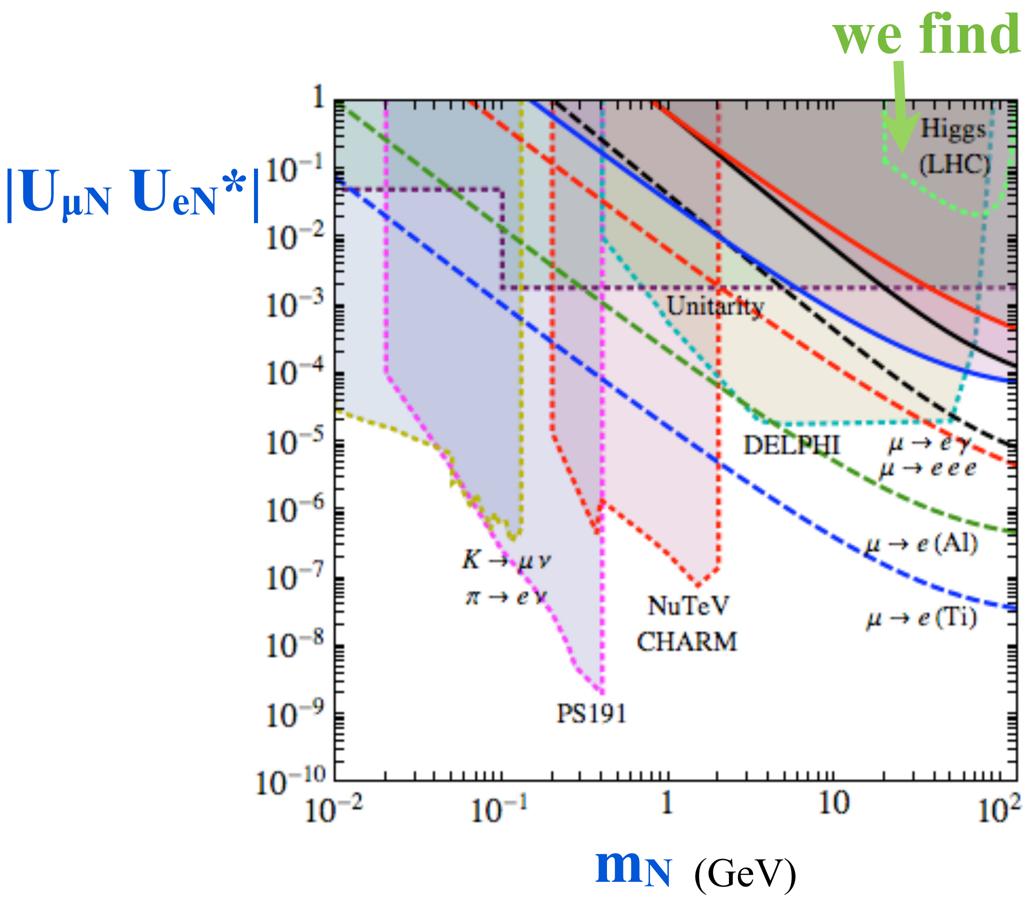
Pilaftsis92....Chen et al.10, Dev+Franceschini+Mohapatra 12, Cely+Ibarra+Molinaro+Petcov

We get for the model-independent rate:

$$Br(h \to \nu N) = \frac{\alpha_W}{8 M_W^2 \Gamma_h^{tot}} \sum_i^k \left(|U_{eN_i}|^2 + |U_{\mu N_i}|^2 + |U_{\tau N_i}|^2 \right) m_h m_{N_i}^2 \left(1 - \frac{m_{N_i}^2}{m_h^2} \right)^2$$

and using $|\Sigma_i U_{eN_i} U^*_{\mu N_i}| < \Sigma_{i,\alpha} |U_{\alpha N_i}|^2$

* Low mass regime eV << m_N << m_W



Absolute bound from Higgs, from absence of

H --> ν N

↘ ν $l^+ l^-$

at LHC:

Br (H -> ν N) < 0.4

(Espinosa, Grojean, Muhlleitner, Trott, 12 Dev+Franceschini+Mohapatra12, Cely+ Ibarra+Molinaro+Petcov 12)

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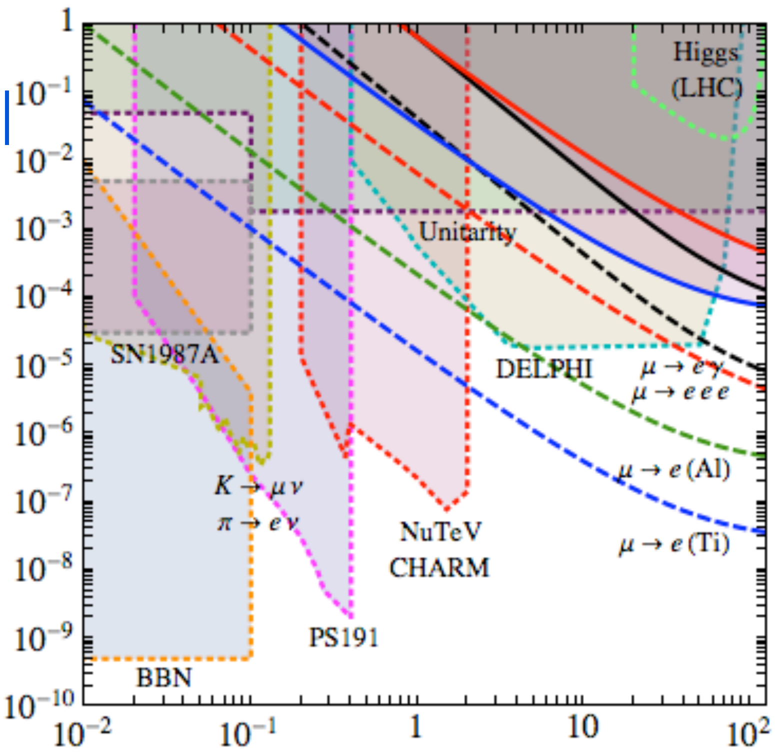

m_N (GeV)

BBN and SN: Kainulainen+Maalampi+Peltoniemi91, Kusenko+Pascoli+Semikoz 05, Mangano+Serpico 11, Ruchaysiliy +Ivashko 12,
Kufflick+McDermott+Zurek 12

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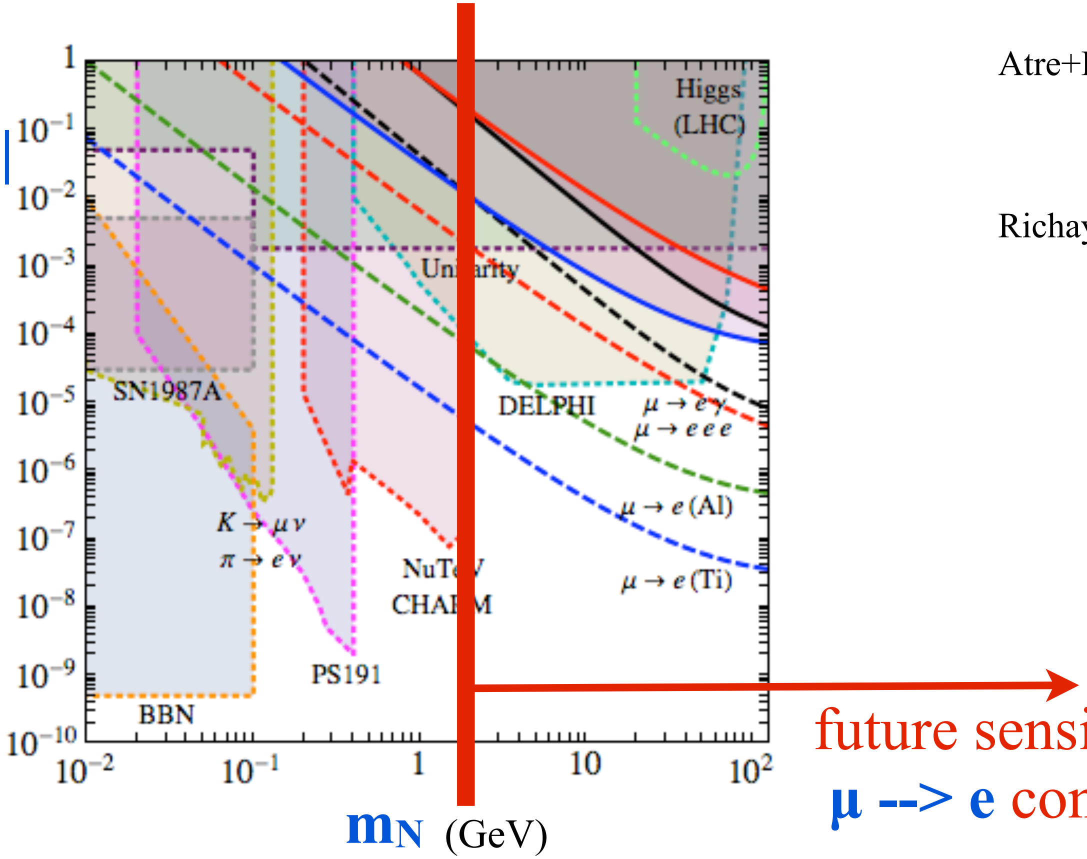

Atre+Han+Pascoli+Zhang 09

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.
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Richayskiy+Ivashko 12

future sensitivity reach of **μ --> e** conversion:

2 Gev --- 6000 TeV !

model-independent

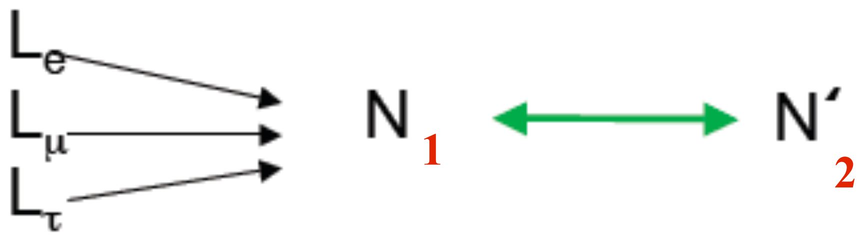
Improved bounds from Higgs decay are model-dependent:

For instance, in L-conserving seesaw:

$$\mathscr{L}_{M_\nu} = \begin{array}{c} \\ \\ \\ \end{array} \begin{array}{ccc} \bar{L}_i & \bar{N}^c & \bar{N}'^c \\ \end{array}$$

$$\mathscr{L}_{M_\nu} = \begin{pmatrix} 0 & Y^T v & 0 \\ Y\, v & 0 & M^T \\ 0 & M & 0 \end{pmatrix}$$

Lepton number conserved



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Lepton number conserved

U(1)

$$\Lambda_{flavour} = M$$

$$\Lambda_{LN} = \infty$$

e.g.

$$\mathscr{L}_{M_\nu} = \begin{pmatrix} & \bar{L}_i & \bar{N}^c & \bar{N}'^c \\ & 0 & Y^T v & Y'^T v \\ & Y\,v & \mu' & \mathbf{M}^T \\ & Y'\,v & \mathbf{M} & \mu \end{pmatrix}$$

Lepton number violated by any of those 3 entries

Λ may be ~ TeV and Ys ~1, and be ok with m_ν

Just TWO heavy neutrinos

$$\mathcal{L}_{\mathcal{M}_\nu} = (\bar{\ell}_L\,,\, \bar{N}^c\,,\, \bar{N}'^c) \begin{pmatrix} 0 & vY & vY' \\ vY^T & 0 & \mathbf{M} \\ vY'^T & \mathbf{M} & 0 \end{pmatrix} \begin{pmatrix} \ell_L^c \\ N \\ N' \end{pmatrix}$$

--> One massless neutrino and only one Majorana phase α

Gavela, Hambye, Hernandez[2]
Raidal, Strumia, Turszynski

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--> One massless neutrino and only one Majorana phase α

the Yukawas are determined up to their overal magnitude

$$\text{N.H.} \qquad Y = \frac{y}{\sqrt{m_{\nu_2} + m_{\nu_3}}} U_{PMNS} \begin{pmatrix} 0 \\ -i\sqrt{m_{\nu_2}}e^{-i\alpha} \\ \sqrt{m_{\nu_3}}e^{i\alpha} \end{pmatrix}$$

Gavela, Hambye, Hernandez[2]
Raidal, Strumia, Turszynski

Normal hierarchy:

Up to terms of $\mathcal{O}(\sqrt{r}, s_{13})$, we find

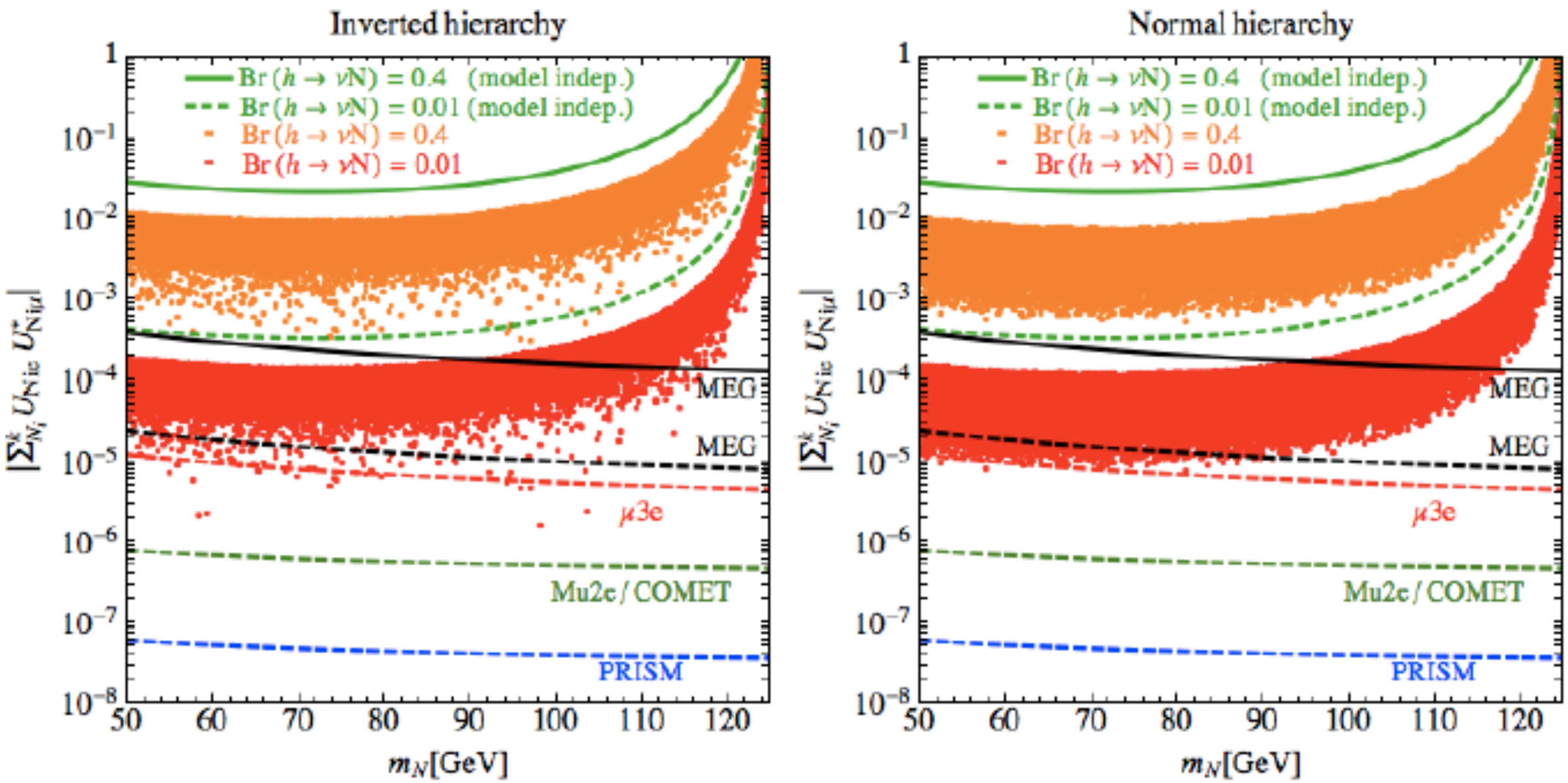
$$r = \frac{|\Delta m^2_{12}|}{|\Delta m^2_{13}|}$$

$$Y_N^T \simeq y \begin{pmatrix} e^{i\delta} s_{13} + e^{-i\alpha} s_{12} r^{1/4} \\ s_{23}\left(1 - \frac{\sqrt{r}}{2}\right) + e^{-i\alpha} r^{1/4} c_{12} c_{23} \\ c_{23}\left(1 - \frac{\sqrt{r}}{2}\right) - e^{-i\alpha} r^{1/4} c_{12} s_{23} \end{pmatrix} .$$

Inverted hierarchy:

$$Y_N^T \simeq \frac{y}{\sqrt{2}} \begin{pmatrix} c_{12} e^{i\alpha} + s_{12} e^{-i\alpha} \\ c_{12}\left(c_{23} e^{-i\alpha} - s_{23} s_{13} e^{i(\alpha-\delta)}\right) - s_{12}\left(c_{23} e^{i\alpha} + s_{23} s_{13} e^{-i(\alpha+\delta)}\right) \\ -c_{12}\left(s_{23} e^{-i\alpha} + c_{23} s_{13} e^{i(\alpha-\delta)}\right) + s_{12}\left(s_{23} e^{i\alpha} - c_{23} s_{13} e^{-i(\alpha+\delta)}\right) \end{pmatrix}$$

Varying the CP phases α and δ, we get:



$|U_{\mu N}\, U_{eN}^*|$ versus m_N

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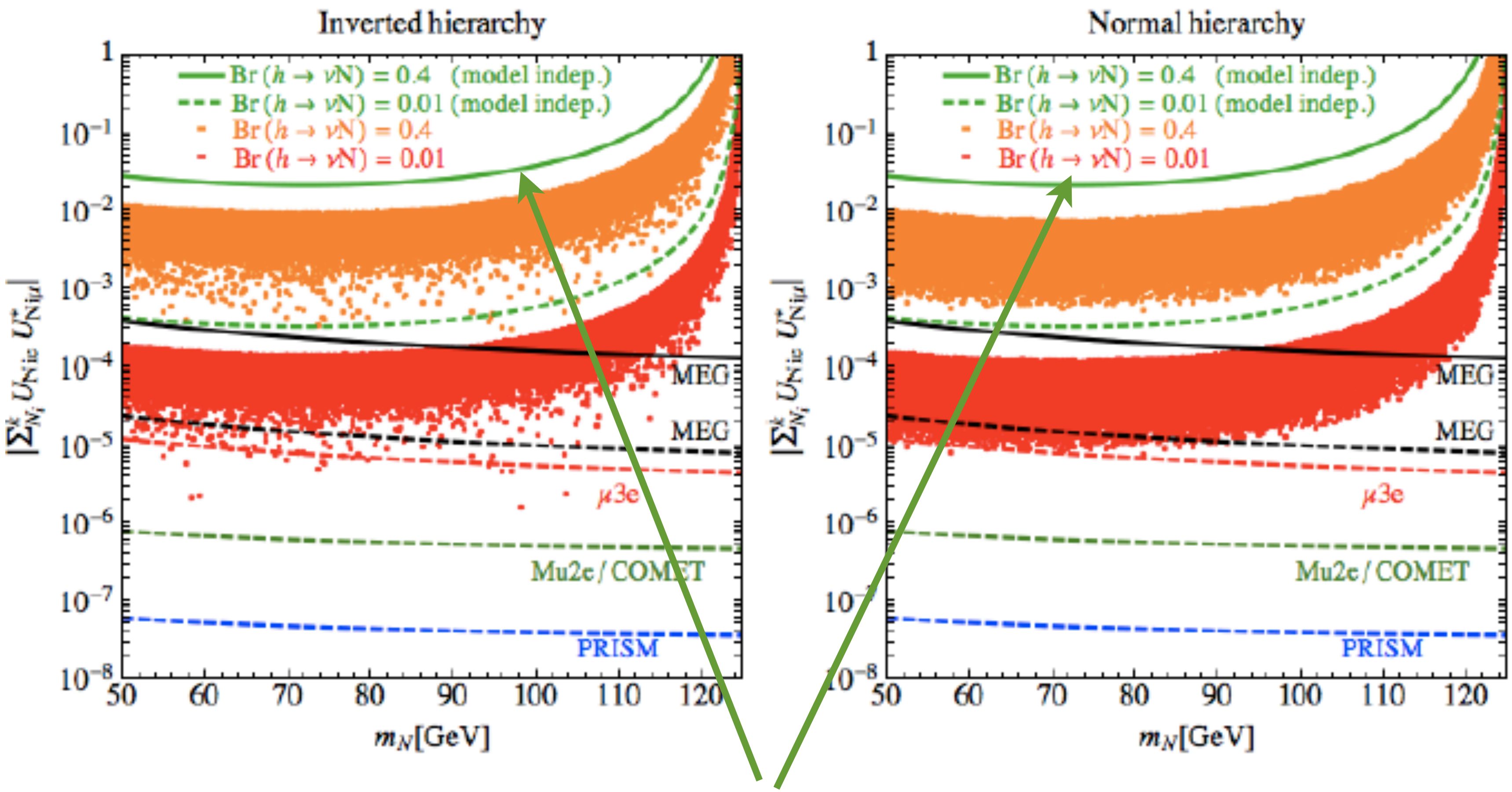
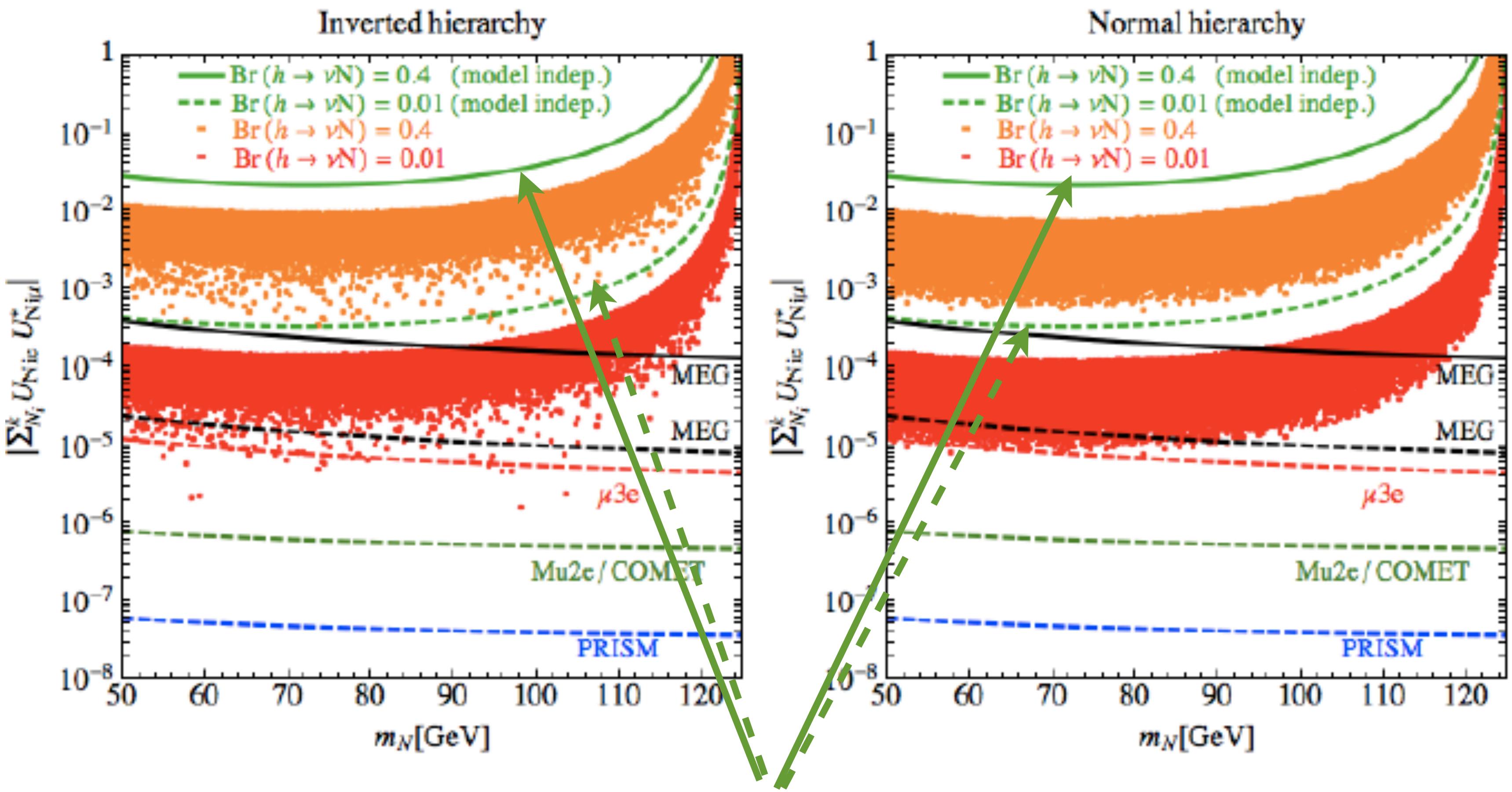


Absolute bound in previous plot

$|U_{\mu N}\, U_{eN}^*|$ versus **m_N**

Varying the CP phases α and δ, we get:



$|U_{\mu N} U_{eN}^*|$ versus m_N

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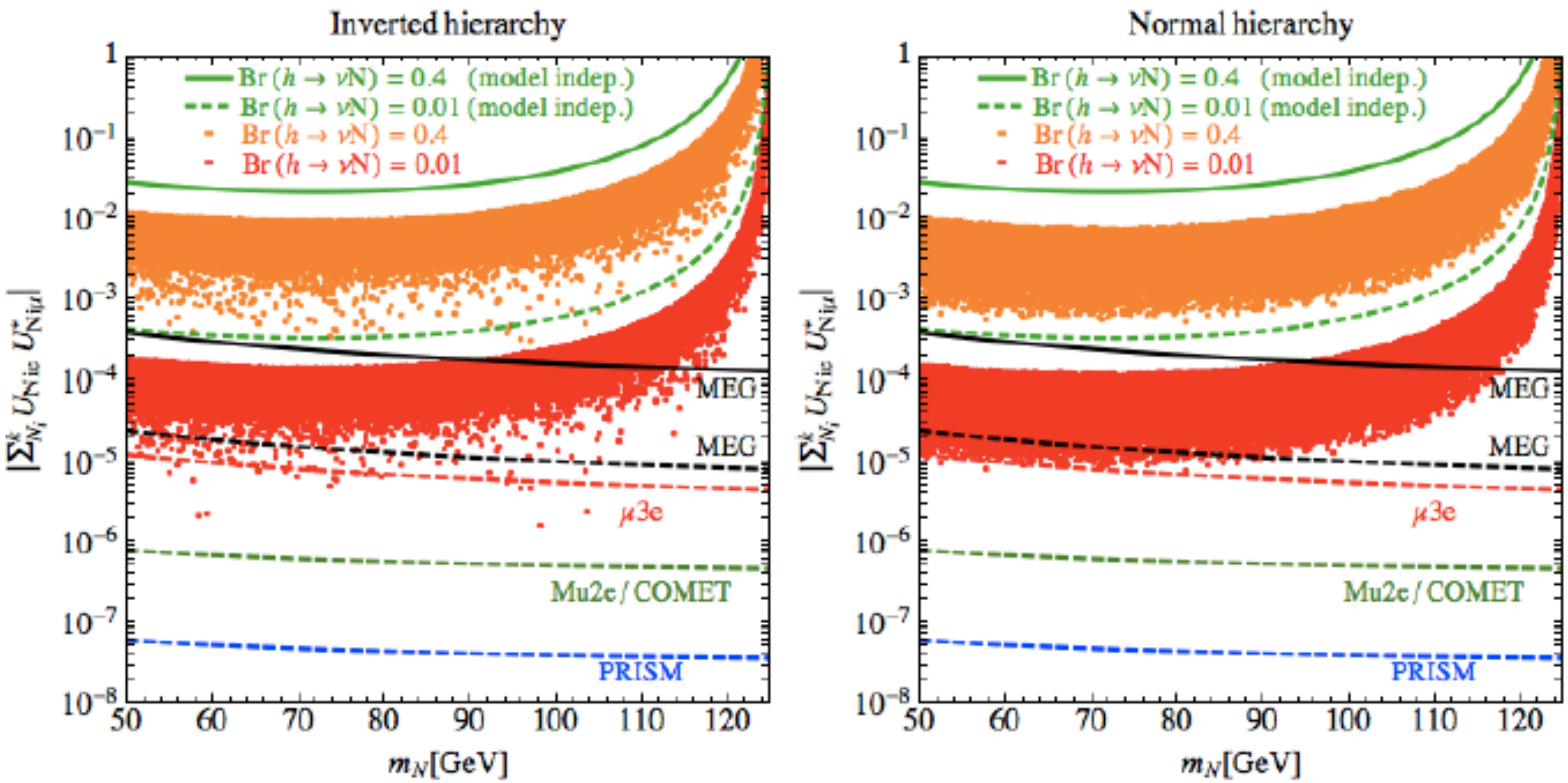


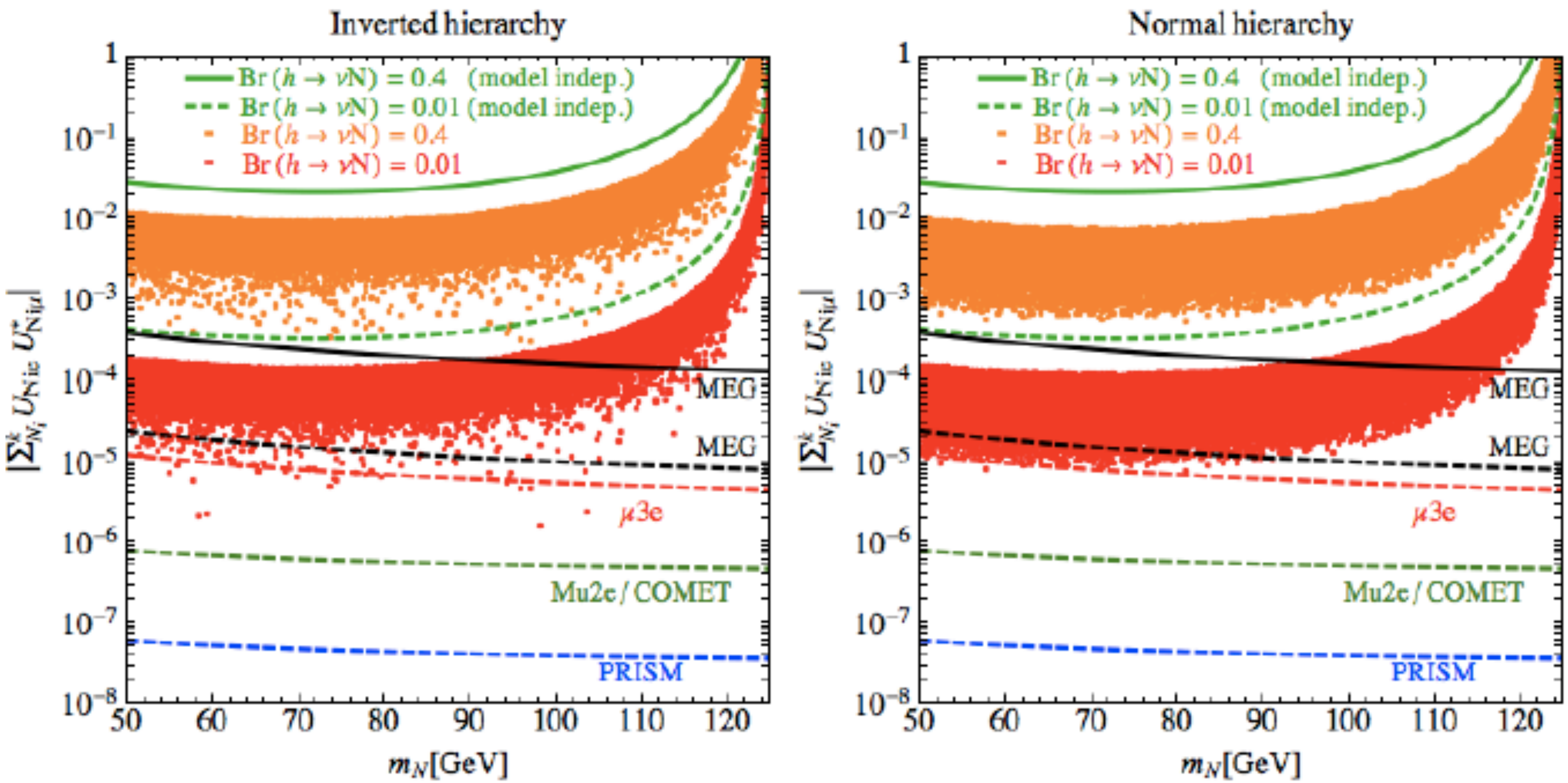
Orange and red model-dependent bounds limited by:

upper boundary: (α= π/2, δ= 0) (α=π, δ=3π/2)

lower boundary: ~(α=-π/2, δ= 0) (α=-π/4, δ=0)

~ it could be consistent with Cely et al. 12, for α~0, δ~ 0

Varying the CP phases α and δ, we get:



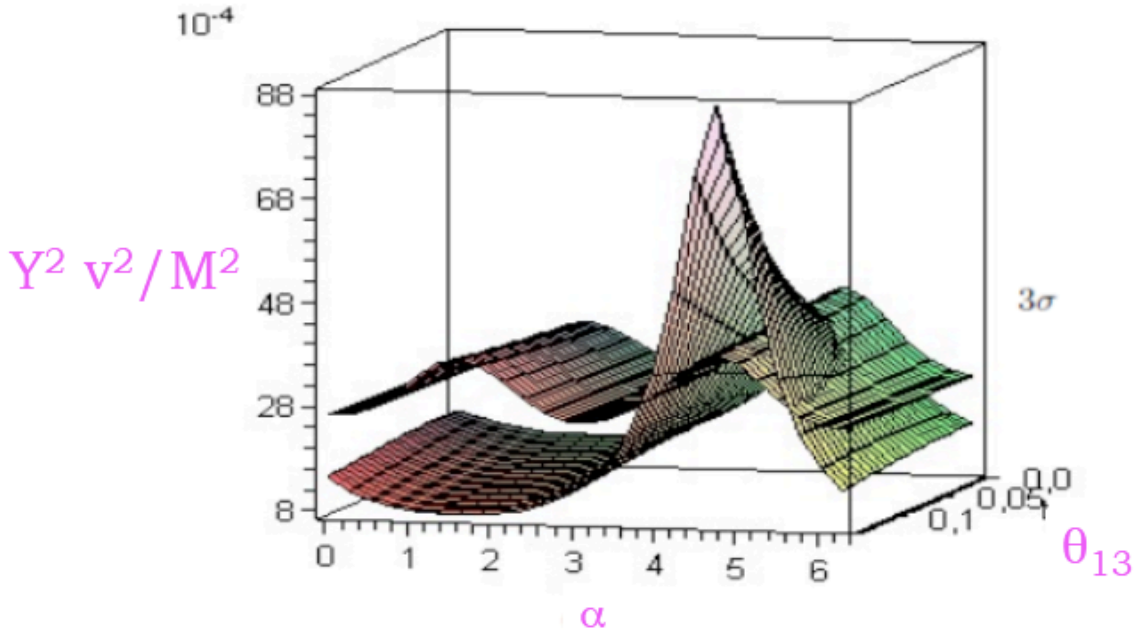
For inverted hierarchy: some very low points for which μ-->e very small, because the Yukawas involved ---> 0 for particular values of α and δ (Alonso et al. 09, Alonso 08, Chu+Dhen+Hambye 11....)

* e-μ, μ-τ etc. oscillations and rare decays studied:

Gavela, Hambye, Hernandez[2] 09; .....

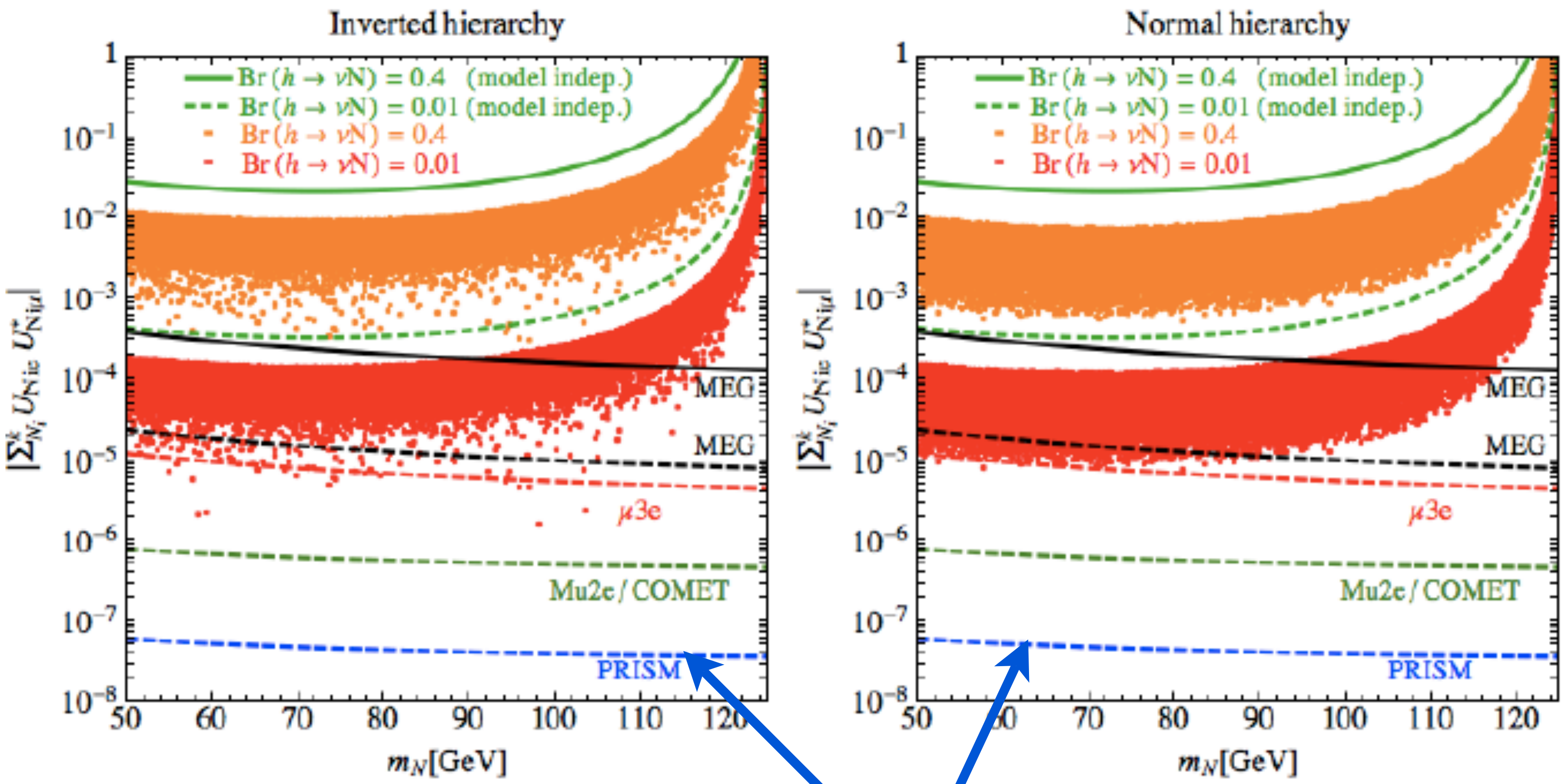
* Alonso + Li, 2010: possible suppresion of μ-e transitions

->important impact of ν_μ - ν_τ at a near detectors



We find that there are regions where an experiment as MINSIS
would improve the present bounds on our Model

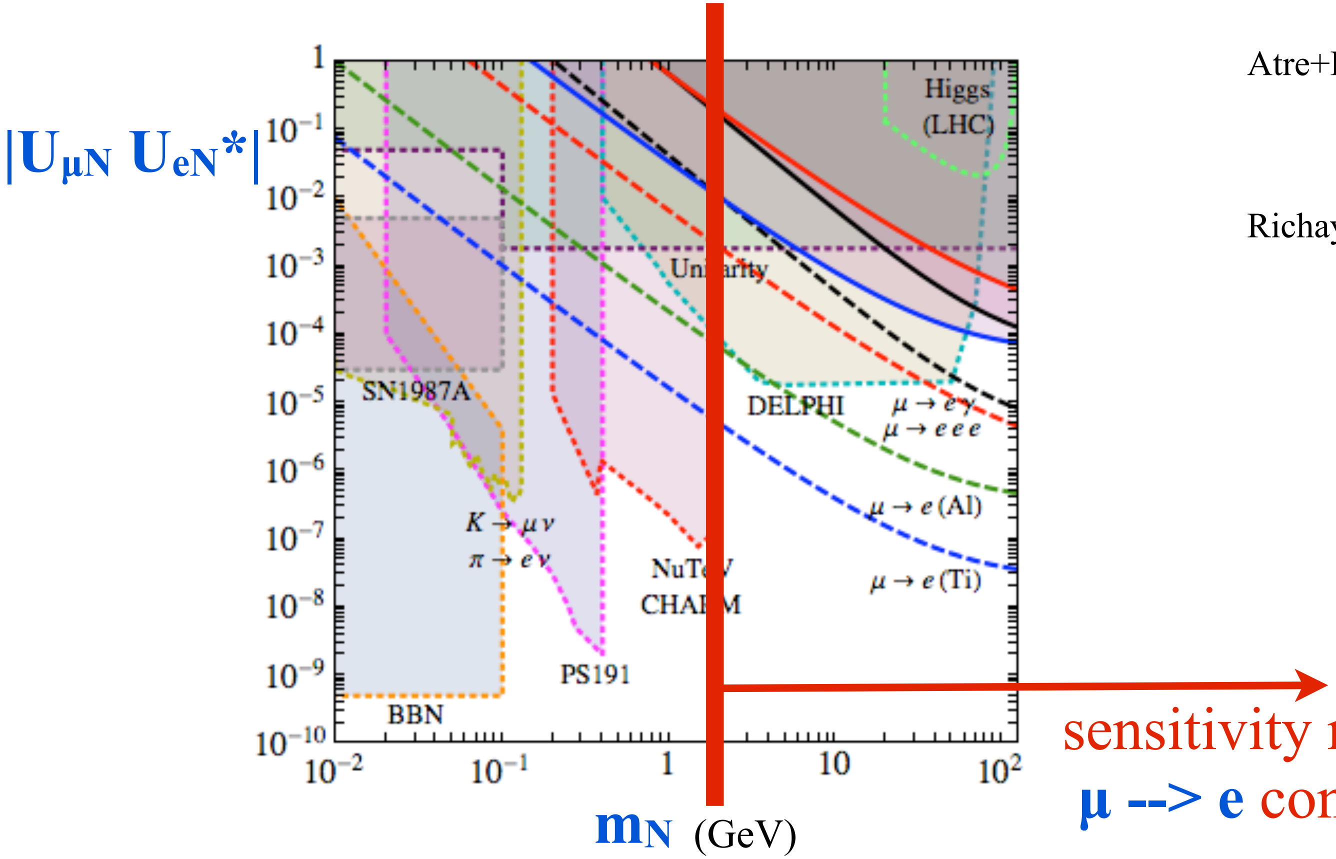
Varying the CP phases α and δ, we get:



In any case, LHC expected sensitivity negligible compared with that of future μ--> e conversion expts.

~ consistent with Cely et al. 12, for α~0, δ~ 0

* Low mass regime eV << m_N << m_W



Atre+Han+Pascoli+Zhang 09

.

.

.

Richayskiy+Ivashko 12

sensitivity reach of
μ --> e conversion:

2 Gev --- 6000 TeV !

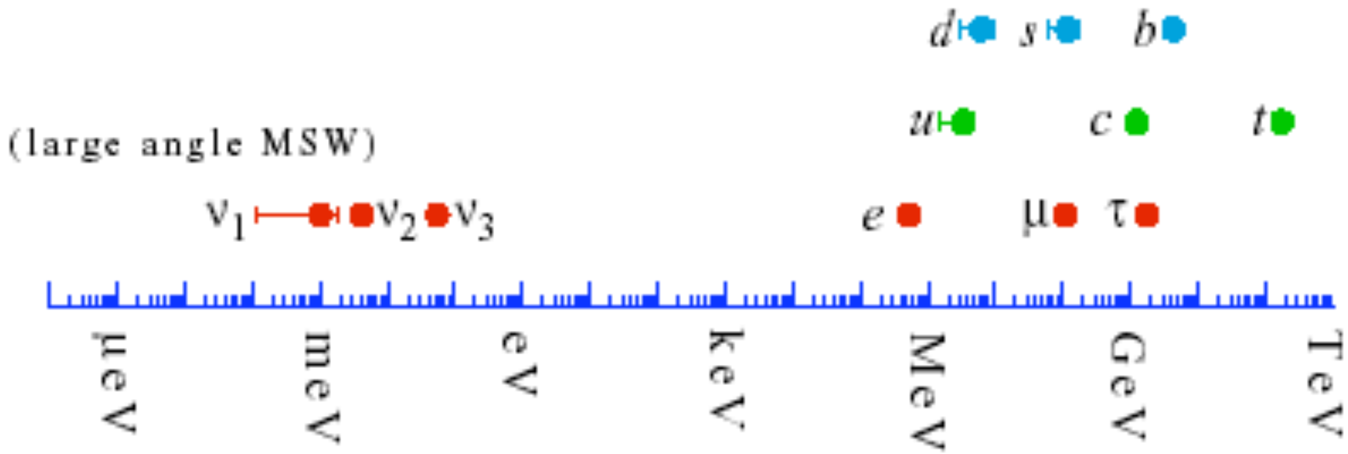
In summary

Future **μ --> e conversion** experiments in nuclei (in particular Ti) **will detect or constraint sterile neutrino scenarios** in an impressive mass range :

2 Gev --- 6000 TeV !

- Dynamical Yukawas

Why quark and neutrino mass hierarchies so different?



Neutrinos lighter because Majorana?

Leptons

$$V_{PMNS} = \begin{pmatrix} 0.8 & 0.5 & \sim 9^o \\ -0.4 & 0.5 & -0.7 \\ -0.4 & 0.5 & +0.7 \end{pmatrix}$$

Quarks

$$V_{CKM} = \begin{pmatrix} \sim 1 & \lambda & \lambda^3 \\ \lambda & \sim 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & \sim 1 \end{pmatrix} \quad \lambda \sim 0.2$$

Why so different?

More wood for the Flavour Puzzle

Leptons

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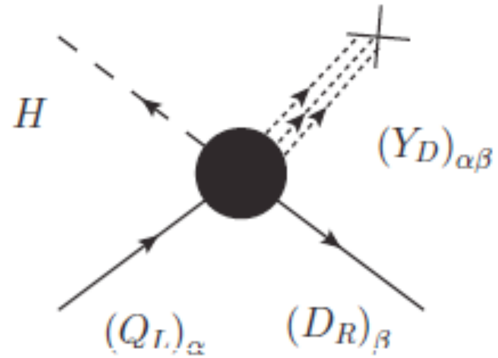
Maybe because of Majorana neutrinos?

May the Yukawas Y_U, Y_D, Y_E, Y_N... have a dynamical origin at high energies .... ?

(Anselm+Berezhiani 96; Berezhiani+Rossi 01)

$Y \sim <\varphi>$ or $Y \sim 1/<\varphi>$ or ......



(Alonso+Gavela+Merlo+Rigolin 11)

***What is the scalar potential for those fields ?**

May the Yukawas Y_U, Y_D, Y_E, Y_N... have a dynamical origin at high energies .... ?

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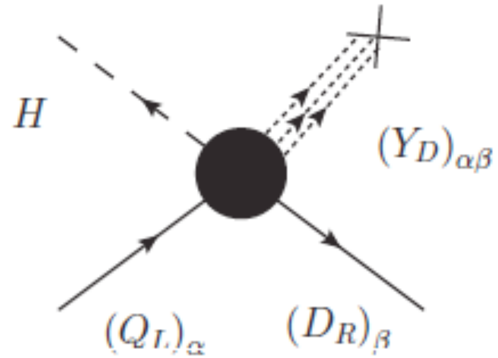

(Alonso+Gavela+Merlo+Rigolin 11)

***Does the minimum of its scalar potential justify the observed masses and mixings?**

Use the continuous flavour symetry of the SM for Y=0

The global Flavour symmetry of the SM with massless fermions:

$$G_f = \overbrace{SU(3)_{Q_L} \times SU(3)_{D_R} \times SU(3)_{U_R}}^{Quark} \times \overbrace{SU(3)_{\ell_L} \times SU(3)_{E_R}}^{Lepton} \times \cdots$$

$$D_R = \begin{pmatrix} d_R \\ s_R \\ b_R \end{pmatrix} \qquad D_R \sim (1, 3, 1 \cdots)$$

.... a symmetry also at the basis of Minimal Flavour Violation

(Chivukula+Georgi 87;; Hall+Randall; D'Ambrosio+Giudice+Isidori+Strumia; Cirigliano+Isidori+Grisntein +Wise; Davidson+Pallorini..)

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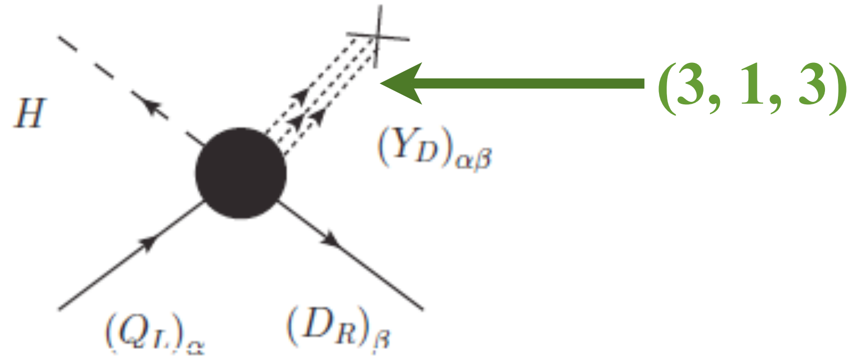
$Y \sim \langle \varphi \rangle$ or $Y \sim 1/\langle \varphi \rangle$ or ......



(Alonso+Gavela+Merlo+Rigolin 11)

***Does the minimum of its scalar potential justify the observed masses and mixings?**

For this talk:

each Y -- >one single field $\mathcal{Y}$

$$\mathbf{Y} \sim \frac{<\mathcal{Y}>}{\Lambda_{fl}}$$

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each Y -- >one single field $\mathcal{Y}$

$$Y \sim \frac{< \mathcal{Y} >}{\Lambda_{fl}}$$

¿$V(\mathcal{Y}_d, \mathcal{Y}_u)$?

$V(\mathcal{Y}_d, \mathcal{Y}_u)$

Construction of the Potential

* two families: 5 invariants at renormalizable level:
  (Feldman, Jung, Mannel)

$$\mathrm{Tr}\,(\mathcal{Y}_u \mathcal{Y}_u^{+}) \qquad \det(\mathcal{Y}_u)$$

$$\mathrm{Tr}\,(\mathcal{Y}_d \mathcal{Y}_d^{+}) \qquad \det(\mathcal{Y}_d)$$

$$\mathrm{Tr}\,(\mathcal{Y}_u \mathcal{Y}_u^{+} \mathcal{Y}_d \mathcal{Y}_d^{+})$$ **<-- mixing**

* non-renormalizable terms are simply functions of those !

$$V(\mathcal{Y}_u, \mathcal{Y}_u) = \Sigma_i [- \mu_i^2 \, \mathrm{Tr}(\mathcal{Y}_i \, \mathcal{Y}_i^+) - \tilde{\mu}_i^2 \det(\mathcal{Y}_i)]$$

$$+ \Sigma_{i \neq j} [\lambda_{ij} \, \mathrm{Tr}(\mathcal{Y}_i \, \mathcal{Y}_i^+ \, \mathcal{Y}_j \, \mathcal{Y}_j^+)] + \ldots$$

it only relies on G_f symmetry

and analyzed its minima

(Alonso, Gavela, Merlo, Rigolin, arXiv 1103.2915)

$V(\mathcal{Y}_d, \mathcal{Y}_u)$

Construction of the Potential

* two families: 5 invariants at renormalizable level:
(Feldman, Jung, Mannel)

$\mathrm{Tr}\,(\mathcal{Y}_u \mathcal{Y}_u^+)$ $\qquad$ $\det\,(\mathcal{Y}_u)$

$\mathrm{Tr}\,(\mathcal{Y}_d \mathcal{Y}_d^+)$ $\qquad$ $\det\,(\mathcal{Y}_d)$

$\mathrm{Tr}\,(\mathcal{Y}_u \mathcal{Y}_u^+ \mathcal{Y}_d \mathcal{Y}_d^+)$ <-- **mixing**

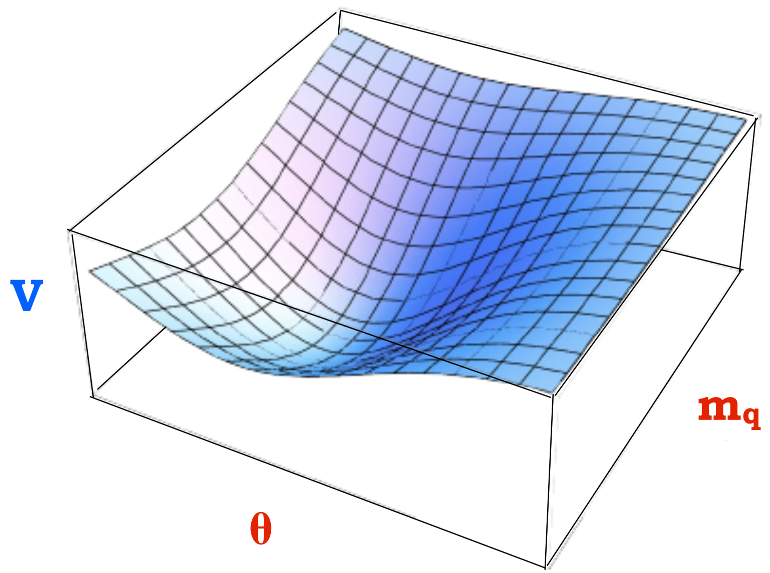
* non-renormalizable terms are simply functions of those !

Quarks

The minimum of $\mathbf{V}(\mathcal{Y}_{\mathbf{d}}, \mathcal{Y}_{\mathbf{u}})$ fixes $< \mathcal{Y}_{\mathbf{d,u}} > = V_{CKM}$ & m_q

At leading order, no mixing can come out of this potential

e.g. 2 families, at the minimum: $$(m_c^2 - m_u^2)(m_s^2 - m_d^2)\sin 2\theta = 0$$



same conclusion for 3 families

Berezhiani-Rossi; Anselm, Berezhiani; Alonso, Gavela, Merlo, Rigolin

Leptons

Just TWO heavy neutrinos

$$\mathcal{L}_{\mathcal{M}_\nu} = (\bar{\ell}_L\,,\, \bar{N}^c\,,\, \bar{N}'^c) \begin{pmatrix} 0 & vY & vY' \\ vY^T & 0 & \mathbf{M} \\ vY'^T & \mathbf{M} & 0 \end{pmatrix} \begin{pmatrix} \ell_L^c \\ N \\ N' \end{pmatrix}$$

the Yukawas are determined up to their overal magnitude

$$\text{N.H.} \qquad Y = \frac{y}{\sqrt{m_{\nu_2} + m_{\nu_3}}} U_{PMNS} \begin{pmatrix} 0 \\ -i\sqrt{m_{\nu_2}}e^{-i\alpha} \\ \sqrt{m_{\nu_3}}e^{i\alpha} \end{pmatrix}$$

The flavour symmetry is $G_f = SU(3)_{\ell_L} \times SU(3)_{E_R} \times O(2)_N$

(Alonso, Gavela, D. Hernandez, Merlo in preparation)

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(Alonso+Gavela+D. Hernandez+Merlo 12)

The mixing terms in **V** is now:

$$\mathrm{Tr}(\mathcal{Y}_E\,\mathcal{Y}_E^{+}\,\mathcal{Y}_\nu\,\mathcal{Y}_\nu^{+}) \propto \Big\{ \sum_{l,i} |U^{li}_{PMNS}|^2 m_l^2 m_{\nu_i} +$$

$$\Big[i\, e^{2i\alpha} \sum_{l,i<j} U^{li}_{PMNS} (U^{lj}_{PMNS})^* m_l^2 \sqrt{m_{\nu_i} m_{\nu_j}} + c.c. \Big] \Big\}$$

while for quarks it was:

$$\mathrm{Tr}(\mathcal{Y}_u\,\mathcal{Y}_u^{+}\,\mathcal{Y}_d\,\mathcal{Y}_d^{+}) \propto \sum_{i,j} |U^{ij}_{CKM}|^2 m_{u_i}^2 m_{d_j}^2$$

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extra term because of Majorana character

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e.g., **for 2 generations**, the mixing terms in $\mathbf{V}(\mathcal{Y}_\mathbf{E}, \mathcal{Y}_\mathbf{\nu})$ is :

Leptons

$$\mathrm{Tr}(\mathcal{Y}_E \mathcal{Y}_E^+ \mathcal{Y}_\nu \mathcal{Y}_\nu^+) \propto (m_\mu^2 - m_e^2)\left((m_{\nu_2} - m_{\nu_1}) \cos 2\theta + 2\sqrt{m_{\nu_2} m_{\nu_1}} \sin 2\alpha \sin 2\theta\right)$$

Quarks

$$\mathrm{Tr}(\mathcal{Y}_u \mathcal{Y}_u^+ \mathcal{Y}_d \mathcal{Y}_d^+) \propto (m_c^2 - m_u^2)(m_s^2 - m_d^2) \cos 2\theta$$

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Renormalizable level: $\partial_\theta V = 0$ yields:

$$\mathrm{tg}2\theta = \sin 2\alpha \frac{2\sqrt{m_{\nu_2} m_{\nu_1}}}{m_{\nu_2} - m_{\nu_1}}, \qquad \sin 2\theta \cos 2\alpha = 0, \quad \alpha = \pi/4 \text{ or } 3\pi/4$$

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Leptons

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new term: related with the fact that for neutrinos there is non-trivial mixing even for $m_{\nu 1} = m_{\nu 2}$, for non-zero Majorana phase e.g., for 2 generations, the mixing terms in $\mathbf{V}(\mathcal{Y}_{\mathbf{E}}, \mathcal{Y}_{\mathbf{\nu}})$ is :

Leptons

$$\mathrm{Tr}(\mathcal{Y}_E\, \mathcal{Y}_E^+\, \mathcal{Y}_\nu\, \mathcal{Y}_\nu^+) \propto (m_\mu^2 - m_e^2)\left((m_{\nu_2} - m_{\nu_1})\cos 2\theta + 2\sqrt{m_{\nu_2} m_{\nu_1}}\sin 2\alpha \sin 2\theta\right)$$

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It leads to: *** large angles correlated with degenerate masses**

*** maximal Majorana phase**

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It leads to: *** large angles correlated with degenerate masses**

*** maximal Majorana phase**

(i.e. relative phase is $2\alpha = \pi/2$: NO Majorana CP viol.)

Preliminary:

* Relation for general 2-family model with degenerate heavy neutrinos

1) Take arbitrary 2-family neutrino model:

2) Use Casas-Ibarra parametrization $\mathbf{Y} = \mathbf{U}_{PMNS}\, \mathbf{m}_\nu^{1/2}\, \mathbf{R}\, \mathbf{M}^{1/2}$

→ for degenerate heavy neutrinos **R** one depends only on one parameter

$$\mathbf{R} = \begin{pmatrix} \text{ch}\,\omega & -i\,\text{sh}\,\omega \\ i\,\text{sh}\,\omega & \text{ch}\,\omega \end{pmatrix}$$

leading to

$$\text{tg}2\theta \propto \sin 2\alpha \frac{2\sqrt{m_{\nu_2} m_{\nu_1}}}{m_{\nu_2} - m_{\nu_1}} \sin 2\omega$$

and still $\alpha = \pi/4$ or $3\pi/4$

Preliminary:

*** Relation for general 2-family model with degenerate heavy neutrinos**

***3 families: ongoing**

Data could point to $m_q \sim Y^{-1}$...
as in gauged-flavour realizations of MFV
(Grinstein, Redi, Villadoro; Feldman; Guadagnoli, Mohapatra, Suhn)

Conclusions

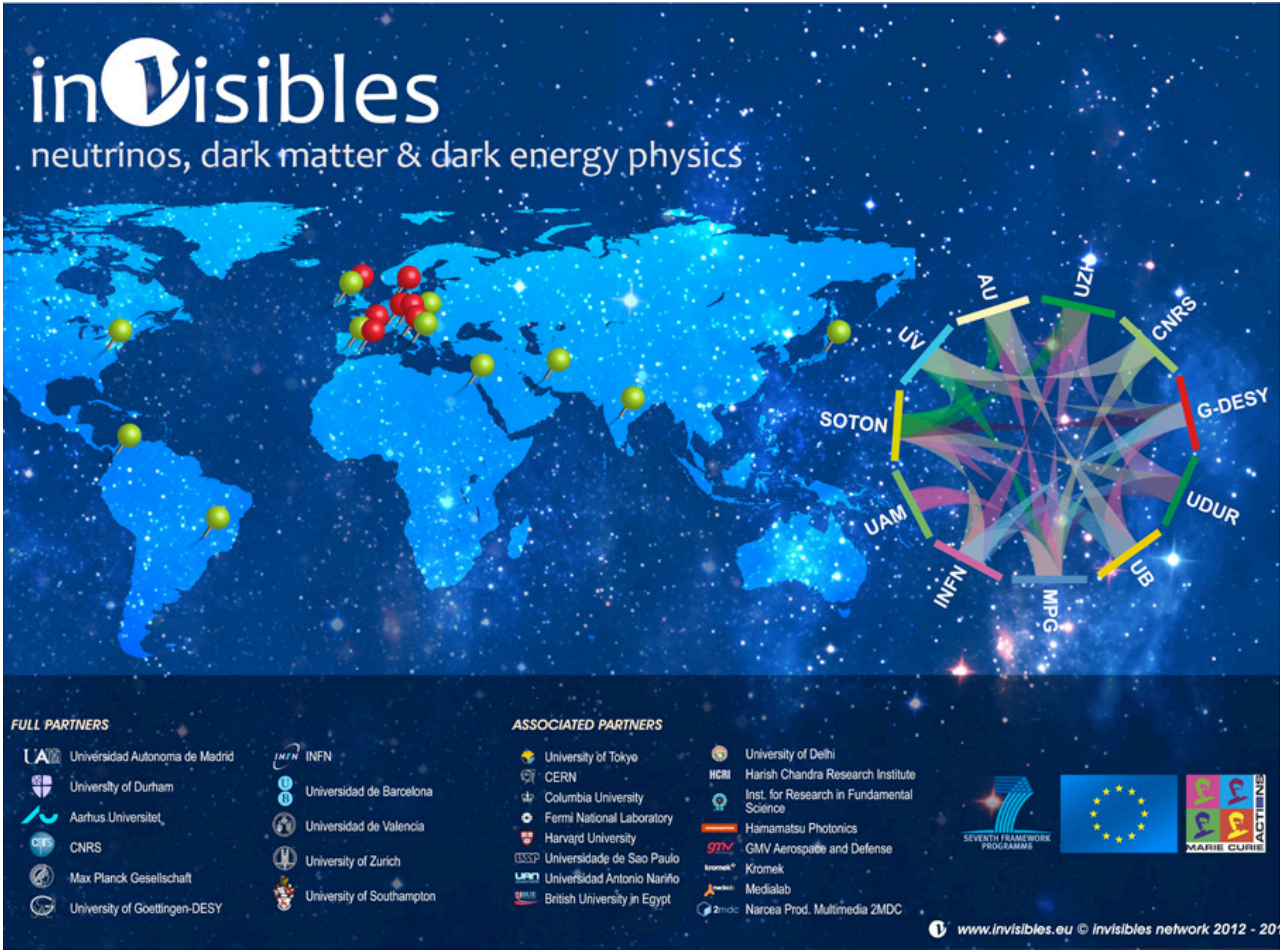
* Planned **μ-e conversion** experiments, taken by themselves, are sensitive to sterile neutrino masses as low as 2 MeV.
They may detect or constraint sterile neutrinos in the **2 GeV- 6000 TeV** mass range !

* The idea of **dynamical origin for the Yukawa** couplings, based on the continuous flavour symmetry of the SM:

a) allows to tackle flavour for both quarks and leptons and for both masses and mixings

b) Simplest case of one Yukawa<-> one field, at leading order:
-- **Vanishing mixing for quarks**; and for Dirac neutrinos.
-- **For Majorana neutrinos**:
predicts **large Majorana phase $2\alpha= \pi/4$**
large/small mixings <----> degenerate/hierarchical masses

Back-up slides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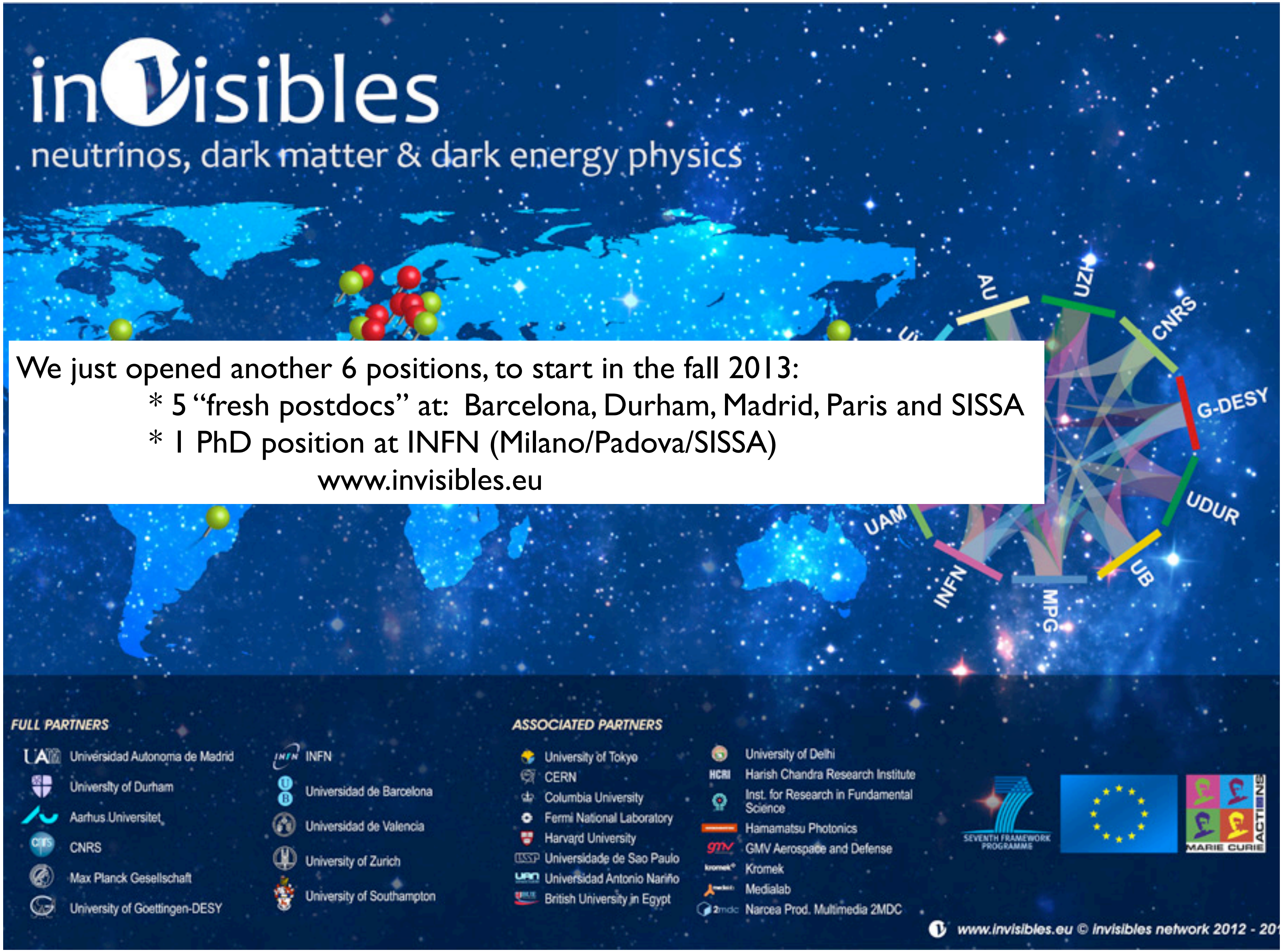

We just opened another 6 positions, to start in the fall 2013:

* 5 "fresh postdocs" at: Barcelona, Durham, Madrid, Paris and SISSA
* 1 PhD position at INFN (Milano/Padova/SISSA)

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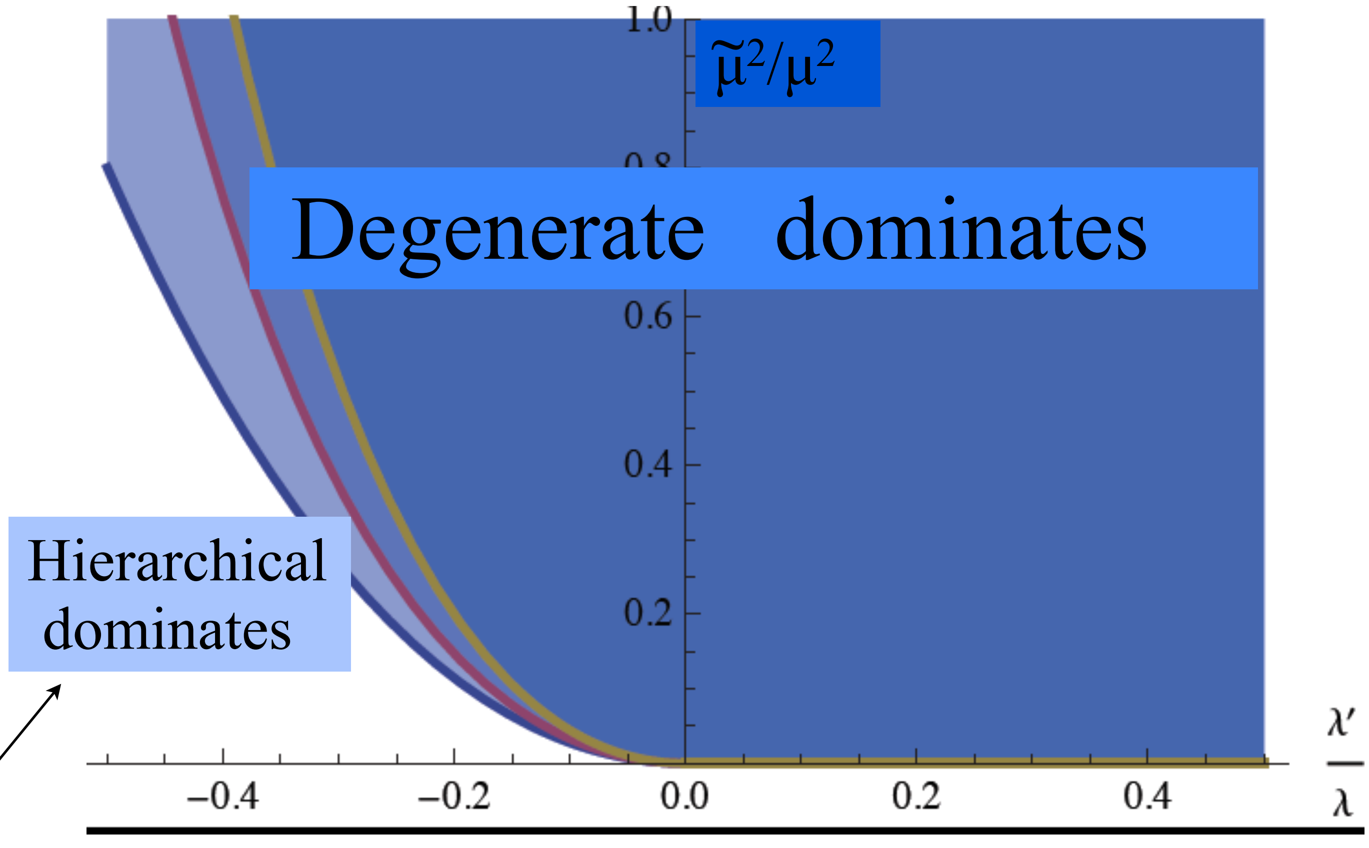
Spectrum: the hierarchical solution is unstable in most of the parameter space.

Stability: $\frac{\tilde{\mu}^2}{\mu^2} < \frac{2\lambda'^2}{\lambda}$

$$V^{(4)} = \sum_{i=u,d} \left(-\mu_i^2 A_i + \tilde{\mu}_i B_i + \lambda_i A_i^2 + \lambda_i' A_{ii}\right) + g_{ud} A_u A_d + \lambda_{ud} A_{ud}\,.$$

ie, the u-part: $V^{(4)} = -\mu_u^2 A_u + \tilde{\mu}_u B_u + \lambda_u A_u^2 + \lambda_u' A_{uu}$



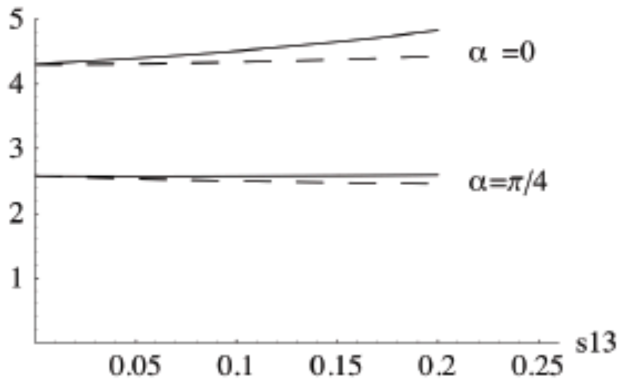
Nardi emphasized this solution (and extended the analysis to include also U(1) factors)

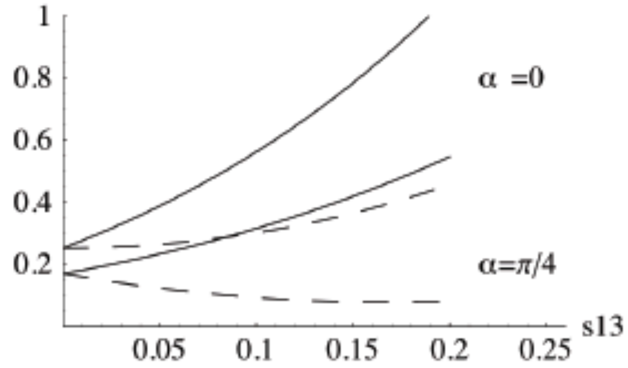
* e-μ, μ-τ etc. oscillations and rare decays studied:

Gavela, Hambye, Hernandez[2] ; .....

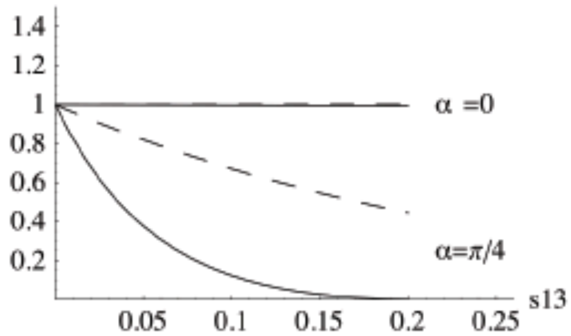
$Br(\mu \to e\gamma)/Br(\tau \to e\gamma)$ $\qquad$ $Br(\mu \to e\gamma)/Br(\tau \to \mu\gamma)$

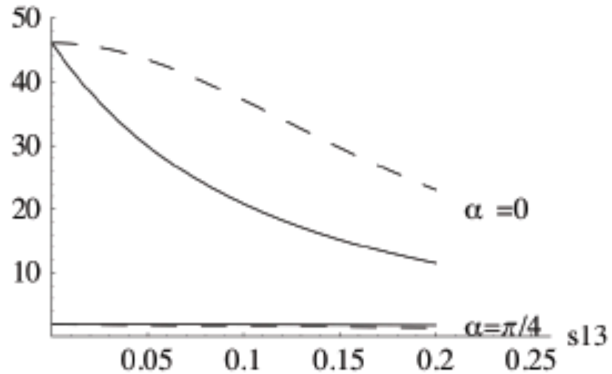
NH





IH





Gavela, Hambye, Hernandez[2] ;

Degeneracy in the Majorana phase α

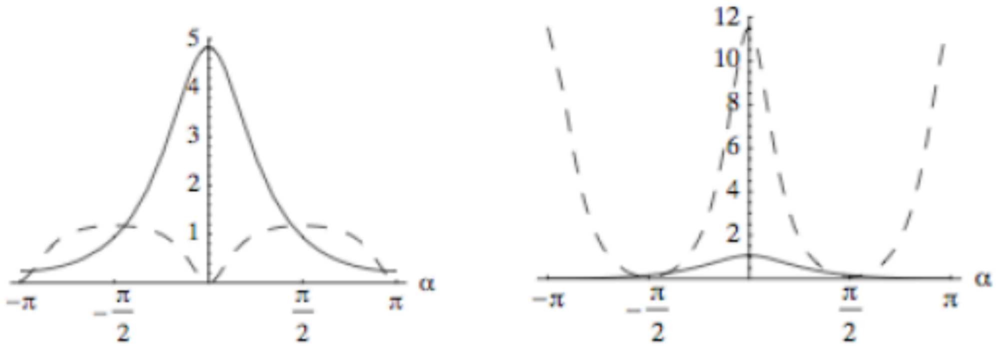
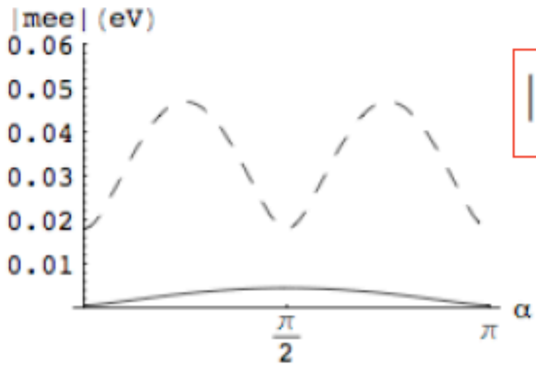


Figure 3: Left: Ratio $B_{e\mu}/B_{e\tau}$ for the normal hierarchy (solid) and the inverse hierarchy (dashed) as a function of α for $(\delta, s_{13}) = (0, 0.2)$. Right: the same for the ratio $B_{e\mu}/B_{\mu\tau}$.



$$|m_{ee}|_{IH} \simeq |s_{12}^2 e^{-2i\alpha} - c_{12}^2 e^{2i\alpha}|$$

Figure 5: m_{ee} as a function of α for the normal (solid) and inverted (dashed) hierarchies, for $(\delta, s_{13}) = (0, 0.2)$.

Gavela, Hambye, Hernandez[2] ;

$$B_{\mu\to e\gamma} \propto |Y_{N_e} Y_{N_\mu}|^2$$

cancellations
for large θ_{13}

i.e.

$$Y_N^T \simeq y \begin{pmatrix} e^{i\delta} s_{13} + e^{-i\alpha} s_{12} r^{1/4} \\ s_{23}\left(1 - \frac{\sqrt{r}}{2}\right) + e^{-i\alpha} r^{1/4} c_{12} c_{23} \\ c_{23}\left(1 - \frac{\sqrt{r}}{2}\right) - e^{-i\alpha} r^{1/4} c_{12} s_{23} \end{pmatrix} \qquad r = \frac{|\Delta m^2_{12}|}{|\Delta m^2_{13}|}$$

Normal hierarchy

* Alonso + Li, 2010, MINSIS report:
possible suppresion of μ-e transitions for large θ_{13}

* e-μ, μ-τ etc. oscillations and rare decays studied:

Gavela, Hambye, Hernandez[2] 09 ; .....

* Alonso + Li, 2010: possible suppresion of μ-e transitions

->important impact of ν_μ - ν_τ at a near detectors

$$B_{\mu\to e\gamma} \propto |Y_{N_e} Y_{N_\mu}|^2$$

i.e.

$$Y_N^T \simeq y \begin{pmatrix} e^{i\delta} s_{13} + e^{-i\alpha} s_{12} r^{1/4} \\ s_{23}\left(1 - \frac{\sqrt{r}}{2}\right) + e^{-i\alpha} r^{1/4} c_{12} c_{23} \\ c_{23}\left(1 - \frac{\sqrt{r}}{2}\right) - e^{-i\alpha} r^{1/4} c_{12} s_{23} \end{pmatrix} \qquad r = \frac{|\Delta m_{12}^2|}{|\Delta m_{13}^2|}$$

Normal hierarchy

QUARKS

$$G_f = \overbrace{SU(3)_{Q_L} \times SU(3)_{D_R} \times SU(3)_{U_R}}^{Quark}$$

$$\mathcal{Y}_d \sim (3, \bar{3}, 1) \qquad \mathcal{Y}_u \sim (3, 1, \bar{3})$$

$$\boxed{\frac{<\mathcal{Y}_d>}{\Lambda_f} = Y_D = V_{CKM} \begin{pmatrix} y_d & 0 & 0 \\ 0 & y_s & 0 \\ 0 & 0 & y_b \end{pmatrix}}, \quad \boxed{\frac{<\mathcal{Y}_u>}{\Lambda_f} = Y_U = \begin{pmatrix} y_u & 0 & 0 \\ 0 & y_c & 0 \\ 0 & 0 & y_t \end{pmatrix}}.$$

¿V($\mathcal{Y}_d$, $\mathcal{Y}_u$)?

QUARKS

$$G_f = \overbrace{SU(3)_{Q_L} \times SU(3)_{D_R} \times SU(3)_{U_R}}^{Quark}$$

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mixing--> $\mathrm{Tr}\left(\mathcal{Y}_u \, \mathcal{Y}_u^{+} \, \mathcal{Y}_d \, \mathcal{Y}_d^{+} \right)$

$$SU(3)_{\ell_L} \times SU(3)_{E_R} \times O(2)_N$$

$$Y_E = \frac{< \mathcal{Y}_E >}{\Lambda_f} \sim (3\,,\,\bar{3}\,,1)\,; \quad (Y\,,\,Y') = \frac{< \mathcal{Y}_\nu >}{\Lambda} \sim (3\,,\,1\,,2)$$

$$< \mathcal{Y}_E > \propto \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix} \quad < \mathcal{Y}_\nu > \propto U_{PMNS} \begin{pmatrix} 0 & 0 \\ \sqrt{m_{\nu_2}} & 0 \\ 0 & \sqrt{m_{\nu_3}} \end{pmatrix} \begin{pmatrix} -iy & iy' \\ y & y' \end{pmatrix}$$

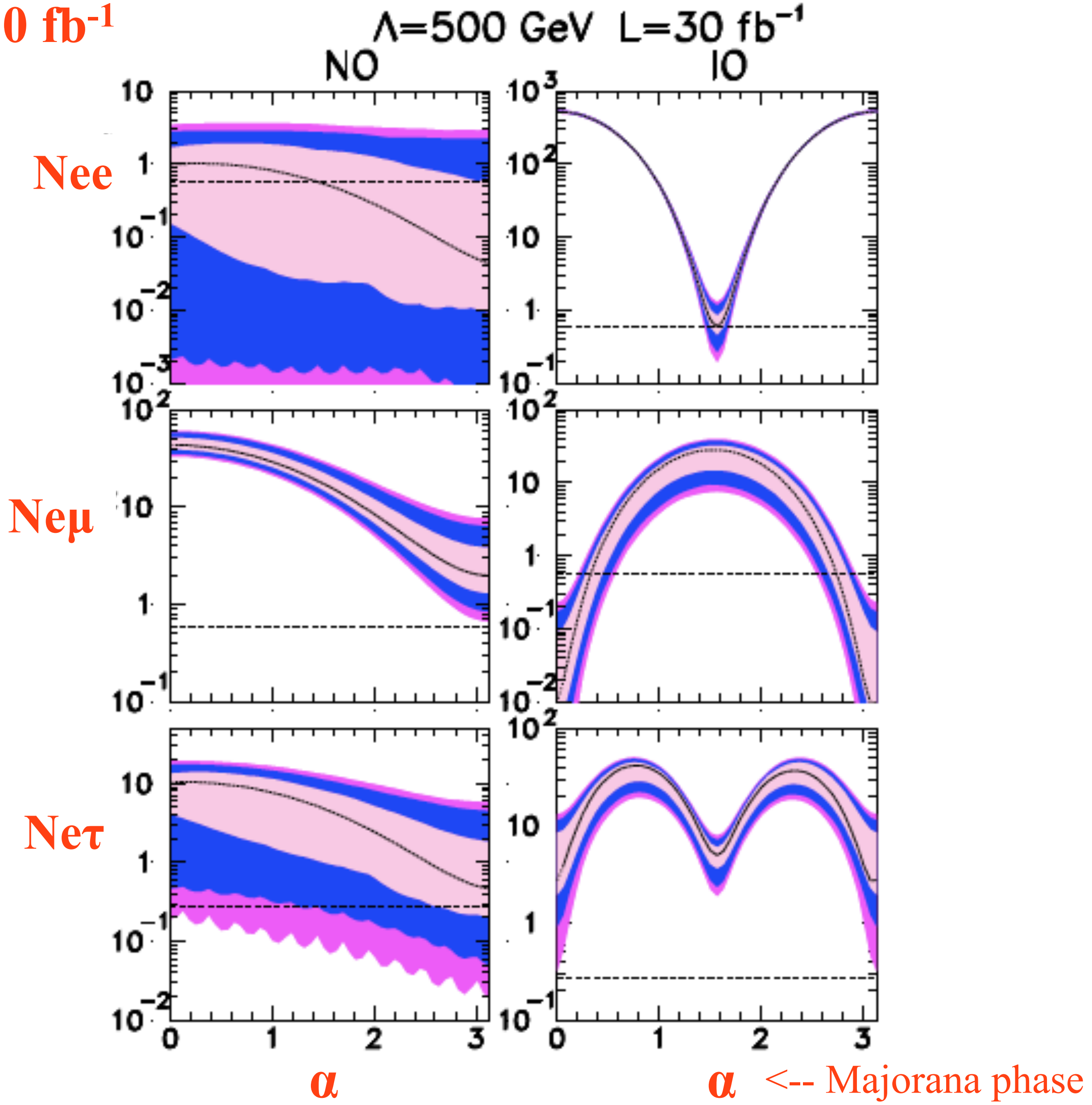
$$\mathscr{L}_{M_\nu} = \begin{array}{c} \\ \end{array} \begin{array}{ccc} \bar{L}_i & \bar{N}^c & \bar{N}'^c \end{array}$$

$$\mathscr{L}_{M_\nu} = \begin{pmatrix} 0 & Y^T v & 0 \\ Y v & 0 & \Lambda^T \\ 0 & \Lambda & 0 \end{pmatrix}$$

For type III version of our 2 N model, signals observable at LHC up to $\Lambda \sim$ 500 GeV for 30 fb^{-1}

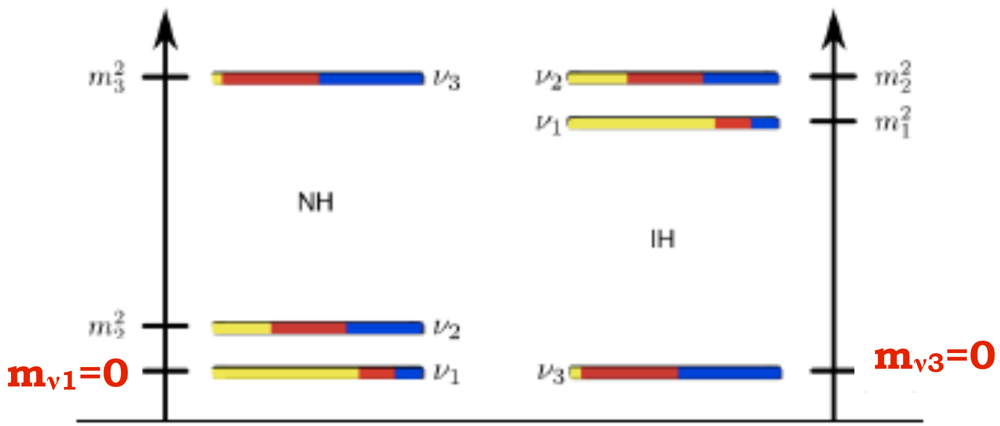
Eboli,
Gonzalez-Fraile,
Gonzalez-Garcia

$$pp \to \ell_a^{\pm}\ell_b^{\mp} jjjj$$



For 3 generations:

* In this particular model, the fact that $m_{\nu 3}=0$ imposes strong hierarchies with $m_{\nu 1}$, $m_{\nu 2}$.



* Data could point to $m_q \sim Y^{-1}$
… as in gauged-flavour realizations of MFV

(Grinstein, Redi, Villadoro; Feldman; Guadagnoli, Mohapatra, Suhr

* There are several possibilities... under exploration

Alonso, D. Hernandez, Melo, B.G.

The scalar potential of MFV

Quark sector: with R. Alonso, L. Merlo, S. Rigolin and J. Yepes

Lepton sector: with R. Alonso and D. Hernandez

QUARKS

The Dynamics Behind MFV

MFV suggests that Y_U & Y_D have a dynamical origin at high energies ....

$$Y \sim \langle \varphi \rangle \quad \text{or} \quad \langle \varphi\, \chi \rangle \quad \text{or} \quad \langle (\quad)^n \rangle \ldots$$

Spontaneous breaking of flavour symmetry dangerous

--> i.e. gauge it (Grinstein, Redi, Villadoro, 2010)
(Feldman, 2010)
(Guadagnoli, Mohapatra, Sung, 2010)

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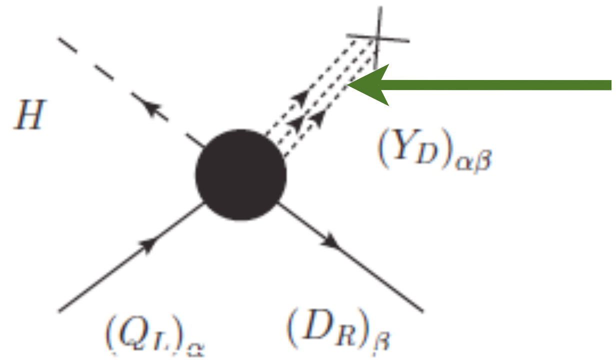

$(3, 1, \bar{3})$

(Alonso, Gavela, Merlo, Rigolin, arXiv 1103.2915)

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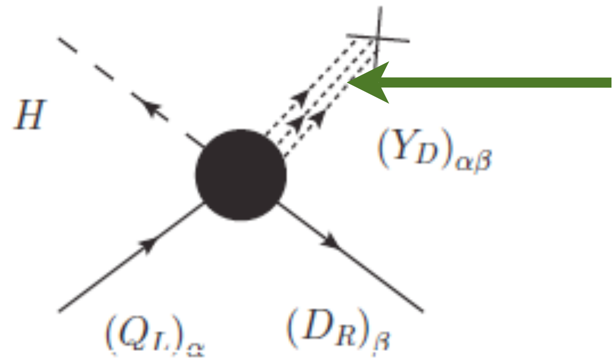


$(3, 1, \bar{3})$

That scalar field or aggregate of fields may have a potential

(Alonso, Gavela, Merlo, Rigolin, arXiv 1103.2915)

The Dynamics Behind MFV

MFV suggests that Y_U & Y_D have a dynamical origin at high energies ....

$$Y \sim \langle \varphi \rangle \quad \text{or} \quad \langle \varphi\, \chi \rangle \quad \text{or} \quad \langle (\quad)^n \rangle \ldots$$

***What is the potential of Minimal Flavour Violation ?**

(Alonso, Gavela, Merlo, Rigolin, arXiv 1103.2915)

The Dynamics Behind MFV

MFV suggests that Y_U & Y_D have a dynamical origin at high energies ....

$$Y \sim \langle \varphi \rangle \quad \text{or} \quad \langle \varphi\, \chi \rangle \quad \text{or} \quad \langle (\quad)^n \rangle \ldots$$

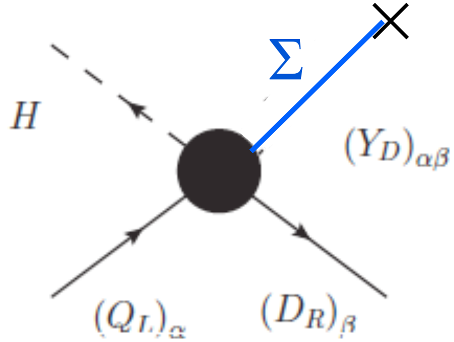
*What is the potential of Minimal Flavour Violation ?

*Can its minimum correspond **naturally** to the observed masses and mixings?

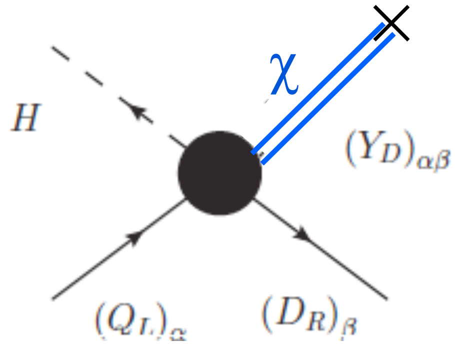
(Alonso, Gavela, Merlo, Rigolin, arXiv 1103.2915)

We constructed the scalar potential for both 2 and 3 families, for scalar fields:

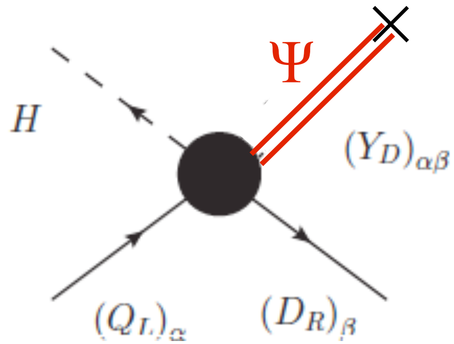
1) Y -- > one single scalar $\Sigma \sim (3, 1, \overline{3})$



2) Y -- > two scalars $\chi\, \chi^+ \sim (3, 1, \overline{3})$



3) Y -- > two fermions $\overline{\Psi}\Psi \sim (3, 1, 3)$



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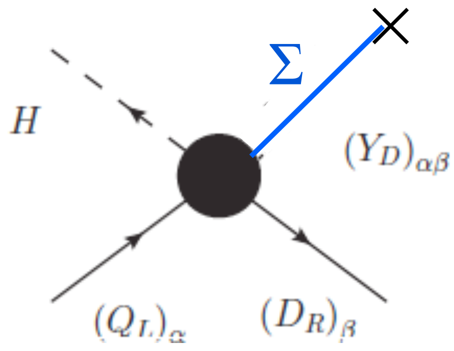
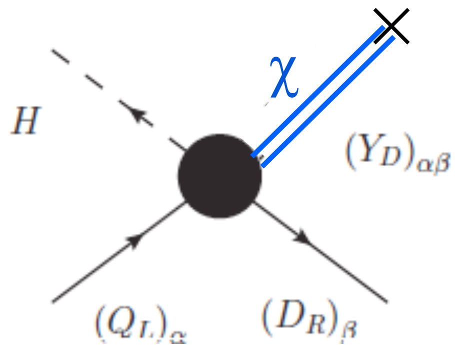
d=5 operator



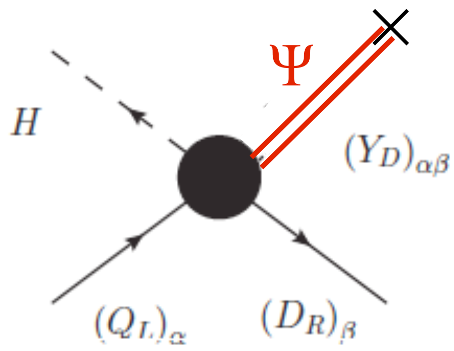
2) Y -- > two scalars $\chi\,\chi^+ \sim (3, 1, \overline{3})$

d=6 operator



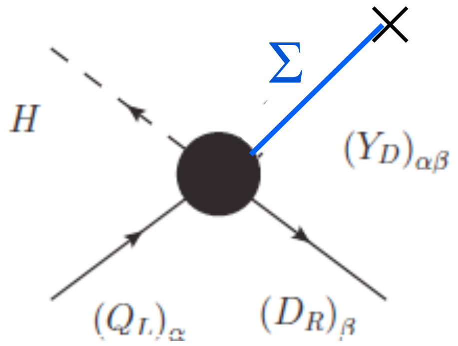
3) Y -- > two fermions $\overline{\Psi}\Psi \sim (3, 1, 3)$

d=7 operator



1) Y --> one single field Σ

$$Y \sim \frac{\langle \Sigma \rangle}{\Lambda_f}$$



* What is the general potential V(Σ , H) invariant under SU(3)xSU(2)xU(1) and G_f ?

$$Y_u \longleftrightarrow \langle \Sigma_u \rangle \; ; \quad Y_d \longleftrightarrow \langle \Sigma_d \rangle$$

Construction of the Potential

* two families: 5 invariants at renormalizable level:
  (Feldman, Jung, Mannel)

$$\mathrm{Tr}\,(\Sigma_u \Sigma_u^{+}) \qquad \det(\Sigma_u)$$

$$\mathrm{Tr}\,(\Sigma_d \Sigma_d^{+}) \qquad \det(\Sigma_d)$$

$$\mathrm{Tr}\,(\Sigma_u \Sigma_u^{+} \Sigma_d \Sigma_d^{+})$$

* non-renormalizable terms are simply functions of those !

Y --> one single field Σ

We constructed the most general potential :

$$V(\Sigma_u, \Sigma_d) = \sum_i \left[-\mu_i^2 \,\mathrm{Tr}(\Sigma_i \Sigma_i^+) - \tilde{\mu}_i^2 \det(\Sigma_i) \right]$$

$$+ \sum_{i,j} \left[\lambda_{ij} \,\mathrm{Tr}(\Sigma_i \Sigma_i^+)\,\mathrm{Tr}(\Sigma_j \Sigma_j^+) + \tilde{\lambda}_{ij} \det(\Sigma_i) \det(\Sigma_j) \right] + \ldots$$

it only relies on G_f symmetry

and analyzed its minima

(Alonso, Gavela, Merlo, Rigolin, arXiv 1103.2915)

Y --> one single field Σ

The invariants can be written in terms of masses and mixing

* two families:

$$< \Sigma_d > = \Lambda_f \,.\, \mathrm{diag}\,(y_d)\,; \qquad < \Sigma_u > = \Lambda_f \,.\, V_{Cabibbo}\, \mathrm{diag}(y_u)$$

$$Y_D = \begin{pmatrix} y_d & 0 \\ 0 & y_s \end{pmatrix}, \qquad Y_U = \mathcal{V}_C^\dagger \begin{pmatrix} y_u & 0 \\ 0 & y_c \end{pmatrix} \qquad V_{Cabibbo} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

$$<\mathrm{Tr}\,(\Sigma_u \Sigma_u^+)> = \Lambda_f^{\,2}\,(y_u^2 + y_c^2)\,; \quad <\det(\Sigma_u)> = \Lambda_f^{\,2}\, y_u y_c$$

$$<\mathrm{Tr}\,(\Sigma_u \Sigma_u^+ \Sigma_d \Sigma_d^+)> = \Lambda_f^4\, [(y_c^2 - y_u^2)(y_s^2 - y_d^2)\cos 2\theta + \ldots]/2$$

(Alonso, Gavela, Merlo, Rigolin, arXiv 1103.2915)

Y --> one single field Σ

Minimum of the Potential

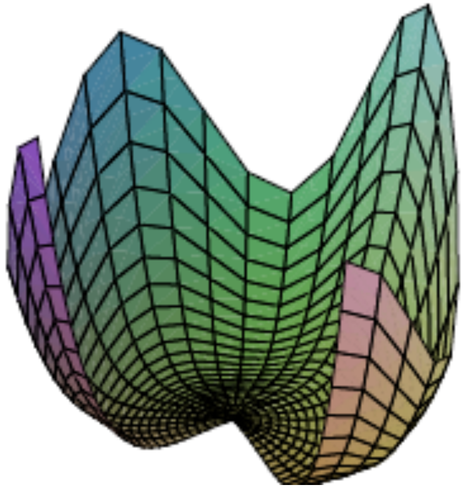
Dimension 5 Yukawa Operator

The minimum of the Potential is given by:

$$\frac{\partial V}{\partial y_i} = 0 \qquad \frac{\partial V}{\partial \theta_i} = 0$$

Take the angle for example:

$$\frac{\partial V}{\partial \theta_c} \propto (y_c^2 - y_u^2)\,(y_s^2 - y_d^2)\,\sin 2\theta_c = 0$$



Non-degenerate masses $\longrightarrow$ $\sin 2\theta_c = 0$ No mixing !

Notice also that $\frac{\partial V^{(4)}}{\partial \theta} \sim \sqrt{J}$ (Jarlskog determinant)

(Alonso, Gavela, Merlo, Rigolin, arXiv 1103.2915)

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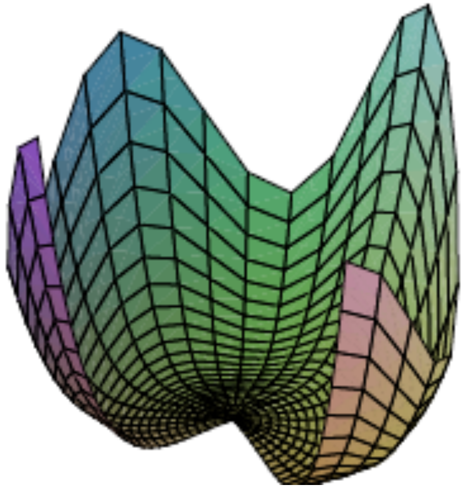

Take the angle for example:

$$\frac{\partial V}{\partial \theta_c} \propto \left(y_c^2 - y_u^2\right)\left(y_s^2 - y_d^2\right) \sin 2\theta_c = 0$$

Non-degenerate masses $\longrightarrow \sin 2\theta_c = 0$ No mixing !

Can the actual masses and mixings fit naturally in the minimum of the Potential? e.g. adding non-renormalizable terms... **NO**

(Alonso, Gavela, Merlo, Rigolin, arXiv 1103.2915)

Y --> one single field Σ

Minimum of the Potential

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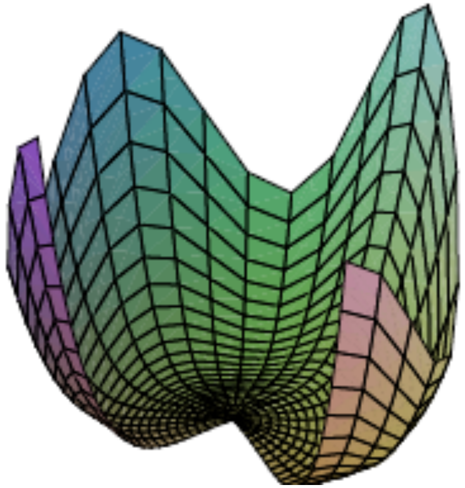

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Non-degenerate masses $\sin 2\theta_c = 0$ No mixing !

Can the actual masses and mixings fit naturally in the minimum of the Potential? e.g. adding non-renormalizable terms...

NO

* Without fine-tuning, for two families the spectrum is degenerate

* To accomodate realistic mixing one must introduce wild fine tunnings of $O(10^{-10})$ and nonrenormalizable terms of dimension 8

(Alonso, Gavela, Merlo, Rigolin)

Y --> one single field Σ

three families

* at renormalizable level: 7 invariants instead of the 5 for two families

$$\mathrm{Tr}\left(\Sigma_u\Sigma_u^\dagger\right) \overset{vev}{=} \Lambda_f^2\left(y_t^2+y_c^2+y_u^2\right)\,,\qquad Det\left(\Sigma_u\right) \overset{vev}{=} \Lambda_f^3 y_u y_c y_t\,,$$

$$\mathrm{Tr}\left(\Sigma_d\Sigma_d^\dagger\right) \overset{vev}{=} \Lambda_f^2\left(y_b^2+y_s^2+y_d^2\right)\,,\qquad Det\left(\Sigma_d\right)\Big| \overset{vev}{=} \Lambda_f^3 y_d y_s y_b\,,$$

$$\mathrm{Tr}\left(\Sigma_u\Sigma_u^\dagger\Sigma_u\Sigma_u^\dagger\right) \overset{vev}{=} \Lambda_f^4\left(y_t^4+y_c^4+y_u^4\right)\,,$$

$$\mathrm{Tr}\left(\Sigma_d\Sigma_d^\dagger\Sigma_d\Sigma_d^\dagger\right) \overset{vev}{=} \Lambda_f^4\left(y_b^4+y_s^4+y_d^4\right)\,,$$

$$\mathrm{Tr}\left(\Sigma_u\Sigma_u^\dagger\Sigma_d\Sigma_d^\dagger\right) \overset{vev}{=} \Lambda_f^4\left(P_0+P_{int}\right)\,,$$

Interesting angular dependence:

$$P_0 \equiv -\sum_{i<j}\left(y_{u_i}^2-y_{u_j}^2\right)\left(y_{d_i}^2-y_{d_j}^2\right)\sin^2\theta_{ij}\,,$$

$$\begin{aligned} P_{int} \equiv \sum_{i<j,k} & \left(y_{d_i}^2-y_{d_k}^2\right)\left(y_{u_j}^2-y_{u_k}^2\right)\sin^2\theta_{ik}\sin^2\theta_{jk}+ \\ & -\left(y_d^2-y_s^2\right)\left(y_c^2-y_t^2\right)\sin^2\theta_{12}\sin^2\theta_{13}\sin^2\theta_{23}+ \\ & +\frac{1}{2}\left(y_d^2-y_s^2\right)\left(y_c^2-y_t^2\right)\cos\delta\,\sin 2\theta_{12}\sin 2\theta_{23}\sin\theta_{13}\,, \end{aligned}$$

Y --> one single field Σ

Spectrum for flavons Σ in the bifundamental:

* **3 generations: for the largest fraction of the parameter space, the stable solution is a degenerate spectrum**

$$\begin{pmatrix} y_u & & \\ & y_c & \\ & & y_t \end{pmatrix} \sim \begin{pmatrix} y & & \\ & y & \\ & & y \end{pmatrix}$$

instead of the observed hierarchical spectrum, i.e.

$$\begin{pmatrix} y_u & & \\ & y_c & \\ & & y_t \end{pmatrix} \sim \begin{pmatrix} 0 & & \\ & 0 & \\ & & y \end{pmatrix}$$

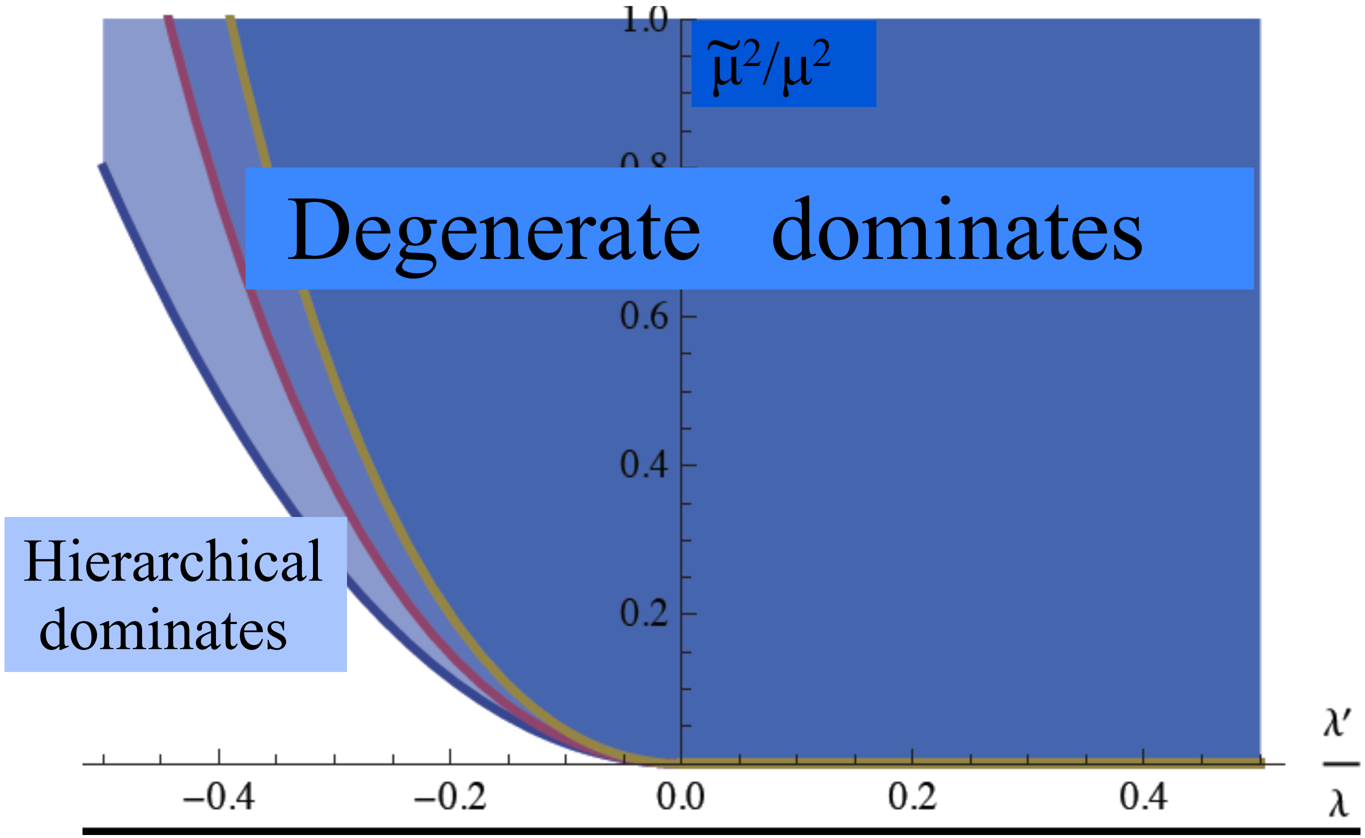
(at leading order)

Spectrum: the hierarchical solution is unstable in most of the parameter space.

Stability: $\frac{\tilde{\mu}^2}{\mu^2} < \frac{2\lambda'^2}{\lambda}$

$$V^{(4)} = \sum_{i=u,d} \left(-\mu_i^2 A_i + \tilde{\mu}_i B_i + \lambda_i A_i^2 + \lambda_i' A_{ii}\right) + g_{ud} A_u A_d + \lambda_{ud} A_{ud} \, .$$

ie, the u-part: $V^{(4)} = -\mu_u^2 A_u + \tilde{\mu}_u B_u + \lambda_u A_u^2 + \lambda_u' A_{uu}$



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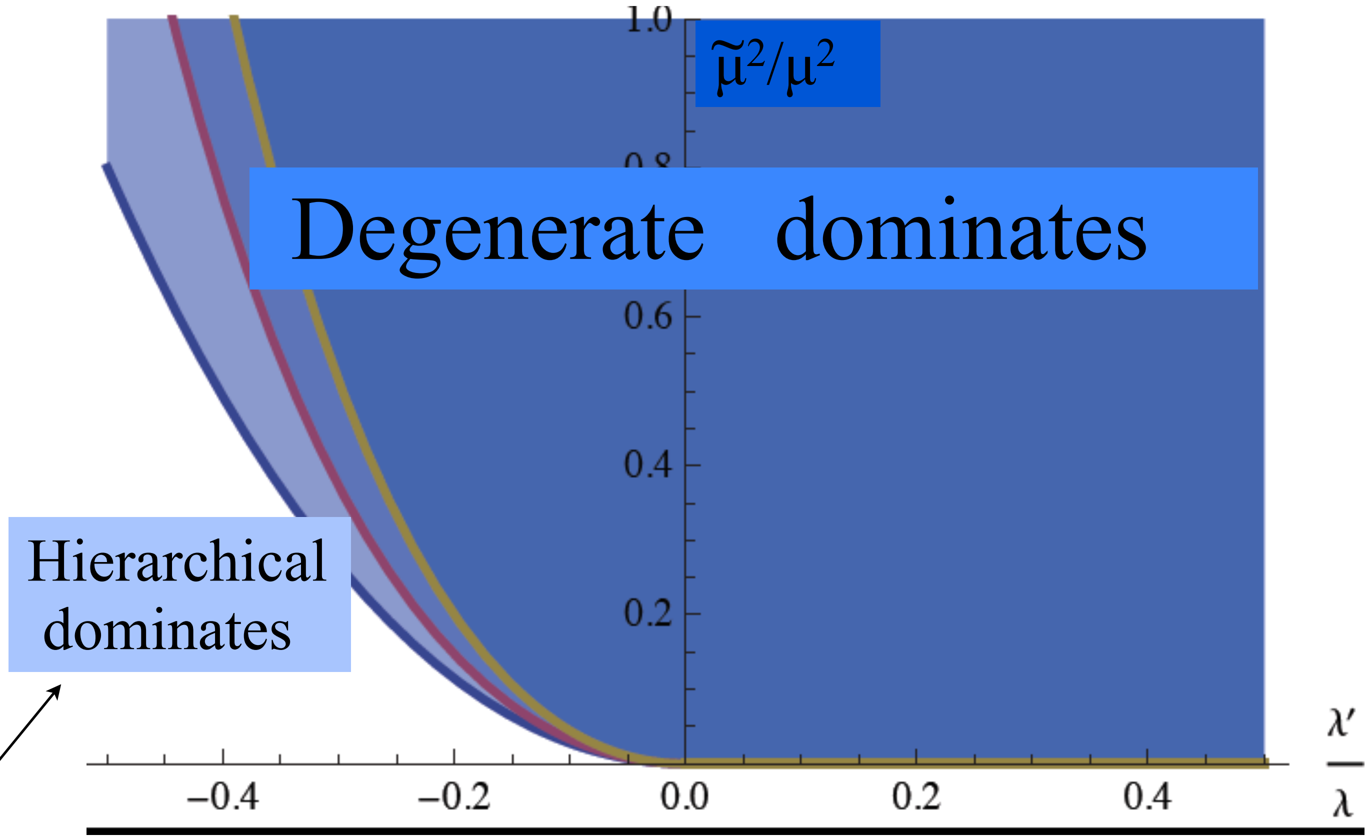


Nardi emphasized this solution (and extended the analysis to include also U(1) factors)

3 (or any number of) families

* Only one invariant contains mixing, at renormalizable level, with general form

$$S \propto \sum_{i,j} \left| U_{CKM}^{ij} \right|^2 m_{u_i}^2 m_{d_j}^2$$

The real, unavoidable, problem is again mixing:

* Just one source:

$$\mathrm{Tr}\,(\Sigma_u \Sigma_u^+ \Sigma_d \Sigma_d^+) = \Lambda_f^4 (P_0 + P_{int})$$

P_0 and P_{int} encode the angular dependence,

$$P_0 \equiv -\sum_{i<j} \left(y_{u_i}^2 - y_{u_j}^2\right)\left(y_{d_i}^2 - y_{d_j}^2\right)\sin^2\theta_{ij}\,,$$

$$P_{int} \equiv \sum_{i<j,k} (y_{d_i}^2 - y_{d_k}^2)\left(y_{u_j}^2 - y_{u_k}^2\right)\sin^2\theta_{ik}\sin^2\theta_{jk} +$$

$$-\,(y_d^2 - y_s^2)\,(y_c^2 - y_t^2)\sin^2\theta_{12}\sin^2\theta_{13}\sin^2\theta_{23} +$$

$$+\,\frac{1}{2}\,(y_d^2 - y_s^2)\,(y_c^2 - y_t^2)\cos\delta\,\sin 2\theta_{12}\sin 2\theta_{23}\sin\theta_{13}$$

Sad conclusions as for 2 families:

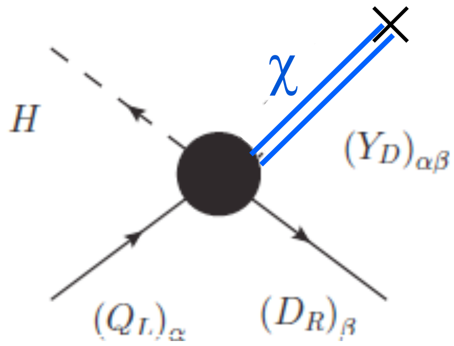
needs non-renormalizable + super fine-tuning

Summary

--> **Dynamical** MFV scalars in the bifundamental of G_f do not provide realistic masses and mixings (at least in the minimal realization)

2) Y --> quadratic in fields χ

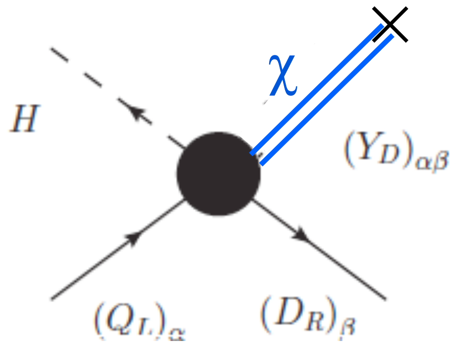
$$Y \sim \frac{\langle \chi \chi^\dagger \rangle}{\Lambda_f^2}$$



$$\text{i.e. } Y_D \sim \frac{\chi^L{}_d \, (\chi^R{}_d)^+}{\Lambda_f^2} \sim (3, 1, 1)\,(1, 1, \bar{3}) \sim (3, 1, \bar{3})$$

2) Y --> quadratic in fields χ

$$Y \sim \frac{\langle \chi \chi^{\dagger} \rangle}{\Lambda_f^2}$$



→ **Automatic strong mass hierarchy and one mixing angle ! already at the renormalizable level**

Holds for 2 and 3 families !

Y --> quadratic in fields χ

It is very simple:

- a square matrix built out of 2 vectors

$$\begin{pmatrix} d \\ e \\ f \\ \vdots \end{pmatrix} (a, b, c \ldots\ldots)$$

has only one non-vanishing eigenvalue

→ strong mass hierarchy at leading order:

-- only 1 heavy "up" quark

-- only 1 heavy "down" quark

only $|\chi|$´s relevant for scale

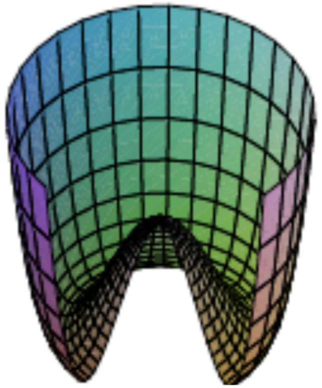
Y --> quadratic in fields χ

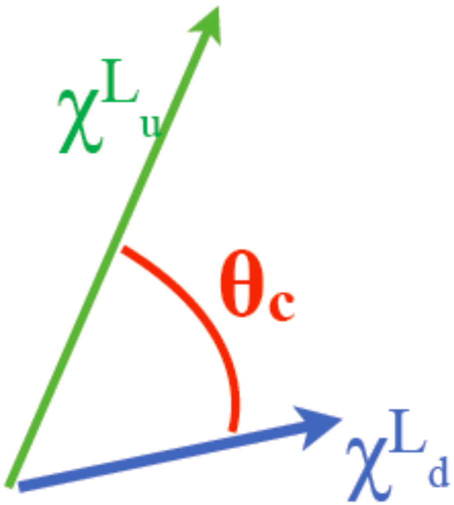
Minimum of the Potential

Dimension 6 Yukawa Operator

The invariants are:

$$\chi_u^{L\dagger}\chi_u^L\,,\qquad \chi_u^{R\dagger}\chi_u^R\,,\qquad \chi_d^{L\dagger}\chi_d^L\,,$$
$$\chi_d^{R\dagger}\chi_d^R\,,\qquad \chi_u^{L\dagger}\chi_d^L = |\chi_u^L|\,|\chi_d^L|\cos\theta_c\,.$$





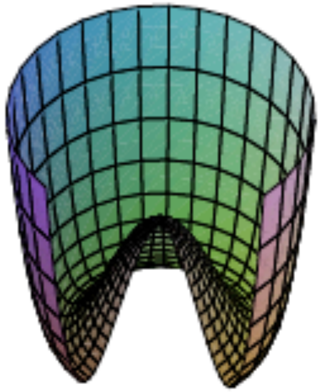
θ_c is the angle between up and down L vectors

Y --> quadratic in fields χ

Minimum of the Potential

Dimension 6 Yukawa Operator

The invariants are:



$$\chi_u^{L\dagger}\chi_u^L\,,\qquad \chi_u^{R\dagger}\chi_u^R\,,\qquad \chi_d^{L\dagger}\chi_d^L\,,$$
$$\chi_d^{R\dagger}\chi_d^R\,,\qquad \chi_u^{L\dagger}\chi_d^L = |\chi_u^L|\,|\chi_d^L|\cos\theta_c\,.$$

We can fit the angle and the masses in the Potential; as an example:

$$V' = \lambda_u\left(\chi_u^{L\dagger}\chi_u^L - \frac{\mu_u^2}{2\lambda_u}\right)^2 + \lambda_d\left(\chi_d^{L\dagger}\chi_d^L - \frac{\mu_d^2}{2\lambda_d}\right)^2$$
$$+\lambda_{ud}\left(\chi_u^{L\dagger}\chi_d^L - \frac{\mu_{ud}^2}{2\lambda_{ud}}\right)^2 + \cdots$$

Whose minimum sets (2 generations):

$$y_c^2 = \frac{\mu_u^2}{2\lambda_u\Lambda_f^2}\quad y_s^2 = \frac{\mu_d^2}{2\lambda_d\Lambda_f^2}\quad \cos\theta = \frac{\mu_{ud}^2\sqrt{\lambda_u\lambda_d}}{\mu_u\mu_d\lambda_{ud}}$$

Y --> quadratic in fields χ

Towards a realistic 3 family spectrum

e.g. replicas of χ^{L} , χ^{R}_{u} , χ^{R}_{d}

???

Y --> quadratic in fields χ

Towards a realistic 3 family spectrum

e.g. replicas of χ^L , χ^R_u , χ^R_d

???

Suggests sequential breaking:

$$SU(3)^3 \xrightarrow{\text{mt, mb}} SU(2)^3 \xrightarrow{\text{mc, ms, } \theta_C} \cdots\cdots\cdots$$

$$Y_u \equiv \frac{\langle\chi^L\rangle\,\langle\chi_u^{R\dagger}\rangle}{\Lambda_f^2} + \frac{\langle\chi_u'^L\rangle\,\langle\chi_u'^{R\dagger}\rangle}{\Lambda_f^2} = \begin{pmatrix} 0 & \sin\theta\, y_c & 0 \\ 0 & \cos\theta\, y_c & 0 \\ 0 & 0 & y_t \end{pmatrix}$$

$$Y_d \equiv \frac{\langle\chi^L\rangle\,\langle\chi_d^{R\dagger}\rangle}{\Lambda_f^2} + \frac{\langle\chi_d'^L\rangle\,\langle\chi_d'^{R\dagger}\rangle}{\Lambda_f^2} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & y_s & 0 \\ 0 & 0 & y_b \end{pmatrix} .$$

Y --> quadratic in fields χ

Towards a realistic 3 family spectrum

e.g. replicas of χ^L , χ^R_u , χ^R_d

???

Suggests sequential breaking:

$SU(3)^3$ ⟶ **$SU(2)^3$** ⟶ …………

mt, mb   **mc, ms, θ_C**

$$\frac{\cdots}{\Lambda_f^2} + \frac{\cdots}{\Lambda_f^2} \quad \begin{pmatrix} 0 & \sin\theta\, y_c & 0 \\ \cdots & & \\ 0 & 0 & y_t \end{pmatrix}$$

Maybe some connection to: Berezhiani+Nesti; Ferretti et al., Calibbi et al. ??

$$Y_d \equiv \frac{\langle \chi^L \rangle \langle \chi_d^{R\dagger} \rangle}{\Lambda_f^2} + \frac{\langle \chi_d^{\prime L} \rangle \langle \chi_d^{\prime R\dagger} \rangle}{\Lambda_f^2} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & y_s & 0 \\ 0 & 0 & y_b \end{pmatrix} .$$

Y --> linear + quadratic in fields

Towards a realistic 3 family spectrum

Combining fundamentals and bi-fundamentals

i.e. combining d=5 and d =6 Yukawa operators

$$\Sigma_u \sim (3,\overline{3},1)\ , \qquad \Sigma_d \sim (3,1,\overline{3})\ , \qquad \Sigma_R \sim (1,3,\overline{3})\ ,$$

$$\chi_u^L \in (3,1,1)\ , \qquad \chi_u^R \in (1,3,1)\ , \qquad \chi_d^L \in (3,1,1)\ , \qquad \chi_d^R \in (1,1,3)\ .$$

The Yukawa Lagrangian up to the second order in $1/\Lambda_f$ is given by:

$$\mathscr{L}_Y = \overline{Q}_L \left[\frac{\Sigma_d}{\Lambda_f} + \frac{\chi_d^L \chi_d^{R\dagger}}{\Lambda_f^2} \right] D_R H + \overline{Q}_L \left[\frac{\Sigma_u}{\Lambda_f} + \frac{\chi_u^L \chi_u^{R\dagger}}{\Lambda_f^2} \right] U_R \tilde{H} + \text{h.c.}\ ,$$

* From bifundamentals: $<\Sigma_u> = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_t \end{pmatrix}$

$$<\Sigma_d> = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_b \end{pmatrix}$$

* From fundamentals χ : y_c, y_s and θ_C

* At leading (renormalizable) order:

$$Y_u \equiv \frac{\langle \Sigma_u \rangle}{\Lambda_f} + \frac{\langle \chi_u^L \rangle \langle \chi_u^{R\dagger} \rangle}{\Lambda_f^2} = \begin{pmatrix} 0 & \sin\theta_c \, y_c & 0 \\ 0 & \cos\theta_c \, y_c & 0 \\ 0 & 0 & y_t \end{pmatrix},$$

$$Y_d \equiv \frac{\langle \Sigma_d \rangle}{\Lambda_f} + \frac{\langle \chi_d^L \rangle \langle \chi_d^{R\dagger} \rangle}{\Lambda_f^2} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & y_s & 0 \\ 0 & 0 & y_b \end{pmatrix}.$$

without unnatural fine-tunings

* The masses of the first family and the other angles from non-renormalizable terms or other corrections or replicas ?

....under exploration

LEPTONS

Can its minimum correspond naturally to the observed masses and mixings?

i.e. with all dimensionless λ's ~ 1

and dimensionful μ's $\leqq \Lambda_f$