Nonclassical states (and reservoir engineering) in cavity optomechanics

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Outline of the talk


2. Proposal for generating nonclassical mechanical states in a quadratic MIM setup

3. Controlling light with cavity optomechanics: Experiments on: i) optomechanically induced transparency (OMIT); ii) noise reduction for ponderomotive squeezing

4. Proposal for a quantum optomechanical interface between microwave and optical signals
A large variety of cavity optomechanical devices: few examples

1. Fabry-Perot cavity with a moving micromirror
   - Micropillar mirror (Heidmann, Paris)
   - Monocrystalline Si cantilever, (Aspelmeyer, Vienna)

2. Silica toroidal optical microcavities
   - Spoke-supported microresonator (Kippenberg, EPFL)
   - With electronic actuation, (Bowen, Brisbane)
Evanescent coupling of a SiN nanowire to a toroidal microcavity (Kippenberg, EPFL)

Radiation-pressure or dipole gradient coupling

“membrane in the middle” scheme: Fabry-Perot cavity with a thin SiN membrane inside (Yale, Caltech, JILA, Camerino)

Silicon nanobeam photonic optomechanical cavity (Painter, Caltech)
Many cavity modes (standard Gaussian \( \text{TEM}_{mn} \) if membrane aligned and close to the waist)

\[
H_{\text{cav}} = \sum_k \hbar \omega_k (z_0) a_k^+ a_k
\]

Many vibrational modes \( u_{mn}(x,y) \) of the membrane

\[
u_{mn}(x,y) = \sin \frac{n \pi x}{d} \sin \frac{m \pi y}{d}
\]

Vibrational frequencies

\[
\Omega_{nm} = \sqrt{\frac{\pi T}{\rho L_d d^2}} \left( m^2 + n^2 \right)
\]

T = surface tension
\( \rho \) = SiN density,
\( L_d \) = membrane thickness
\( d \) = membrane side length
\( m,n = 1,2... \)

(Thompson et al., Nature 2008)
General multimode optomechanical interaction due to radiation pressure

$$H_{\text{int}} = -\int dx dy \, P_{\text{rad}}(x, y) \, z(x, y)$$

(at first order in $z$)

$$P_{\text{rad}}(x, y) = \varepsilon_0 \left( n_M^2 - 1 \right) \int_{z_0 - L_d / 2}^{z_0 + L_d / 2} \, dz \begin{pmatrix} \hat{E}(x, y, z) \times \hat{B}(x, y, z) \end{pmatrix}_z$$

Radiation pressure field

$$z(x, y) = \sum_{n,m} \sqrt{\frac{\hbar}{M \Omega_{nm}}} q_{nm} u_{nm}(x, y)$$

Membrane axial deformation field

$$\hat{H}_{\text{int}} = -\frac{2\hbar}{L} \sum_{l,k,n,m} \sqrt{\frac{\hbar \omega_l \omega_k}{M \Omega_{nm}}} \Theta_{nmlk} \Lambda_{lk} q_{nm} a_l^+ a_k$$

Multimode trilinear coupling describing photon scattering between cavity modes caused by the vibrating membrane

Possible when:

- We **drive only a single cavity mode** \( a \) (and scattering into the other cavity modes is negligible, no frequency close mode)

- A **bandpass filter** in the detection scheme can be used, isolating a single mechanical resonance

\[
\hat{H} = \frac{\hbar \omega_m}{2} (p^2 + q^2) + \hbar \omega(q) a^+ a + H_{\text{drive}}
\]

**Cavity optomechanics Hamiltonian**

\[
\hat{H}_{\text{drive}} = i\hbar \left( E e^{-i\omega_L t} a^+ - E^* e^{i\omega_L t} a \right)
\]

\[
E = \sqrt{\frac{2\kappa P_L}{\hbar \omega_L}}
\]

amplitude of the driving laser with input power \( P_L \)
Usual radiation pressure force interaction
⇔ first order expansion of \( \omega(q) \)

\[
\omega(q) = \omega_c - G_0 q
\]

**Poor approximation at nodes and antinodes** (where the dependence is quadratic) (Thompson et al., Nature 2008)

\[
\omega(q) = \omega_0 + (-1)^p \arcsin \left( \sqrt{R} \cos \left( 2k_0 z_0(q) \right) \right)
\]

General periodic dependence for a **perfectly aligned membrane** with reflectivity \( R \), placed close to the waist
**Membrane misalignment** (and shift from the waist) couples the TEM_{mn} cavity modes

Splitting of degenerate modes and avoided crossings
linear combinations of nearby TEM_{mn} modes become the new cavity modes: \( \omega(q) \) is changed significantly: tunable optomechanical interaction

(J.C. Sankey et al., Nat. Phys. 2010)

Crossing between the TEM_{00} singlet and the TEM_{20} triplet

**Experimental cavity frequencies with 0.21 mrad misalignment**
Enhanced quadratic interaction at an avoided crossing


\[
\frac{\omega''(q)}{2\pi} = 4.46\text{MHz} / \text{nm}^2
\]

Avoided crossing between

\[
\left[ |\text{TEM}_{20,p-1}\rangle + |\text{TEM}_{02,p-1}\rangle - |\text{TEM}_{00,p}\rangle \right] / \sqrt{3}, \quad \text{and}
\]

\[
|\phi\rangle_{\text{gray}} = \left[ 2|\text{TEM}_{02,p-1}\rangle - |\text{TEM}_{20,p-1}\rangle + |\text{TEM}_{00,p}\rangle \right] / \sqrt{6}
\]

quadratic “dispersive” coupling

\[
H_{\text{int}} = \hbar \omega''(q) q^2 a^+ a
\]

It allows the nondemolition measurement of mechanical energy: possible detection of “phonon quantum jumps”
Results in a very short fiber-based cavity setup at Yale.

Flowers-Jacobs et al. APL, 2012

\[ \frac{\omega''(q)}{2\pi} = 20 \text{GHz} / \text{nm}^2 \]
WHAT TO DO WITH A QUADRATIC HAMILTONIAN?

Proposals for the generation of MECHANICAL NONCLASSICAL STATES

1. Schrodinger cat states through reservoir engineering

2. Stationary squeezed state with “bang-bang” open loop controls

(M. Asjad and D. Vitali, arXiv:1308.0259)
Dynamics driven by an effective dissipative generator, with a nonclassical steady state $\rho_\infty$ (target state) (Poyatos et al., 1993, generalization in Verstraete et al., 2009, Diehl et al., 2008)

Typical solution = Lindblad generator

$$\frac{\partial}{\partial t}\rho = \mathcal{L}\rho \quad \mathcal{L}\rho_\infty = 0$$

Engineered dynamics which must dominate over undesired ones

"Target state"

such that

$$\rho_\infty = |\psi_\infty\rangle\langle \psi_\infty|$$

$$C|\psi_\infty\rangle = 0$$
Generation of **entangled two-mode squeezed state of two bosons**:

1. Entangled atomic ensembles through **engineered optical reservoir** (experiment by Krauter et al., PRL 2011)
2. Entangled cavity modes through **engineered atomic reservoir** (Pielawa et al., PRL 2007)
3. Entangled mechanical resonators in cavity optomechanics (Tan et al. PRA 2013)

Generation of **single-mode squeezed state**

1. Motion of Trapped ions (Carvalho, et al., PRL 2011)
2. Squeezed mechanical resonators in cavity optomechanics (Kronwald et al. , 2013)
Here we engineer the “optical mode reservoir” in cavity optomechanics for robust generation of a mechanical Schrödinger cat states with amplitude $\beta$

$$|\psi_\infty\rangle \approx |\beta\rangle + |-\beta\rangle$$

with $C = \hat{b}^2 - \beta^2$

(M. Asjad and D. Vitali, arXiv:1308.0259)

(see also in trapped ions motion (Carvalho et al. 2001) and also H. Tan et al., PRA 2013)
One needs **quadratic optomechanical coupling** and a **bichromatic driving** (pump on the second red sideband)

![Diagram](image)

rotating wave approximation

\[
H_{\text{eff}} = \hbar g_2 \alpha_s^* \delta a \left( b^{\dagger^2} - i E_1 / g_2 \alpha_s^* \right) + \text{H.C.},
\]

\[
\beta^2 = i \frac{E_1}{g_2 \alpha_s^*}
\]

Adiabatic elimination of cavity mode \(\Rightarrow\) **effective dissipative dynamics for the mechanical resonator**
One has to **beat the undesired standard thermal reservoir** coupled with damping rate $\gamma_m$

$$\frac{\partial}{\partial t} \rho = \Gamma \mathcal{D}(C) \rho + \frac{\gamma_m}{2} (\bar{n} + 1) \mathcal{D}(b) \rho + \frac{\gamma_m}{2} \bar{n} \mathcal{D}(b^\dagger) \rho$$

The cat generation is possible at transient time $t \approx 1/\Gamma$, if $\Gamma \gg \gamma_m n$

$$\Gamma = g_s^2 |\alpha_s|^2 / \kappa_T$$

$$C = \hat{b}^2 - \beta^2$$

“Metastable” cat state $Q_m = 10^7$
Worser performance if starting from a thermal state with $n = 2$, rather than from the ground state $n = 0$. 

![Diagrams and graph showing different states and fidelity over time.](image)
Time evolution very well approximated by a “decohering cat” state

\[
\rho_{\text{app}}(t) = e^{-\Gamma t} \rho(0) + \left(1 - e^{-\Gamma t}\right) \rho_{\text{dec}}^{\text{cat}}(t)
\]

\[
\rho_{\text{dec}}^{\text{cat}}(t) = \mathcal{N}(t)^{-1} \left\{ |\beta\rangle\langle\beta| + | - \beta\rangle\langle - \beta| \\
+ e^{-(1+2\bar{n})\gamma_m t} [ |\beta\rangle\langle - \beta| + | - \beta\rangle\langle \beta|]\right\},
\]

Such a state is ideal to test decoherence models (i.e., environmental decoherence versus collapse models...) on nanomechanical resonators

Macroscopic tests of quantum mechanics

(M. Asjad and D. Vitali, arXiv:1308.0259)
EFFECT OF THE MECHANICAL RESONATOR ON THE OPTICAL FIELD

I. EXPERIMENTS ON OPTOMECHANICALLY INDUCED TRANSPARENCY (OMIT)

The optomechanical analogue of electromagnetically-induced transparency (EIT)
The optomechanical analogue of EIT occurs when

1. **an additional weak probe field is sent into the cavity**
2. **blue sideband of the laser is resonant with the cavity,** $\Delta = \omega_m$

Agarwal & Huang, PRA 2010

The probe at resonance is perfectly transmitted by the cavity instead of being fully absorbed: **destructive interference between the probe and the anti-Stokes sideband of the laser**
Width of the transparency window

\[ \gamma_{\text{m}}^{\text{eff}} = \gamma_{\text{m}} + \Gamma \]

\[ \gamma_{\text{m}}^{\text{eff}} = \gamma_{\text{m}} (1 + C) \]

when \( \Delta = \omega_{\text{m}} \)

cooperativity

EIT is strongly reduced out of the resonant condition \( \Delta = \omega_{\text{m}} \)

Concomitant with transparency, one has **strong group dispersion ⇔ slow light and superluminal effects**

⇒ EIT can be used for **tunable optical delays**, for stopping, storing and retrieving classical and quantum information.

Teufel et al., *Nature* 471, 204 (2011)  
(OMIT with an electromechanical system)

OMIT versus atomic EIT

1. it does not rely on naturally occurring resonances ⇒ applicable to inaccessible wavelengths;
2. a single optomechanical element can already achieve unity contrast
3. Long optical delay times achievable, since they are limited only by the mechanical decay time

OUR OMIT EXPERIMENT WITH THE MEMBRANE

1. Room temperature
2. Significantly lower frequencies (~ 350 kHz) rather than Ghz ⇒ longer delay times
3. Free space (rather than guided) optics

Karuza et al., PRA 88, 013804 (2013)
Estimated group delay $\tau \approx 670 \text{ ns}$ (from the derivative of the phase shift)

Cooperativity ($\sim 160$ here)

Karuza et al., PRA 88, 013804 (2013)
When the red sideband of the laser is resonant with the cavity, $\Delta = -\omega_m$, one has instead constructive interference and “optomechanically induced amplification”

One has the optomechanical analogue of a parametric oscillator below threshold

Karuza et al., PRA 88, 013804 (2013)
EFFECT OF THE MECHANICAL RESONATOR ON THE OPTICAL FIELD

II. PONDEROMOTIVE SQUEEZING

Radiation pressure fluctuations prevails over thermal fluctuations, ⇒ correlations between the mechanical motion and the optical field ⇒ suppress fluctuations on an interferometer’s output optical field below the shot-noise level

Squeezed amplitude of the cavity output correlated with resonator fluctuations
Ponderomotive squeezing of cavity output recently demonstrated in the MHz range (predicted by Tombesi, Fabre, PRA1994)

Waveguide-coupled zipper optomechanical cavity

A, Safavi-Naeini et al., Nature 2013
With a MIM setup, Purdy et al, PRX, August 2013, **1.7 dB of squeezing**
However, for gravitational wave detection, \textit{squeezed light injection} would be useful at lower frequencies, $< \text{kHz band}$

Enhanced sensitivity of the LIGO gravitational wave detector by using squeezed states of light, Nat Phot. 2013

Injection of a low noise – low frequency OPO source of squeezed vacuum with nonlinear crystal (Valhbruch et al GEO600)
At lower frequencies, ponderomotive squeezing is more difficult, mainly due to higher frequency noise (from cavity and lasers)

We have recently experimentally demonstrated a frequency noise cancelation in the kHZ band, in a Fabry-Perot cavity with a micromechanical mirror. It can be useful for pond. squeezing.

Camerino-Trento-Florence collaboration, A. Pontin et al., arXiv:1308.5176v1

The cancelation is just around the bare mechanical frequency, $\omega_m$, and is due to the destructive interference between the frequency noise directly affecting the cavity and the same frequency noise transduced by the resonator

$\alpha_1^{\text{out}}(\omega) \propto \frac{\chi_{\text{eff}}(\omega)}{\chi_0(\omega)} \phi(\omega)$

Cancelation when $Q_m$ is large

$\chi_0(\omega_m) \approx \infty$
Experimental spectrum of the cavity output with added large frequency noise (room T, $Q = 10^4$)

A. Pontin et al., arXiv:1308.5176v1

Micromechanical mirror mode
This frequency noise cancelation facilitates the detection of ponderomotive squeezing at low frequencies.

Theory predictions at lower $T, Q = 10^5$, lower freq noise

Squeezing due to cancelation

Output spectrum at $\omega = \omega_m$ versus homodyne phase and detuning.
How to use a nanomechanical resonator as a quantum interface between optics and microwaves

Based on:

1. Establishing stationary, continuous wave, strong continuous variable (CV) entanglement between the optical and microwave output field
2. High-fidelity CV optical-to-microwave teleportation of nonclassical states

Why an optical-microwave transducer?

Light is optimal for quantum communications between nodes, while microwaves are used for manipulating solid state quantum processors.

⇒ a quantum interface between optical and microwave photons would be extremely useful.

Quantum interface between optical and microwave photons based on a nanomechanical resonator in a superconducting circuit, simultaneously interacting with the two fields.
The membrane resonator is coupled capacitively with the microwave cavity and by radiation pressure with the optical cavity. Possible implementations:


Schematic of a Fabry-Perot cavity with a nanomechanical membrane inserted in the waist of the cavity; the membrane, in turn, is part of a parametric capacitor; (right) Equivalent circuit with a dc voltage bias describing the coupled electromechanical system.

J. Taylor, Sorensen, Marcus, Polzik, PRL 107, 273601 (2011), T. Bagci et al., 2013

See also A. Cleland talk, with electrically actuated AlN optomechanical crystal
Both cavities are driven coherently: 
⇒ the dynamics of the quantum fluctuations around the stable steady state well described by Quantum Langevin Equations (QLE) for optical and microwave operators \( a \) and \( b \)

The nanomechanical resonator mediates a retarded interaction between the two cavity fields (exact QLE), with a kernel

\[
\chi_M(t) = e^{-\gamma_m t/2} \sin \omega_m t
\]

\[
\begin{align*}
\dot{a} &= -\kappa_c \hat{a} + \sqrt{2\kappa_c} \hat{a}_{in}(t) e^{i\Delta_c t} + \frac{i}{2} \int_{-\infty}^{t} ds \chi_M(t - s) \left\{ G_c \hat{\xi}(s) e^{i\Delta_c t} ight. \\
&\left. + G_c^2 \left[ \delta \hat{a}(s) e^{i\Delta_c (t-s)} + \delta \hat{a}^\dagger(s) e^{i\Delta_c (t+s)} \right] + G_c G_w \left[ \delta \hat{b}(s) e^{i\Delta_c t - i\Delta_w s} + \delta \hat{b}^\dagger(s) e^{i\Delta_c t + i\Delta_w s} \right] \right\} \\
\dot{b} &= -\kappa_w \hat{b} + \sqrt{2\kappa_w} \hat{b}_{in} e^{i\Delta_w t} + \frac{i}{2} \int_{-\infty}^{t} ds \chi_M(t - s) \left\{ G_w \hat{\xi}(s) e^{i\Delta_w t} \\
&+ G_w^2 \left[ \delta \hat{b}(s) e^{i\Delta_w (t-s)} + \delta \hat{b}^\dagger(s) e^{i\Delta_w (t+s)} \right] + G_c G_w \left[ \delta \hat{a}(s) e^{i\Delta_w t - i\Delta_c s} + \delta \hat{a}^\dagger(s) e^{i\Delta_w t + i\Delta_c s} \right] \right\}
\end{align*}
\]

Beamsplitter-like optical-microwave interaction ⇒ state transfer term

parametric optical-microwave interaction ⇒ entangling term
One can resonantly select one of these processes by appropriately adjusting the two cavity detunings:

- Equal detunings: \( \Delta_c = \Delta_w \Rightarrow \text{state transfer} \) between optics and microwave (see other proposals, Tian et al., 2010, Taylor et al., PRL 2011, Wang & Clerk, PRL 2011)

Opposite detunings: \( \Delta_c = -\Delta_w \Rightarrow \text{two-mode squeezing and entanglement} \)

Interaction kernel = mechanical susceptibility

\[
\chi_M(t) = e^{-\gamma_m t/2} \sin \omega_m t
\]

Here we choose \( \Delta_c = -\Delta_w = \pm \omega_m \Rightarrow \text{two-mode squeezing is resonantly enhanced} \) (because the interaction kernel does not average to zero)

The mechanical interface realizes an effective parametric oscillator with an optical signal (idler) and microwave idler (signal) \( \Leftrightarrow \) microwave-optical two mode squeezing
ENTANGLEMENT BETWEEN MECHANICS AND THE INTRACAVITY MODES IS NOT LARGE

$E_N$ of the three bipartite subsystems (OC-MC full black line, OC-MR dotted red line, MC-MR dashed blue line) vs the normalized microwave cavity detuning at fixed temperature $T = 15 \text{ mK}$, and at three different MR masses: $m = 10 \text{ ng (a), } m = 30 \text{ ng (b), } m = 100 \text{ ng.}$ The optical cavity detuning has been fixed at $\Delta_c = \omega_m$.

BUT, similarly to single-mode squeezing, entanglement can be very strong for the OUTPUT cavity fields by properly choosing the central frequency $\Omega_j$ and the bandwidth $1/\tau$ of the output modes, one can optimally filter the entanglement between the two output modes.

normalized causal filter function

\[
g_j(t) = \sqrt{\frac{2}{\tau}} \theta(t) e^{-\frac{1}{\tau} + i\Omega_j} t \quad j = c, w
\]
LARGE ENTANGLEMENT FOR NARROW-BAND OUTPUTS

LogNeg at four different values of the normalized inverse bandwidth \( \epsilon = \tau \omega_m \)
vs the normalized frequency \( \Omega_w/\omega_m \), at fixed central frequency of the optical output mode \( \Omega_c = -\omega_m \).

Optical and microwave cavity detunings fixed at \( \Delta_c = -\Delta_w = -\omega_m \)
Other parameters: \( \omega_m/2\pi = 10 \) MHz, \( Q = 1.5 \times 10^5 \), \( \omega_w/2\pi = 10 \) GHz, \( \kappa_w = 0.04\omega_m \), \( P_w = 42 \) mW, \( m = 10 \) ng, \( T = 15 \) mK. This set of parameters is analogous to that of Teufel et al.
Optical cavity of length \( L = 1 \) mm and damping rate \( \kappa_c = 0.04\omega_m \), driven by a laser with power \( P_c = 3.4 \) mW.

Entanglement is robust wrt to temperature
The common interaction with the nanomechanical resonator establishes quantum correlations which are strongest between the output Fourier components *exactly at resonance* with the respective cavity field.

Such a large stationary entanglement can be exploited for continuous variable (CV) optical-to-microwave quantum teleportation:
TELEPORTATION FIDELITY OF A CAT STATE

Input cat state

\[ |\psi\rangle = N(|\alpha\rangle + | - \alpha\rangle) \]

(a) Plot of the teleportation fidelity \( F \) at four different values of \( \epsilon = \tau \omega_m \) versus \( \Omega_w/\omega_m \) and for the Schrödinger cat-state amplitude \( \alpha = 1 \).

(b) Plot of \( F \) for the same values of \( \epsilon \) vs temperature at a fixed central frequency of the microwave output mode \( \Omega_w = \omega_m \).

The fidelity behaves as the logneg: large and robust \( F \) for narrow output bandwidths.
Through teleportation we realize a high-fidelity optical-to-microwave quantum state transfer assisted by measurement and classical communication.

The selected narrow-band microwave and optical output modes possess (EPR) correlations that can be optimally exploited for teleportation.

\[ F_{opt} = \frac{1}{1 + e^{-E_N}} \]

(see A. Mari, D. Vitali, PRA 78, 062340 (2008)).
CONCLUSIONS

1. Proposals for generating mechanical Schrodinger cat and squeezed states in quadratically coupled optomechanical system

2. Controlling light with cavity optomechanics:
   Experiments on: i) optomechanically induced transparency (OMIT); ii) noise reduction for ponderomotive squeezing

3. CV optics-to-microwave interfaces at the classical and quantum level realizable with opto-electro-mechanical systems