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INTERFEROMETRY: CONCEPTS AND APPLICATIONS

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INTERFEROMETRY: CONCEPTS AND APPLICATIONS





Outline of Presentation

- **1. Principles of Interferometry**
- 2. Some examples of Interferometric Systems
- 3. ESPI and Digital Holographic Interferometry

Recommended Literature:

- 1) Optical Metrology, K.J. Gasvik
- 2) Optics, M.V. Klein

3) Fundamentals of Optics and Modern Physics, H.D. Young

4) T. Kreis, "Handbook of Holographic Interferometry" WILEY-VCH Verlag GmbH & Co.KGaA, 2005



Interferometry: superposition of n E-M beams in space. The result of interefrence depends on the phase relations between the beams.

Interefrence relates to the interaction betwen propagating beams,

while refraction, scattering and diffraction depend on the interaction between a beam and matter.

A short history of interferometry

- 1. XVIIc. R. Boyle, R. Hook, observation and analysis of interference effects in a thin air layer limited by two glass plates which demonstrated the wave nature of light
- 1690 C. Huygens, Huyghens theorem (beginning of the wave theory of light)
 Each element of a wavefront may be regarded as the center of a secondary disturbance which gives rise to spherical wavelets and the position of the wevefront at any latter time is the envelope of such wavelets
- 3. 1738 T. Young experiment confirmed Huyghens' hipothesis and gave the basis to modern theory of light coherence
- 4. 1818 A. Fresnel extension of Huyghens theorem, leading to so-called Huyghens-Fresnel principle great importance in the diffraction theory and the basic postulate of the wave theory of light, development of stellar interferometry

1a. 1874 Lord Rayleigh used for the first time moire phenomenon⁴

A short history of interferometry

- 5. 1881 Michelson experiment (speed of light) and his further works on interferometry, stellar interferometry, high resolution interferometric spectroscopy - he is considered as the father of interferometry (Nobel prize 1907)
- 6. 1916 F. Twyman modifications of Michelson ineterferometer
- 7. 1960 invention of laser: Schawlow, Maiman, Townes, Prochorow....
- 8. 1948 Gabor principles of holography
- 9. 1962 -Leith and Upatnieks off-axis holography and development of holographic interferometry (works of Burch, Brooks, Collier, Stetson...)
- 10. 1970 Archbold, Leendertz speckle interferometry and speckle photography
- 11. 1982- .. Development of phase based interferogram analysis methods
- 11. 1995-....Rapid progress in digital holography
- 12. 2000-...Rapid progress in active interferometry and holography

Fundamentals of interferometry

Vector of electric field

 $\overline{E}_{i}(\overline{r},t) = \overline{E}_{i0} exp[i(\phi_{i}(\overline{r}) - \omega_{i}t)]$

Resultant vector in two beam interferometry

$$\overline{\mathbf{E}}(\overline{\mathbf{r}},\mathbf{t}) = \sum_{i} \overline{\mathbf{E}}_{i}(\overline{\mathbf{r}},\mathbf{t}); \quad i = 1,2$$

Result of two beam interference (E field intensity):

$$\begin{split} I(\mathbf{r})\alpha \ \left|\overline{\mathbf{E}}\right|^2 &= \left|\overline{\mathbf{E}}_1 + \overline{\mathbf{E}}_2\right|^2 = \left(\overline{\mathbf{E}}_1 + \overline{\mathbf{E}}_2\right)\left(\overline{\mathbf{E}}_1 + \overline{\mathbf{E}}_2\right)^* = \overline{\mathbf{E}}_1\overline{\mathbf{E}}_1^* + \overline{\mathbf{E}}_2\overline{\mathbf{E}}_2^* + \overline{\mathbf{E}}_1\overline{\mathbf{E}}_2^* + \overline{\mathbf{E}}_1^*\overline{\mathbf{E}}_2 \\ I(\overline{\mathbf{r}}) &= I_1 + I_2 + I_{12} = I_1 + I_2 + 2\sqrt{I_1I_2}\cos\left[\left(\phi_1(\overline{\mathbf{r}}) - \phi_2(\overline{\mathbf{r}})\right) - \left(\omega_1\mathbf{t} - \omega_2\mathbf{t}\right)\right] \end{split}$$

Conditions for stationary interference field:

$$\begin{split} &\omega_1 = \omega_2 \\ &\phi_1(\bar{r}) - \phi_2(\bar{r}) = const. \end{split}$$

Recommended: parallel polarization of beams

Fundamentals of interferometry

For $\omega_1 = \omega_2$ (usually one source applied)

 $I(\bar{r}) = I_1 + I_2 + 2\sqrt{I_1I_2}\cos\left(\phi_1(\bar{r}) - \phi_2(\bar{r})\right) = a(\bar{r}) + b(\bar{r})\cos\phi(\bar{r}) \cong 1 + \gamma(\bar{r})\cos\phi(\bar{r})$



The observable physical quantity is the intensity,

$$I = |a|^{2} = (a_{1} + a_{2}) (a_{1}^{*} + a_{2}^{*}) =$$

$$A_{1}^{2} + A_{2}^{2} + A_{1}A_{2} e^{i(\varphi_{2} - \varphi_{1})} + A_{1}A_{2} e^{-i(\varphi_{2} - \varphi_{1})} =$$

$$I_{1} + I_{2} + 2\sqrt{I_{1}I_{2}} \cos \Delta \varphi \quad (3)$$
where $\Delta \varphi = \varphi_{1} - \varphi_{2}$.

Output: interferogram

Modifications of interferograms help to retrieve phase

 $I(x, y, t) = a(x, y) + b(x, y)\cos[2\pi [(f_{ox}x + f_{oy}y) + v_o(t)] + \alpha(t) + \phi(x, y)]$

Required controlled modifications of phase in FP:

- $v_0(t)$ introduces temporal heterodyning (running fringes)
- α (t) introduces controled phase shifts
- f_{0x},f_{0y} introduce spatial carrier fringes (spatial heterodyning)

This will be discussed on Thursday

However the requirement to get a high quality interferogram: Source with spatial and temporal coherence:

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Let's use the Michelson interferometer to determine the coherence length of a laser source







Wavefront Division



if $z \gg D$, then

$$\frac{d}{D} = \frac{x}{z} \rightarrow d = \frac{Dx}{z} :$$

$$\Delta \theta = \frac{2\pi \mathrm{Dx}}{\lambda z}$$

On substituting into eq (3)

$$I(x) = 2I(1 + \cos\left(\frac{2\pi Dx}{\lambda z}\right)) \quad (8)$$

This equation represents a fringe pattern parallel to the y axis, with period $\left(\frac{D}{\lambda z}\right)^{-1}$.

The Thomas Young interferometer is being used in FEL to measure its coherence!



An adaptation of Young's interferometer is used in Michelson' stellar interferometer to measure star diameters.

Other types of wavefront division interferometer are:

- Fresnel biprism
- Lloyd's mirror
- Michelson stellar





if mirror M2 is translated a distance x, the optical path difference with respect to mirror M1 is 2x, thus

$$\Delta\theta = \left(\frac{2\pi}{\lambda}\right) \, 2x$$

That gives a total intensity on the detector,

$$I(x) = 2I(1 + \cos\left(\frac{4\pi x}{\lambda}\right)) \quad (9)$$



As the mirror 2 is translated a certain distance *d*, and by counting the number of maxima (bright fringe) per unit time it is possible to measure the speed of an object.

The Michelson Interferometer used with a low coherence source gives way to: low coherence reflectometry, LCR, and Optical Coherence Tomography, OCT. Study of semitransparent materials, biological tissues!



Other good examples of amplitude dividing interferometers are:

- Mach-Zehnder...QO Applications, NEW: 3 arms (as proposed by Prof. Mataloni)!;
 Microchannel fabrication (Prof. Ramponi)
- Twyman-Green...optical shop testing
- Fizeau...flat mirror in FEL

Challanges for interferometry Example: M(O)EMS Characterisation







deformation dimensions/shape vibration (time average (laser interferometry) (white light interferometry)^{laser} interferometry)



Main challenge in M(O)EMS inspection

- Wafer size increases current diameters 6", 8", 12"
- Up to several thousands structures on one wafer (time issues)
- Feature size decreases typically <10mm²
- Inspection ratio: 10⁻³ to 10⁻⁷
- 100% M(O)EMS inspection



Parallel Inspection concept



J. Micromech. Microeng. 22, 015018, 2012





Instrument plattform



2

Holography and speckle techniques Holographic and speckle interferometry



Registration of optical hologram basic setup



1. Need to have equal optical paths of reference and object beams (within coherence length of laser)

2. During recording the phase between object and reference beams cannot change more than $\Delta \phi_{\rm max} < 0.2\pi$



Basic Theoretical Considerations

Two wave addition: Object and Reference

$\mathbf{U}_{\mathrm{T}} = \mathbf{U}_{\mathrm{o}} + \mathbf{U}_{\mathrm{r}} \qquad (1)$

Intensity/Irradiance on the CCD sensor is proportional to

$$I_{\tau} = U_{o}^{*}U_{o} + U_{r}^{*}U_{r} + U_{o}U_{r}^{*} + U_{r}U_{o}^{*}$$
(2)

Apart from a scale factor, the third term is identical to the original object Gabor D., A new microscopic principle, Nature 161, 777-778 (1948).

Registration of digital hologram

Photographic (analog) material is replaced by CCD or CMOS matrix



Scheme for numerical reconstruction of digital hologram Reconstruction wavefront Reconstruction plane Reconstruction plane wave at hologram is given by Fresnel-Kirchoff integral.

Due to small size of CCD camera versus the distance camera (hologram)object we can use here Fresnel approximation given by:

$$U(\xi,\eta;d) = \frac{iU_0}{\lambda d} \exp\left[-i\frac{\pi}{\lambda d}(\xi^2 + \eta^2)\right] \int_{-\infty-\infty}^{+\infty+\infty} h(x,y) \exp\left[-i\frac{\pi}{\lambda d}(x^2 + y^2)\right] \times \exp\left[i\frac{2\pi}{\lambda d}(x\xi + y\eta)\right] dxdy$$

where : (x,y), (ζ , η) are coordinates, h(x,y) is the amplitude transmittance of hologram, U_0 – is the real amplitude of reconstructing wavefront

Sampled version of the reconstructed field

The hologram function is sampled by CCD camera in MxN points with sampling periods Δx and Δy . Then the numeric representation of the previous equation is given as:

$$U(m,n,d) = \exp\left[-i\frac{\pi}{\lambda d}\left(m^{2}\Delta\xi^{2} + n^{2}\Delta\eta^{2}\right)\right]\sum_{k=0}^{M-1}\sum_{l=0}^{N-1}h(k,l) \times \exp\left[-i\frac{\pi}{\lambda d}\left(k^{2}\Delta x^{2} + l^{2}\Delta y^{2}\right)\right]\exp\left[i2\pi\left(\frac{km}{M} + \frac{\ln}{N}\right)\right]$$

Where $\Delta\zeta$ and $\Delta\eta$ are the sizes of pixels in reconstructed images.

Intensity of the object field at distance d

$$I(\xi, n; d) = \operatorname{Re}^{2}[U(m, n, d)] + \operatorname{Im}^{2}[U(m, n, d)]$$

Phase of the object field at distance d

$$\phi(\xi,\eta;d) = \arctan \frac{\text{Im}[U(m,n;d)]}{\text{Re}[U(m,n;d)]}$$

So we can reconstruct numerically all information about an object

Limitations of digital holography

The recording medium has to fulfil the Nyquist condition ! Each fringe has to be sampled by at least two pixels of CCD matrix

CCD cameras;

- resolution 1024x1534;
- Pixel size $\Delta = 9\mu m$; for 4.5 μm

Spatial resolution - ap. 111 lines/mm, 220l/mm
 holographic materials (plates) - >3000 lines/mm).

Conclusion: SMALL angular size of object (a few degrees) i.e. SMALL OBJECT or Object SITUATED FAR from CAMERA


HOLOGRAPHIC INTERFEROMETRY

- Classical - Digital
 - C. M. Vest, Holographic interferometry, J. Wiley and Sons, New York, 1979
 - P. K. Rastogi (ed), Holographic interferometry, Springer, 1994
 - I. Yamaguchi, T. Zhang, Phase shifting digital holography, Opt. Lett., 22, 1268-1270, 1997
 - T. Kreis, "Handbook of Holographic Interferometry" WILEY-VCH Verlag GmbH & Co.KGaA, 2005

Classical holographic interferometry

Each holographic system can be used to compare optical wavefronts formed by an object in different states (physical conditions)

Let initial state of an object is:

 $E_{p1}(P) = A_{p1}(P) \exp[i\phi(P)]$

After providing load (or other changes)

 $E_{p2}(P) = A_{p2}(P) \exp i[\phi(P) + \Delta \phi(P)]$

If the object (load) is stationary we may compare these wavefronts by: -double exposure holographic interferometry, -real time holographic interferometry.

If the object is vibrating we use: -Time averaged holographic interferometry -Stroboscopic holographic interferometry

In the case of dynamic object investigation we use impulse double exposure HI

PRINCIPLES OF DIGITAL HOLOGRAPHIC INTERFEROMETRY



No intermediate state: sequential monitoring by phase substraction

X, Y, Z, Displacement Acquisition

see for instance: S. Schedin, et.al., Appl Opt, **38**, pp. 7056-7062 (1999), y Mendoza, et. al., Meas. Sci. And Tech., **10**, pp. 1305-1308 (1999).



$$k_1 = P - F_1$$
$$k_0 = C - P$$

P, punto en el Objeto

F₁, posición de la fuente

C, Posición de la CCD

Note: Object has to be illuminated from three different positions

Sensitivity vector



Phase difference due to surface displacement

 $\Delta \phi = \frac{2\pi}{\lambda} \, \mathbf{S} \cdot \, \mathbf{\vec{d}}$

Sensitivity vector as a function of the unity illumination and observation vectors



2D measurements (an in plane and an out of plane component) 2D evaluation requires two independent sensitivity vectors Out of pane sensitivity $\mathbf{S} + \mathbf{S}_2 = \mathbf{S}_2$ Х Ŝ Ŝ, Ŝ From equation (7) $\phi_z = \phi_2 + \phi_1 = \frac{2\pi}{\lambda} \vec{d} \cdot \vec{S}_z$ (9) α In plane displacements Ô In plane sensitivity Ŝ, Out of plane (10)displacements Sensitivity vectors From eq. (7) $\begin{pmatrix} d_x \\ d_z \end{pmatrix} = \frac{\lambda}{2\pi} \begin{pmatrix} -s \, \epsilon n \alpha & 1 + \cos \alpha \\ s \, \epsilon n \alpha & 1 + \cos \alpha \end{pmatrix}^{-1} \begin{pmatrix} \Delta \phi_1 \\ \Delta \phi_2 \end{pmatrix}$ $\vec{S} = (-\sin \alpha, 1 + \cos \alpha)$ (11) $\vec{S}_2 = (\sin \alpha, 1 + \cos \alpha)$

3D Sensitivity

3D evaluation requires three sensitivity vectors



 $= \frac{\lambda}{2\pi} \begin{pmatrix} -\sin\alpha & 0 & 1+\cos\alpha \\ 0 & -\sin\alpha & 1+\cos\alpha \\ \sin\alpha & 0 & 1+\cos\alpha \end{pmatrix}^{-1} \begin{pmatrix} \Delta\phi_1 \\ \Delta\phi_2 \\ \Delta\phi_3 \end{pmatrix} (12)$

Deformation components are evaluated at each point of the object's surface ν

The displacement information must now be drawn on the *Object Contour (shape)*, which may be found using any of several different/complementary (optical and nonoptical) techniques, viz.,

Pedrini, et. al., App Opt, **38**, pp. 3460-3467 (1999) Rodriguez, et. al. JOSA, **A 9**, pp. 2000-2008 (1992)



ESPI

- A) Underwater Sonar at 3KHz
- B) Traveling wave
- C) In-plane harmonic oscillation at 33 and 37 KHz



Journal of Sound and Vibration, 172 (4), pp. 433-448 (1994).





DHI

- A) 2D/3D set up, Dedicated software
- B) 3D component separation
- C) Tympanic membrane
- D) Tumor detection
- E) Vocal Chords
- F) Biomechanics

Experimental set up



Figure No. 1: Forma y Deformaciones en Tres-Dimensiones...FMS



File Edit View Insert Tools Window Help

Forma y Deformacion: Localizar Imagenes, Dar Coor., etc.				GRAFICAS				
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UFF: dataset 55 💿	Evaluar		Perspectiva	tangencial	tangencial	Defx	Defx	



Separation of individual displacement components











Tympanic Membrane



Characteristics of human TM

Semitransparent

Cone shaped

Endoscopic image of the TM

The depth of the cone is about 1.5 mm

Diameter is 8-10 mm

Thickness varies between 55 and 140 µm



Otoscope optics head (OH)

Parameters	Value
FOV	φ =10 mm
Magnification	0.4X
DOF	5 mm
Contrast	0.82
MTF	52.0 lp/mm

Field of View (FOV)	
10.0 mm	
8.5 mm	
6.7 mm	



CAD model of compact (OH)



Hear Res 253. 83-96 (2009)





softer....classical



The TM surface height-change may be found from the difference between the reconstructed phases which are recorded before and after the small tilt of the object illuminating beam by the follow equation:

$$\Delta \varphi = 2K \sin \frac{\Delta \theta}{2} \left[x \cos \left(\theta + \frac{\Delta \theta}{2} \right) - h(x) \sin \theta + \frac{\Delta \theta}{2} \right]$$









INHOMOGENEITIES DETECTION (TUMORS)

- Input sound power of approximately 661 mW, equivalent to a pressure of 2.3 x 105 pa.
- Laser pulse separation 14 ms, at 532 nm, 15 ns pulse width, 20 mJ/pulse, average power of 0.639 µW/cm² at the surface, and 6 m of coherence length.
- CCD with 1024 by 1280 pixels at 12 bits.
- Phantom is a semi sphere with an 8.4 cm in diameter and 4 cm height.




With inhomogeneity: Malignant tumor 10 mm diameter



With 3D data the depth of the tumor may be found









Vocal chords displacements (Pig's Larinx)



VOCAL CHORDS MOVEMENT

La amplitudes de vibración y las velocidades depende de la fonación. Así como la edad y sexo, determinan sus características fisiológicas de las cuerdas.



FRINGE PATTERN





VOCAL CHORDS MOTION

FRINGE PATTERN



WRAPPED PHASE







Research on complex organical surfaces

Thermal deformations (transistor)





Fringe pattern



Wrapped phase map





Comparison among 4 butterflies









Summary

-Interferometry is "not such an old" subject. The first ever experiment, the two slit T. Young's interferometer, is used in today's state of the art lasers (FEL).

- Interferometry allows the non contact more accurate measurements known today. Novel approaches are all the time on the way
- Interferometric systems are being used in many areas of Optics and Photonics, from optical shop testing to writing of Bragg gratings, to being incorporated in lab in a chip.
 Electronic Speckle Pattern Interferometry have been successfully used over the last 40 years.
- Digital Holographic Interferometry brought solutions to new challenges



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Thank you for your kind attention