The Abdus Salam
International Centre
for Theoretical Physics

2445-07

Advanced Workshop on Nanomechanics

9 - 13 September 2013

Nanomechanics:
a brief overview

Florian Marquardt
Erlangen (Germany)

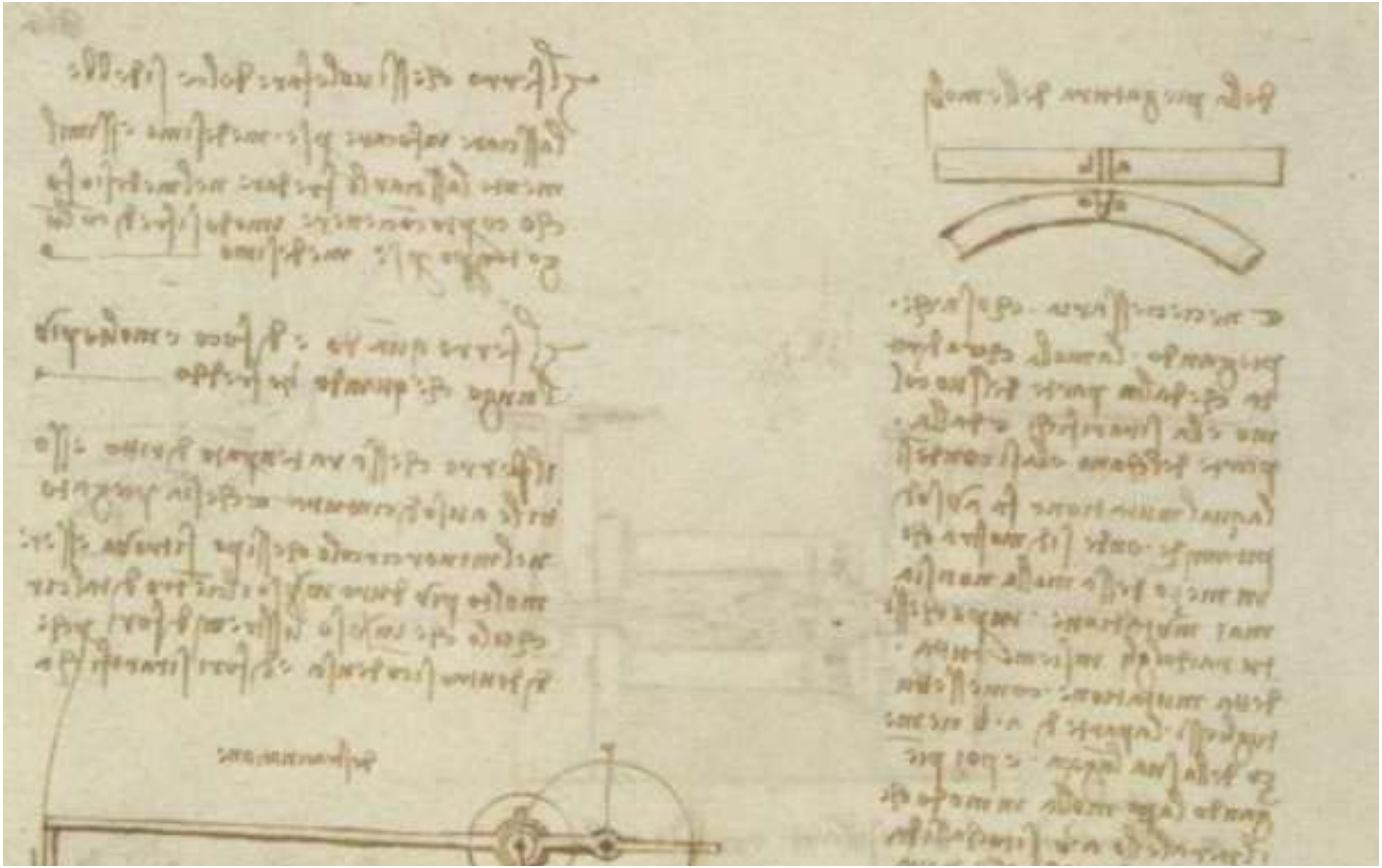
Nanomechanics: a brief overview

Frontiers of Nanomechanics / Trieste 2013

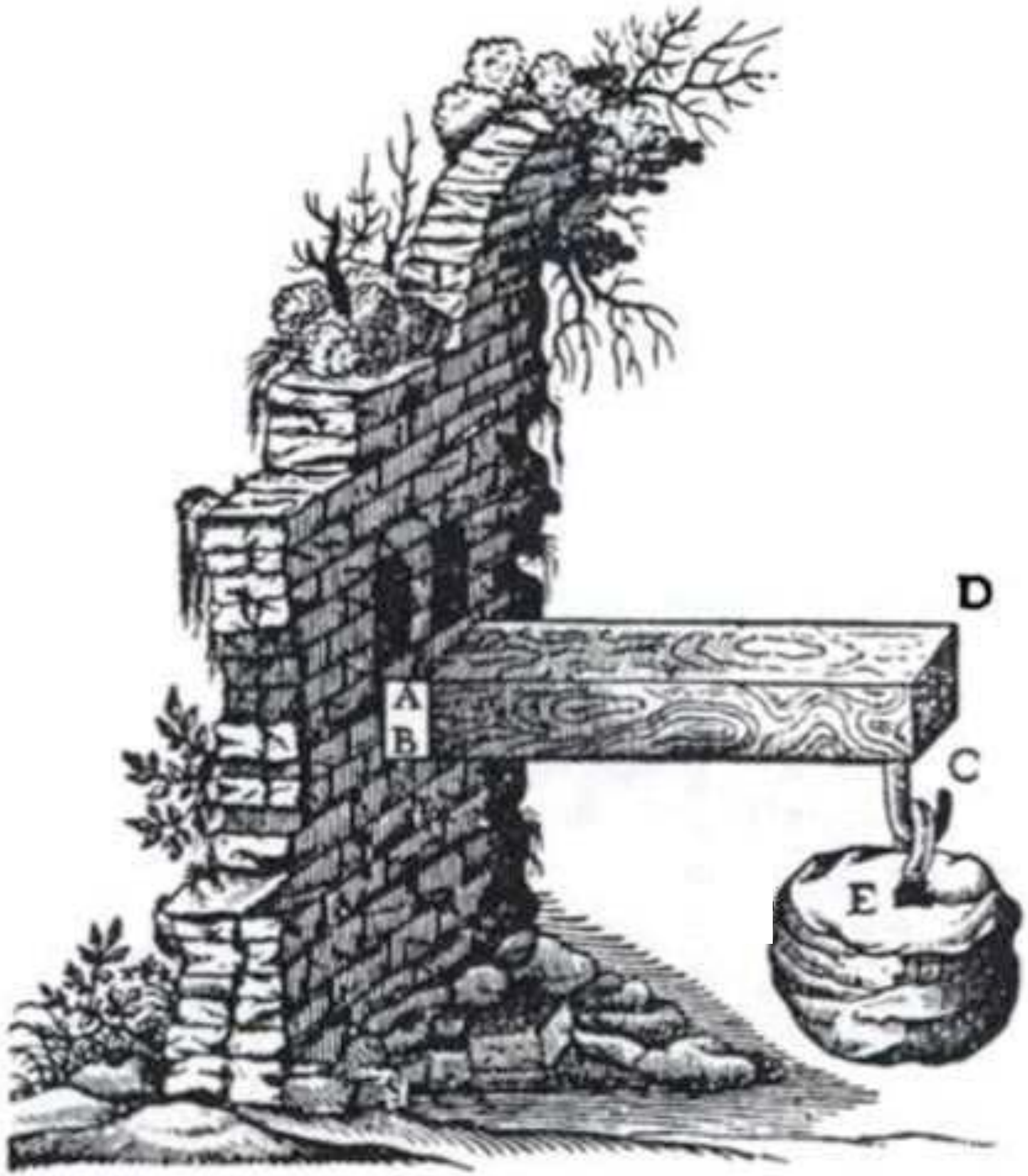
Florian Marquardt, Erlangen (Germany)

Of bending (nano-)beams

Leonardo da Vinci 1493



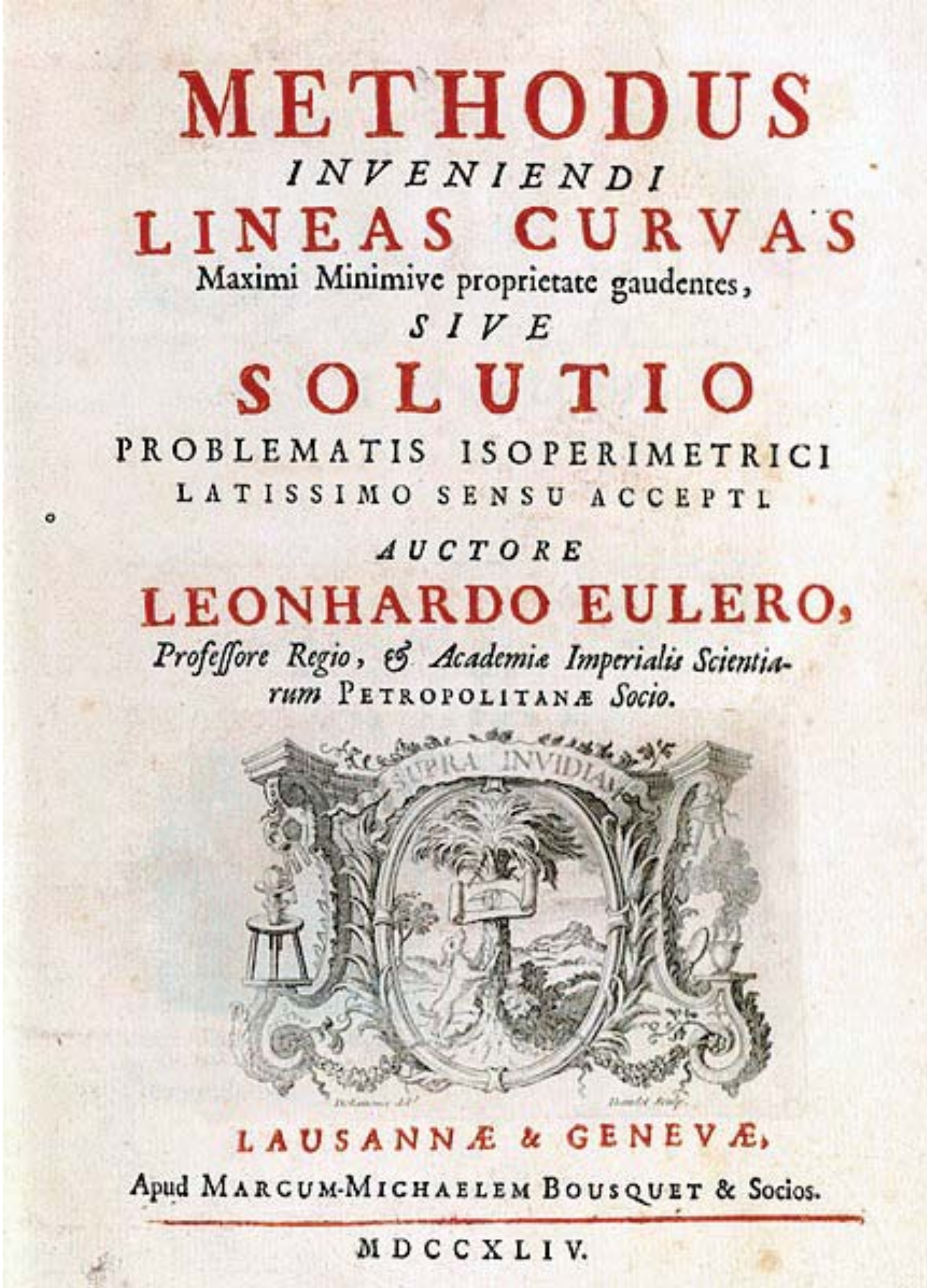
Galileo Galilei 1638



Daniel Bernoulli & Leonhard Euler 1744

Euler

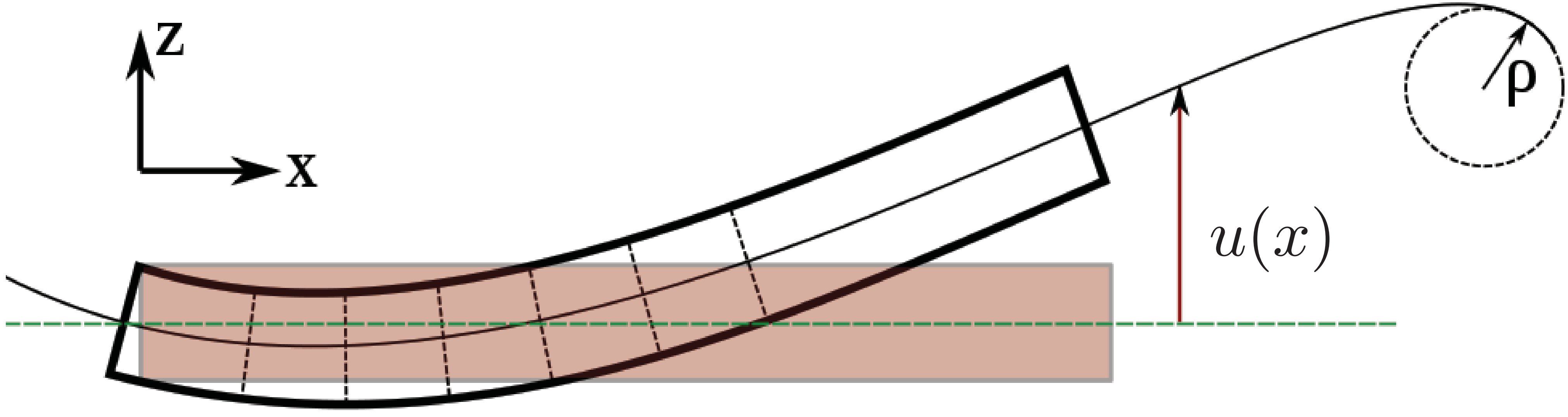
Elasticity theory & energy approach



METHODUS
INVENIENDI
LINEAS CURVAS
Maximi Minimive proprietate gaudentes,
SIVE
SOLUTIO
PROBLEMATIS ISOPERIMETRICI
LATISSIMO SENSU ACCEPTI.
AUCTORE
LEONHARDO EULERO,
Professore Regio, & Academiæ Imperialis Scientiarum PETROPOLITANÆ *Socio.*

LAUSANNÆ & GENEVÆ,
Apud MARCUM-MICHAELEM BOUSQUET & Socios.
MDCCXLIV.

Euler-Bernoulli theory for beam bending



energy: bending energy stretching energy

$$U \approx \int_0^L dx \left[\frac{EI}{2}(u''(x))^2 + \frac{F}{2}(u'(x))^2 \right]$$

E: elastic modulus

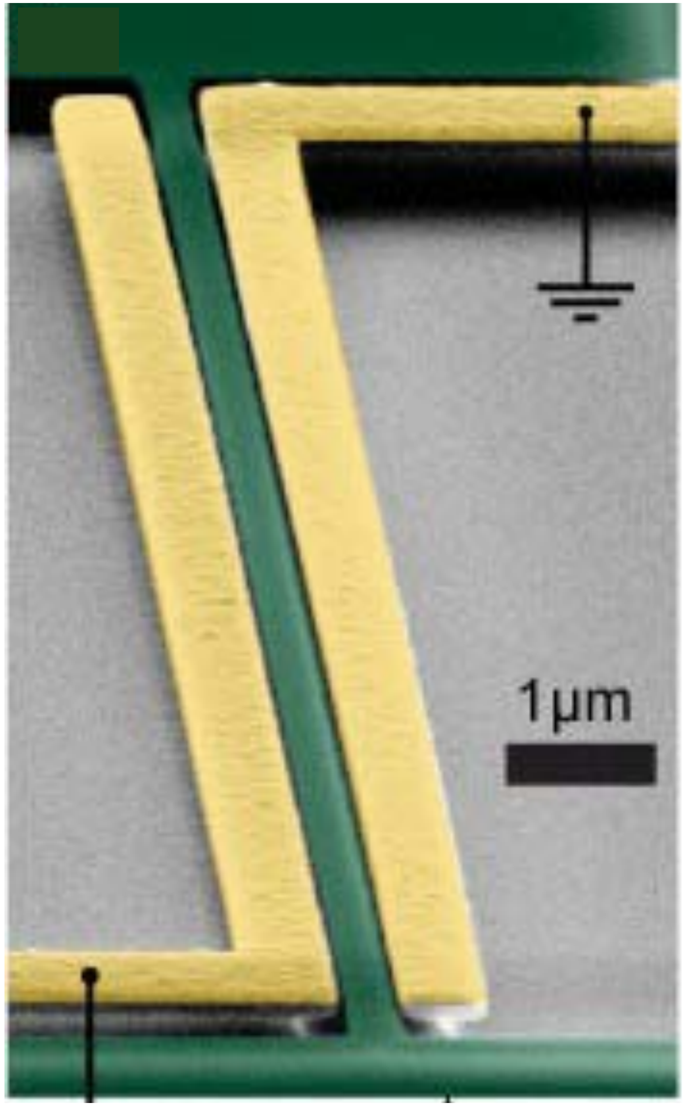
I: moment of inertia $\quad I = \int dz dy z^2$

F: applied force
(doubly clamped beam)

Wikipedia

Elasticity theory still works well on the nanometer scale!

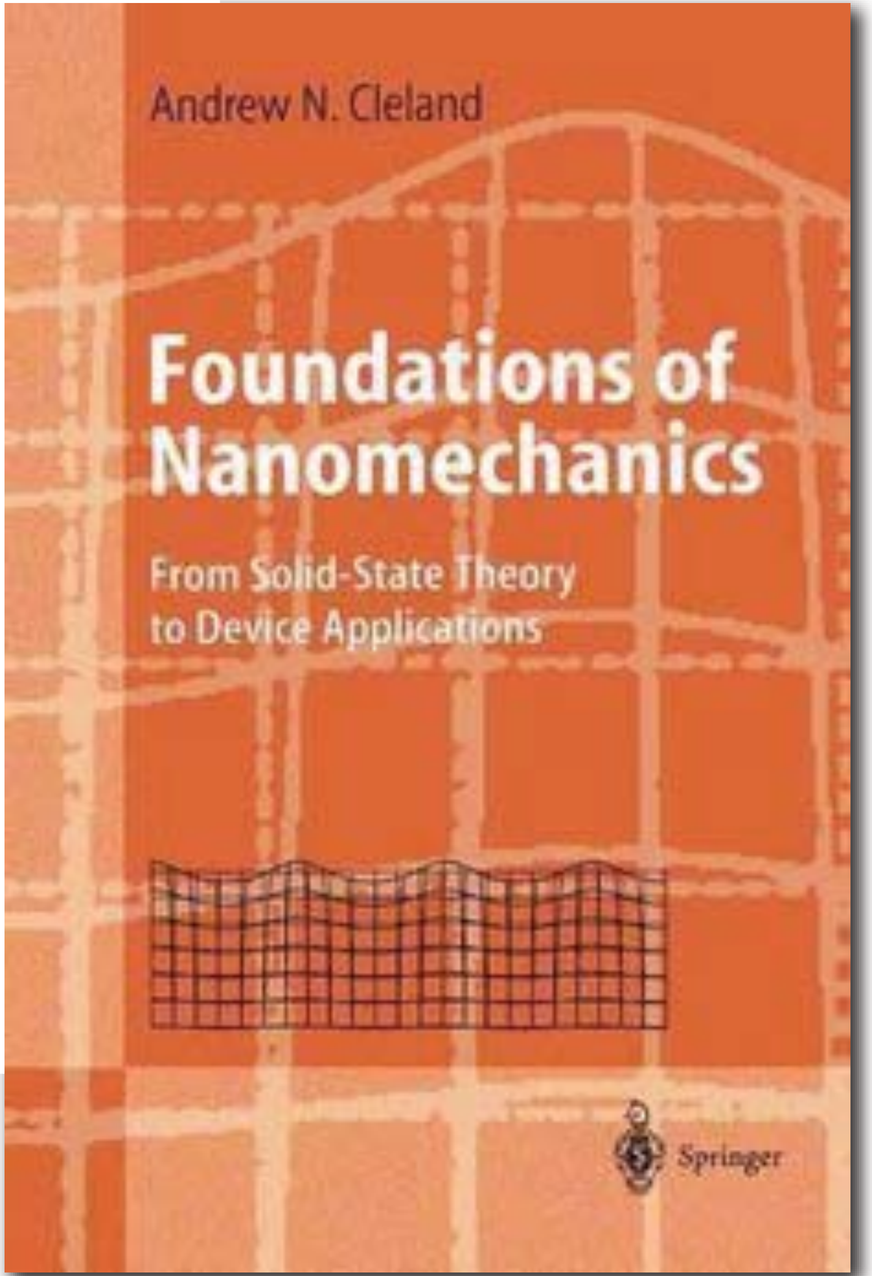
1,000,000 times smaller!



(Weig)

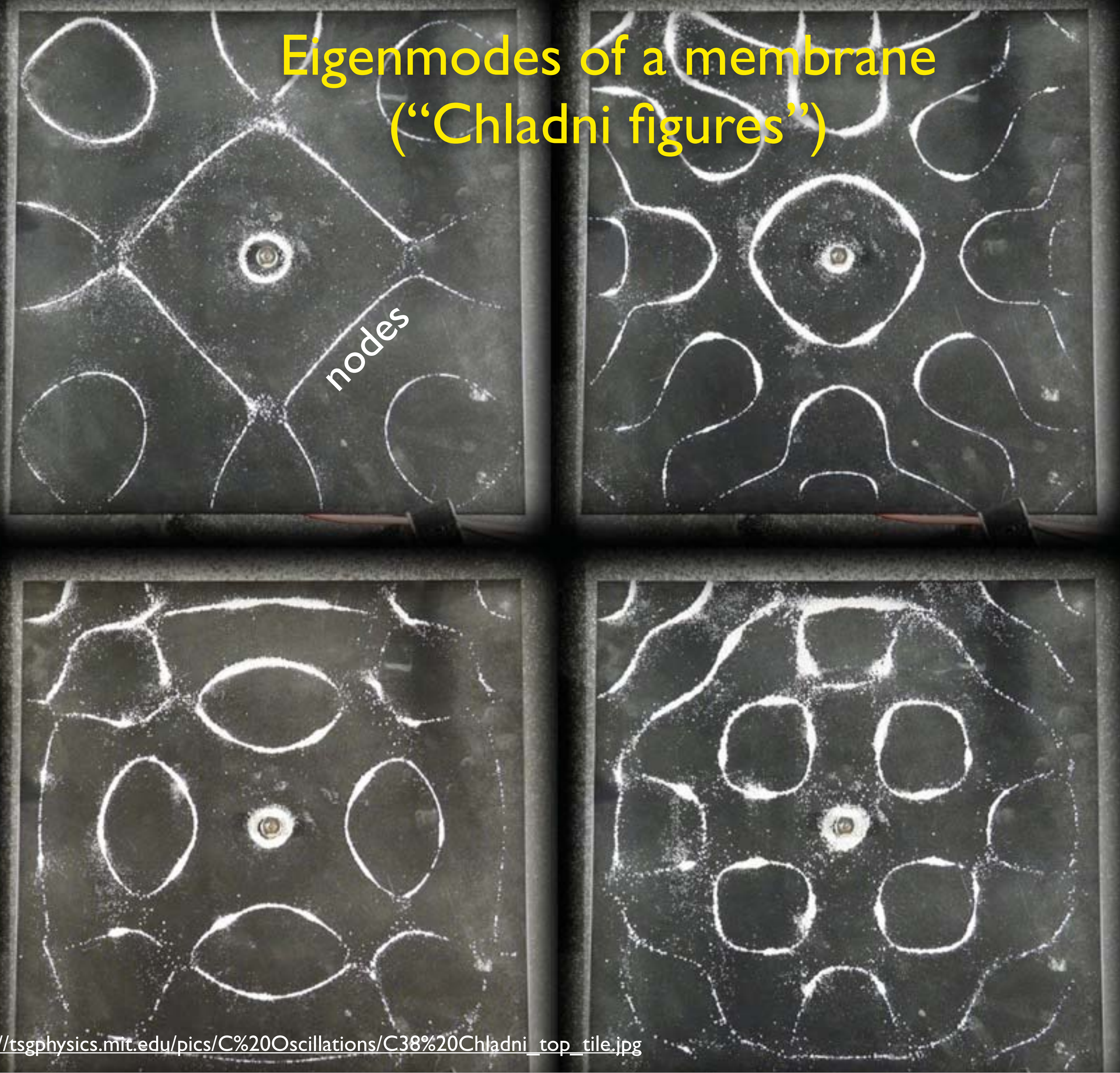
Elasticity theory still works well on the nanometer scale!

1,000,000 times
smaller!



Mechanical vibrations

Eigenmodes of a membrane
(“Chladni figures”)



http://tsgphysics.mit.edu/pics/C%20Oscillations/C38%20Chladni_top_tile.jpg

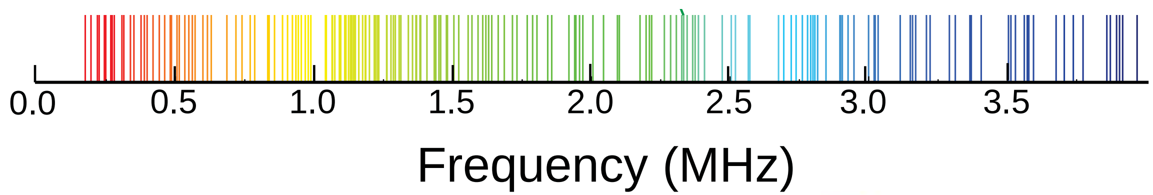
Small vibrations of any mechanical structure described by:

Eigenfrequencies Eigenmodes

$$\Omega_n\,,\,\vec{u}_n(\vec{r})$$

deflection from equilibrium

(Yamaguchi)



Linear superposition of vibrations

$$\vec{u}(\vec{r},t) = \sum_n x_n(t)\vec{u}_n(\vec{r})$$

displacement field

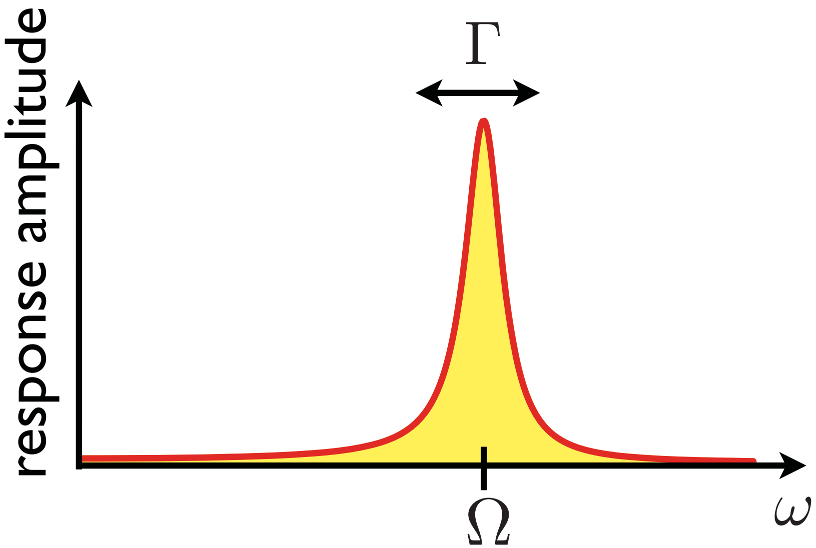
Each eigenmode is a harmonic oscillator

$$m\frac{d^2x}{dt^2} = -m\Omega^2 x - m\Gamma\frac{dx}{dt} + F(t)$$

restoring force (linear)

damping

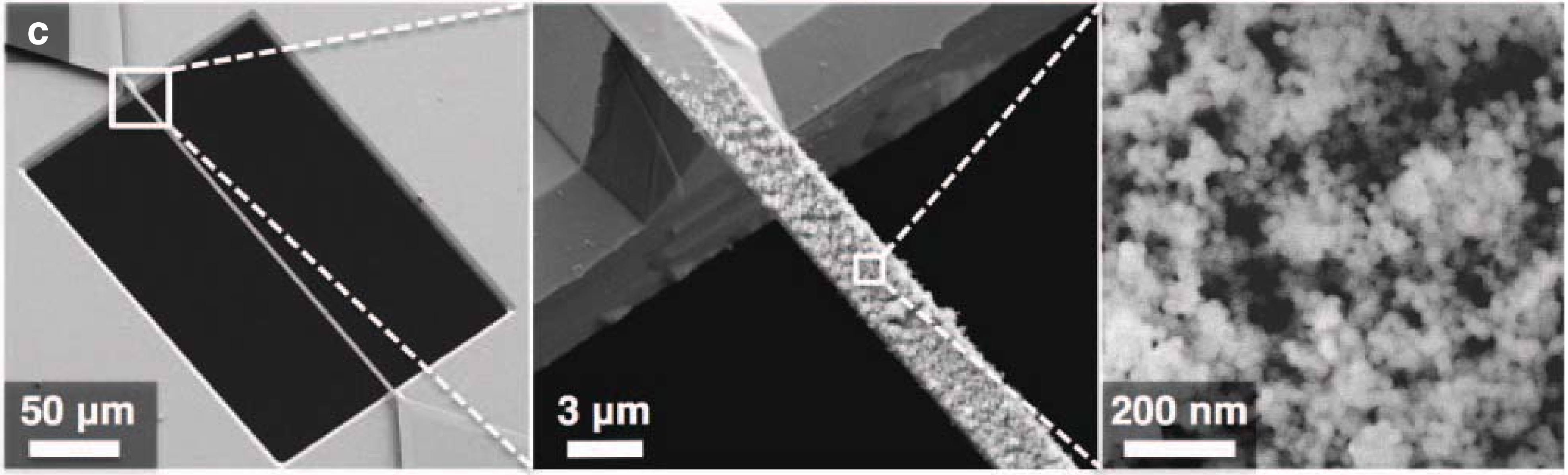
external force

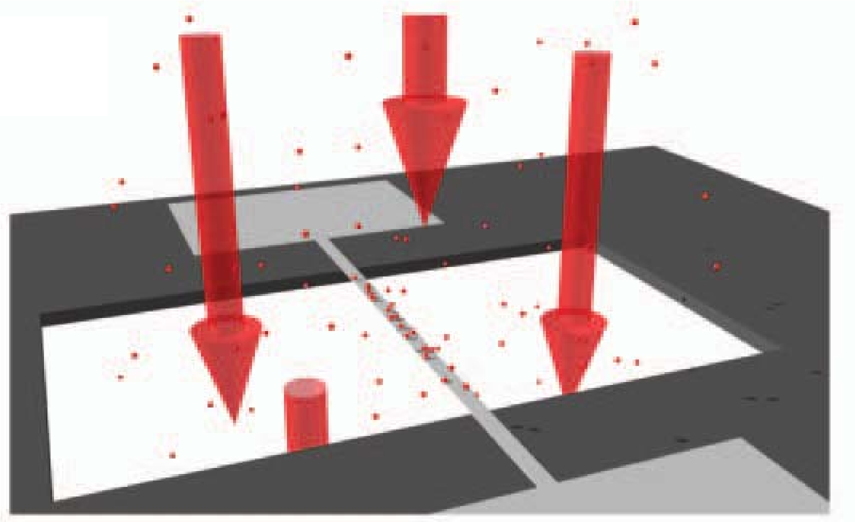


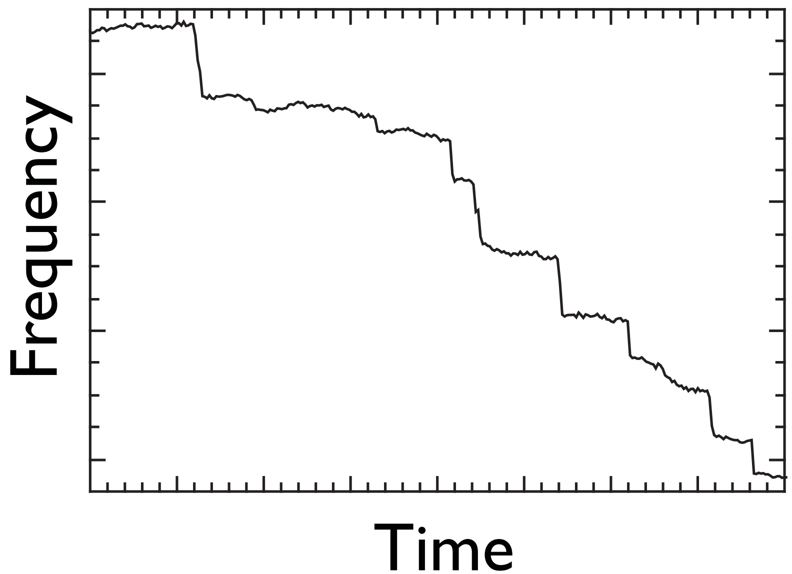
Mass sensing via a shift of the eigenfrequency

$$\Omega = \sqrt{\frac{k}{m}}$$

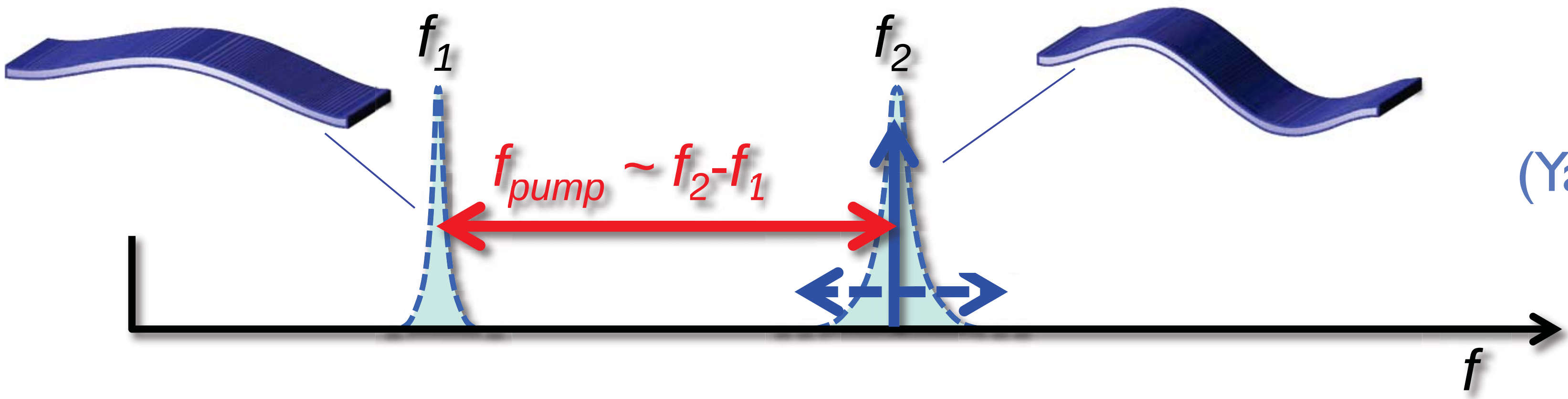
Silvan Schmid (Friday)





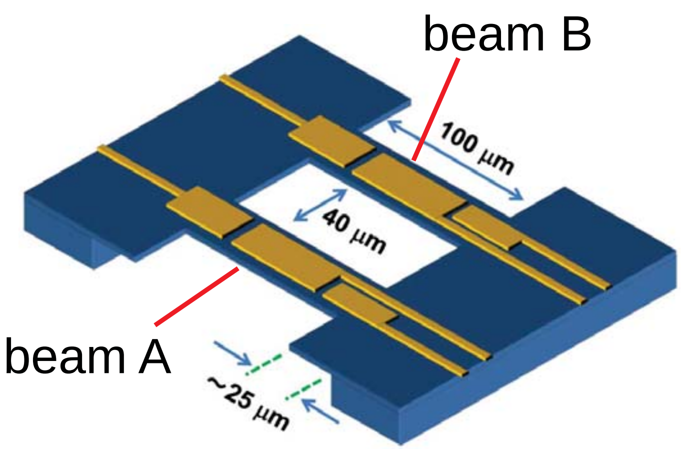


Usually focus on **one** mechanical mode
...but interesting effects for multiple coupled modes!



(Yamaguchi)

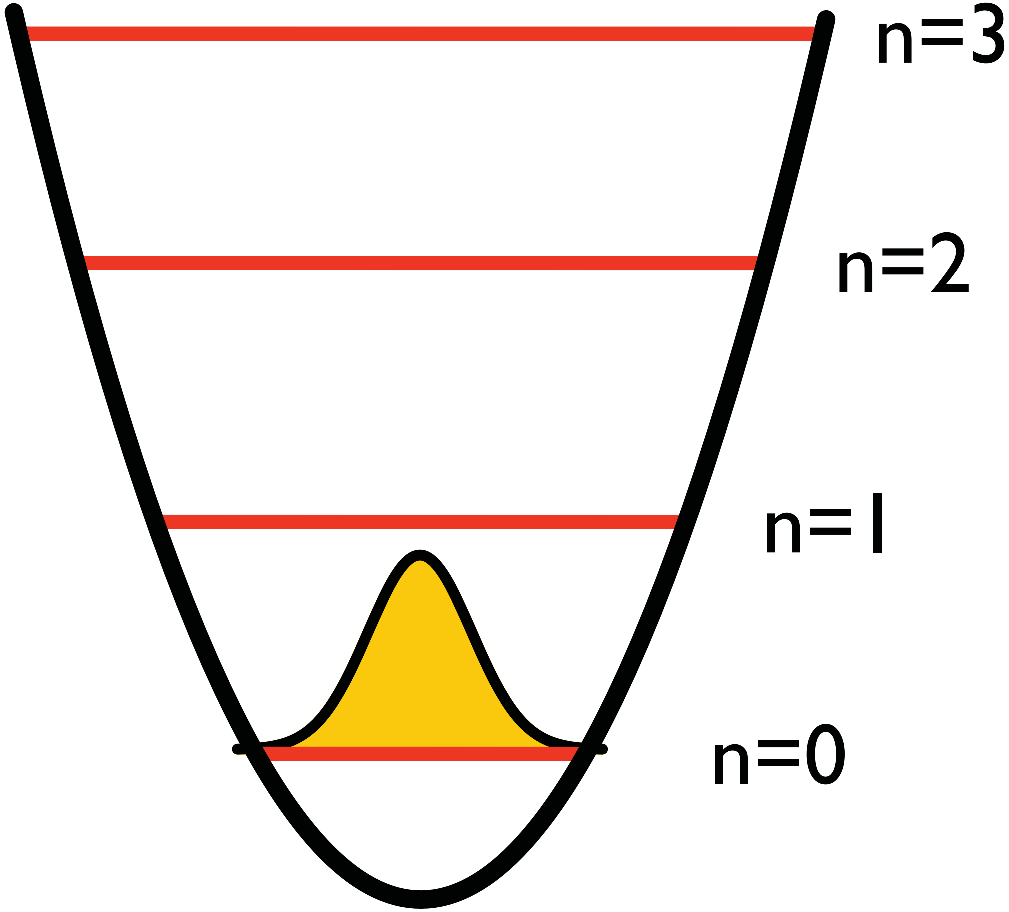
Parametric drive of coupling: $F_1 = K\cos(2\pi f_{\mathrm{pump}} t) x_2 + \ldots$



...leads to "Rabi oscillations" of mechanical energy between the two modes

Hiroshi Yamaguchi (Thursday)

Quantum-mechanical mechanical harmonic oscillator



$$\hat{H} = \frac{\hat{p}^2}{2m} + \frac{m\Omega^2}{2}\hat{x}^2$$

$$E_n = \hbar\Omega(n + \frac{1}{2})$$

phonon number

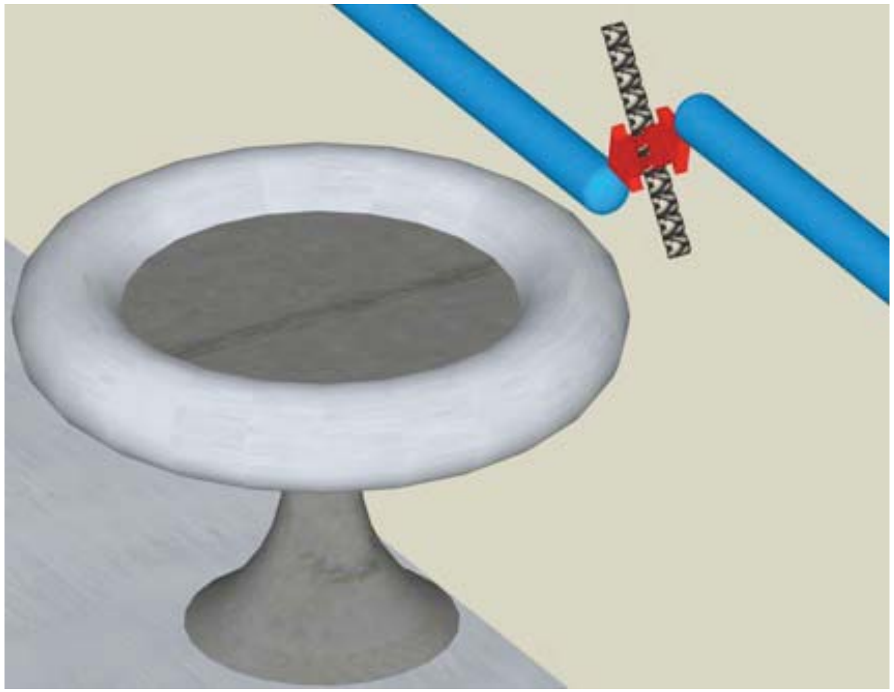
$$\hat{H} = \hbar\Omega(\hat{b}^\dagger\hat{b} + \frac{1}{2})$$

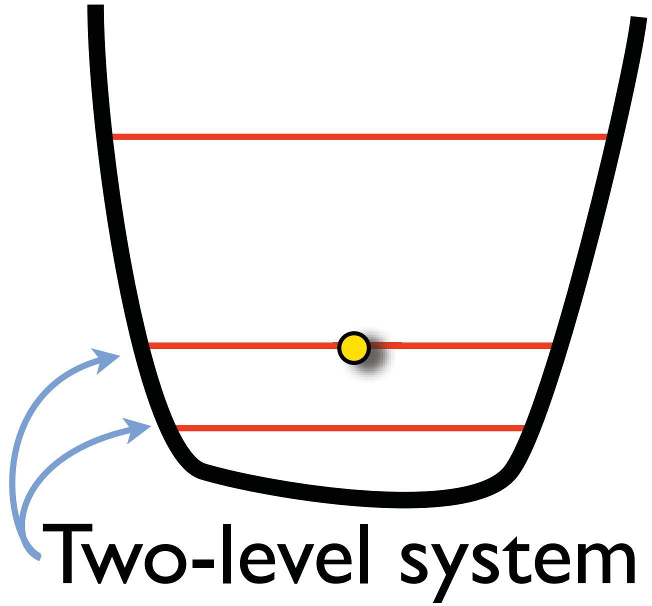
$$\hat{x} = x_{\mathrm{ZPF}}(\hat{b} + \hat{b}^\dagger)$$

mechanical zero-point fluctuations (ground state width) $x_{\mathrm{ZPF}} = \sqrt{\hbar/2m\Omega}$

Usually: mechanical modes are **harmonic** oscillators
(typically very good approximation for small vibrations, e.g. near the single-phonon level)

But: Potential use as qubits if anharmonicity (nonlinearity) can be made strong enough!





Michael Hartmann (Tuesday)

Mechanical damping

Mechanical damping

Damping rate

$$\Gamma$$

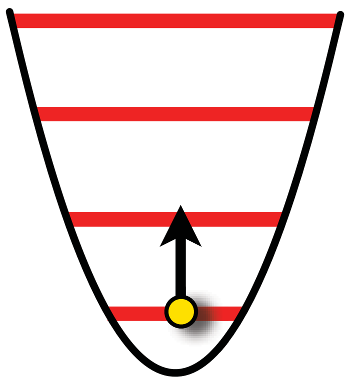
(different for each mode)

Quality factor

$$Q = \frac{\Omega}{\Gamma}$$

~ number of oscillations during damping time

e.g. 10^5



Excitation of ground state

(due to thermal fluctuations)

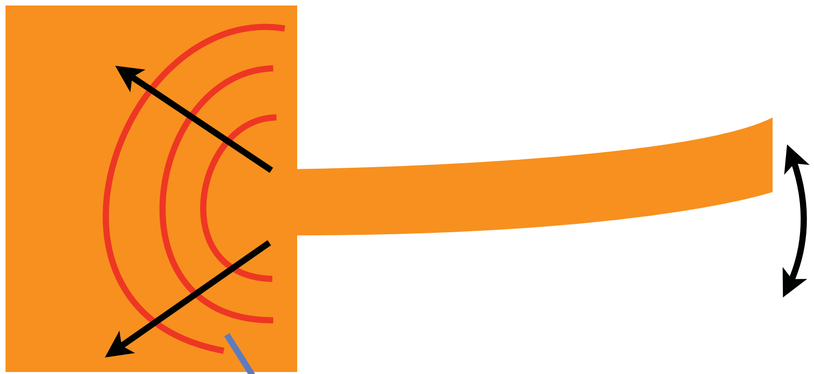
$$\Gamma_{0\to 1} = \Gamma \bar{n}_{\mathrm{th}} \approx \frac{k_B T}{\hbar Q}$$

thermal occupation

...sets limits on quantum coherence!

Common sources of mechanical damping

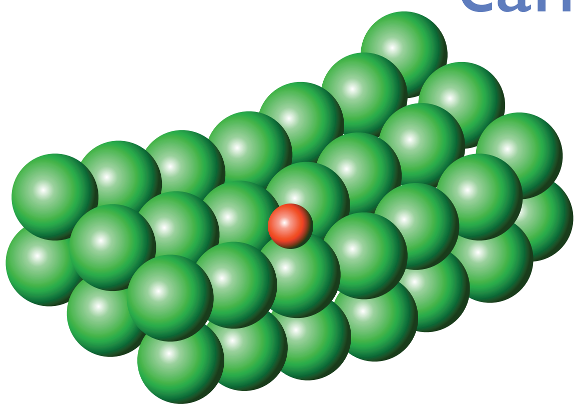
"Clamping losses": Beam attached to structure

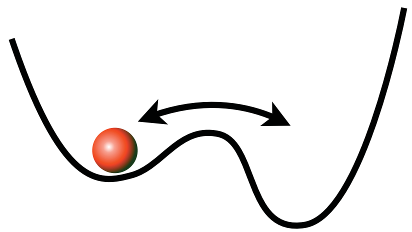


Sound waves radiated into structure

Structural losses; e.g. due to two-level fluctuators

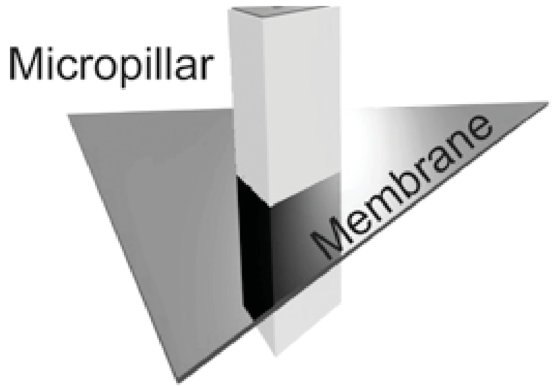
can be excited by vibrations

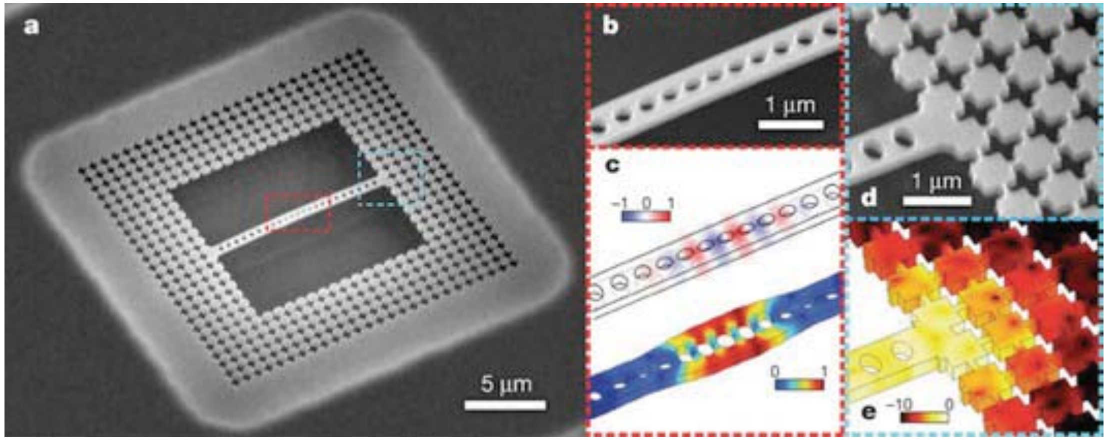




How to prevent...

"Clamping losses": Engineer mode shape or surroundings





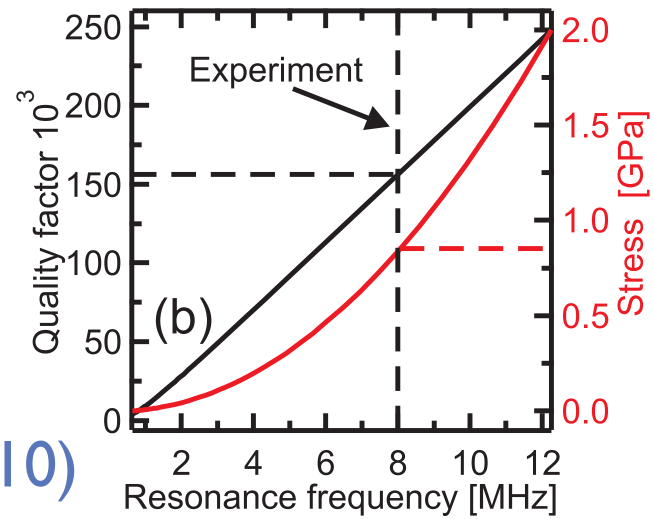
Antisymmetric mode (LKB group)

Samuel Deleglise (Thursday)

"Phonon shield" (Painter group)

Amir Safavi-Naeini (Wednesday)

Structural losses: increase tension (oscillation energy)

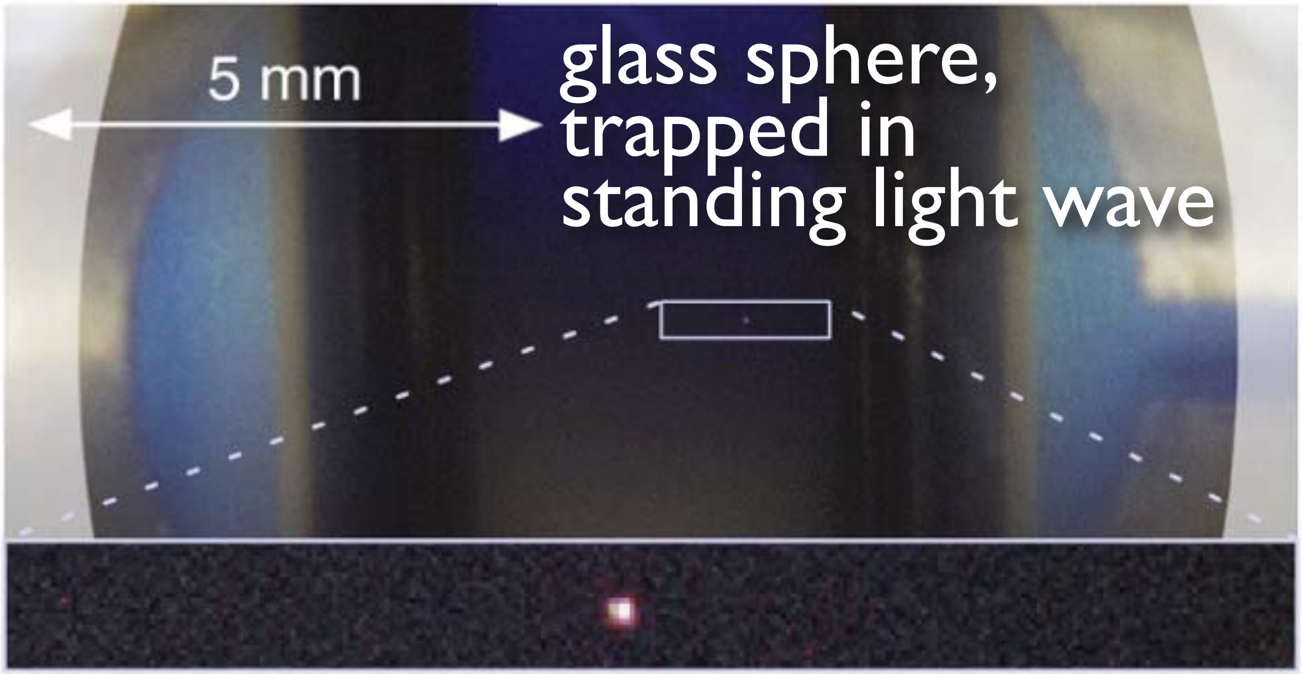


(Unterreithmeier, Faust, Kotthaus, 2010)

How to prevent...

"Clamping losses": levitate mechanical object!

Nicolai Kiesel (Thursday)

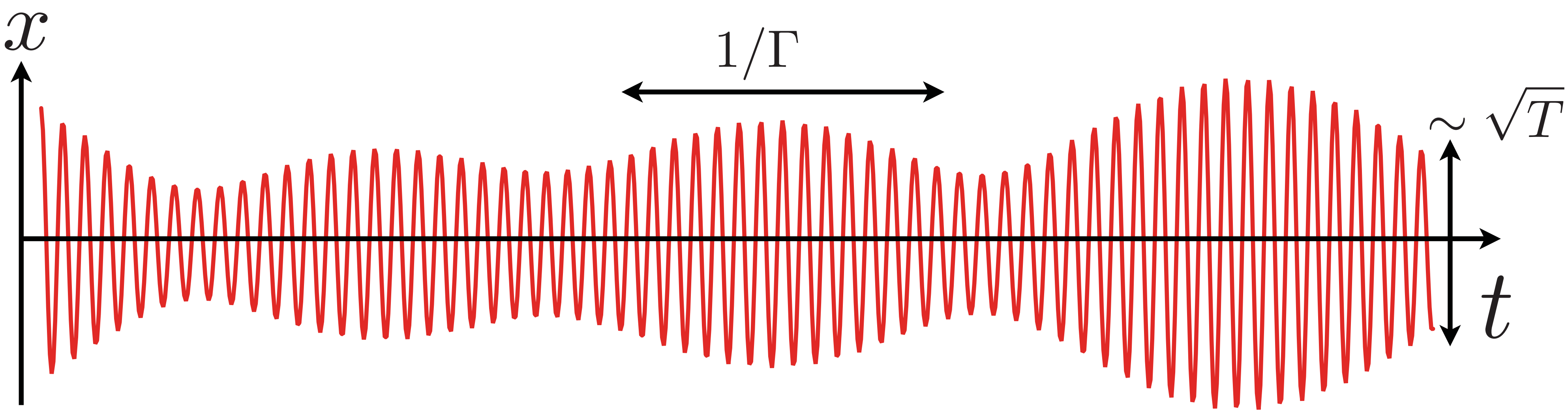


Levitate drop of
superfluid helium
(surface waves!)

Jack Harris (Tuesday)

The mechanical fluctuation spectrum

Thermal fluctuations of a harmonic oscillator



Classical equipartition theorem:

$$\frac{m\omega_M^2}{2}\left\langle x^2 \right\rangle = \frac{k_B T}{2} \quad \Rightarrow \quad \left\langle x^2 \right\rangle = \frac{k_B T}{m\omega_M^2}$$

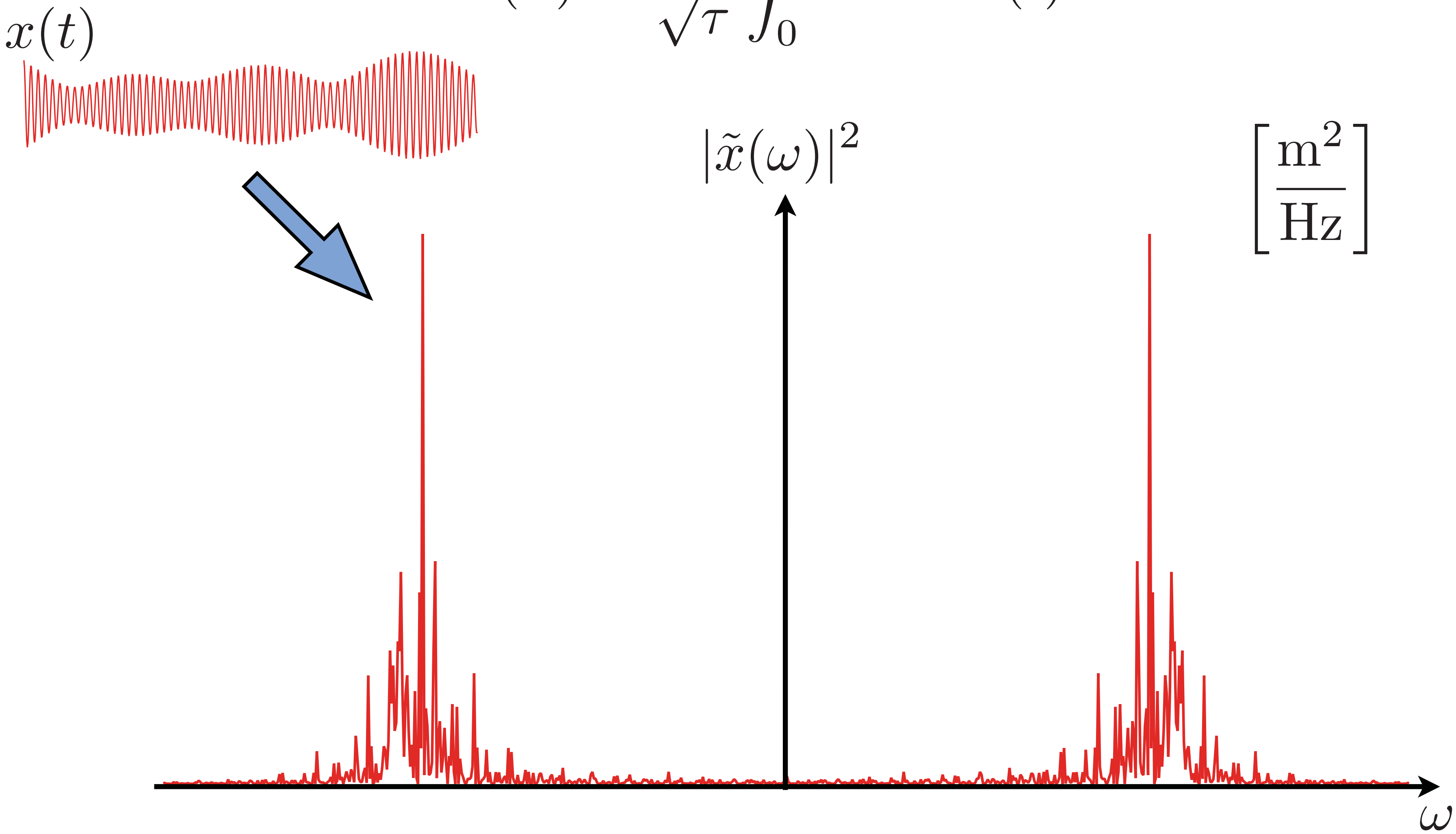
extract temperature!

Possibilities:

- Direct time-resolved detection
- Analyze **fluctuation spectrum of x**

The fluctuation spectrum

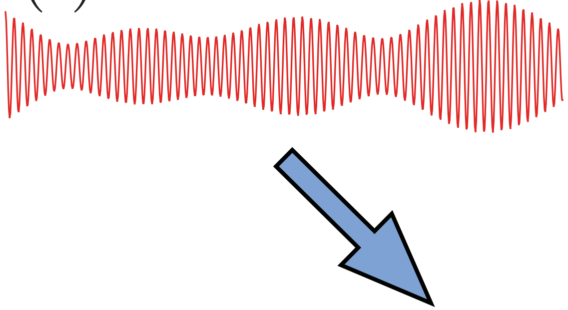
$$\tilde{x}(\omega) = \frac{1}{\sqrt{\tau}} \int_0^\tau dt e^{i\omega t} x(t)$$



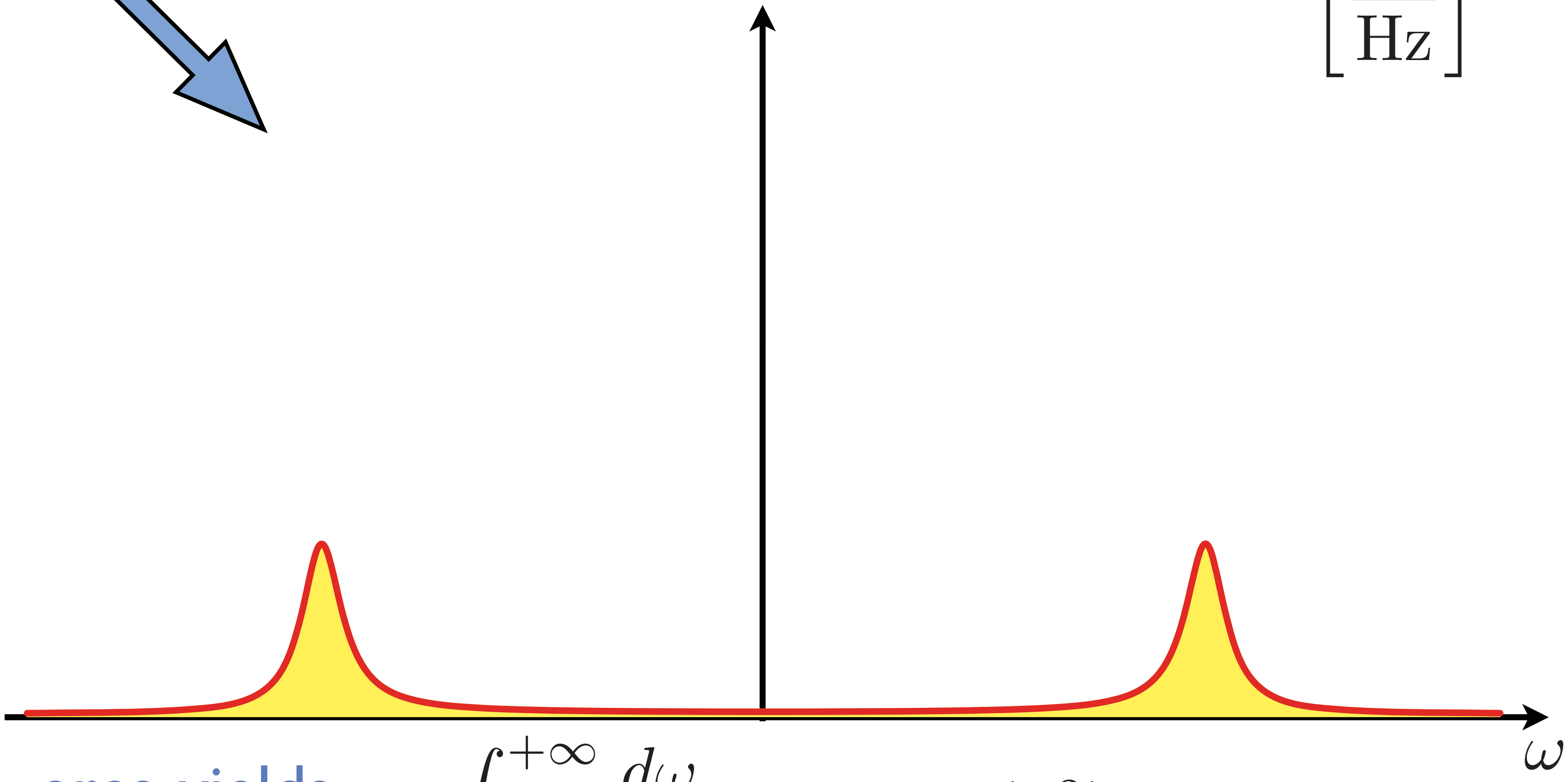
The fluctuation spectrum

$$\tilde{x}(\omega) = \frac{1}{\sqrt{\tau}} \int_0^{\tau} dt e^{i\omega t} x(t)$$

$x(t)$



$$\langle |\tilde{x}(\omega)|^2 \rangle \equiv S_{xx}(\omega) \qquad \left[\frac{\mathrm{m}^2}{\mathrm{Hz}}\right]$$



area yields variance of x: $$\int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} S_{xx}(\omega) = \langle x^2 \rangle$$

Fluctuation spectrum from the susceptibility: Fluctuation-dissipation theorem

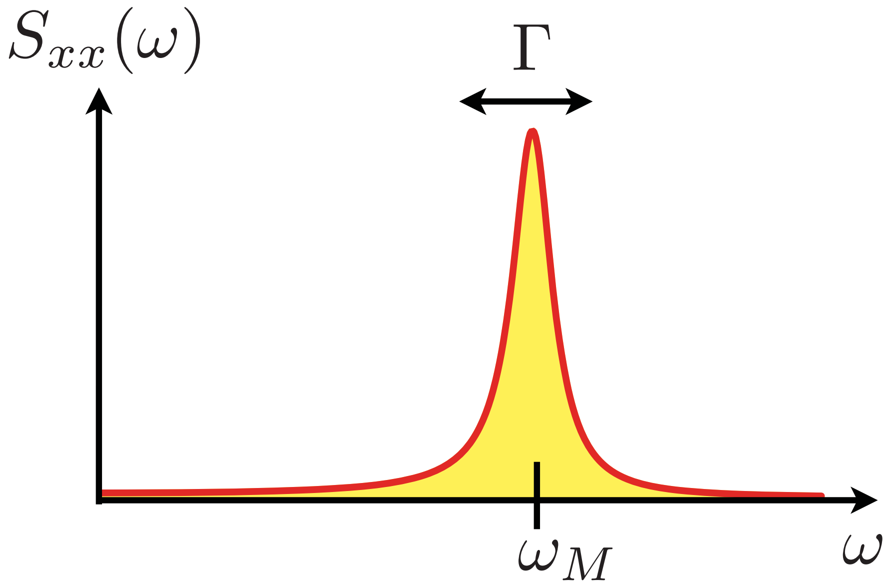
$$\underbrace{\langle \delta x \rangle (\omega)}_{\text{response}} = \underbrace{\chi_{xx}(\omega)}_{\text{susceptibility}} \underbrace{F(\omega)}_{\text{force}}$$

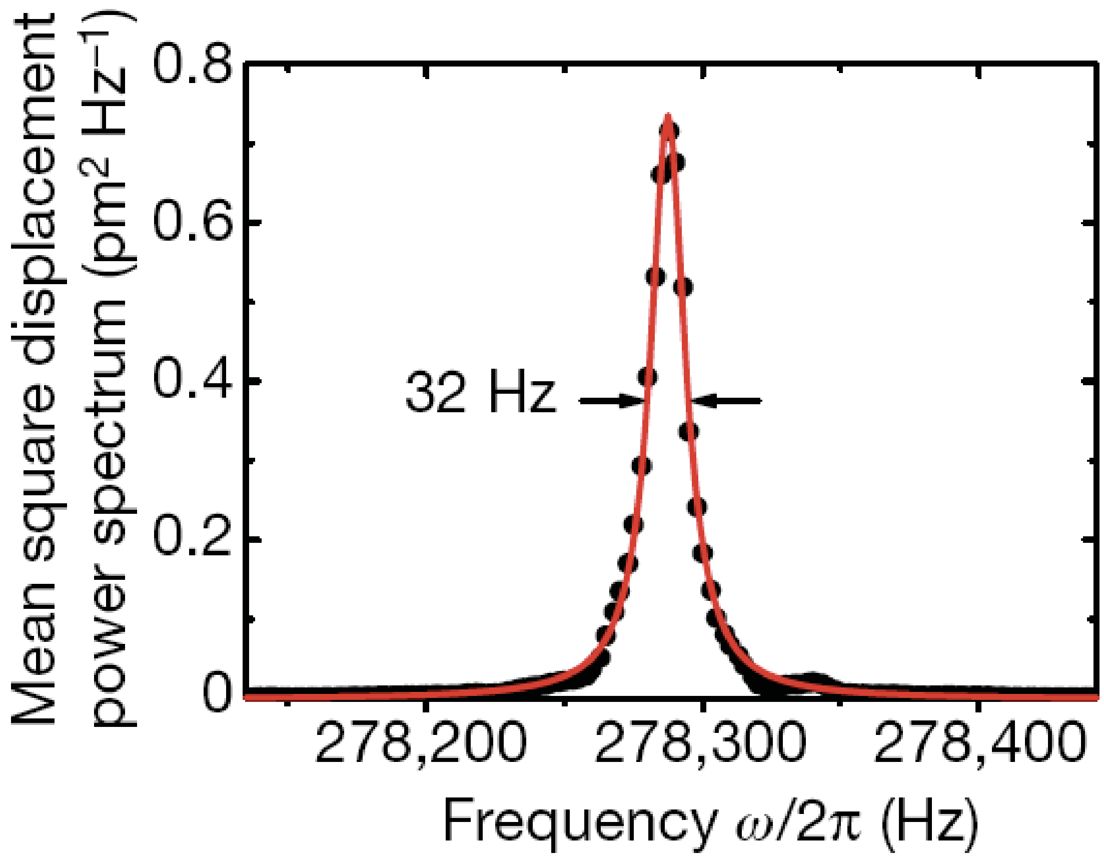
$$\boxed{S_{xx}(\omega) = \frac{2k_B T}{\omega}\mathrm{Im}\chi_{xx}(\omega)}$$ (classical limit)

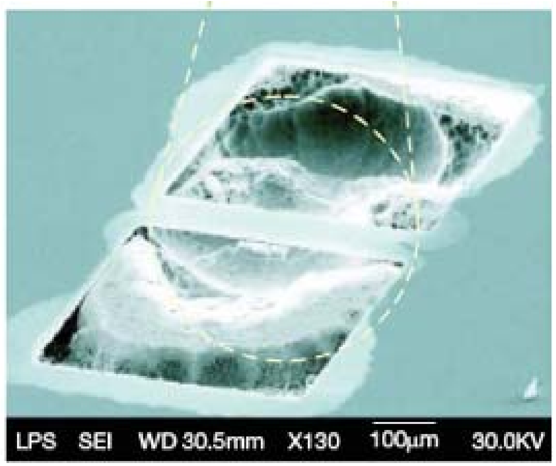
for the damped oscillator:

$$m\ddot{x} + m\omega_M^2 x + m\Gamma\dot{x} = F$$

$$x(\omega) = \underbrace{\frac{1}{m(\omega_M^2 - \omega^2) - im\Gamma\omega}}_{\chi_{xx}(\omega)} F(\omega)$$



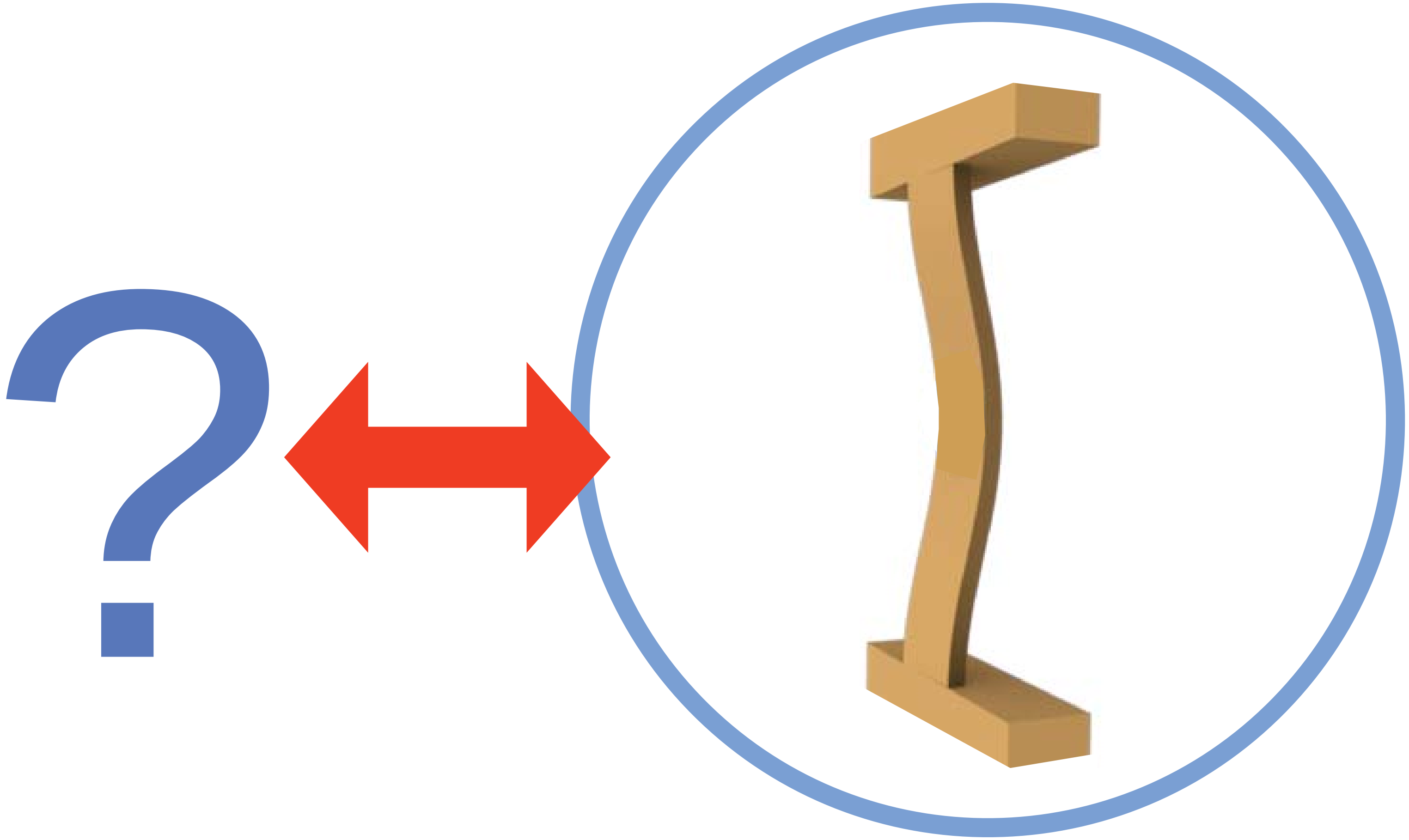


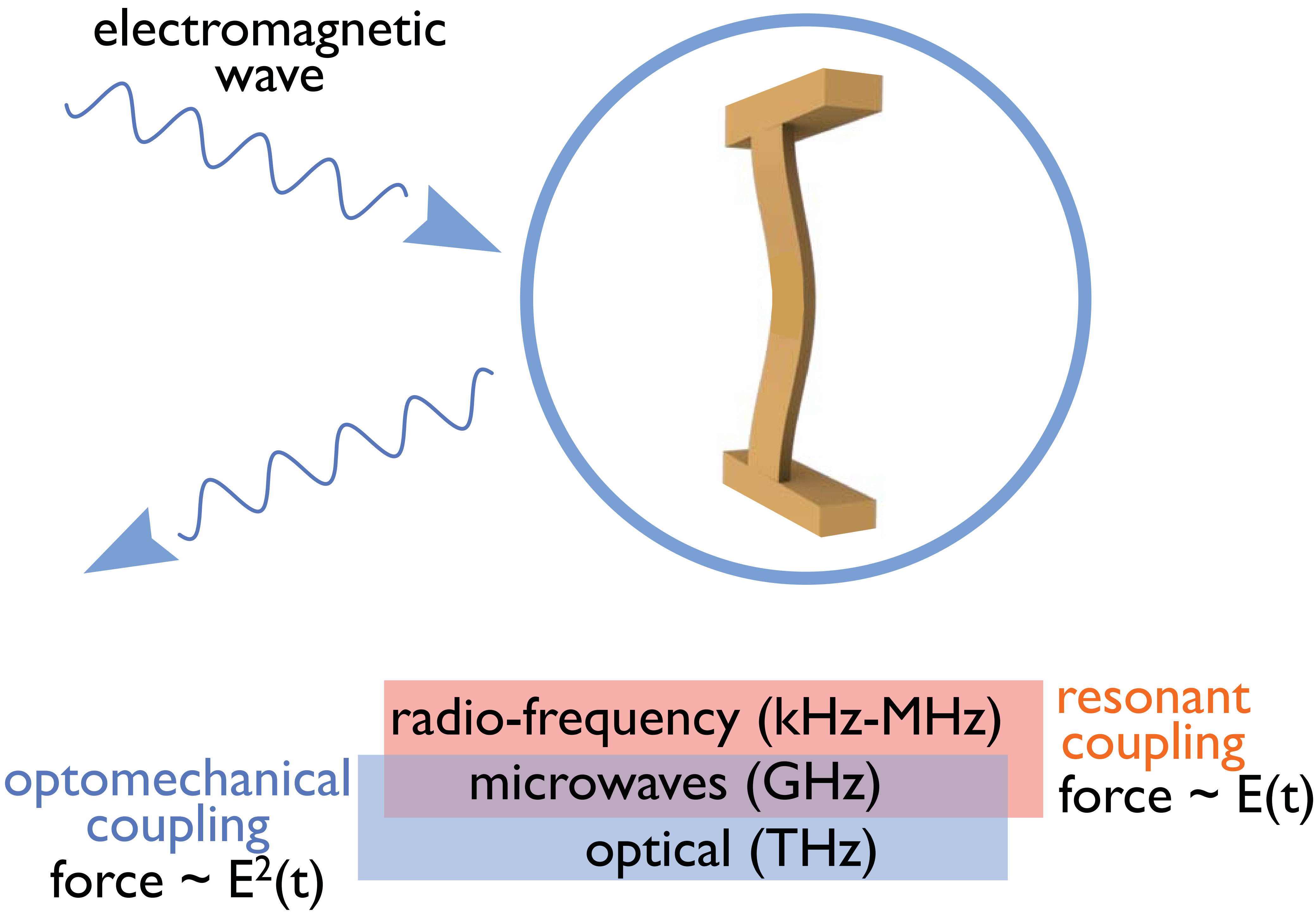


T=300 K

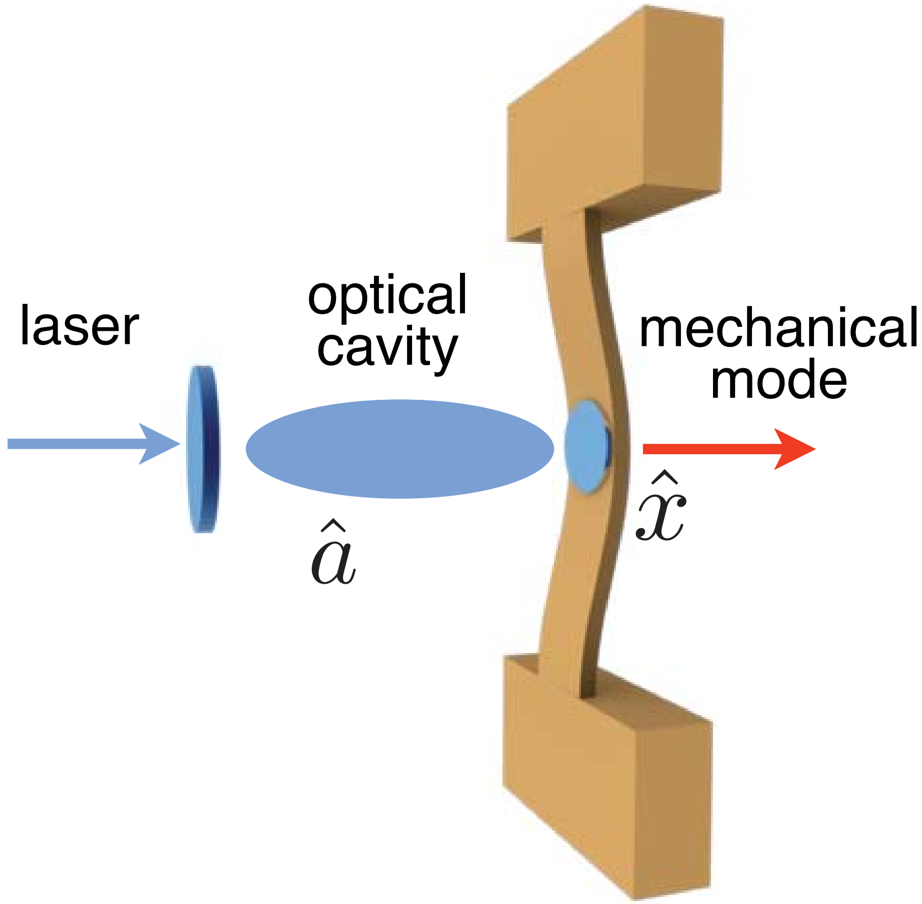
Experimental curve:
Gigan et al., Nature 2006

Coupling radiation to a mechanical resonator




electromagnetic wave
radio-frequency (kHz-MHz)
microwaves (GHz)
optical (THz)
resonant coupling
force ~ E(t)
optomechanical coupling
force ~ E^2(t)

The standard optomechanical setup



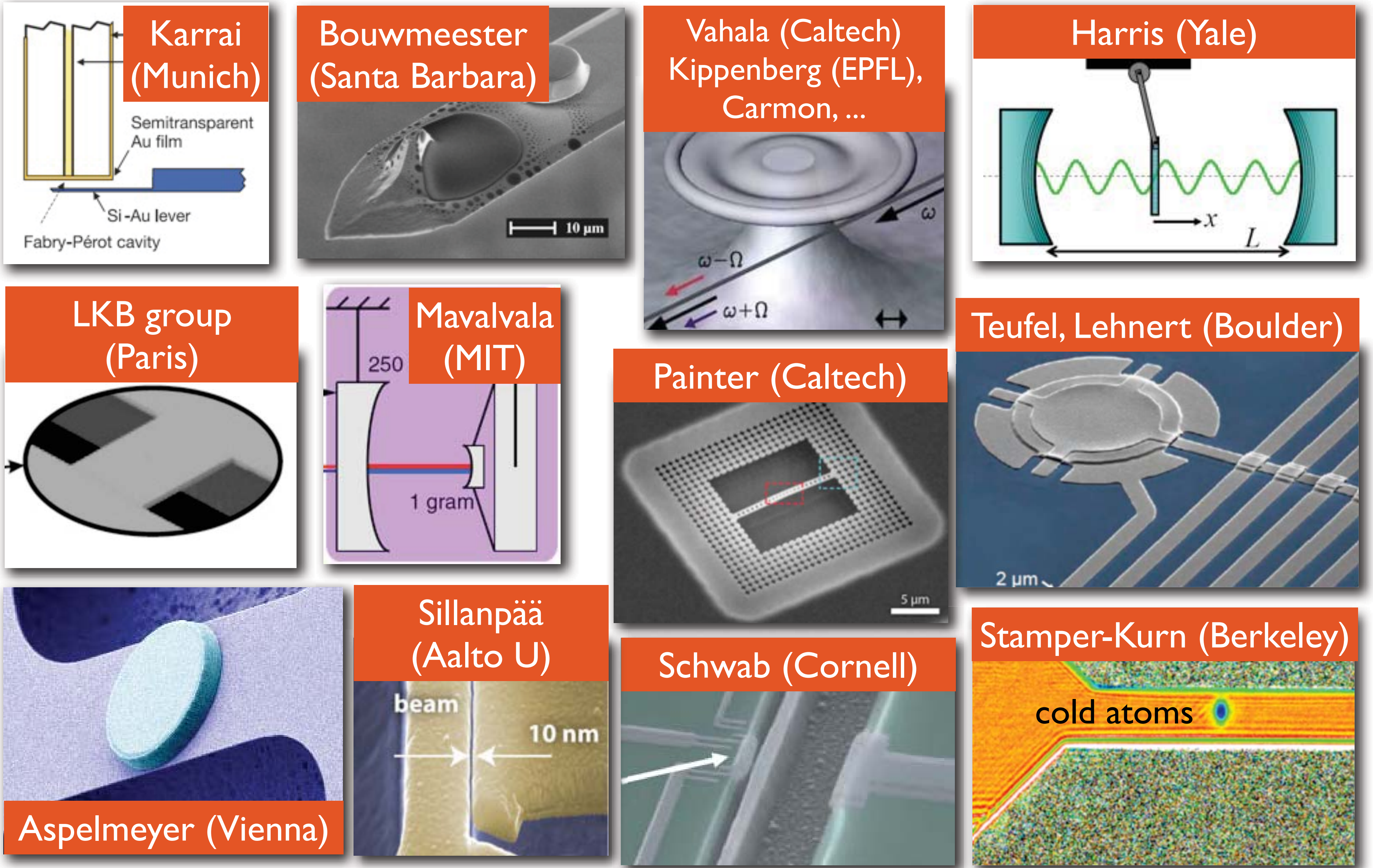
$$\hat{H} = \hbar\omega_{\mathrm{cav}} \cdot (1 - \hat{x}/L)\hat{a}^\dagger\hat{a} + \hbar\Omega\hat{b}^\dagger\hat{b} + \ldots$$

$$\hat{x} = x_{\mathrm{ZPF}}(\hat{b} + \hat{b}^\dagger)$$

$$x_{\mathrm{ZPF}} = \sqrt{\hbar/2m\Omega}$$

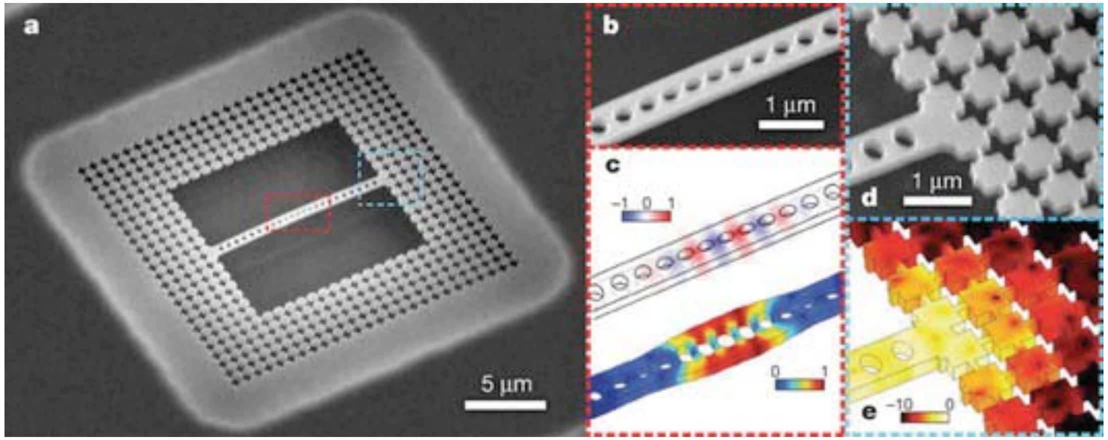
Recent Review "Cavity Optomechanics":
M. Aspelmeyer, T. Kippenberg, FM; arXiv 2013

Optomechanical experiments (selection)

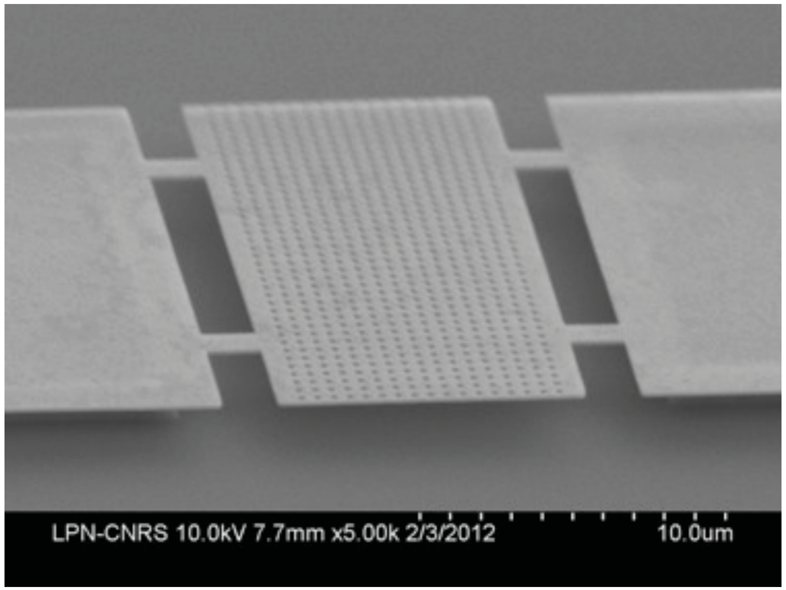


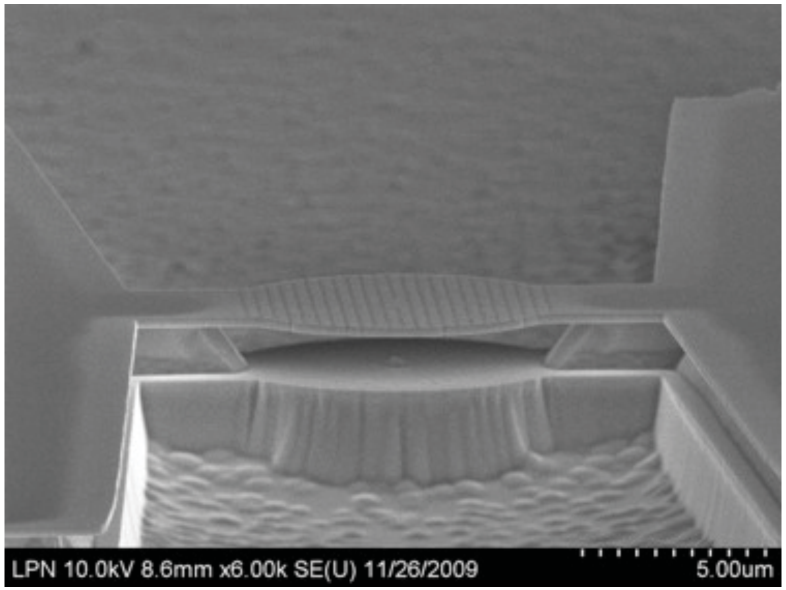
Photonic crystals: Very strong coupling between localized vibrational and optical modes

Amir Safavi-Naeini (Wednesday)

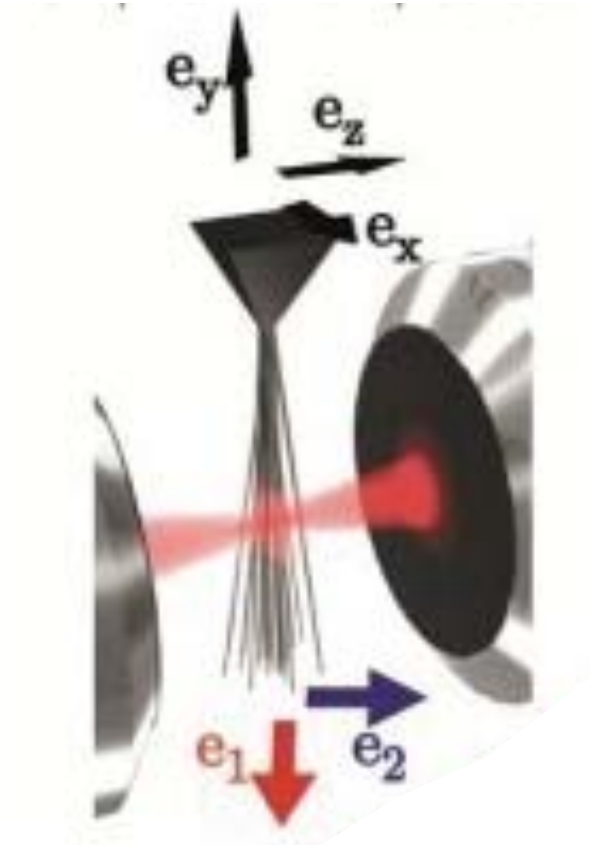


Isabelle Robert (Thursday)





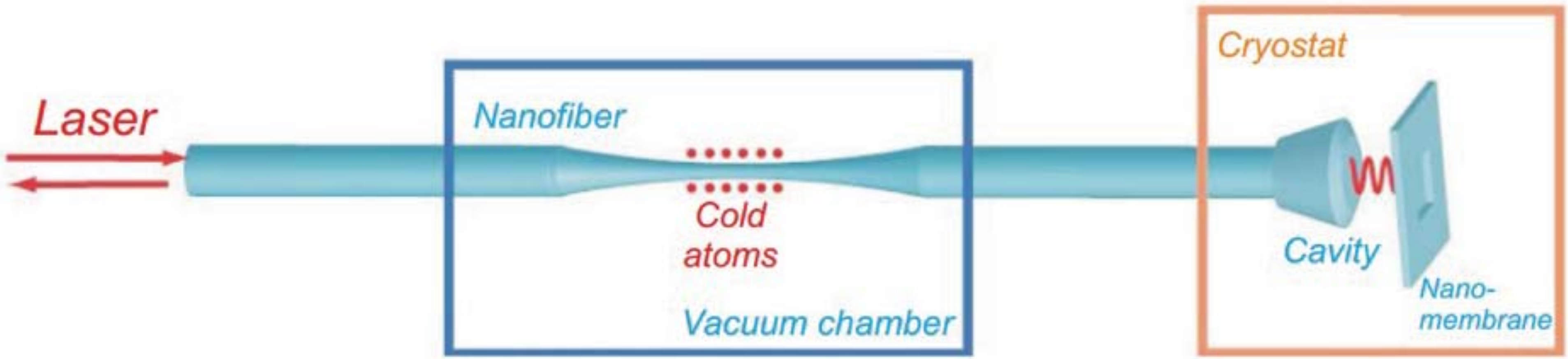
Nano-Optomechanics: Nanowire in a light field



- Ultra-sensitive nano-optomechanical detection of a bi-dimensional nanomechanical degree of freedom
- Topological structure of the radiation force in a focused laser beam

Pierre Verlot (Friday)

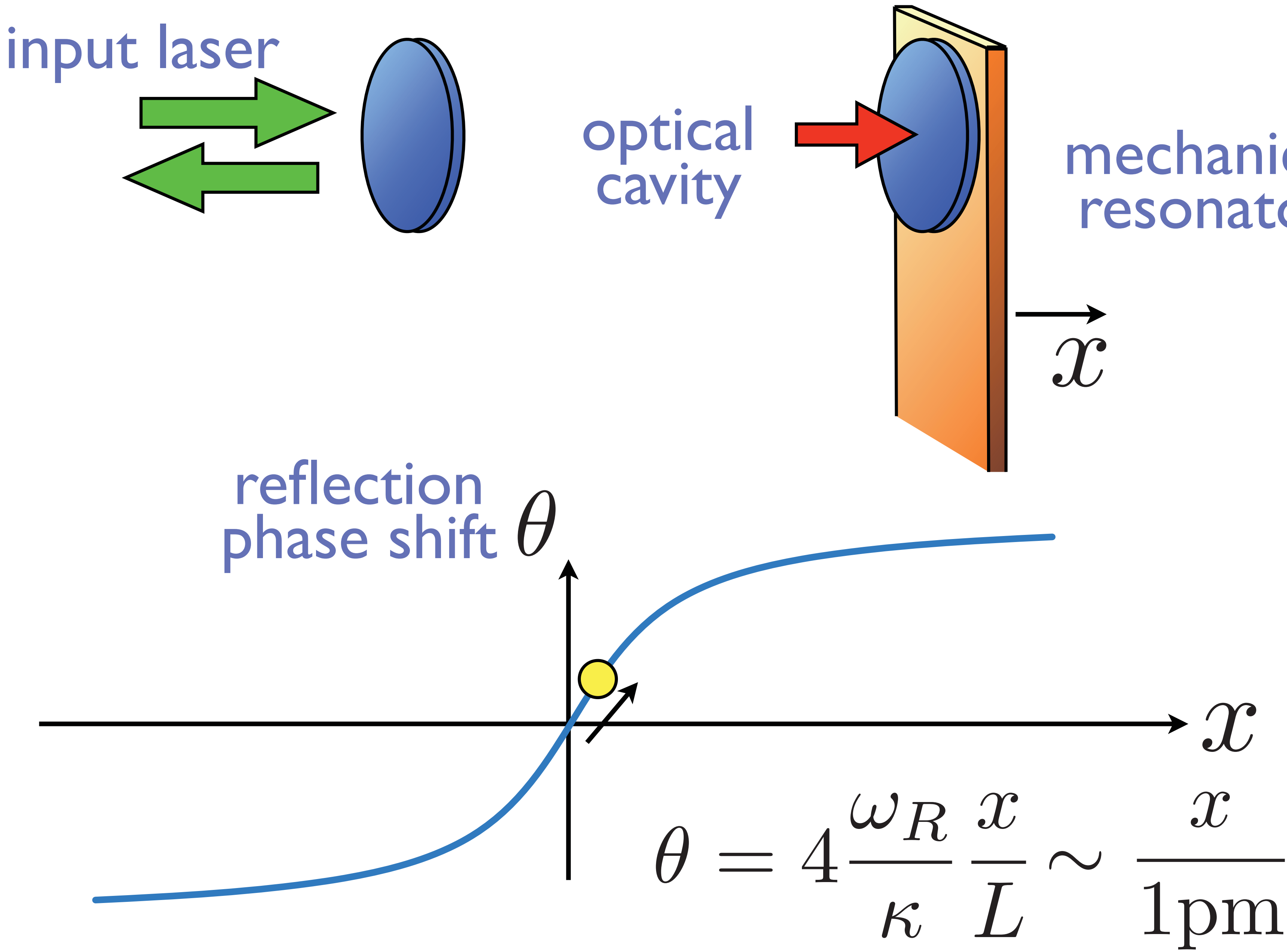
Coupling to atoms

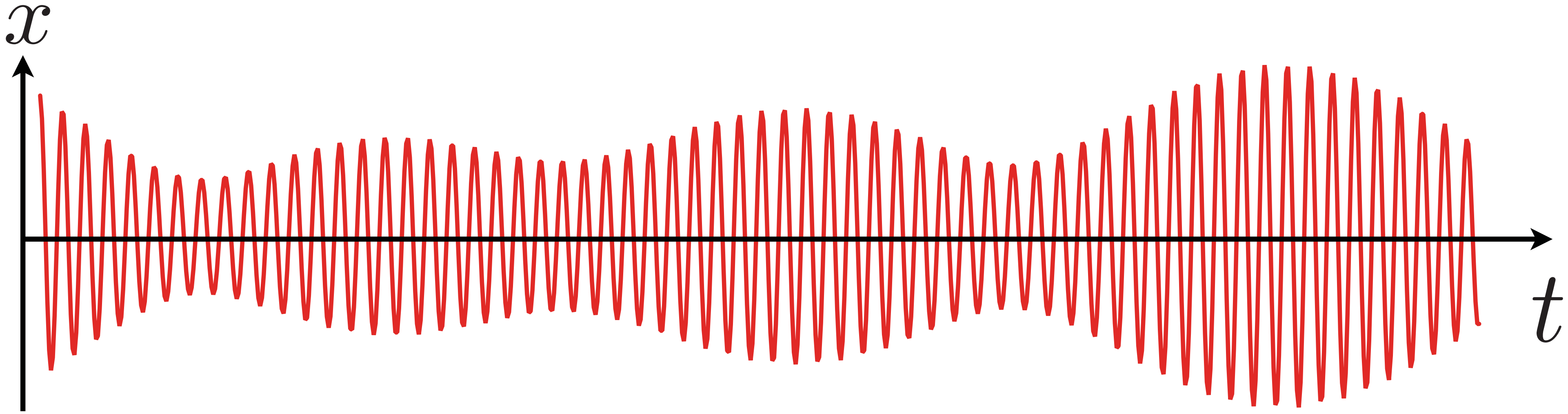


Samuel Deleglise (Thursday)

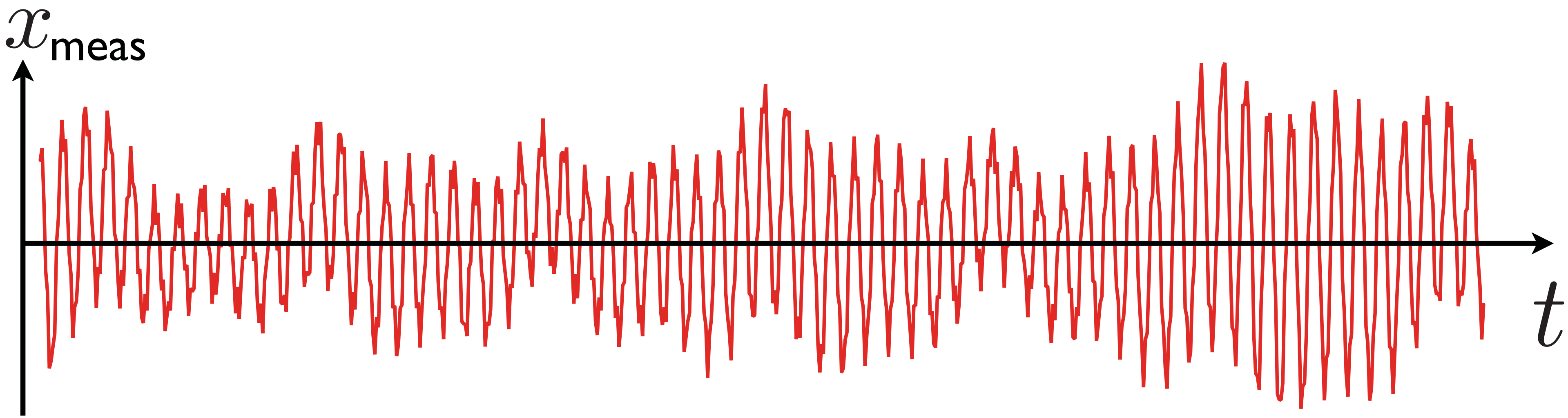
Measuring mechanical motion

Optical detection of mechanical motion





x
t



$$x_{\text{meas}}(t) = x(t) + x_{\text{noise}}(t)$$

Two contributions to $x_{\text{noise}}(t)$

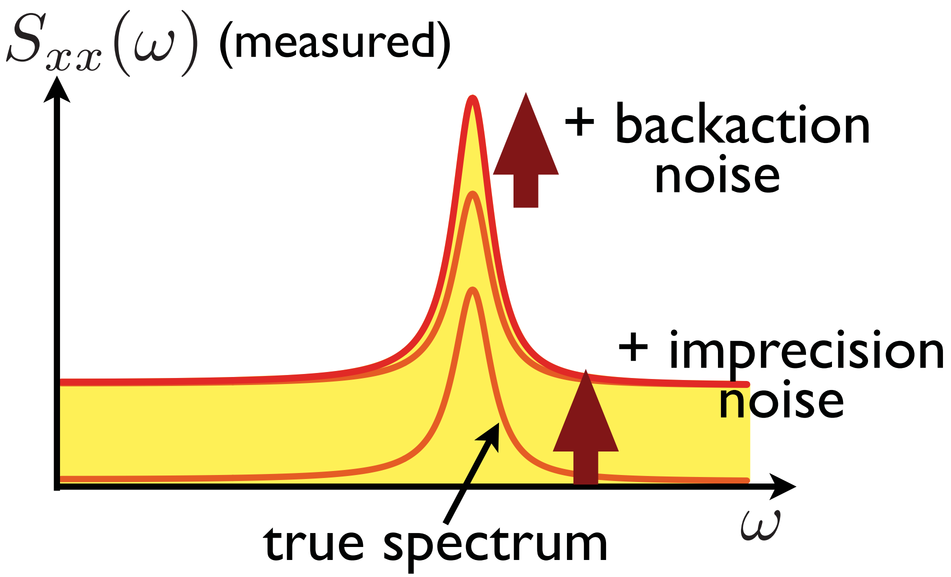
1. measurement imprecision

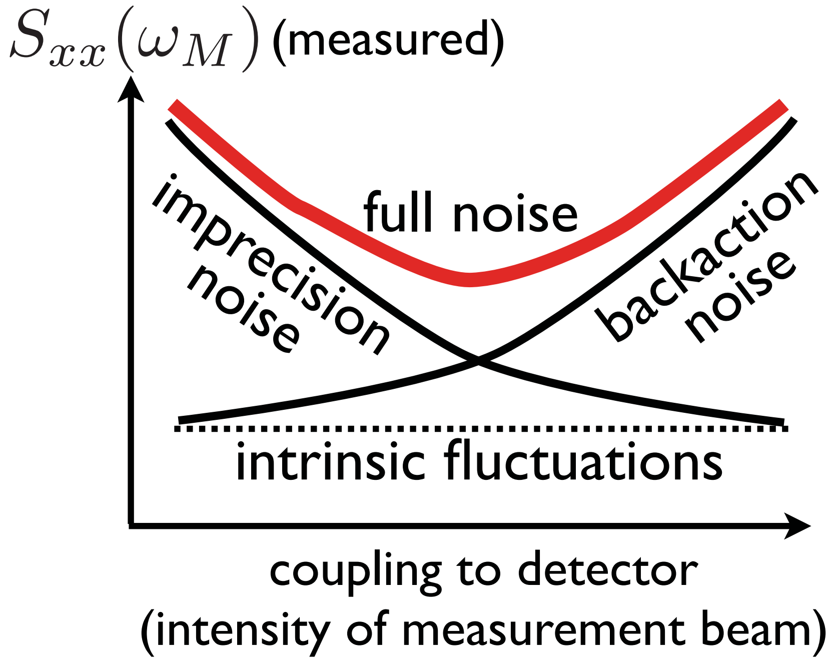
phase noise of laser beam (shot noise limit!)

2. measurement back-action: fluctuating force on system

noisy radiation pressure force

“Standard quantum limit” of displacement detection





Best case allowed by quantum mechanics:

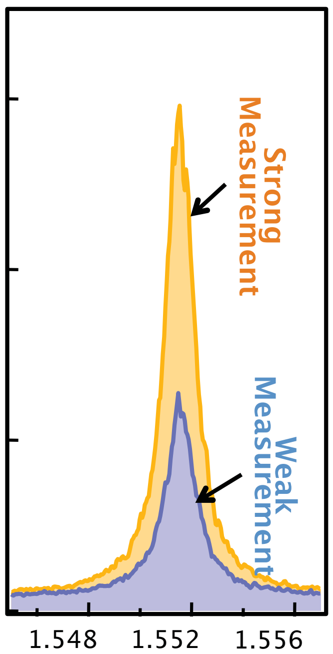
$$S_{xx}^{(\mathrm{meas})}(\omega) \geq 2 \cdot S_{xx}^{T=0}(\omega)$$

“Standard quantum limit (SQL) of displacement detection”

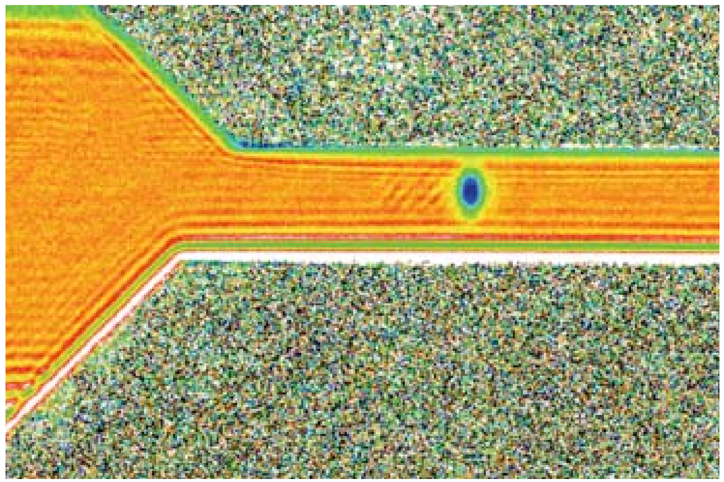
Challenge: Reach optimal regime (where backaction becomes important)

Recent experimental results:

Solid state: Membrane resonator



Cold atoms



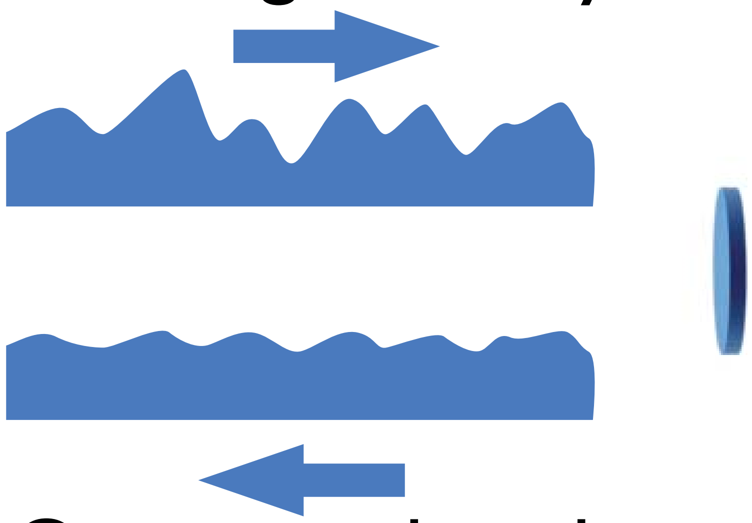
(Berkeley group)

Thomas Purdy (Monday)

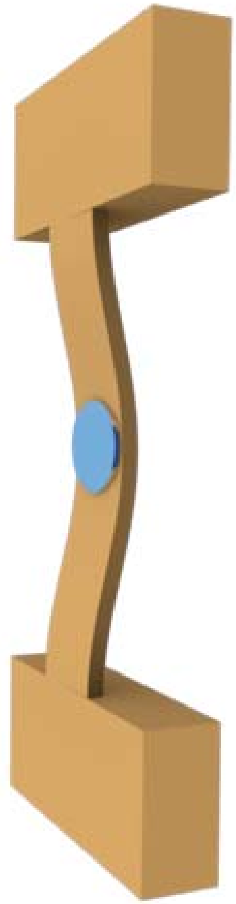
Sydney Schreppler (Tuesday)

Strong backaction induces squeezing of radiation field!

Input: laser field with
fluctuating intensity



Output: reduced
("squeezed") noise



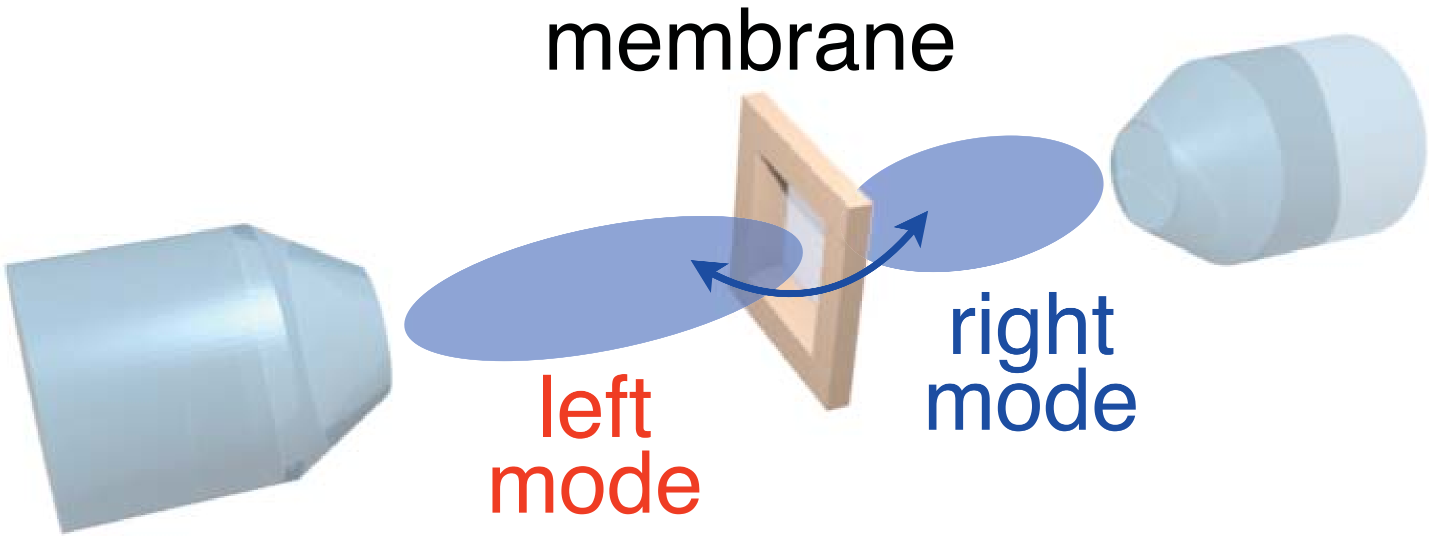
Optomechanical system:
Intensity-dependent
optical resonance
(=Kerr medium)

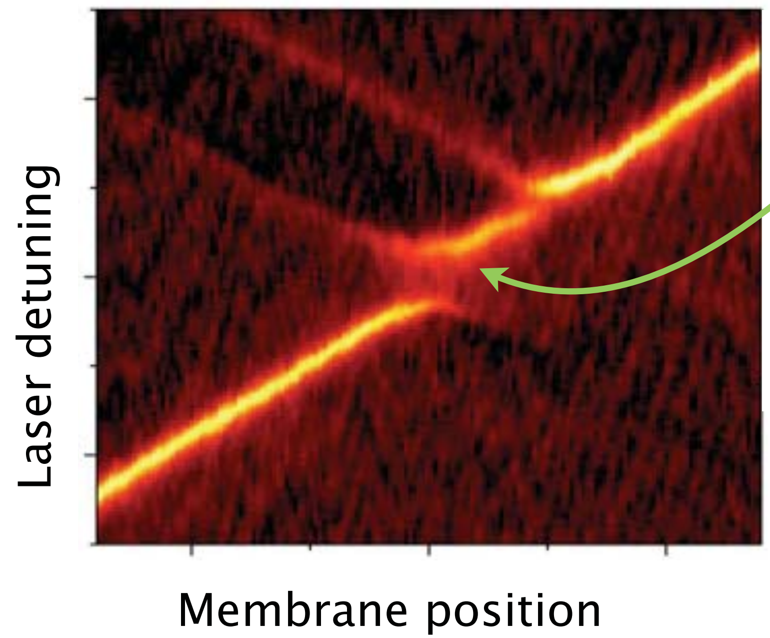
Amir Safavi-Naeini (Wednesday)

Thomas Purdy (Monday)

Measuring x^2 instead of x

(Ultimate goal: phonon number detection)





$$\delta\omega_{\rm cav} \sim x^2$$

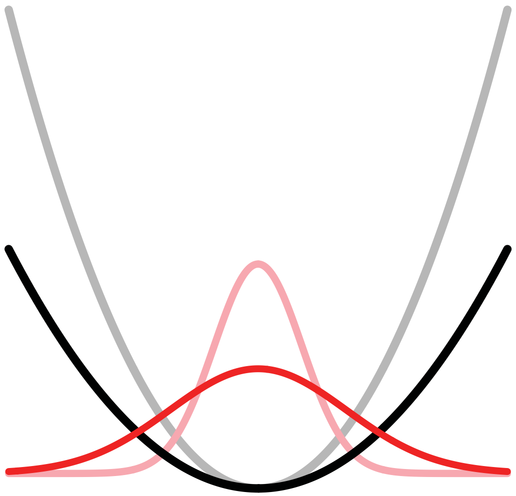
$$\hat{H} = \hbar\omega_{cav}\hat{a}^{+}\hat{a} + \hbar\omega_{m} b^{\dagger} b + g_2 \hat{b}^{\dagger}\hat{b}\hat{a}^{+}\hat{a}$$

Shift in membrane frequency per intracavity photon

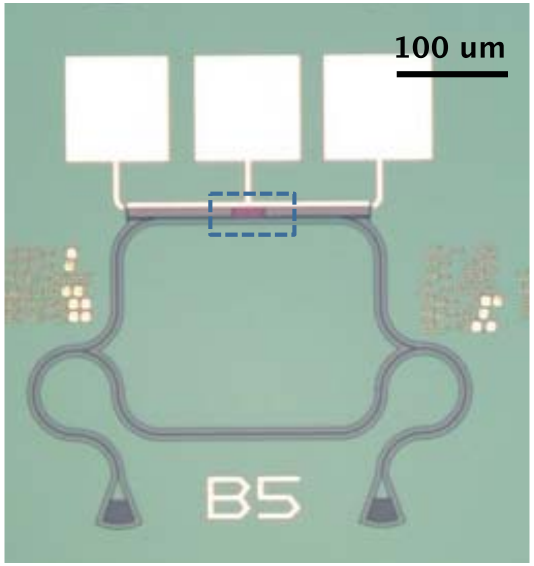
Jack Harris (Tuesday)

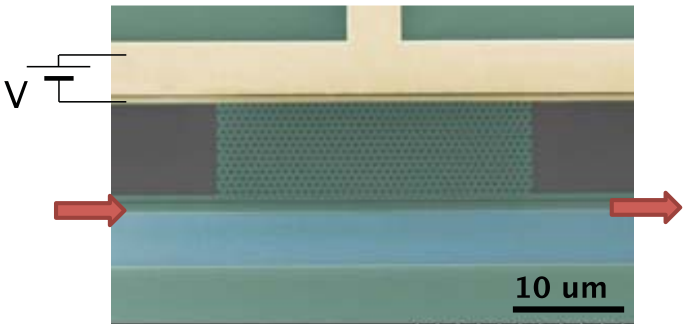
Sensitive measurement can be used for feedback!

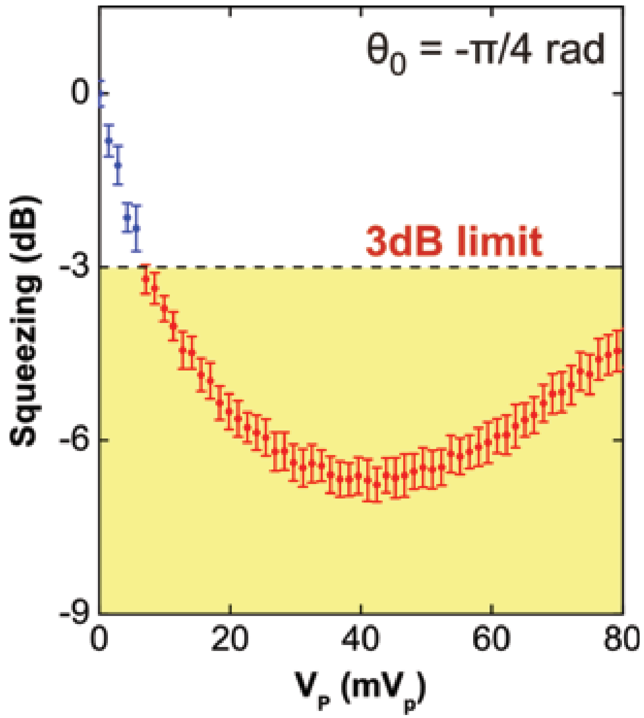
here: use feedback to optimize squeezing of a thermal mechanical state



general trick: time-dependent modulation of spring constant produces squeezing

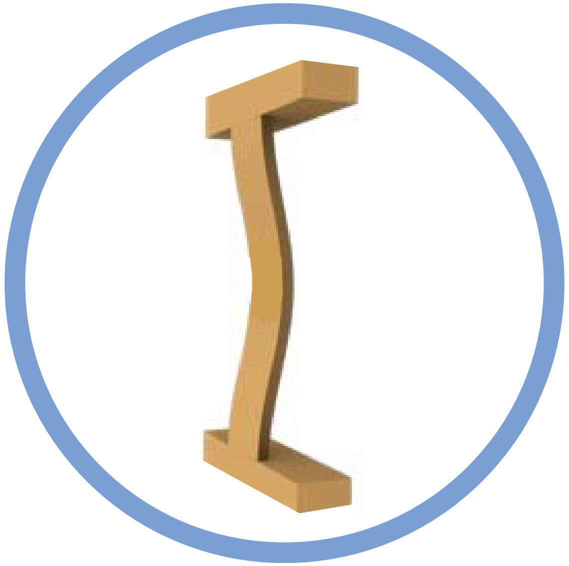






Menno Poot (Friday)

Mechanical resonators from carbon



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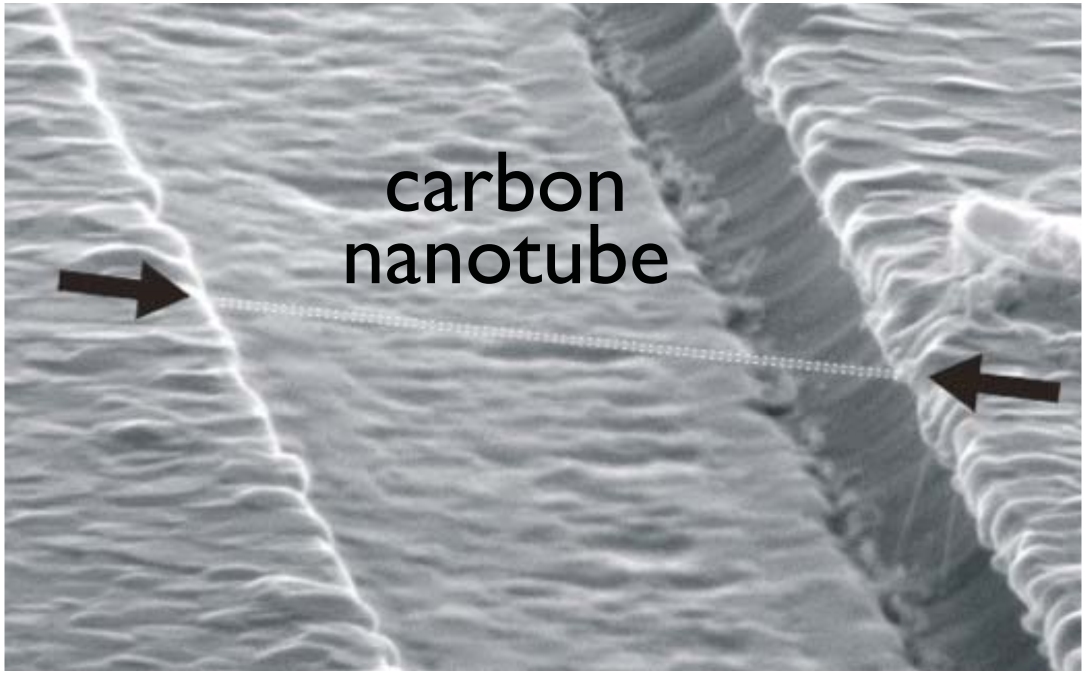
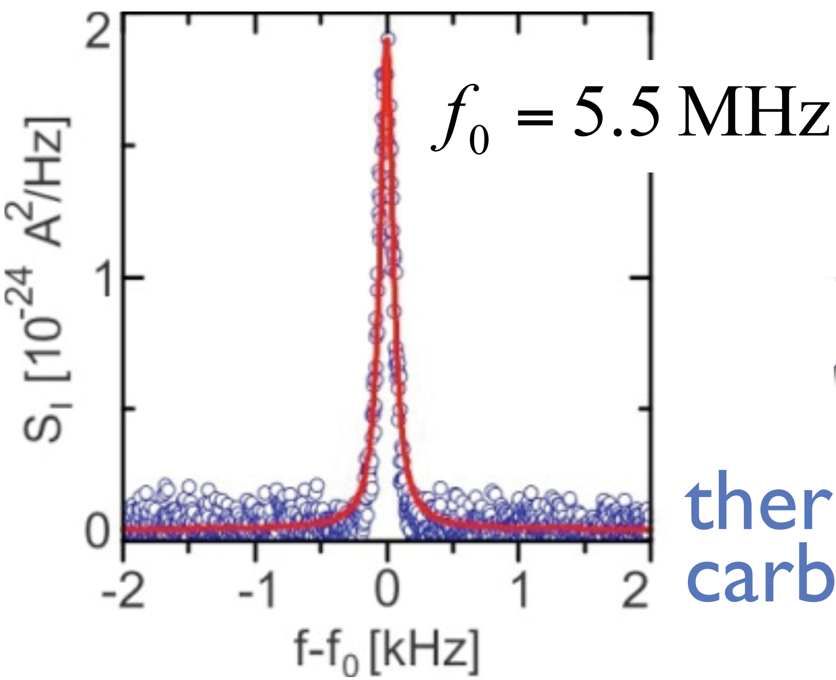
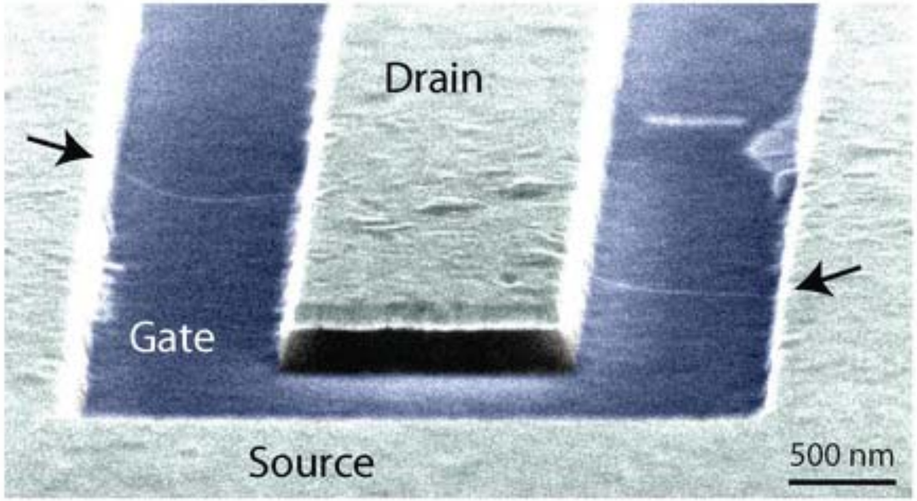


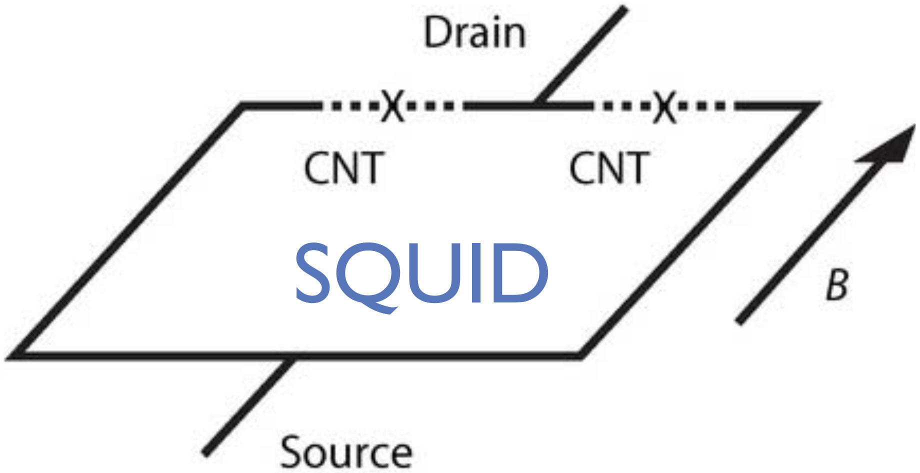


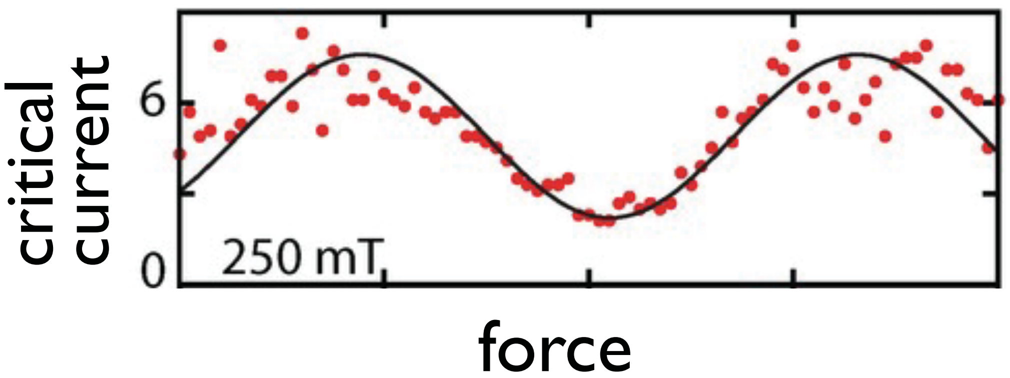
Adrian Bachtold (Wednesday)

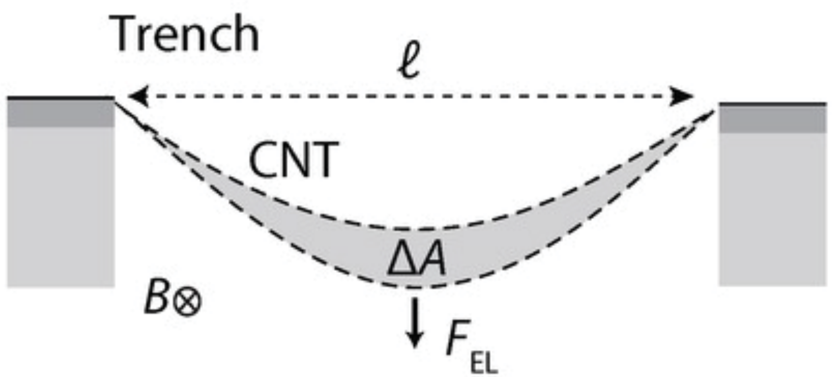
thermal motion of a carbon nanotube

Carbon nanotubes: very low mass, strong quantum zero-point fluctuations – couple to other quantum devices!







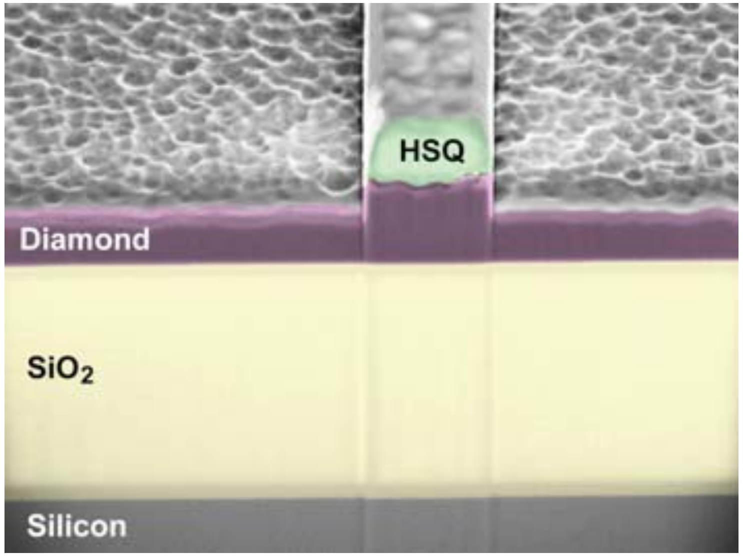


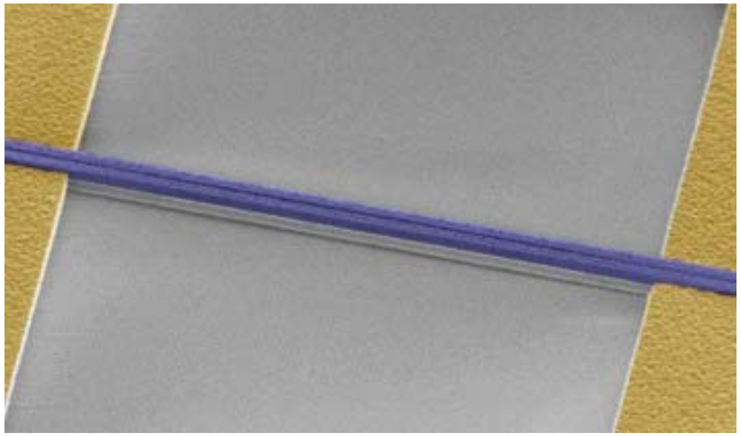
carbon nanotube motion coupled to a superconducting circuit (SQUID)

Gary Steele (Wednesday)

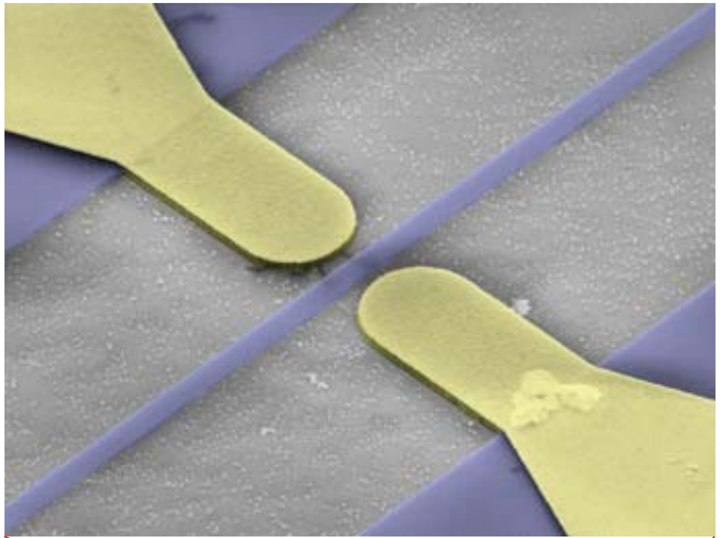
Carbon nanotubes or diamond in photonic circuits

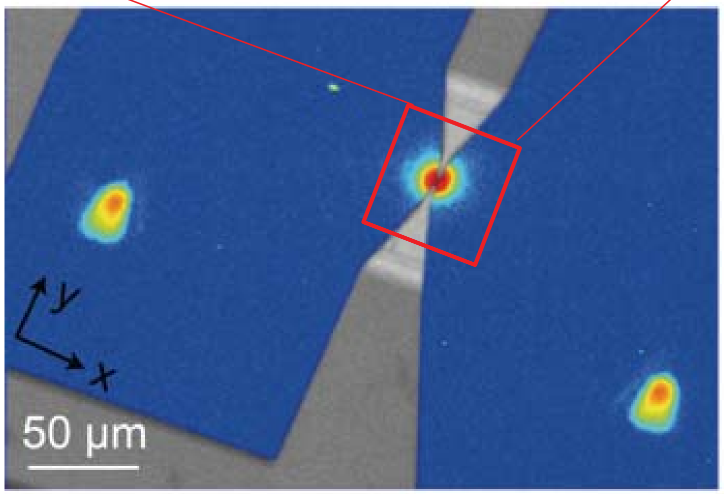
Diamond nanophotonic circuits





Waveguide integrated carbon nanotubes

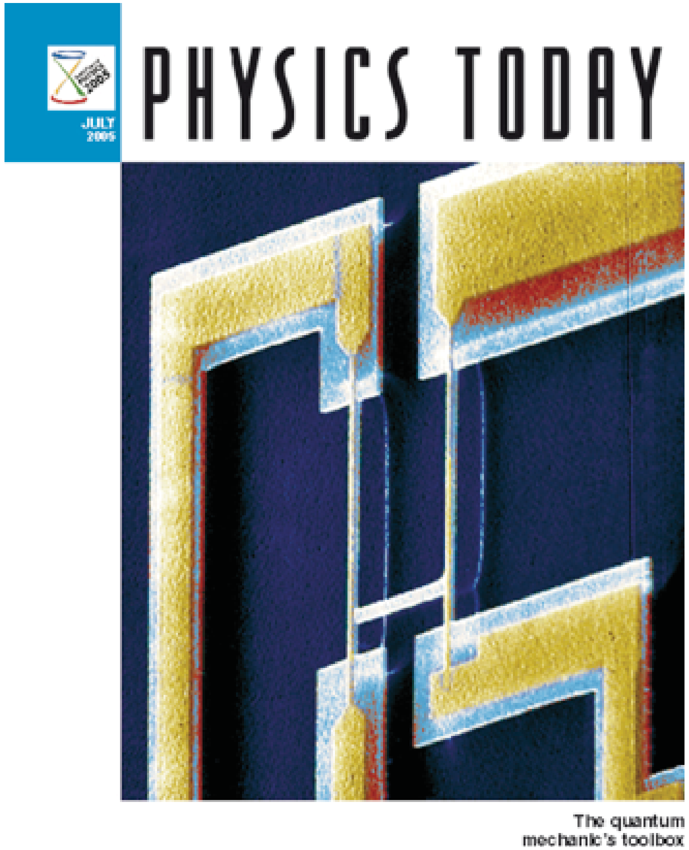


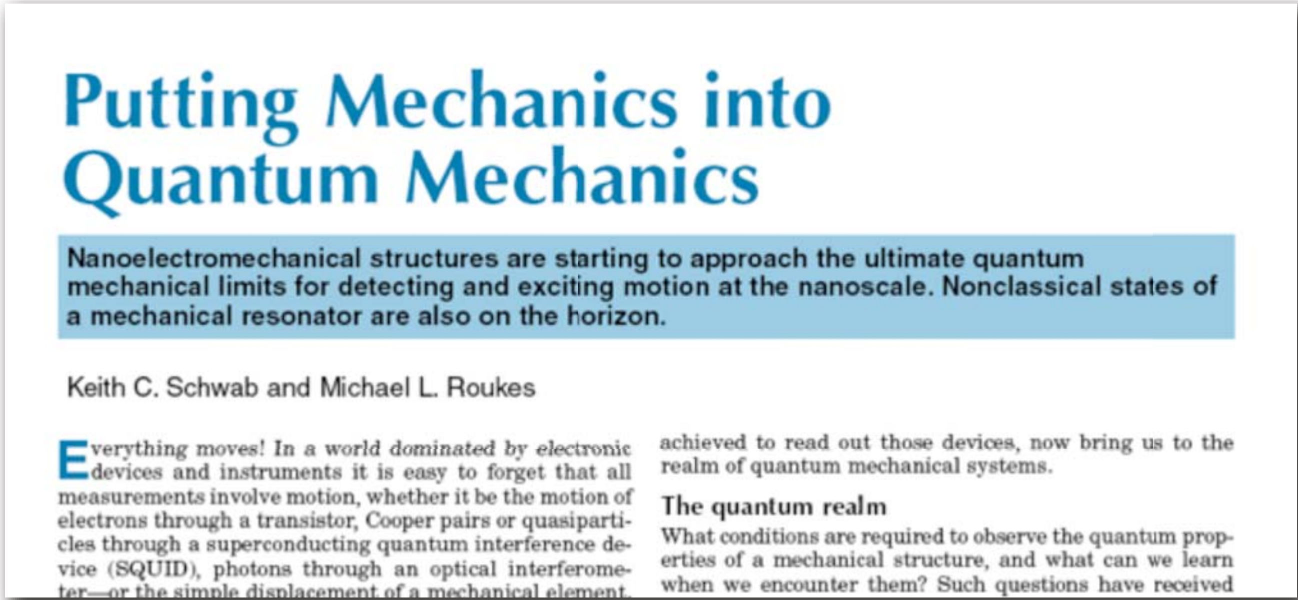


Wolfram Pernice (Tuesday)

The Quantum Regime

(still mostly theory, but first experiments exist)





Putting Mechanics into Quantum Mechanics

Nanoelectromechanical structures are starting to approach the ultimate quantum mechanical limits for detecting and exciting motion at the nanoscale. Nonclassical states of a mechanical resonator are also on the horizon.

Keith C. Schwab and Michael L. Roukes

Everything moves! In a world dominated by electronic devices and instruments it is easy to forget that all measurements involve motion, whether it be the motion of electrons through a transistor, Cooper pairs or quasiparticles through a superconducting quantum interference device (SQUID), photons through an optical interferometer—or the simple displacement of a mechanical element. achieved to read out those devices, now bring us to the realm of quantum mechanical systems.

The quantum realm

What conditions are required to observe the quantum properties of a mechanical structure, and what can we learn when we encounter them? Such questions have received

Schwab and Roukes, Physics Today 2005

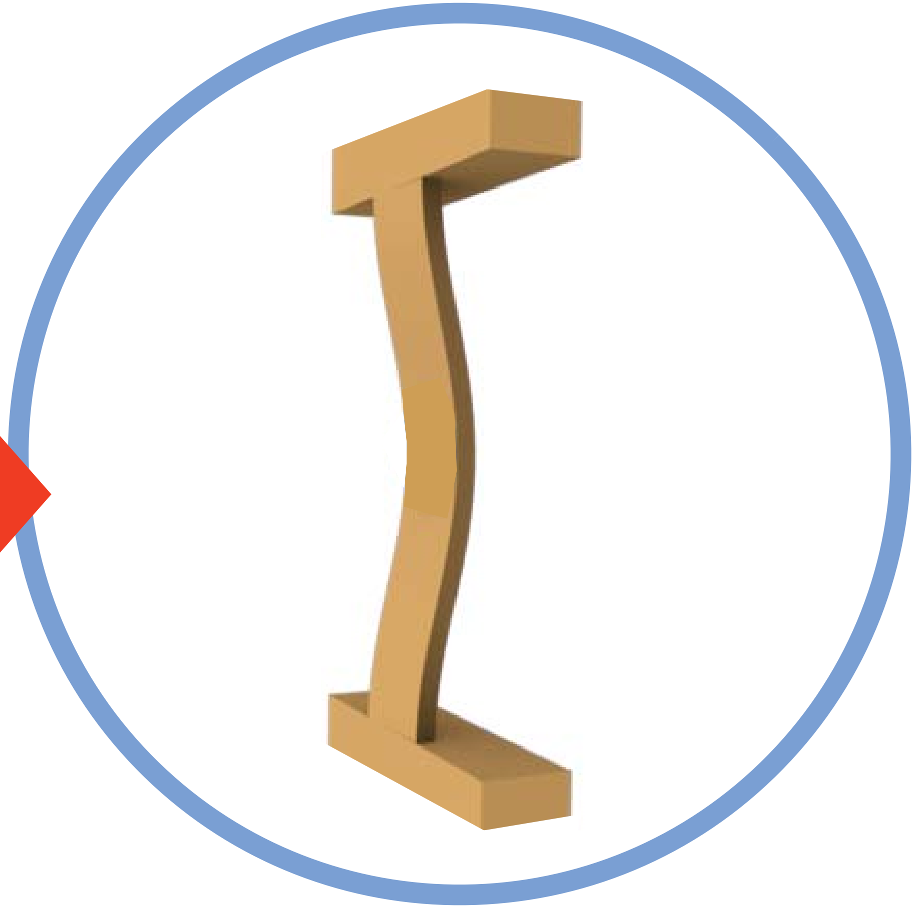
- nano-electro-mechanical systems

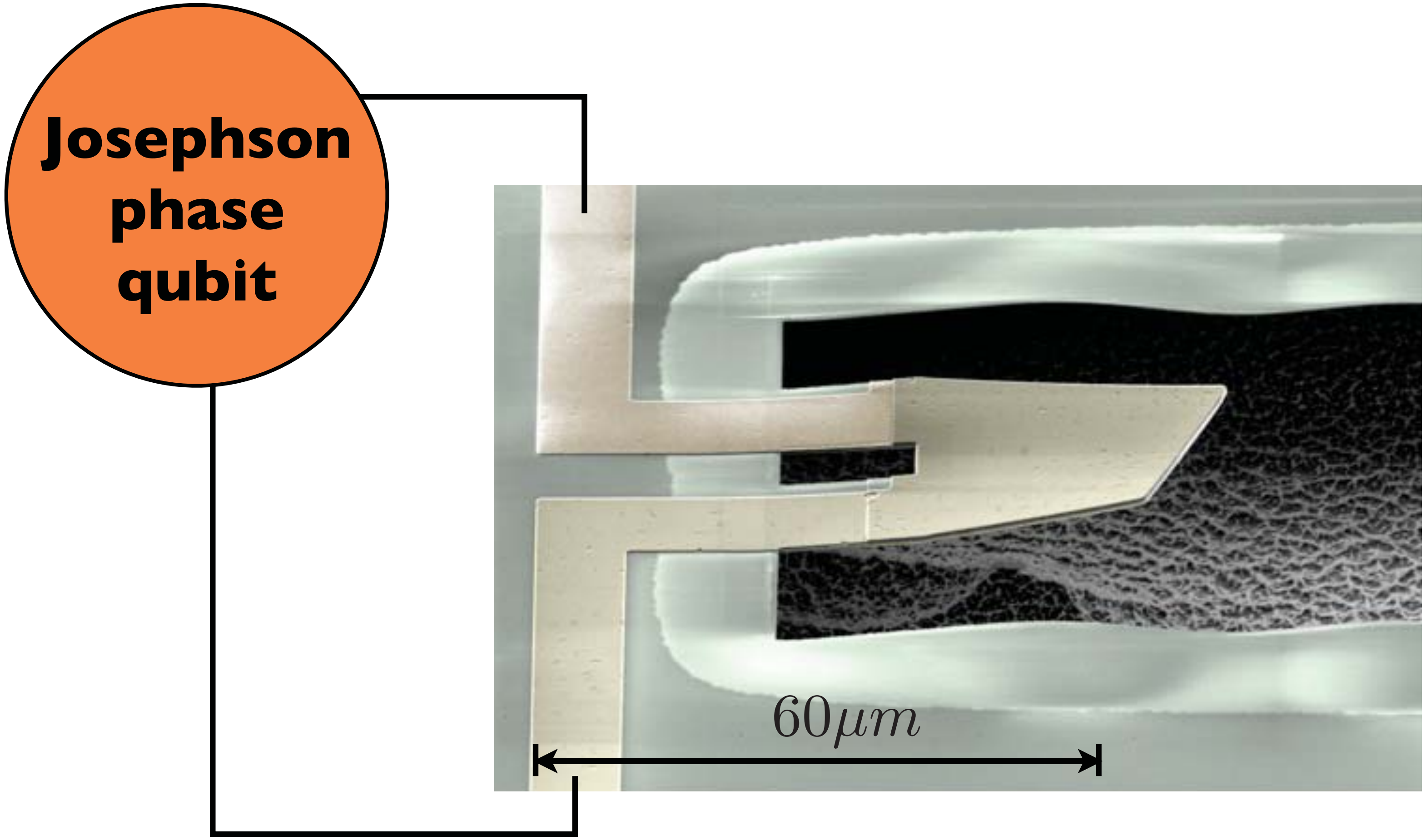
Superconducting qubit coupled to nanoresonator: Cleland & Martinis 2010

- optomechanical systems

Laser-cooled to ground state: Teufel et al in microwave circuit 2011,
Painter group in photonic crystal 2011

two-level system (qubit)





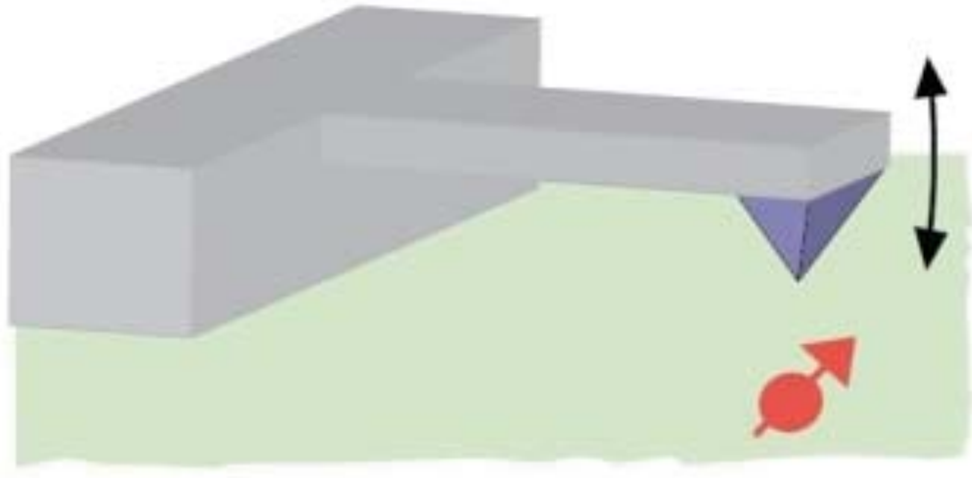
piezoelectric nanomechanical resonator

(GHz @ 20 mK: ground state!)

swap excitation between qubit and mechanical resonator in a few ns!

Andrew Cleland (Tuesday, ICTP Coll.)

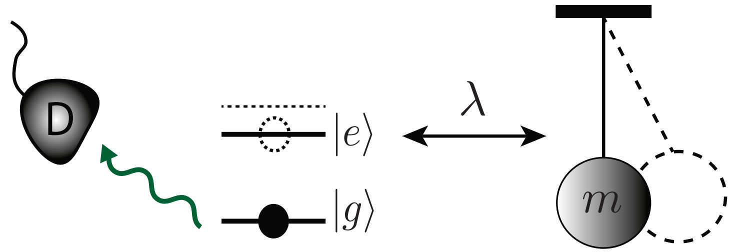
Nanomechanical resonator coupled to spin



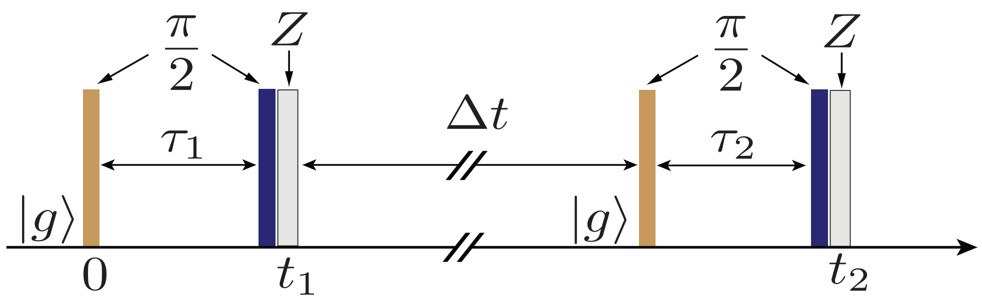
(Rabl)

Mikhail Lukin (Tuesday)

Two-level system as a probe of a mechanical resonator



probe quantum superpositions of a macroscopic resonator via multiple Ramsey measurements:



$$\Rightarrow \quad C(t_1, t_2) = \langle Z(t_2) Z(t_1) \rangle$$

Correlations between subsequent measurement outcomes violate the Leggett-Garg inequality

$$C(t_1, t_2) + C(t_2, t_3) - C(t_1, t_3) \leq 1$$

and can be used for other fundamental tests of quantum mechanics !

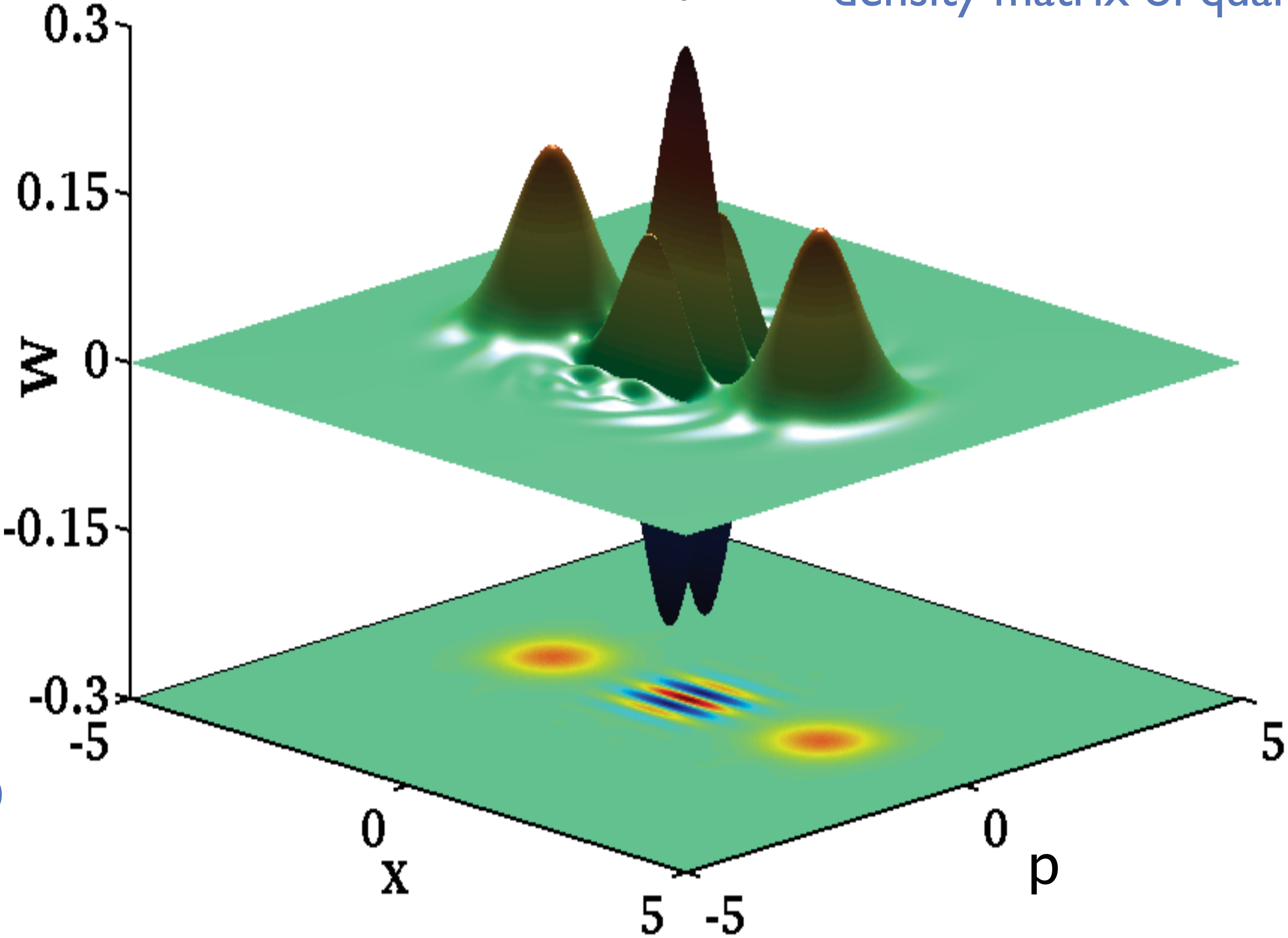
Peter Rabl (Tuesday)

Nonclassical mechanical states

Wigner phase space density:

$$W(x,p) \propto \int dy e^{ipy/\hbar} \rho(x-y/2, x+y/2)$$

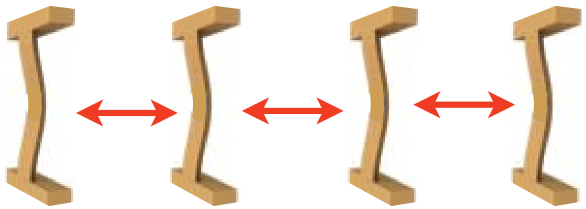
density matrix of quantum state

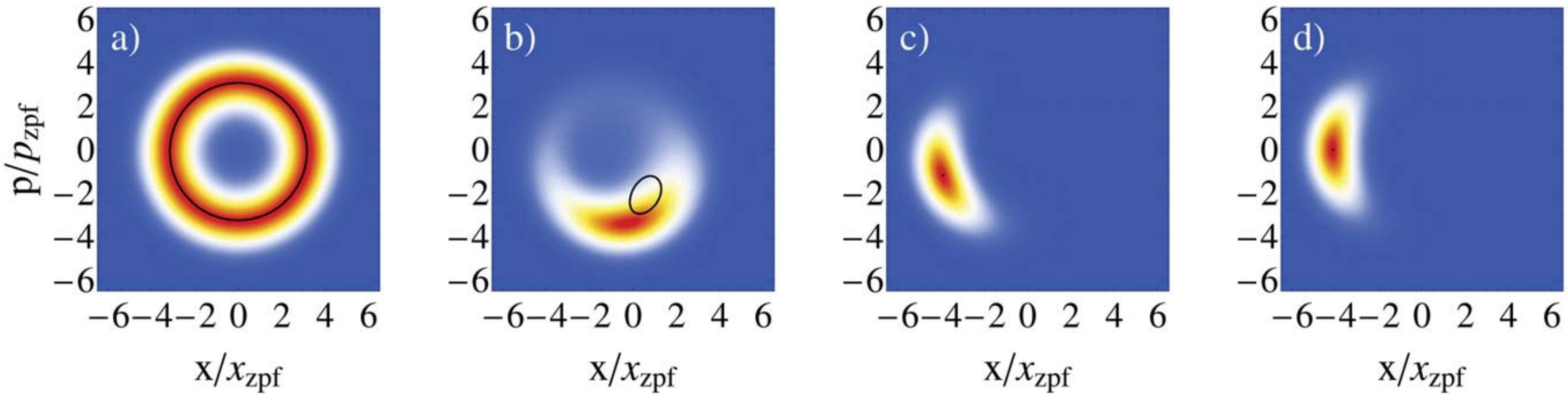


negative values!

David Vitali (Monday)

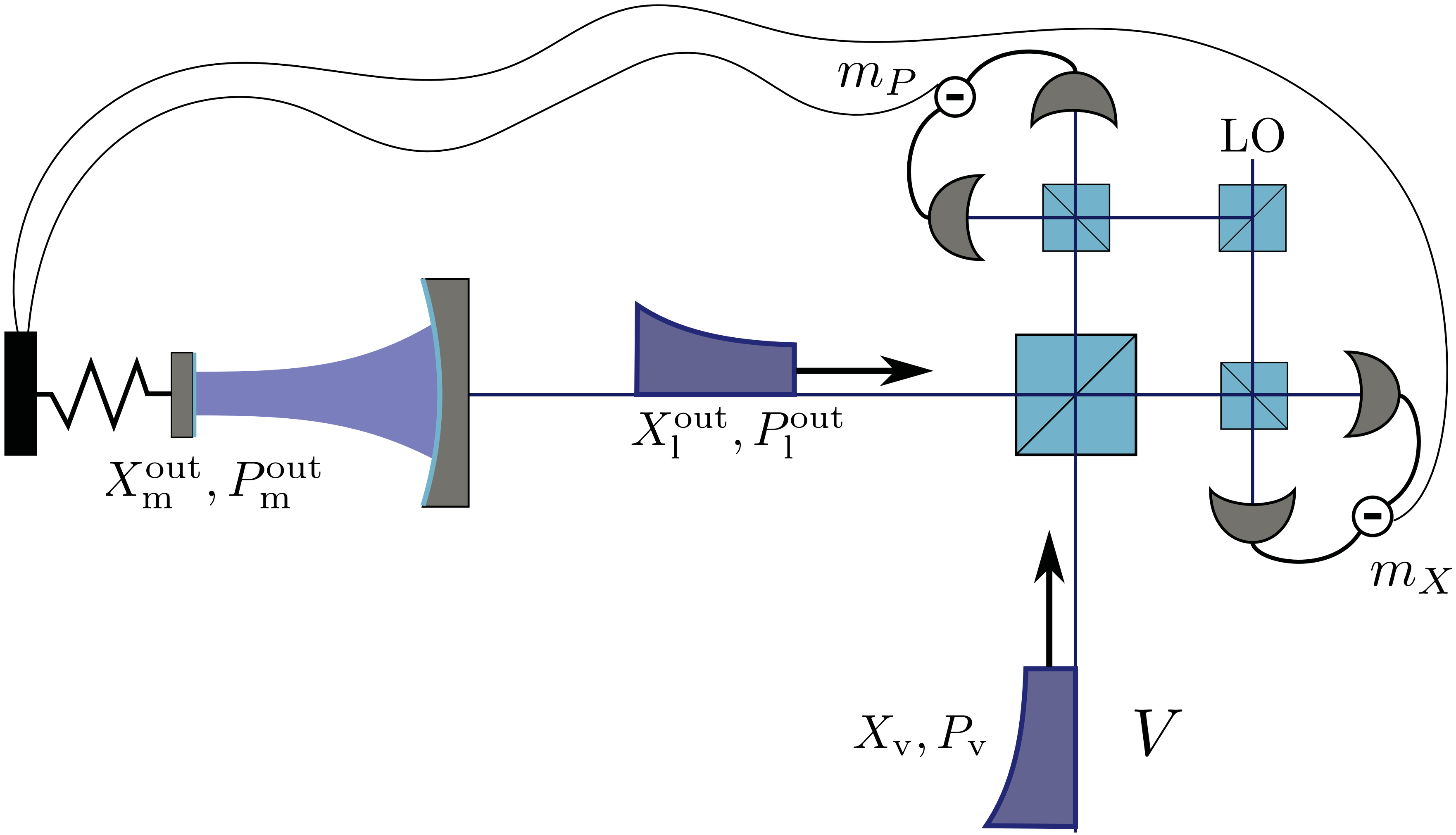
Synchronization between multiple resonators in the quantum regime





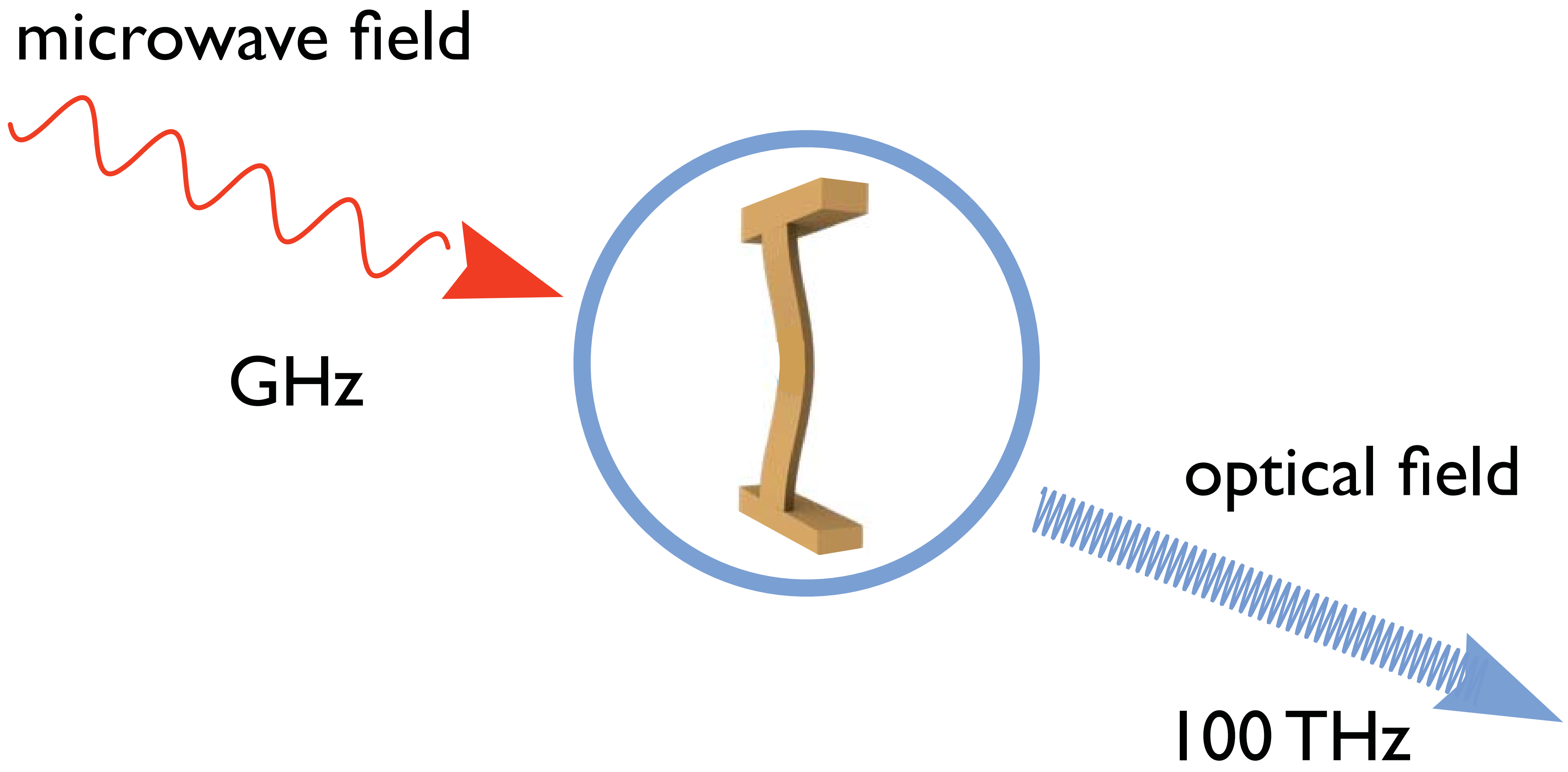
Andreas Nunnenkamp (Thursday)

Optomechanical control & entanglement with light pulses

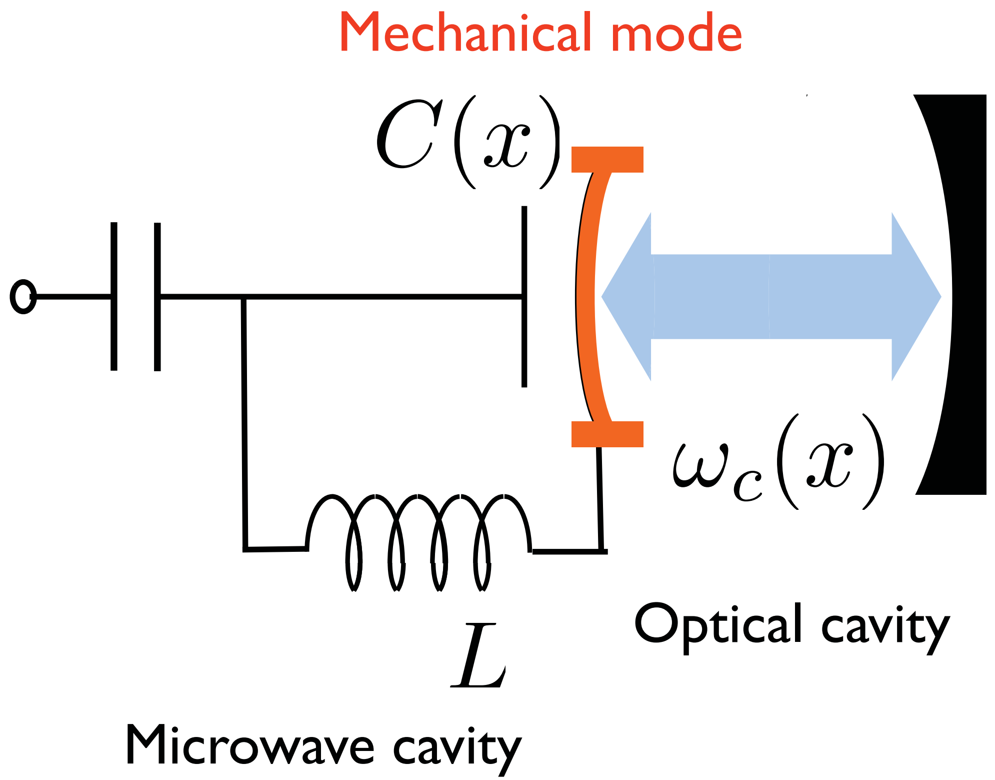


Klemens Hammerer (Friday)

A quantum interface:
Taking quantum information from microwave to optical



Mechanical mode connects resonators with different frequencies



(Lin Tian)

Connect different parts of a hybrid quantum network
Achieve quantum operations through the mechanical mode

Experiment: Andrew Cleland (Monday)

Theory: Lin Tian (Wednesday)

Summary

build quantum interfaces

sensitive readout

couple several

nanomechanical resonators

couple to radiation field

coupling to
spins, qubits, atoms,
quantum dots, SQUIDs etc.:
“hybrid systems”

interesting
mechanical
quantum states