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IAEA International Atomic Energy Agency

2443-7

Winter College on Optics: Trends in Laser Development and Multidisciplinary Applications to Science and Industry

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Pulse characterization

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Pulse Characterization

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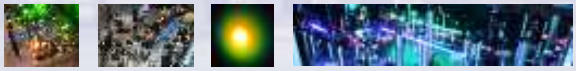
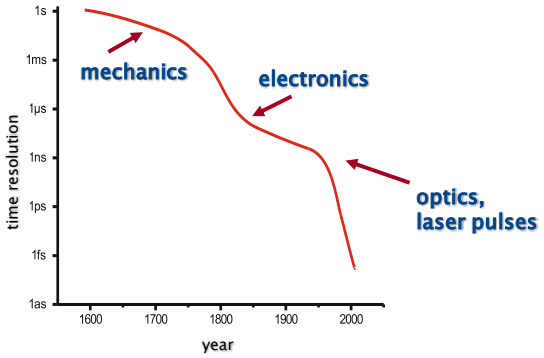
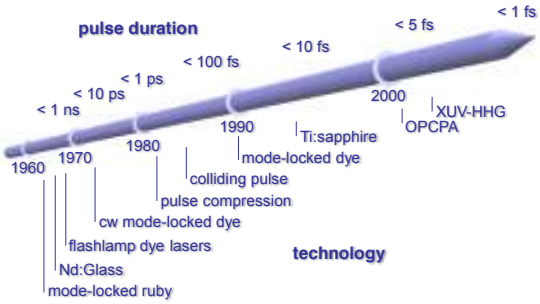


Technical time resolution



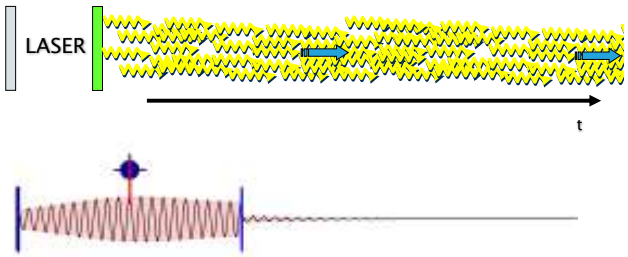
Herrmann/Wilhelmi: Lasers for Ultrashort Light Pulses, 1987

History of short pulse lasers

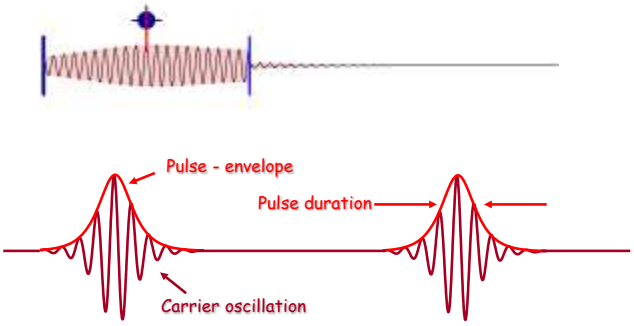


source: P. Dietrich

Pulsed laser



Intracavity amplitude modulation – active mode-locking



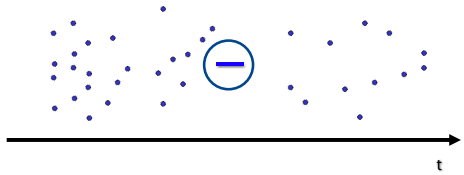
The Dilemma

In order to measure an event in time, you need a *shorter* one.

Photograph taken by Harold Edgerton, MIT

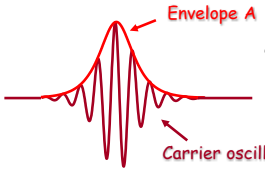
How do you measure the *shortest* events?

Ultrashort electron pulses for detection?



Optical tools required!

Some definitions

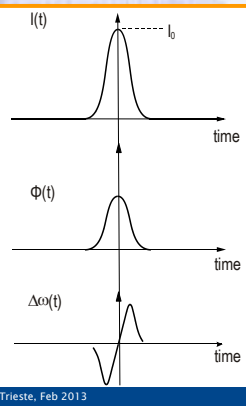


$$E(t) = Re\left\{\bar{E}(t)\right\} = Re\left\{\bar{A}(t)\, e^{i(\omega_0 t + \Delta\varphi)}\right\}$$

$$E(t) = Re\left\{A(t)\, e^{i(\omega_0 t + \phi(t) + \Delta\varphi)}\right\}$$

Intensity envelope: $I(t) = \frac{1}{2} n c \varepsilon_0 |E(t)|^2 = \frac{1}{2} n c \varepsilon_0 |A(t)|^2$

Chirp

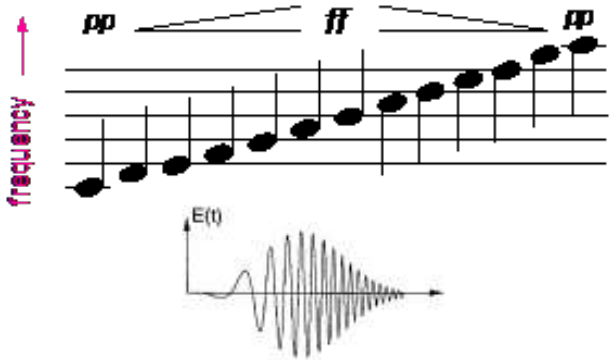


Intensity envelope

Temporal phase (example)

Local frequency shift: $\Delta\omega(t) = -\frac{d\phi(t)}{dt}$

Linear chirp = parabolic phase



Fourier transform

$$\bar{E}(t) = A(t)\, e^{i(\omega_0 t + \phi(t) + \Delta\varphi)}$$

temporal phase

$|A(t)|^2 \sim I(t) \rightarrow$ intensity envelope

FT

$$\tilde{E}(\omega) = S(\omega)\, e^{i(\varphi(\omega) + \Delta\varphi)}$$

spectral phase

$|S(\omega)|^2 \sim \rightarrow$ power spectrum

TBP

$\Delta\tau$ = pulse duration (FWHM) [s]

$\Delta\nu$ = spectral bandwidth (FWHM) [Hz]

$$\Delta\tau \cdot \Delta\nu \geq TBP$$

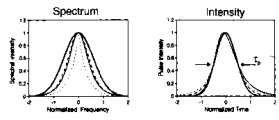
$$\Delta\nu \cong \frac{c}{\lambda^2} \cdot \Delta\lambda$$

TBP = time – bandwidth product

Common pulse envelopes (with τ_p = Intensity FWHM):

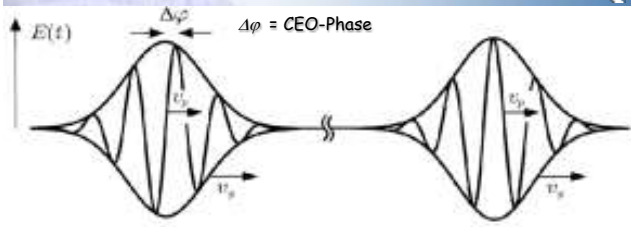
Gaussian pulse	$\mathcal{E}(t) \propto$	$\exp[-1.385(t/\tau_p)^2]$
sech - pulse	$\mathcal{E}(t) \propto$	$\mathrm{sech}[1.763(t/\tau_p)]$
Lorentzian pulse	$\mathcal{E}(t) \propto$	$[1 + 1.656(t/\tau_p)^2]^{-1}$
asymm. sech pulse	$\mathcal{E}(t) \propto$	$[\exp(t/\tau_p) + \exp(-3t/\tau_p)]^{-1}$

Spectra for pulses with the same pulse width



Field envelope	Intensity profile	τ_p (FWHM)	Spectral profile	$\Delta\omega_p$ (FWHM)	TBP
Gauss	$e^{-2(t/\tau_G)^2}$	$1.177\tau_G$	$e^{-(\omega\tau_G)^2/2}$	$2.355/\tau_G$	0.441
sech	$\mathrm{sech}^2(t/\tau_s)$	$1.763\tau_s$	$\mathrm{sech}^2(\pi\omega\tau_s/2)$	$1.122/\tau_s$	0.315
Lorentz	$[1 + (t/\tau_L)^2]^{-2}$	$1.287\tau_L$	$e^{-2\lvert\omega\rvert\tau_L}$	$0.693/\tau_L$	0.142
asymm. sech	$[e^{t/\tau_a} + e^{-3t/\tau_a}]^{-2}$	$1.043\tau_a$	$\mathrm{sech}(\pi\omega\tau_a/2)$	$1.677/\tau_a$	0.278
rectang.	1 for $\lvert t/\tau_r\rvert \leq \frac{1}{2}$, 0 else	τ_r	$\mathrm{sinc}^2(\omega\tau_r)$	$2.78/\tau_r$	0.443

Carrier-Envelope Phase



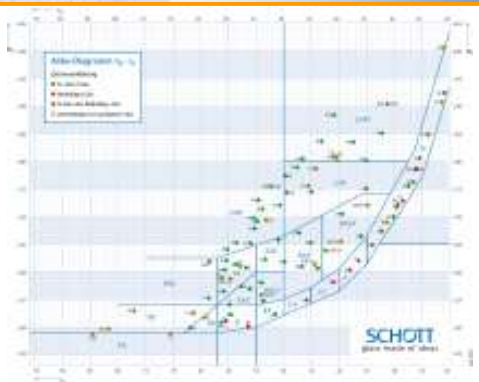
v_p = phase velocity $\quad v_p = \frac{c}{n}$

v_g = group velocity $\quad v_g = \frac{c}{n_g} = \frac{c}{n + \omega \frac{dn}{d\omega}}$

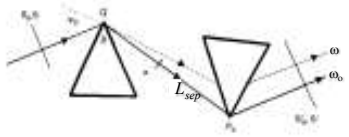
Dispersion acts on the spectral phase

- Influences only $\varphi(\omega)$:
- group delay GD: $\frac{d\varphi(\omega)}{d\omega}$
- group delay dispersion GDD: $\frac{d^2\varphi(\omega)}{d\omega^2}$
- Third order dispersion TOD: $\frac{d^3\varphi(\omega)}{d\omega^3}$
- …

Dispersive glasses



Negative GVD from prisms

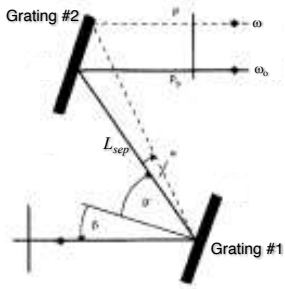


$$\left.\frac{d^2\varphi}{d\omega^2}\right|_{\omega_0} \approx -4L_{sep}\frac{\lambda_0^3}{2\pi c^2}\left(\left.\frac{dn}{d\lambda}\right|_{\lambda_0}\right)^2 + L_{prism}\frac{\lambda_0^3}{2\pi c^2}\left.\frac{d^2n}{d\lambda^2}\right|_{\lambda_0}$$

Prism compressor

Grating compressor

$$\left.\frac{d^2\varphi}{d\omega^2}\right|_{\omega_0} \approx -\frac{\lambda_0^3}{2\pi c^2 d^2}\frac{L_{sep}}{\cos^2(\beta)}$$



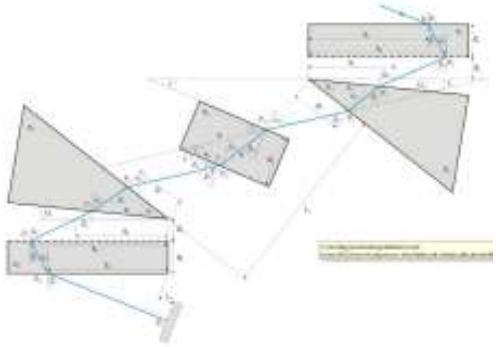
2nd- and 3rd-order phase terms for prism and grating pulse compressors

Grating compressors offer more compression than prism compressors.

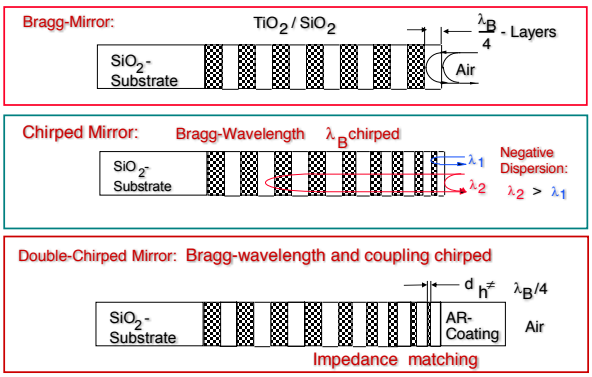
Device	λ_f [nm]	ω_f [fs^{-1}]	φ'' [fs^{-2}]	φ''' [fs^{-3}]
SQ1 ($L = 1$ cm)	620	3.04	550	240
Piece of glass	800	2.36	362	280
Brewster prism pair, SQ1	620	3.04	-760	-1300
$\ell = 50$ cm	800	2.36	-523	-612
grating pair $b = 20$ cm; $\beta = 0°$	620	3.04	$-8.2\ 10^4$	$1.1\ 10^5$
$d = 1.2\ \mu m$	800	2.36	$-3\ 10^6$	$6.8\ 10^6$

Note that the relative signs of the 2nd and 3rd-order terms are opposite for prism compressors and grating compressors.

GRISM = grating prism combination

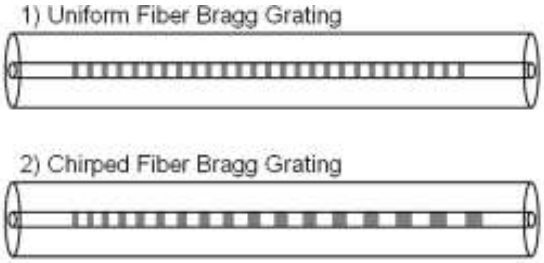


Dispersion compensation with double-chirped mirrors



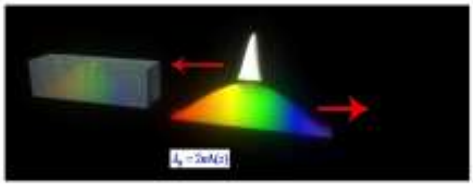
Szipöcs et al, OL 19, 201 (1994) Kärtner et al, OL 33, 831 (1997)

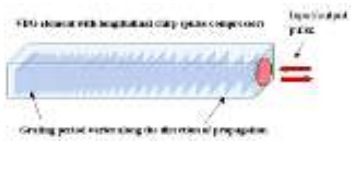
CFBG = chirped fiber Bragg grating



http://en.wikipedia.org/wiki/Fiber_Bragg_grating

CVBG = chirped volume Bragg grating





Diss Matthew A. Rever, Michigan 2010, www.pd-ld.com

Pulse characterization

Measure either $|A(t)|^2$ and $\phi(t)$ in time domain

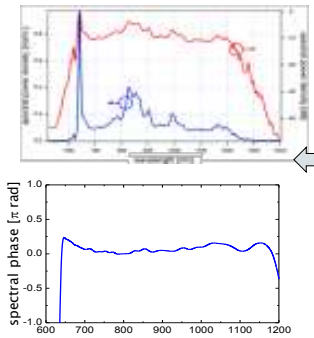
or measure $|S(\omega)|^2$ and $\varphi(\omega)$ in frequency domain

(and measure the CEO-phase ϕ_0)

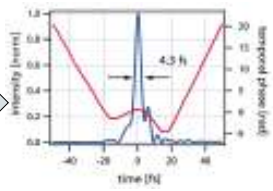
→ Then you know everthying about the pulse!

Pulses in time and in Fourier domain

$|S(\omega)|^2$ and $\varphi(\omega)$ in spectral domain



$|A(t)|^2$ and $\phi(t)$ in time domain



Spectrometer

measure $|S(\omega)|^2$

Pulse characterization methods

Measure $|A(t)|^2$ and $\phi(t)$ in time domain

- Autocorrelation, PICASSO
- FROG
- Attosecond streaking

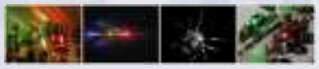
or measure $|S(\omega)|^2$ and $\varphi(\omega)$ in spectral domain

- SPIDER, 2DSI
- MIIPS

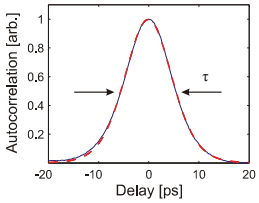
Autocorrelation

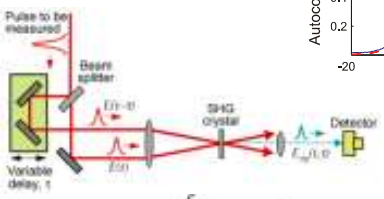
References:

- J. A. Armstrong, Measurement of picosecond laser pulse widths, Appl. Phys. Lett. 10, 16 (1967)
- D. J. Bradley, G. H. C. New, Ultrashort Pulse Measurement, Proc. of the IEEE 62, 313 (1974)
- C. Spielmann, L. Xu, F. Krausz, Measurement of interferometric autocorrelations: comment, Appl. Opt. 36, 2523-5 (1997)



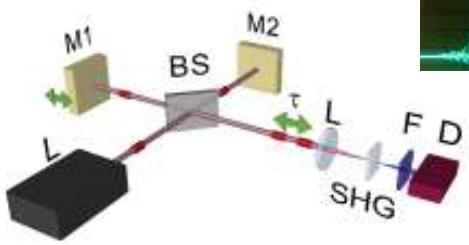
Intensity autocorrelation





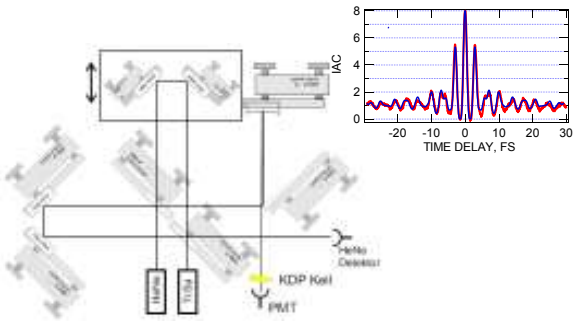
$$\mathrm{AKF}(\tau) = \int \left[|E(t)|^2 + |E(t+\tau)|^2\right]^2 dt$$

Interferometric autocorrelation



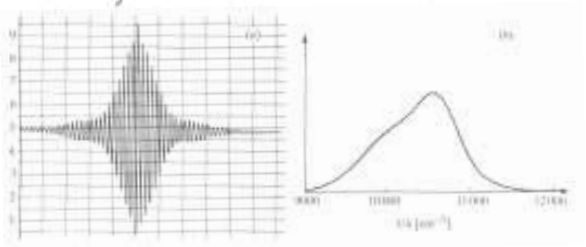
$$\mathrm{IAC}(\tau) = \int |E(t) + E(t+\tau)|^4 dt$$

Balanced interferometric autocorrelation



Why do we need the nonlinearity?

$$I(\tau) = \int |E(t)+E(t+\tau)|^2 dt$$



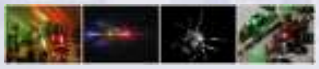
$$= I_{const} + FT\left\{|E(\omega)|^2\right\}$$

PICASO

"phase and intensity from correlation and spectrum only"

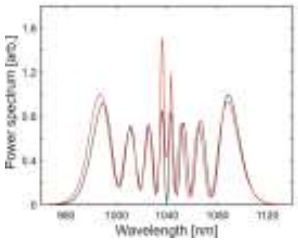
References:

- J.W. Nicholsen, J.Jasapara, W.Rudolph, F.G.Omennetto and A.J.Tylor, "Full field characterization of femtosecond pulses by spectrum and cross correlation measurements," Opt. Lett. 24, 1774-1776 (1999).
- J. W. Nicholson and W. Rudolph , Noise sensitivity and accuracy of femtosecond pulse retrieval by phase and intensity from correlation and spectrum only (PICASO), JOSA B19 330-339 (2002)

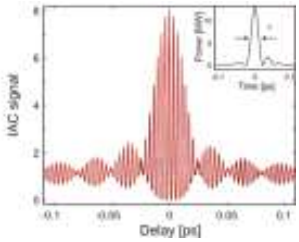


PICASO

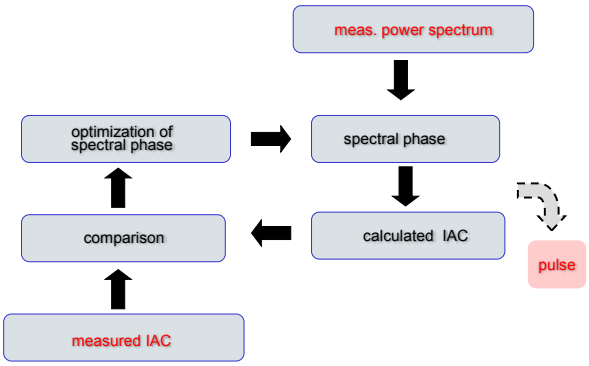
$|S(\omega)|^2$



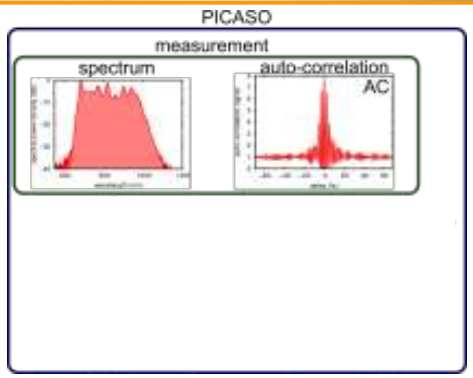
$$IAC(\tau) = \int |E(t)+E(t+\tau)|^4 dt$$



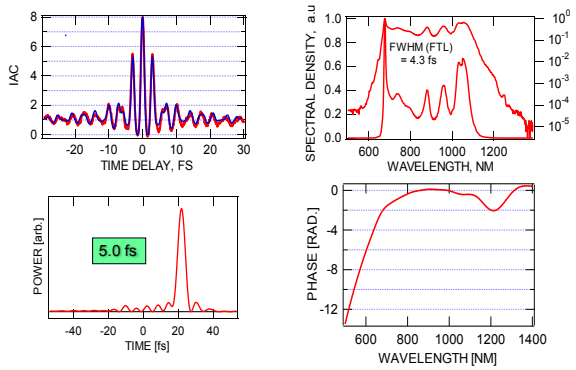
Phase retrieval from IAC and Spectrum



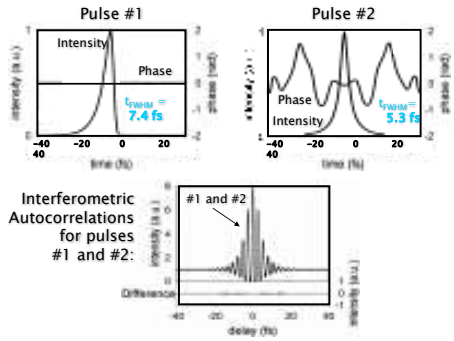
PICASO – algorithm



PICASO results



Example of autocorrelation ambiguity



Chung and Weiner, IEEE JSTQE, 2001

FROG

Frequency resolved optical gating

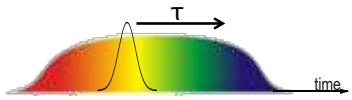
References:

Rick Trebino (2002). Frequency-Resolved Optical Gating: The Measurement of Ultrashort Laser Pulses. Springer. ISBN 1-4020-7066-7 (2002)

R. Trebino, K. W. DeLong, D. N. Fittinghoff, J. N. Sweetser, M. A. Krumbügel, and D. J. Kane, "Measuring Ultrashort Laser Pulses in the Time-Frequency Domain Using Frequency-Resolved Optical Gating", Review of Scientific Instruments 68, 3277-3295 (1997)

http://frog.gatech.edu/

The spectrogram

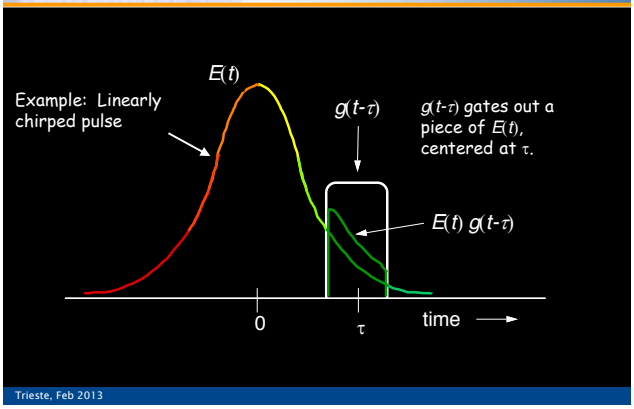


$g(t-\tau)$: variable-delay gate function

$$\Sigma_E(\omega,\tau) \equiv \left| \int_{-\infty}^{\infty} E(t)\, g(t-\tau) \exp(-i\omega t)\, dt \right|^2$$

Set of spectra of temporal slices of $E(t)$.

The Spectrogram of a waveform E(t)



Spectrogram

Delta-function gate pulse:

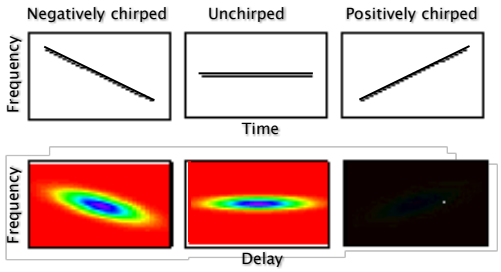
$$\left| \int_{-\infty}^{\infty} E(t)\, \delta(t-\tau) \exp(-i\omega t)\, dt \right|^2 = \left| E(\tau) \exp(-i\omega\tau) \right|^2$$

$$= \left| E(\tau) \right|^2$$ = local intensity.

No phase information!

The gate need not be—and should not be—much shorter than $E(t)$.

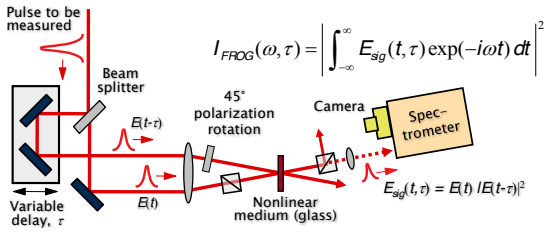
Spectrograms for Linearly Chirped Pulses



Frequency-Resolved Optical Gating (FROG)

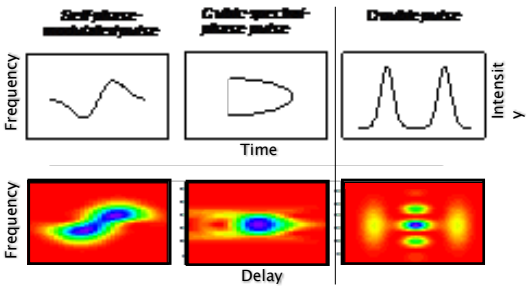
Gating the pulse with a variably delayed replica of itself!

"Polarization Gate" Geometry



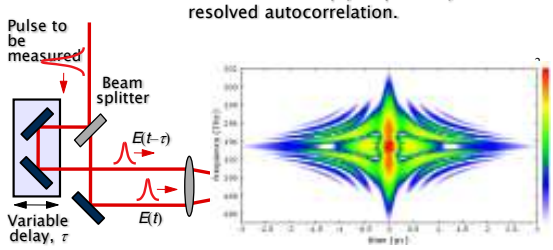
$$I_{FROG}(\omega,\tau) = \left| \int_{-\infty}^{\infty} E_{sig}(t,\tau)\exp(-i\omega t)\,dt \right|^2$$

FROG Traces for More Complex Pulses



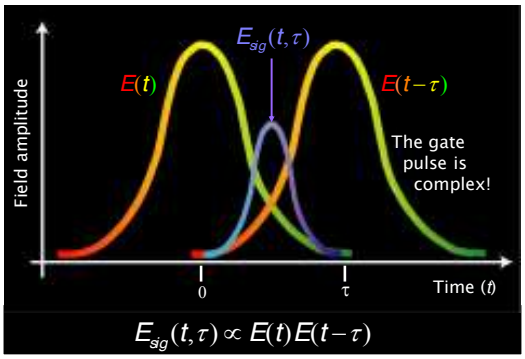
Second-harmonic- FROG

SHG FROG is simply a spectrally resolved autocorrelation.

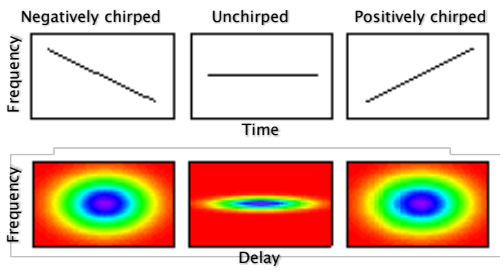


SHG FROG is the most sensitive version of FROG.

SHG FROG

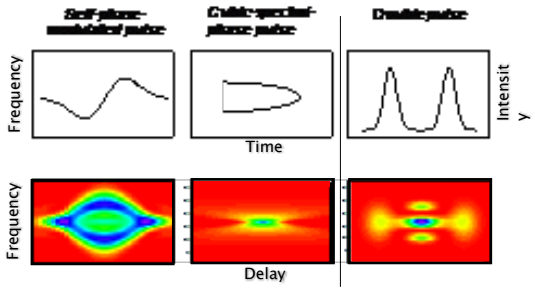


SHG FROG traces



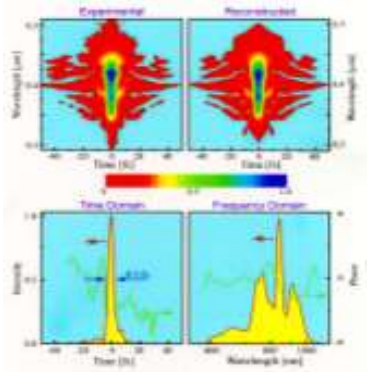
symmetrical with respect to delay.

SHG FROG traces for complex pulses



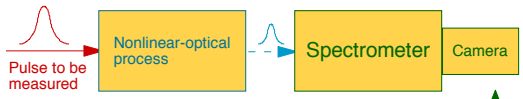
SHG FROG traces are symmetrized PG FROG traces.

FROG Measurements of a 4.5-fs Pulse



J. Quant. Electron., 35, 459 (1999)

FROG with arbitrary nonlinear interactions



$E_{sig}(t,\tau) \propto$
- $E(t)\,E(t-\tau)$ — SHG
- $E(t)\,|E(t-\tau)|^2$ — polarization gate
- $E(t)^2\,E(t-\tau)^*$ — self diffraction
- $E(t)^2\,E(t-\tau)$ — THG

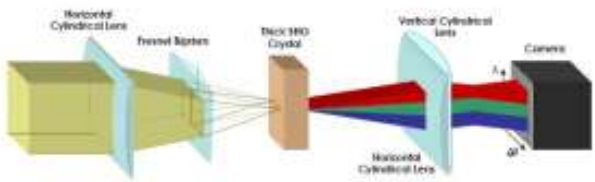
$$I_{FROG}(\omega,\tau) = \left| \int_{-\infty}^{\infty} E_{sig}(t,\tau)\exp(-i\omega t)\,dt \right|^2$$

Obtaining E(t) from the FROG trace is equivalent to the 2D phase-retrieval problem.

FROG offspring

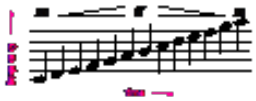
- XFROG, cross-correlation FROG
- IFROG, interferometric FROG
- TG-FROG, transient-grating FROG
- PG-FROG, polarization-gated FROG
- SD-FROG, self-diffraction FROG
- GRENOUILLE, Grating-eliminated no-nonsense observation of ultrafast incident laser light e-fields

GRENOUILLE



en.wikipedia.org

FROG-Conclusions



Advantages:
- Easy setup (AC with spectrometer instead of PD)
- Complete pulse characterization (intensity + phase)
- Pulse durations: sub10fs – ~ps
- Pulse energies: <nJ – mJ
- Wavelength range: UV-IR

Disadvantages:
- (in most cases) scanning required
- Complex algorithm

Attosecond streaking

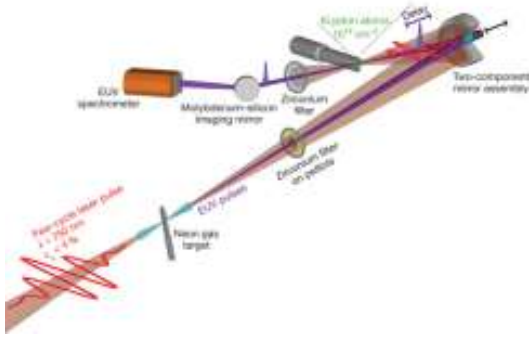
References:

Goulielmakis, E., Yakovlev, V. S., Cavalieri, A. L., Uiberacker, M., Pervak, V., Apolonski, A., ... & Krausz, F. (2007). Attosecond control and measurement: lightwave electronics. Science, 317(5839), 769 (2007)

Goulielmakis, E., Schultze, M., Hofstetter, M., Yakovlev, V. S., Gagnon, J., Uiberacker, M., ... & Kleineberg, U. Single-cycle nonlinear optics. Science, 320(5883), 1614 (2008)

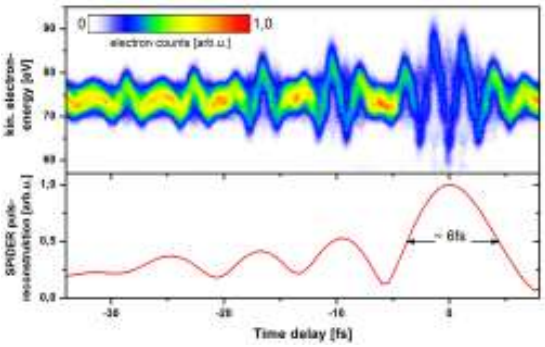
http://www.attoworld.de/

Attosecond streaking for sub-two cycle pulses



Nature 466, 739-743 (2010)

Streaking



(courtesy of Walter Pfeiffer)

Pulse characterization methods

Measure $|A(t)|^2$ and $\phi(t)$ in time domain

- Autocorrelation, PICASO
- FROG
- Attosecond streaking

or measure $|S(\omega)|^2$ and $\varphi(\omega)$ in spectral domain

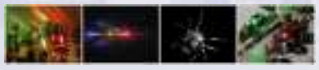
- SPIDER, 2DSI
- MIIPS

SPIDER

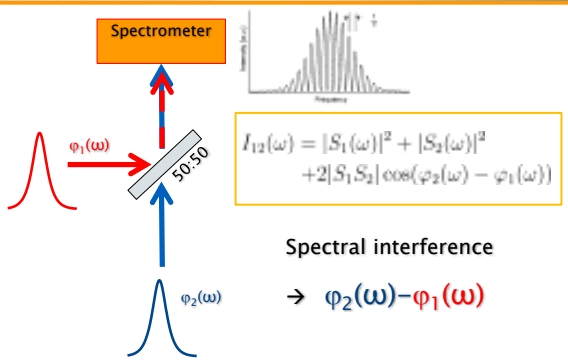
Spectral Phase Interferometry for Direct Electric-Field Reconstruction

References:

- [Iac98] C. Iaconis, I. A. Walmsley, *Spectral phase interferometry for direct electric-field reconstruction of ultrashort optical pulses*, Optics Letters 23, 792-794 (1998)
- [Iac99] C. Iaconis, I. Walmsley, *Self-referencing Spectral interferometry for Measuring Ultrashort Optical Pulses*, IEEE Journ. of Quantum Electron. QE 35, 501-9 (1999)
- [Gal01] L. Gallmann, *Generation and Characterization of Few-Femtosecond Optical Pulses* (Dissertation, ETH Zürich (Swiss), 2001)
- [Tak82] M. Takeda, H. Ina, S. Kobayashi, *Fourier-transform method of fringe-pattern analysis for computer-based topography and interferometry*, J. Opt. Soc. Amer. 72, 156 (1982)



Spectral interferometry



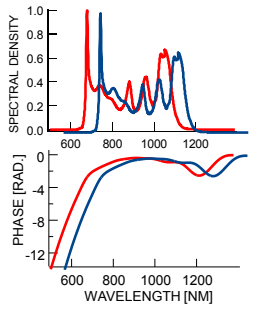
$$I_{12}(\omega) = |S_1(\omega)|^2 + |S_2(\omega)|^2 + 2|S_1 S_2| \cos(\varphi_2(\omega) - \varphi_1(\omega))$$

Spectral interference

→ $\varphi_2(\omega) - \varphi_1(\omega)$

Takeda et al., Josa 72, 156 (1982)

SPIDER principle



$|S(\omega)|^2$ and $\varphi(\omega)$ in spectral domain

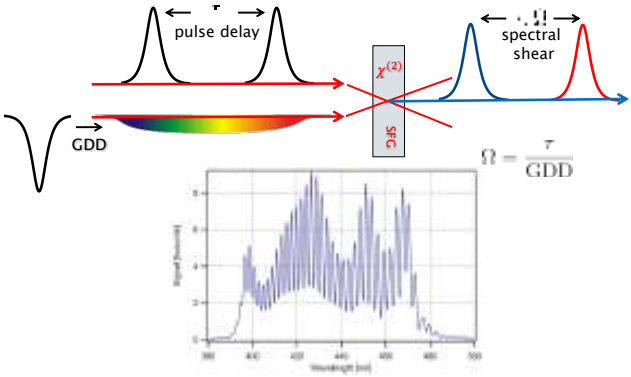
Sheared: $|S(\omega+\Omega)|^2$, $\varphi(\omega+\Omega)$

Spectral interference

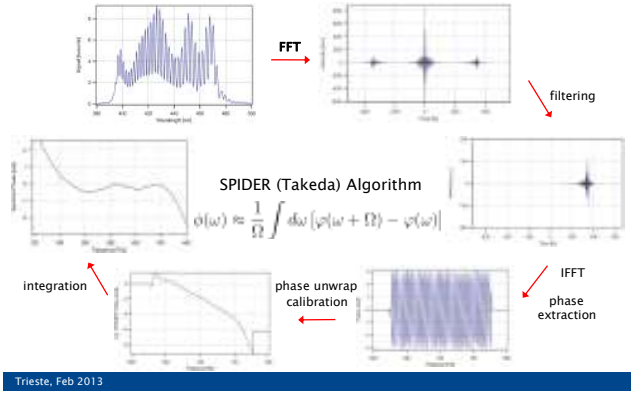
→ $\varphi(\omega+\Omega)-\varphi(\omega)$

$$\frac{\varphi(\omega+\Omega)-\varphi(\omega)}{\Omega} = GD(\omega)$$

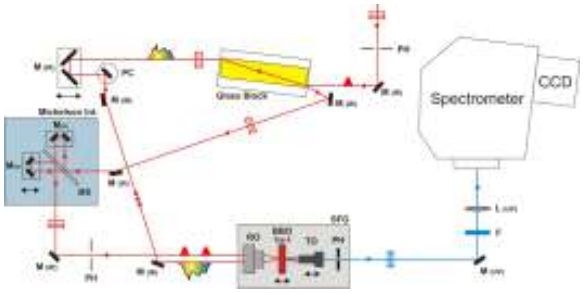
SPIDER shearing



Phasen Rekonstruktion



SPIDER Set-up



BS = beam splitter, CM = concave mirror, F = filter, L = lens, M = mirror, PC = periscope, PH = pinhole, RO = reflective objective, TO = transmittive objective

SPIDER pros and cons

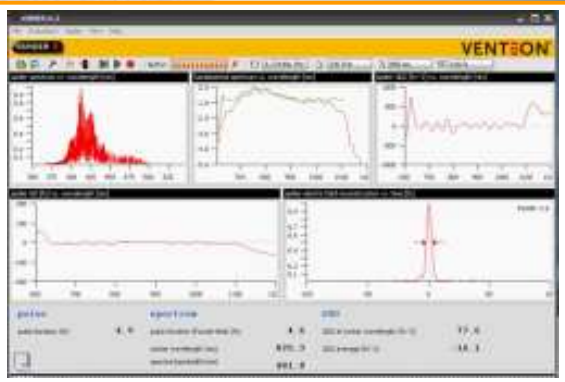
pro

- suited for few-cycle pulses
- unambiguous pulse reconstruction
- 1D measurement
- single-shot, no mechanical movements
- quick (up to 1kHz)
- information just in the fringe spacing, not in the amplitudes

con

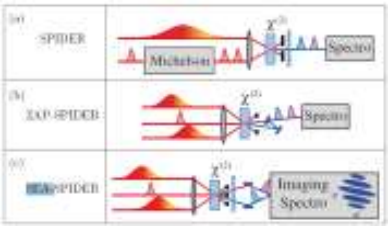
- exact calibration (τ) necessary
- complex set-up

SPIDER live



SPIDER offspring

- ZAP-SPIDER, zero additional phase SPIDER
- SEA-SPIDER, spatially encoded arrangement SPIDER
- M-SPIDER, modified SPIDER
- 2DSI, 2D spectral interferometry



J. Phys. B: At. Mol. Opt. Phys. **43** 103001 (2010)

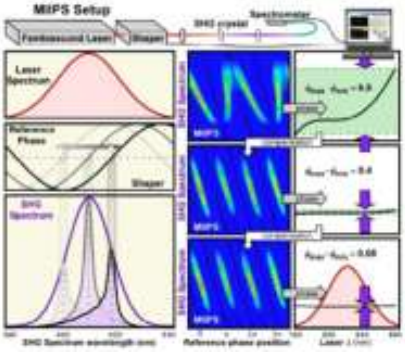
MIIPS

Multiphoton intrapulse interference phase scan

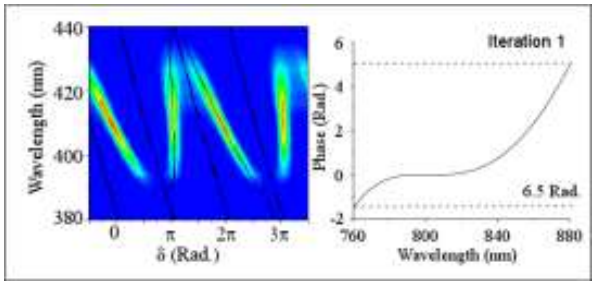
References:

V. V. Lozovoy, I. Pastirk, and M. Dantus, "Multiphoton intrapulse interference 4: Characterization and compensation of the spectral phase of ultrashort laser pulses." Optics Letters 29, 775 (2004)

B. Xu, J. M. Gunn, J. M. Dela Cruz, V. V. Lozovoy, M. Dantus, "Quantitative investigation of the MIIPS method for phase measurement and compensation of femtosecond laser pulses," J. Optical Society of America B 23, 750 (2006)



MIIPS



en.wikipedia.org

The zoo of methods

Intensity Autocorrelation, Interferometric Autocorrelation, PICASO, FROG, Attosecond streaking, SPIDER, 2DSI, MIIPS, ZAP-SPIDER, SEA-SPIDER, M-SPIDER, 2DSI, XFROG, IFROG, TG-FROG, PG-FROG, SD-FROG, GRENOUILLE, TADPOLE, FROGCRAB

Which one ist the right one?

It depends…

Thank you very much for your attention!