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Lectures on Fresnel formulae and total reflection

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RIFLECTION AND REFRACTION OF PLANE WAVES
FRESNEL FORMULAE

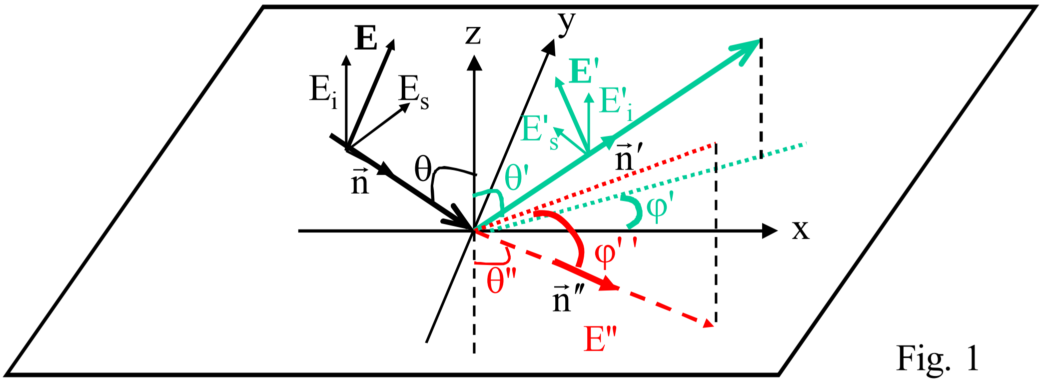

Fig. 1

The electric field is written as

$$\vec{E} = \vec{E}_m \sin\omega\left(t - \frac{\vec{n}\cdot\vec{r}}{v_1}\right) \qquad \text{incident wave} \qquad (1a)$$

$$\vec{E}' = \vec{E}'_m \sin\left[\omega'\left(t - \frac{\vec{n}'\cdot\vec{r}'}{v_1}\right) - \delta'\right] \qquad \text{Reflected wave} \qquad (1b)$$

$$\vec{E}'' = \vec{E}''_m \sin\left[\omega''\left(t - \frac{\vec{n}''\cdot\vec{r}''}{v_2}\right) - \delta''\right] \qquad \text{Refracted wave} \qquad (1c)$$

We write

$$\vec{n}' = \sin\theta'\cos\varphi'\vec{i} + \sin\theta'\sin\varphi'\vec{j} + \cos\theta'\vec{k} \qquad (2a)$$

$$\vec{n}'' = \sin\theta''\cos\varphi''\vec{i} + \sin\theta''\sin\varphi''\vec{j} + \cos\theta''\vec{k} \qquad (2b)$$

Due to continuity relations on the plane z = 0

$$E_x + E'_x = E''_x \qquad \frac{B_x}{\mu_1} + \frac{B'_x}{\mu_1} = \frac{B''_x}{\mu_2}$$
$$E_y + E'_y = E''_y \qquad \frac{B_y}{\mu_1} + \frac{B'_y}{\mu_1} = \frac{B''_y}{\mu_2} \qquad (3)$$

which automatically implies

$$\omega = \omega' = \omega'' \qquad \delta' \text{ e } \delta'' = 0 \text{ o } \pi$$

For the points for which z = 0

$$\frac{\vec{n}\cdot\vec{r}}{v_1} = \frac{\vec{n}'\cdot\vec{r}'}{v_1} = \frac{\vec{n}''\cdot\vec{r}''}{v_2} \qquad (4)$$

from which the reflection and refraction laws are immediately obtained.

To obtain the amplitudes of the field we rewrite eqs.(3) considering the field components in the incidence plane (E_p, B_p), called **parallel component** (p wave), and orthogonal (E_s, B_s), **perpendicular** (wave s from German senkrecht). So each wave is due to the superposition of two polarized waves

p wave

$$E_t = E_p \cos\theta$$
$$E'_t = E'_p \cos\theta \qquad (5)$$
$$E''_t = E''_p \cos\theta''$$

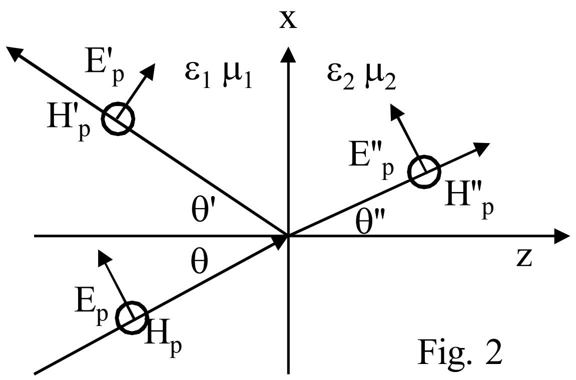

Fig. 2

Due to (3)

$$E_p \cos\theta - E'_p \cos\theta = E''_p \cos\theta'' \qquad (6)$$

Similarly for B remembering that $B_y = E_x / v$ putting $n_1 = c/v_1$, $n_2 = c/v_2$,

$$\frac{B_y}{\mu_1} + \frac{B'_y}{\mu_1} = \frac{B''_y}{\mu_2} \qquad (7)$$

$$\frac{E_p}{\mu_1 v_1} + \frac{E'_p}{\mu_1 v_1} = \frac{E''_p}{\mu_2 v_2} \quad \therefore \quad n_1(E_p + E'_p) = n_2 E''_p \qquad (8)$$

if

$$\mu_{r1} = \mu_{r2} \; .$$

s wave

$$E_s + E'_s = E''_s \; . \qquad (9)$$

For the magnetic field

$$B_t = B_x \cos\theta$$
$$B'_t = -B'_x \cos\theta \qquad (10)$$
$$B''_t = B''_x \cos\theta''$$

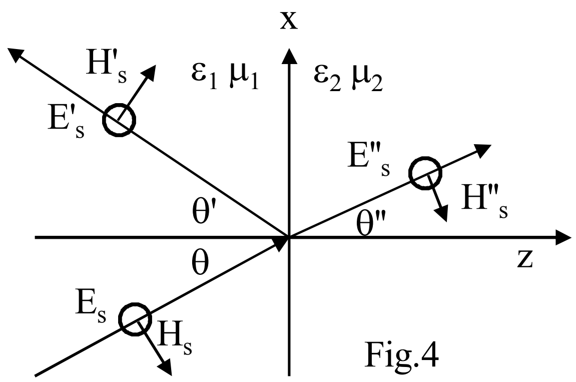

Fig.4

so

$$\frac{E_s \cos\theta}{\mu_1 v_1} - \frac{E'_s \cos\theta}{\mu_1 v_1} = \frac{E''_s \cos\theta''}{\mu_2 v_2}$$

$$n_1(E_s - E'_s)\cos\theta = n_2 E''_s \cos\theta'' \qquad (11)$$

if $\mu_1 = \mu_2$.

So we have two equations with E_p and two equations with E_s. The two waves **parallel** and **normal** to the incident plane are therefore **independent** one from the other.

Eqs. (6), (8), (9), (11) may be solved.
From (6) we have

$$E'_p = E_p - E''_p \frac{\cos\theta''}{\cos\theta} \qquad (12)$$

which, substituting into (8), tking into account that

$$\frac{n_1}{n_2} = \frac{\sin\theta''}{\sin\theta} \qquad (13)$$

gives

$$E''_p = E_p \frac{2\sin\theta''\cos\theta}{\sin\theta\cos\theta + \sin\theta''\cos\theta''} = E_p \frac{2\sin\theta''\cos\theta}{\sin(\theta+\theta'')\cos(\theta-\theta'')} . \qquad (14)$$

Substituting into (12)

$$E'_p = E_p \frac{\mathrm{tg}(\theta-\theta'')}{\mathrm{tg}(\theta+\theta'')} . \qquad (15)$$

Similarly from (9) and (11)

$$E'_s = -E_s \frac{\sin(\theta-\theta'')}{\sin(\theta+\theta'')} \qquad (16)$$

$$E''_s = E_s \frac{2\sin\theta''\cos\theta}{\sin(\theta+\theta'')} \qquad (17)$$

which, with $n = n_2/n_1$,can be written

$$E'_s = E_s \frac{\cos\theta - \sqrt{n^2 - \sin^2\theta}}{\cos\theta + \sqrt{n^2 - \sin^2\theta}} \qquad (16a)$$

$$E'_p = E_p \frac{n^2\cos - \sqrt{n^2 - \sin^2\theta}}{n^2\cos\theta + \sqrt{n^2 - \sin^2\theta}} \qquad (15a)$$

These expressions are the **Fresnel relations**.

REFRACTION AND REFLECTION FROM A LESS DENSE MEDIUM TO A DENSER ONE

Assume medium 1 be optically less dense, for ex. air (n_1 = 1) and medium 2 be denser, f.e. water or glass, with refractive index n_2 and put n= n_2/n_1.

We define a coefficient

$$r = -\frac{E'}{E} \tag{18}$$

that we call **amplitude reflection coefficient**. The reflection coefficient for intensity is

$$R = |r|^2 = \frac{|E'|^2}{|E|^2} = \frac{I'}{I}.$$

Let us consider eparately the two cases

A. Parallel case:

$$r_p = -\frac{E'_p}{E_p} = -\frac{\mathrm{tg}(\theta - \theta'')}{\mathrm{tg}(\theta + \theta'')} = -\frac{n^2\cos\theta - \sqrt{n^2 - \sin^2\theta}}{n^2\cos\theta + \sqrt{n^2 - \sin^2\theta}} \tag{19}$$

A plot of r_p is a parabole starting from

$$-\frac{n-1}{n+1}$$

ending with a positive value for $\theta = \pi/2$

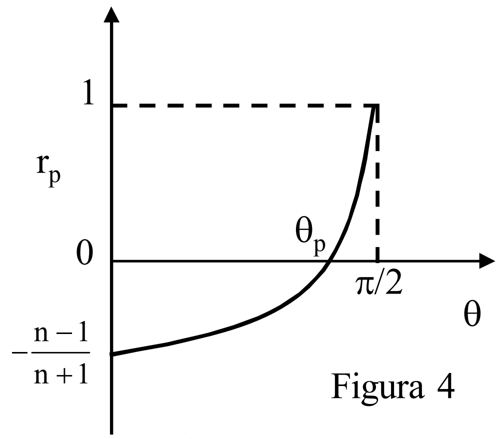

Figura 4

$$r_p = \frac{-\mathrm{tg}\left(\frac{\pi}{2} - \theta''\right)}{\mathrm{tg}\left(\frac{\pi}{2} + \theta''\right)} = +1 \,. \qquad (20)$$

The curve of r_p cut the abscissa at an angle θ_p. From (19) the denominator should change from $+\infty$ to $-\infty$ and therefore

$$\theta_p + \theta'' = \frac{\pi}{2} \qquad \therefore \qquad \theta'' = \frac{\pi}{2} - \theta_p$$

$$\sin\theta'' = \cos\theta_p \,. \qquad (21)$$

From the refraction law

$$\sin\theta'' = \frac{1}{n}\sin\theta_p = \cos\theta_p$$

from which

$$\mathrm{tg}\,\theta_p = n \,. \qquad (22)$$

For glass (n = 3/2) and water (n = 4/3), $\theta_p = 57°$ and $53°$ respectively. Because $r_p = 0$ at that angle, the reflected light is completely polarized with the electric field vibrating **perpendicularly** to the incident plane (**Brewster law**). Similarly we may define an **amplitude transmission coefficient:**

$$t_p = \frac{E''_p}{E_p} = \frac{2\sin\theta''\cos\theta}{\sin(\theta+\theta'')\cos(\theta-\theta'')} \,.$$

B. Perpendicular case:

In this case:

$$r_s = \frac{E'_s}{E_s} = \frac{sin(\theta-\theta'')}{sin(\theta+\theta'')} = -\frac{\cos\theta - \sqrt{n^2 - sin^2\theta}}{\cos\theta + \sqrt{n^2 - sin^2\theta}} \qquad (23)$$

For small θ (nearly normal incidence):

$$r_s = \frac{\theta - \frac{\theta}{n}}{\theta + \frac{\theta}{n}} = \frac{n-1}{n+1} \ . \qquad (24)$$

For ex. For water (n = 4/3), $r_s = 1/7$ $\qquad R_s = |r_s|^2 = \frac{I'_s}{I_s} = 2\%$.

For glass (n = 3/2), $r_s = 1/5$, $R_s = 4\%$.

Nor water nor glass may be suitable as **mirrors** for **perpendicular** incidence because they have very low reflectivity. In glass mirrors the reflectivity is provided by the silver covering. The glass is only a protection.

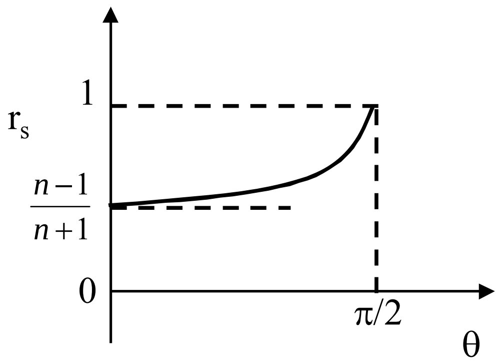

Figura 5

A plot of r_s versus θ starts from θ=0 with an horizontal tangent and increases parabolically with θ. At θ = π/2, Snell law becomes

$$\sin\theta'' = \frac{1}{n} \qquad (25)$$

$$\sin(\theta \pm \theta'') = \sin\left(\frac{\pi}{2} \pm \theta''\right) = \cos\theta'' = \frac{\sqrt{n^2-1}}{n} \qquad (26)$$

therefore $r_s = 1$.

At grazing incidence reflection is **cull**. This is the reason for the beautiful specular image of the opposite side in a lake waters, or of the sunset specular image over a quiet sea: this image has an intensity near the one of the sun itself.

The expressions for E′ and E″ are real. Therefore the phases are either 0 or π with respect to the phase of the incident wave and are simply indicated by the amplitude sign. Figs. 4 and 5 show that for all incidence angles a phase shift of π is produced for s polarization (remember that $r_s = - E'_s/E_s$). On the contrary E'_p is in phase with E_p for all incidence angles lower than θ_p and is phase shifted by π for larger angles.

In the interpretation of this result, attention should be given to which is the positive vibration direction for the vibration of the field parallel to the incidence plane (s. fig. 6).

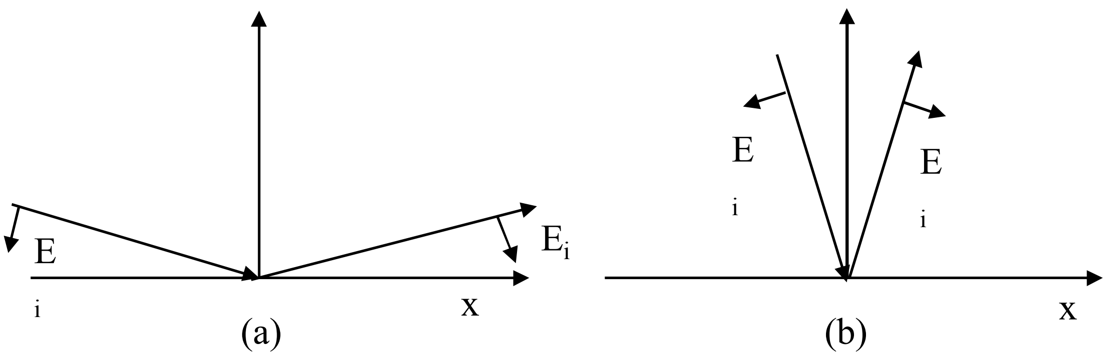

Fig. 6

For grazing light the positive direction of E_p for incident and reflected field are mostly parallel as shown in fig.6° . Therefore a phase shift of π means that to an observer on the x-axis the direction of E_p appear suddenly inverted due to the reflection. This behaviour is important for the interference phenomena.

If the incident light is not polarized it could be considered the sum of two p and s polarized lights. Fof normal incidence the intensity reflection coefficient $R = |r|^2 = I'/I$ is the same for the two waves. This is obvious because for normal incidence an incidence plane cannot be defined so p and s cannot be distinguished. Therefore reflected and refracted wave are not polarized. For all the other incidence angle the intensity reflection coefficients for the two components are different.

The amplitude transmission coefficient is:

$$t_s = \frac{E''_s}{E_s} = \frac{2\sin\theta''\cos\theta}{\sin(\theta+\theta)} = \frac{2\cos\theta}{\cos\theta + \sqrt{n^2 - \sin^2\theta}}$$

The **reflecting power** R, is defined as:

$$R = |r|^2 \ .$$

The **transmission power** T is

$$R + T = 1$$

Because in absence of absorption the light is only reflected or transmitted

$$T = \frac{n_2 \cos\theta''}{n_1 \cos\theta}\left(\frac{E''}{E}\right)^2 \quad .$$

This expression is due to the fact that the cross section of the transmitted beam has a different area of the incident and reflected beam

Reflection and refraction from a denser medium to a less dense one:

For example medium2 is air (n_2 = 1). Let us put n′ = n_1/n_2.. The formulae previously studied are still valid if 1/n′ is put to replace n. For the refraction law f.e. it is

$$n \rightarrow \frac{1}{n'} \qquad (27) \qquad\qquad \frac{\sin\theta}{\sin\theta''} = \frac{1}{n'} \; . \qquad (28)$$

From this relation θ′′>θ for small θ, but there is no solution for

$$n' \sin\theta > 1 \quad . \qquad (29)$$

In this case the coefficients for E_p and E_s become complex.

In fig. 7 viene the amplitude reflection coefficient is plottedversus θ for 0 < θ < π/2.

In this case we consider the coefficient:

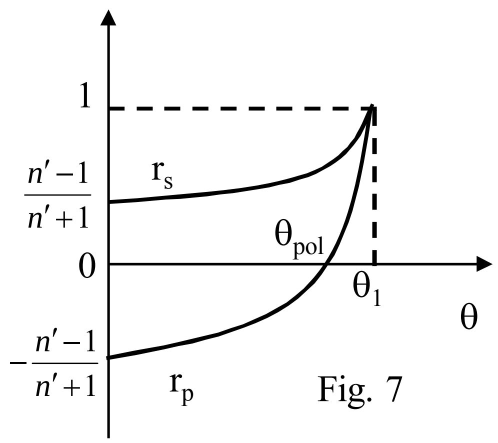

Fig. 7

$$r = \frac{E'}{E} \qquad (30)$$

That is

$$r_s = \frac{\sin(\theta'' - \theta)}{\sin(\theta'' + \theta)} = \frac{n'\cos\theta - \cos\theta''}{n'\cos\theta + \cos\theta''} \qquad (31)$$

and

$$r_p = -\frac{tg(\theta'' - \theta)}{tg(\theta'' + \theta)} = \frac{-n'\cos\theta'' + \cos\theta}{n'\cos\theta'' + \cos\theta} \quad . \qquad (32)$$

Pay attension that for r we choose the opposite sign of (18). In fig. 7 the limit angle θ_1 for total reflection is marked

$$n'\sin\theta_1 = 1 \quad . \qquad (33)$$

The plots for r_p and r_s are stil similar to the ones of figs.4 and 5, but now the ordinate 1 is reached not at $\theta = \pi/2$ but for $\theta = \theta_1$. Infact, because in that point $\theta'' = \pi/2$, we have:

$$r_s = \frac{\sin\left(\frac{\pi}{2} - \theta\right)}{\sin\left(\frac{\pi}{2} + \theta\right)} = \frac{\cos\theta}{\cos\theta} = 1 \qquad (34)$$

$$r_p = -\frac{tg\left(\frac{\pi}{2} - \theta\right)}{tg\left(\frac{\pi}{2} + \theta\right)} = 1 \, . \qquad (35)$$

Before that point the plot for r_p cut the abscissa at the polarization angle

$$\theta_{pol} + \theta'' = \frac{\pi}{2} \quad \therefore \quad tg\theta_{pol} = \frac{1}{n'} \quad . \qquad (36)$$

What happens for $\theta > \theta_1$?.

In this case $n' \sin\theta$ is greater than 1 and we may write

$$\cos\theta'' = \sqrt{1 - \sin^2\theta''} = \sqrt{1 - (n')^2\sin^2\theta} = i\sqrt{(n')^2\sin^2\theta - 1} \, . \quad (37)$$

Substituting in (31) and (32)

$$r_s = \frac{n'\cos\theta - i\sqrt{(n')^2\sin^2\theta - 1}}{n'\cos\theta + i\sqrt{(n')^2\sin^2\theta - 1}} \qquad (38)$$

$$r_p = \frac{\cos\theta - in'\sqrt{(n')^2\sin^2\theta - 1}}{\cos\theta + in'\sqrt{(n')^2\sin^2\theta - 1}} \quad . \qquad (39)$$

Another way to look at the problem is to think that for $\theta > \theta_1$ the θ'' angle becomes complex: that is

$$sin\theta'' = sin\left(\frac{\pi}{2} \pm i\beta\right) = \cos(\pm i\beta) = \cosh\beta > 1 \qquad (40)$$

As required by the refraction law (*)

$$\sin\theta'' = n'\sin\theta > 1 . \qquad (41)$$

Using (40)

$$r_s = \frac{\sin\left(\frac{\pi}{2} \pm i\beta - \theta\right)}{\sin\left(\frac{\pi}{2} \pm i\beta + \theta\right)} = \frac{\cos(\theta \mp i\beta)}{\cos(\theta \pm i\beta)} = \frac{\cos\theta\cosh\beta \pm i\sin\theta\sinh\beta}{\cos\theta\cosh\beta \mp i\sin\theta\sinh\beta} . \qquad (42)$$

Deriving the modulus of r, we have

$$|r_s|^2 = r_s r_s^* = \frac{(\cos\theta\cosh\beta \pm i\sin\theta\sinh\beta)(\cos\theta\cosh\beta \mp i\sin\theta\sinh\beta)}{(\cos\theta\cosh\beta \mp i\sin\theta\sinh\beta)(\cos\theta\cosh\beta \pm i\sin\theta\sinh\beta)} = 1 .$$

This means that r_s may be written as

$$r_s = e^{i\gamma} . \qquad (43)$$

Similarly for r_p

$$r_p = -\frac{tg\left(\frac{\pi}{2} - \theta \pm i\beta\right)}{tg\left(\frac{\pi}{2} + \theta \pm i\beta\right)} = \frac{\cos\theta - in'\sqrt{(n')^2\sin^2\theta - 1}}{\cos\theta + in'\sqrt{(n')^2\sin^2\theta - 1}} = e^{i\delta} . \qquad (44)$$

(*) Because $e^{i\alpha} = \cos\alpha + i\sin\alpha$

It is $\cos\alpha = \dfrac{e^{i\alpha} + e^{-i\alpha}}{2}$

And if $\alpha = i\beta$,

$$\cos i\beta = \frac{e^{-\beta} + e^{+\beta}}{2} = \cosh\beta$$

The absolute values of r_p and r_s are easily seen to have absolute value equal to one. The quantities γ and δ are real phase angles. They may be derived observing that

$$r_s = \frac{\cos\theta\cosh\beta \pm i\sin\theta\sinh\beta}{\cos\theta\cosh\beta \mp i\sin\theta\sinh\beta} = \frac{n'\cos\theta - i\sqrt{n'^2\sin^2\theta - 1}}{n'\cos\theta + i\sqrt{n'^2\sin^2\theta - 1}} = e^{i\gamma} = \frac{ae^{i\alpha}}{ae^{-i\alpha}} \,. \quad (45)$$

From (45)

$$\gamma = 2\alpha$$

so

$$\mathrm{tg}\alpha = \mathrm{tg}\frac{\gamma}{2} = \frac{\sin\theta\sinh\beta}{\cos\theta\cosh\beta} = -\frac{\sqrt{(n')^2\sin^2\theta - 1}}{n'\cos\theta} \qquad (46)$$

$$\text{e} \qquad \mathrm{tg}\frac{\delta}{2} = -\frac{n'\sqrt{(n')^2\sin^2\theta - 1}}{\cos\theta} \,. \qquad (47)$$

The relative phase between the two waves

$$\Delta = \gamma - \delta$$

is

$$\mathrm{tg}\frac{\Delta}{2} = \frac{\cos\theta\sqrt{(n')^2\sin^2\theta - 1}}{n'\sin^2\theta} \,. \quad (48)$$

It is shown in fig.8 for n = 1.5. The amplitudes of the reflected fields are

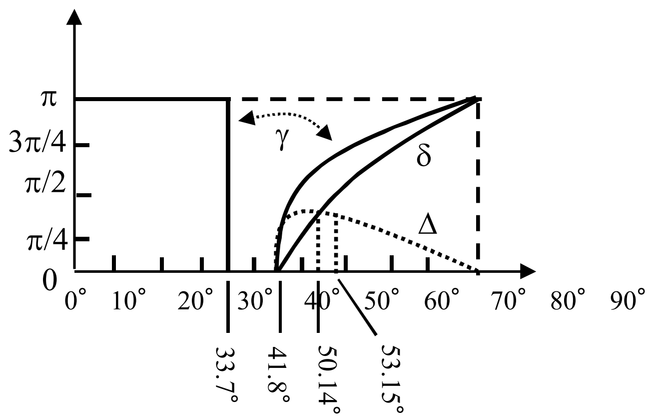

Figura 8

$$E'_s = E_s e^{i\gamma} \text{ e } E'_p = E_p e^{i\delta} \,. \qquad (49)$$

Or

$$E'_s = A_s e^{i[\omega t - k(x\sin\theta - z\cos\theta) + \gamma]} \qquad (50)$$

$$E'_p = A_i e^{i[\omega t - k(x\sin\theta - z\cos\theta) + \delta]} \,. \qquad (51)$$

The phase shift Δ between the two components of the reflected field brings a change in the properties of the totally reflected light.

Let us consider a specific example. Let us take a polarized light incident on a glass-air interface with the vibration plane tilted by an angle 45° with respect to the incidence plane. In this case $E_p = E_s$ and therefore also $E'_p = E'_s$. It is not so for the phases. From (48) for n' = 1.5 the phase difference between the two reflected components is 45° when the incidence angle is 53° 15' or 50° 14'. The reflected light is therefore elliptically polarized.

More generally, because

$$\frac{r_p}{r_s} = e^{i(\gamma-\delta)} = \frac{\cos(\theta \mp i\beta)}{\cos(\theta \pm i\beta)} \cdot \frac{ctg(\theta \mp i\beta)}{ctg(\theta \pm i\beta)} = \frac{\sin(\theta \mp i\beta)}{\sin\theta \pm i\beta)} \qquad (52)$$

for $\theta = \theta_l$, because $\theta'' = \pi/2$, that is $\beta = 0$, the ratio

$$\frac{\sin(\theta \mp i\beta)}{\sin(\theta \pm i\beta)} = \frac{\sin\theta}{\sin\theta}$$

Becomes real (and equal to one) and therefore $\gamma = \delta$. The same is valid for $\theta = \pi/2$ because in this case

$$\sin\left(\frac{\pi}{2} - i\beta\right) 0 \sin\left(\frac{\pi}{2} + i\beta\right) = \cosh\beta \ .$$

Between these two points there must be a point for which $\Delta = \gamma - \delta$ is a maximum. This can be seen making the derivative of (52) and putting it to zero

$$0 = \frac{\cos(\theta_m - i\beta)}{\sin(\theta_m + i\beta)}\left(1 - i\frac{d\beta}{d\theta}\right) - \frac{\sin(\theta_m - i\beta)}{\sin^2(\theta_m + i\beta)}\cos(\theta_m + i\beta)\left[1 + i\frac{d\beta}{d\theta}\right]. \quad (53)$$

but

$$n\sin\theta = \cos i\beta \qquad (54)$$

The derivattive

$$\frac{d\beta}{d\theta} = \frac{in\cos\theta}{\sin i\beta} \ . \qquad (55)$$

Substituting (55) into (53)

$$0 = \sin 2i\beta + n\frac{\sin 2\theta_m \cos\theta_m}{\sin i\beta}$$

From (54)

$$0 = 2n\frac{\sin\theta_m}{\sin i\beta}\left\{2-(n^2+1)\sin^2\theta_m\right\}$$

or

$$\sin^2\theta_{max} = \frac{2}{n^2+1} \quad . \qquad (56)$$

The (52) may be written now as

$$\frac{e^{i\Delta_{max}}-1}{e^{i\Delta_{max}}+1} = -\frac{\cos\theta\sin i\beta}{\sin\theta\cos i\beta}$$

or

$$i\,tg\frac{\Delta_{max}}{2} - ctg\theta\, tg i\beta \quad .$$

Because

$$ctg\theta = \sqrt{\frac{n^2-1}{2}}$$

$$tg i\beta = \frac{i}{n}\sqrt{\frac{n^2-1}{2}}$$

It is

$$tg\frac{\Delta_{max}}{2} = \frac{n^2-1}{2n} \quad . \qquad (57)$$

Let us apply this result to a glass with n= 1.5. It is

$$tg\frac{(\gamma-\delta)_{max}}{2} = 0.417 \quad \therefore \quad (\gamma-\delta)_{max} = 45°24'$$

$$\sin\theta_{max} = 0.7845 \quad \therefore \quad \theta_{max} = 51°67' \quad .$$

It is not possible to obtain **circular** polarization ($\Delta = 90°$) through a single total reflection because γ - $\delta \leq 45°$ 24' for all incidence angles

However circular polarization may be obtained with two reflections. For 53° 15' or 50° 14' a phase shift of 45° may be obtained. If now there is a second internal reflection at one of these angles an additional shift of 45° is obtained so that the total phase difference is now 90° .

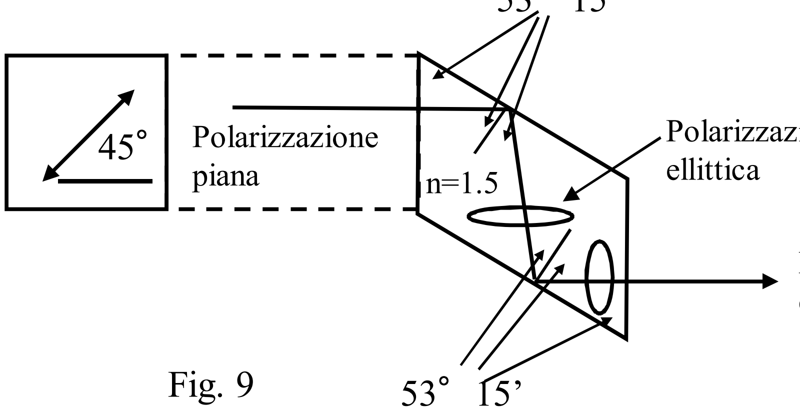

Fig. 9

Because the components have equal amplitude the light is circularly polarized. Fig.9, shows a **Fresnel romb**. In the example is used glass with n = 1.5 and incidence angles 53° 15'. The light impinges normally on the first face with the vibration plane at 45° with respect to the figure plane.

Evanescent waves:

Even if the incident energy is totally reflected for n' sin θ>1, a wave is propagating in the second medium. Let us write the complete expression of the electric field of the transmitted wave

$$\begin{aligned} E'' &= E''_m \exp i(\vec{k}'' \cdot \vec{r} - \omega t) \\ &= E''_m \exp i(k'' x \sin\theta'' + k'' y \cos\theta'' - \omega t) \quad . \qquad (58) \end{aligned}$$

Substituting from the refraction law (37)

$$\cos\theta'' = i\sqrt{(n')^2 \sin^2\theta - 1}$$

with

$$n' = \frac{n_1}{n_2}$$

one has

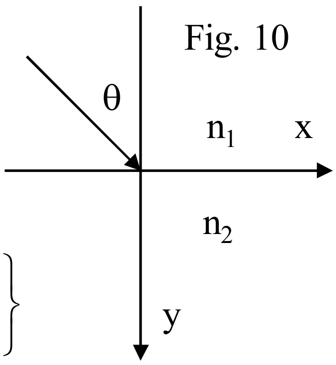

Fig. 10

$$E'' = E_m \exp\left\{-k'' y \sqrt{(n')^2 \sin^2\theta - 1}\right\} \exp i \left\{k'' x \frac{n_1}{n_2} \sin\theta - \omega t\right\}$$

$$= E''_m e^{-\alpha y} e^{i(kx - \omega t)} \qquad (59)$$

where

$$\alpha = k'' \sqrt{\left(\frac{n_1}{n_2}\right)^2 \sin^2\theta - 1}$$

$$\text{e} \qquad k = \frac{n_1}{n_2} k'' \sin\theta \quad . \qquad (60)$$

The first exponential function is real and represents a sharp decreasing of the amplitude of the wave while it penetrates in the less dense medium. If there is no absorption the Poynting vector turs and comes back in the denser medium.

Looking at eq.(59) one may see that this wave has an entirely different structure from a plane wave and that we call **homogeneous**. It is called **inhomogeneous** and it propagates like an ordinary planbe wave along x, in the plane z = 0, while its amplitude decreases perpendicularly along y. Because k" = 2π/λ the amplitude of the wave is sensibly different from zero only at distances of the order of a few wavelengths from the limit surface.

The magnetic field of the wave (59) has the same inhomogeneous structure and may be derived through Maxwell equation

$$\text{rot}\,\vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad .$$

Once B is derived the Poynting vector may be calculated

$$\vec{P} = -\frac{\vec{E} \times \vec{B}}{\mu_0} \quad .$$

One finds that the component of $\vec{P}$ parallel to the limit surface is allways positive. The flux of energy in the direction perpendicular to the limit surface chnges instead periodically its sign.Its mean value is therefore zero. There is an effective flux of energy parallel to the limit surface. The energy does not penetrate in the second medium but flows along the separation plane. This analysis refers to the stationary state. At the start before the stationary state is settled on a small quantity of energy goes in the second medium

That the wave penetrates in the less dense medium may be demonstrated experimentally with two prisms as shown in fig.11. This disposition may be used to built optical couplers. The passage of the light through the air layer in these conditions is called **optical tunnelling** or **frustrated total reflection**

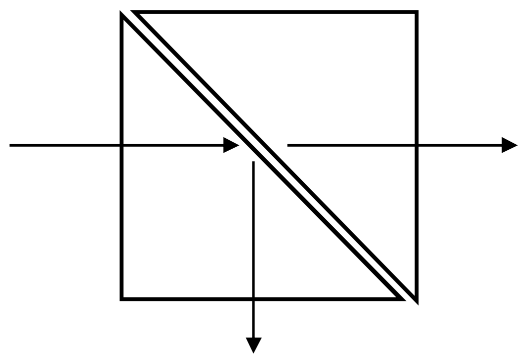

Fig. 11