First ICO-ICTP-TWAS Central American Workshop in Lasers, Laser Applications and Laser Safety Regulations

Laser Resonators

Dr. Eric Rosas

Centro Nacional de Metrología

México

International Commission for Optics
Commission Internationale d'Optique

The Abdus Salam
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Contents

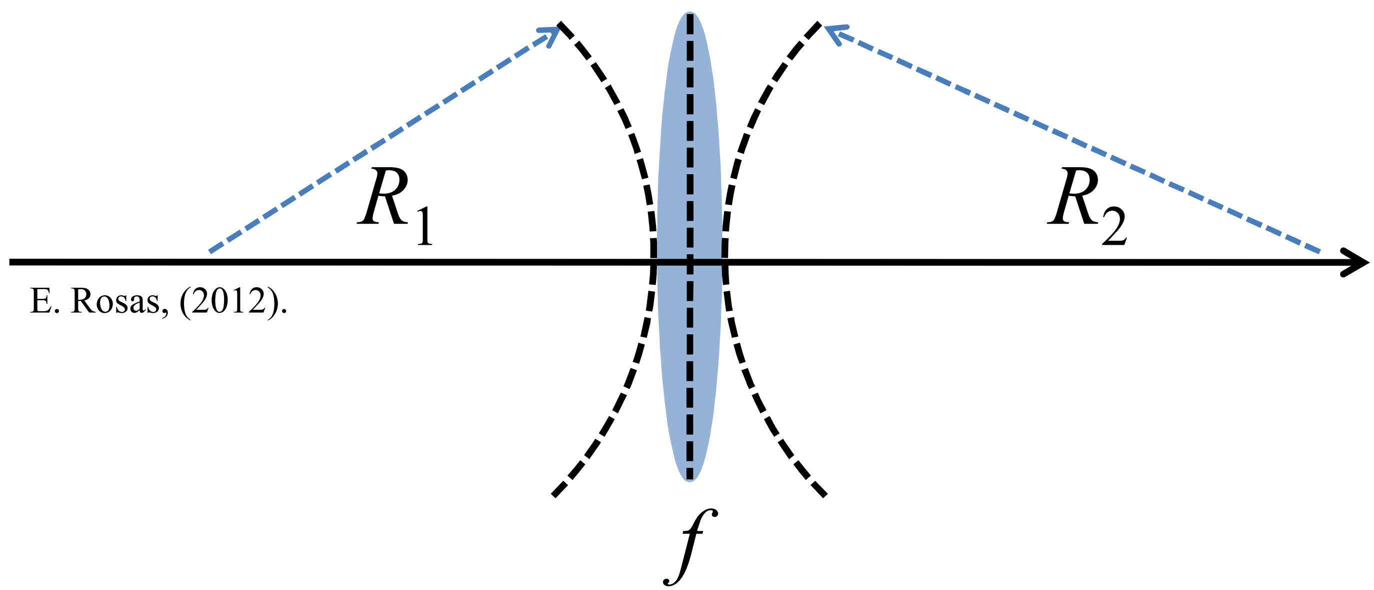
Beam Transformation by a Lens.

A lens can be used to focus a laser beam or to produce a beam of suitable diameter and phase front curvature.

Ideally, a lens does not change the transverse field distribution of the beam mode.

A lens does change the beam parameters $R(z)$ and $w(z)$.

Since $R(z)$ and $w(z)$ are the same for modes of all orders, then the following discussion is valid for all orders.



$R(z)$ changes immediately to the left of the lens.
$w(z)$ remains the same.

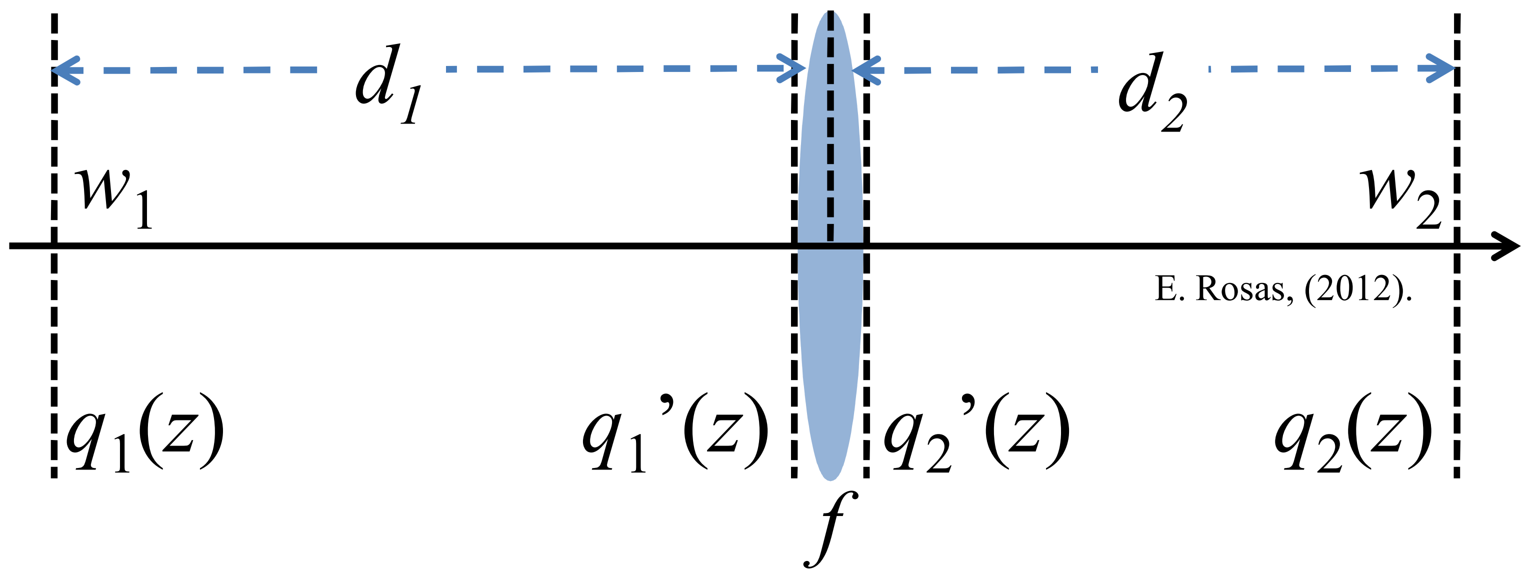
Thus: $\frac{1}{R_2(z)} = \frac{1}{R_1(z)} - \frac{1}{f}$ and: $\frac{1}{w_2(z)} = \frac{1}{w_1(z)}$

$$\frac{1}{R_2(z)} + j\frac{\lambda}{\pi\, w_2^2(z)} = \left(\frac{1}{R_1(z)} - \frac{1}{f}\right) + j\frac{\lambda}{\pi\, w_1^2(z)}$$

$$\Rightarrow \quad \frac{1}{q_2(z)} = \frac{1}{q_1(z)} - \frac{1}{f}$$

where $q_1(z)$ and $q_2(z)$ are measured just at the lens.

If $q_1(z)$ and $q_2(z)$ are measured at distances d_1 and d_2 from the lens:



where: $$q_o(z) = q_i(z) + z$$

is used for the intermediate planes propagation.

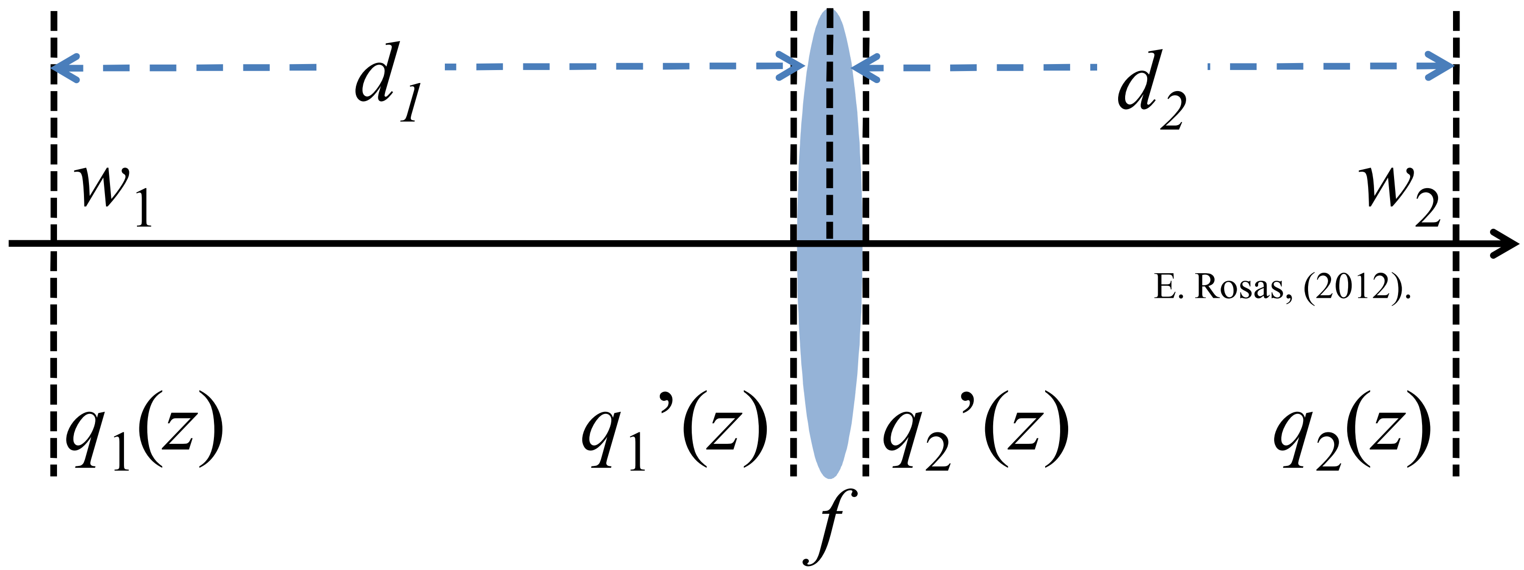
$$q_1'(z) = q_1(z) + d_1$$

$$\frac{1}{q_2'(z)} = \frac{1}{q_1'(z)} - \frac{1}{f}$$

$$q_2(z) = q_2'(z) + d_2$$

$$q_2(z) = \frac{\left(1 - d_2/f\right)\cdot q_1(z) + \left(d_1 + d_2 - d_1\, d_2/f\right)}{-\left(1/f\right)\cdot q_1(z) + \left(1 - d_1/f\right)}$$

Using matrices:



$$\begin{bmatrix} 1 & d_2 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ -1/f & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & d_1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} A_{1\to 2} & B_{1\to 2} \\ C_{1\to 2} & D_{1\to 2} \end{bmatrix}$$

where:

$$= \begin{bmatrix} 1 & d_2 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} (1\times 1)+(0\times 0) & (1\times d_1)+(0\times 1) \\ (-1/f\times 1)+(1\times 0) & (-1/f\times d_1)+(1\times 1) \end{bmatrix}$$

$$= \begin{bmatrix} 1 & d_2 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & d_1 \\ -1/f & 1-d_1/f \end{bmatrix}$$

$$= \begin{bmatrix} (1\times 1)+\left[d_2\times(-1/f)\right] & (1\times d_1)+\left[d_2\times(1-d_1/f)\right] \\ (0\times 1)+\left[1\times(-1/f)\right] & (0\times d_1)+\left[1\times(1-d_1/f)\right] \end{bmatrix}$$

$$= \begin{bmatrix} 1-d_2/f & d_1+d_2-d_1\,d_2/f \\ -1/f & 1-d_1/f \end{bmatrix}$$

then:

$$\begin{bmatrix} A_{1\to 2} & B_{1\to 2} \\ C_{1\to 2} & D_{1\to 2} \end{bmatrix} = \begin{bmatrix} 1-d_2/f & d_1+d_2-d_1\,d_2/f \\ -1/f & 1-d_1/f \end{bmatrix}$$

$$q_2(z) = \frac{\left(1-d_2/f\right)\cdot q_1(z) + \left(d_1+d_2-d_1\,d_2/f\right)}{-\left(1/f\right)\cdot q_1(z) + \left(1-d_1/f\right)}$$

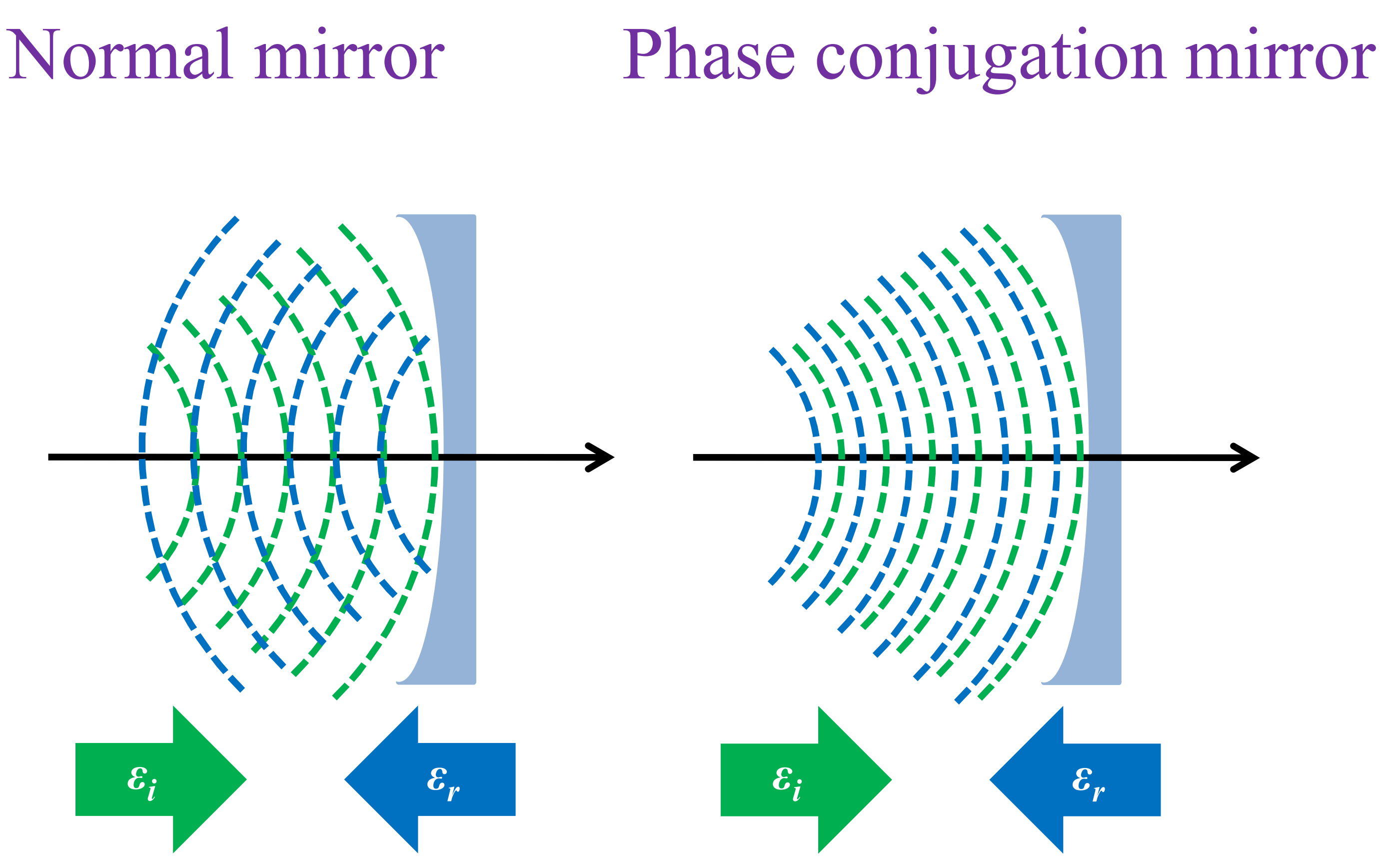
$$\Rightarrow \quad q_2(z) = \frac{A_{1\to 2}\cdot q_1(z) + B_{1\to 2}}{C_{1\to 2}\cdot q_1(z) + D_{1\to 2}}$$

In general, if the *ABCD Transfer Matrix* for paraxial rays is known, then the $q(z)$ parameter of the output beam can be calculated from the *ABCD Law*:

$$q_2(z) = \frac{A \cdot q_1(z) + B}{C \cdot q_1(z) + D}$$

Which works for any combination of ABCD matrices representing fundamental optical structures, like lenses, mirrors, apertures, media, etc.

The Phase Conjugation (Time Reversal).



E. Rosas, (2012).

For an incident field:

$$\varepsilon_i(x, y, z, t) = \mathrm{Re}\left\{\widetilde{E}(x, y, z) \cdot \exp\left[j(\omega_0 t - kz)\right]\right\}$$

the conjugated reflected field is:

$$\varepsilon_r(x, y, z, t) = \mathrm{Re}\left\{\kappa\, \widetilde{E}^*(x, y, z) \cdot \exp\left[j(\omega_0 t + kz)\right]\right\}$$

where κ is the “mirror” reflection coefficient.

As a consequence of the phase conjugation, a Gaussian beam reflecting from a phase conjugation mirror (PCM) will have its radius of curvature $R(z)$ reversed in sign, but its spot size $w(z)$ inchanged.

Thus, if the incident beam parameter on a PCM is:

$$\frac{1}{q_i(z)} = \frac{1}{R_1(z)} - j\frac{\lambda}{\pi\, w_1^2(z)}$$

Then the reflected beam parameter will be:

$$\frac{1}{q_r(z)} = -\frac{1}{R_1(z)} - j\frac{\lambda}{\pi\, w_1^2(z)} = -\left[\frac{1}{R_1(z)} + j\frac{\lambda}{\pi\, w_1^2(z)}\right]$$

Then $\frac{1}{q_r(z)} = -\frac{1}{q_i^*(z)}$ or $q_r(z) = -q_i^*(z)$

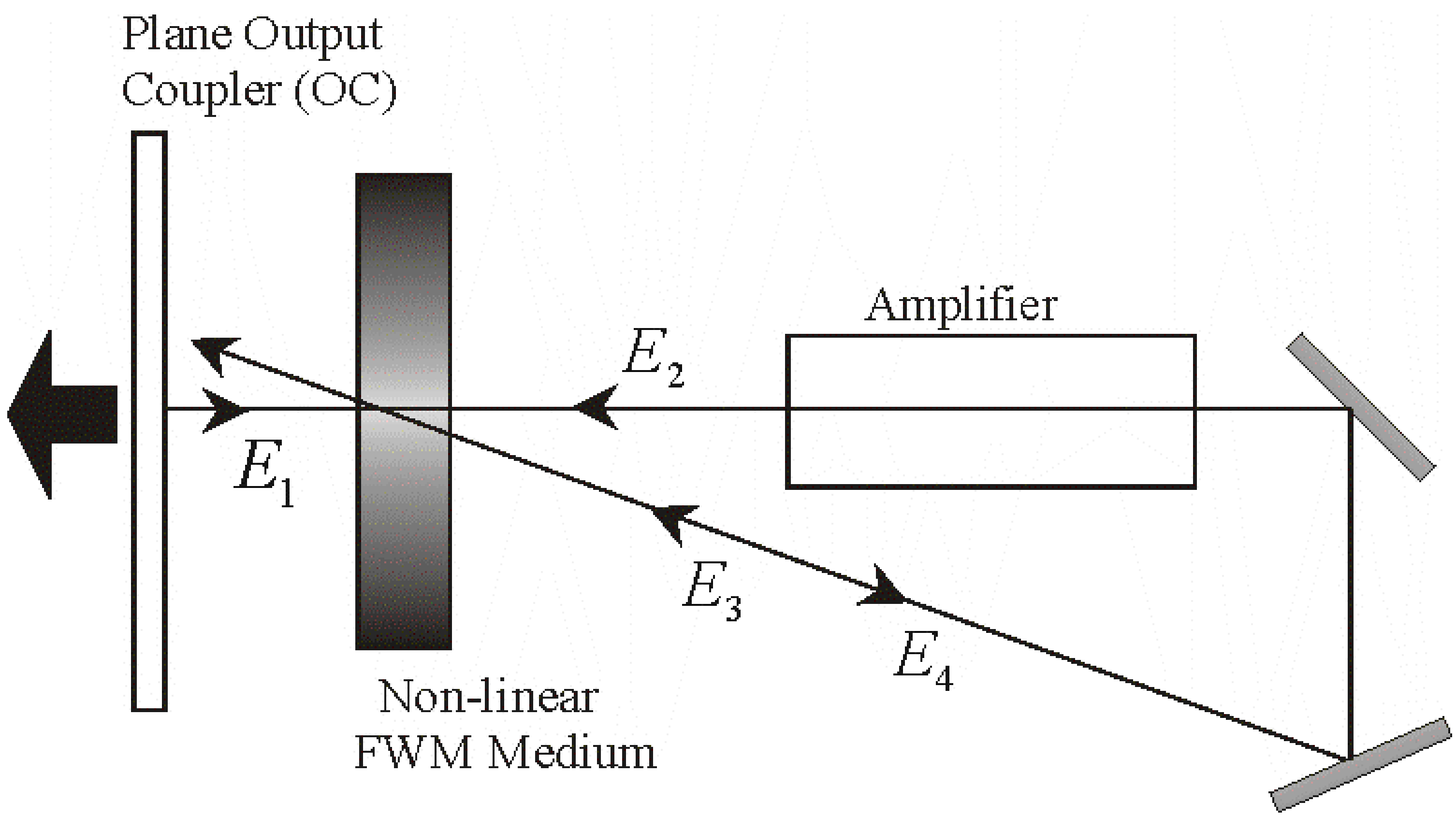
where * denotes complex conjugation.

By comparison with the ABCD law:

$$q_2(z) = \frac{A \cdot q_1(z) + B}{C \cdot q_1(z) + D} \quad \Rightarrow \quad \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

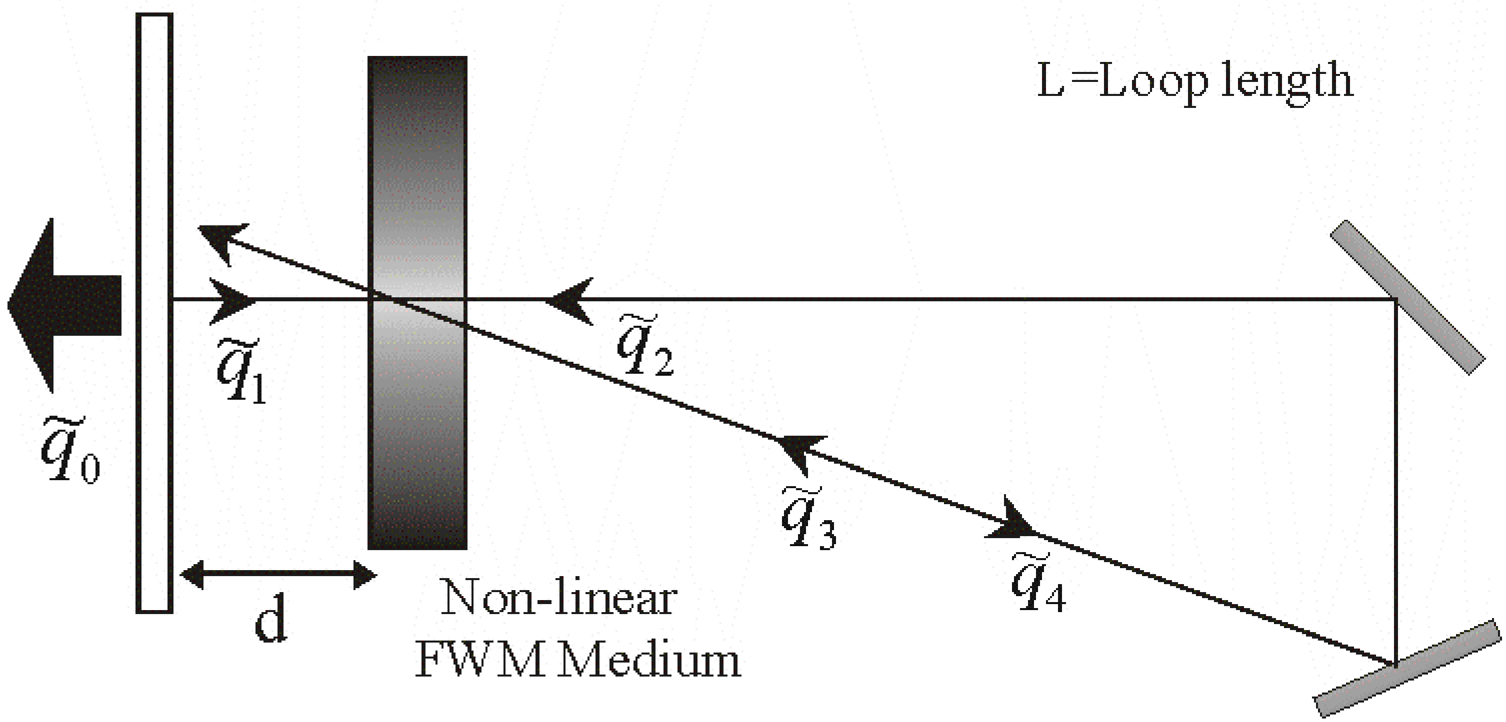
is the ABCD Transfer Matrix for a PCM.

Four-Wave Mixing Phase Conjugation Resonators.



E. Rosas *et al*, J. Opt., **29**, (1998).

Four-Wave Mixing Interaction ABCD Transfer Matrix.



E. Rosas *et al*, Opt. Commun., **174**, (2000).

To produce a self-consistent oscillation inside an ALO (Self-Adaptive or Hoographic Laser Oscillator):

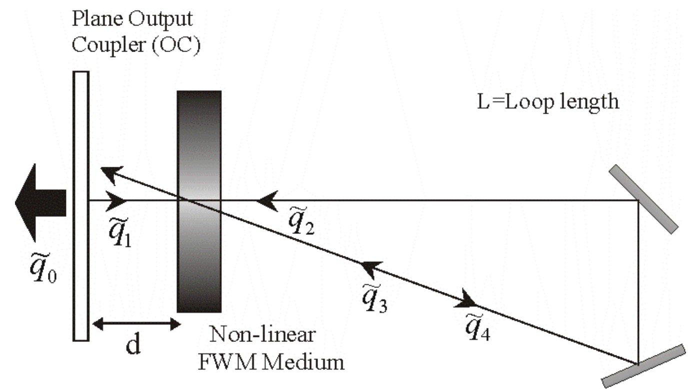
$$\frac{1}{q_o(z)} = \frac{1}{q_{\mathrm{ALO}}(z)} - \frac{1}{q_i(z)}$$

Which leads to an ABCD Transfer Matrix of the form:

$$\frac{1}{q_0(z)} = \frac{C_{\mathrm{ALO}} + D_{\mathrm{ALO}}\dfrac{1}{q_i(z)}}{A_{\mathrm{ALO}} + B_{\mathrm{ALO}}\dfrac{1}{q_i(z)}}$$

$$\Rightarrow \begin{bmatrix} A_{\mathrm{ALO}} & B_{\mathrm{ALO}} \\ C_{\mathrm{ALO}} & D_{\mathrm{ALO}} \end{bmatrix} \equiv \begin{bmatrix} 1 & 0 \\ 1/q_{\mathrm{ALO}}(z) & 1 \end{bmatrix}$$

Then the ABCD Transfer Matrix for the FWM Interaction can be derived from:



E. Rosas *et al*, Opt. Commun., **174**, (2000).

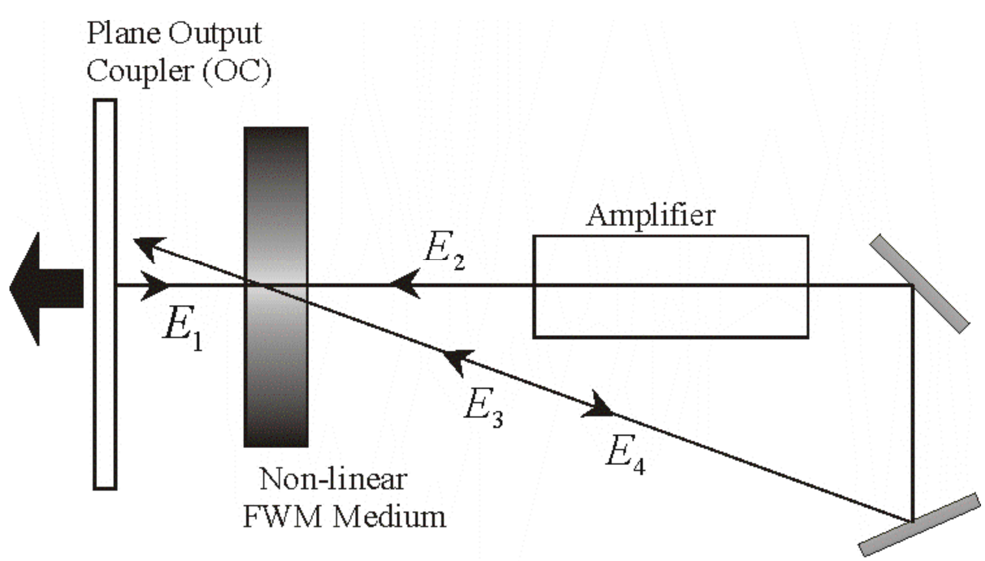
$$\begin{bmatrix} A_{\mathrm{ALO}} & B_{\mathrm{ALO}} \\ C_{\mathrm{ALO}} & D_{\mathrm{ALO}} \end{bmatrix} \equiv \begin{bmatrix} 1 & 0 \\ L & 1 \end{bmatrix} \begin{bmatrix} A_{\mathrm{FWM}} & B_{\mathrm{FWM}} \\ C_{\mathrm{FWM}} & D_{\mathrm{FWM}} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ L & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} A_{\mathrm{FWM}} & B_{\mathrm{FWM}} \\ C_{\mathrm{FWM}} & D_{\mathrm{FWM}} \end{bmatrix} \equiv \begin{bmatrix} 1 - \dfrac{L}{q_{\mathrm{ALO}}(z)} & \dfrac{L^2}{q_{\mathrm{ALO}}(z)} - 2L \\ \dfrac{1}{q_{\mathrm{ALO}}(z)} & 1 - \dfrac{L}{q_{\mathrm{ALO}}(z)} \end{bmatrix}$$

Transient Evolution, Spatial Mode Size Analysis, Spatial Mode Size Control, Laser Resonator Design.

$$\left.\begin{aligned}
\frac{1}{q_4(z)} &= \frac{1}{q_1(z)} + \frac{1}{q_2(z)} - \frac{1}{q_3{}^*(z)} \\
\frac{1}{R_4(z)} &= \frac{1}{R_1(z)} + \frac{1}{R_2(z)} - \frac{1}{R_3(z)} \\
\frac{1}{w_4^2(z)} &= \frac{1}{w_1^2(z)} + \frac{1}{w_2^2(z)} + \frac{1}{w_3^2(z)}
\end{aligned}\right\} \quad q_o(z) = \frac{A \cdot q_i(z) + B}{C \cdot q_i(z) + D}$$

To evaluate the transient dynamics:



(a)

E. Rosas *et al*, Opt. Commun., **156**, (1998).

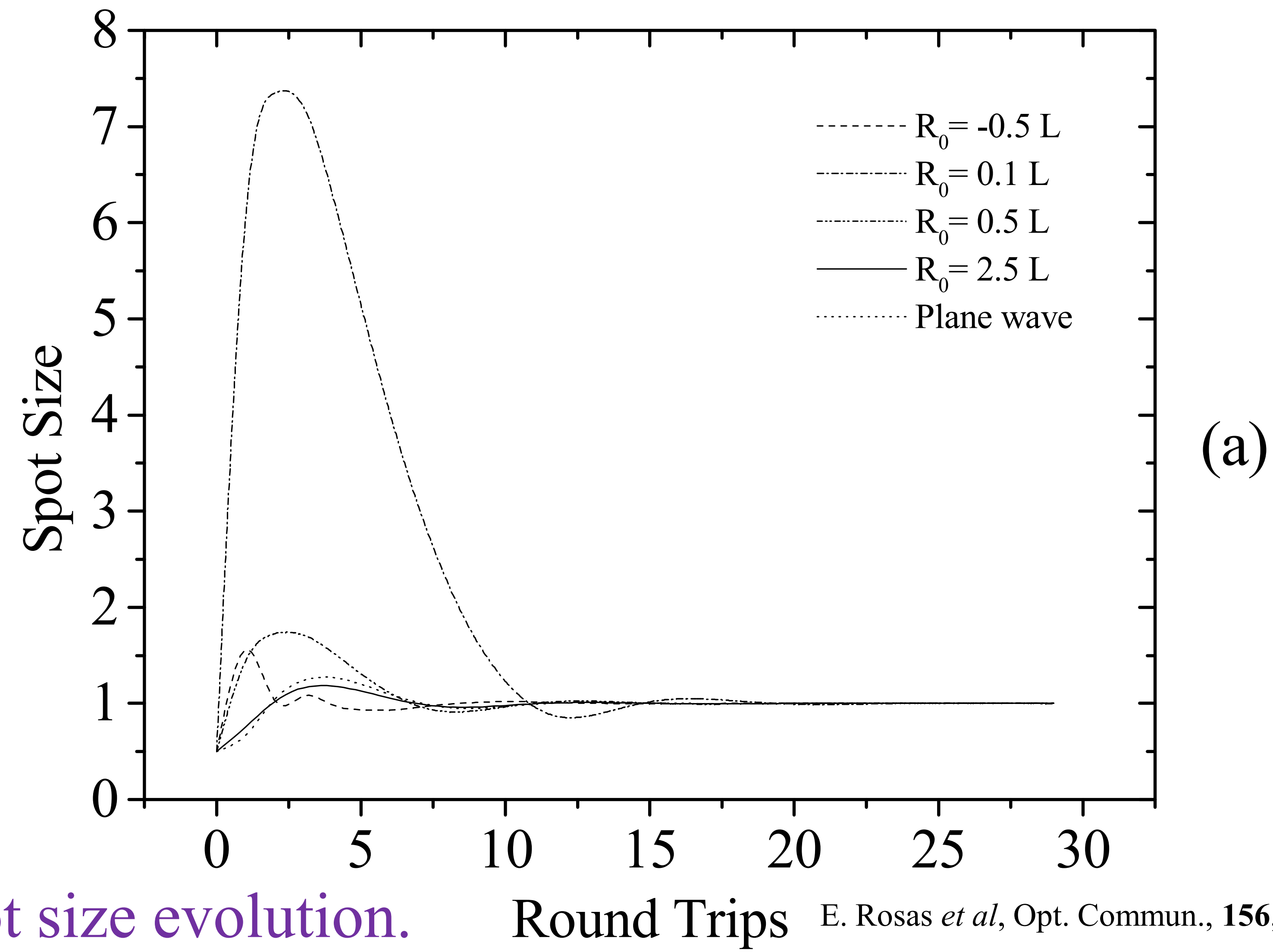
$$q_0^{(n)} = q_2^{(n)} + d$$

$$q_1^{(n)} = q_0^{(n)} + d$$

$$q_3^{(n)} = q_1^{(n-1)} + L$$

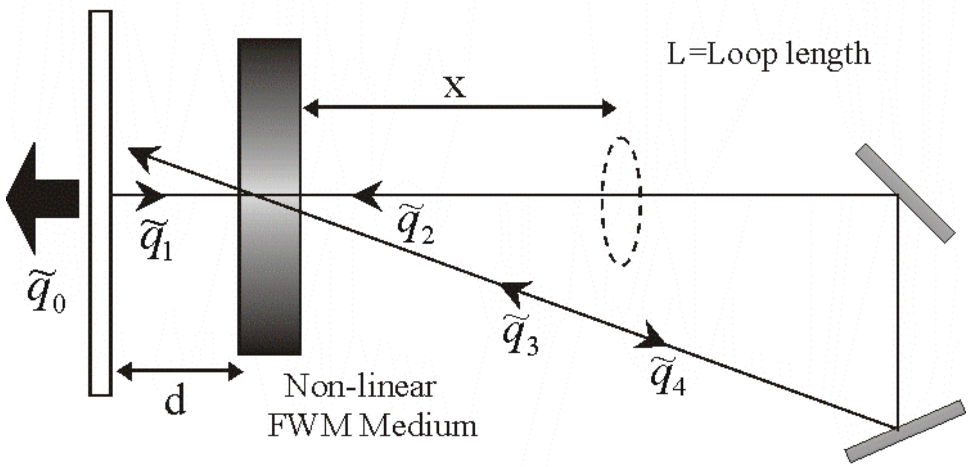
$$\frac{1}{q_4^{(n)}} = \frac{1}{q_1^{(n)}} + \frac{1}{q_2^{(n)}} - \frac{1}{q_3^{*(n)}}$$

$$q_2^{(n+1)} = q_4^{(n)} + L$$



Spot size evolution.

E. Rosas *et al*, Opt. Commun., **156**, (1998).



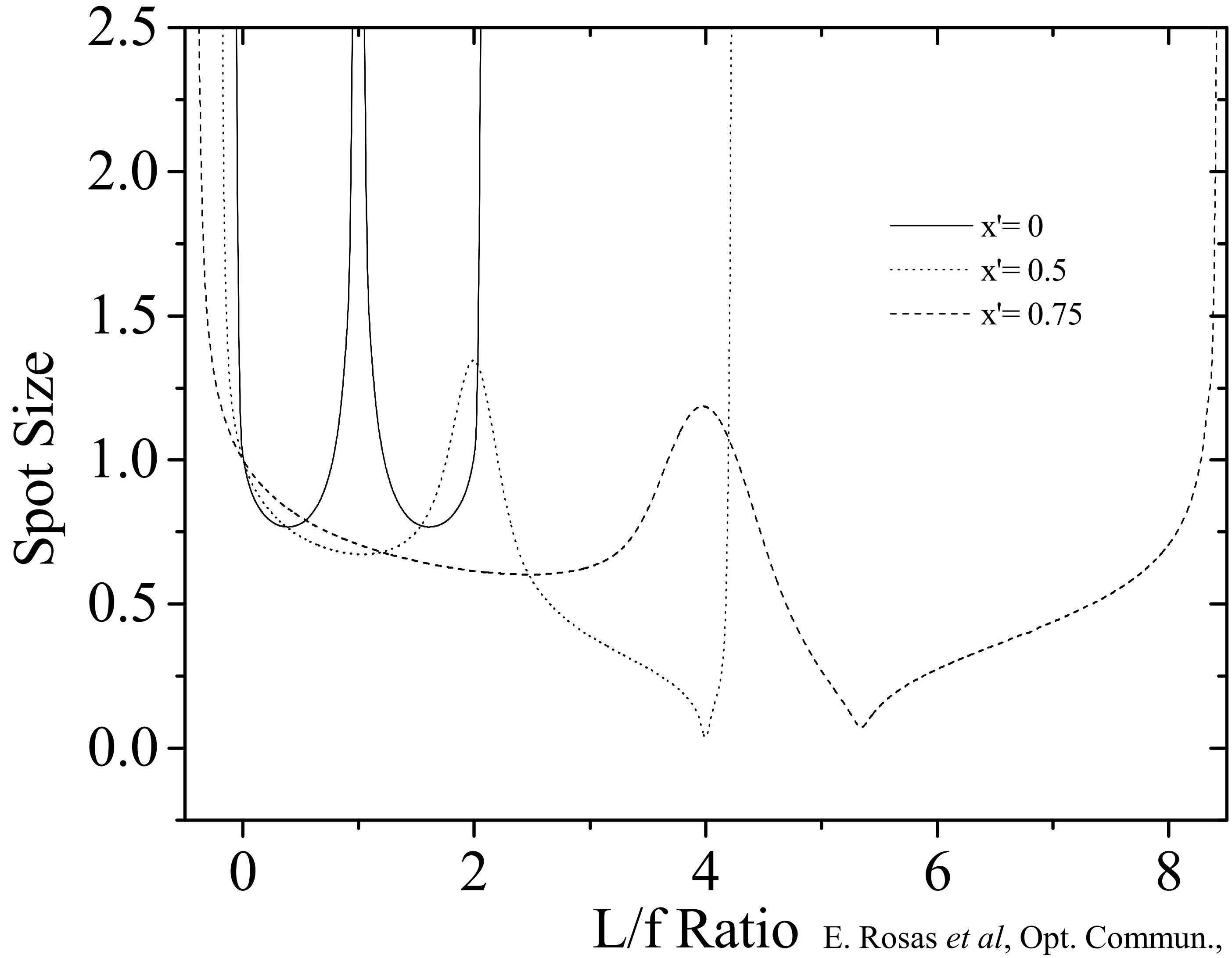
(b)

E. Rosas *et al*, Opt. Commun., **156**, (1998).

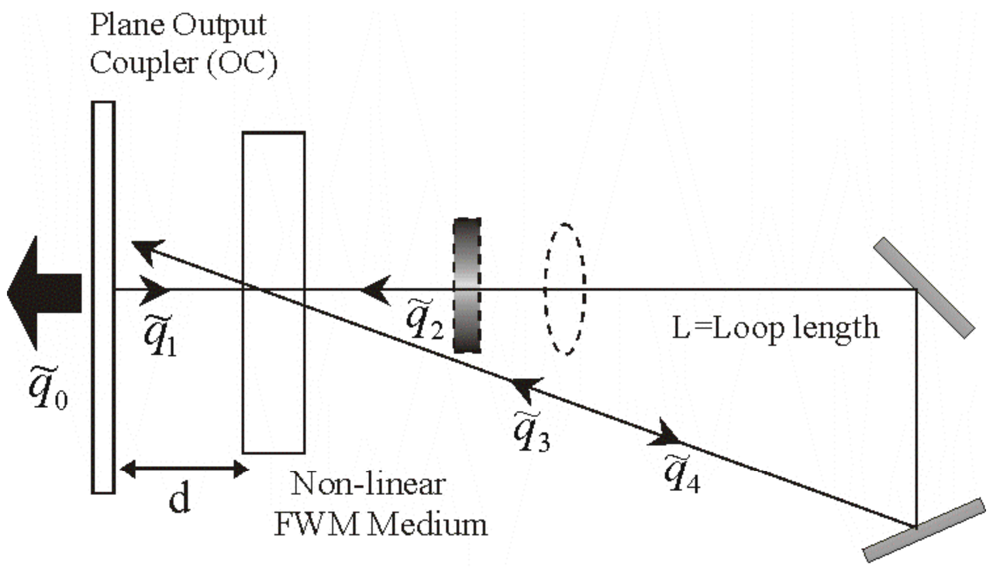
To evaluate the effect the lens position inside the resonator loop, has on the spot size:

$$q_3^{(n)} = \frac{(1 - L/f + x/f) \cdot q_1^{(n\text{-}1)} + (L - xL/f + x^2/f)}{(-1/f) \cdot q_1^{(n\text{-}1)} + (1 - x/f)}$$

$$q_2^{(n+1)} = \frac{(1 - x/f) \cdot q_4^{(n)} + (L - Lx/f + x^2/f)}{(-1/f) \cdot q_4^{(n)} + (1 - L/f + x/f)}$$



E. Rosas *et al*, Opt. Commun., **156**, (1998).



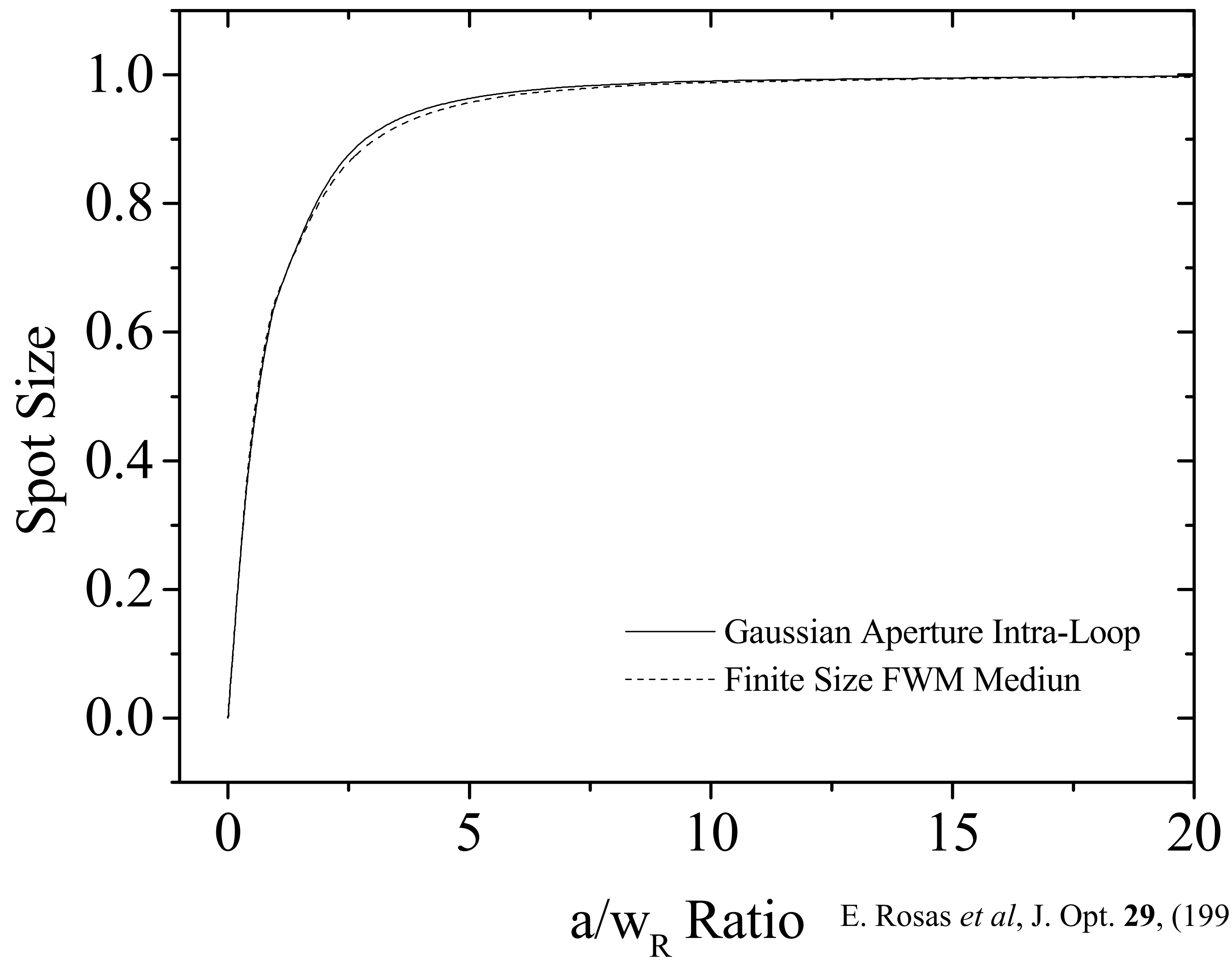
(a)

E. Rosas *et al*, J. Opt., **29**, (1998).

To evaluate the effect a finite-size Gaussian aperture inside the resonator loop, has on the spot size:

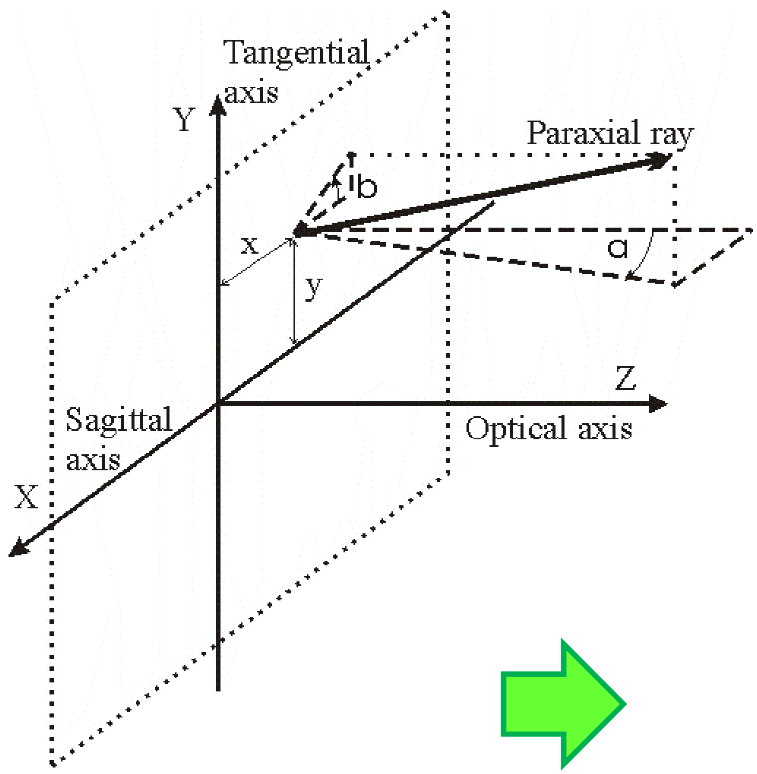
$$q_3^{(n)} = \frac{(1 - j\lambda L / \pi a^2) \cdot q_1^{(n-1)} + L}{(-j\lambda / \pi a^2) \cdot q_1^{(n-1)} + 1}$$

$$q_2^{(n+1)} = \frac{q_4^{(n)} + L}{(-j\lambda / \pi a^2) \cdot q_4^{(n)} + (1 - j\lambda L / \pi a^2)}$$



E. Rosas *et al*, J. Opt. **29**, (1998).

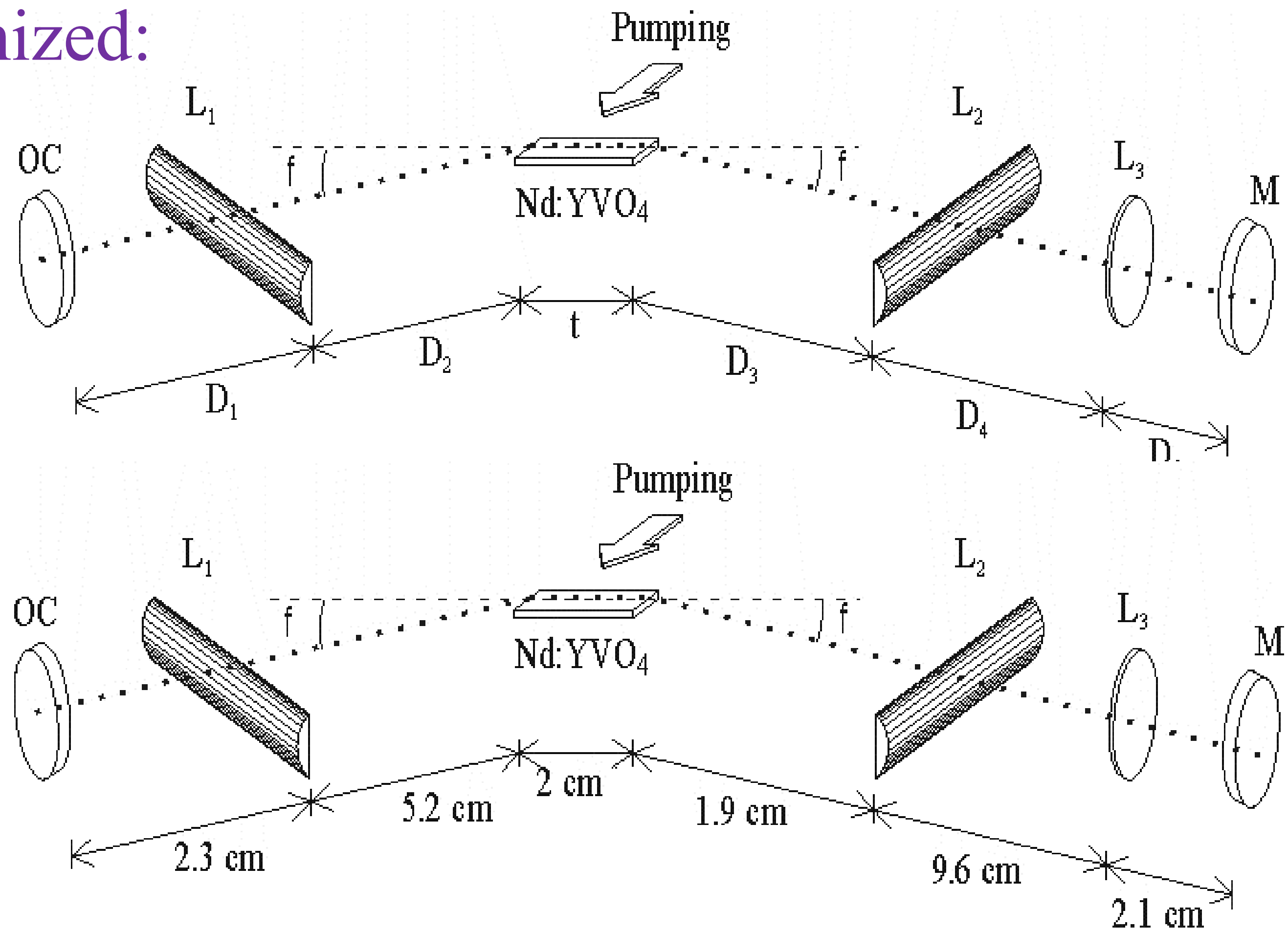
To design laser resonators with non rotational symmetry, in which case the ABCD Transfer matrix takes the form:



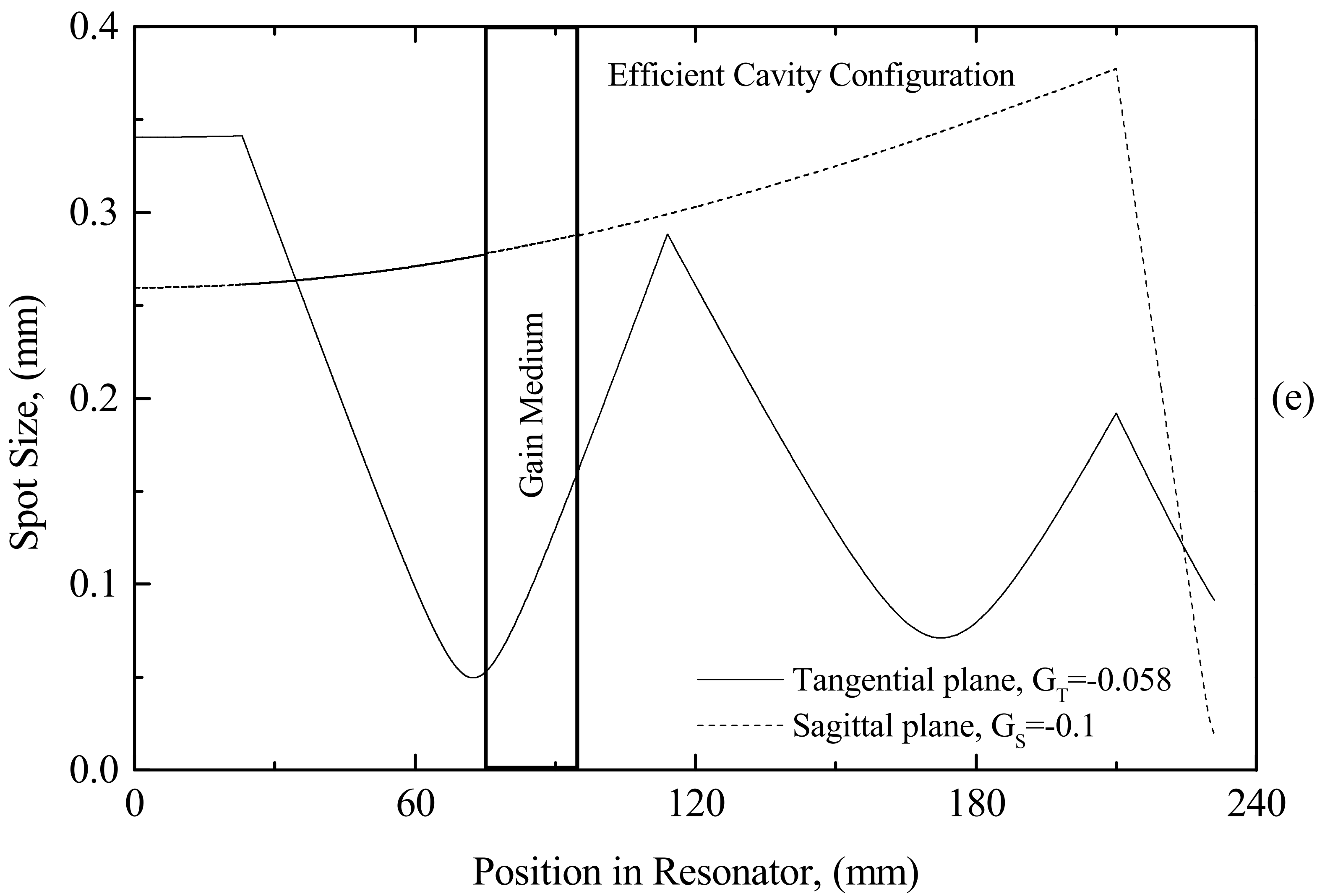
$$\begin{pmatrix} x' \\ y' \\ a' \\ b' \end{pmatrix} = \begin{pmatrix} A_{ss} & A_{st} & B_{ss} & B_{st} \\ A_{ts} & A_{tt} & B_{ts} & B_{tt} \\ C_{ss} & C_{st} & D_{ss} & D_{st} \\ C_{ts} & C_{tt} & D_{ts} & D_{tt} \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ a \\ b \end{pmatrix}$$

$$q'_{s,t}(z) = \frac{A_{ss,\,tt} \cdot q_{s,t}(z) + B_{ss,\,tt}}{C_{ss,\,tt} \cdot q_{s,t}(z) + D_{ss,\,tt}}$$

Thus the laser resonator configuration design can be optimized:



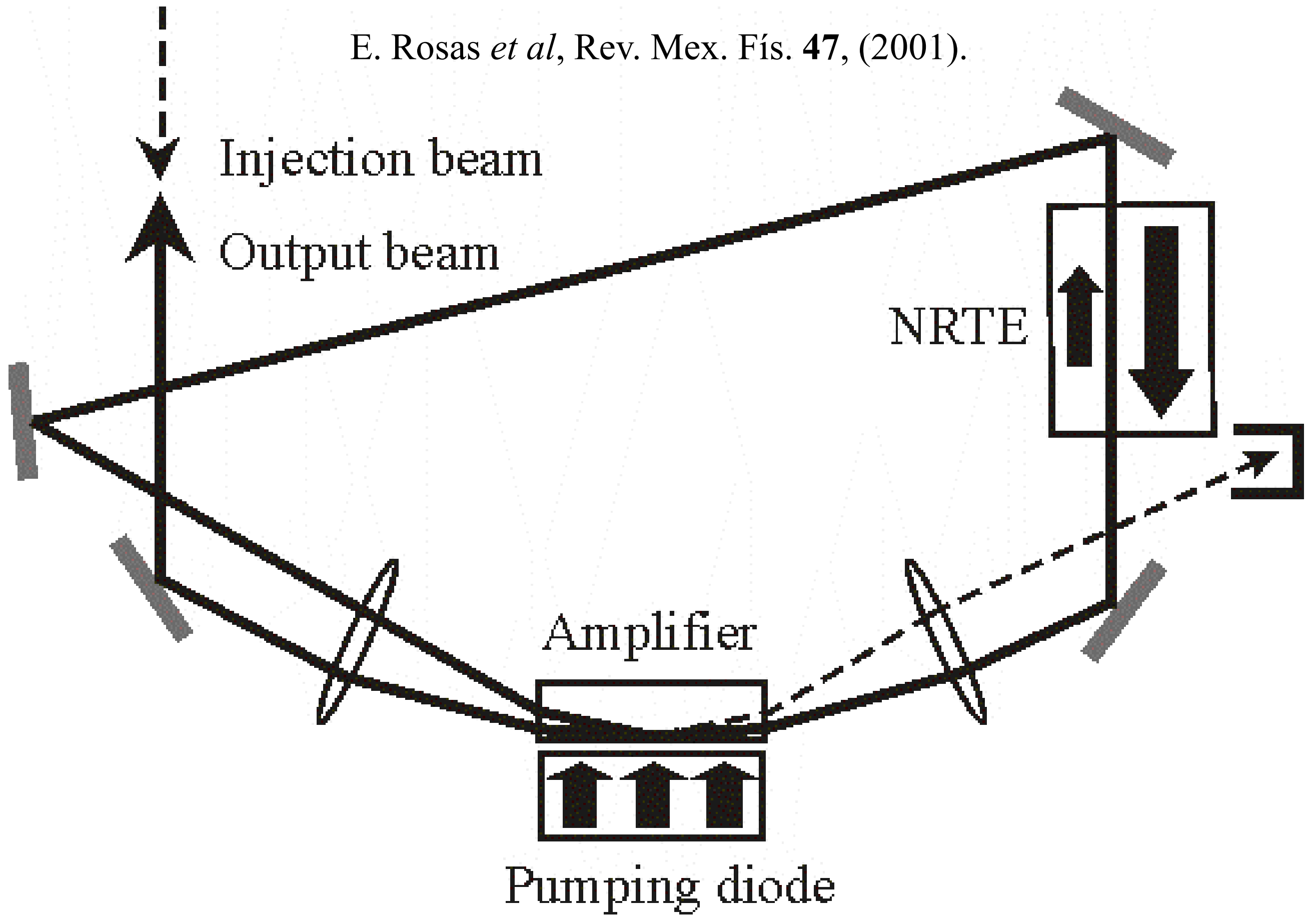
J. J. Soto-Bernal *et al*, Opt. Commun, **184**, (2000).

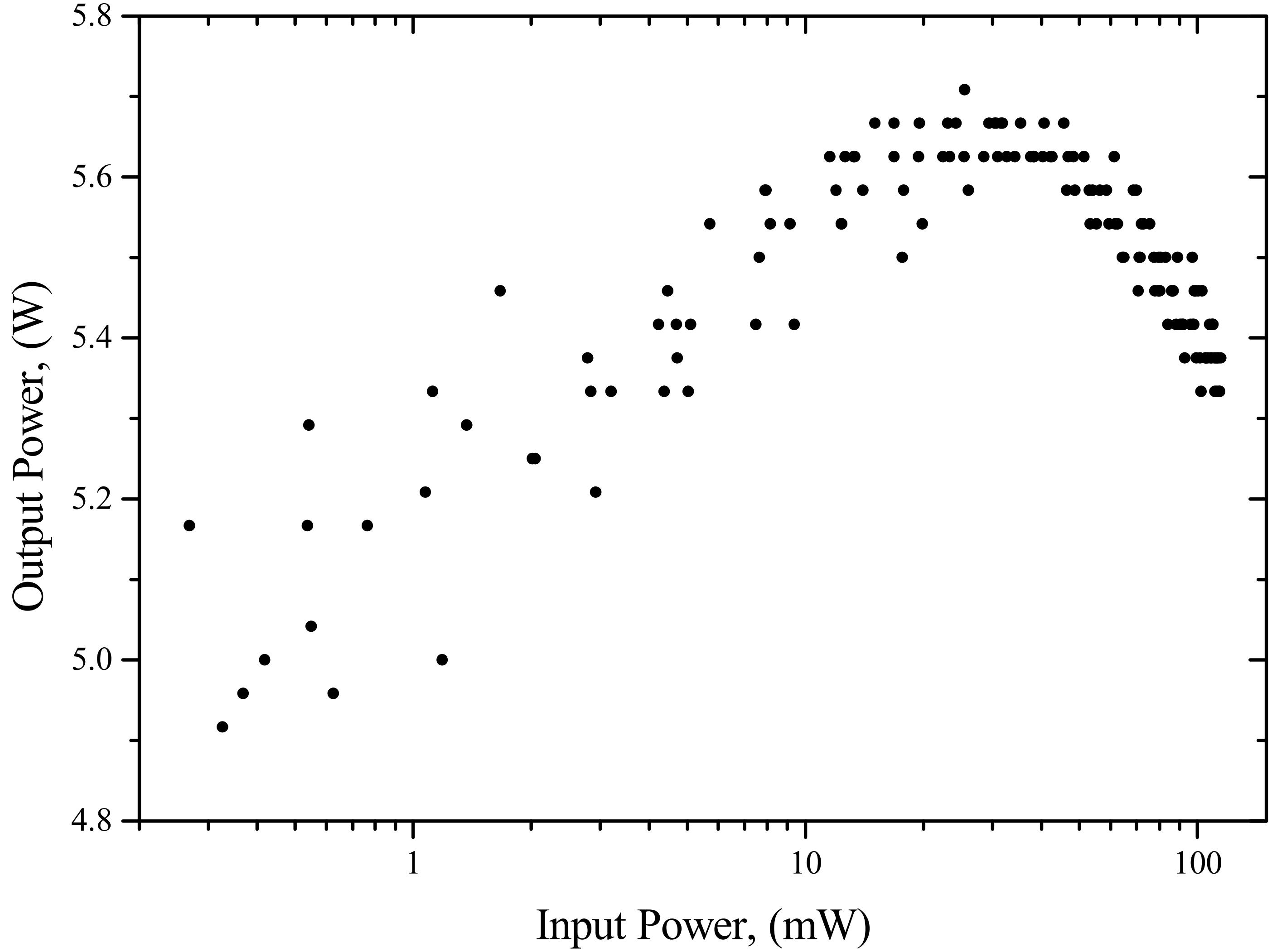


J. J. Soto-Bernal *et al*, Opt. Commun. **184**, (2000).

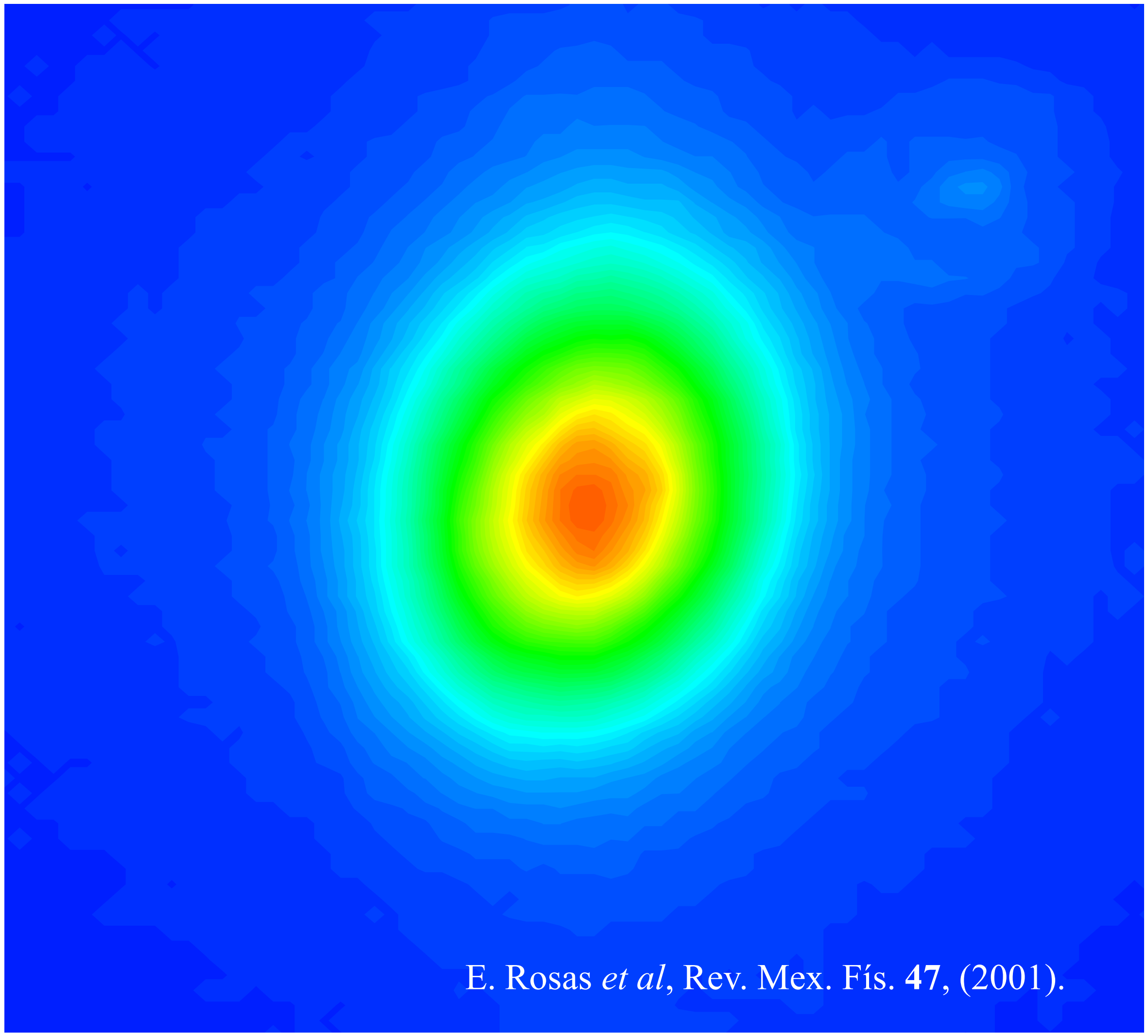
To reduce the thermal lensing, shape the Gaussian mode and increase the energy extraction:

E. Rosas *et al*, Rev. Mex. Fís. **47**, (2001).

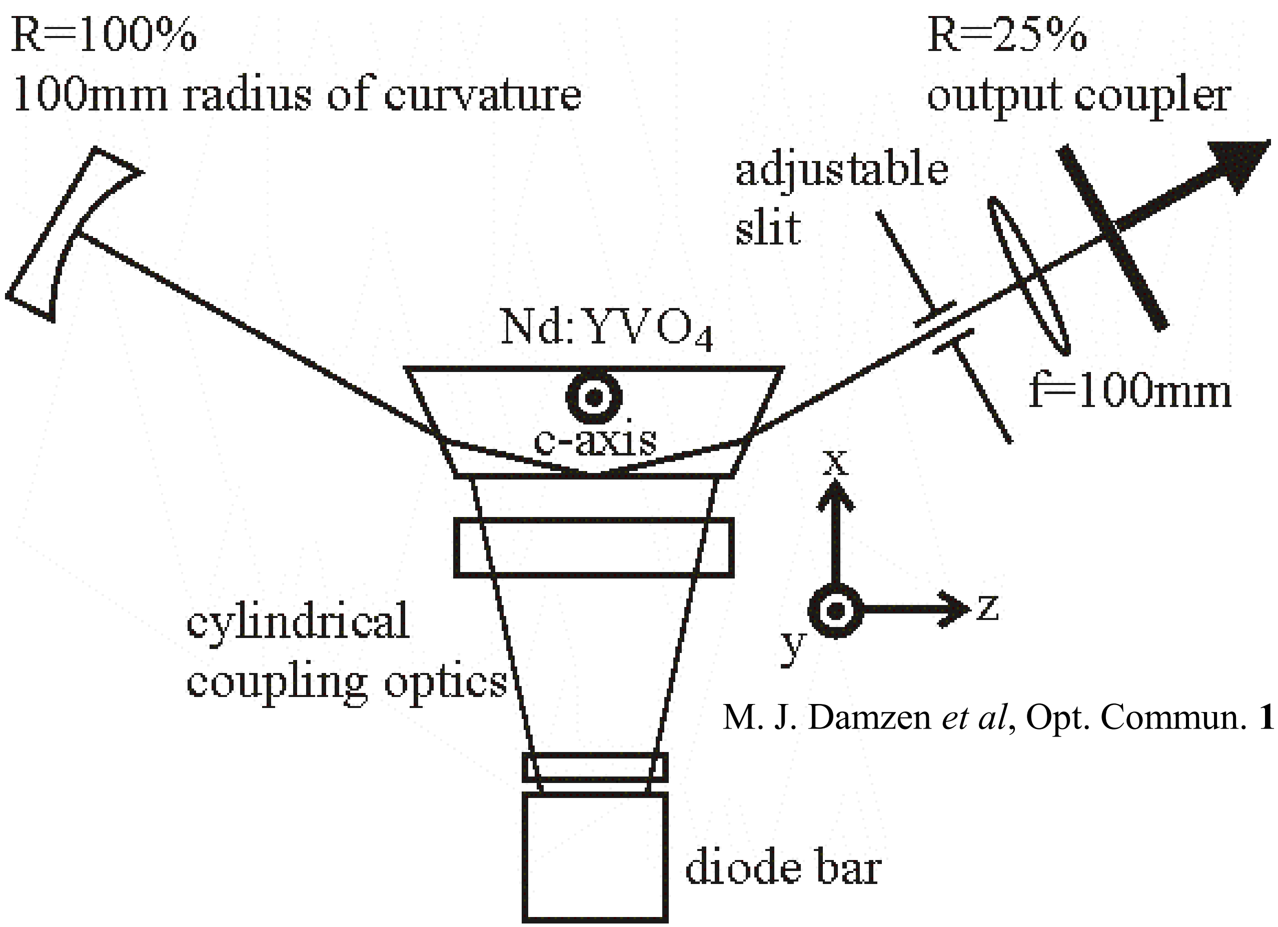




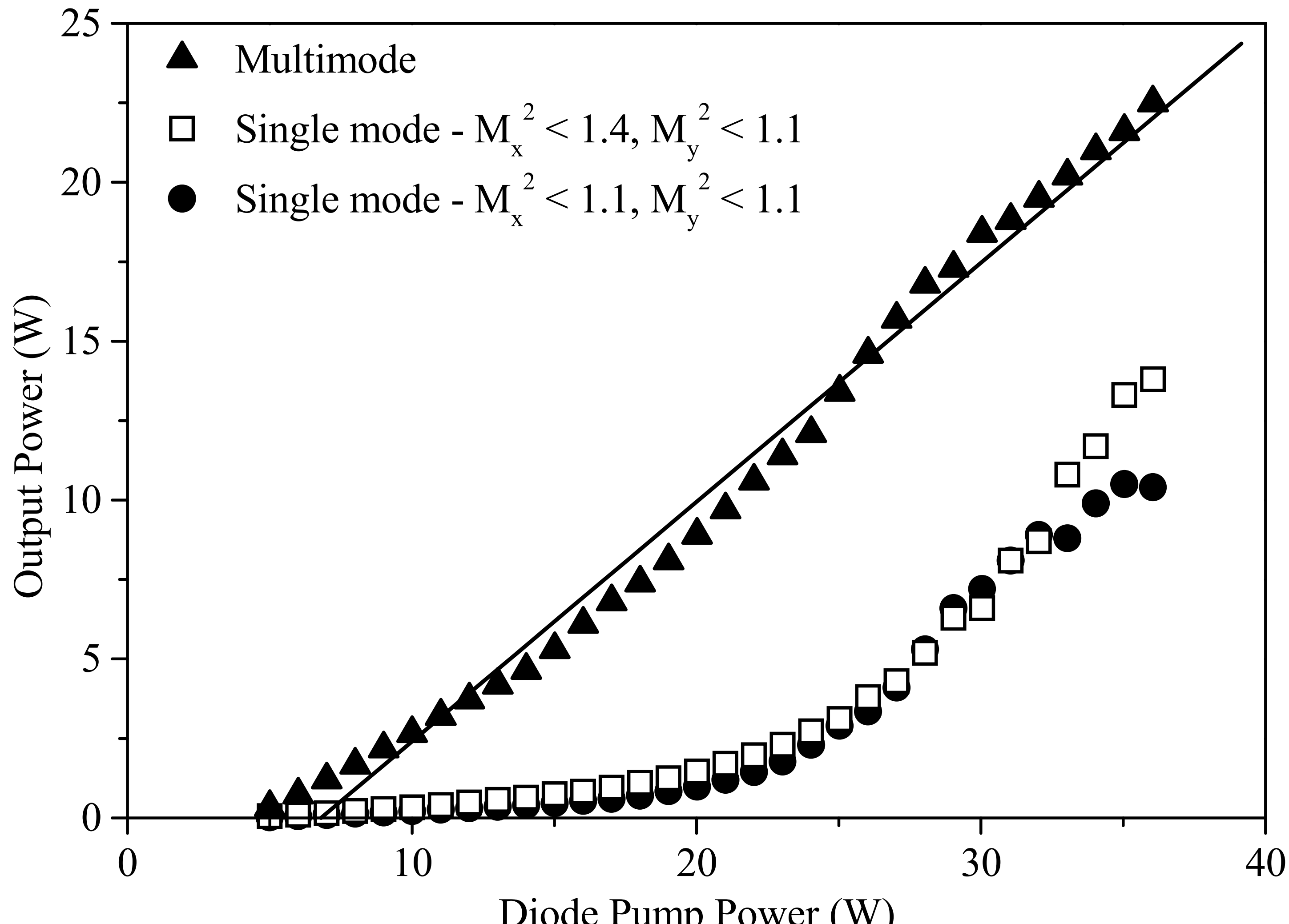
E. Rosas *et al*, Rev. Mex. Fís. **47**, (2001).


E. Rosas *et al*, Rev. Mex. Fís. **47**, (2001).

To optimize the energy extraction from the amplifying medium, as well the laser conversion efficiency:



M. J. Damzen *et al*, Opt. Commun. **196**, (2001).



M. J. Damzen *et al*, Opt. Commun. **196**, (2001).

End of Session Four.

Thank you very much for your attendance.