The Abdus Salam International Centre for Theoretical Physics

2458-4

Workshop on GNSS Data Application to Low Latitude Ionospheric Research

6 - 17 May 2013

TEC estimation from GNSS observations

Luigi Ciraolo
ICTP

Strada Costiera, 11 - 34151 - Trieste - Italy • Tel. +39 0402240111 • Fax. +39 040224163 • sci_info@ictp.it • www.ictp.it
ICTP is governed by UNESCO, IAEA, and Italy, and it is a UNESCO Category 1 Institute

TEC estimation from GNSS observations

Luigi Ciraolo, ICTP

Workshop on GNSS Data Application

to Low Latitude Ionospheric Research

Trieste, May 6-17, 2013

Objective of this presentation is

how getting from ***GNSS*** observations (Pseudo-range, Phase) Ionospheric parameters (***Total Electron Content***)

GNSS observations have been already extensively presented in this Workshop: so actual presentation will start from the ionospheric quantities derived from them.

Find anyway several slides repeating what already presented (terminology, different point of view, sometimes a more “tutorial” approach)

Navigation, Positioning:

Enabling the **user** to determine its own coordinates in some given

REFERENCE SYSTEM

Observing ***distances*** (or ***angles***) to ***points of known coordinates***

Artificial satellites provide with a powerful tool for positioning:

1. Newtonian mechanics enable to know and forecast very precisely the coordinates of satellites

2. ➔ ***points of known coordinates***

2. Exchange of electro-magnetic (e.m.) signals between the user and the satellites enables the estimation of their distance ***d.*** GPS basic observable are ***delays*** which are converted (multiplying by ***c***, the velocity of light) to

   ➔ ***distances***.

3. The knowledge of a sufficient number ***K*** of distances from satellites of coordinates $x_k, y_k, z_k, k=1..K$ enables the user to determine his coordinates ξ, η, ζ

Properly designing the constellation of satellites, one can build GLOBAL systems, i.e. able to work at any time of day at any place.

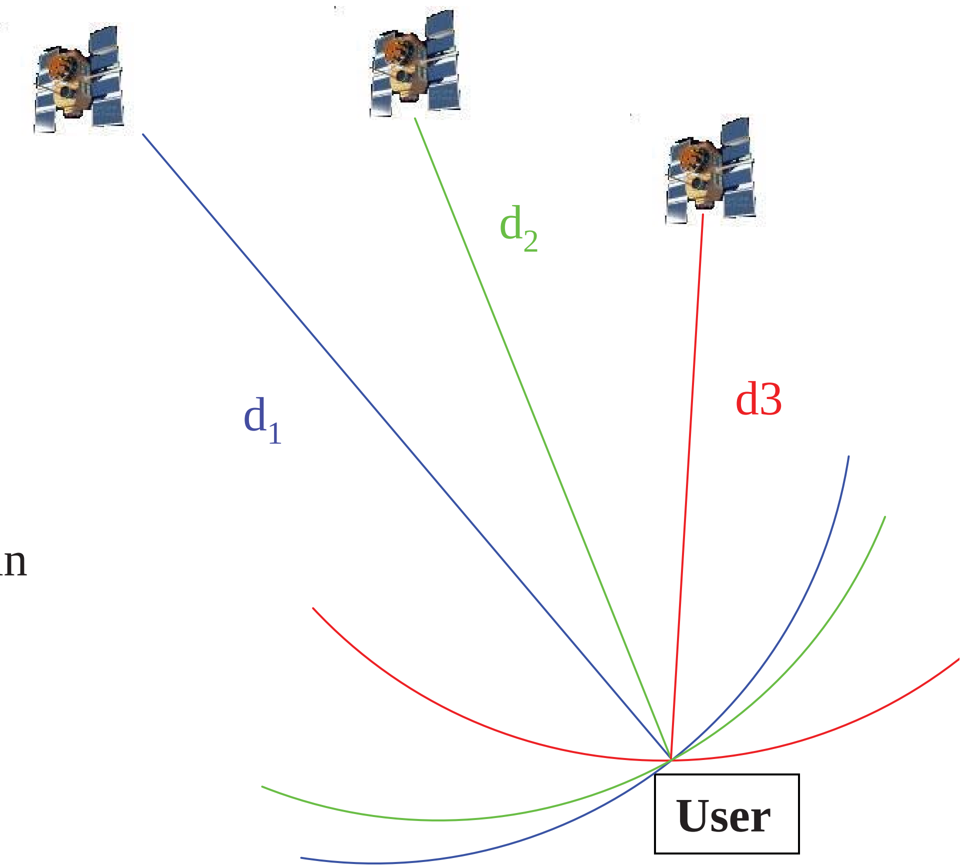
At a 0th order approximation

E.m. signals propagate in **vacuum** with constant velocity $\boldsymbol{c}$ = 2299792500 m/s.

Measuring the propagation delay δ is the same as measuring distance $\boldsymbol{d}$, as

$$\boldsymbol{d} = \boldsymbol{c} \cdot \boldsymbol{\delta}$$

The knowledge of the distances $\boldsymbol{d_i}$, ***i=1..3*** provides with the position of the user as intersection of the **3 spheres** with center in satellites and radius $\boldsymbol{d_i}$

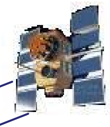


$$\boldsymbol{d_k = \sqrt{(x_k - \xi)^2 + (y_k - \eta)^2 + (z_k - \zeta)^2}}$$

$$\mathbf{k = 1,\ldots,K \qquad K \geq 3}$$

The e.m. measurement of distance between user and satellite

Standard methods (RADAR, LIDAR): **Two Way**

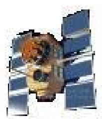


***d* = *c* · *δ*/2**

Method commonly used in positioning: **One Way**

E.m. signals propagate from satellite to user (or vice versa)

How does this work?



DORIS

GNSS:

NNSS, PRARE, GPS, GLONASS, GALILEO

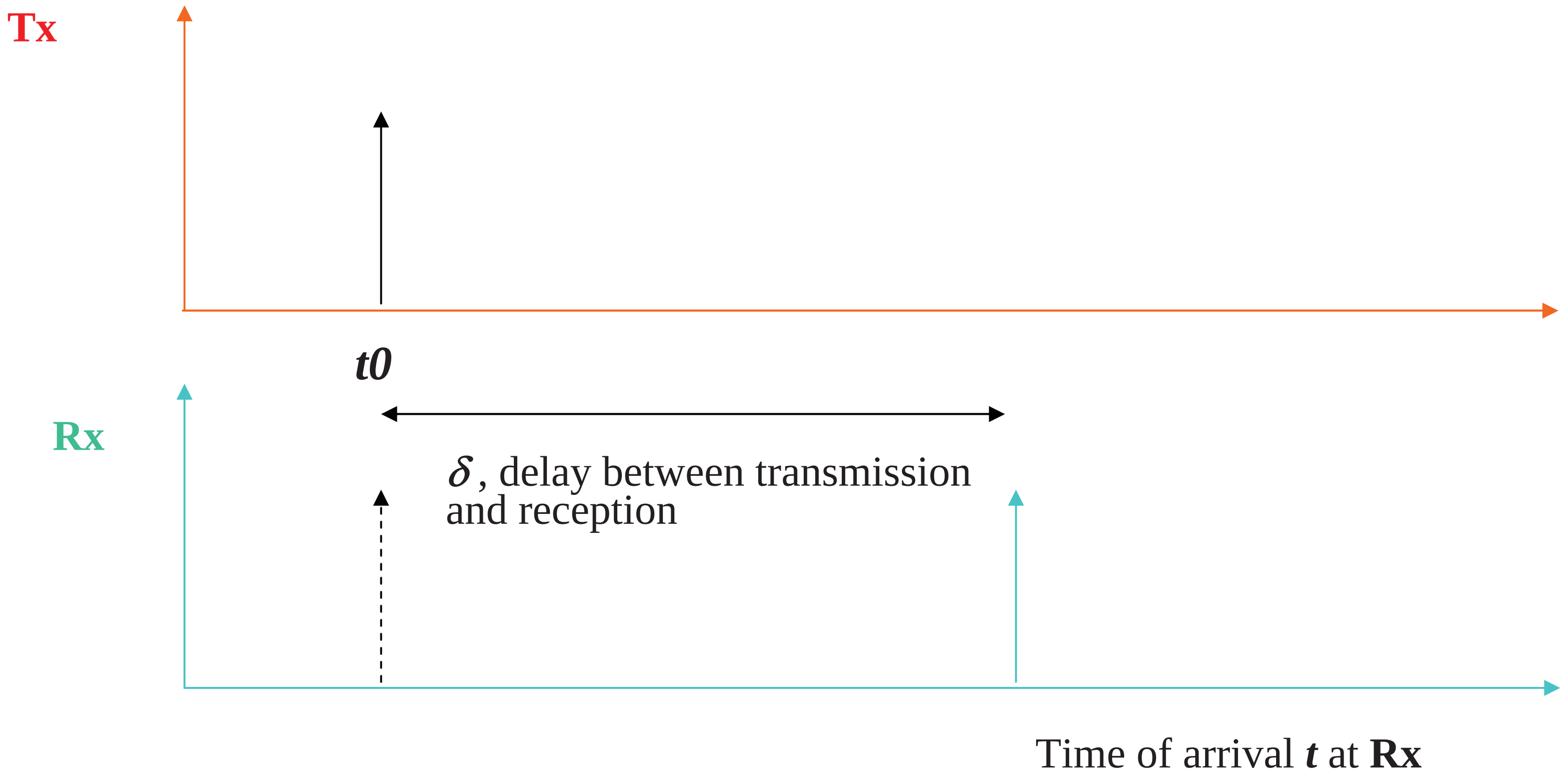
User

The One Way measurement

Satellite: transmits a pulse at a given time $\boldsymbol{t_0}$, arriving at $\boldsymbol{t_0 + \delta}$

User: knows the pulse starts at $\boldsymbol{t_0}$, arrival time is $\boldsymbol{t}$

$$\delta = t - t_0$$

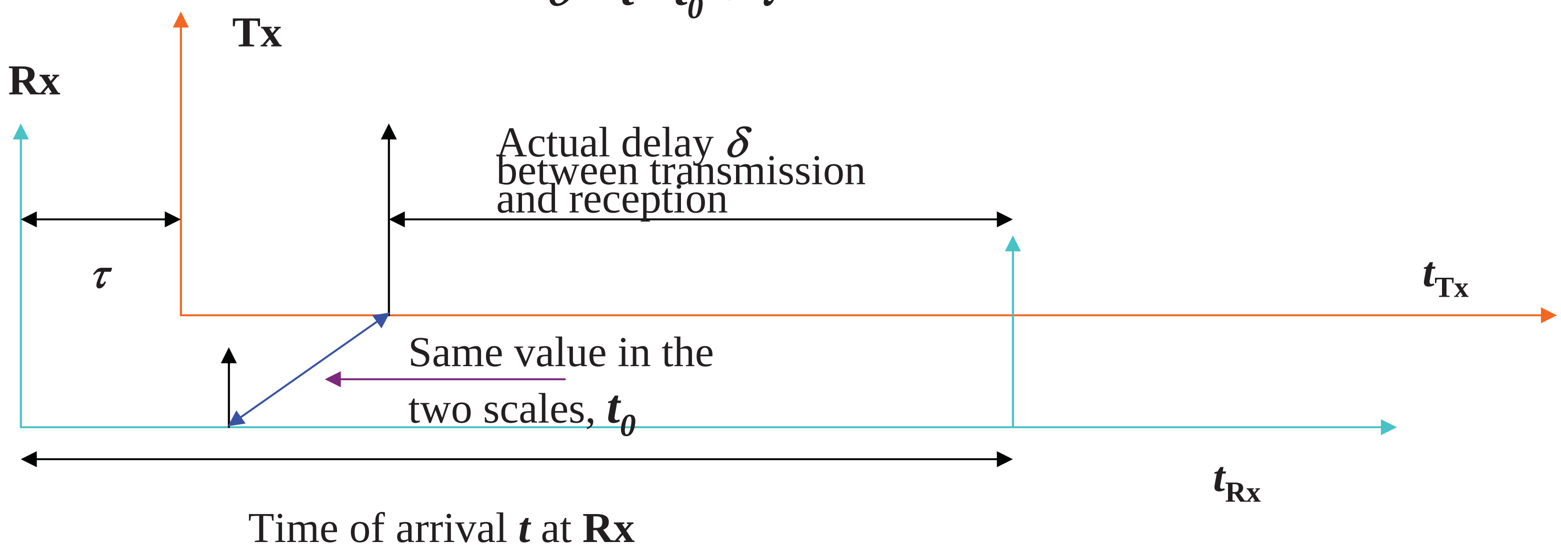


But:

Tx and **Rx** operate in their own time scale, affected by an offset τ.

Time t_0 for user is actually $t_0 + \tau$, so the measured value is

$$\delta = t - t_0 + \tau$$



The observation is affected by the time scale offset, so instead of measuring a ***range*** user actually gets a ***pseudo-range*** (to be better specified)

Using ***pseudo-range***

Positioning require the knowledge of at least three distances to solve for the three unknowns coordinates.

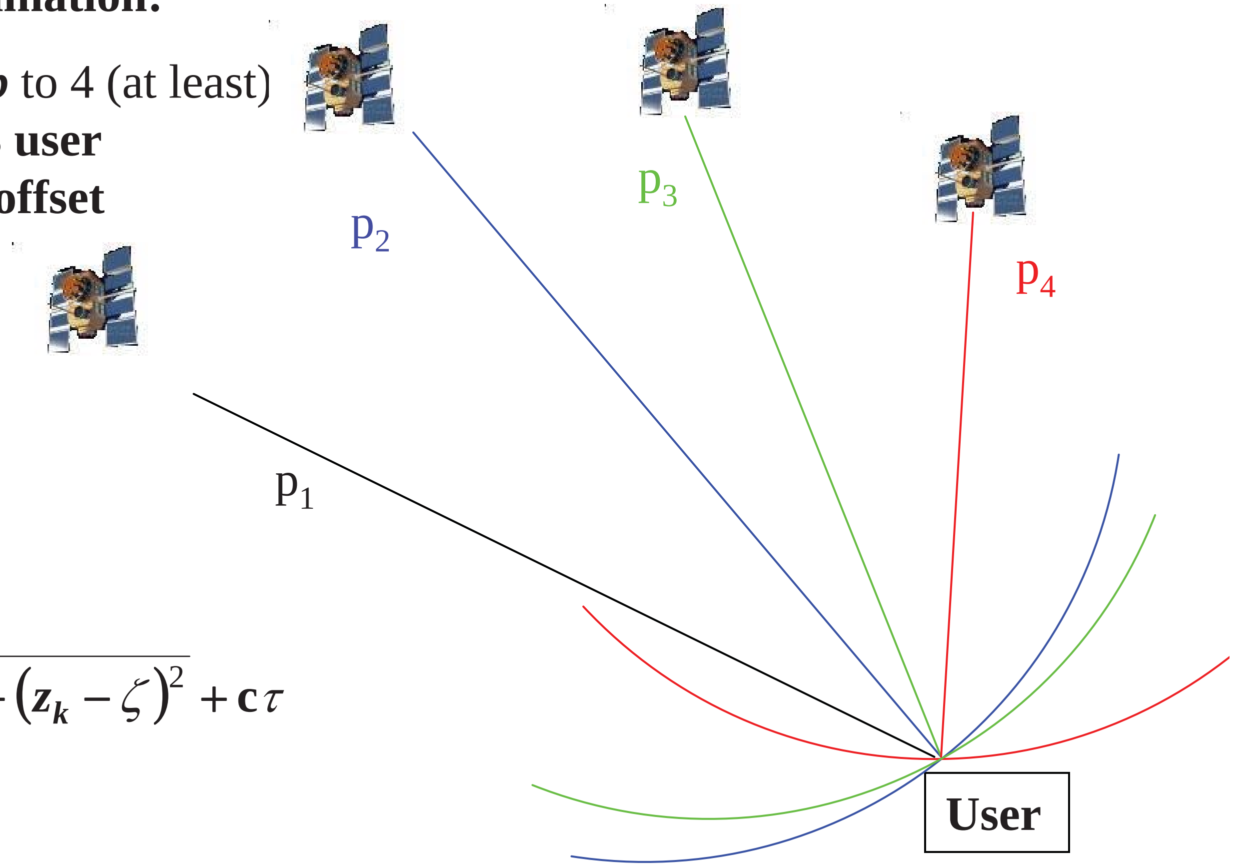
How proceeding with ***pseudo-distances***?

Provided all satellites operate in the same time scale (responsibility of Control Segment), only the user clock offset τ is unknown: **one unknown more**.

It is sufficient using at least four pseudo-range observations ***p***, solving for the three unknown coordinates ξ, η, ζ plus the unknown user clock offset τ

At a 1st order approximation:

Observation of pseudo-ranges ***p*** to 4 (at least) satellites enable estimation of **3 user coordinates** plus **1 user clock offset**



$$\boldsymbol{p_k = d_k + c\tau =}$$

$$= \sqrt{(x_k - \xi)^2 + (y_k - \eta)^2 + (z_k - \zeta)^2} + c\tau$$

$$\mathbf{k = 1,\dots,K \quad K \geq 4}$$

Actual pseudo-range measurements

Instead of pulses, satellites use special codes modulating the carrier

Some Radio terminology:

Satellites transmit sinusoidal signals

$$S(t) = A \cos [\, \phi(t) \,]$$

Instantaneous Phase $\phi(t) = \omega t + \phi_0$

Amplitude A (Volts, Volts/m)

Angular frequency $\omega = 2 p f$ (Radians/s)

Carrier frequency f (Cycles/s, Hertz)

From the super-position of sinusoidal monochromatic signals an amplitude and/or phase modulated signal S_M can be obtained

$$S_M = A_M(t) \cos [\omega t + \varphi_M(t)]$$

$A_M(t)$, $\varphi_M(t)$ amplitude and phase modulation

Phase Delay and Optical Path

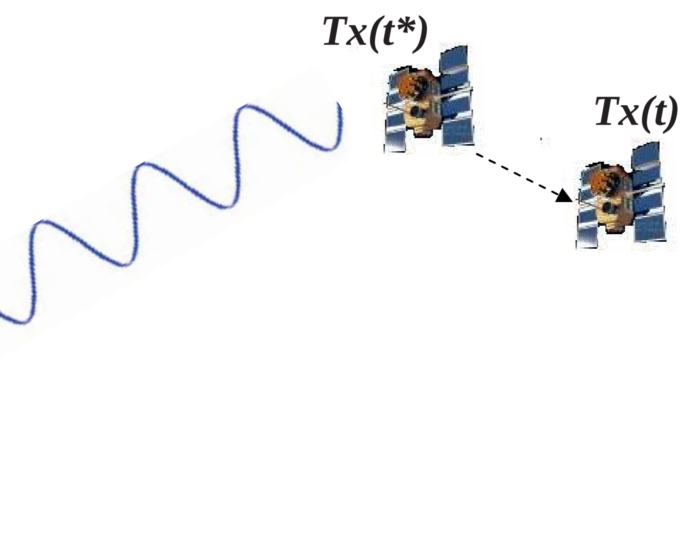
Sinusoidal signal (or super-position of …) is used.

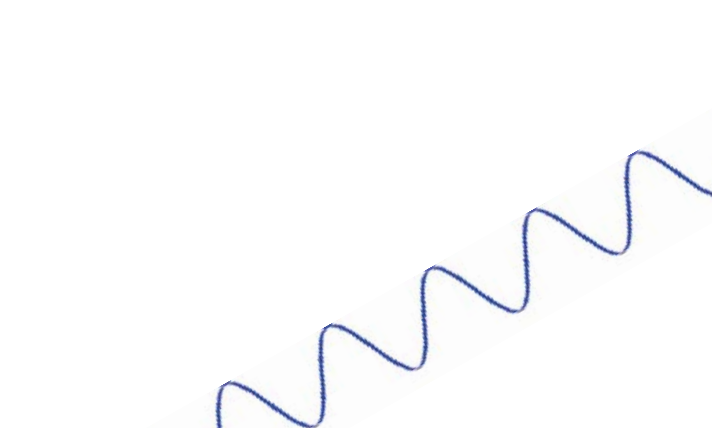
$$\boldsymbol{E_{Tx}(t) = A_{Tx}\cos(\omega t)},$$

Introducing the Optical path Λ and the wavelength λ in vacuum ($\lambda f = c$)

$$E_{Rx}(t) = A_{Rx}\cos\left[\omega\left(t - \frac{\Lambda(t^*)}{c}\right)\right] = A_{Rx}\cos\left[\omega\, t - 2\pi\frac{\Lambda(t^*)}{\lambda}\right]$$

$$t = t^* + \frac{\Lambda(t^*)}{c}$$





Phase delay in meters Λ **(Optical path)**

Phase delay in seconds $\tau = \dfrac{\Lambda}{c}$

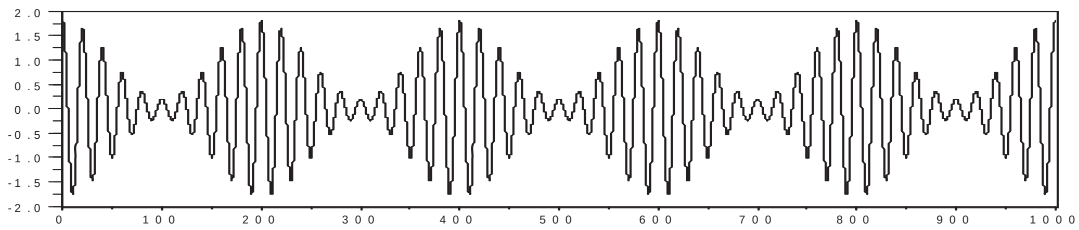
Phase delay in cycles $L = \dfrac{\Lambda}{\lambda}$

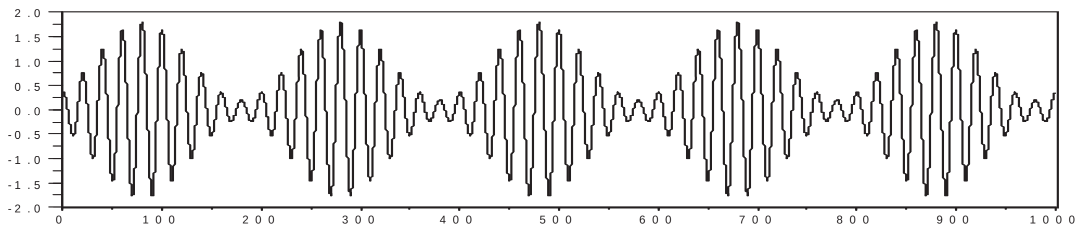
Group delay

Modulation of carrier is described as super-position of sinusoidal signals at different frequency

The delay affecting modulation is different from phase delay in dispersive propagation media

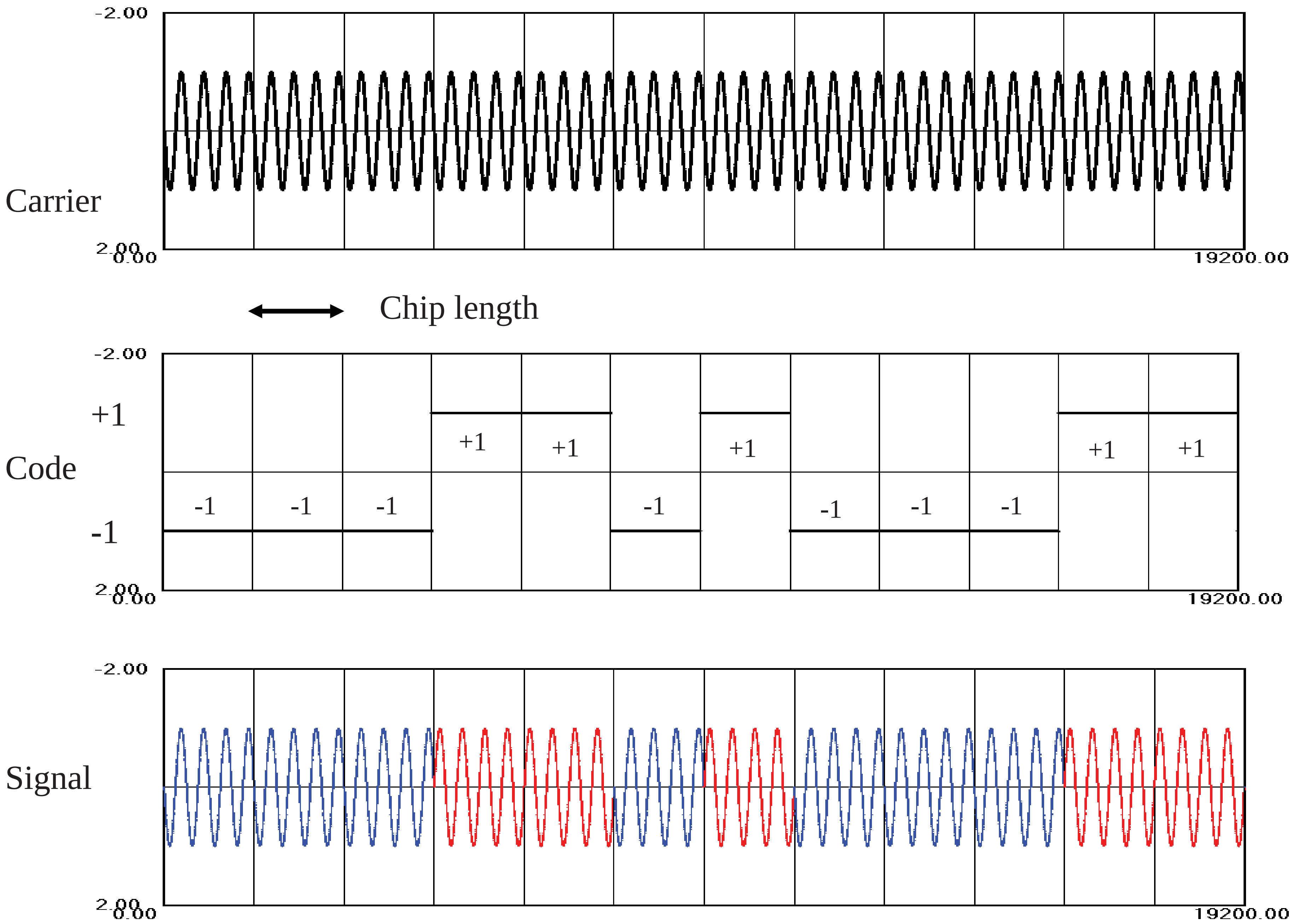
$$\tau_G = \frac{d\phi}{d\omega} = \frac{dL}{df}$$





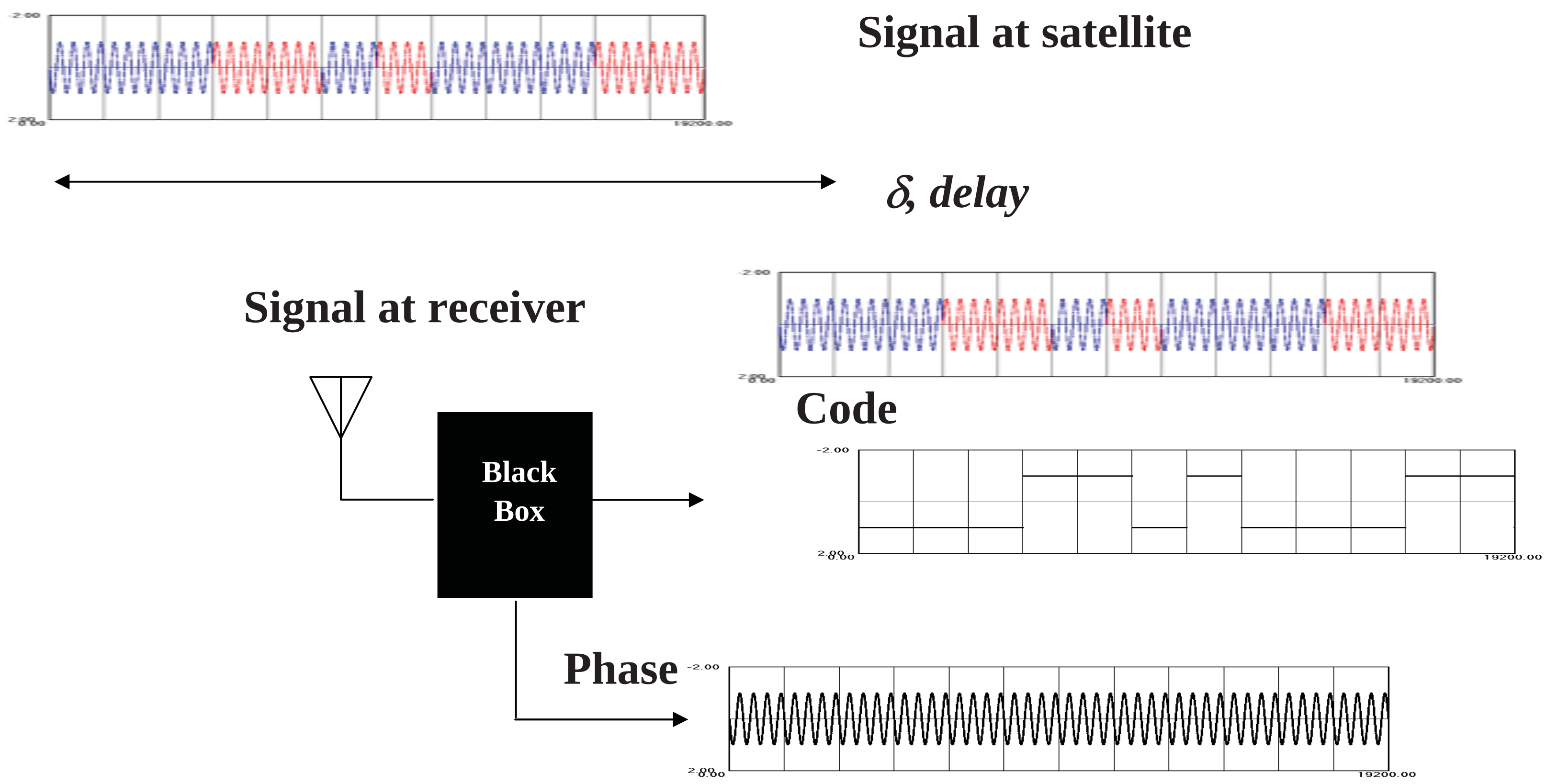
Group delay in seconds τ_G

Group delay in meters $\Lambda_G = c\,\tau_G$



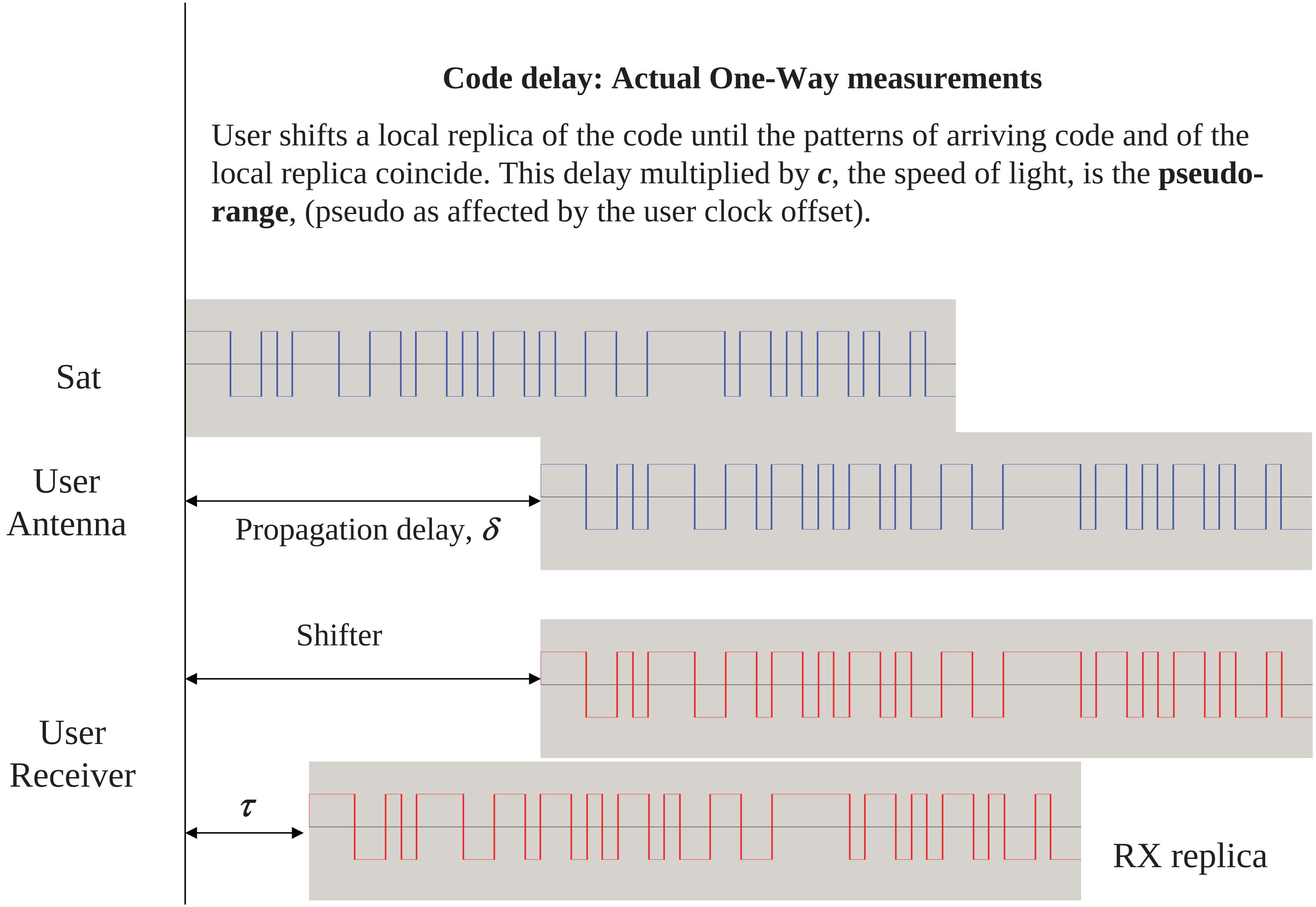
-2.00
Carrier
2.00
0.00
19200.00
Chip length
-2.00
+1
Code
-1
-1 -1 -1 +1 +1 -1 +1 -1 -1 -1 +1 +1
2.00
0.00
19200.00
-2.00
Signal
2.00
0.00
19200.00

Signal (delayed by the propagation time) is split into its original components, **Phase** and modulating **Code**



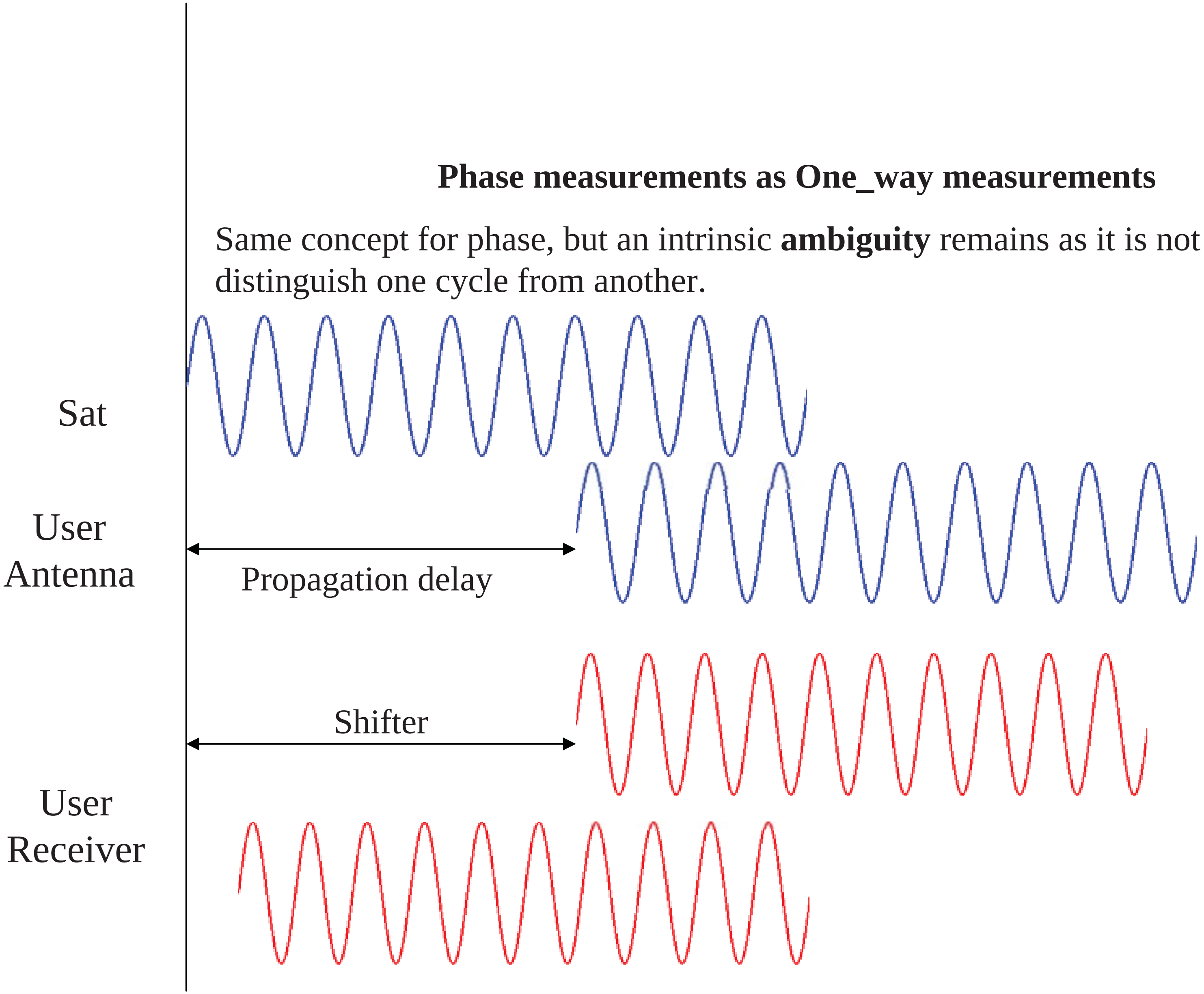
Code delay: Actual One-Way measurements

User shifts a local replica of the code until the patterns of arriving code and of the local replica coincide. This delay multiplied by ***c***, the speed of light, is the **pseudo-range**, (pseudo as affected by the user clock offset).



Phase measurements as One_way measurements

Same concept for phase, but an intrinsic **ambiguity** remains as it is not possible to distinguish one cycle from another.

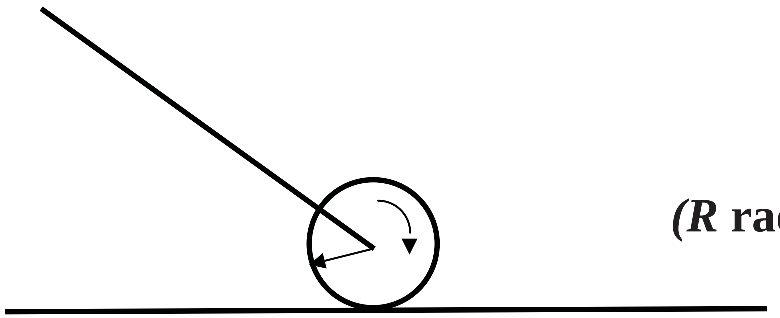


Phase ambiguity

Measuring phase is like measuring distance with an odometer

Apart the initial ambiguity $\boldsymbol{\Omega}$, the user can cumulate the cycles (***L***) of the incoming signal achieving very high resolution in the measurement of the distance

$$\boldsymbol{D = \Omega + L \cdot \lambda}$$



$$\boldsymbol{\lambda = 2\ \pi \cdot R}$$

(*R* radius of odometer)

Still increasing the order of approximation

The delays so far obtained are not actual distances to the satellites

Propagation occurs in not vacuum media

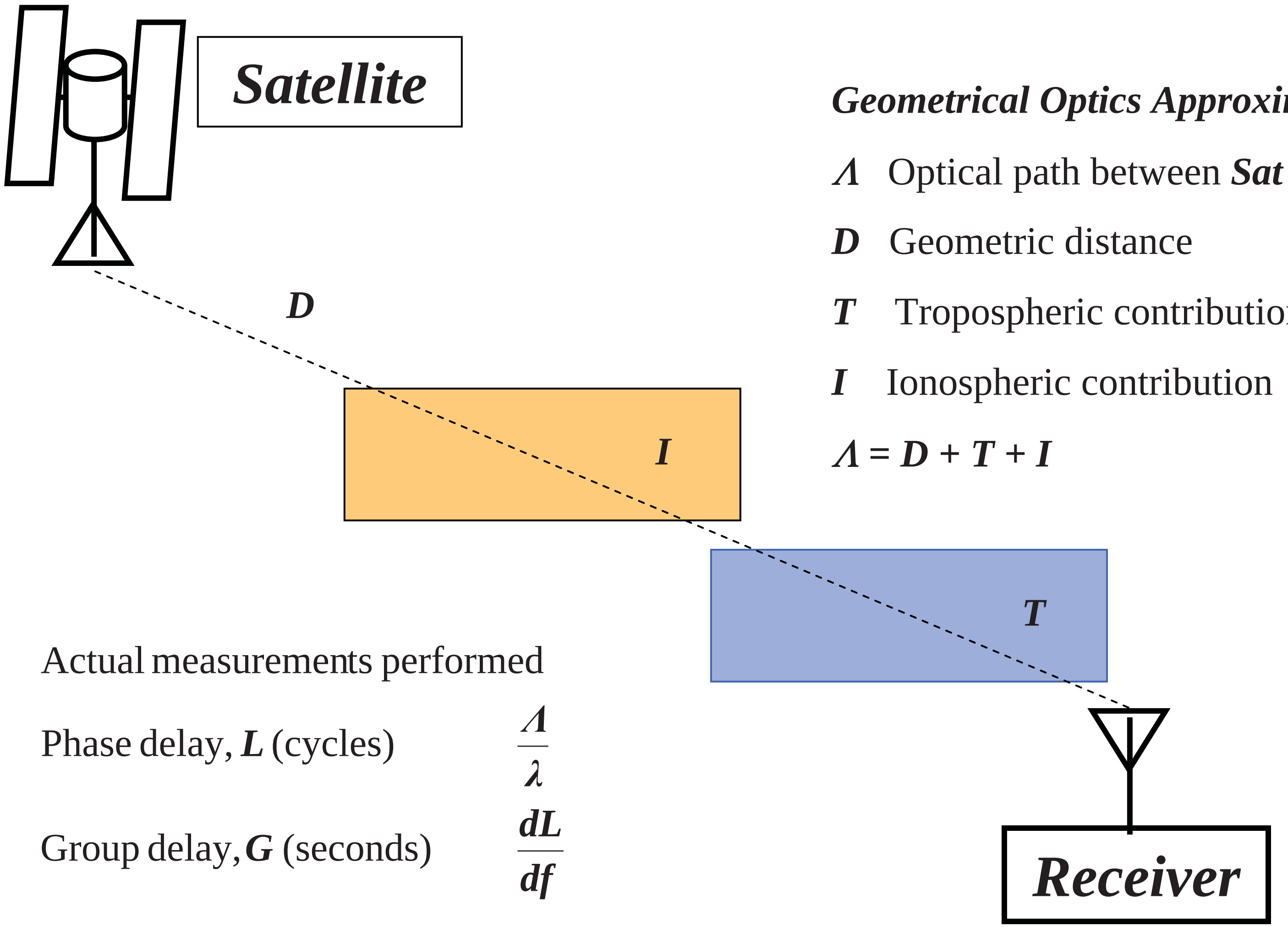
If not propagating in vacuum, signals excite the matter present, mainly free electrons in the ionosphere (contribution $\boldsymbol{I}$); molecules of N_2, O_2 and H_2O in the troposphere (contribution $\boldsymbol{T}$), which generate secondary waves introducing errors.

$$\boldsymbol{p_k = d_k + c\tau + T_k + I_k}$$

Some further step is needed in order to get the $\boldsymbol{d_k}$

Still, we shall see that propagation medium affects code and phase delays in a different way.

Propagation delays are derived by the Optical path



Geometrical Optics Approximation

Λ Optical path between ***Sat*** and ***Rec***

D Geometric distance

T Tropospheric contribution

I Ionospheric contribution

$\Lambda = D + T + I$

Actual measurements performed

Phase delay, L (cycles) $\quad \dfrac{\Lambda}{\lambda}$

Group delay, G (seconds) $\quad \dfrac{dL}{df}$

Aim of navigators ➔ ***D***

Magnitude of tropospheric (***T***) and ionospheric (***I***) contributions must be evaluated in order to correct for them

Ionospheric investigators ➔ ***I***

Geometric (***D***) and tropospheric (***T***) contributions must be eliminated

How to deal with above corrections/eliminations will be show in the following, but focusing on the ionospheric terms. For the ionospheric investigator it will be very easy to get rid of ***D*** and ***T***

The basic Physics needed is **Propagation of Electromagnetic Waves**

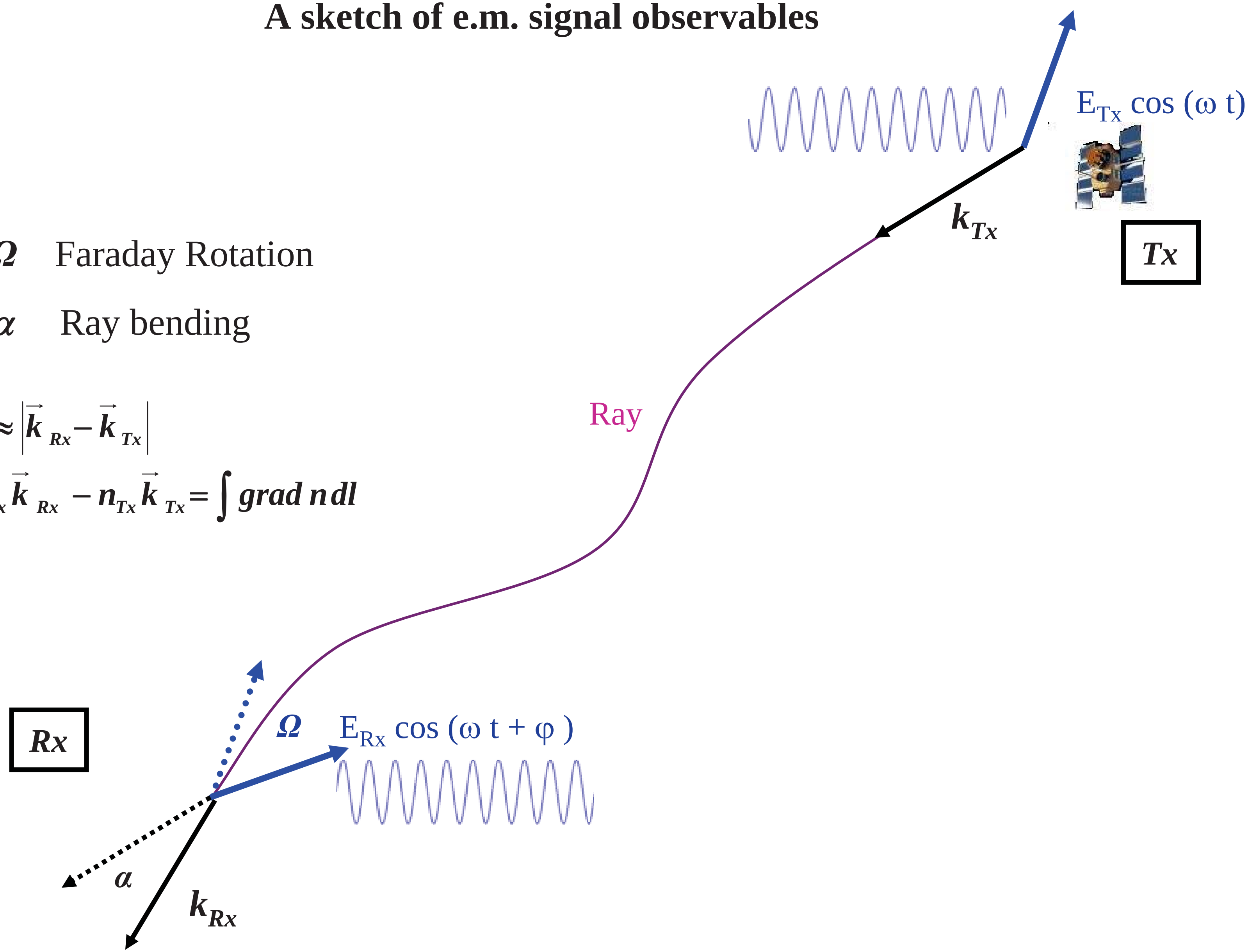
A sketch of e.m. signal observables

Ω Faraday Rotation

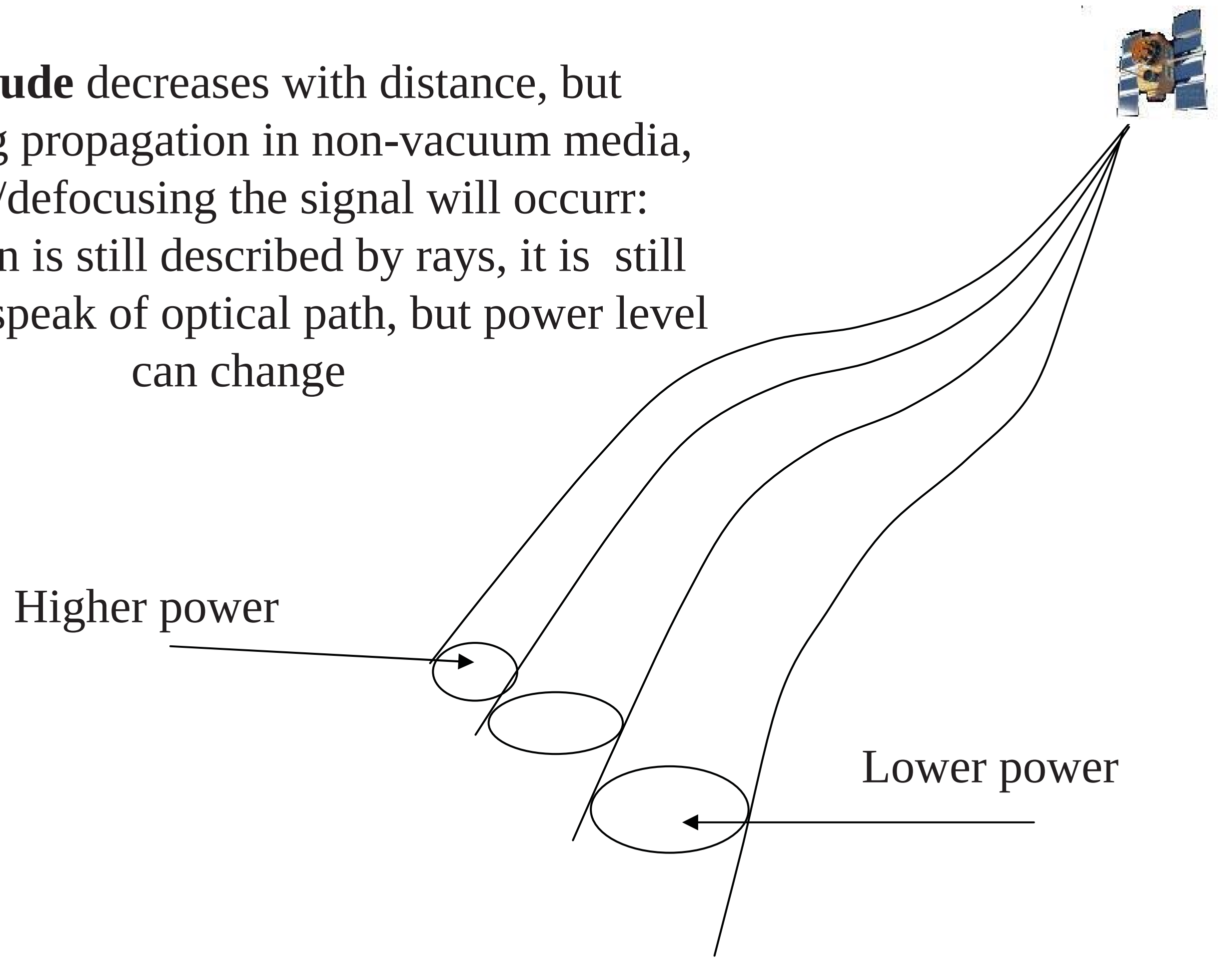
α Ray bending

$$\alpha \approx \left| \vec{k}_{Rx} - \vec{k}_{Tx} \right|$$

$$n_{Rx}\vec{k}_{Rx} - n_{Tx}\vec{k}_{Tx} = \int grad\, n\, dl$$



Amplitude decreases with distance, but considering propagation in non-vacuum media, focusing/defocusing the signal will occurr: propagation is still described by rays, it is still possible to speak of optical path, but power level can change



Signal undergoes random changes in amplitude (**Scintillation**)

Decrease of signal power does not affect in principle positioning performance: but receiver can lose lock

Related problem

Propagation can be described at several levels of approximation

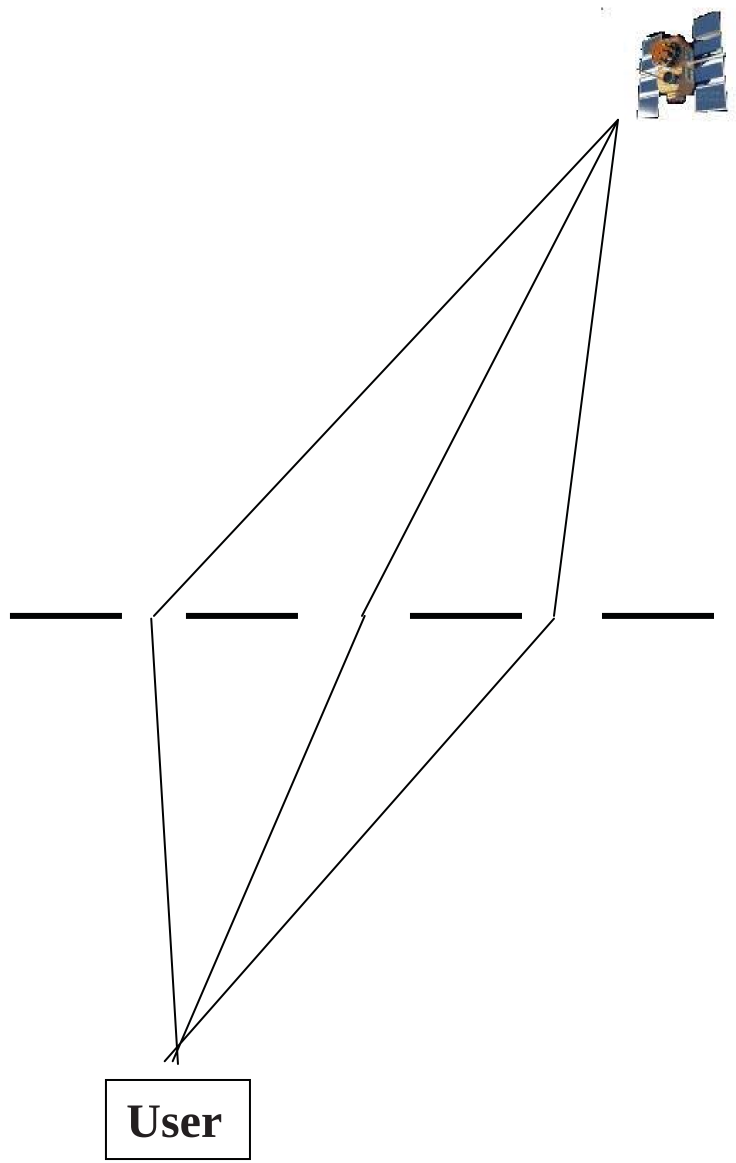
Geometrical Optics

Diffraction

Full wave solution

In presence of structures or turbulence, Geometrical Optics is no more suitable and the other two approaches must be used. In these situations, strong amplitude scintillation will be present increasing loss of lock.

But also phase scintillation will occur: how phase measurements are now related to the quantity needed, I.e. satellite-user distance?



Propagation delays

In order that things work properly model must be

Geometrical Optics

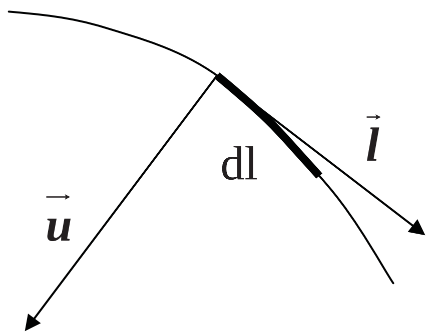
According to Geometrical Optics, the propagation of a **sinusoidal** e.m. signal can be described as occurring along **rays**, paths from the source to the target, defining the local direction of propagation: in vacuum these paths are straight lines).

Among all the paths from source to target, the ray is the one that minimizes the value of the Optical Path ***Λ*** defined as the line integral between source and target of the refraction index ***n***, a proper function of the place and the medium

$$\Lambda = \int_{Tx}^{Rx} n\,ds$$

The equation of the ray is

$$\frac{d(n\vec{l})}{dl} = grad\ n$$



Geometrical Optics and Propagation Delays

Once determined the actual ray path (**ray-tracing**), the line integral of refraction index ***n*** is computed, obtaining the optical path $\boldsymbol{\Lambda}$

$$\Lambda = \int_{Tx}^{Rx} n\,ds$$

The phase ***L*** (cycles) of the e.m. field along the path is given by the Optical Path $\boldsymbol{\Lambda}$ divided by the wavelength λ (in vacuum) of the carrier frequency

$$L = \frac{\Lambda}{\lambda} = \frac{f}{c}\Lambda$$

The code delay, affecting the modulation, is the group delay, given by the derivative of phase with respect to frequency

$$\delta = \frac{dL}{df}$$

What must be known at this point, is

The index of refraction *n* of the ionosphere

The index of refraction ***n*** of the ionosphere can be theoretically computed from the motion of the free electrons excited by the incoming signal: this computation has been performed by **Appleton and Hartree**

$$n^2 = 1 - \frac{X}{1 - jZ - \left(\frac{Y_T^{\,2}}{2(1 - X - jZ)}\right) \pm \left(\frac{Y_T^4}{4(1 - X - jZ)^2} + Y_L^2\right)^{\frac{1}{2}}}$$

$$X = \frac{\omega_N^2}{\omega^2} \qquad Y = \frac{\omega_B}{\omega} \qquad Y_L = \frac{\omega_B \cos\vartheta}{\omega} \qquad Y_T = \frac{\omega_B \sin\vartheta}{\omega} \qquad Z = \frac{\nu}{\omega}$$

$$\omega_N = \sqrt{\left(\frac{Ne^2}{\varepsilon_0 m}\right)} \qquad \omega_B = \frac{Be}{m}$$

Parameters of A-H formula

$\omega = 2\pi f$ angular frequency, radians/s (f signal frequency, cycles/s = hertz)

ω_N angular plasma frequency

ω_B angular gyromagnetic frequency

ν collision frequency

ε_0 dielectric constant of vacuum / permittivity of free space

e, m electron charge and mass

N_e electron density, electrons/m^3

B magnetic field

ϑ local angle between B and ray

The steps needed to evaluate propagation delays

Ray tracing using ***n*** from the Appleton-Hartree formula

Computation of Optical path along the ray

Computation of related quantities (phase and group delays)

How do actually look these computations for

Global Navigation Satellite Systems (GNSS)

GPS GLONASS GALILEO

Global Positioning System (GPS)

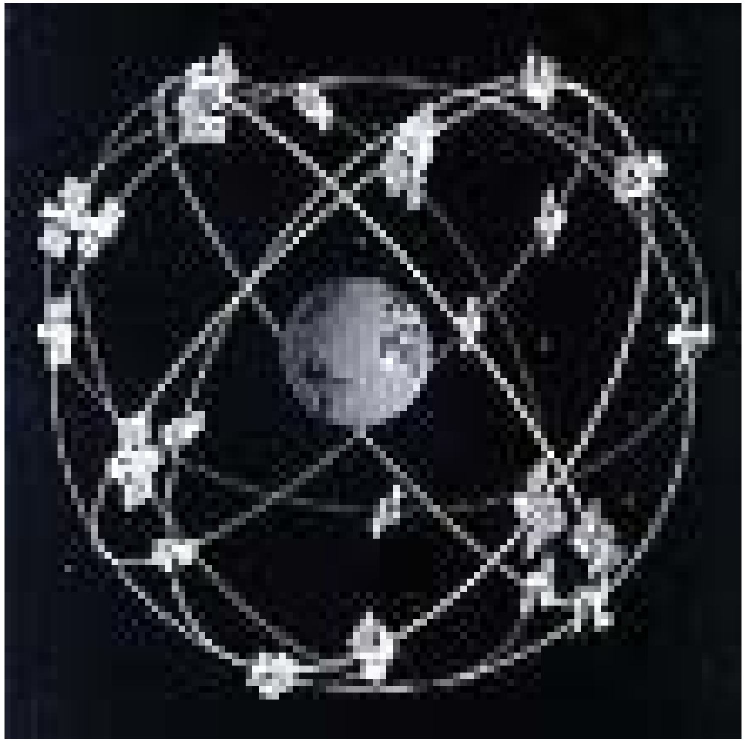
≈30 Satellites orbiting at 20.200 km

6 orbital planes

55º inclination circular orbits

12 sidereal hours period

Global: at any time, in any place at least 4 satellite visible in a geometry enabling good positioning



GALILEO

30 Satellites orbiting at 23 222 km

3 orbital planes

56º inclination circular orbits

14 hours period

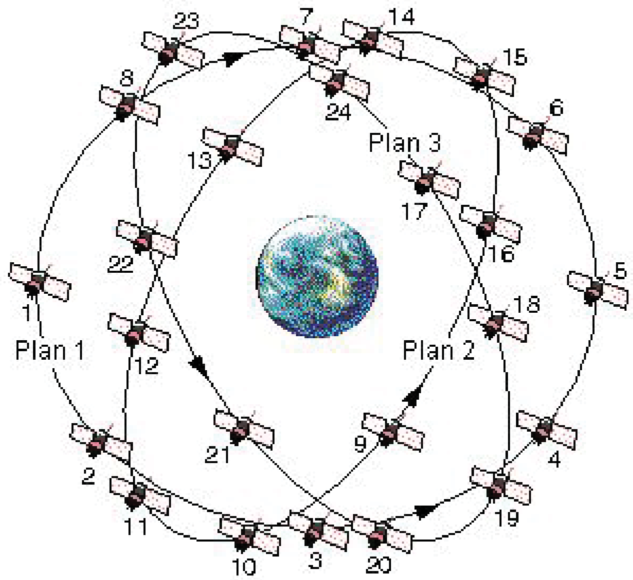
Claimed

better coverage in polar areas

Inter-operability with GPS

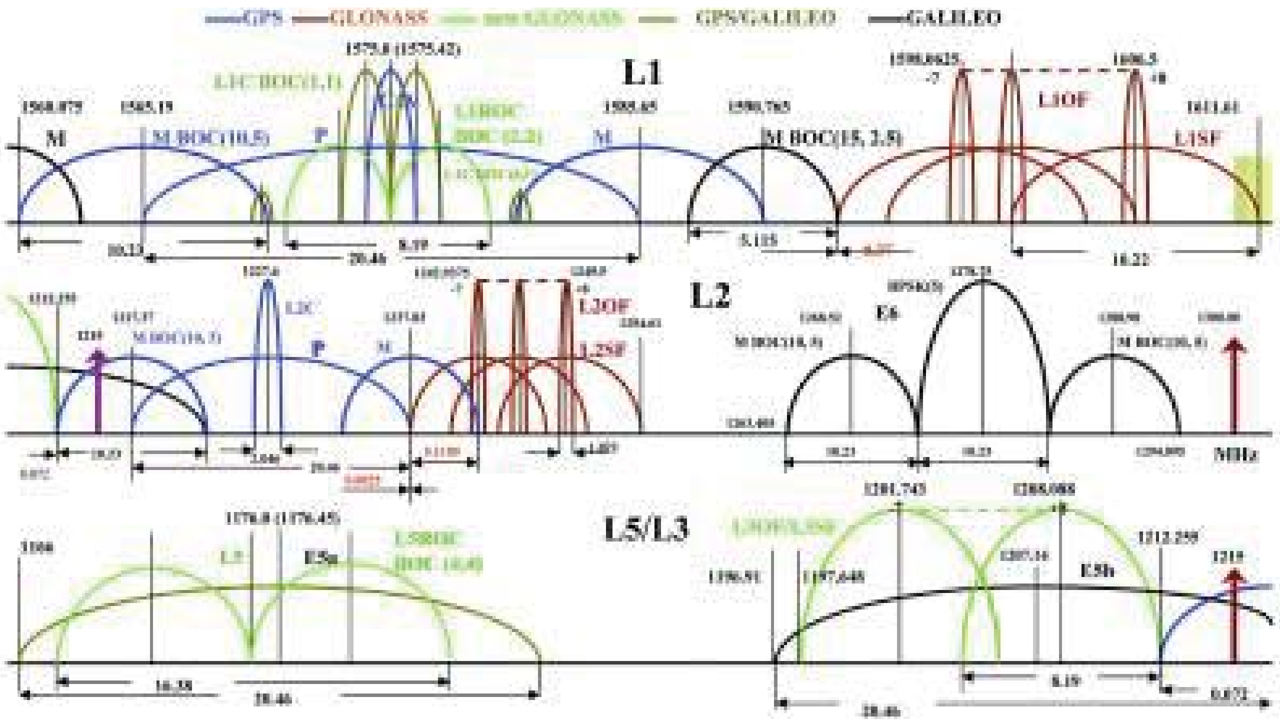
Carriers: see spectrum in next slide

A fully operational GLONASS constellation consists of 24 satellites. The three orbital planes' ascending nodes are separated by 120° with each plane containing eight equally spaced satellites. The orbits are roughly circular, with an inclination of about 64.8°, and orbit the Earth at an altitude of 19,100 km (11,868 mi), which yields an orbital period of approximately 11 hours, 15 minutes. GNSS use a spectrum centered mainly around the same frequencies: no significant difference will be noticed as concerns ionospheric effects.



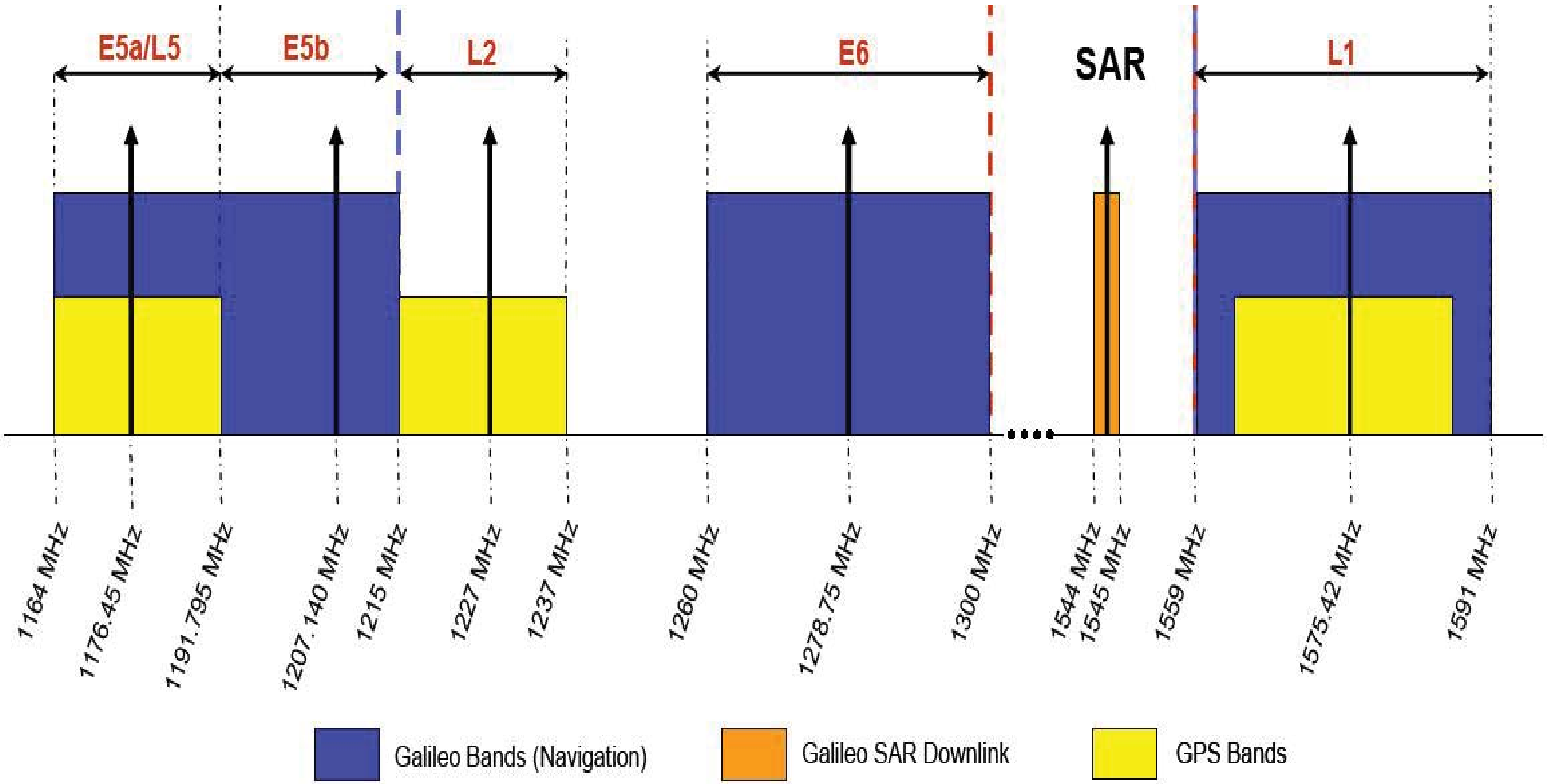
GNSS spectrum

www.positim.com/gnss_signals.html



GNSS spectrum

www.positim.com/gnss_signals.html



Some order of magnitude

The frequency of signals used for positioning has to be selected in order to make $\boldsymbol{n^2}$ as close as possible to unity (compatibly with international rules and status of art of technology).

Consider $\boldsymbol{N_e}$ = 10^{12} electrons/m^3 (a rather strong value)

Consider $\boldsymbol{f}$ = 1.5 GHz (good representative of GNSS frequencies)

It results X = 1 - $2 \cdot 10^{-5}$

For frequencies used in positioning, it can be used a first order approximation of the Appleton-Hartree formula :

$$\boldsymbol{n^2 = 1 - X}$$

Or expanding the square root at the first order

$$\boldsymbol{n = 1 - \frac{X}{2} = 1 - \frac{40.3 N_e}{f^2}}$$

***TEC*, the Total Electron Content**

Using the 1st order expansion,the Optical Path $\boldsymbol{\Lambda}$ becomes

$$\Lambda = \int \boldsymbol{nds} = \int \left(1 - \frac{40.3\boldsymbol{N_e}}{\boldsymbol{f}^2}\right)\boldsymbol{ds} =$$

$$= \boldsymbol{S} - \frac{40.3}{\boldsymbol{f}^2}\int \boldsymbol{N_e ds} = \boldsymbol{S} - \frac{40.3}{\boldsymbol{f}^2}\boldsymbol{TEC}$$

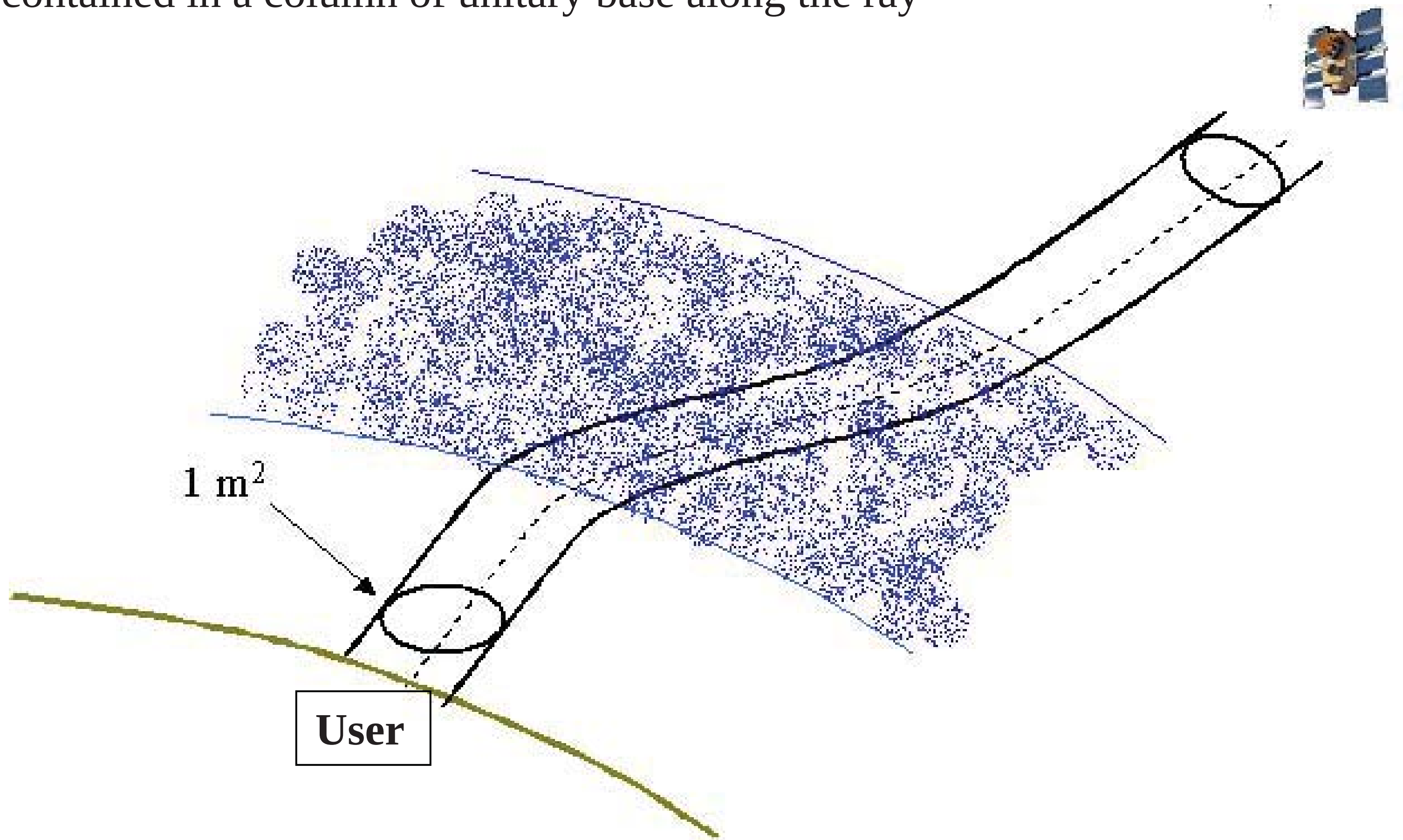
$$\boldsymbol{TEC} = \int \boldsymbol{N_e ds}$$

The optical path in the ionosphere is given by the geometrical length of the ray, ***S***, plus the ionospheric contribution proportional to the

Total Electron Content (***TEC***)

Given by the total number of free electrons contained in a column of unitary base along the ray.

The classical interpretation for ***TEC*** as the numbers of electrons contained in a column of unitary base along the ray



TEC is the ionospheric contribution to optical path

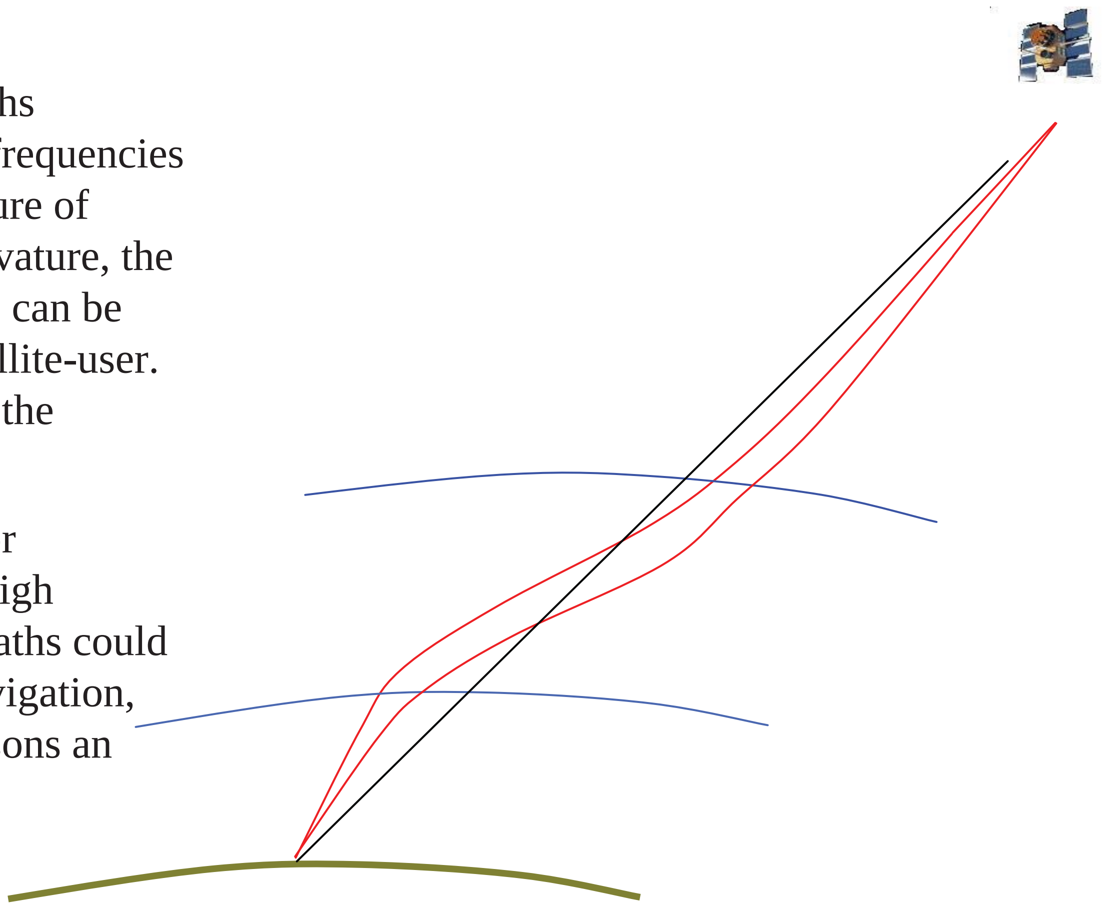
(apart the sign)

In the above approximation, the curvature of the ray

$$\frac{1}{R} = \vec{u} \cdot \frac{grad\, n}{n}$$

is a very small quantity too, so paths corresponding to different GNSS frequencies differ very little. Given the departure of chord from arc is of order 3 in curvature, the computation of line integrals for L can be performed on the straight line satellite-user. In simulations one can take safely the straight line for both paths.

Only at very low elevations and for applications requiring extremely high accuracy the non coincidence of paths could have some impact: nothing for navigation, when for completely different reasons an elevation mask of 10º is used.



Ionospheric Phase (L) and Code (δ) Delays for GNSS

Phase (<u>cycles</u>)

$$L = \frac{\Lambda}{\lambda} = \frac{1}{\lambda}\left(S - \frac{40.3}{f^2} TEC \right) = \frac{S}{\lambda} - \frac{40.3}{\lambda f^2} TEC = \frac{f}{c} S - \frac{40.3}{cf} TEC$$

Code (<u>seconds</u>) $\left(\delta = \frac{\partial L}{\partial f} \right)$

$$\delta = \frac{S}{c} + \frac{40.3 TEC}{cf^2}$$

Preferred measurement unit in positioning: ***length***

Instead of using cycles and seconds better using ***paths***

Phase: Optical path = $\boldsymbol{L \cdot \lambda}$

Code: Range = $\boldsymbol{c \cdot \delta}$

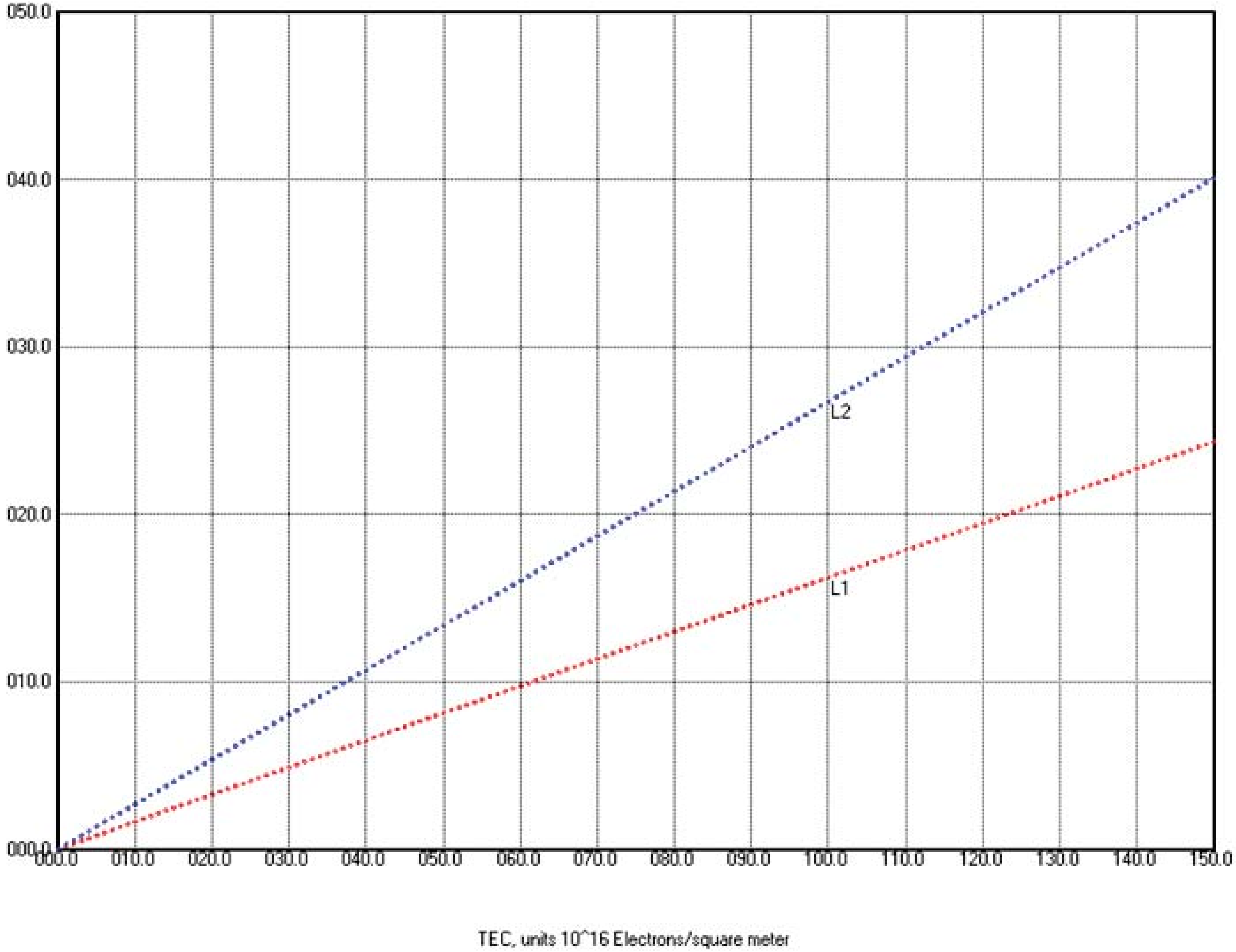
Phase path = Optical path $\qquad \boldsymbol{S - \frac{40.3}{f^2} TEC}$

Code path = $\boldsymbol{c}$ · Code delay $\qquad \boldsymbol{S + \frac{40.3}{f^2} TEC}$

Under the assumed approximation of the Appleton-Hartree formula:

distance measurements using phase delay and code delay provide with an estimation of actual distance $\boldsymbol{S}$ plus a ionospheric contribution which in absolute value is the same for phase and code, but with opposite sign.

Computing the ionospheric contribution at ***L1*** and ***L2***
(GPS)



Keeping in mind the order of magnitude of the ionospheric contribution, it depends on the accuracy needed by a given application deciding to take it in account or not.

Designing the GNSS's, the accuracy required was of few meters in real time: a rather low electron content of 10 ***TEC*** units produces at ***L*** band a ionospheric error of 1.63 m, which can become more than 10 times larger at high solar activity.

GNSS's must be able to correct for ionosphere

Ionospheric correction is achieved taking simultaneous measurements at two different frequencies

Since the beginning of satellite positioning, all satellite navigation systems were provided with two (at least) carriers

Ionosphere is a **dispersive** medium: index of refraction depends on frequency in a perfectly known way (Appleton-Hartree). Using two frequencies

$$M_1 = S \pm \frac{40.3TEC}{f_1^2}$$

$$M_2 = S \pm \frac{40.3TEC}{f_2^2}$$

One gets a system of two observation equations, (***M1, M2,*** code or phase, just select the proper sign) in the two unknowns ***S*** and ***TEC***, which can be easily solved (also in real time by the receiver).

Warning: equations for ionospheric correction should be written

$$M_1 = S_1 \pm \frac{40.3Tec_1}{f_1^2}$$

$$M_2 = S_2 \pm \frac{40.3Tec_2}{f_2^2}$$

As the ray paths at different frequencies are different, but in the approximation used this can be disregarded.

The ionospheric corrections and the practical situation for GPS

The correction of ionospheric contribution by the two frequency method seems to allow users to state that ionosphere is not a problem:

True, but only if the user is U.S.A. military personnel,

or Anti-spoofing (AS) is off

P codes on ***L1*** and ***L2*** may be encrypted: evaluation of ***P1*** and ***P2*** is still possible to civil users using sophisticated wide band receivers equipped with non linear correlators, which imply

- significant increase of Signal to Noise Ratio
- long integration times to extract reconstructed carrier phases and pseudo-ranges.

This makes real-time operation at dual-frequency un-practical

and un-reliable for the civil user

When real time is not a problem, these "civil receivers" allow very accurate positioning to Geodesists and Geophysicists, as performed by the International Global Navigation Satellite System (**IGS**, formerly International GPS Service) for tectonic and crustal movements. Their use is essential in the "augmentation" techniques

The situation for civil users

According to ***DoD*** (U.S.A. Department of Defense) rules, civil users can freely access ***C/A*** code (the ***C1*** pseudo-range) present on ***L1*** only (phase too)*. This enables real-time positioning too.

How behaving with ionospheric error?

1. Neglecting it
2. Using a broadcast model
3. Using **Satellite Based Augmentation Systems** (SBAS)

* Possible degradation introducing Selective Availability (SA)

1. **Neglecting ionospheric error**

Single-point Positioning

Satisfactory for accuracy up to 25÷30 m, possible (and unpredictable) decreasing (> 70 m ?) during high solar activity and SA. Acceptable at the "excursion" level (cars, boats, ..)

Differential GPS (Translocation)

The ionospheric error is the same for users operating in a small area (up to 10 km)

One reference station of known coordinates evaluates the ionospheric error in the form "computed-known" coordinates. Other user subtract this error from their computed coordinates obtaining much higher accuracy. The technique is able to operate in real time.

In the same conditions, surveyors (using phase and "interferometric" observables) succeed to achieve centimeter accuracy

Warning:

Both “Using a broadcast model” and “ Using **Satellite Based Augmentation Systems** (SBAS)” assume that the system is able to provide with reliable forecast values of ionospheric parameters, i.e. ***TEC***.

This anticipates the need to be able to compute ***TEC***, or to perform

Calibration

the topic which will be developed in the following.

2. **Using a broadcast model**

(Single point positioning, global performance)

Control Segment implements evaluation and forecasting of ***TEC*** based on some model of the ionosphere and its expansion through a proper set of coefficients.

The set of coefficients is transmitted to the satellites and stored in the satellite message

The user gets the coefficients from the message and evaluates the ionospheric correction

Interesting to report how GPS and GALILEO solve the problem

GPS

GPS uses the Klobuchar model, based on the thin shell approximation.

The ionosphere is confined into an infinitesimally thin shell located at some reference height. On this surface it is defined a Vertical TEC, ***V(P)***

V(P) is globally mapped through a trigonometric expansion (8 coeffs) transmitted to the satellites.

User computes V(P) from the expansion and gets ***TEC*** through the mapping function ***sec*** χ

Reduction of ionospheric error at 50% minimum is claimed, but sometimes updating the coefficients by the GPS Ground Segment not sufficiently fast.

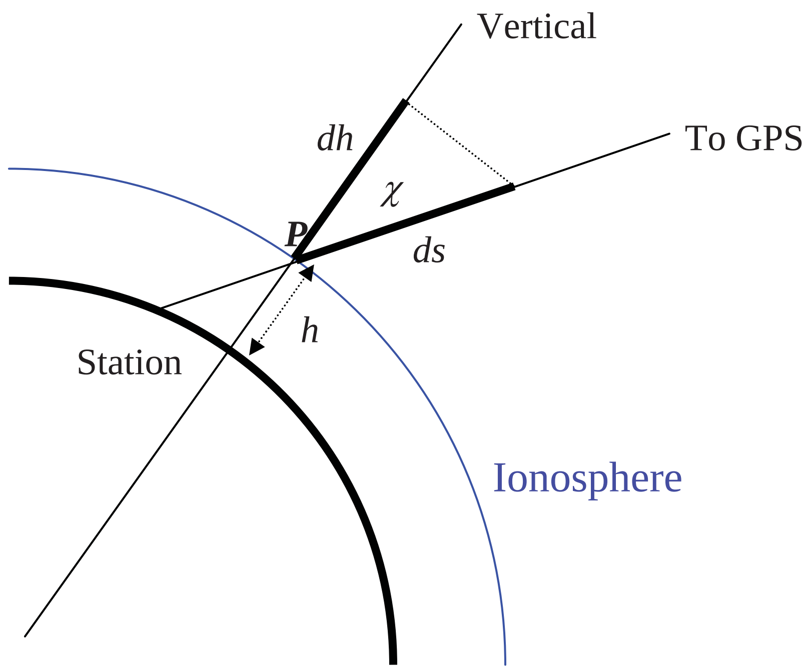
Still, the mapping function assumptions introduces errors which can be significant

The thin-shell approximation

$$\boldsymbol{TEC = V(P)\ sec\ \chi + \beta + \gamma}$$

V(P) is the ***TEC*** along the vertical to the ionospheric point ***P***

V(P) is a 2D function of horizontal coordinates



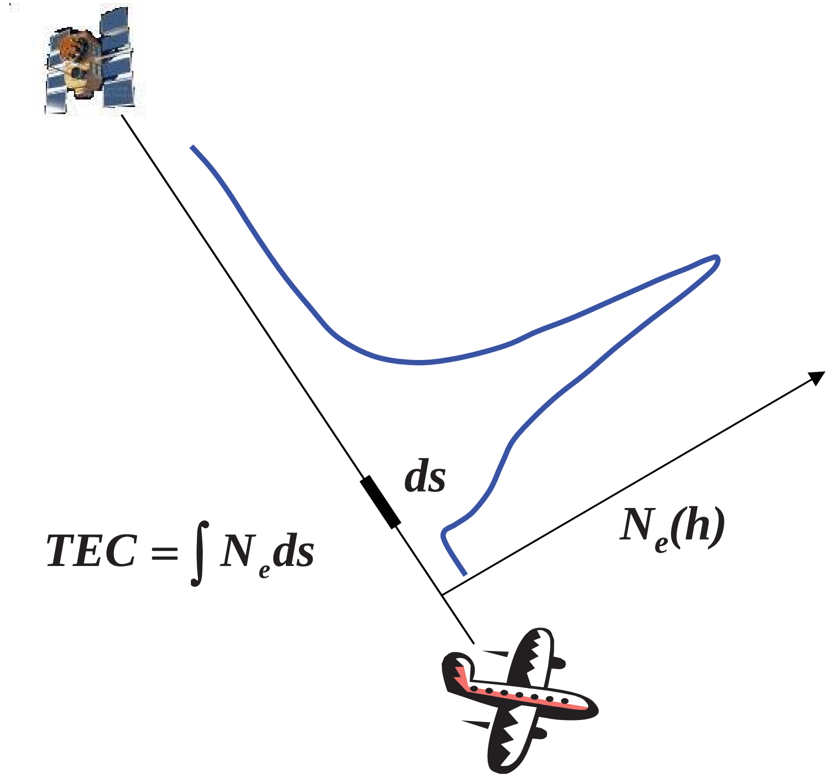
GALILEO

Ionospheric correction is evaluated integrating a 3D model of electron density. The model used is **NeQuick**, driven by one global parameter described by an expansion of very low (2nd) order.

Ground segment estimates the coefficients of the expansion which are transmitted to the satellites.

User receives the coefficients and avails (firmware) the **NeQuick** model computing the correction avoiding the problems of mapping function.

This method has been proposed by Prof. Sandro Radicella (ICTP, Trieste), to whom is also due the NeQuick model implementation together with Dr. Reinhart Leitinger (University of Graz, Austria)



Note

GALILEO operates on 3 frequencies.

Why caring so much a correction needed for single frequency operation?

Possible explanation

GALILEO is a civil GNSS: probably different levels of accuracy will be available, some free of charge, some paying.

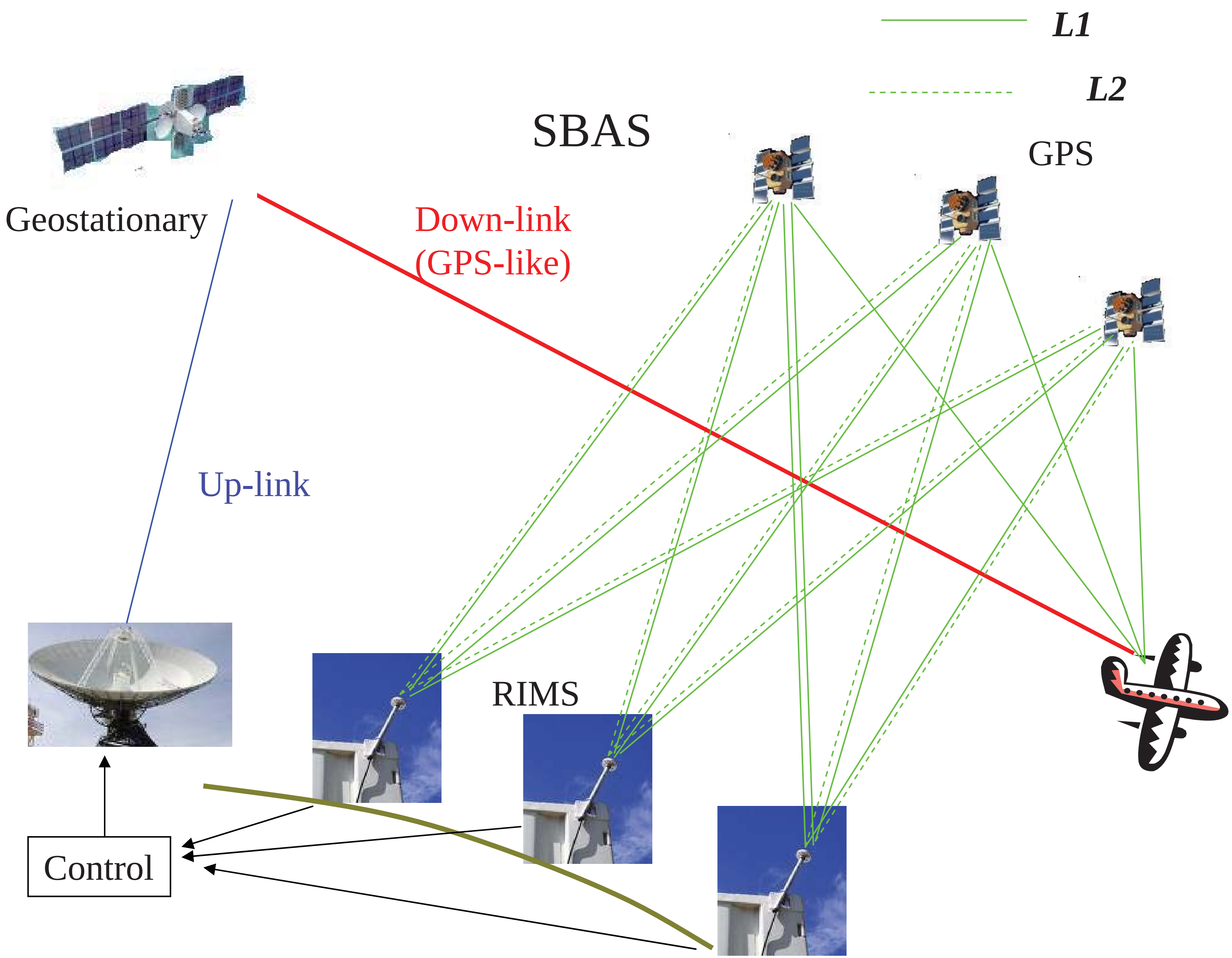
A single-frequency system is very simple and cheap, but it requires ionospheric correction

GALILEO is required to perform very high integrity level: if for any reason two carriers are lost, the user will be able to get still good accuracy

3. **Using SBAS**

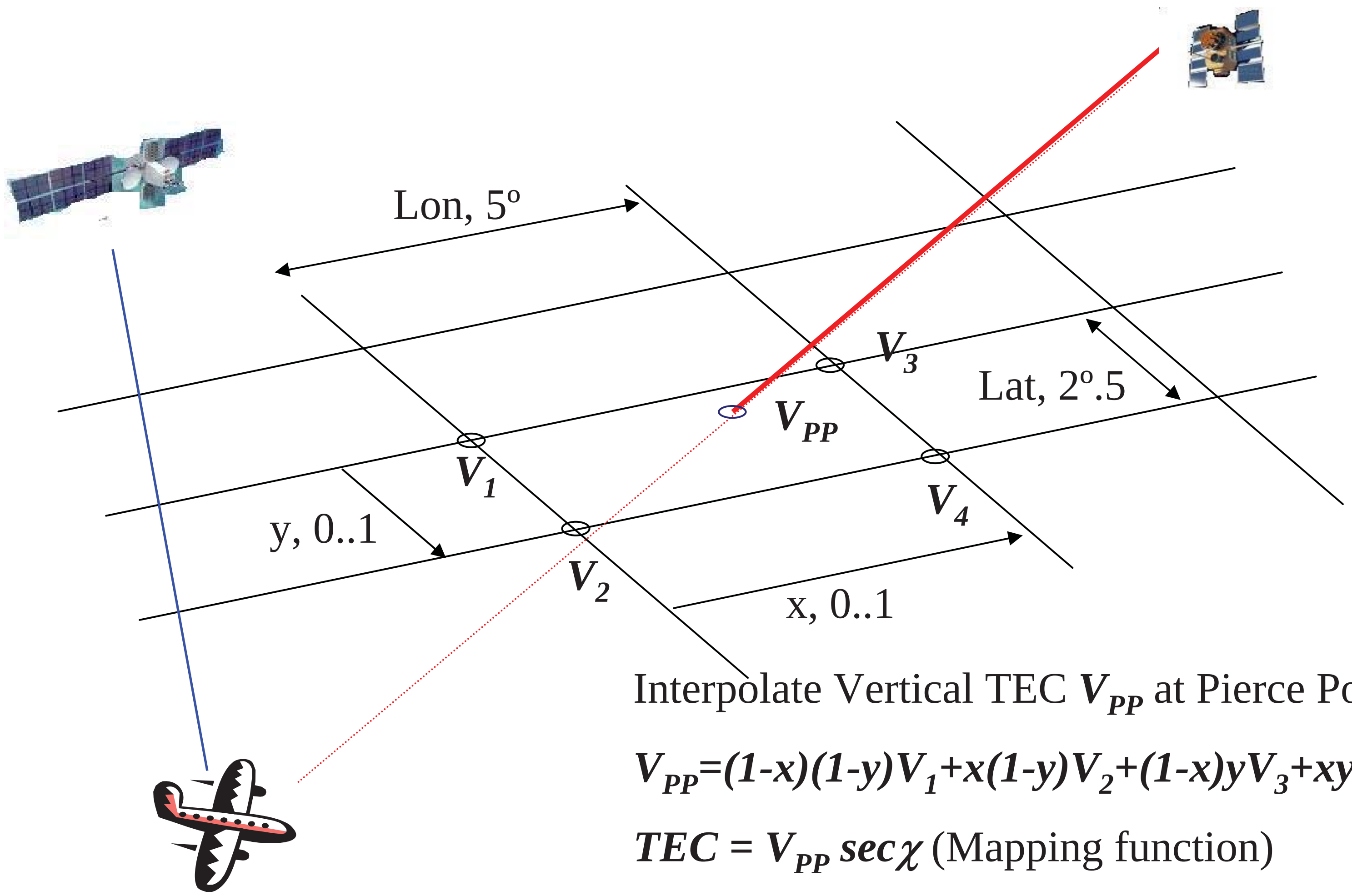
A ground network of stations (RIMS) equipped with dual-frequency ("civil") GPS receivers processes the GPS signals in order to evaluate ionospheric content in quasi-real time. The information is transmitted every ≈10^{th} minute to a Geo-stationary satellite

User receive GPS and GEO signals and evaluate them with proper algorithms the needed ionospheric corrections.



L1
L2
SBAS
GPS
Geostationary
Down-link
(GPS-like)
Up-link
RIMS
Control

The actual evaluation of Iono correction in SBAS



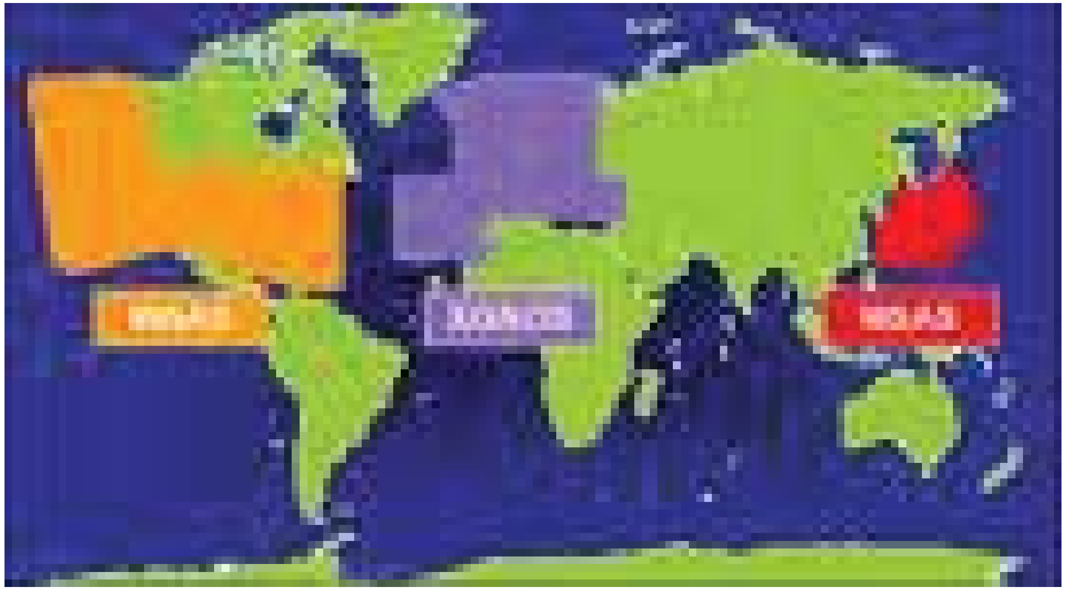
Interpolate Vertical TEC V_{PP} at Pierce Point ***PP***

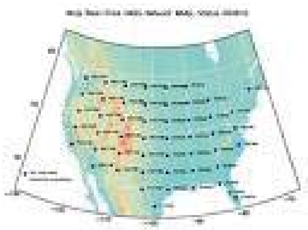
$$V_{PP}=(1-x)(1-y)V_1+x(1-y)V_2+(1-x)yV_3+xyV_4$$

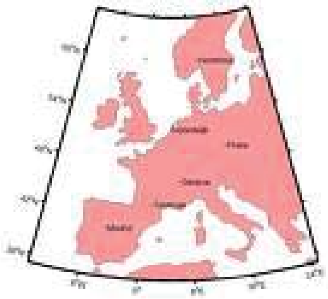
$$TEC = V_{PP}\sec\chi \text{ (Mapping function)}$$

SBAS implementation

WAAS EGNOS MSAS



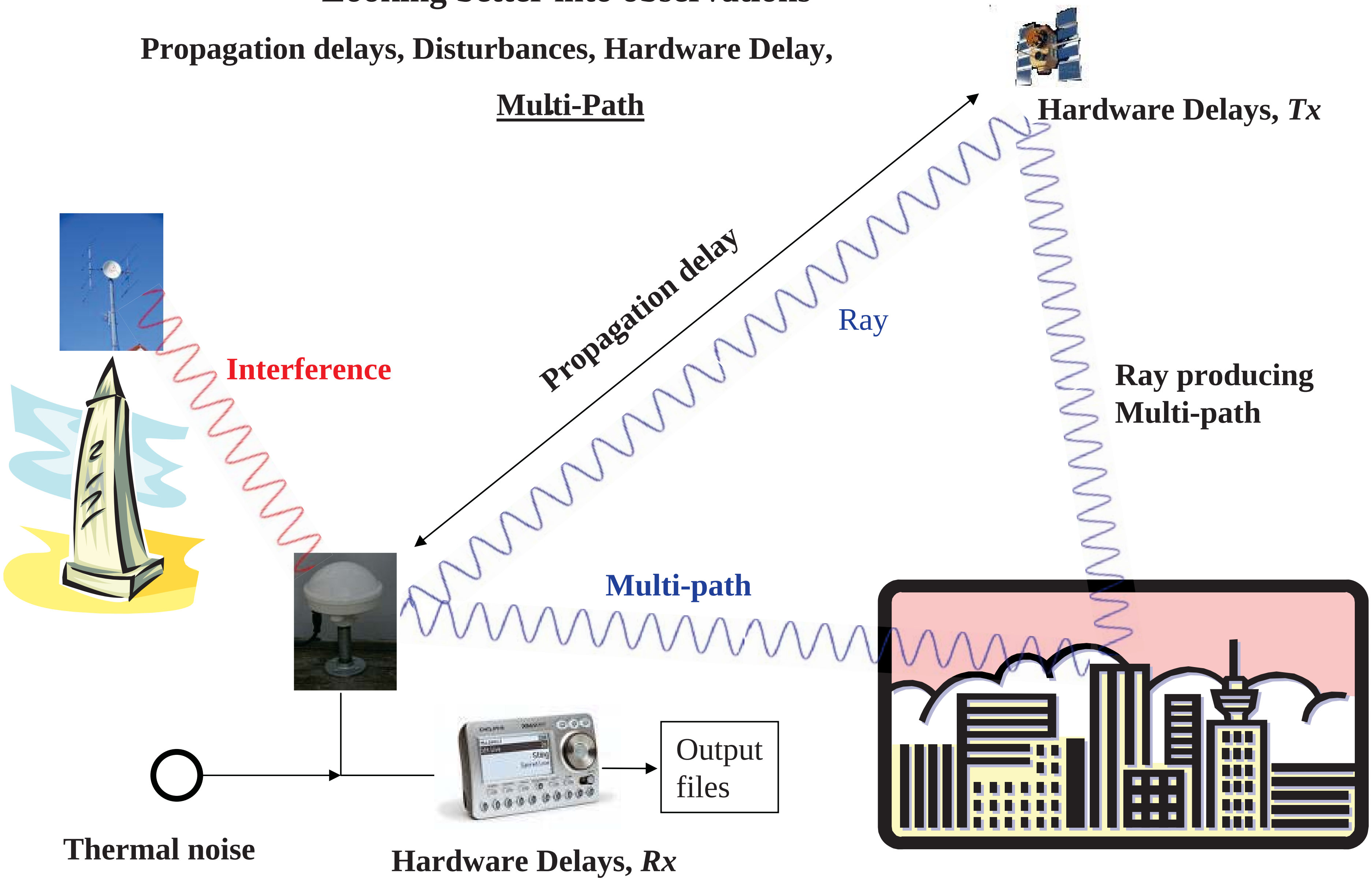




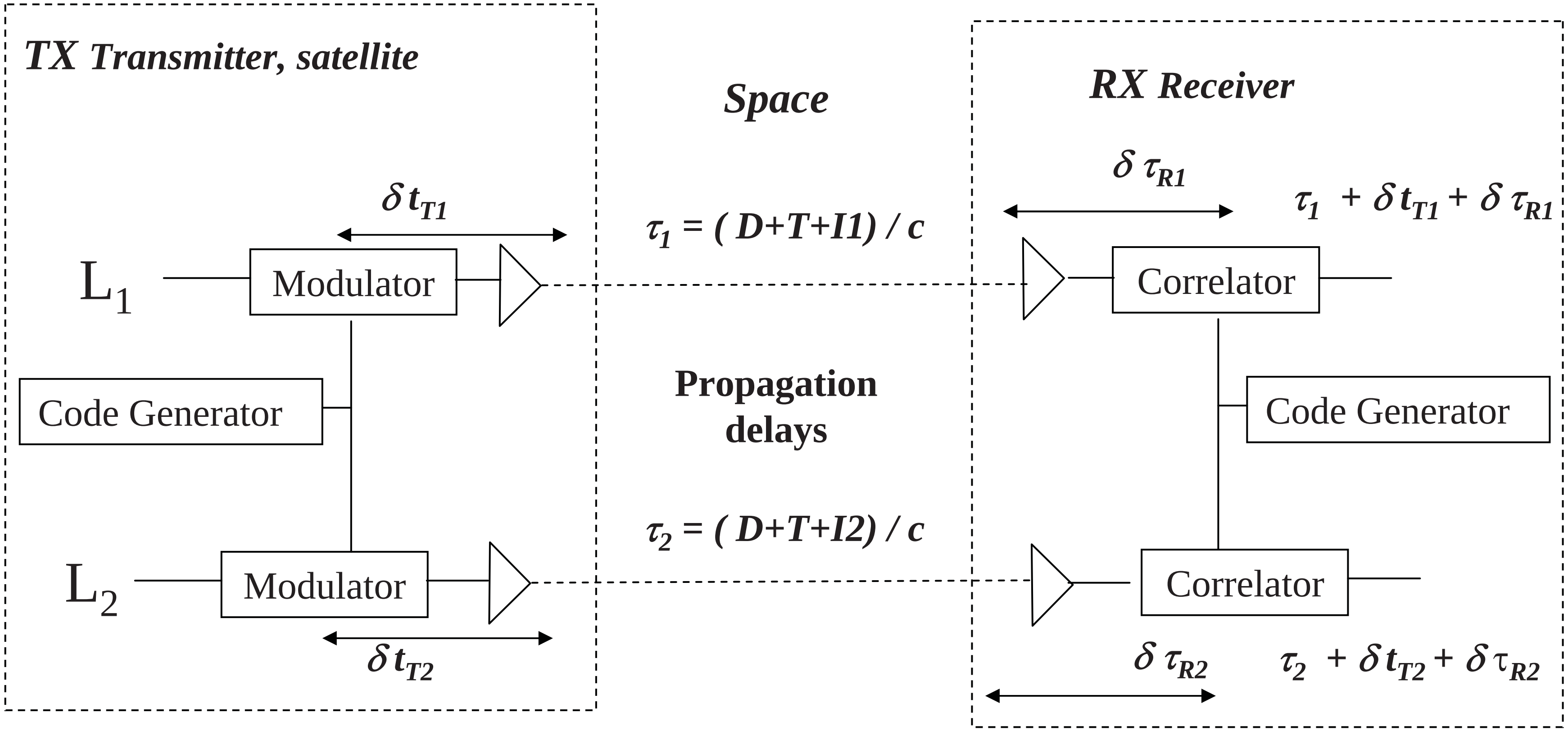
Looking better into observations

Propagation delays, Disturbances, Hardware Delay,

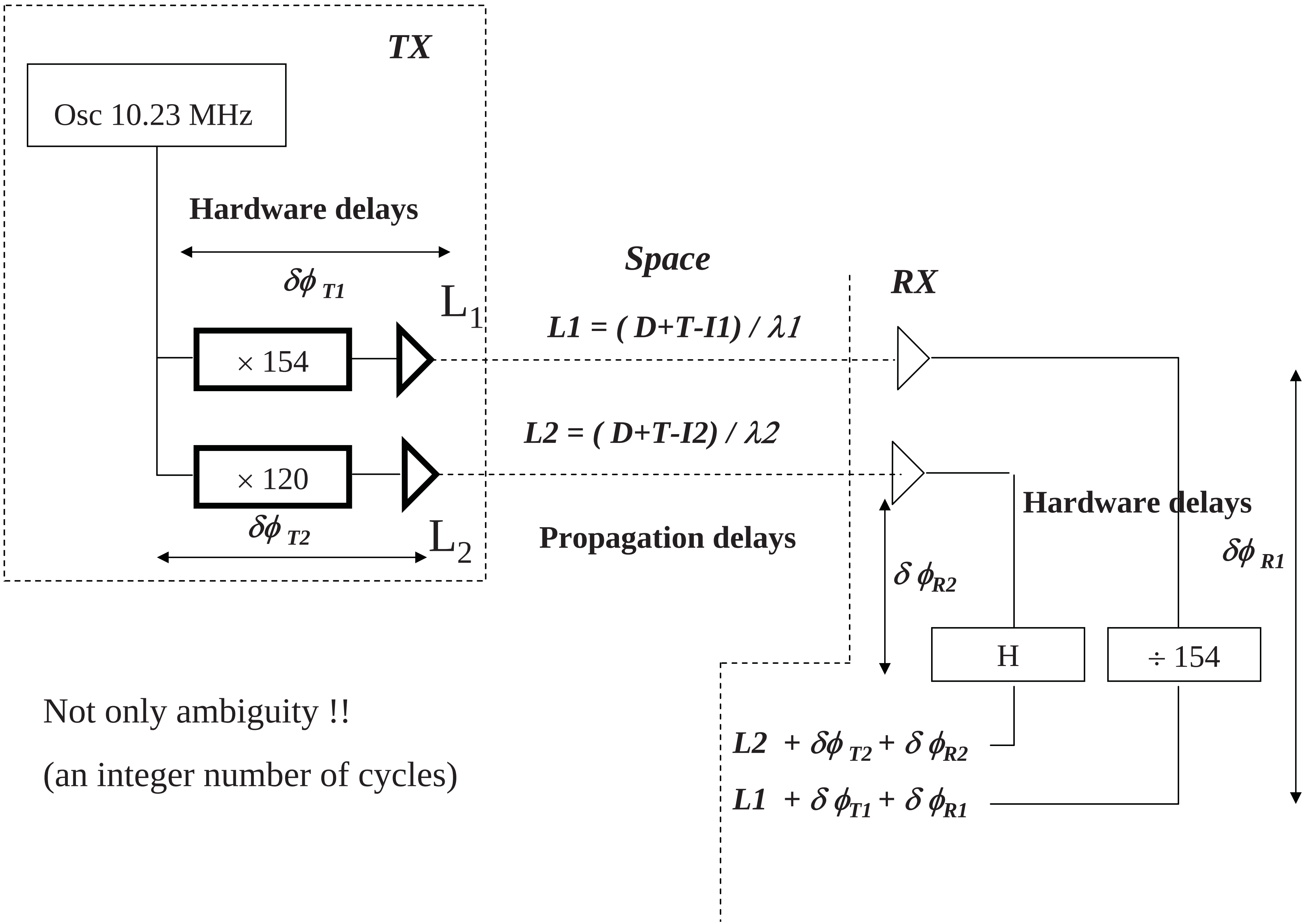
<u>Multi-Path</u>



Hardware delays: Code



Hardware Delays: Phase



Not only ambiguity !!

(an integer number of cycles)

The extraction of ionospheric information

Once all the contributions/disturbances to observables are individuated,

how isolating ionospheric information?

Remind

$$M_1 = S \pm \frac{40.3TEC}{f_1^2}$$

$$M_2 = S \pm \frac{40.3TEC}{f_2^2}$$

Solving the system provided by measurements at two frequencies ***f1*** and ***f2*** at advantage of the ionospheric investigator

For ionospheric investigation, the solution of the system is very simple: differential delays from the GPS observables will be computed

The classical solution for **propagation delays**:

I depends on frequency: using two coherent frequencies f_1, f_2

($f_1 > f_2$) **differential delays** of optical paths are formed,

both for Phase (***DPD***) and Group (***DGD***)

$$DPD = P_2 - P_1 = I_1 - I_2$$

$$DGD = L_1 - L_2 = I_1 - I_2$$

$$I_1 - I_2 = k\ TEC$$

$$k = 40.3 \cdot \left(\frac{1}{f_2^2} - \frac{1}{f_1^2} \right)$$

Receiver needed: dual frequency "civil" receiver

Summary of delays

Propagation	$\boldsymbol{D + T \pm I}$
Hardware electronic delays originating in satellite and receiver,	$\boldsymbol{B}, \boldsymbol{\Gamma}$
Offset (delay, ambiguity) for phase	$\boldsymbol{\Phi}$
Noise	$\boldsymbol{N}$
Multi-path	$\boldsymbol{M}$
User clock offset	$\boldsymbol{\tau}$

Code delay affected by user clock offset is *pseudo-range*

$$\boldsymbol{P = D + T - I + B + \Gamma + N + m + \tau}$$

For following discussion, noise and multipath can be neglected for phase delays. Hardware delays for phase are included in $\boldsymbol{\Omega}$

$$\boldsymbol{\Lambda = D + T + I + \Phi}$$

$$P = D + T - I + B + \Gamma + N + m + \tau$$

$$\Lambda = D + T + I + \Phi$$

The GPS differential delays are obtained taking the difference of paths Λ e P at the frequencies f_1 e f_2

Code: $P2 - P1$

Phase = $\Lambda 1 - \Lambda 2$

Contributions $\underline{D, T, \tau}$ cancel out

With the position $\beta = B2 - B1$, $\gamma = \Gamma 2 - \Gamma 1$, $\Omega = \Phi 1 - \Phi 2$

$$P2 - P1 = k\,TEC + \beta + \gamma + n + m$$

$$\Lambda 1 - \Lambda 2 = L1 \cdot \lambda 1 - L2\, \lambda 2 = k\,Tec + \Omega$$

<u>Dividing by $k \cdot 10^{16}$ one works in ***TEC units***</u>.

Differential delays are known as code and phase **slants** S_G and S_Φ

$$S_G = TEC + \beta + \gamma + n + m$$

$$S_\Phi = TEC + \Omega$$

In the following only differential delays will be considered

Ionospheric observables:

$$\boldsymbol{S_G = TEC + \beta + \gamma + n + m}$$

$$\boldsymbol{S_\Phi = TEC + \Omega}$$

are instantaneous measurements: various contributions have different time behaviors. Before getting in more detail, remind the meaning of **Arc** in radio observations, as a series of observations carried out with *continuity* from one station to one satellite**.** *Continuity***:** presence of satellite over the horizon of the station (astronomical arc) **,** no loss of lock for phase or code.

If not recoverable loss of lock occurs, two distinct arcs will be considered also if observations belong to the same "astronomical" arc.

Behaviour vs time

TEC, varies from observation to observation.As regards other terms

Phase

Offset $\boldsymbol{\Omega}$: constant along one arc, arbitrarily variable from arc to arc.

Code

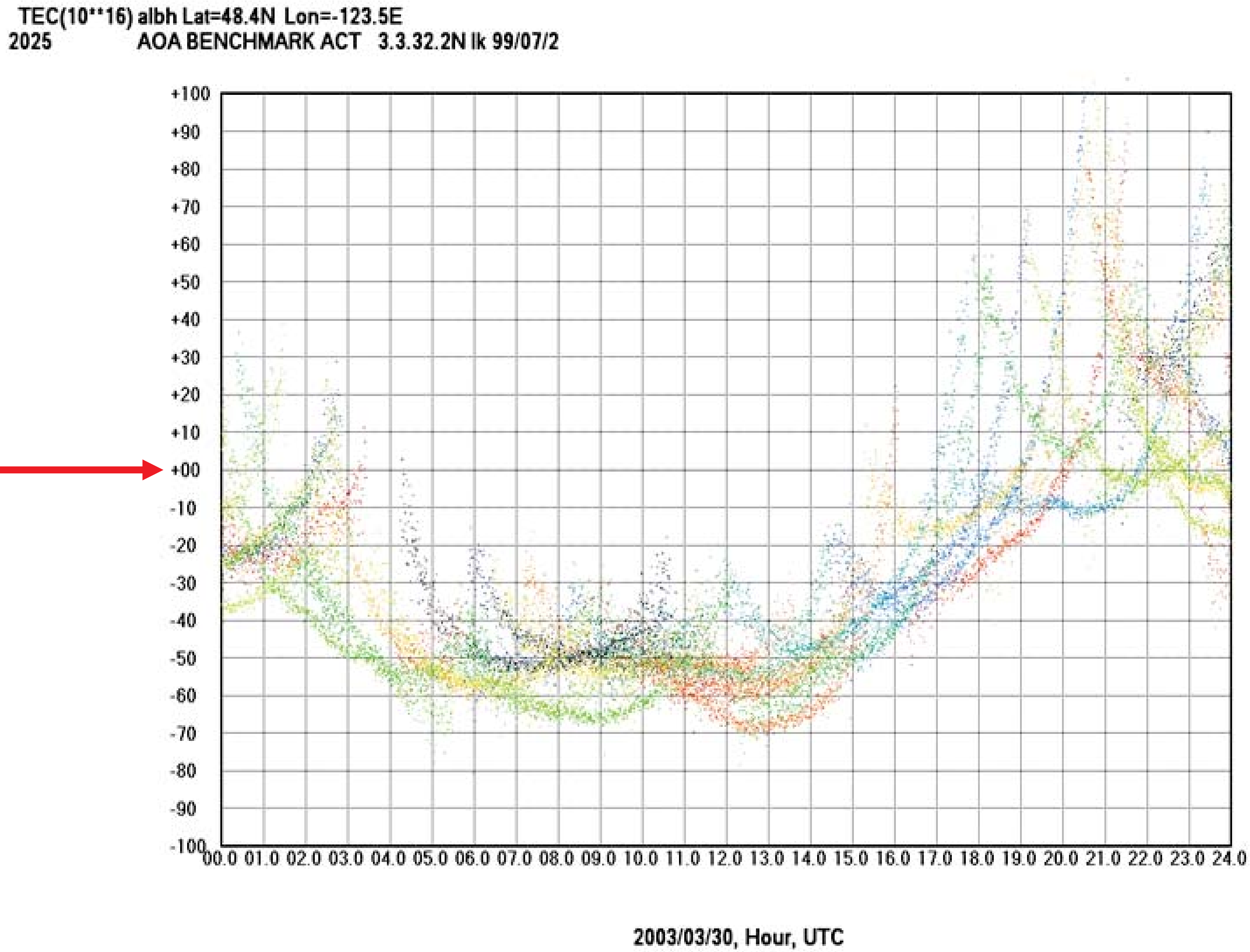
Noise ***n***: stochastic variable. Signal to noise ratio *S/N* is severely degraded by the non linear techniques used to overcome the effects of W code (Anti-Spoofing).

Multi-path ***m***: not stochastic, but unpredictable. To reduce effect, one must care antenna environment. It may be very strong at low elevations. If environment does not change, ***m*** repeats its behaviuor day by day with a shift of ≈ 4 minutes.

Hardware biases $\boldsymbol{\beta}$ and $\boldsymbol{\gamma}$ produced by electronic circuits may be subject to aging and thermal drift. For cared environments and not long time spans they can be considered as constants. In this assumption, we have one $\boldsymbol{\beta}$ per satellite ed one $\boldsymbol{\gamma}$ per receiver.

Better now looking at some practical case

Plot of S_C arcs for one day



*** Evidence that calibration is needed: TEC is a positive quantity**

Code slants: $\boldsymbol{S_G = TEC + n + m + \beta + \gamma}$

Advantages: the electronic delays are physical quantities, stable or undergoing slow aging in controlled environmental conditions: they are generally considered constants over long times (up to 1 month).

One $\boldsymbol{\beta}$ per satellite, one $\boldsymbol{\gamma}$ for station: a favorable unknowns/observations budget.

$\boldsymbol{n}$: strong radio noise (non linear techniques used to evaluate pseudo-ranges:does this result into consistent estimations?)

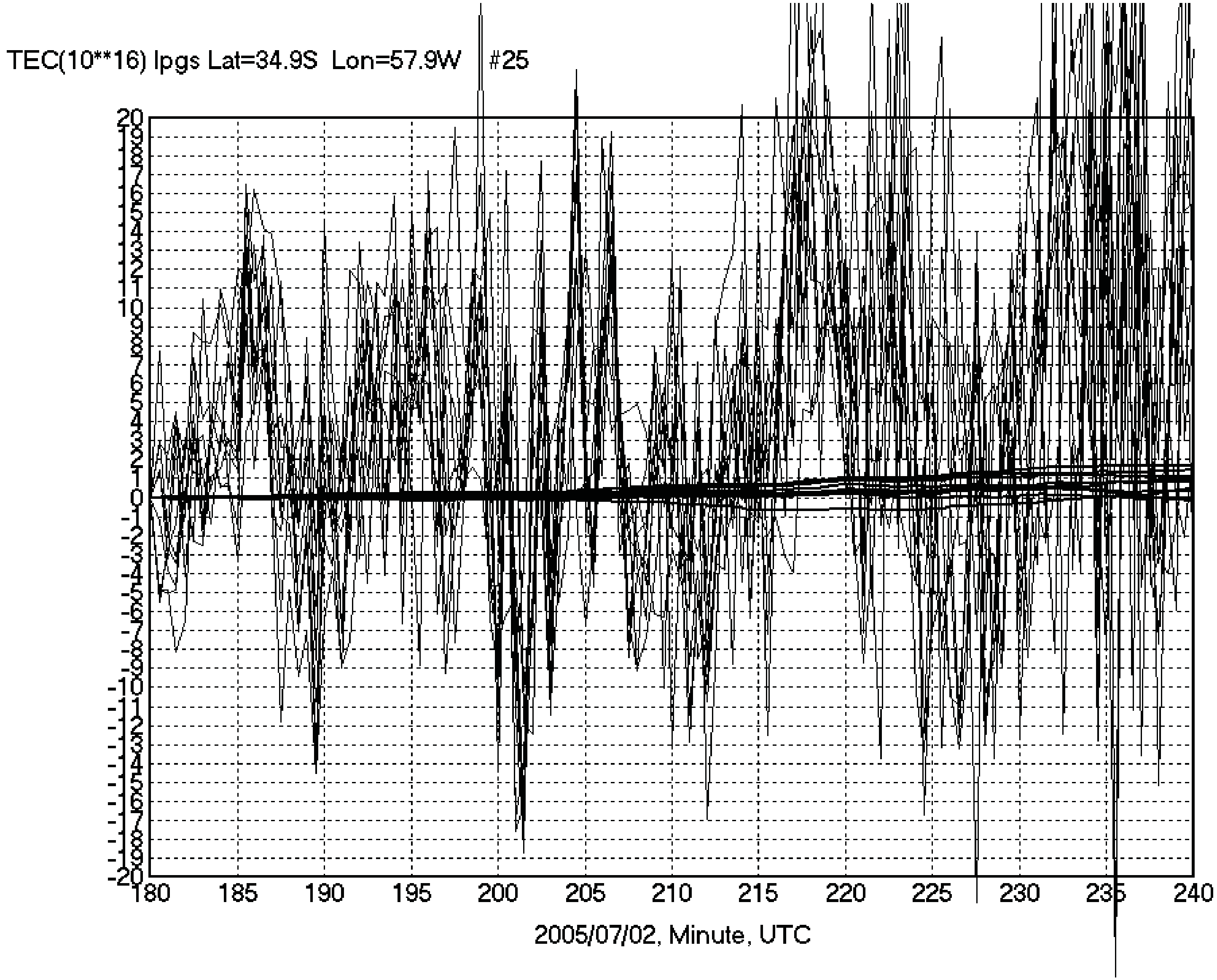
Can multi-path be considered a disturbance?

How to distinguish it from noise? Recipe follows.

Period of GPS orbits is 12 sidereal hours: day after day the same satellite will occupy the same position with an advance of ≈ 4 minutes: if the antenna environment does not change day after day, $\boldsymbol{m}$ will advance by the same amount.

Plot a fraction of arc of the same satellite day by day with an advance of ≈ 4 minutes

Note: to avoid ***TEC*** variability, what is plotted for each arc is $\boldsymbol{TEC(t) - TEC(t_0)}$, $\boldsymbol{t_0}$ being the beginning of each arc. Both $\boldsymbol{S_G}$ and $\boldsymbol{S_\Phi}$ relative to the same arc are plotted .


TEC(10**16) lpgs Lat=34.9S Lon=57.9W #25
20
19
18
17
16
15
14
13
12
11
10
9
8
7
6
5
4
3
2
1
0
-1
-2
-3
-4
-5
-6
-7
-8
-9
-10
-11
-12
-13
-14
-15
-16
-17
-18
-19
-20
180
185
190
195
200
205
210
215
220
225
230
235
240
2005/07/02, Minute, UTC

Phase slants

No significant noise and multi-path (above slides)

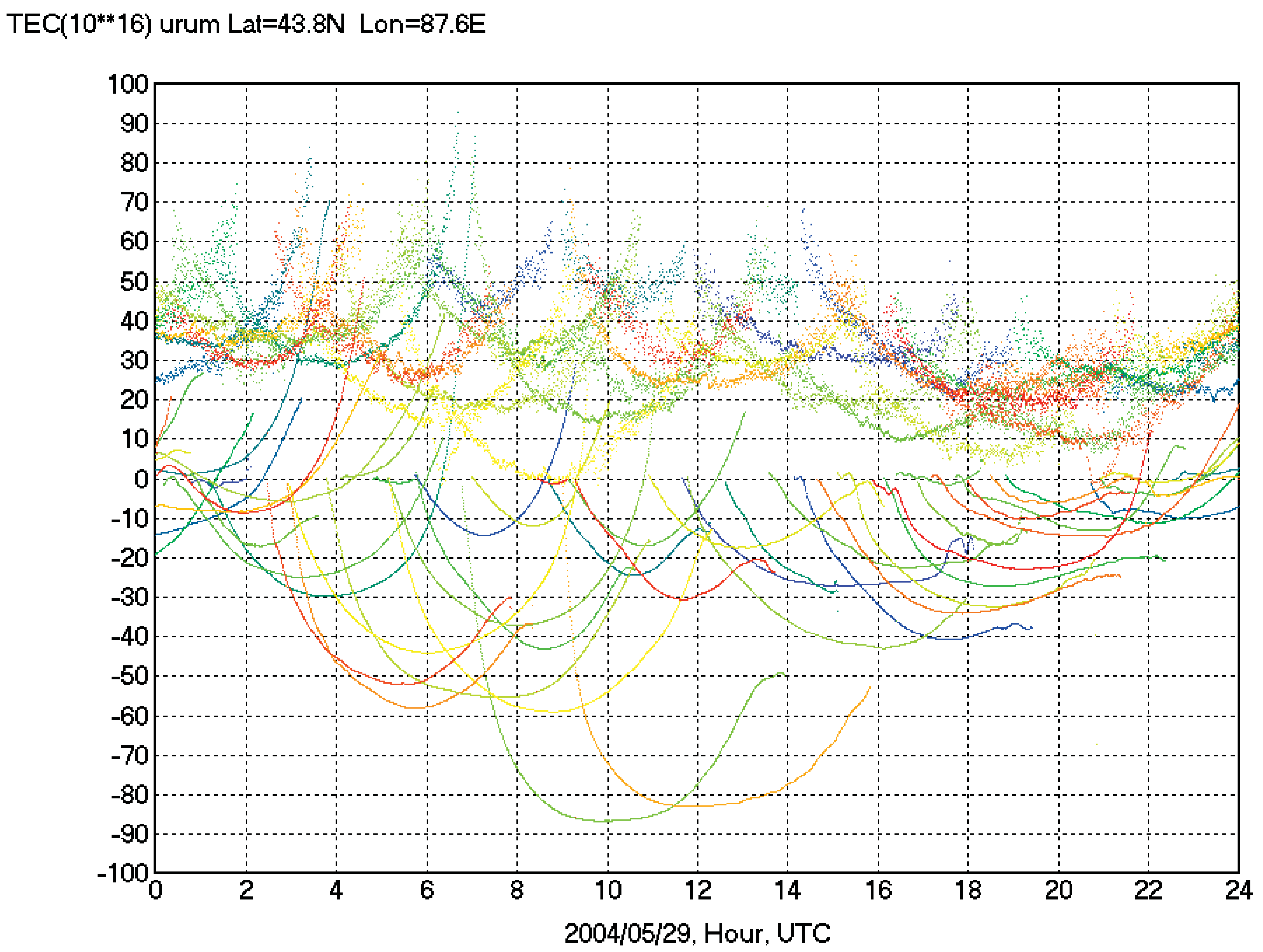
Modest equations/unknown budget: one unknown per arc

Global single day solution, 200 stations

Unknowns: than 1000 unknown offsets, compared to 200+32 hardware biases.

Possibility to use first differences (in time) of the observations of one arc. Only TEC coefficients remain: calibration relies entirely on the model used for the expansion.

Other possibility: solving by geodetic techniques for the ambiguities and therefore for the offsets.


TEC(10**16) urum Lat=43.8N Lon=87.6E
100
90
80
70
60
50
40
30
20
10
0
-10
-20
-30
-40
-50
-60
-70
-80
-90
-100
0
2
4
6
8
10
12
14
16
18
20
22
24
2004/05/29, Hour, UTC

Code Slants are very noisy, ***Phase Slants*** are affected by offsets

How getting clean data without phase ambiguities?

Building

Differential Phase Delays leveled to Differential Group Delays

In few words: change offset of each phase arc in order to overlap it to the corresponding code arc.

The operation is

Leveling

The new set of observables

Operator $<\cdot>$ is a properly selected weighted average

Note: slants, multi-path, noise are functions of time

Offsets are constant during one arc, biases are constant for each satellite-station pair

Average of a constant quantity gives the quantity itself.

Build, <u>**arc by arc**</u>, the <u>leveled</u> slants ***S***

$$S = S_{\Phi} - <S_{\Phi} - S_G>$$

$$<S_{\Phi} - S_{G}> = \Omega - <n> - <m> - \beta - \gamma$$

$$S = TEC + <n> + <m> + \beta + \gamma$$

Properties of ***S***

Offset has been cancelled

Noise is the same (neglected) of phase slants

Biased as code slants

But: an arc dependent constant leveling error $\lambda = <n> + <m>$ appears

Why arc by arc?

Because to perform leveling the Offset Ω must be constant.

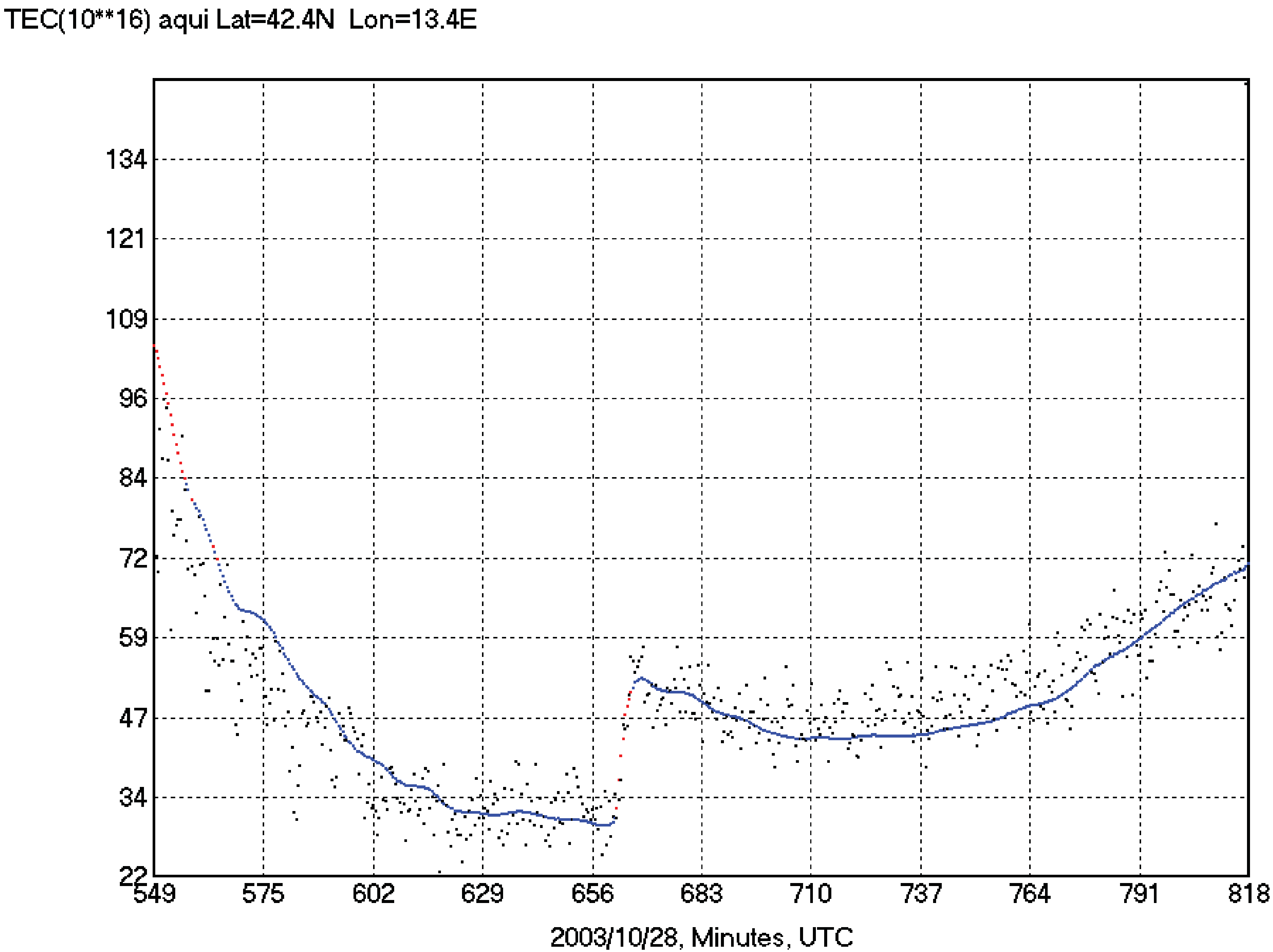
This requires that possible phase jumps are individuated and possibly corrected before leveling is carried out:

a very difficult task

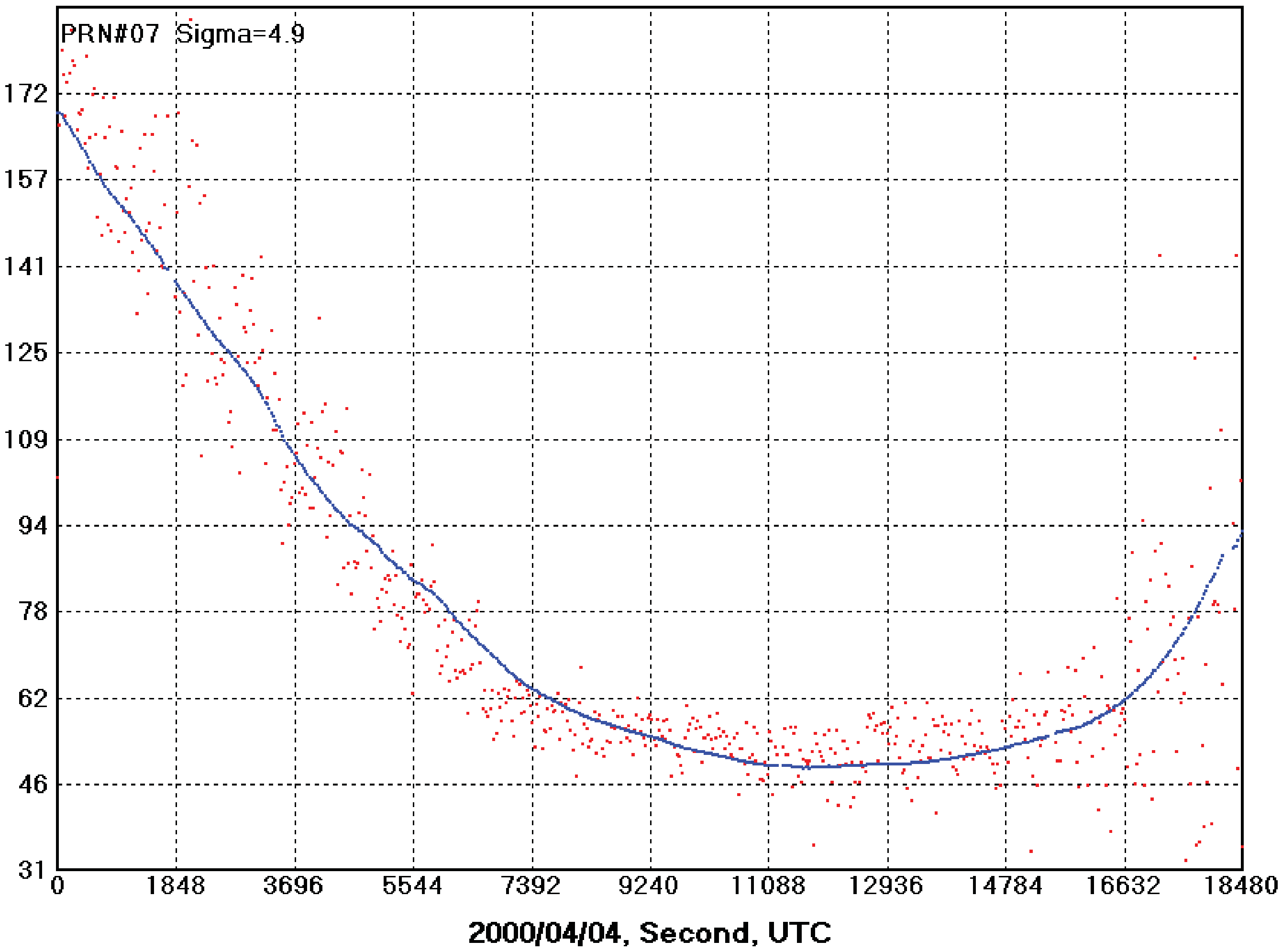
Possible methods: if consecutive slants are very different, there is a jump.

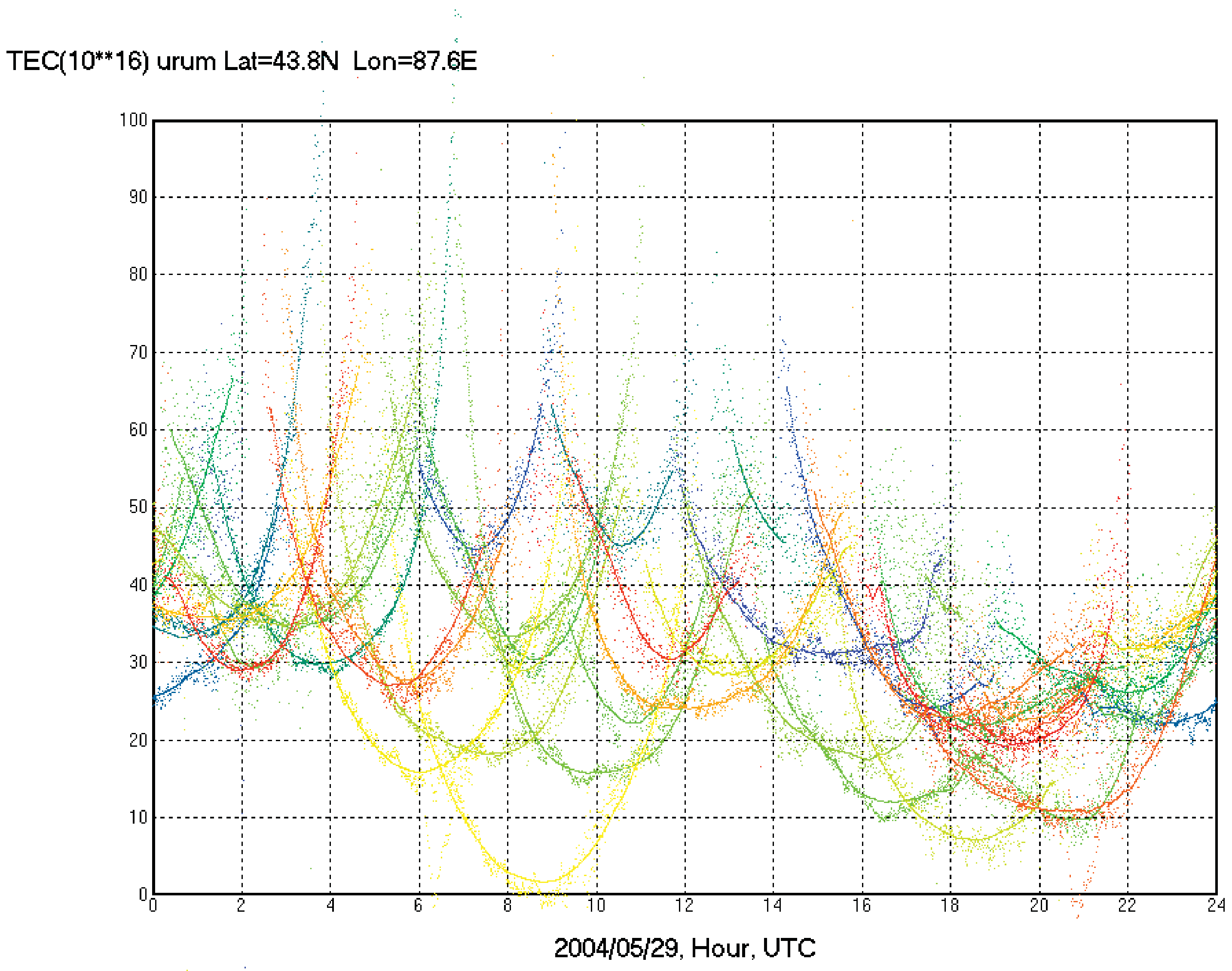
Practically: consider the **first differences**. If some of them is larger than the adjacent ones, substitute it with some interpolated value; reconstruct the arc from the new set of differences.

Problems: reality shows always unpleasant effects. Look at following slide in which an event looking like a phase jump is actually a radiation burst.


TEC(10**16) aqui Lat=42.4N Lon=13.4E
134
121
109
96
84
72
59
47
34
22
549
575
602
629
656
683
710
737
764
791
818
2003/10/28, Minutes, UTC

TEC(10**16) nklg Lat=00.4N Lon=9.7E



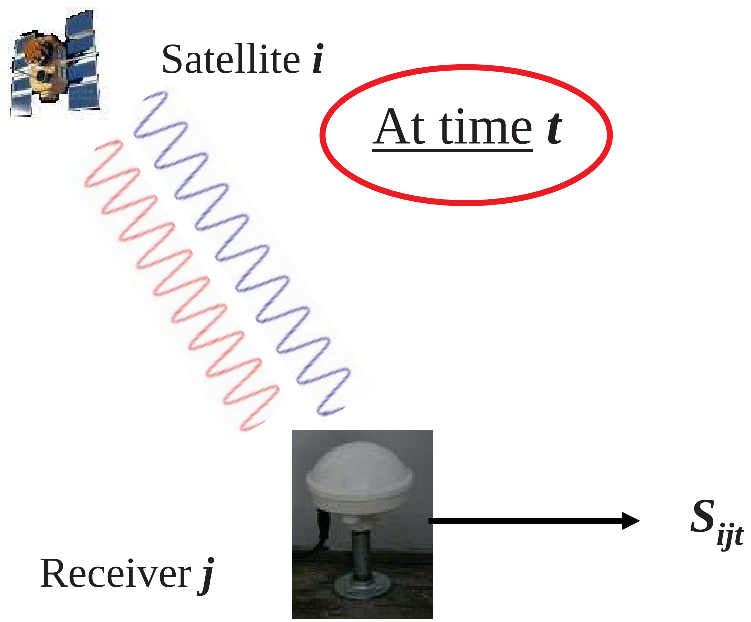

TEC(10**16) urum Lat=43.8N Lon=87.6E
100
90
80
70
60
50
40
30
20
10
0
0
2
4
6
8
10
12
14
16
18
20
22
24
2004/05/29, Hour, UTC

CALIBRATION

Rewriting the full set of observations

As already shown, properly processing GPS measurements, forming differential delays (dual frequency receiver), combining them to obtain '**leveled slants**', one gets slant Total Electron Content (***TEC)*** measurements affected by biasing terms $\beta_i, \gamma_j, (\lambda_{Arc})$

$$S_{ijt} = TEC_{ijt} + \beta_i + \gamma_j + (\lambda_{Arc})$$



i = 1, 2 , …,32 available GPS satellites

j = 1, .., available receivers

t all the available observation epochs (in one day or fraction, or many days)

Arc = common to all continuous observations performed by receiver ***j*** on satellite ***i*** at times contiguos to ***t***

We bracket λ_{Arc} because this term is disregarded in the traditional approach but basic for the proposed "arc offset" solution.

Description of the biasing terms

$$S_{ijt} = TEC_{ijt} + \beta_{\iota} + \gamma_{j} + (\lambda_{Arc})$$

β differential hardware delays in satellite electronic circuitry

γ the same for receiver circuitry

λ the average contribution of differential multi-path along an arc

All biasing terms can be considered as constants

For ionospheric investigation and its applications (ionospheric corrections) an algorithm is needed able to estimate the biasing terms in order to have only ***TEC***

$$TEC_{ijt} = S_{ijt} - \beta_{\iota} - \gamma_{j} - (\lambda_{Arc})$$

This algorithm is known as

CALIBRATION or DE-BIASING

Red: unknowns

Blue: estimates

Note:

Satellite and receiver hardware delays appear always through their sum $\boldsymbol{S_{ijt} = TEC_{ijt} + \underline{\beta_i + \gamma_j} + \ldots}$

If we add some quantity to satellite biases and subtract the same from receiver biases, nothing will change: the solution is undetermined.

Indetermination disappears if we are able to determine only one bias (satellite or receiver), i.e. by some measurement.

Satellite biases are in some way measured in pre-flight phase and possibly updated during satellite operation (TGD’s, transmitted in the satellite message)

For receivers, some instrument was built claiming to determine receiver bias: results were very poor.

The calibration or de-biasing of GPS leveled slants

The system of the equations of observation is linear in all unknown terms

$$S_{ijt} = TEC_{ijt} + \beta_{\iota} + \gamma_{j} + (\lambda_{Arc})$$

but contains more unknowns than equations.

Number of unknowns = number of TEC**s *plus number of unknown (constant) terms***

$$\beta_{\iota} , \gamma_{j} , (\lambda_{Arc})$$

How is it possible performing the calibration?

TEC's are not actually uncorrelated: at some location, at some time they depend on the electron density distribution N_e.

Assume electron density N_e can be written as a function of position P, time t and a set of K parameters $Z_1, Z_2, \ldots$

$$TEC = \int N_e(P, Q, t, Z_1, Z_2, \ldots) ds$$

Calibration is performed finding the values $Z_1, Z_2, \ldots, \beta_{\iota}, \gamma_j, (\lambda_{Arc})$ which minimize the sum of the square of the residuals

$$\varepsilon_{ijt} = S_{ijt} - TEC(P, t, Z_1, Z_2, \ldots) - \beta_{\iota} - \gamma_j - (\lambda_{Arc})$$

$$\Sigma \varepsilon^2_{ijt} \Rightarrow Minimum$$

Example: Ionospheric model ***NeQuick*** computes electron density N_e at given point ***P***,

at given time ***t***, as a function of a Solar Flux equivalent parameter ***Az***.

$$\varepsilon_{ijt} = S_{ijt} - TEC\,(P, t, Az) - \beta_{\imath} - \gamma_{j} - (\lambda_{Arc})$$

$$\text{Find } Az,\ \beta_{\imath},\ \gamma_{j},\ (\lambda_{Arc}) \text{ such that } \Sigma\varepsilon^{2}_{ijt} \Rightarrow \text{Minimum}$$

Observations/Unknowns budget: very favorable

Problems

Non linear minimization methods needed / Dependence on parameters is not analytical but numerical

Models provide with excellent median values whereas calibration requires that the model describes very precisely the actual N_e distribution

But:

Excellent perspectives for the future

Writing TEC

Better using formulations in which also actual gradients (not only median ones provided by the Ionospheric Models) can be taken into account, possibly linear in all unknowns.

$$S_{ijt} = TEC_{ijt} + \beta_\iota + \gamma_j + (\lambda_{Arc})$$

Writing TEC ➔ Write the integral $TEC = \int N_e(P,Q,t,Z_1,Z_{2,\ldots})ds$

Possible (linear) expansions of ***TEC***

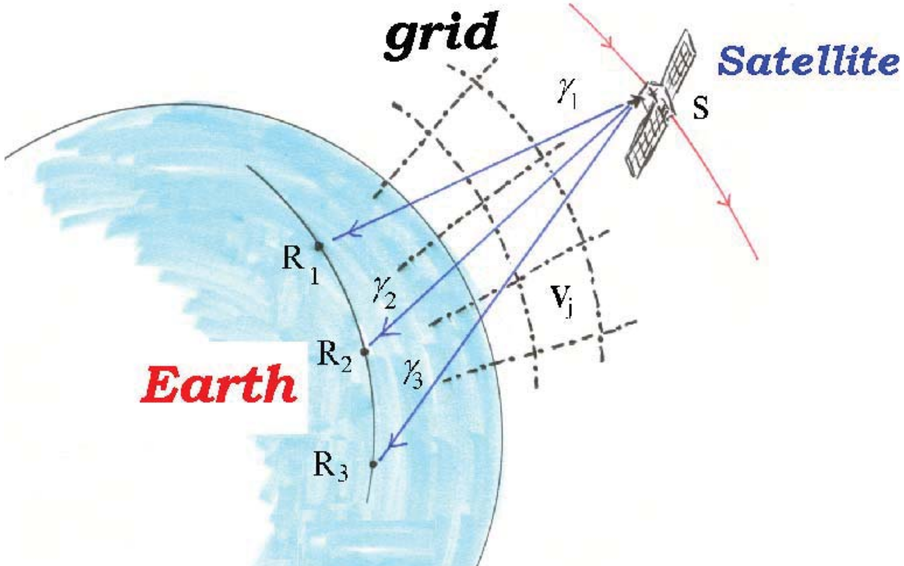
3D (Tomography)

Multi shell

Thin shell

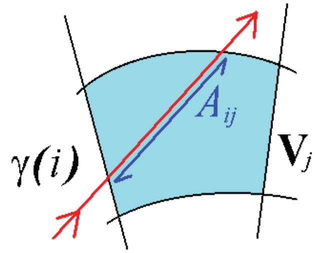
3D-4D approach (Tomography)

the ionosphere is divided in elements of volume (voxels) inside which $\boldsymbol{N_e}$ is constant. $\boldsymbol{N_e}$ of voxels are the unknowns. Evolution with time of $\boldsymbol{N_e}$ is considered to improve the budget unknowns/observations. Vertical behavour of $\boldsymbol{N_e}$ is expanded in Empirical Orthogonal Functions (EOF)



$$d_i = \int_{\gamma(i)} N(\mathbf{r},t)\,ds = \sum_j x_j \int_{\gamma(i)} b_j(\mathbf{r})\,ds \equiv \sum_j A_{ij} x_j$$

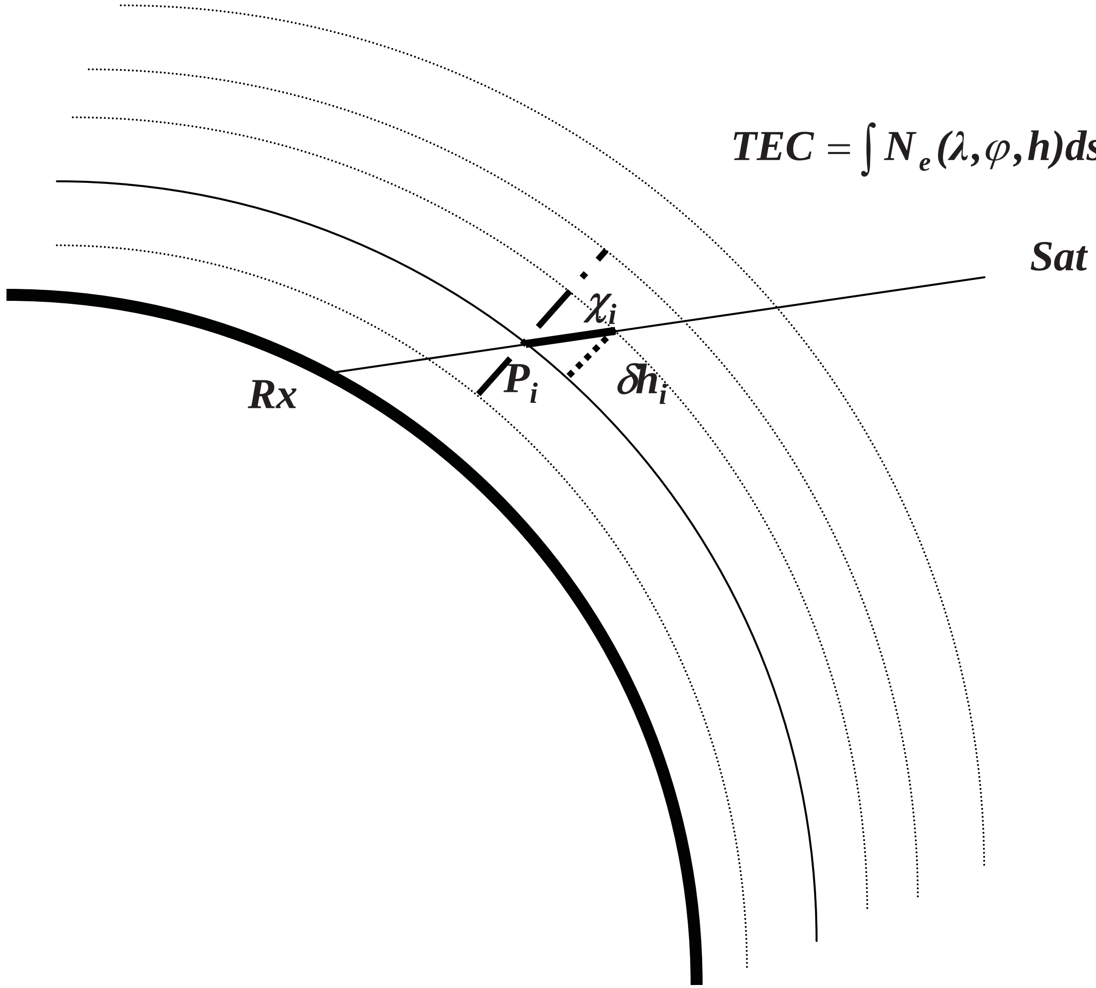
$$\underline{d} = A\underline{x} \quad \Rightarrow \quad \underline{x} = A^{-1}_{SVD}\underline{d}$$



3D: The multishell method

If many shells are used, this is exactly the method by which numerical integration is carried out. For each shell, a suitable 2D expansion in horizontal coordinates is assumed.

$$TEC = \int N_e(\lambda,\varphi,h)ds \approx \sum N_e(P_i)\delta s_i = \sum N_e(P_i)sec\chi_i \delta h_i$$



P_i, point on the generic i^{th} shell

δh_i increment in height

δs_i increment in arc length

$\delta s_i = \delta h_i \sec \chi_i$

The classical thin shell model

Reducing down the number of shells, ***and in principle the expected accuracy***,

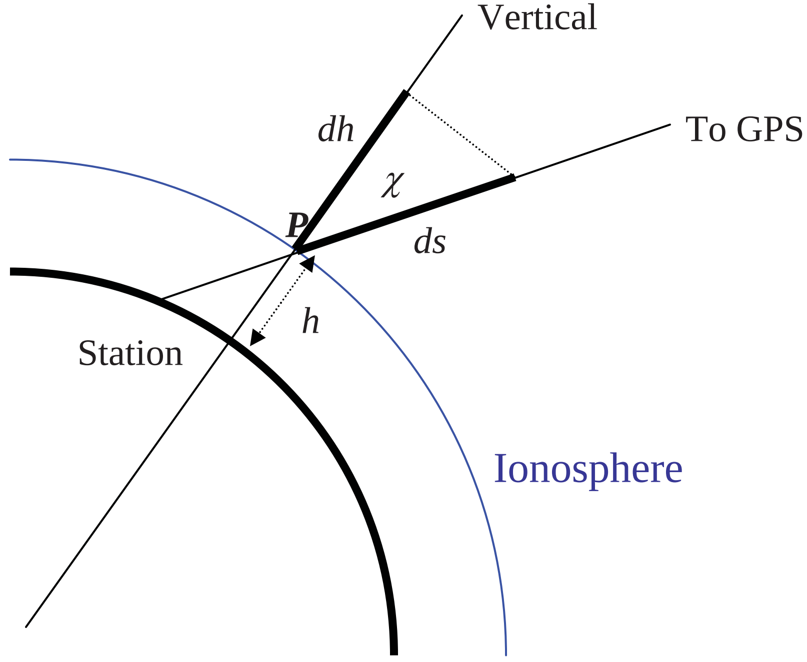
take only one **(thin) shell at some reference height *h***

$$\boldsymbol{TEC = V(P)\ sec\ \chi}$$

V(P) is the ***TEC*** along the vertical of the ionospheric point ***P***

(**V**ertical **E**lectron **C**ontent, ***VEC***)

V(P) is a **2D** function of horizontal coordinates



An intuitive description of the proposed approach to Calibration

What the algorithm performs can be described by the following figures.

Basically it is a quasi-geometric approach assuming that behavior of

$$\boldsymbol{VEC = (S - \beta) \cos \chi}$$

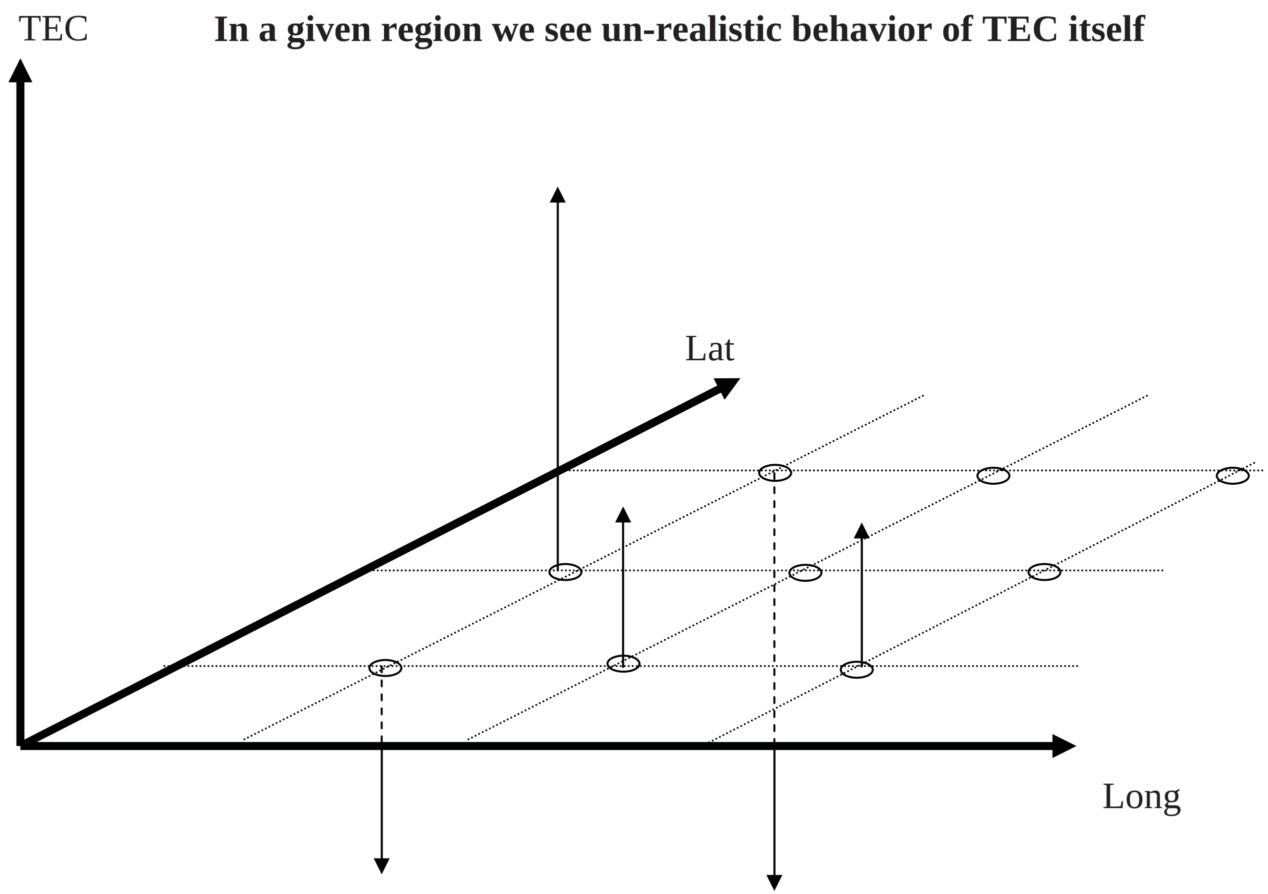
can be represented by some "smooth" expansion of horizontal coordinates.

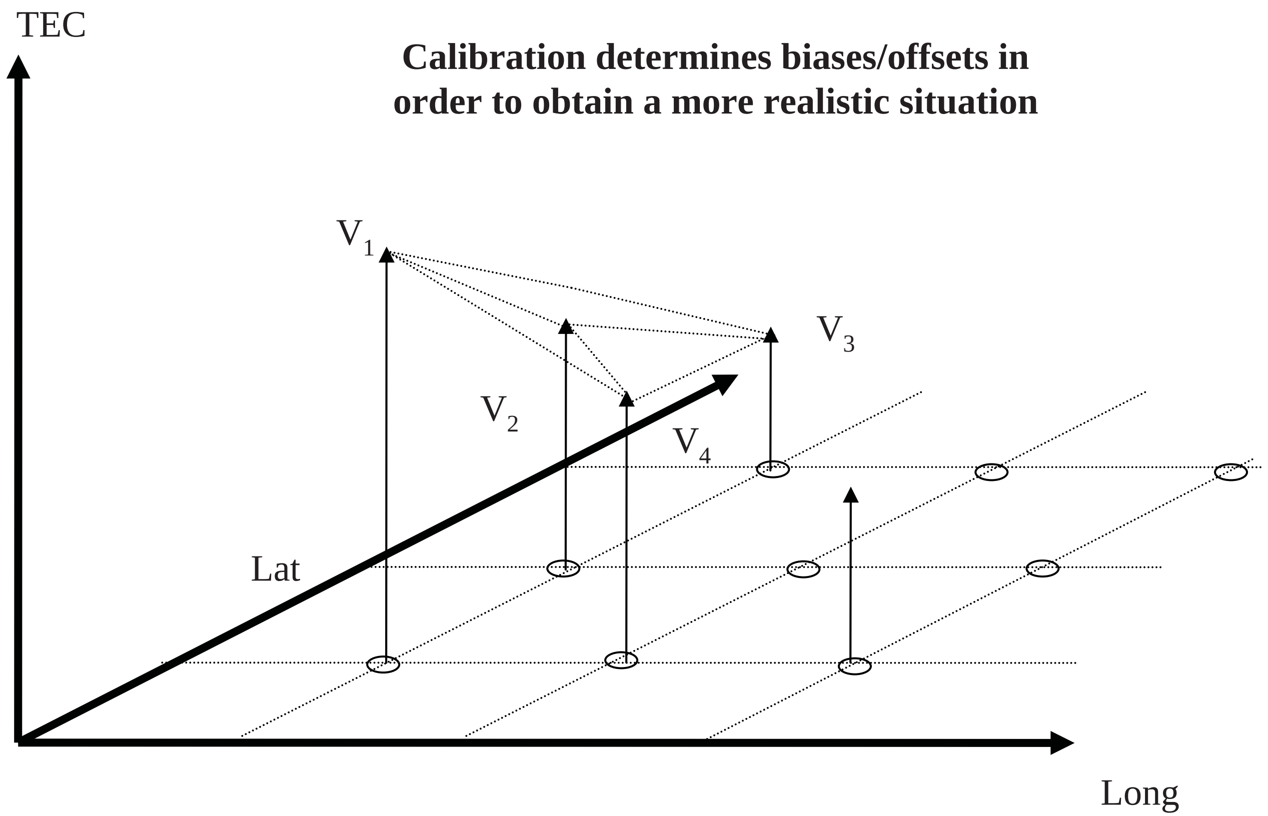
This works satisfactorily in quiet conditions at middle latitudes.

At high and low latitudes this assumption can often fail also during quiet conditions, and is one of the reason of not getting reliable estimations of biases and TEC itself in such regions.

Mapping un-calibrated vertical TEC (Thin shell assumption)

In a given region we see un-realistic behavior of TEC itself




Calibration determines biases/offsets in
order to obtain a more realistic situation
TEC
V1
V2
V3
V4
Lat
Long

Some simple example for ***VEC*** expansion

$$V(P_{ijt}) = \Sigma_n c^{(t)}_n \Psi_n\, (\Phi_{ijt}, \Lambda_{ijt})$$

Single-station: assume, at time ***t***, that ***VEC*** is constant over the station horizon, $VEC = V_0^{(t)}$:

$$V(P_{ijt}) = V_0^{(t)}$$

Single-station : assume ***VEC*** varies linearly with latitude Φ and longitude Λ

$$V(P_{ijt}) = V^{(t)}_0 + a^{(t)}\, (\Phi - \Phi_0) + b^{(t)}\, (\Lambda - \Lambda_0)$$

Which can be improved up to bi-linear, bi-polynomial expansion and the full spherical harmonics expansion for global solutions

Rewrite equations of observation

$$S_{ijt} = TEC_{ijt} + \beta_{\iota} + \gamma_{j} + \lambda_{Arc} = V(P_{ijt}) \sec \chi_{ijt} + \beta_{\iota} + \gamma_{j} + (\lambda_{Arc})$$

$$S_{ijt} = \sec \chi_{ijt} \, \Sigma_{n} c^{(t)}_{n} \Psi_{n} (\Phi_{ijt}, \Lambda_{ijt}) + \beta_{\iota} + \gamma_{j} + (\lambda_{Arc})$$

Symbolically written as

$$S = Ax$$

Unknowns x will be solved using Least Squares or equivalent (and more sophisticated) methods

$$x = (A^{T} A)^{-1} A^{T} S$$

Going back to the equations of observations, knowing solution x means knowing

The coefficients of the expansion of vertical TEC $c^{(t)}_{n}$

The biasing terms β_{ι} , γ_{j} , (λ_{Arc})

After the numerical solution

Having solved for $c^{(t)}_n, \beta_\iota, \gamma_j, (\lambda_{Arc})$, available products are

The calibrated slants

Calibrated slants will be available as $TEC_{ijt} = S_{ijt} - \beta_\iota - \gamma_j - (\lambda_{Arc})$

The Vertical TEC

In addition, as a by-product of calibration, knowledge of the coefficients $c^{(t)}_n$ of ***TEC*** expansion will enable to estimate slants along directions different from the ones of the actual observations.

$$TEC_{ijt} = \sec \chi_{ijt} \, \Sigma_n c^{(t)}_n \Psi_n \, (\Phi_{ijt}, \Lambda_{ijt})$$

The most familiar is vertical ***TEC (VEC)***, the Total Electron Content relative to the zenith of the station of coordinates Φ^*_j, Λ^*_j

$$VEC(j,t) = TEC_{jt} = \Sigma_n c^{(t)}_n \Psi_n \, (\Phi^*_j, \Lambda^*_j)$$

Summary

All solutions for calibration follow the reported scheme

Extraction of un-calibrated slants from GPS observations

Solution of the system in unknown *VEC* coefficients and biasing terms

According to the geographical distribution of stations and the time span in which observations are available, several solutions are possible getting the possible combinations of one solution per line

Hourly / Single-day / Multi-day

Single-station / Regional /Global

Factors affecting the reliability of calibration

Modelling of observations

$$S = VEC \sec \chi + \beta + \gamma + (\lambda_{Arc})$$

Mapping function accuracy, constancy of biases, role of (λ_{Arc})

Adequacy of the model used for the expansion of *VEC*

$$VEC(P, t) = \Sigma c\, \Psi(P, t)$$

Conditioning of the resulting systems of equations

biasing terms and ***VEC*** are strongly correlated

Limitations of the thin shell assumption

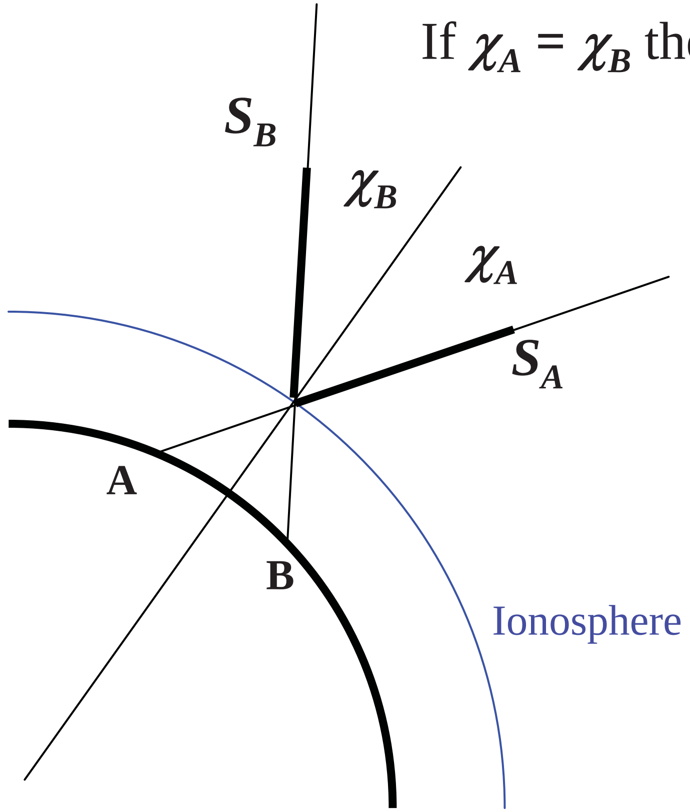
The thin shell assumption is self-evidently poor:

TEC is the same for rays passing through the same ionospheric point for given χ,

disregarding at all gradients

Errors range to few ***TECu*** in normal conditions, but up to 30-40 ***TECu*** under storm (thesis of ***Bruno Nava,*** carried out on super-truth data). This may introduce severe errors in regional and global solutions.

If $\chi_A = \chi_B$ then $S_A = S_B$



Estimation of errors introduced by the mapping function in the thin shell approach

Using a **ionospheric model**, compute the slant ***TEC*** from station to GPS and the vertical content ***VEC*** at the ionospheric point.

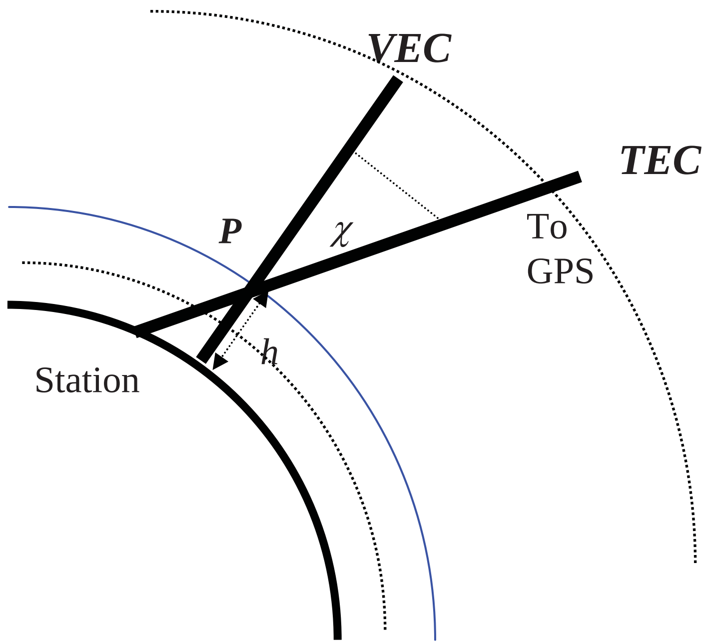
Compute the error

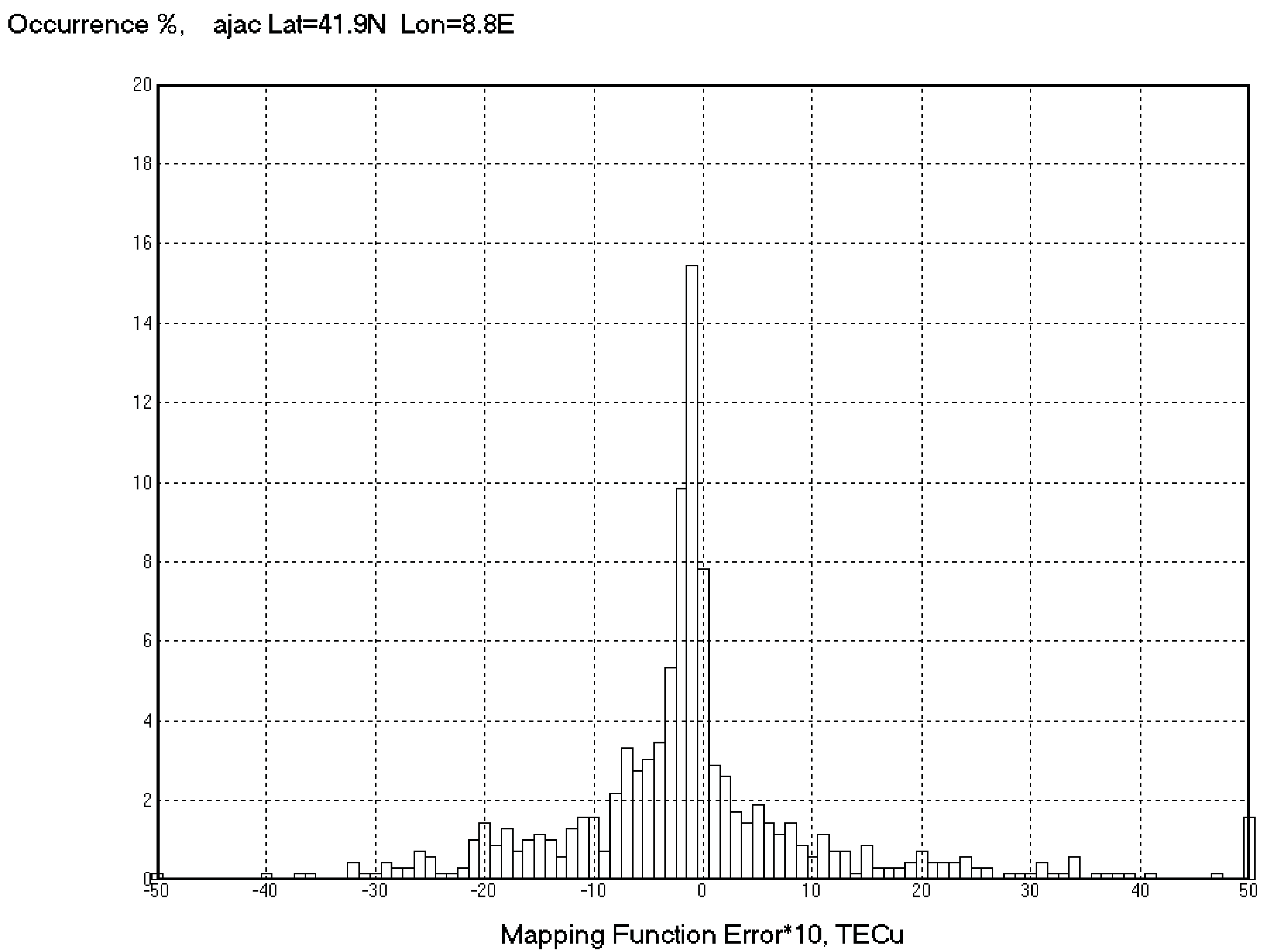
$\boldsymbol{\varepsilon = TEC - VEC \sec \chi}$

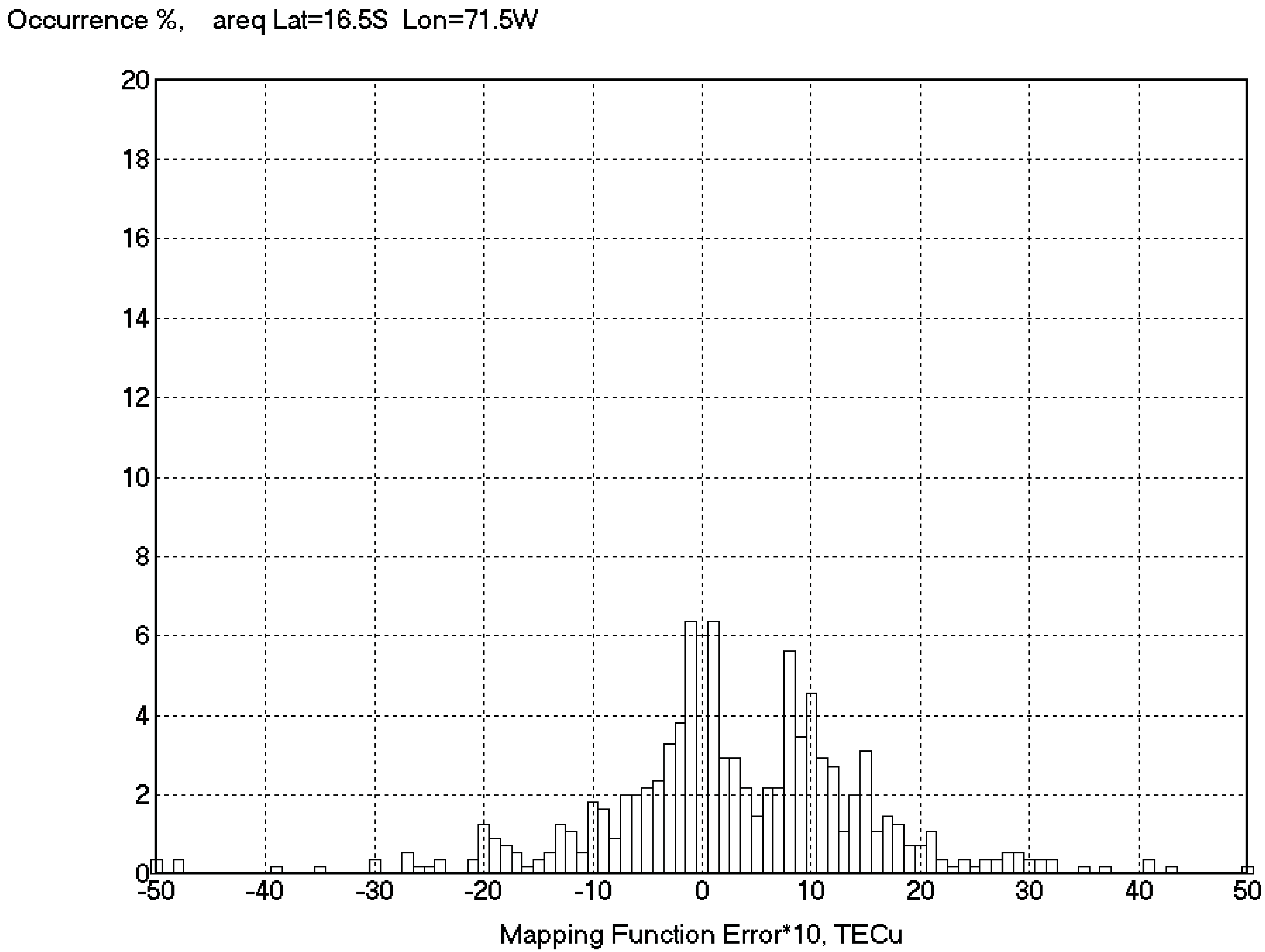
Consider the error distribution:

Acceptable Errors at Middle Latitudes

Strong errors at low latitudes




Occurrence %,   ajac Lat=41.9N Lon=8.8E
20
18
16
14
12
10
8
6
4
2
0
-50
-40
-30
-20
-10
0
10
20
30
40
50
Mapping Function Error*10, TECu


Occurrence %,   areq Lat=16.5S Lon=71.5W
20
18
16
14
12
10
8
6
4
2
0
-50
-40
-30
-20
-10
0
10
20
30
40
50
Mapping Function Error*10, TECu

But shall we discard the thin shell approach?

A new interpretation

For a given ray, rearrange ***TEC*** definition using **sec** χ_{REF} at a given reference height

$$TEC = \int N_e ds = \int N_e sec\chi dh = sec\chi_{REF} \int N_e \frac{sec\chi}{sec\chi_{REF}} dh = sec\chi_{REF} V_{eq}$$

$$V_{Eq} = \int N_e \frac{sec\chi}{sec\chi_{REF}} ds \qquad \boxed{TEC = sec\chi_{REF} V_{eq}}$$

The expression is formally identical to the mapping function approximation,

but it is **exact** provided V_{Eq}, a 2D Function (elevation/azimut or displacement of horizontal coordinates from the station) is not interpreted as the vertical ***TEC***.

V_{Eq} will change for stations in different locations, **so its use is limited to the calibration performed by the single station solution**.

Calibration requires a relationship correlating the various slants: for the single station solution the properly interpreted mapping function does not implies errors other than the capability to map V_{Eq} in satisfactory way.

The traditional method: assumptions

Accept the known limitations of the thin shell approach (which enables global and regional solutions)

Accept the constancy of biases

Disregard the leveling error contribution λ

Solve the system

$$S_{ijt} = \sec \chi_{ijt} \, \Sigma_n c^{(t)}_n \Psi_n (\Phi_{ijt}, \Lambda_{ijt}) + \beta_\iota + \gamma_j$$

In the unknowns $c^{(t)}_n$, β_ι, γ_j

The $\beta + \gamma$ indetermination is avoided assuming some additional condition on the set of unknowns β_ι, γ_j

The traditional method: Advantages

$$S_{ijt} = sec\ \chi_{ijt}\ \Sigma_n c^{(t)}{}_n \Psi_n\ (\Phi_{ijt}, \Lambda_{ijt}) + \beta_\iota + \gamma_j$$

Excellent observations/unknowns budget

Coefficients of ***VEC*** expansion plus one β per satellite, one γ per receiver, both constant.

No need to perform calibration for every new set of data:

just compute the leveled slants and subtract an available set of pre-computed β_ι, γ_j

$$TEC_{ijt} = S_{ijt} - \beta_\iota - \gamma_j$$

Use pre-computed values during storm periods or at extreme latitudes (inadequacy of ***VEC*** expansion)

Use pre-computed values provided by others

Use of pre-computed values

Slants to calibrate

From a set of IGS stations (RINEX files)

Work has been already done by IGS: monthly values biases for satellites and IGS stations are available at

ftp://ftp.unibe.ch/aiub/CODE/

For user owning their own receiver

Use CODE for satellite biases, set up a calibration algorithm to estimate the bias of the receiver γ

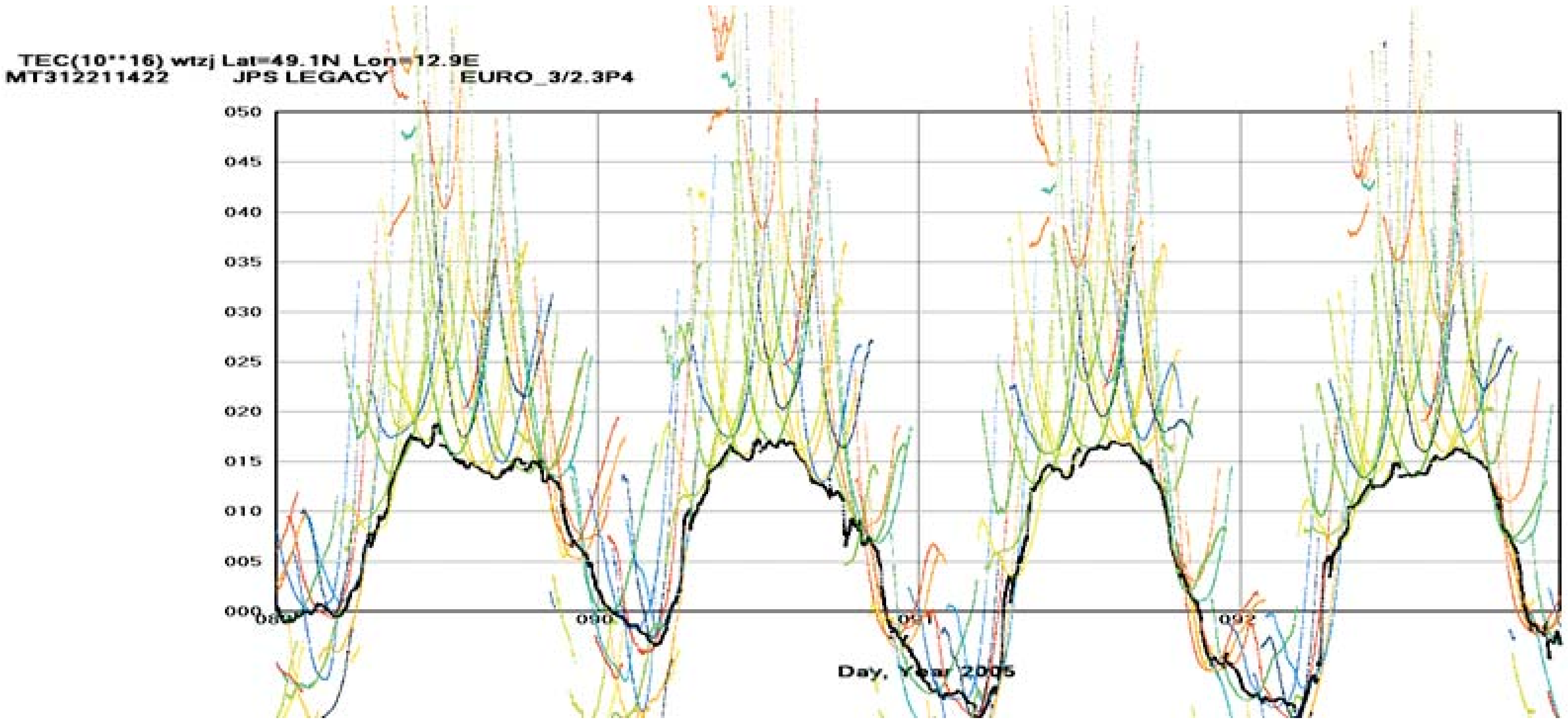
$$S_{ijt} - \beta_\iota = \sec \chi_{ijt} \, \Sigma_n c^{(t)}_n \Psi_n \, (\Phi_{ijt}, \Lambda_{ijt}) + \gamma$$

Why proposing a different solution?

Assuming the traditional approach:

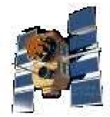
Slants (to the same satellite) of co-located receivers are not the same

Possible occurrence of negative ***TEC***s at middle latitudes



Which of the reported limitations can produce this errors?

Disregarding the multi-path error λ_{Arc} ?



β

The close stations experiment

Station 1

$$S1_{PRN} = TEC + \lambda 1 + \beta_{PRN} + \gamma 1$$

TEC

Station 2

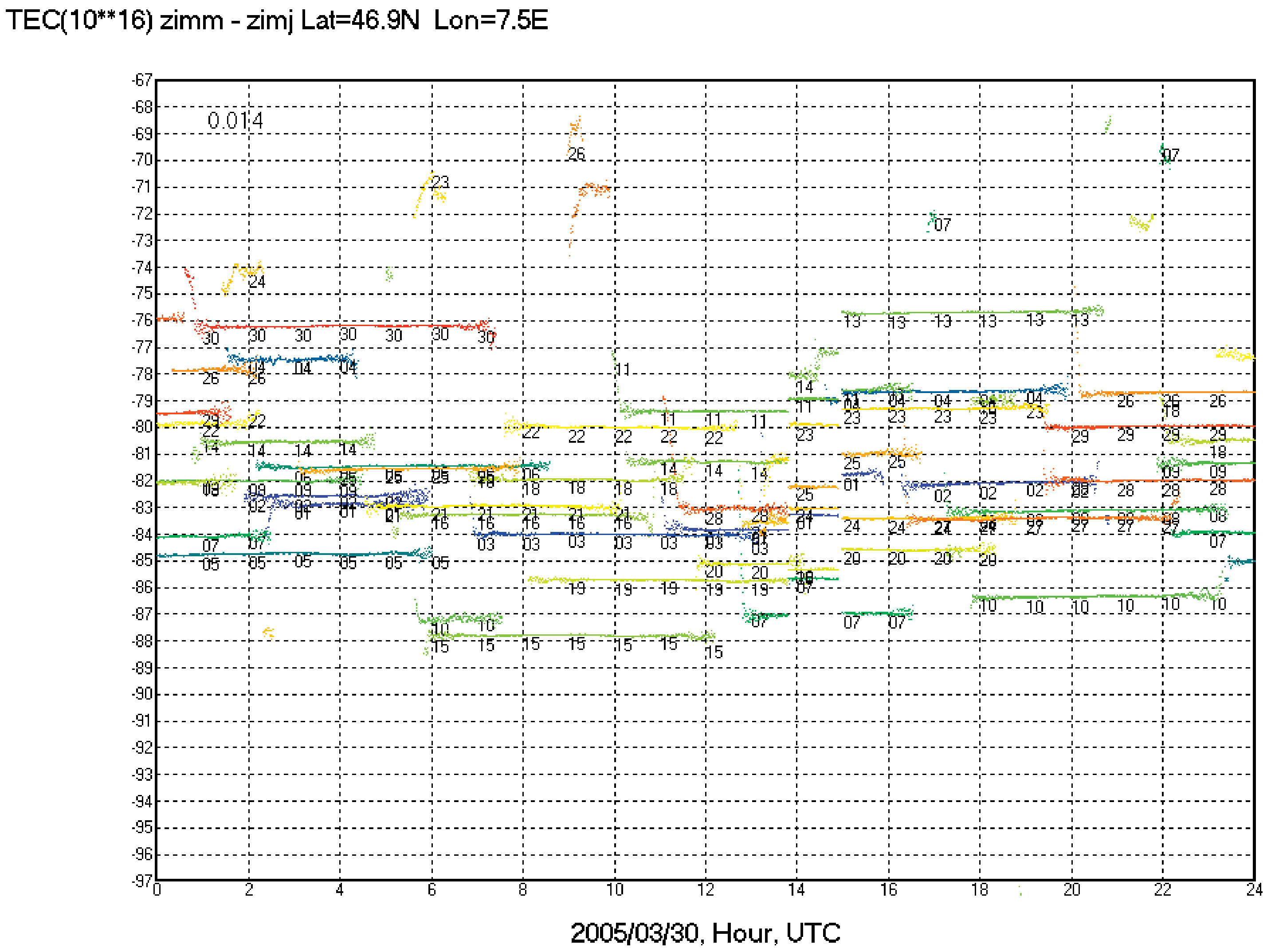
$$S2_{PRN} = TEC + \lambda 2 + \beta_{PRN} + \gamma 2$$

< 100 m

$$S1 - S2 = \gamma 1 - \gamma 2 + \lambda 1 - \lambda 2$$

Not dependent on PRN

$S_1 - S_2$, all satellites



To know more about the topic: look at the recent publication on the

Journal of Geodesy

Calibration Errors on Experimental Slant Total Electron Content (TEC) Determined with GPS

L. Ciraolo, F. Azpilicueta, C. Brunini, A. Meza, S. M. Radicella
(DOI 10.1007/s00190-006-0093-1)

Notes:

having assumed the validity of the thin shell approximation in the single-station solution, in the observations

$$S_{ijt} = sec\ \chi_{ijt}\ \Sigma_n c^{(t)}_n \Psi_n\ (\Phi_{ijt}, \Lambda_{ijt}) + \Omega_{Arc}$$

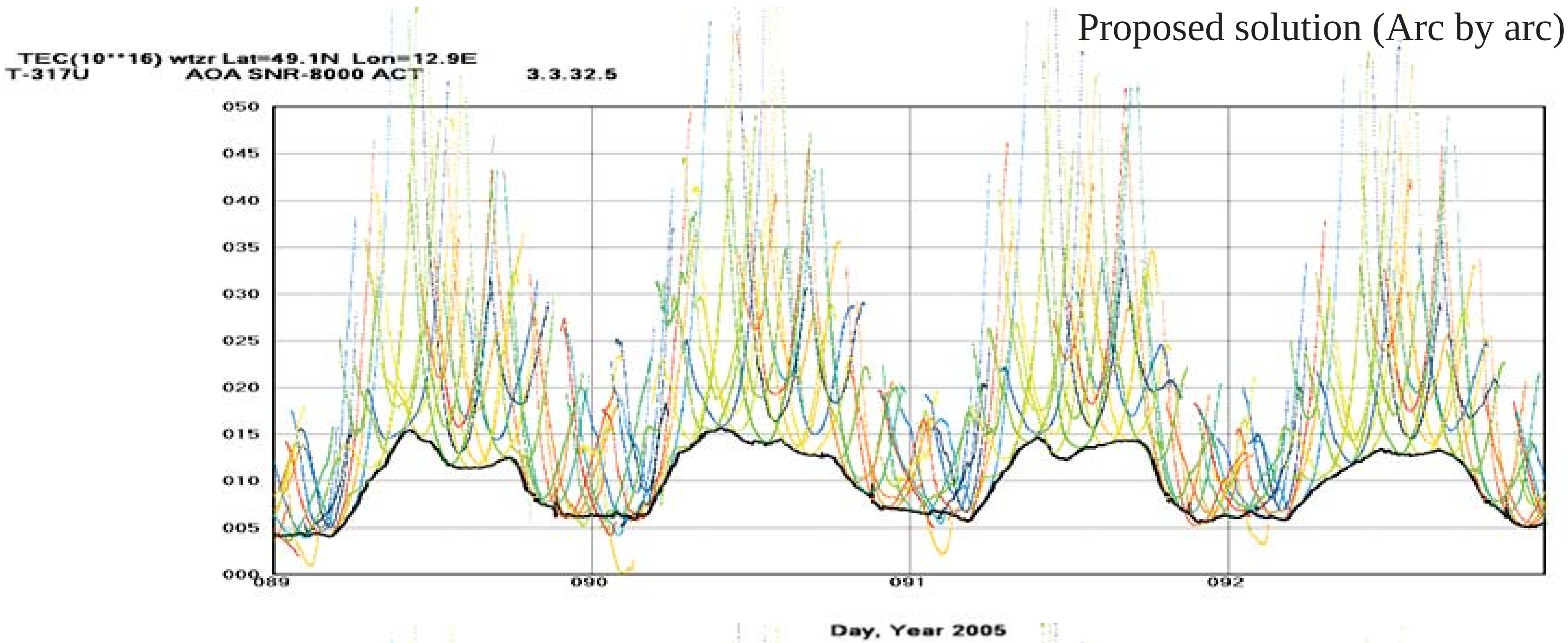
the expansion $\Sigma_n c^{(t)}_n \Psi_n\ (\Phi_{ijt}, \Lambda_{ijt})$ represents the **Vertical Equivalent Content** (***VEq***) and not the actual **Vertical Electron Content** (***VEC***)

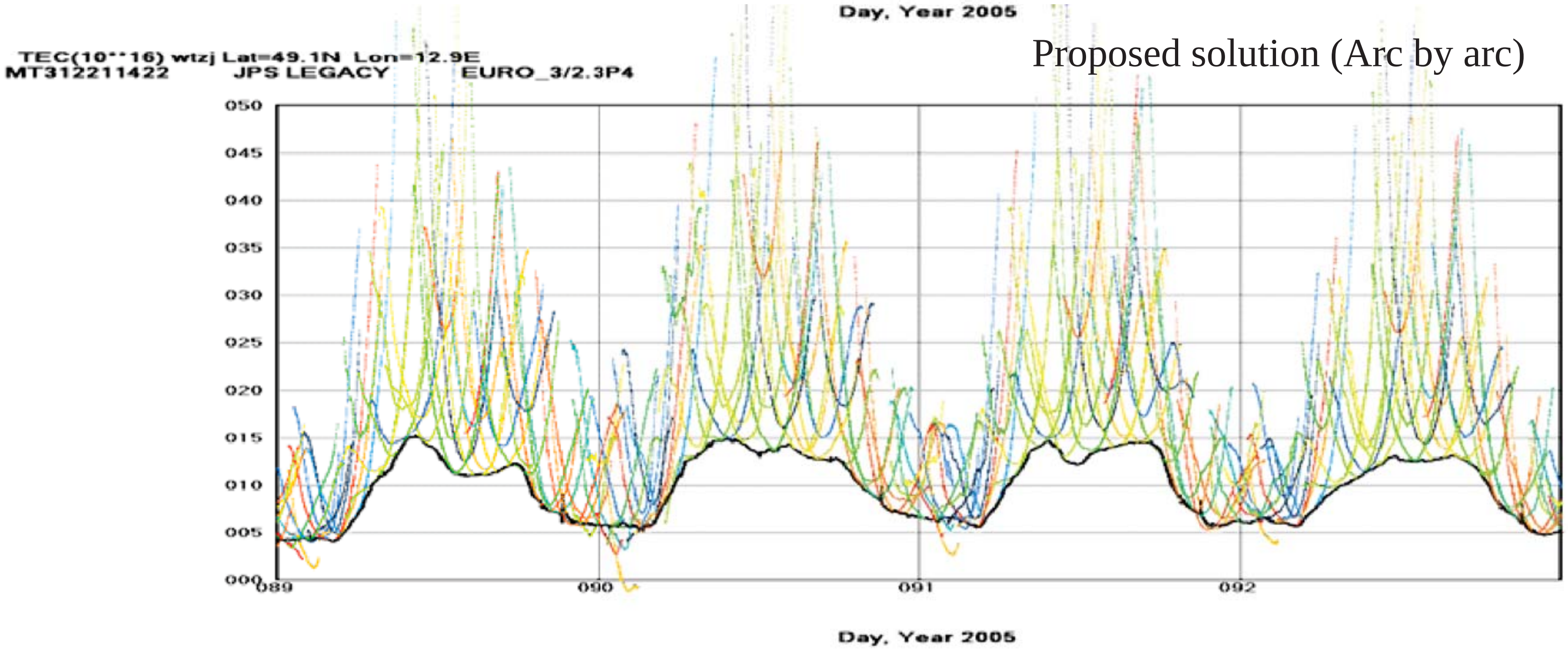
VEq takes automatically into account of plasmaspheric contribution.

Considering Vertical ***TEC*** over the station, nothing will change as ***VEC*** and ***VEq*** coincide.

No possibility to use pre-computed biases

But the solution for co-located receiver will look much more reliable


Proposed solution (Arc by arc)
TEC(10**16) wtzr Lat=49.1N Lon=12.9E
T-317U AOA SNR-8000 ACT 3.3.32.5
050
045
040
035
030
025
020
015
010
005
000
089
090
091
092
Day, Year 2005


Proposed solution (Arc by arc)
TEC(10**16) wtzj Lat=49.1N Lon=12.9E
MT312211422 JPS LEGACY EURO_3/2.3P4
050
045
040
035
030
025
020
015
010
005
000
089
090
091
092
Day, Year 2005

Summary of Proposed Solution characteristics

Observations

Leveled slants or directly phase slants

Assumptions

One thin shell at given height

Elevation mask: 10^o

TEC is expressed through $\boldsymbol{V_{Eq}}$ at the ionospheric point, by the mapping function $\boldsymbol{TEC = V_{Eq} \sec \chi}$

$\boldsymbol{V_{Eq}}$ expressed as a proper expansion of horizontal coordinates ***l, f*** with one set of coefficients at each time $\boldsymbol{V_{Eq}(l, f) = \Sigma_n c_n p_n(l,f)}$

$$\boldsymbol{S_{ijt} = \Sigma_n c^{(t)}_n p_n (l_{ijt}, f_{ijt}) \sec \chi_{ijt} + \Omega_{Arc}}$$

The unknowns are now the coefficients $\boldsymbol{c_n^{(t)}}$ and the offsets $\boldsymbol{\Omega_{Arc}}$

The adopted horizontal coordinates

Using as horizontal coordinates ***Modified Dip Angle*** and ***Local Time***, we can assume that for a set of adjacent epochs (up to ±15 minutes), the coefficients $c_n^{(t)}$ keep constant.

This allows also reducing computing resources during solution using commonly used standard methods for sparse systems.

After the solution of the system, we provide with :

Calibrated slants along the observed rays $TEC_{ijt} = S_{ijt} - \Omega_{Arc}$

“Mapped slants” at given coordinates l_{ijt}, f_{ijt}

Vertical ***TEC*** above the station (ionospheric point at the its zenith)

$$VTec(t) = \sum_n c_n^{(t)} p_n \left(l_{ijt}^{Zenith}, f_{ijt}^{Zenith} \right) \sec \chi_{ijt}$$

Why multi-day solution

A multi-day solution is performed, avoiding day to day discontinuities in calibrated slants, except that at the beginning and the end of the solution.

Still, at the beginning and the end of the set of data, broken arcs occur.

Broken arcs are generally shorter implying

1. worse results during leveling
2. worse numerical conditioning for the solution

To reduce these problems, in order to calibrate ***N*** days, ***N+2*** days are actually processed: first and last day of the ***N+2*** set are discarded.

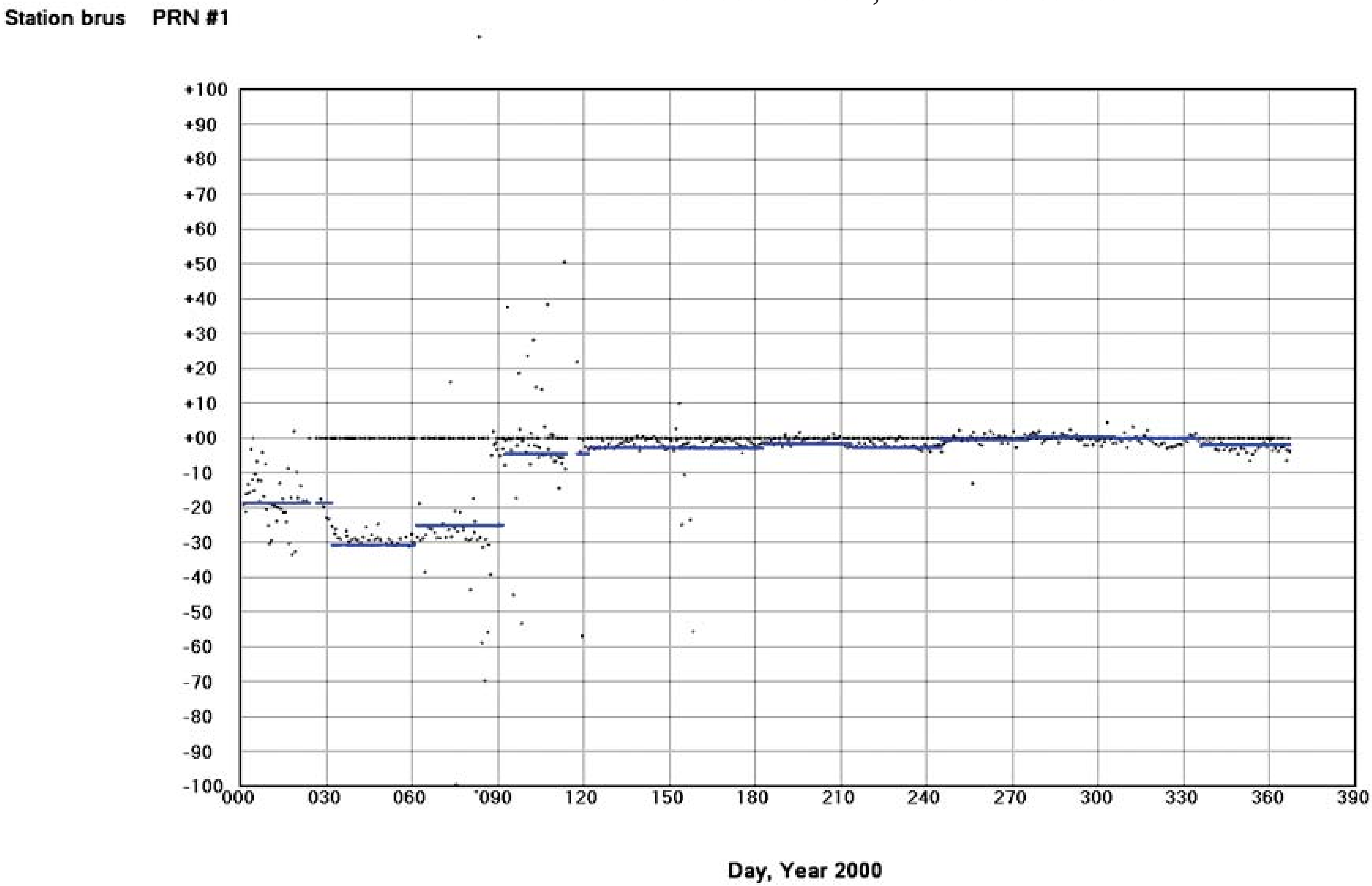
How do traditional and proposed solution compare?

In the following slides it can be seen that the two solutions agree in the average, but the difference in bias can amount to 10 ***TECu***

The pattern of the jumps, similar for different satellites, simply indicates that something has changed in the receiver

CODE Station + Satellite Biases

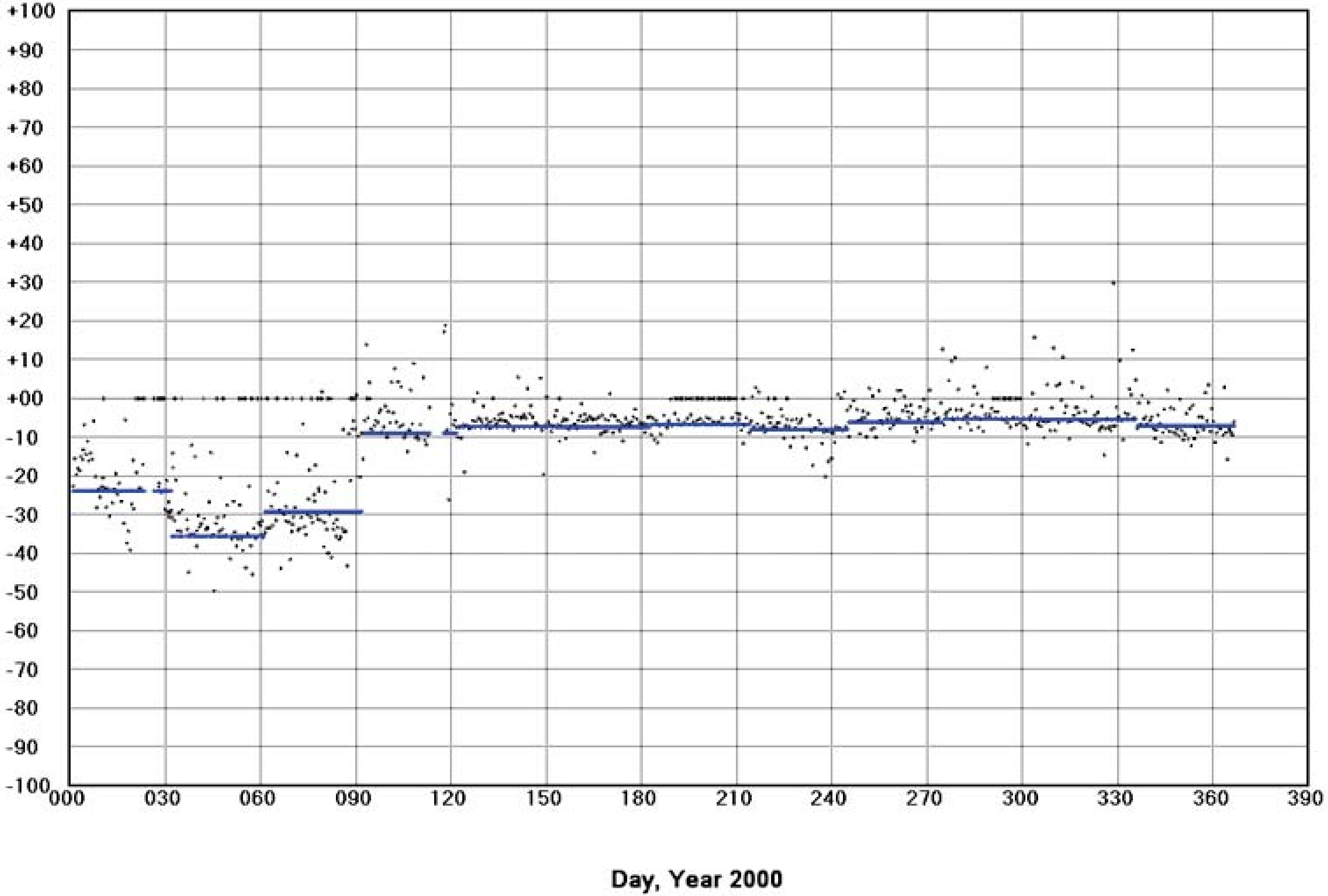
Arc offset solution, individual values



CODE Station + Satellite Biases

Arc offset solution, individual values

Station brus PRN #4



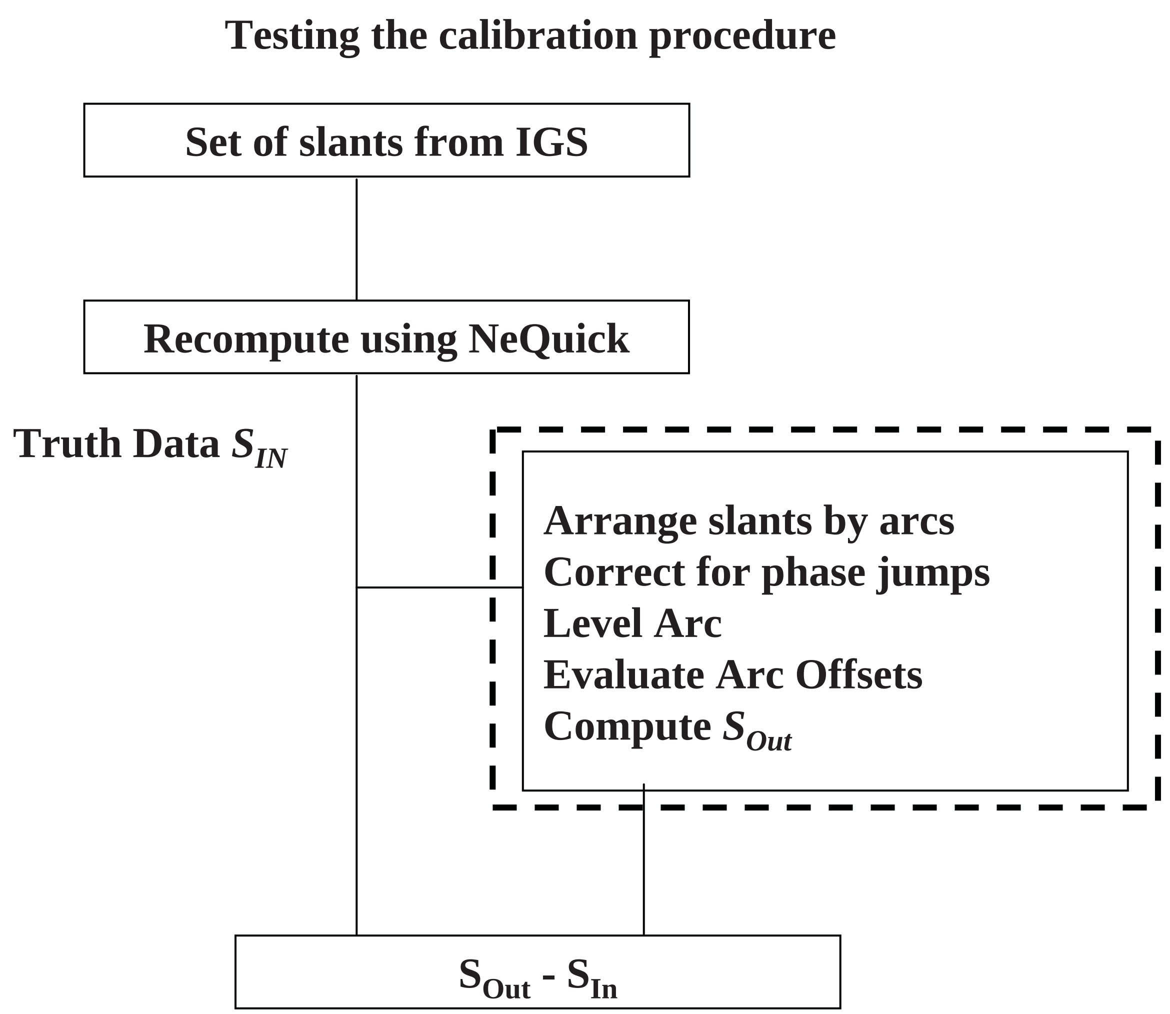
Conclusions for the single-station, multi-day, arc-offset solution

Is it better than the traditional solutions?

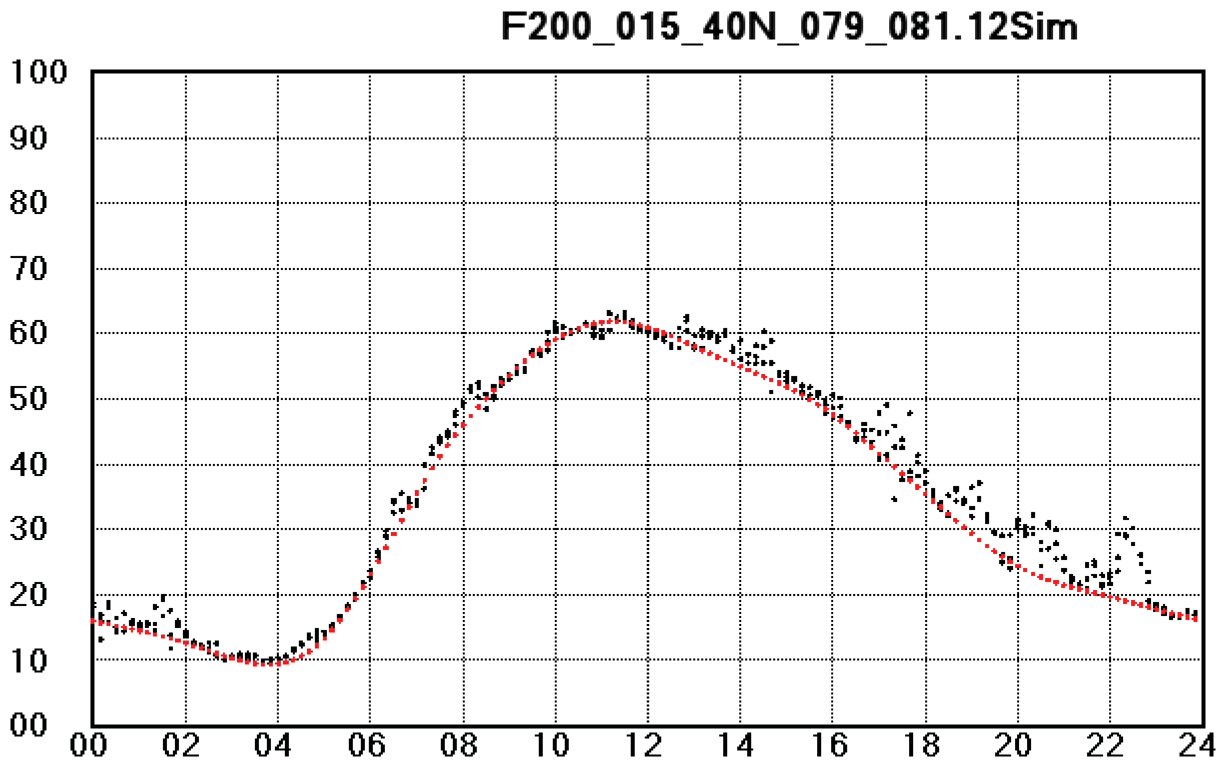
A direct answer is *not* possible because **reliable truth data to perform comparison are not available**.

Models of the electron density can provide with “artificial data” to check the performance of the technique used for the calibration.

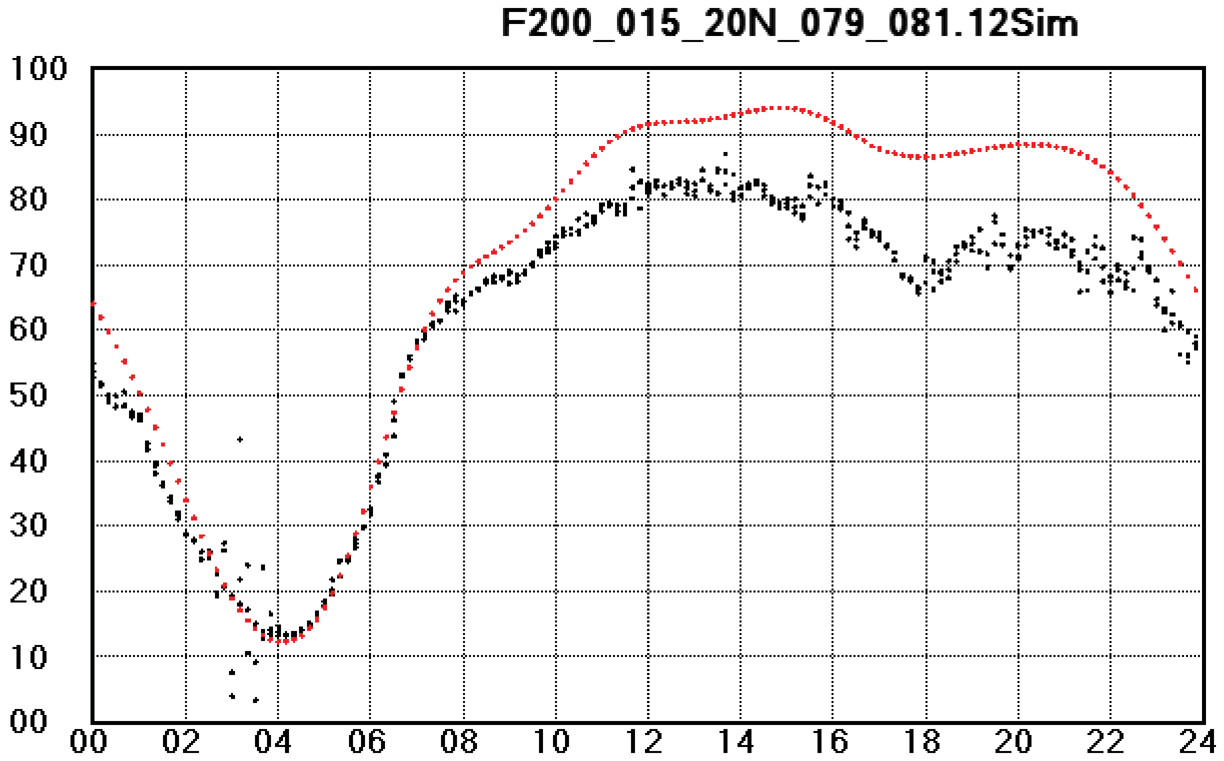
Next slide sketches how reliability of the technique is evaluated.


Testing the calibration procedure
Set of slants from IGS
Recompute using NeQuick
Truth Data S_{IN}
Arrange slants by arcs
Correct for phase jumps
Level Arc
Evaluate Arc Offsets
Compute S_{Out}
S_{Out} - S_{In}

Model VEC (red) vs VEC from calibration (black): middle latitude.



Model VEC (red) vs VEC from calibration (black): low latitude.



Hints on present work

Investigating numerical methods in which assumption is made that not only observations but also the coefficients of ***VEq*** expansion are affected by errors.

Thank you