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Preparatory School to the Winter College on Optics: Advances in Nano-Optics and Plasmonics

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Coherence

M. Bertolotti
University of Rome La Sapienza
Italy

COHERENCE

Atoms or molecules emit a "photon" following a **transition**.

In an incoherent source each particle emits light pulses randomly, **uncorrelated** from each other and from those of other particles.

A macroscopic source is made by a great number of excited atoms each one of which when disexciting emits a short wave-train.

These trains are characterized by a mean frequency characteristic of the transition and have very short duration, tipically of the order of $10^{-8} - 10^{-10}$ s.

COHERENCE

The observed light is the superposition of all wavetrains. Because the emission process is a statistical one the field is a random quantity.

We define a **mutual coherence function** as

$\Gamma(P_1,t_1; P_2,t_2)) = \langle V(P_1,t_1)V^*(P_2,t_2)\rangle$

For ergotic processes

$\Gamma(P_1,t_1;P_2,t_2)=\langle V(P_1,t+\tau)V^*(P_2,t)\rangle$

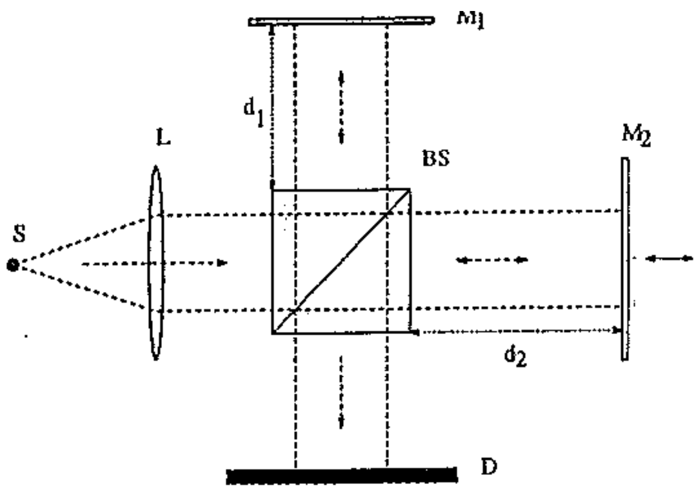
with $\tau = t_1 - t_2$

TEMPORAL COHERENCE

Use a Michelson interferometer and assume the two mirrors M_1 and M_2 are at equal distance from the beam splitter BS.

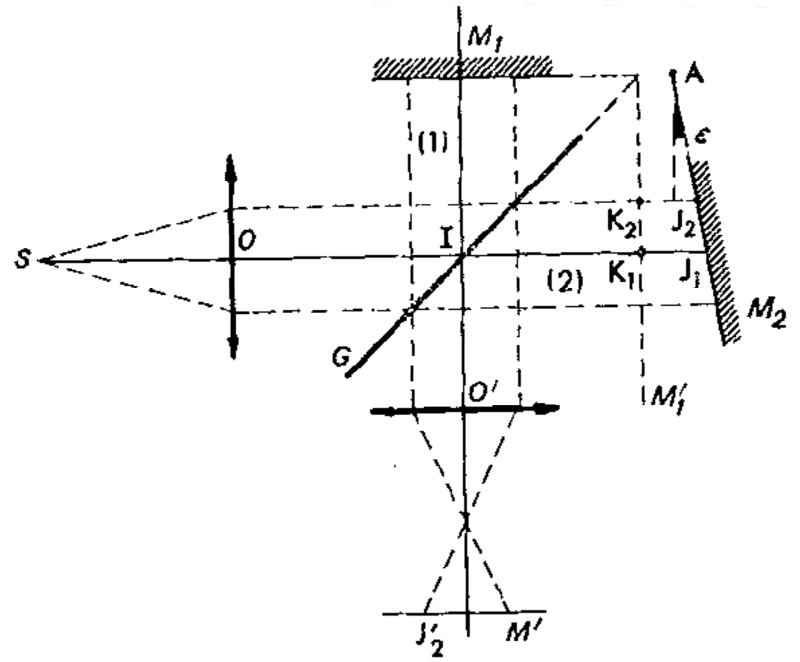
Then we displace for example M_1 increasing the distance. The fringes, very sharp at the beginning, gradually decrease in visibility and eventually **disappear**.

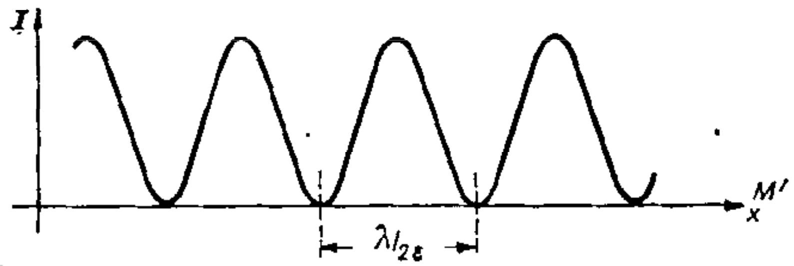
WHY?



SPATIAL COHERENCE

The Michelson interferometer must be slightly misaligned to produce fringes

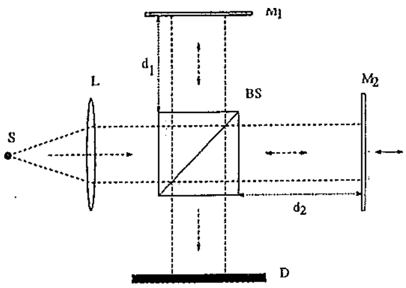




TEMPORAL COHERENCE

The light is made of a great number of randomly emitted wavetrains and in the observation time many of them are observed.

Assume each train has a duration Δt. When entering the interferometer it is splitted into two beams that after reflection by mirrors M_1 and M_2 are observed on the screen D



TEMPORAL COHERENCE

The two trains may interfer on D if one arrives before the other is completely passed across and therefore the path difference between the two interferometer arms should be less than the length of the trains.

If this is not so each train reflected by a mirror (for ex. M_1) superposes on new trains reflected by M_2 that are uncorrelated with the first train

TEMPORAL COHERENCE

Because the phase difference between different trains fluctuates randomly during the time of observation, many different trains with random phase superpose and the net effect is zero

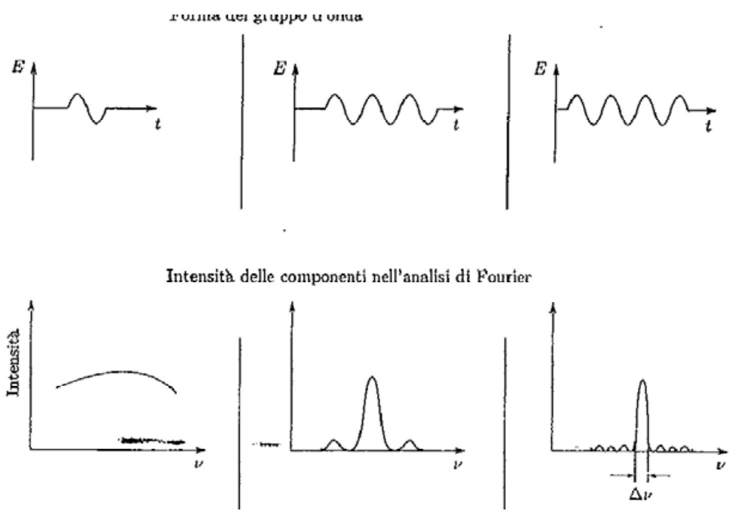
TEMPORAL COHERENCE

Different train shapess are shown in the figure together with their Fourier transform.

The linewidth Δν, defined as the frequency range in which the armonic components are sensibly different from zero is

$$\Delta\nu \simeq 1/\Delta t$$

The time Δt is called coherence time



TEMPORAL COHERENCE

The length

$l = c\Delta t = c/\Delta\nu$

is called **coherence length**.

The coherence time or the coherence length allow to characterize the temporal coherence properties of the light emitted by a source.

TEMPORAL COHERENCE

In ordinary (f.e.thermal) sources Δl is of the order of a few centimeter.

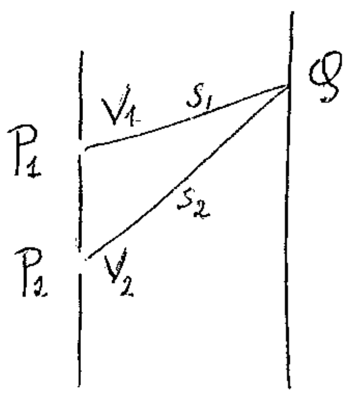
For a laser it could be of the order of 10^7 meters.

SPATIAL COHERENCE

Let us consider the field made by a quasi-mpnochromatic source spatially extended and two points P_1 and P_2 in it.

To study the correlation between the field (disturbance) in P_1 and P_2 a Young interferometer may be used.

The points P_1 and P_2 act as secondary sources for the interferometer..



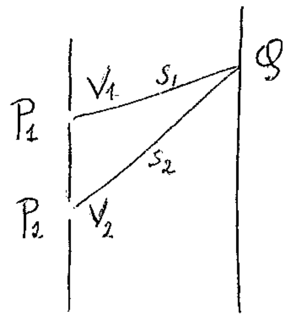
SPATIAL COHERENCE

If the produced fringes are studied one finds that if P_1 and P_2 are sufficiently near to each other, their optical disturbances are not independent and the fringe visibility is high.

When the distance is large the visibility is zero.

In such way a coherent region is defined.

SPATIAL COHERENCE



In Q

$$V(Q) = V_1(t-t_1) + V_2(t-t_2)$$

where

$t_1 = s_1/c$ and $t_2 = s_2/c$ are the times the light employs to travel from P_1 to Q and P_2 to Q..

SPATIAL COHERENCE

The intensity in Q is

$<I(Q)> = <|V(Q|^2)> = <|V_1+V_2|^2 > =$

$<|V_1 |^2> + <|V_2|^2>+ <V_1V_2^*>+ <V_1^*V_2> =$

$<|V_1|^2> + <|V_2|^2 > + 2Re<(V_1V_2^*)>$

If light is stationary

$<V_1(t-t_1)V_1^*(t-t_1)> = <V_1(t)V_1^*(t)>=$

$= <|V_1(Q)|^2> = I_1(Q)$

is the light in Q if P_2 is closed and only P_1 allows the radiation to pass.

Similarly if P_1 is closed and P_2 allows light to pass

$<|V_2(Q)|^2> = I_2(Q)$

SPATIAL COHERENCE

Therefore

$$I(Q) = I_1(Q) + I_2(Q) +$$

$$+ 2\mathrm{Re}\langle V_1(t + \tau)V_2^*(t)\rangle$$

The quantity

$$I_{12} = \langle V_1(t + \tau)V_2^*(t)\rangle$$

is called the **mutual intensity** with

$$\tau = t_1 - t_2$$

We define

$$\gamma_{12} = I_{12} / \sqrt{(I_1 I_2)}$$

and call it the **complex degree of coherence**.

By its definition

$$0 < \gamma_{12} < 1$$

At the end we may write

$$I(Q) = I_1 + I_2 + 2\sqrt{(I_1 I_2)}\mathrm{Re}(\gamma_{12})$$

SPATIAL COHERENCE

When P_1 and P_2 coincide

$$\Gamma_{11}(\tau) = \langle V_1(t + \tau))V_1^*(t)\rangle$$

If $\tau = 0$

$$\Gamma_{11}(0) = I_1$$

The mutual intensity is

$$I_{12} = \Gamma_{12}(\tau) = \Gamma_{12}\{(s_2 - s_1)/\tau\}$$

SPATIAL COHERENCE

In general the light is not monochromatic. However if the frequency varies over a small interval around a mean frequency ν_o one may write

$$\gamma_{12}(\tau) = |\gamma_{12}(\tau)| \exp i[(\alpha_{12}(\tau) - 2\pi\bar{\upsilon}\tau]$$

where

$$\alpha_{12}(\tau) = 2\pi\bar{\upsilon}\tau + \arg\gamma_{12}(\tau)$$

Then

$$I(Q) = I_1 + I_2 + 2\sqrt{(I_1 I_2)}|\gamma_{12}(\tau)|\cos[\alpha_{12}(\tau) - \delta]$$

with

$$\tau = (s_2 - s_1)/c \quad \text{and} \quad \delta = 2\pi\bar{\upsilon}\tau = (2\pi/\lambda)(s_2 - s_1)$$

SPATIAL COHERENCE

If

$|\gamma_{12}(\tau)| = 1$

the intensity in Q is the same as it would be obtained in monochromatic light

$$I(Q)= I_1+I_2+2\sqrt{(I_1I_2)}|\gamma_{12}(\tau)\cos[(2\pi/\lambda)(s_2-s_1)]$$

SPATIAL COHERENCE

In general however

$\gamma_{12}(\tau) \neq 1$

and

$I(Q) = I_1+I_2+2\sqrt{(I_1 I_2)}|\gamma_{12}(\tau)|\cos[\alpha_{12}(\tau)-\delta]$

Therefore I(Q) varies from a maximum

$I_{max} = I_1 + I_2 + 2\sqrt{(I_1 I_2)}|\gamma_{12}(\tau)|$

and a minimum

$I_{min} = I_1 + I_2 - 2\sqrt{(I_1 I_2)}|\gamma_{12}(\tau)|$

SPATIAL COHERENCE

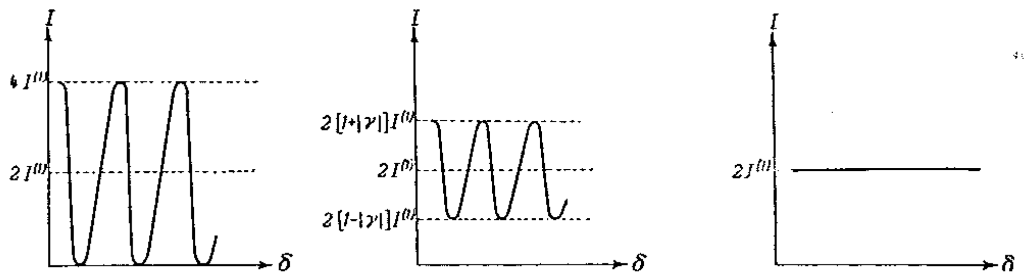
The intensity of the produced fringes varies between the maximum and minimum intensity with a sinusoidal law. One defines the **visibility** of the fringes as

$\theta(Q) = (I_{max} - I_{min})/(I_{max} + I_{min}) =$

$2\sqrt{(I_1 I_2)}|\gamma_{12}(\tau)|/(I_1 + I_2)$

If $I_1 = I_2$ then

$\theta(Q) = |\gamma_{12}(\tau)|$



(a) Coherent superposition ($|\gamma| = 1$) (b) Partially coherent superposition ($0 < |\gamma| < 1$) (c) Incoherent superposition ($\gamma = 0$)

SPATIAL COHERENCE

If τ is small about zero, one defines

$$J_{12} = \Gamma_{12}(0) = \langle V_1(t)V_2^*(t) \rangle$$

which is the **mutual intensity**

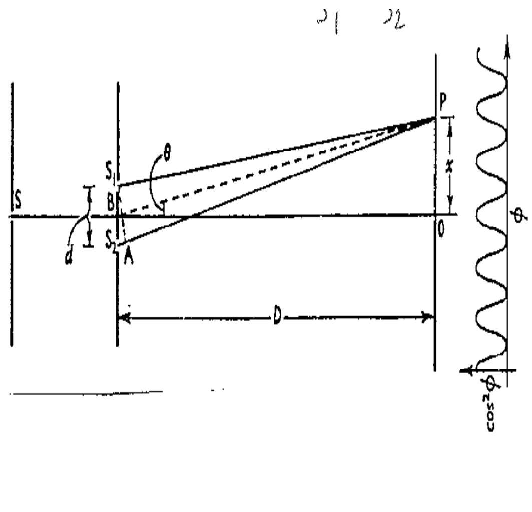
and

$$\mu_{12} = \gamma_{12}(0)$$

which takes the name of **complex degree of coherence**

SPATIAL COHERENCE

We may study the spatial coherence properties of a source with the Young interferometer shown below



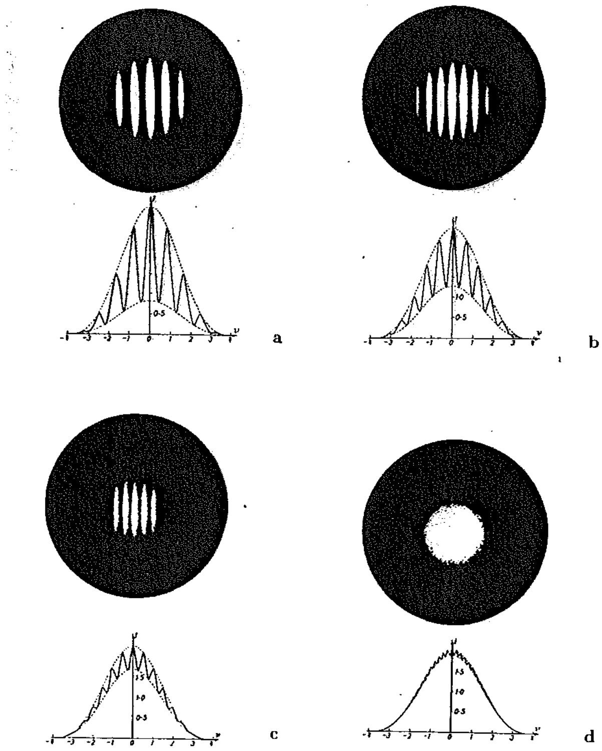


Figura 5.4: Interferenza tra due fasci gaussiani per differenza dei cammini nei due bracci dell'interferometro di Michelson crescenti: a: 0.6 cm, b: 0.8 cm, c: 1. cm, d: 2.3 cm. (Da B.J. Thompson, E. Wolf, Journ. Opt. Soc. Am., vol. 47 (1957), pag. 895)

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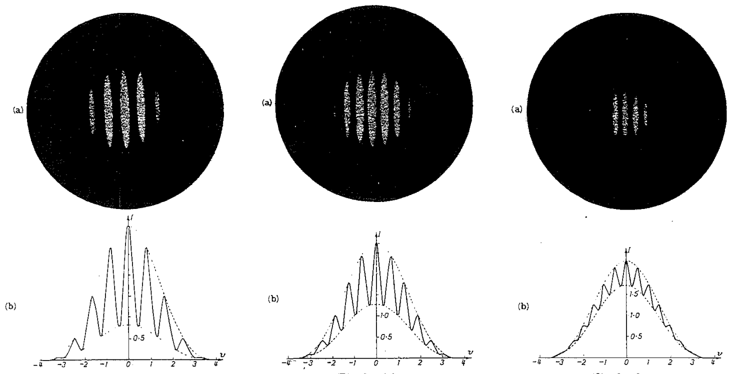

(A): $d = 0{\cdot}6$ cm
$|\mu_{12}| = 0{\cdot}593$, $\beta_{12} = 0$

(B): $d = 0{\cdot}8$ cm
$|\mu_{12}| = 0{\cdot}361$, $\beta_{12} = 0$

(C): $d = 1$ cm
$|\mu_{12}| = 0{\cdot}146$, $\beta_{12} = 0$