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**Dynamical screening effects from first principles: implications for low-energy  
models and application to the iron pnictides**

Michele Casula  
*Université Pierre et Marie Curie*  
*France*

# Dynamical screening effects from first principles: Implications for low-energy models and application to the iron pnictides

M. Casula\*, P. Werner, F. Aryasetiawan, T. Miyake, L. Vaugier,  
A. Rubtsov, A. J. Millis, and S. Biermann

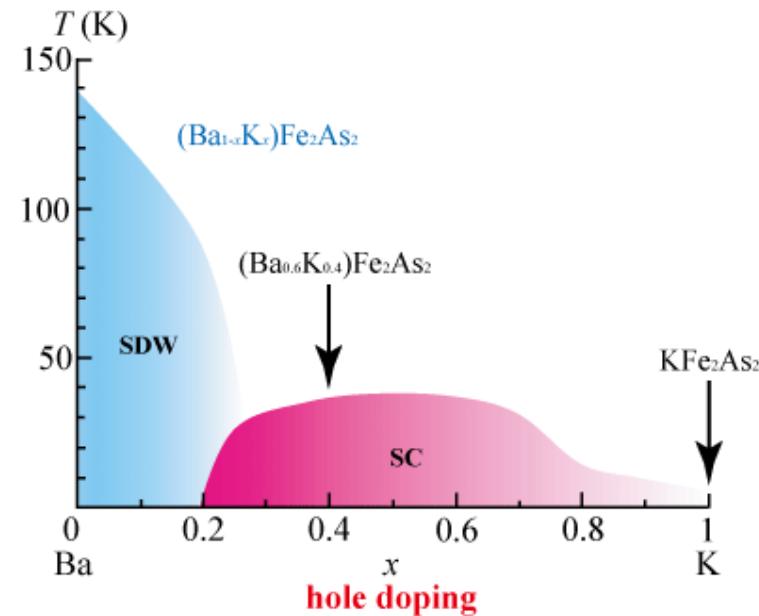
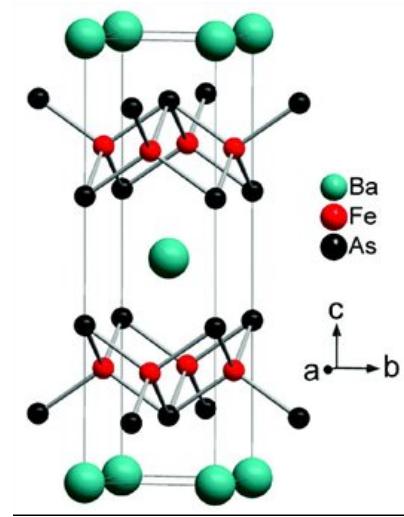


CNRS & Institut de Minéralogie et de Physique des Milieux Condensés,  
Université Pierre et Marie Curie, Paris



# Iron-pnictides: A new family of high- $T_c$ superconductors

- Discovery of high- $T_c$  in pnictides in 2008: new excitement
- Electron-phonon coupling cannot account for  $T_c$
- New family of unconventional high- $T_c$  superconductors: cuprates are not the only ones!
- Proximity of spin ordered phases
- Multi-band physics

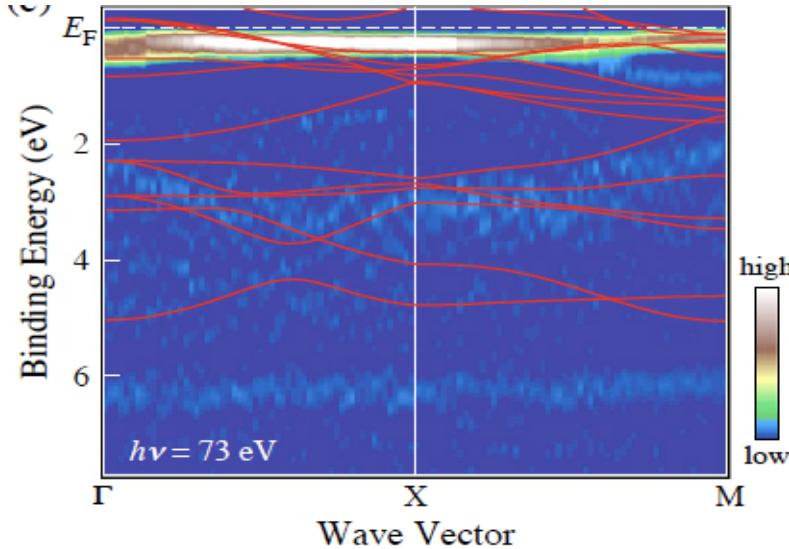


Correlation level?

Pairing symmetry and superconducting mechanism?

Can first principle calculations account for the spectral properties and the phase diagram?

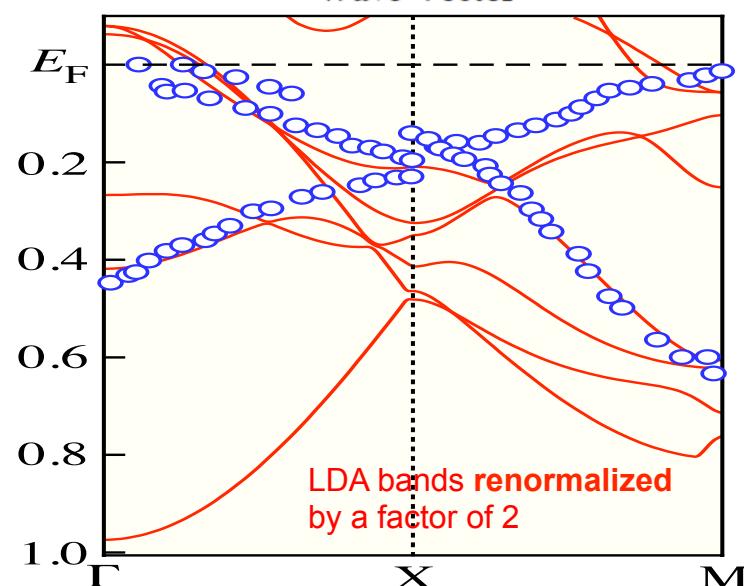
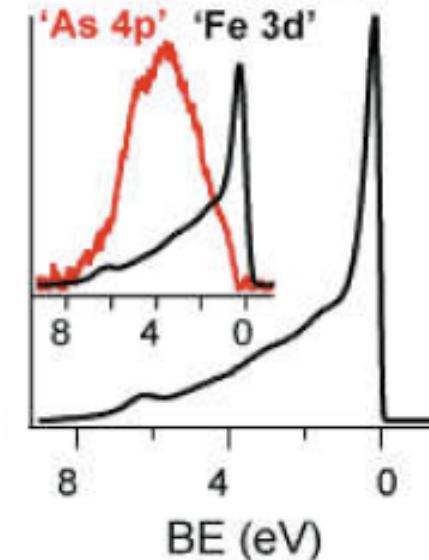
# Angle resolved photoemission spectroscopy (ARPES) in $\text{BaFe}_2\text{As}_2$



Ding et al., J. Phys.: Cond. Matter **23**, 135701 (2011)

de Jong et al. Phys. Rev. B **79**, 115125 (2009)

and many others....



DFT-LDA band structure resembles ARPES only after **band renormalization** (usually a factor in the range of 2-5)

ARPES satellites are missing

Need of improved theories for an *ab-initio* description of correlated materials

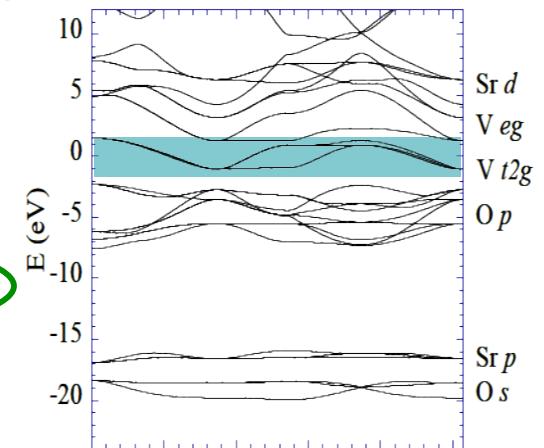
# Outline

- Constructing effective models for correlated materials: dynamical (frequency dependent) onsite interaction  $U$  from reducing the Hamiltonian to correlated bands
- Satellites and low-energy properties of  $\text{BaFe}_2\text{As}_2$  from dynamical mean field theory (DMFT) with frequency dependent  $U$
- Low-energy Hamiltonian for frequency dependent  $U$

# Low-energy modeling of real materials

## Tight-binding model from localized (Wannier) orbitals

$$H = \sum_{\{im\sigma\}} (H_{im,i'm'}^{\text{LDA}} - H_{im,i'm'}^{\text{double counting}}) a_{im\sigma}^\dagger a_{i'm'\sigma} + \frac{1}{2} \sum_{\substack{imm'\sigma \\ (\text{correl. orb.})}} V_{mm'} n_{im\sigma} n_{im'-\sigma} + \frac{1}{2} \sum_{\substack{im \neq m'\sigma \\ (\text{correl. orb.})}} (V_{mm'} - J_{mm'}) n_{im\sigma} n_{im'\sigma}$$

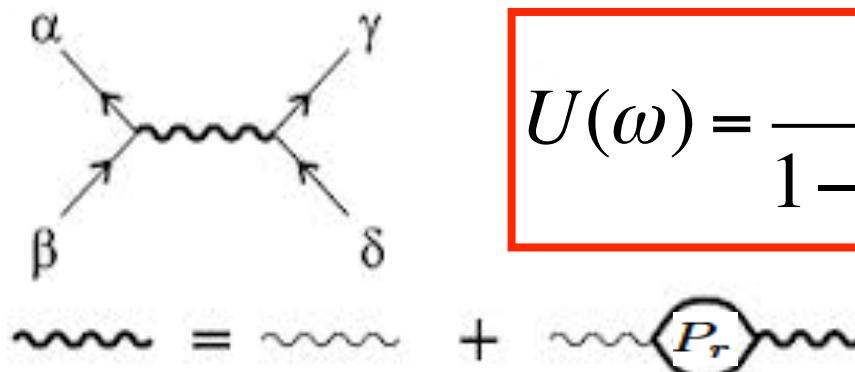


Density-density interactions  
(matrix elements of the Coulomb potential in the Wannier basis)

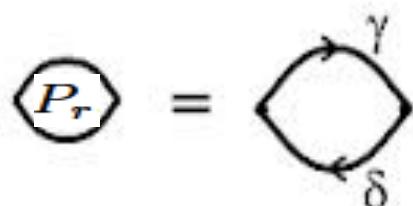
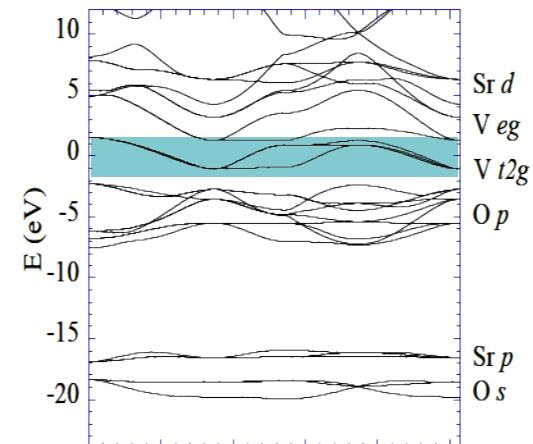
**Those interactions are screened**  
**by the excluded (high-energy) degrees of freedom**

# cRPA screened interaction

- Screening coming from the electrons excluded from the model
- RPA screening:



$$U(\omega) = \frac{v}{1 - P_r(\omega)v}$$



RPA polarization: e-h pair excitations

$$P(\mathbf{r}, \mathbf{r}'; \omega) = \sum_i^{\text{occ}} \sum_j^{\text{unocc}} \psi_i(\mathbf{r}) \psi_i^*(\mathbf{r}') \psi_j^*(\mathbf{r}) \psi_j(\mathbf{r}') \\ \times \left\{ \frac{1}{\omega - \epsilon_j + \epsilon_i + i0^+} - \frac{1}{\omega + \epsilon_j - \epsilon_i - i0^+} \right\}$$

Total polarizability  $P = P_d + P_r$

$P_d$  d-d transitions

$P_r$  other transitions

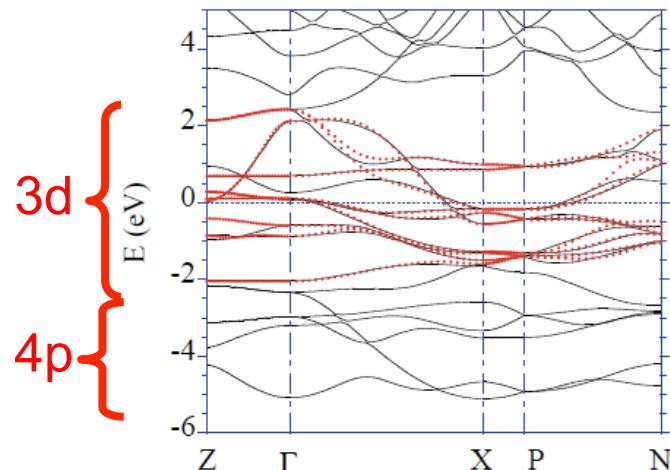
Aryasetiawan et al. PRB **70**, 195104 (2004)

Miyake et al. **80**, 155134 (2009)

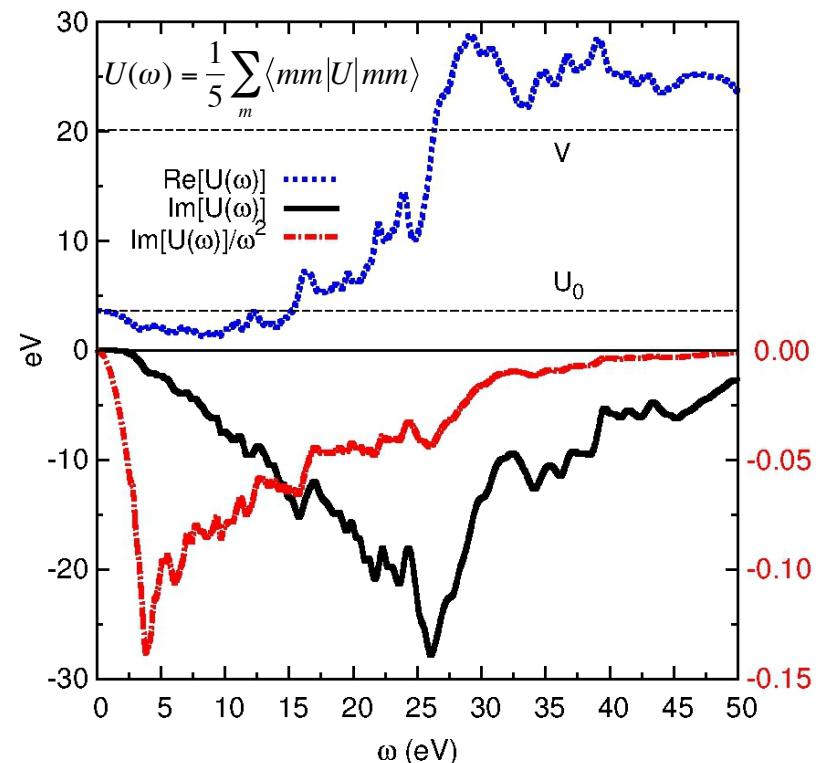
Miyake et al. **61**, 7172 (2000)

# Dynamic U in $\text{BaFe}_2\text{As}_2$

- Downfolding the full LDA Hamiltonian by Fe-3d and As-4p max localized Wannier functions

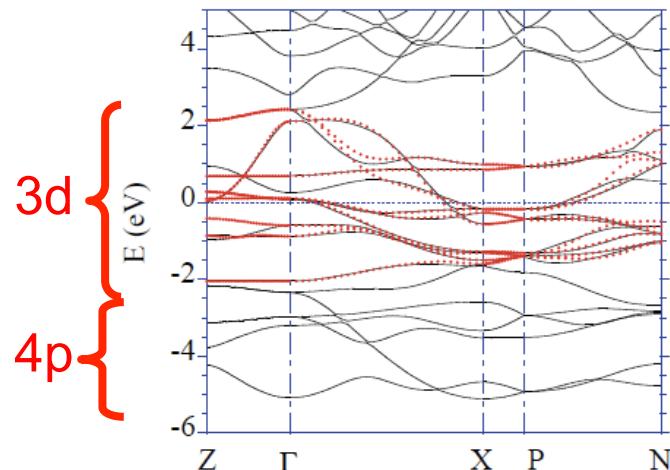


- Coulomb interaction:  
**frequency dependence** from cRPA,  
Slater parameterization for static part of U
- Important observations:
  - Static value very different from the unscreened** (infinite frequency) limit
  - Strong frequency dependence** even at low energy scale

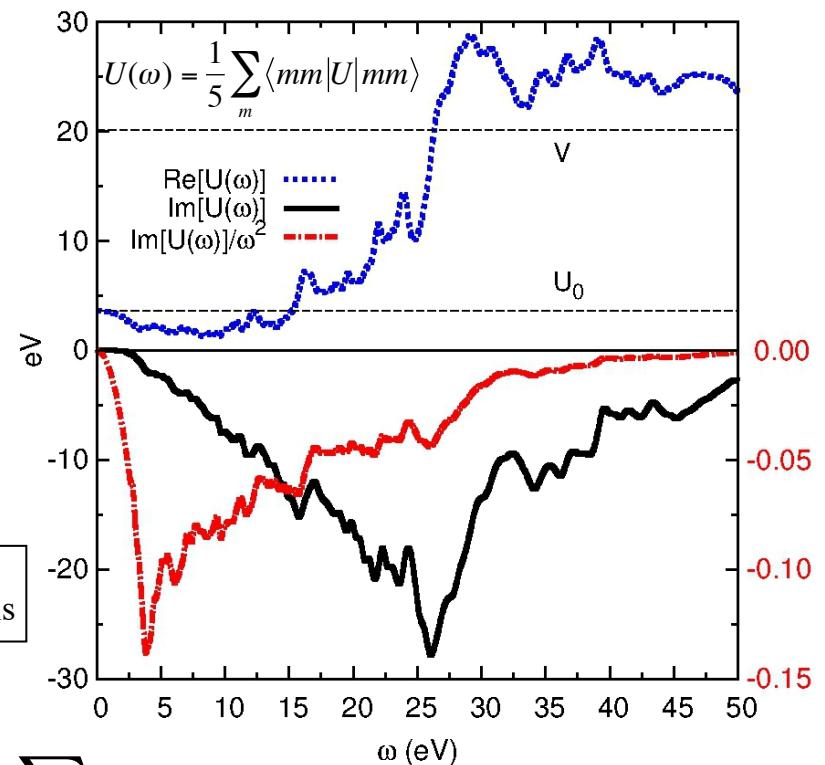


# Dynamic U and mapping to bosons

- Downfolding the full LDA Hamiltonian by Fe-3d and As-4p max localized Wannier functions



$$H = H_{\text{multi-band Hubbard with STATIC } U} + H_{\text{plasmons}}$$

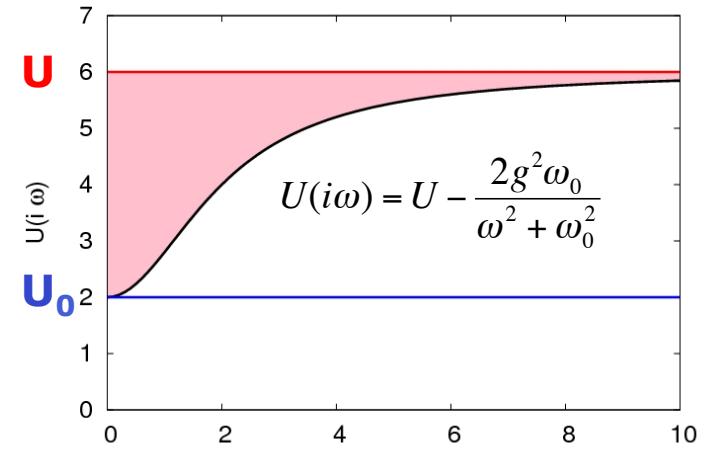


$$H_{\text{plasmons}} = \sum_{i,\nu} \omega_\nu b_{i\nu}^\dagger b_{i\nu} + \sum_{i,\nu} \lambda_\nu (b_{i\nu}^\dagger + b_{i\nu}) \sum_{m,\sigma} n_{im\sigma}$$

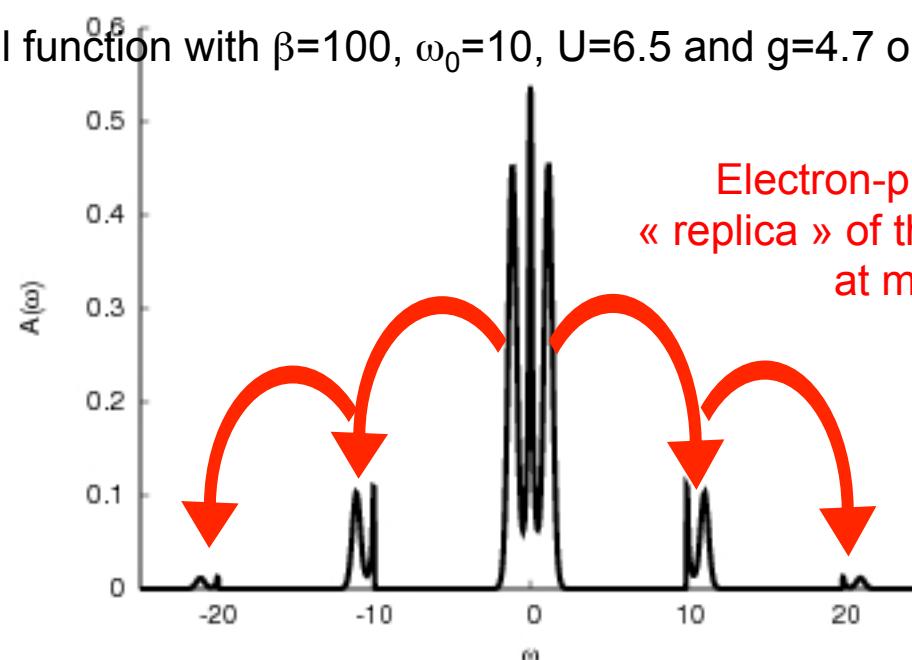
$$\lambda_\nu^2 = -\frac{1}{\pi} \text{Im}[U(\omega_\nu)] \quad \text{Electron-plasmon coupling given by ImU}$$

# Hubbard-Holstein model: plasmons interacting with correlated electrons

$$\begin{aligned}
 H = & \sum_{\vec{k}\sigma} \epsilon(\vec{k}) c_{\vec{k}\sigma}^\dagger c_{\vec{k}\sigma} + U \sum_i n_{i\uparrow} n_{i\downarrow} + \omega_0 \sum_i b_i^\dagger b_i \\
 & + g \sum_i (b_i^\dagger + b_i)(n_{i\uparrow} + n_{i\downarrow} - 1).
 \end{aligned}$$



Spectral function with  $\beta=100$ ,  $\omega_0=10$ ,  $U=6.5$  and  $g=4.7$  on the Bethe lattice



Electron-plasmon excitations:  
 « replica » of the low-energy structure  
 at multiples of  $\omega_0$

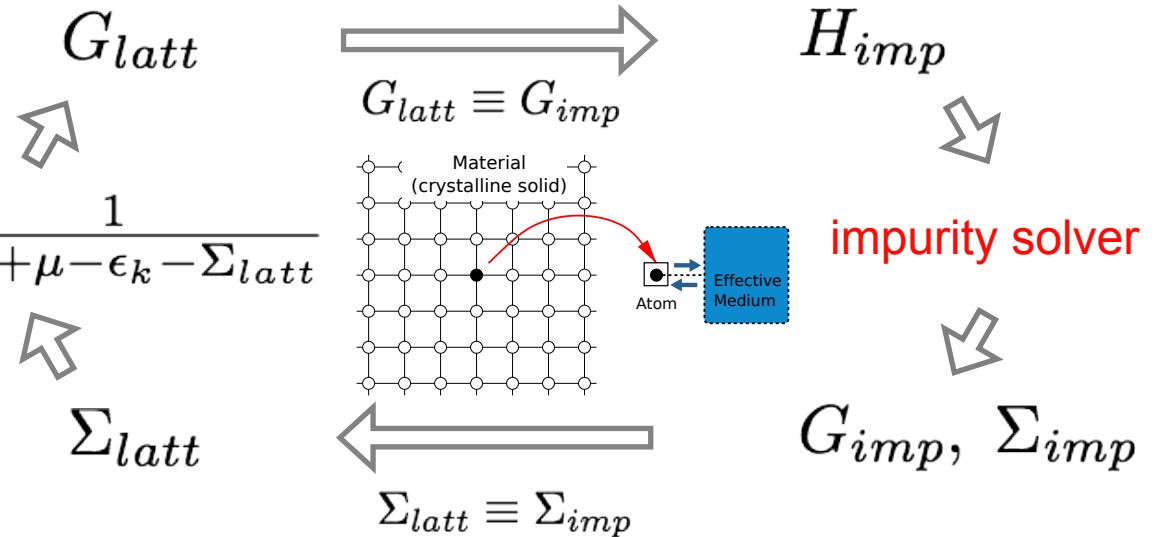
# Dynamical mean-field approach

Accurate description of **local atomic physics** is essential in case of strong correlation

$$\int dk \frac{1}{i\omega_n + \mu - \epsilon_k - \Sigma_{latt}}$$

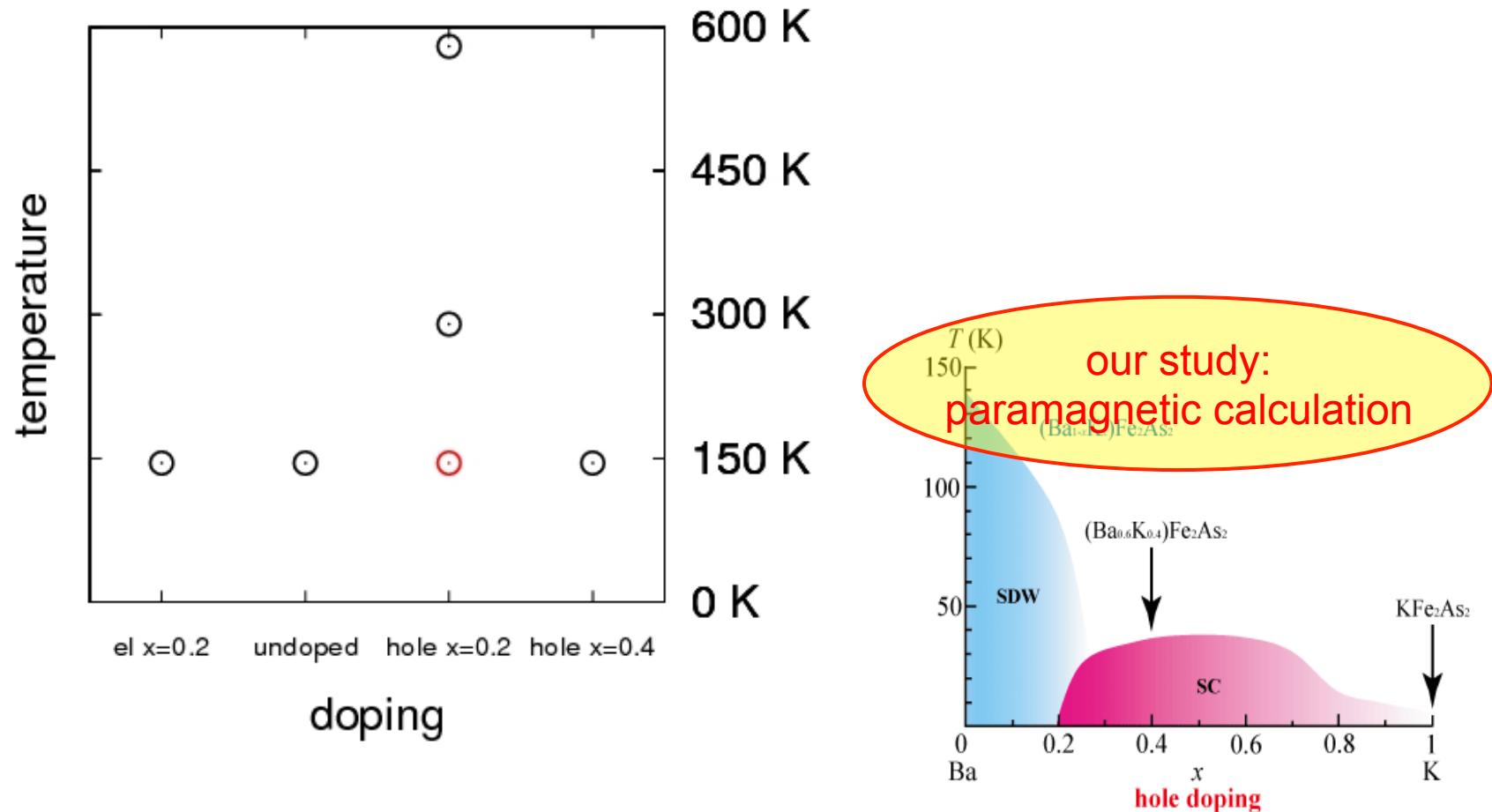
DMFT maps the full lattice problem into a single site Hamiltonian hybridized with a self-consistent bath

A. Georges, G. Kotliar, W. Krauth, M. Rozenberg, RMP **68**, 13 (1996)

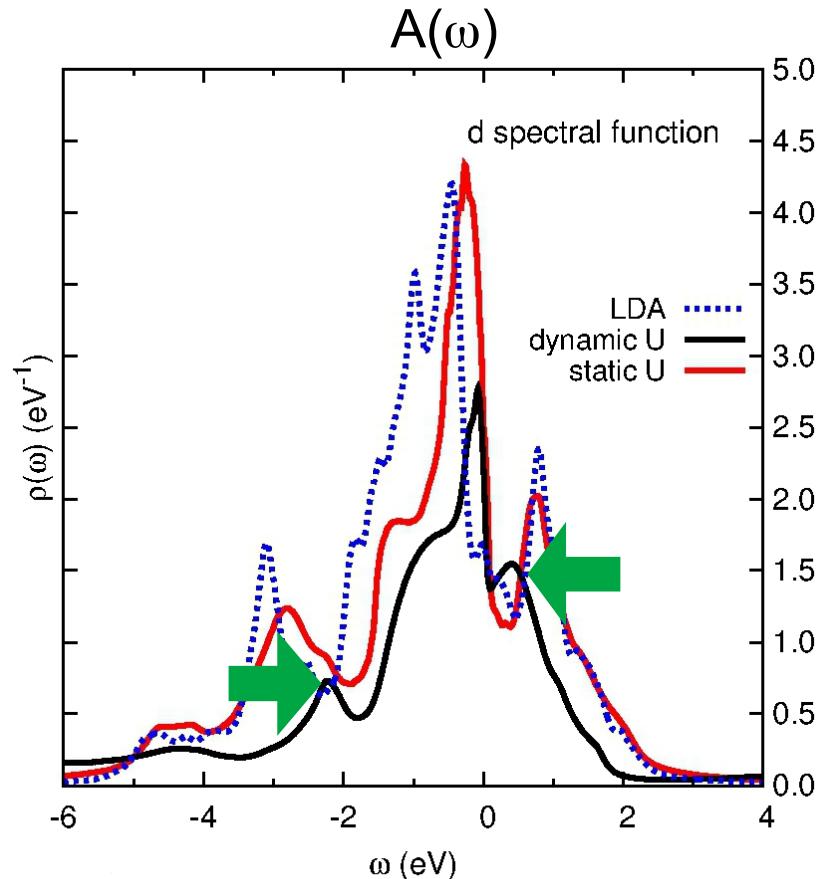


**Resolution of the **impurity problem** by the hybridization expansion CTQMC algorithm which supports retarded interactions (Werner and Millis, PRL 2006)**

# $\text{Ba}_{(1-2x)}\text{K}_{2x}\text{Fe}_2\text{As}_2$ : $x$ - $T$ phase space

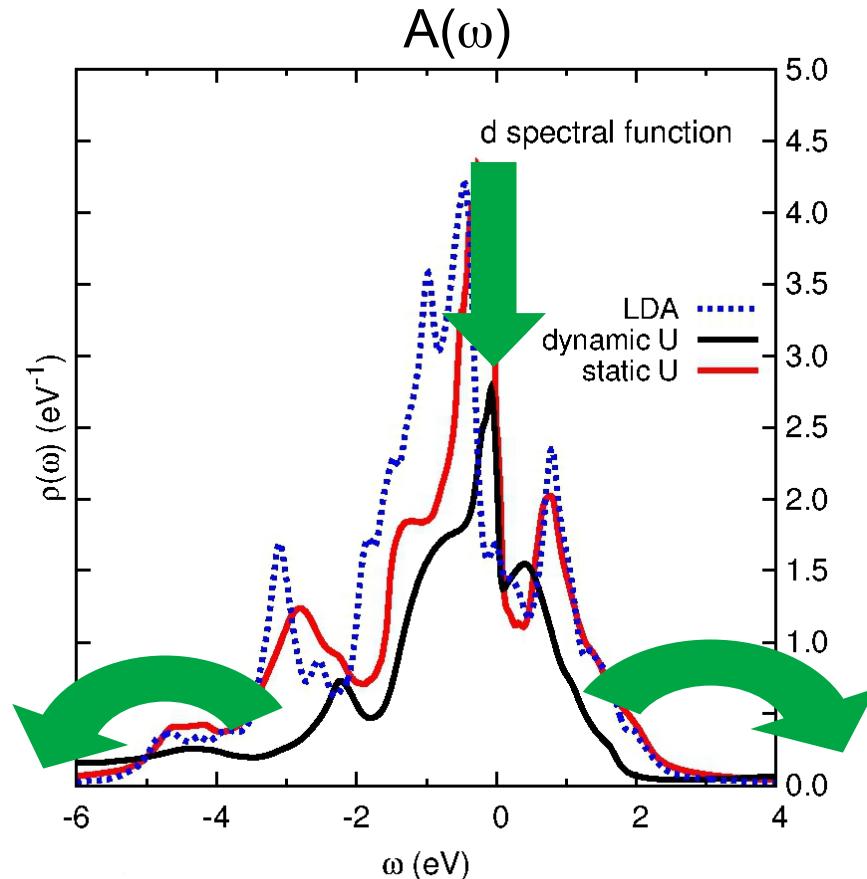


# Dynamic U effects in $\text{Ba}_{0.6}\text{K}_{0.4}\text{Fe}_2\text{As}_2$



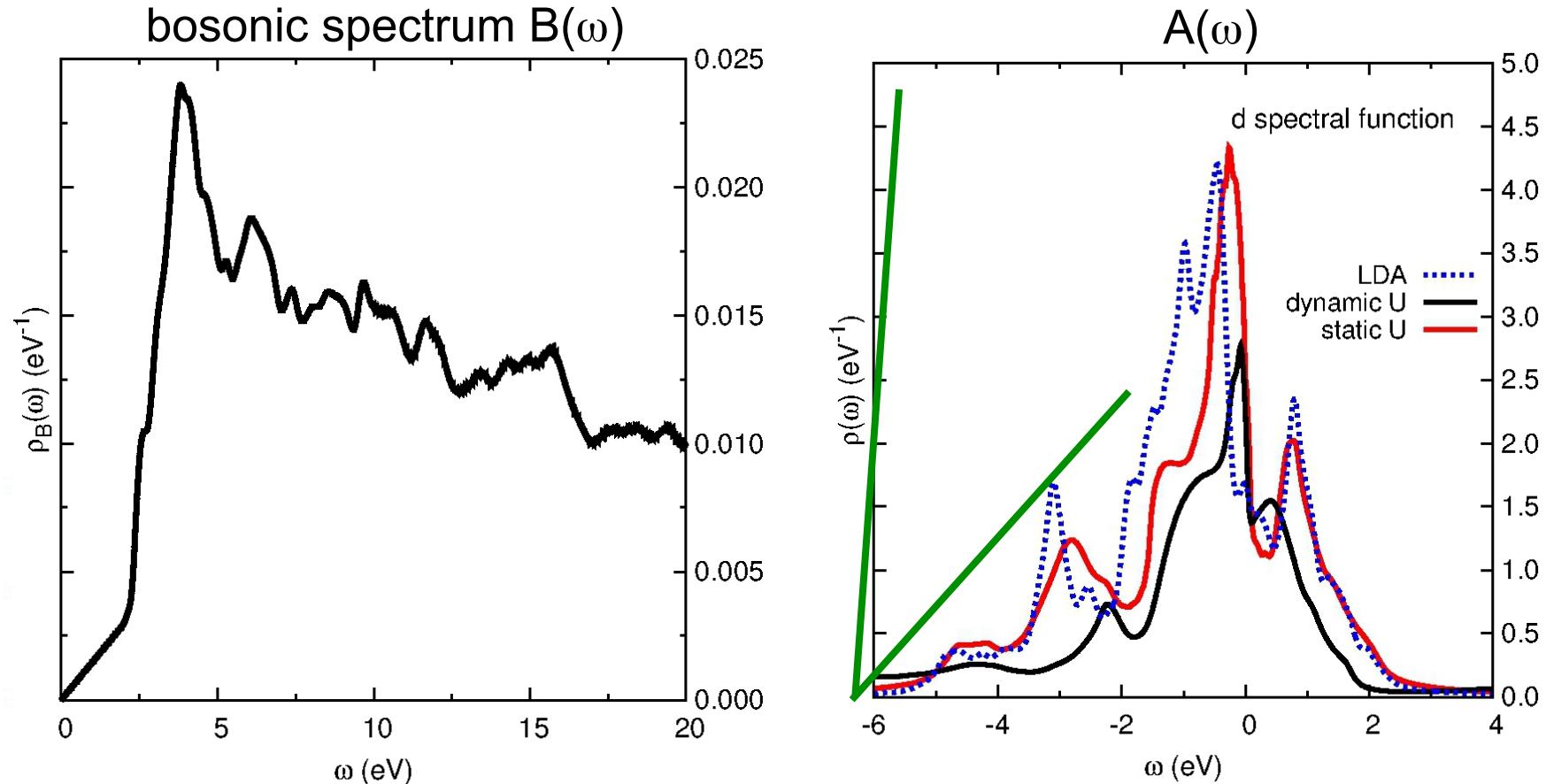
Structures from p-d hybridization squeezed toward the Fermi energy

# Dynamic U effects in $\text{Ba}_{0.6}\text{K}_{0.4}\text{Fe}_2\text{As}_2$



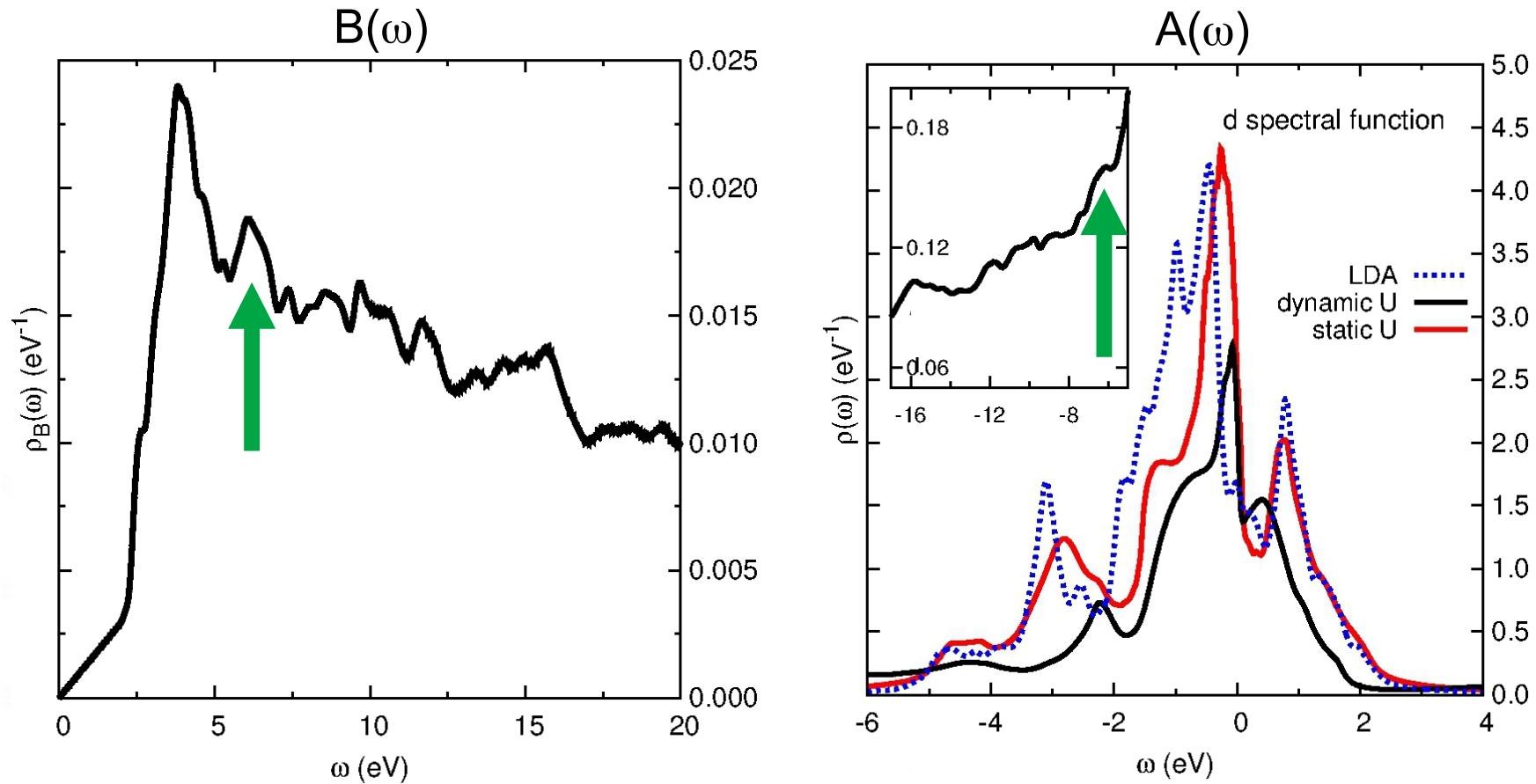
Weight reduction at low-energy and spectral weight transfer to high-energy

# Dynamic U effects in $\text{Ba}_{0.6}\text{K}_{0.4}\text{Fe}_2\text{As}_2$



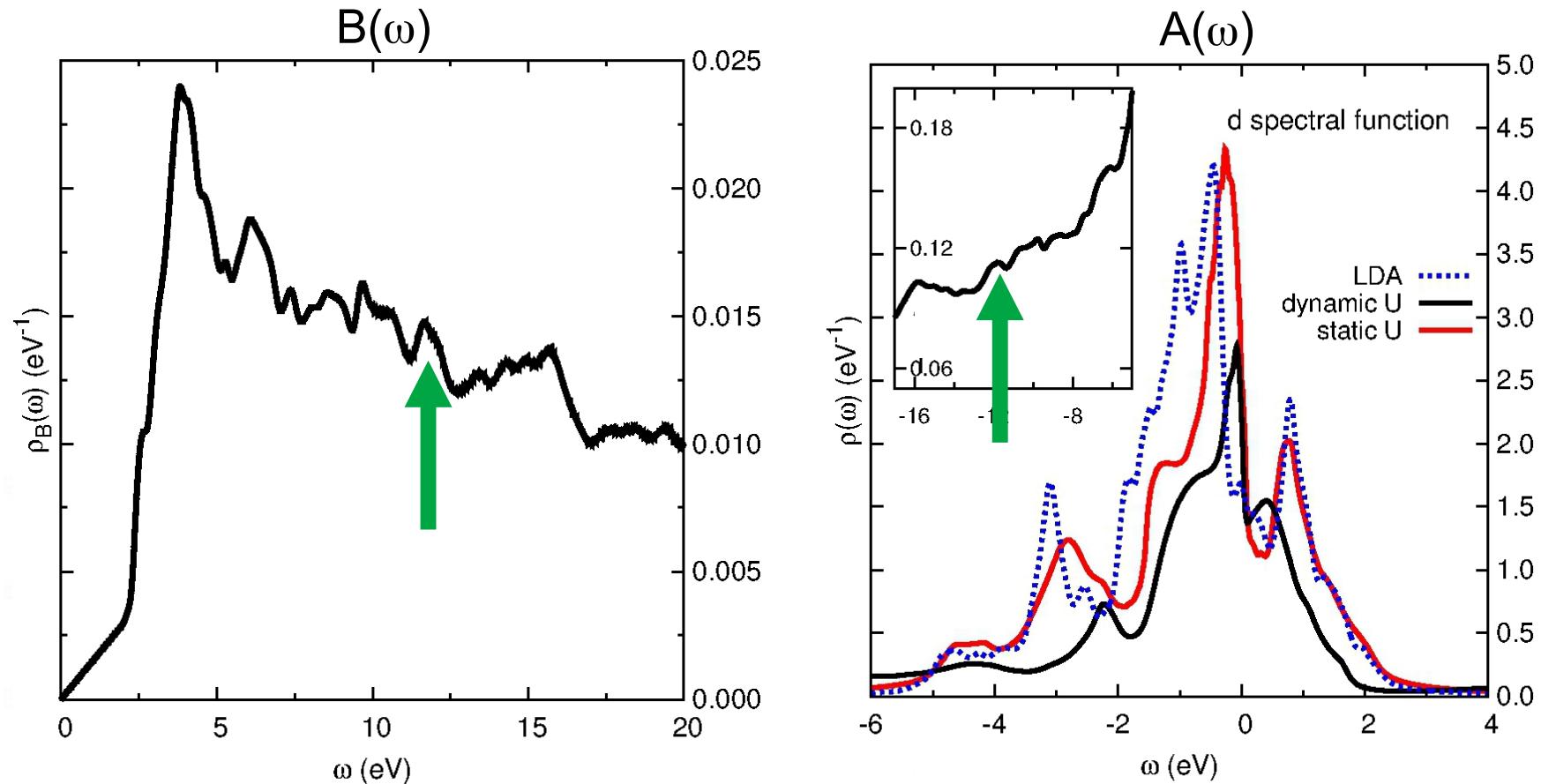
$$A(\omega) = \int d\varepsilon B(\varepsilon) \frac{1 + e^{-\beta\omega}}{(1 + e^{-\beta(\varepsilon-\omega)})(1 - e^{-\beta\varepsilon})} A_0(\omega - \varepsilon)$$

# Dynamic U effects in $\text{Ba}_{0.6}\text{K}_{0.4}\text{Fe}_2\text{As}_2$



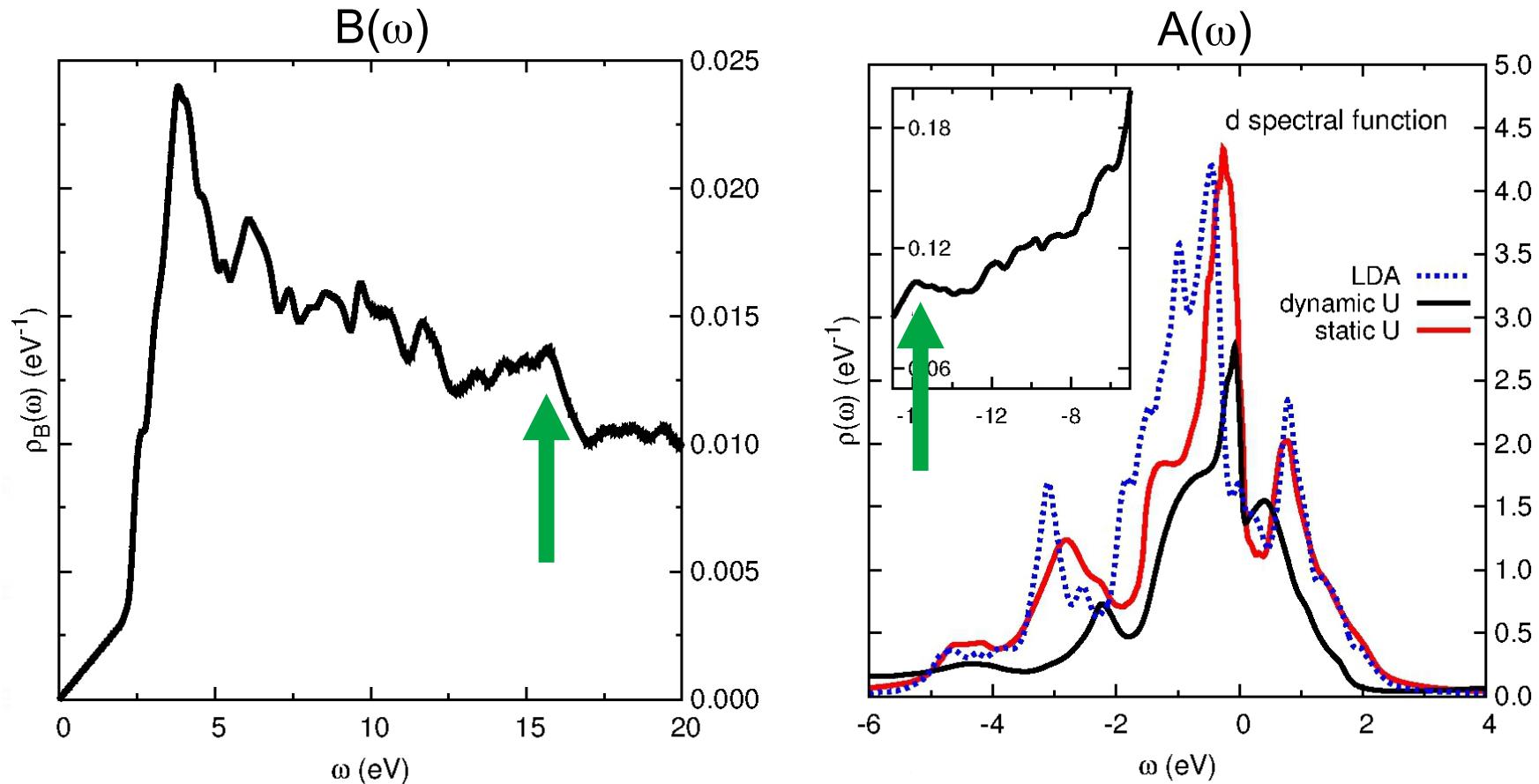
Satellites: at 6.1 eV

# Dynamic U effects in $\text{Ba}_{0.6}\text{K}_{0.4}\text{Fe}_2\text{As}_2$

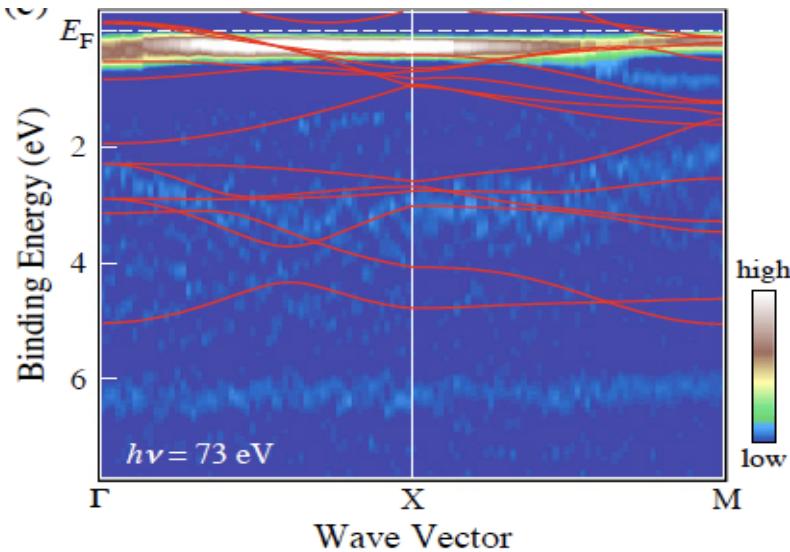


Satellites: at 12 eV

# Dynamic U effects in $\text{Ba}_{0.6}\text{K}_{0.4}\text{Fe}_2\text{As}_2$



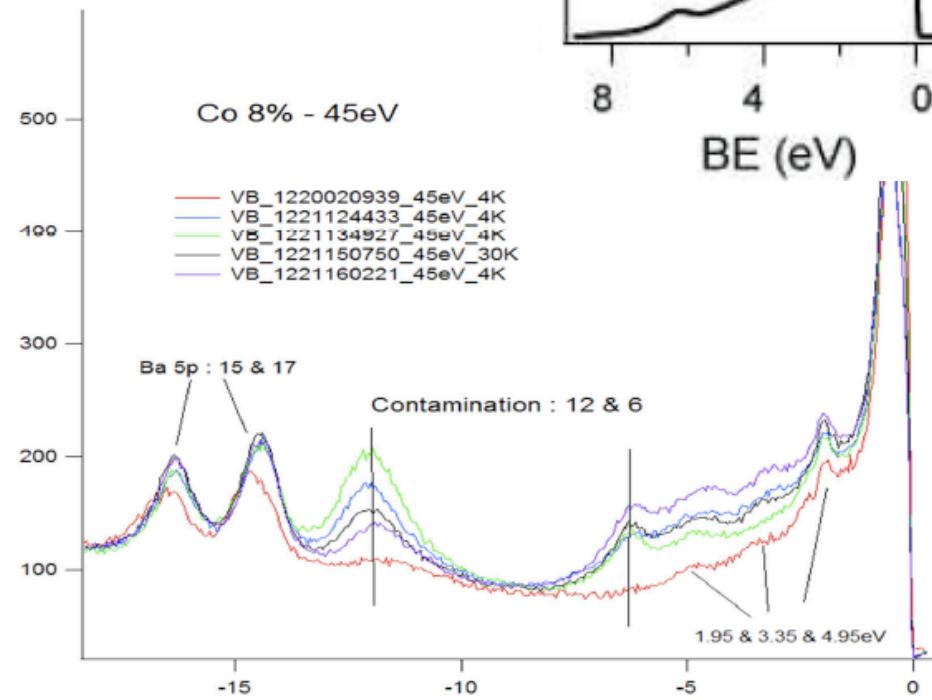
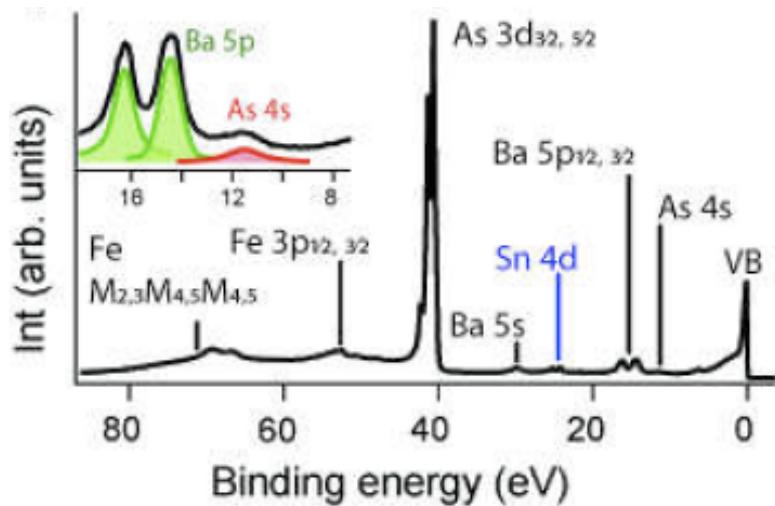
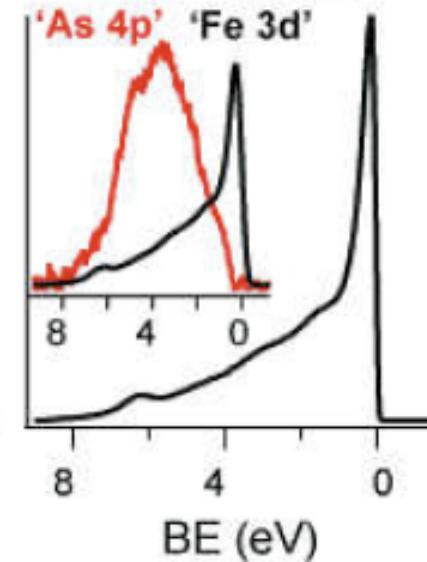
Satellites: at 16 eV



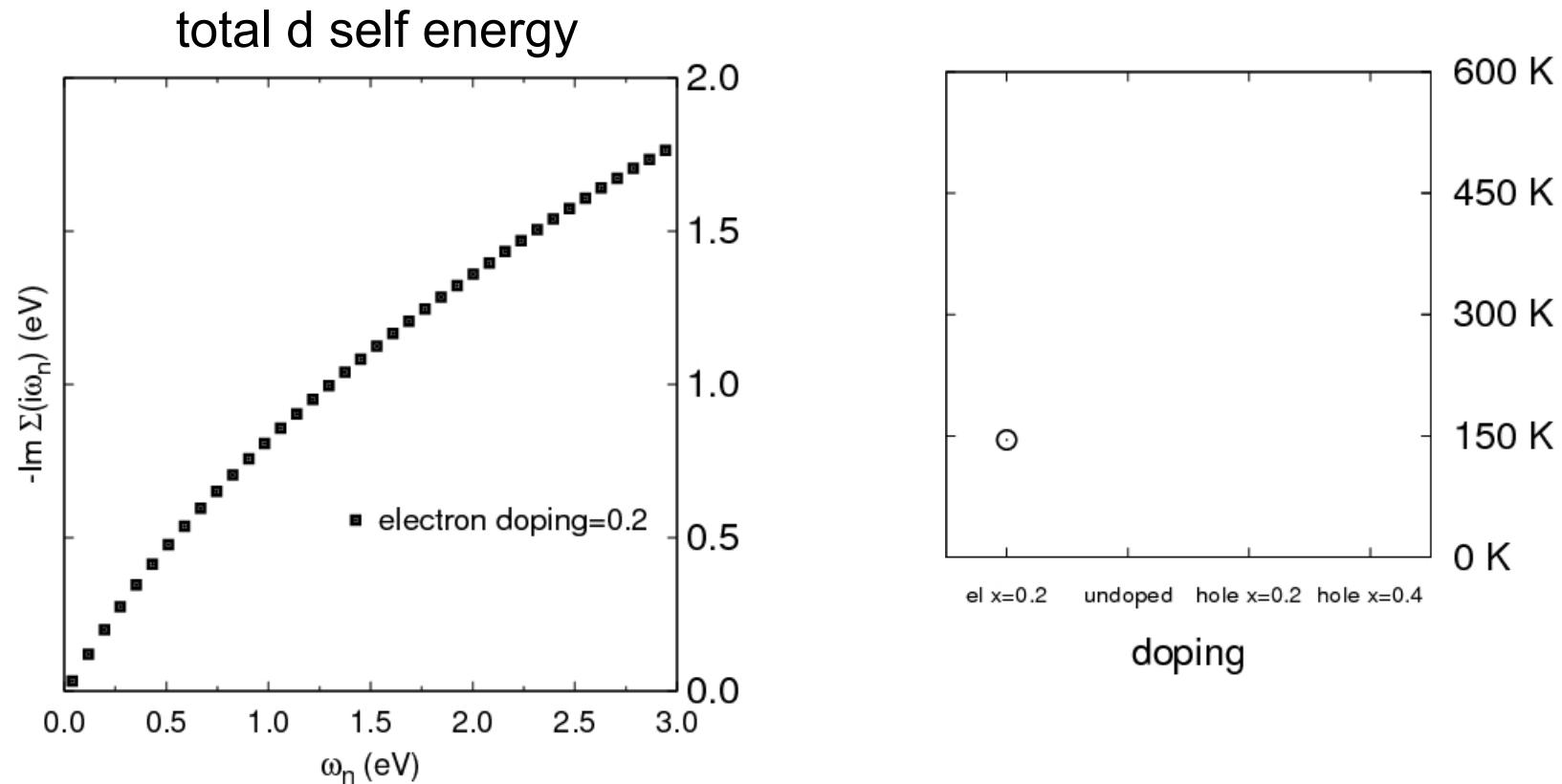
Ding et al., J. Phys.: Cond. Matter  
**23**, 135701 (2011)

de Jong et al. Phys. Rev. B **79**,  
115125 (2009)

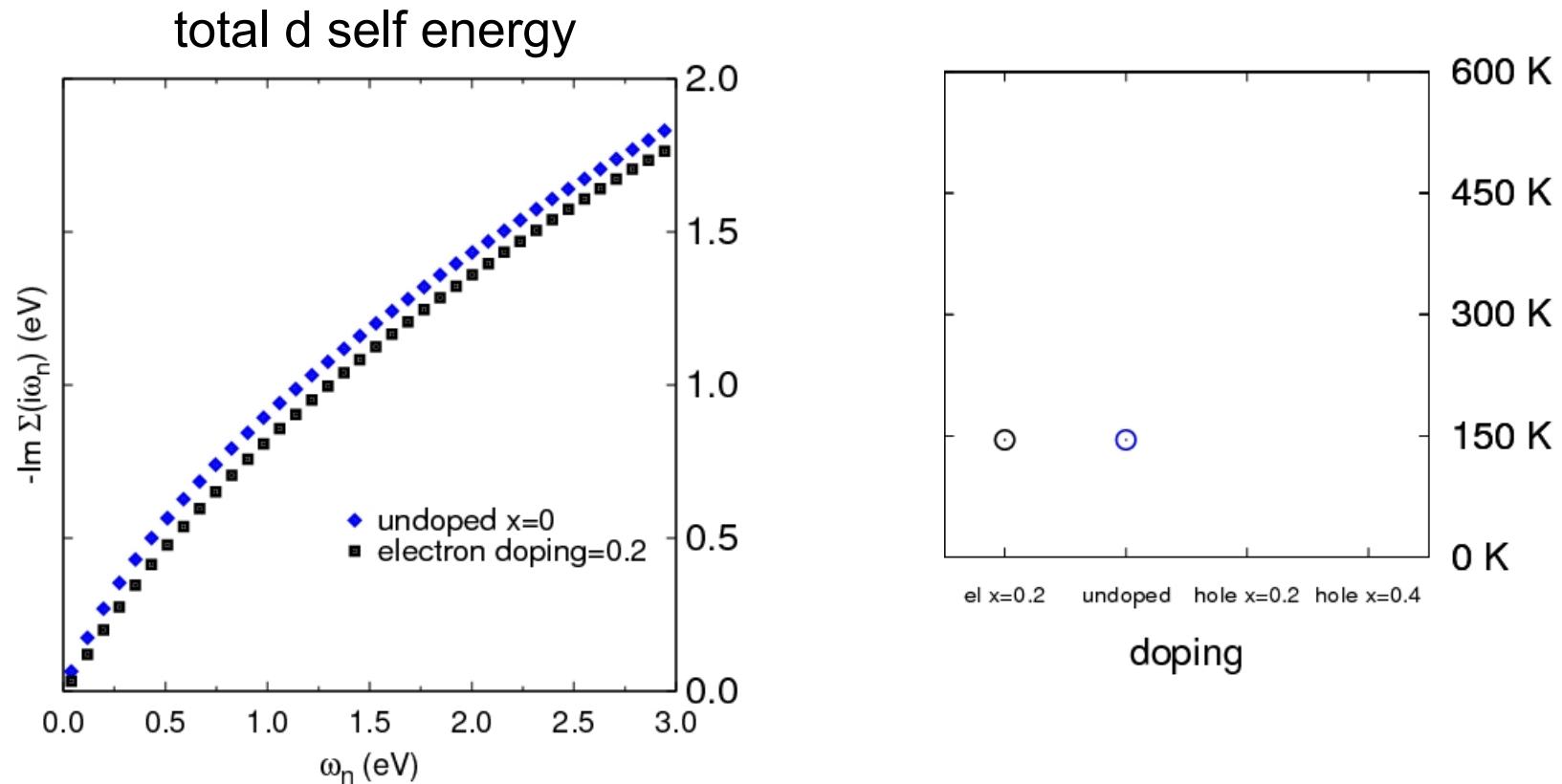
Veronique Brouet, private  
communication



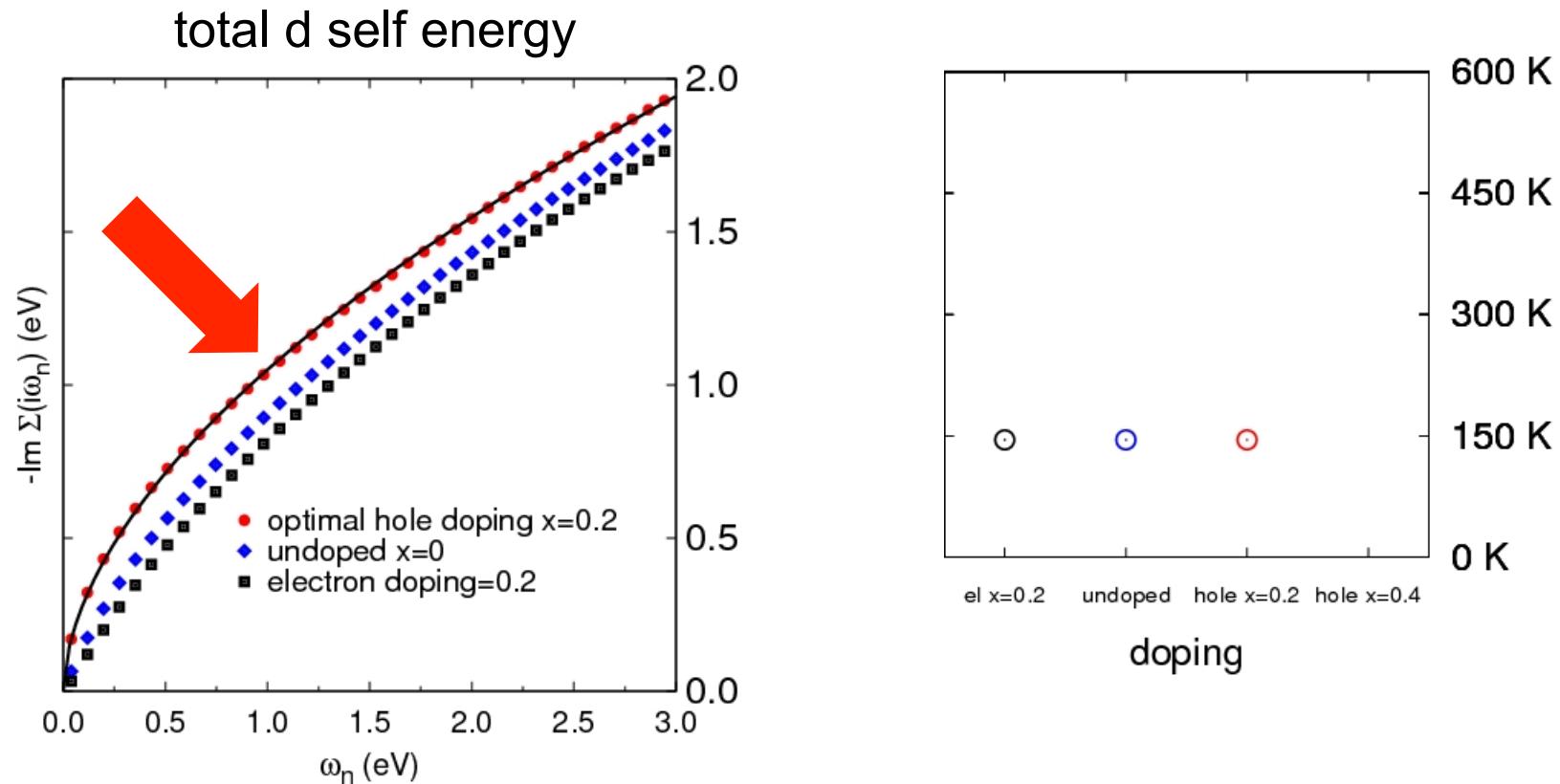
# Doping dependence in the self energy



# Doping dependence in the self energy

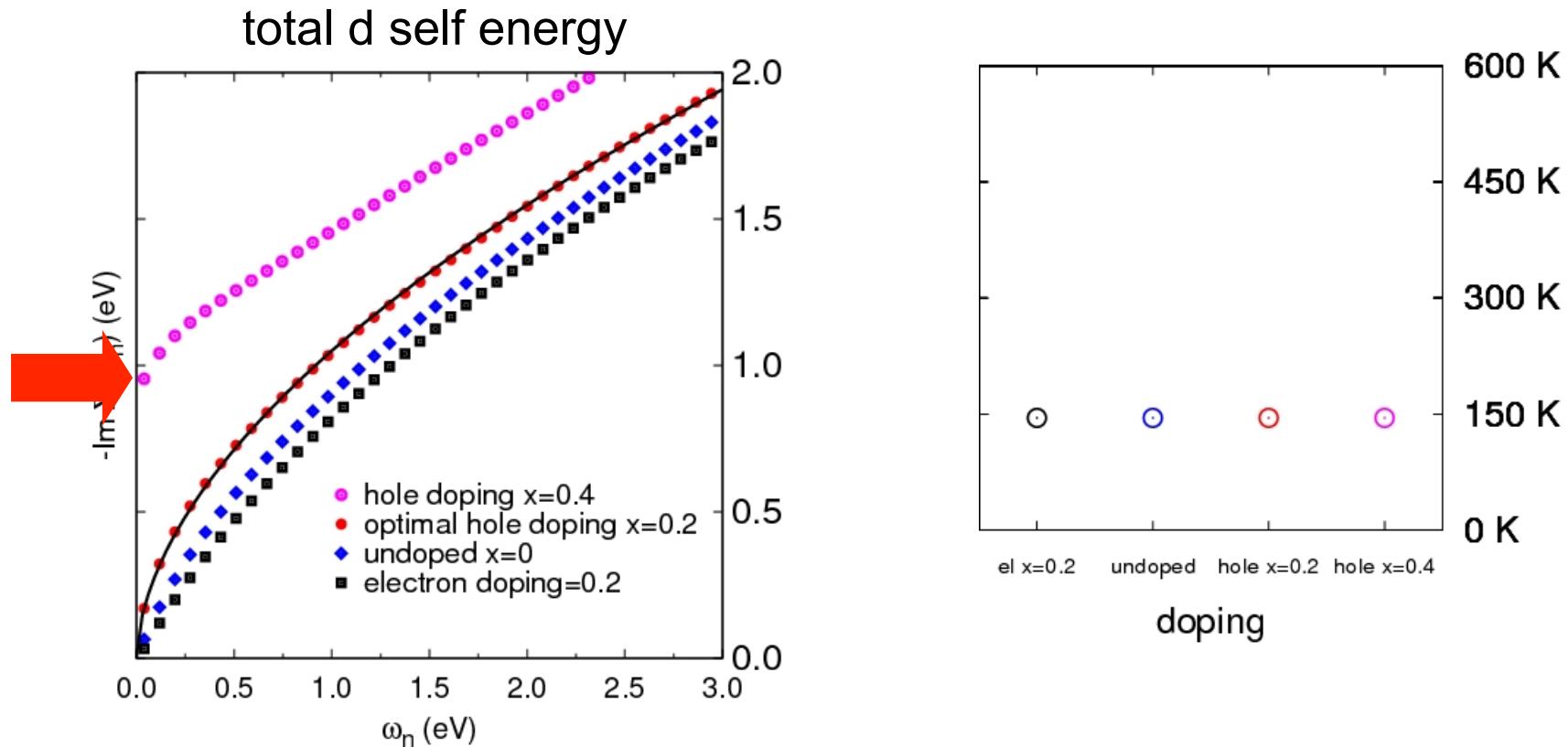


# Doping dependence in the self energy



Onset of the square root behavior at optimal hole doping  
**NON FERMI LIQUID BEHAVIOR**

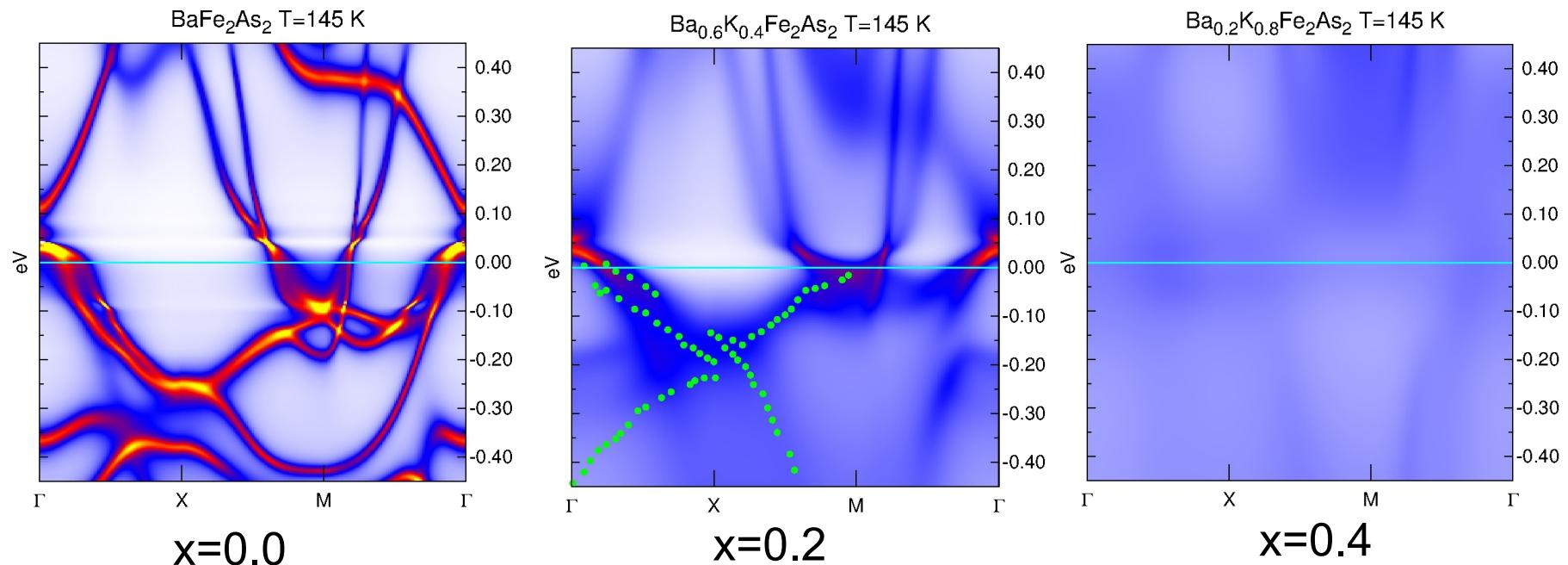
# Doping dependence in the self energy



Close to the full substitution ( $\text{KFe}_2\text{As}_2$ )  
strong non-fermi liquid behavior (finite intercept)

# K-resolved spectral function: hole doping

$\text{Ba}_{(1-2x)}\text{K}_{2x}\text{Fe}_2\text{As}_2$

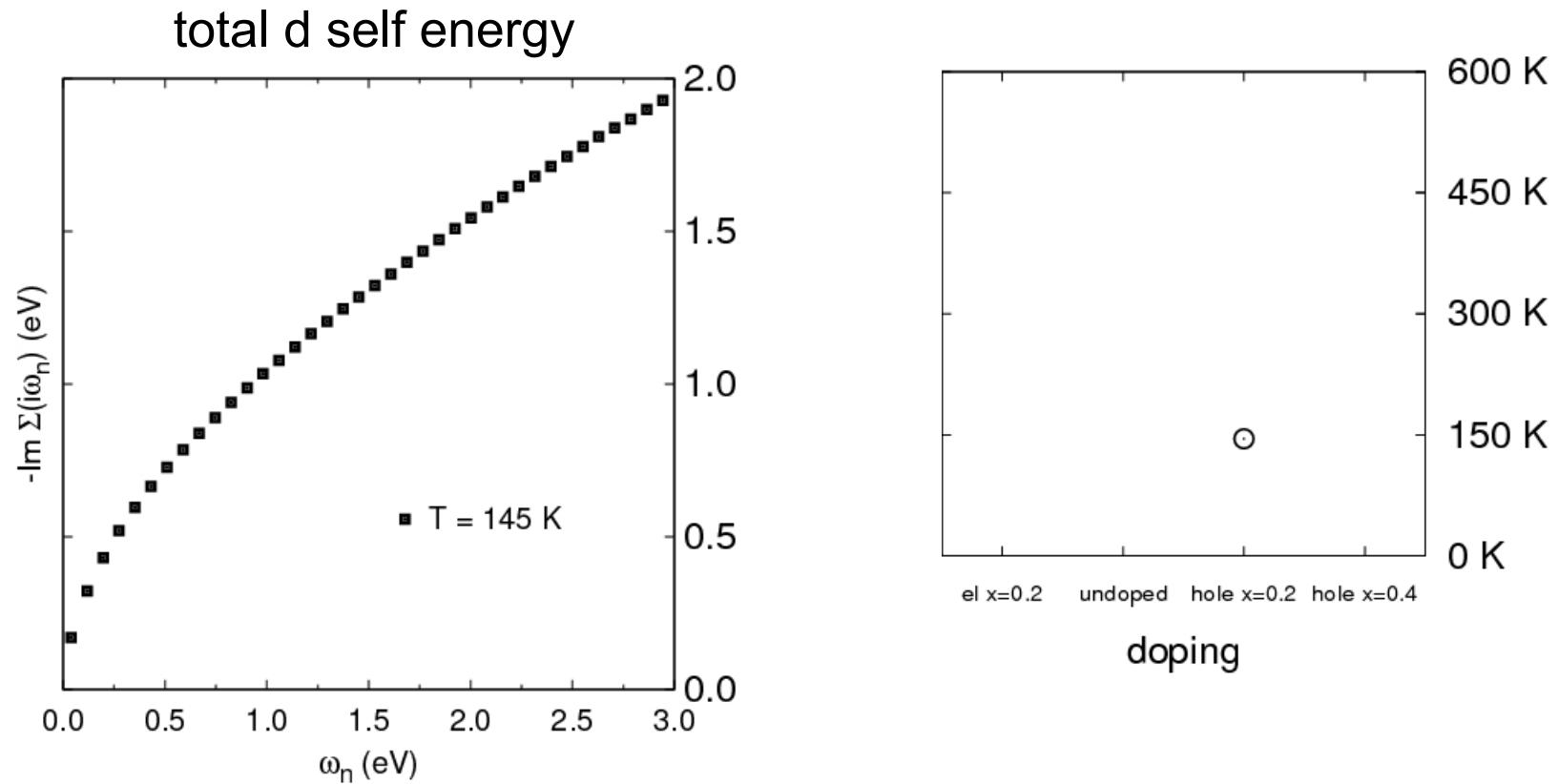


undoped

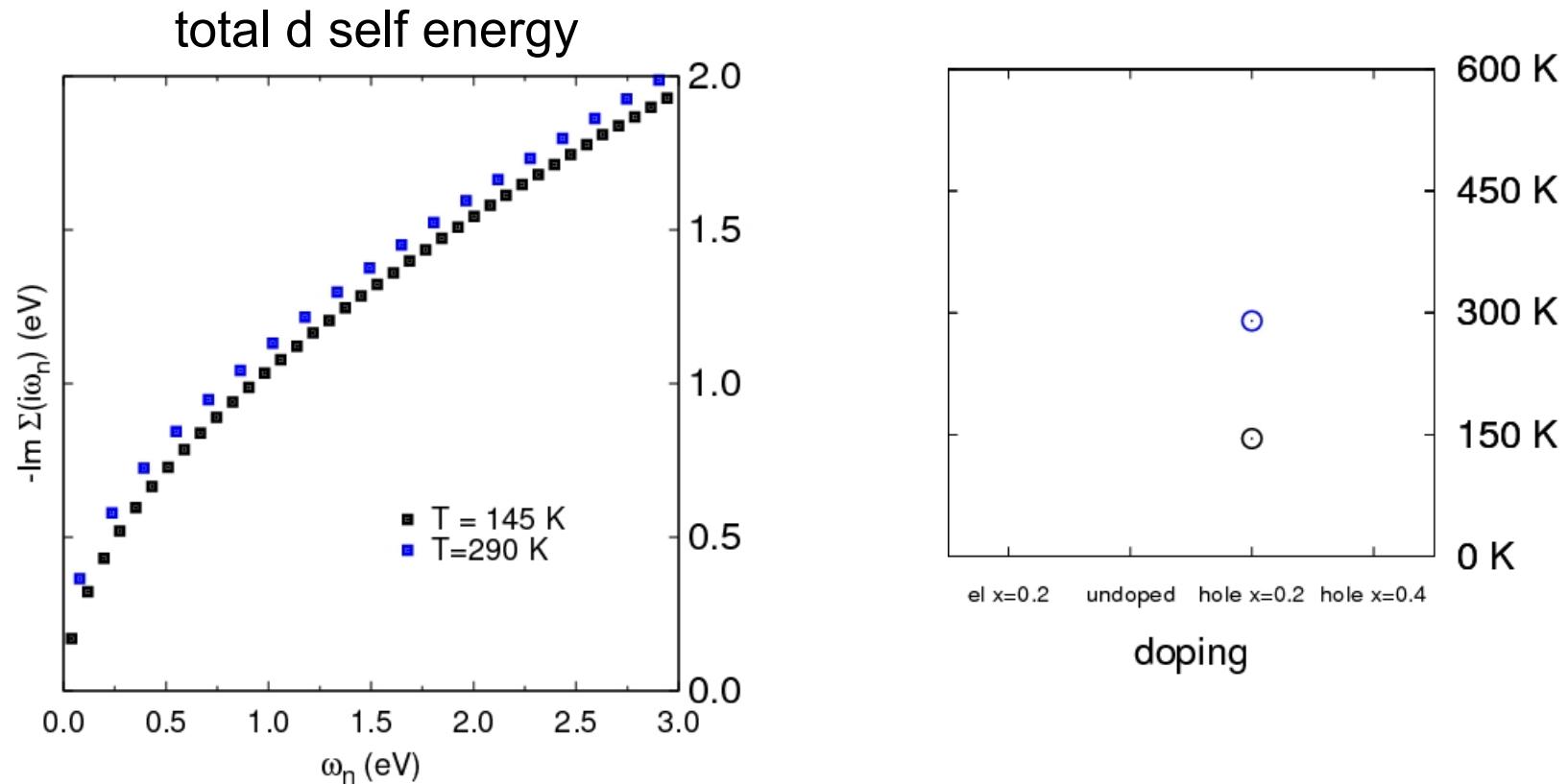
strongly doped

Fermi liquid to non-Fermi liquid crossover

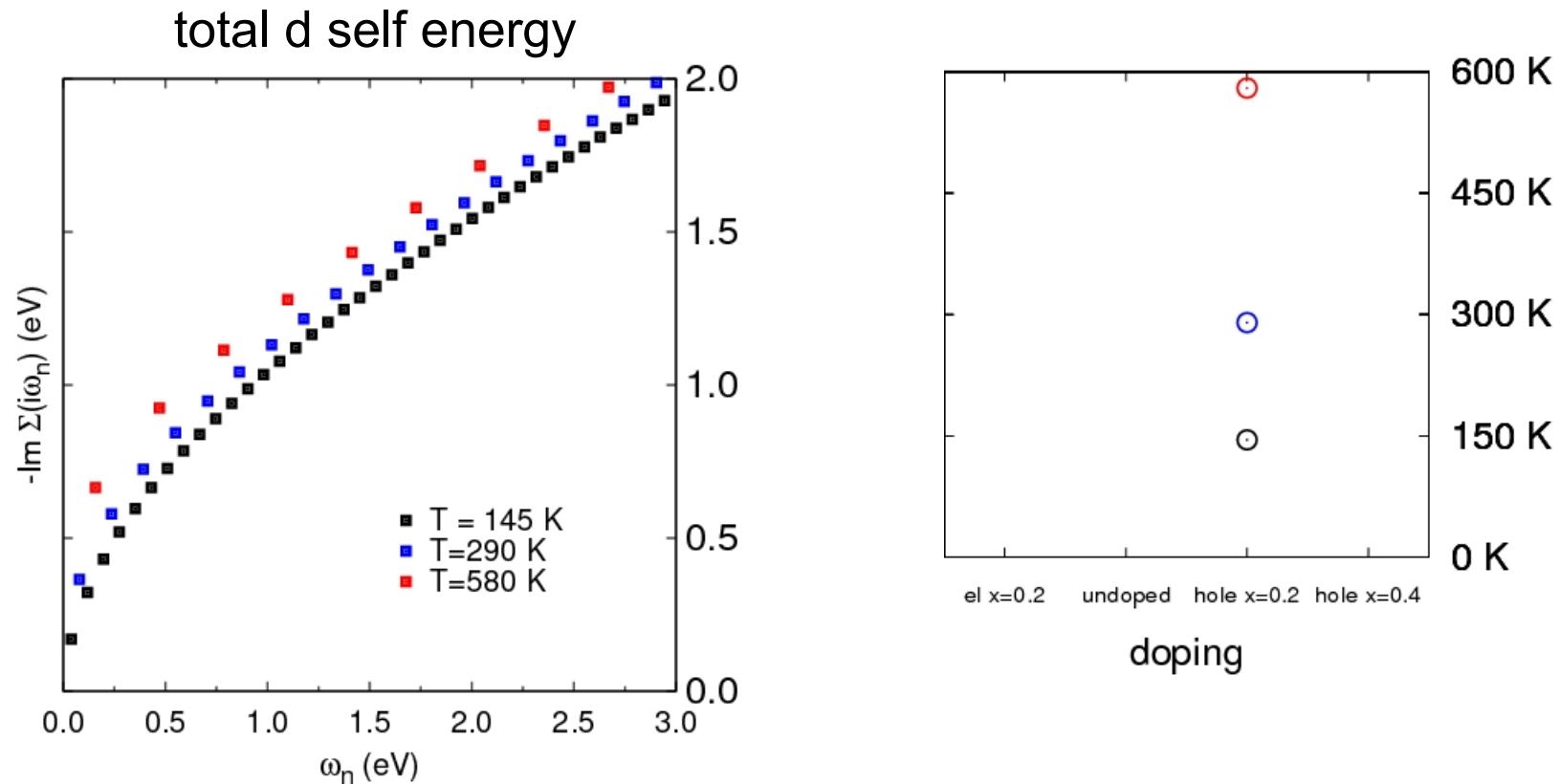
# T dependence in the self energy



# T dependence in the self energy

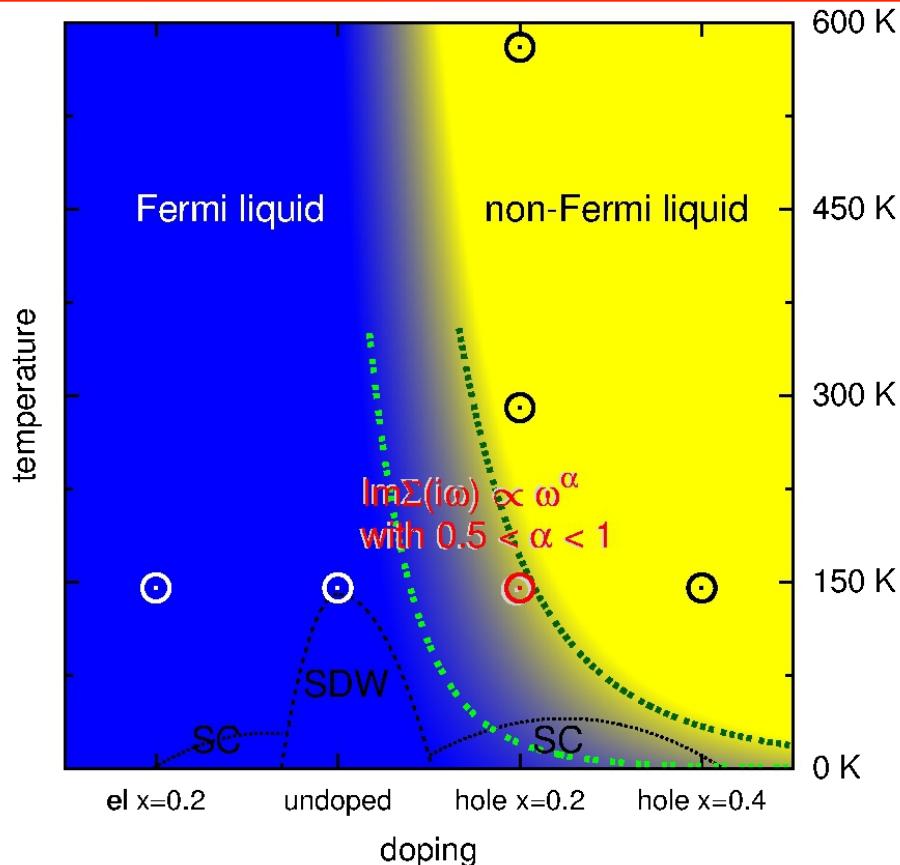


# T dependence in the self energy



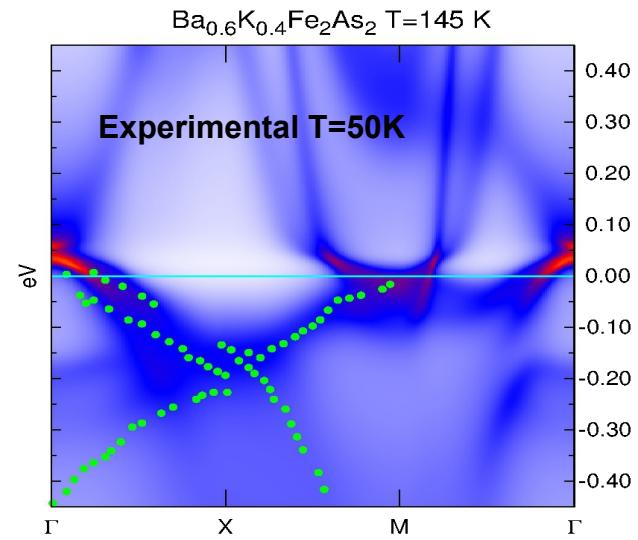
By rising the temperature  
the system goes quickly into a strongly incoherent regime

# Paramagnetic phase diagram

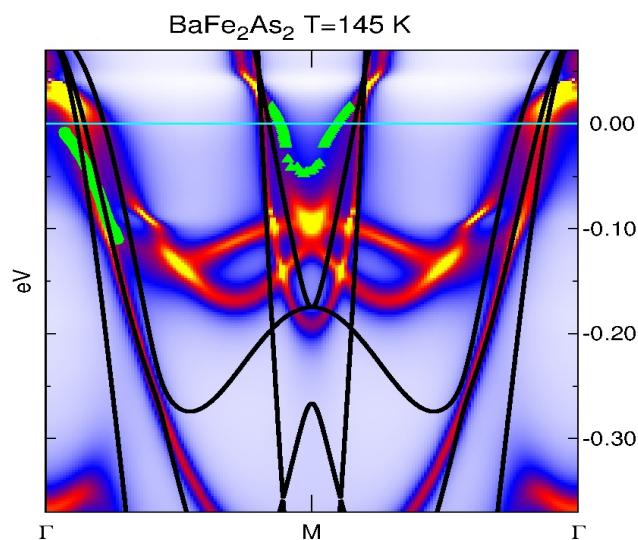
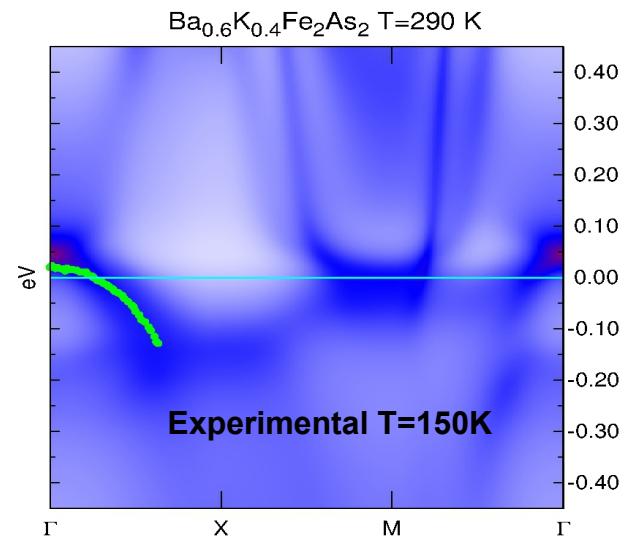


- Electron-hole **asymmetry**
  - Spin-freezing crossover in the hole doped region
- Werner et al. Phys. Rev. Lett. **101**, 166405 (2008), De' Medici et al. PRL **107**, 256401 (2011)
- Doping dependent renormalization of the LDA band structure

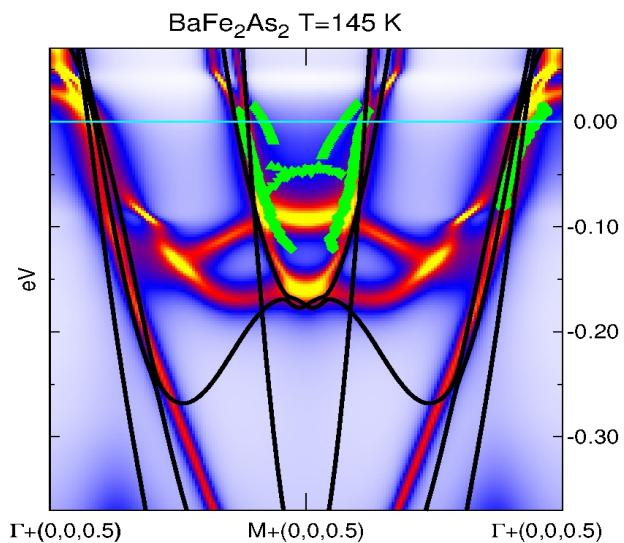
# Comparison to ARPES



Comparison with Ding et al.  
J. Phys.: Cond. Matter **23**,  
135701 (2011)



Comparison with V. Brouet et al.  
arXiv:1105.5604



# Comparison to ARPES

**Agreement within 0.05 eV  
in the low-energy part!**

# Low-energy effective static model

Start from Hubbard-Holstein

$$H = - \sum_{ij\sigma} t_{ij} d_{i\sigma}^\dagger d_{j\sigma} + V \sum_i d_{i\uparrow}^\dagger d_{i\uparrow} d_{i\downarrow}^\dagger d_{i\downarrow} + \mu \sum_{i\sigma} d_{i\sigma}^\dagger d_{i\sigma}$$

$$+ \omega_0 \sum_i b_i^\dagger b_i + \lambda \sum_{i\sigma} d_{i\sigma}^\dagger d_{i\sigma} (b_i + b_i^\dagger).$$

Lang-Firsov  
transformation

$$H \rightarrow H_{LF} = e^S H e^{-S} \quad S = \frac{\lambda}{\omega_0} \sum_{i\sigma} n_{i\sigma} (b_i^+ - b_i^-)$$

$$c_{i\sigma} = \exp(\frac{\lambda}{\omega_0} (b_i^- - b_i^+)) d_{i\sigma}$$

$$c_{i\sigma}^\dagger = \exp(\frac{\lambda}{\omega_0} (b_i^+ - b_i^-)) d_{i\sigma}^\dagger$$

$$H_{LF} = - \sum_{ij\sigma} t_{ij} c_{i\sigma}^\dagger c_{j\sigma} + U_0 \sum_i c_{i\uparrow}^\dagger c_{i\uparrow} c_{i\downarrow}^\dagger c_{i\downarrow} + \omega_0 \sum_i b_i^\dagger b_i,$$

$$U_0 = V - \frac{2\lambda^2}{\omega_0} \quad Z_B = \exp(-\lambda^2/\omega_0^2)$$

Projection on the  
zero-plasmon  
subspace

$$H_{\text{eff}} = \langle 0 | H | 0 \rangle = - \sum_{ij\sigma} Z_B t_{ij} d_{i\sigma}^\dagger d_{j\sigma} + U_0 \sum_i d_{i\uparrow}^\dagger d_{i\uparrow} d_{i\downarrow}^\dagger d_{i\downarrow}$$

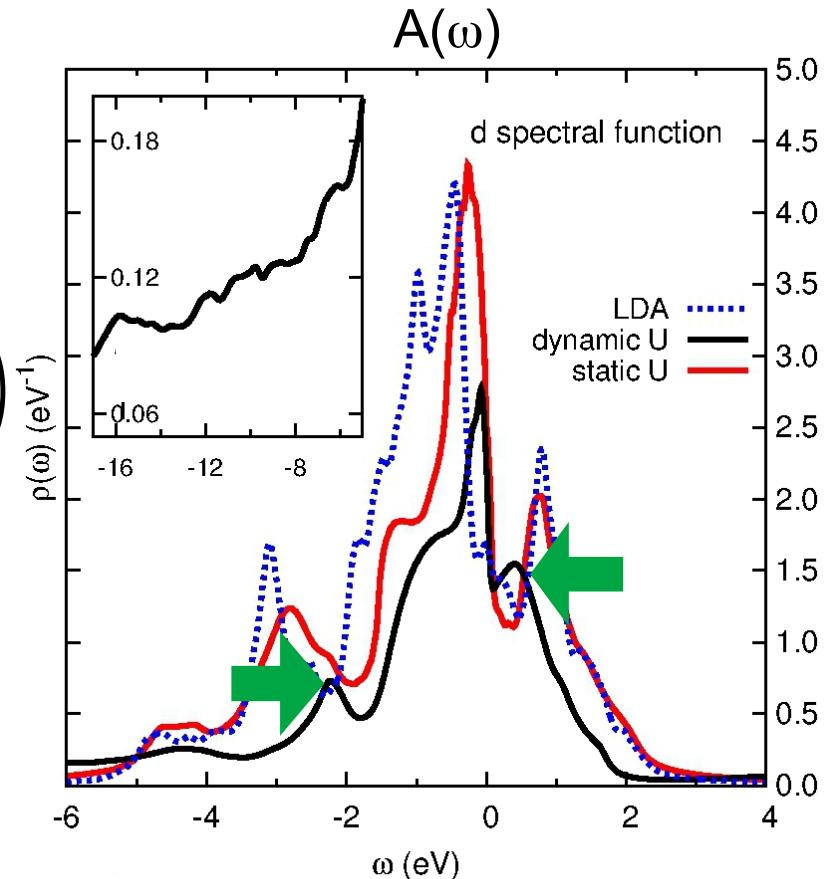
# Dynamic U effects recovered in the effective static model

Bandwidth renormalization

$$(p^\dagger d^\dagger) \begin{pmatrix} \mathcal{T}_{pp} & \sqrt{Z_B} \mathcal{T}_{pd} \\ \sqrt{Z_B} \mathcal{T}_{pd}^\dagger & Z_B \mathcal{T}_{dd} \end{pmatrix} \begin{pmatrix} p \\ d \end{pmatrix}$$

$$Z_B = \exp \left( 1/\pi \int_0 \mathrm{d}\nu \operatorname{Im} U_{\mathrm{ret}}(\nu)/\nu^2 \right)$$

$$Z_B = 0.59$$



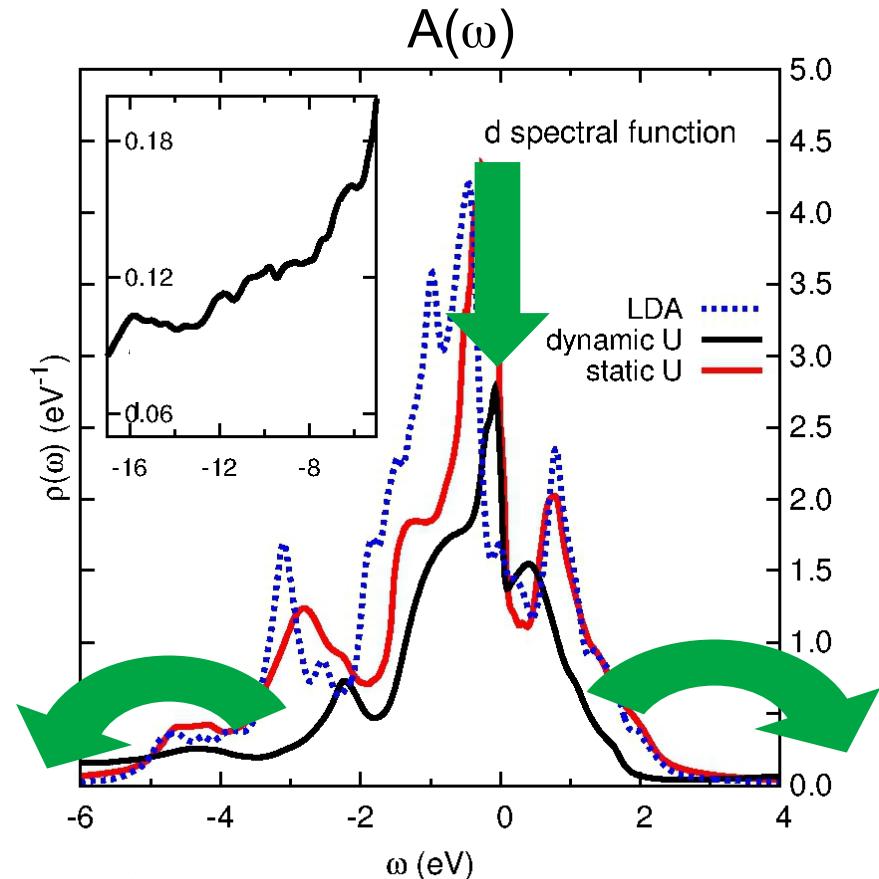
Structures from p-d hybridization squeezed toward the Fermi energy

# Dynamic U effects recovered in the effective static model

Green's function renormalization

$$G_{ij}^{\text{low-energy}}(\tau) = -Z_B \langle T d_i(\tau) d_j^\dagger(0) \rangle_{H_{\text{eff}}}$$

loss of spectral weight due to  
plasmon “shake-up” excitations



Weight reduction at low-energy and spectral weight transfer to high-energy

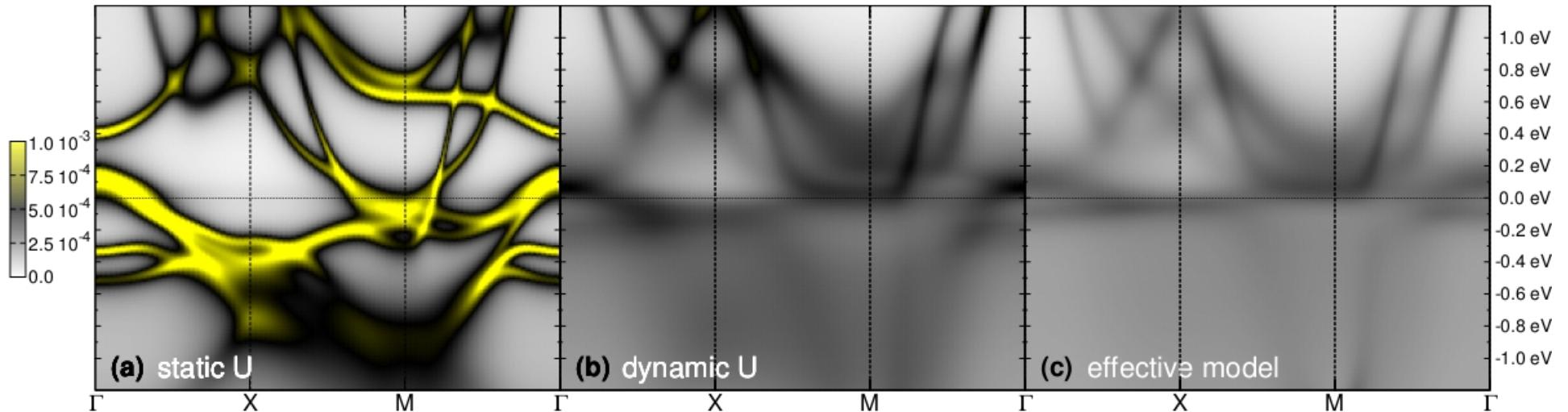
# cRPA U versus “regular” U

- Static U used in literature for correlated compounds compared to effective U

	$Z_B$	$U_0$	$U_{\text{lit}}$
SrVO <sub>3</sub>	0.70	3.3	4 - 5
Sr <sub>2</sub> VO <sub>4</sub>	0.70	3.1	4.2
LaVO <sub>3</sub>	0.57	1.9	5
VO <sub>2</sub>	0.67	2.7	4
TaS <sub>2</sub>	0.79	1.5	
SrMnO <sub>3</sub>	0.50	3.1	2.7
BaFe <sub>2</sub> As <sub>2</sub>	0.59	2.8	5
LaOFeAs	0.61	2.7	3.5 - 5
FeSe	0.63	4.2	4 - 5
CuO	0.63	6.8	7.5

**The bandwidth renormalization from Coulomb screening  
explains the apparent contradiction  
between the static limit of  $U_{\text{RPA}}$  and the U values used so far in literature**

# Spectral function in effective low-energy model



Effective low-energy model spectral function  
in agreement with the one obtained for the fully dynamic U model

M. Casula, P. Werner, L. Vaugier, F. Aryasetiawan, A. J. Millis, S. Biermann  
 PRL **109**, 126408 (2012)

# Conclusions

- First ab-initio DMFT calculations with frequency dependent U
- Capability of DMFT to treat plasmon satellites
- Surprises from the strong low-energy impact of the dynamic U, despite the fact that the unscreened part sets on at relatively high energies ( $\sim$ 15-20 eV)
- Physical outcome from dynamic part in  $\text{BaFe}_2\text{As}_2$ 
  - Renormalization of the quasiparticle width and spin-freezing crossover
  - Spectral weight transfer to higher energies
  - Prediction of plasmon satellites
- Effective low-energy model to include dynamic screening

M. Casula, A. Rubtsov, S. Biermann, PRB **85**, 035115 (2012)

P. Werner, M. Casula, T. Miyake, F. Aryasetiawan, A. J. Millis, S. Biermann  
Nature Physics **8**, 331-337 (2012)

M. Casula, P. Werner, L. Vaugier, F. Aryasetiawan, A. J. Millis, S. Biermann  
PRL **109**, 126408 (2012)