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Fundamentals of Satellite Navigation

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Fundamentals of Satellite Navigation

- Geodesy
- Time and clocks
- Satellite orbits
- Positioning

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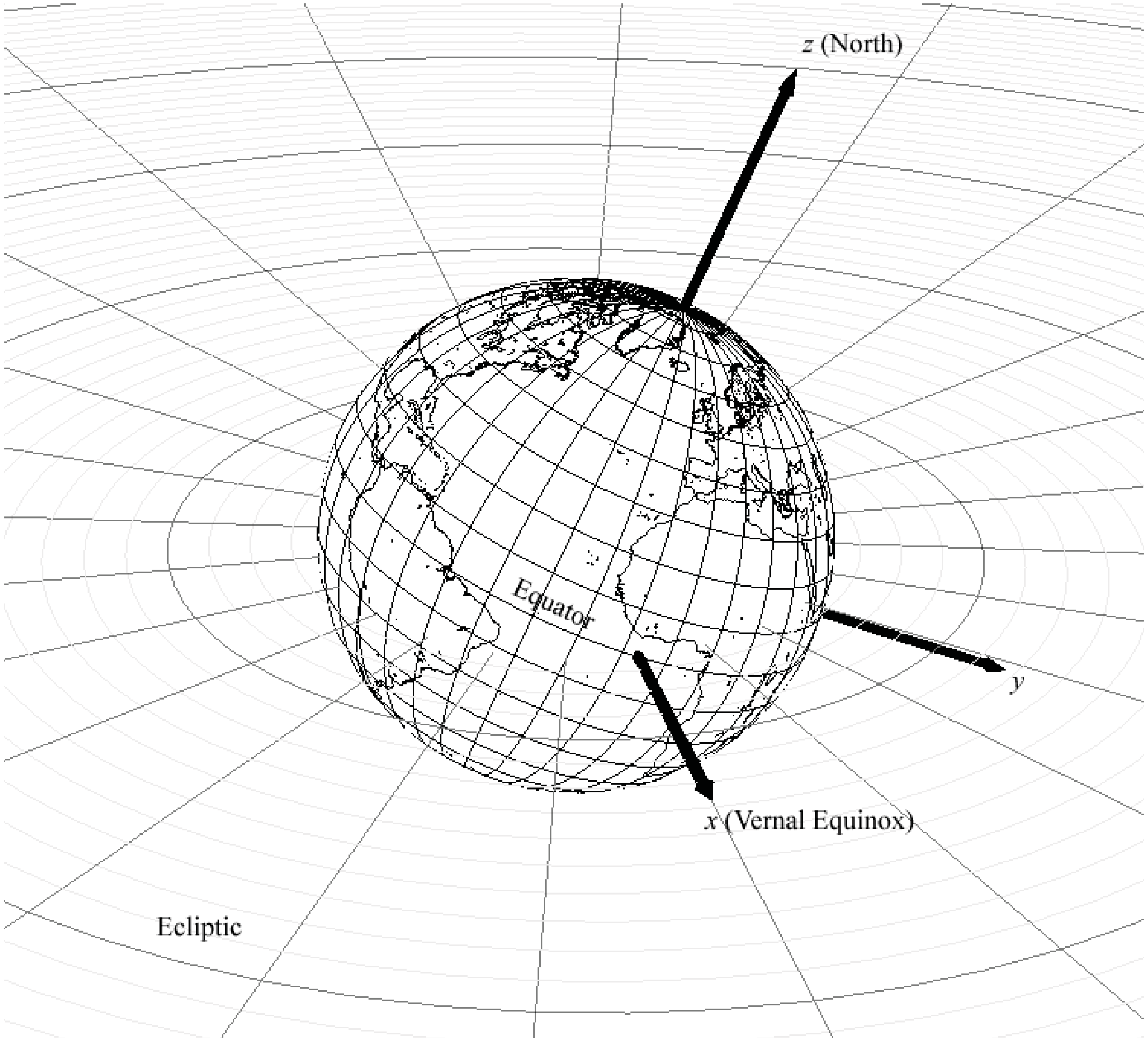
Earth Centered Inertial (ECI) Coordinate System



- Oblateness of the Earth causes direction of axes to move over time
- So that coordinate system is truly “inertial” (fixed with respect to stars), it is necessary to fix coordinates
- J2000 system fixes coordinates at 11:58:55.816 hours UTC on January 1, 2000

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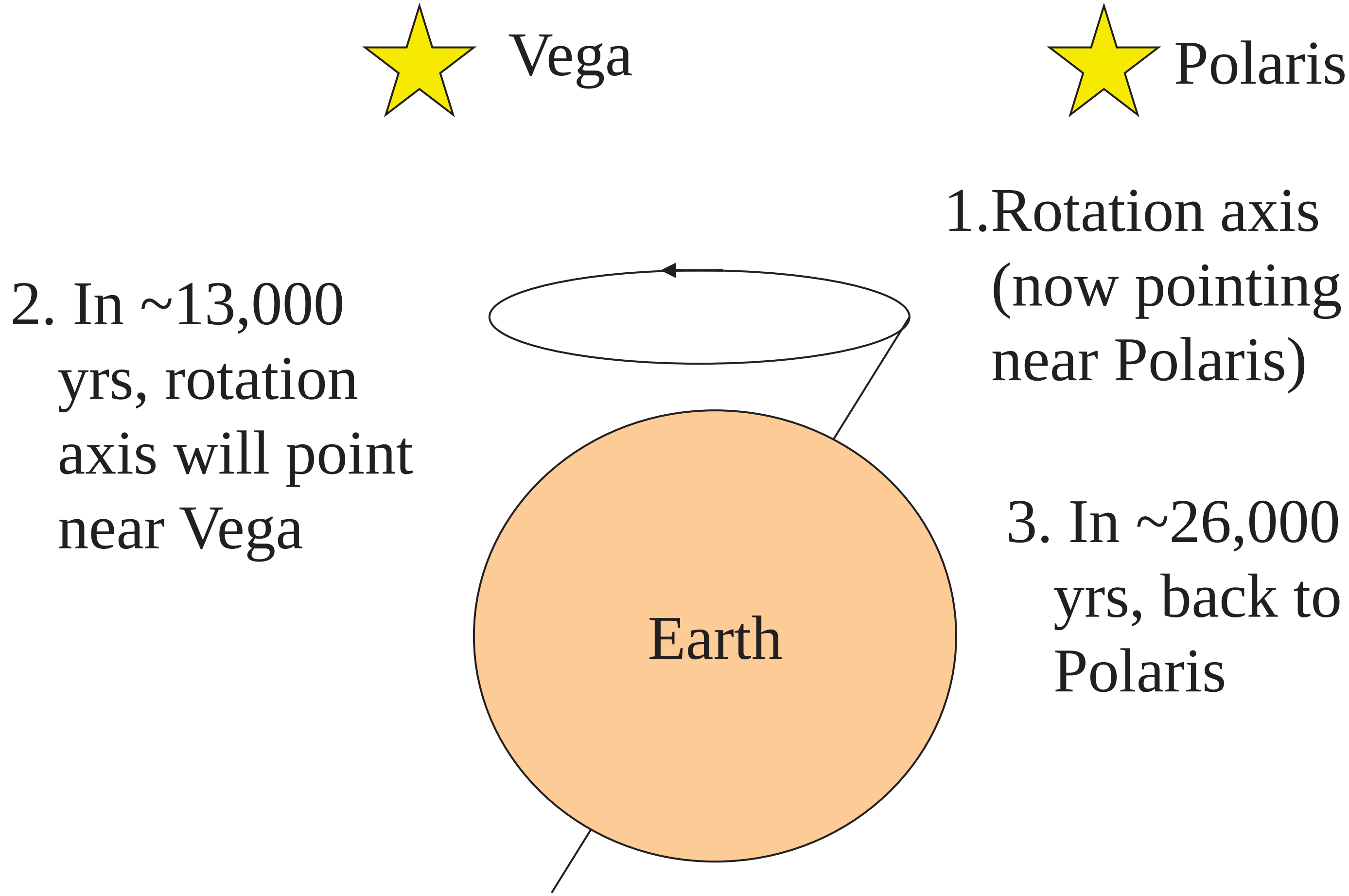
Precession and Nutation



- **Precession is the large (23.5 deg half-angle) periodic motion**
- **Nutation is a superimposed oscillation (~9 arcsec max)**

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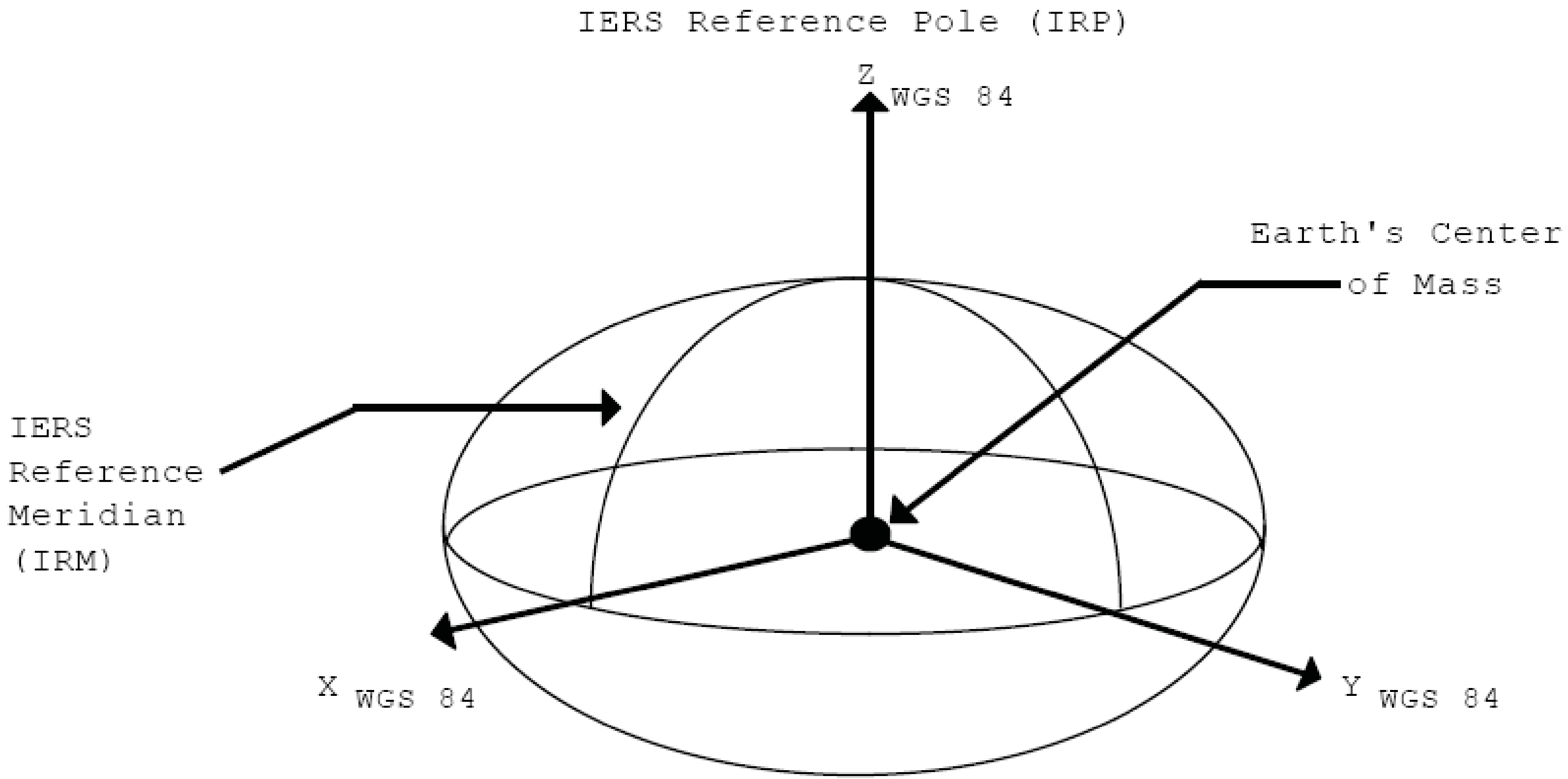
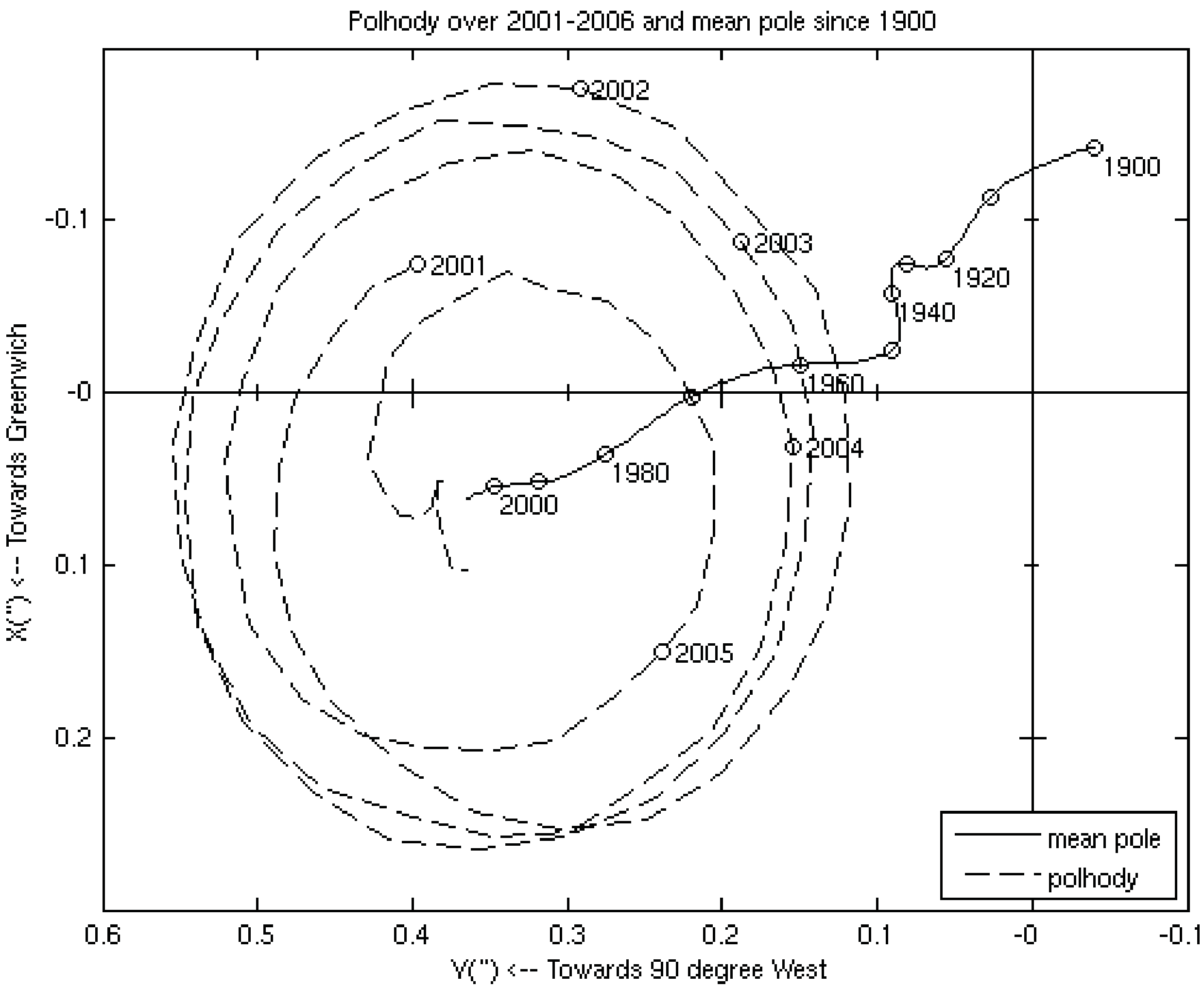
Earth Centered Earth Fixed (ECEF) Coordinate System



Notes: (1) By convention, the z-axis is the mean location of the north pole (spin axis of Earth) for 1900 – 1905, (2) the x-axis passes through 0º longitude, (3) the Earth's crust moves slowly with respect to this coordinate system!

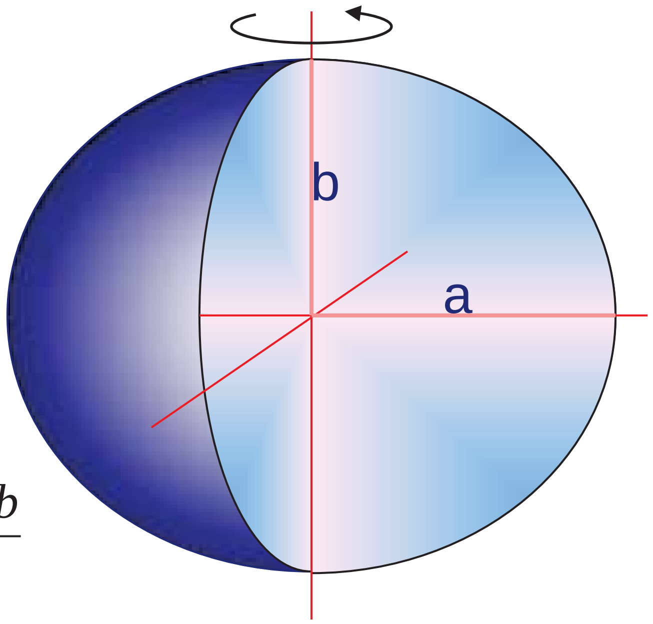
Polar Motion



Source: www.iers.org

Ellipsoid

- Approximation to Earth's shape
  – Flattened at poles
- World Geodetic System (WGS)-84 values:
  – Semimajor axis, a = 6378137.0 m
  – 1/f = 298.257223563

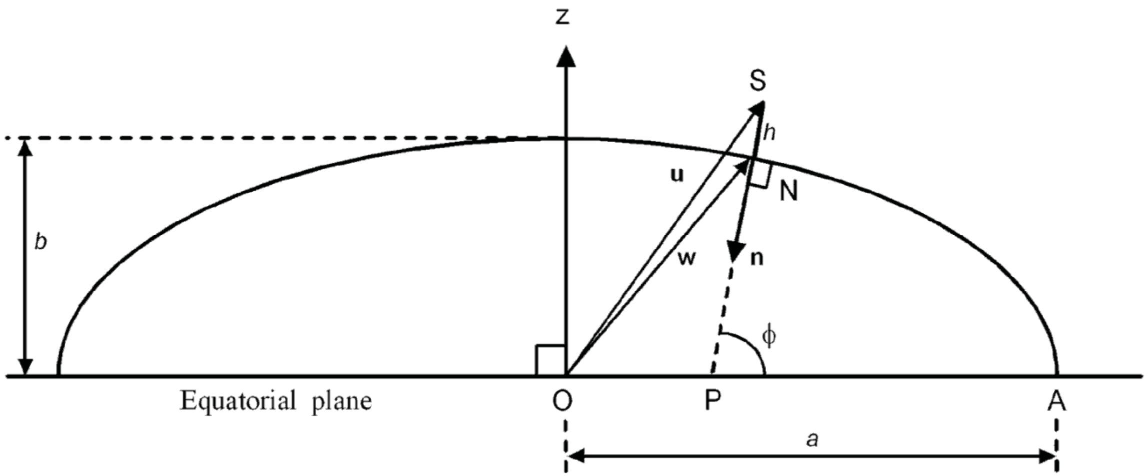


$$f = \frac{a-b}{a}$$

Latitude, Longitude, and Height

Longitude computation

$$\lambda = \begin{cases} \arctan\left(\dfrac{y_u}{x_u}\right), & x_u \geq 0 \\ 180^\circ + \arctan\left(\dfrac{y_u}{x_u}\right), & x_u < 0 \ \text{ and } \ y_u \geq 0 \\ -180^\circ + \arctan\left(\dfrac{y_u}{x_u}\right), & x_u < 0 \ \text{ and } \ y_u < 0 \end{cases}$$



Latitude and height computation

$$p = \sqrt{x^2 + y^2}$$

$$\tan u = \left(\frac{z}{p}\right)\left(\frac{a}{b}\right)$$

Iteration Loop

$$\cos^2 u = \frac{1}{1 + \tan^2 u}$$

$$\sin^2 u = 1 - \cos^2 u$$

$$\tan\varphi = \frac{z + e'^2 b \sin^3 u}{p - e^2 a \cos^3 u}$$

$$\tan u = \left(\frac{b}{a}\right)\tan\varphi$$

until $\tan u$ converges, then

$$N = \frac{a}{\sqrt{1 - e^2 \sin^2 \phi}}$$

$$h = \frac{p}{\cos\phi} - N \qquad \varphi \neq \pm 90^\circ$$

otherwise

$$h = \frac{z}{\sin\phi} - N + e^2 N \qquad \varphi \neq 0$$

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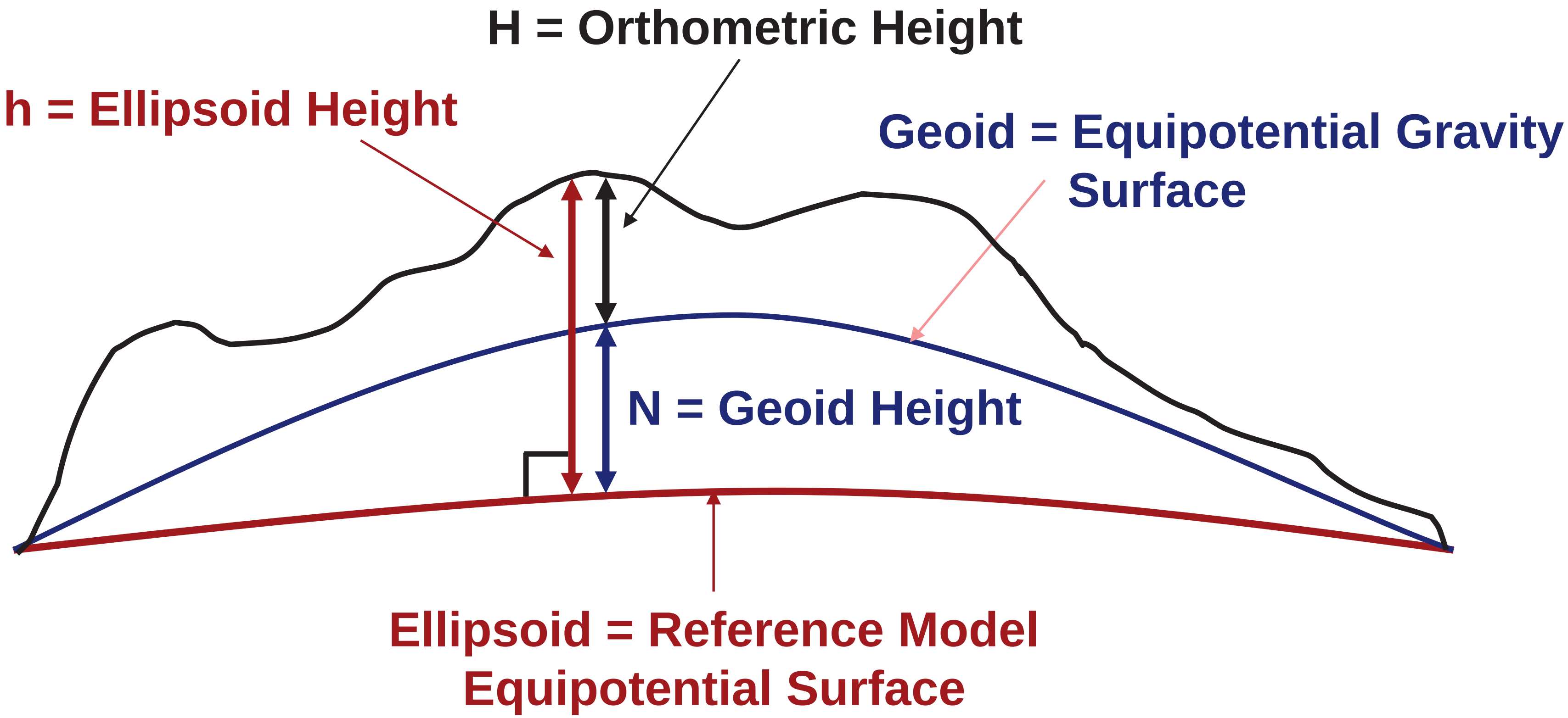
Latitude, Longitude, and Height to ECEF

$$\begin{bmatrix} x_{ecef} \\ y_{ecef} \\ z_{ecef} \end{bmatrix} = \begin{bmatrix} \dfrac{a\cos\lambda}{\sqrt{1+(1-e^2)\tan^2\phi}} + h\cos\lambda\cos\phi \\ \dfrac{a\sin\lambda}{\sqrt{1+(1-e^2)\tan^2\phi}} + h\sin\lambda\cos\phi \\ \dfrac{a(1-e^2)\sin\phi}{\sqrt{1-e^2\sin^2\phi}} + h\sin\phi \end{bmatrix}$$

Geoid

- **Equipotential surface of Earth's gravity field that fits mean sea level on average globally**
- **Common representations:**
  - **Spherical harmonic coefficients**
  - **Grid of values over Earth**
- **Heights measured relative to geoid are referred to as *orthometric heights***

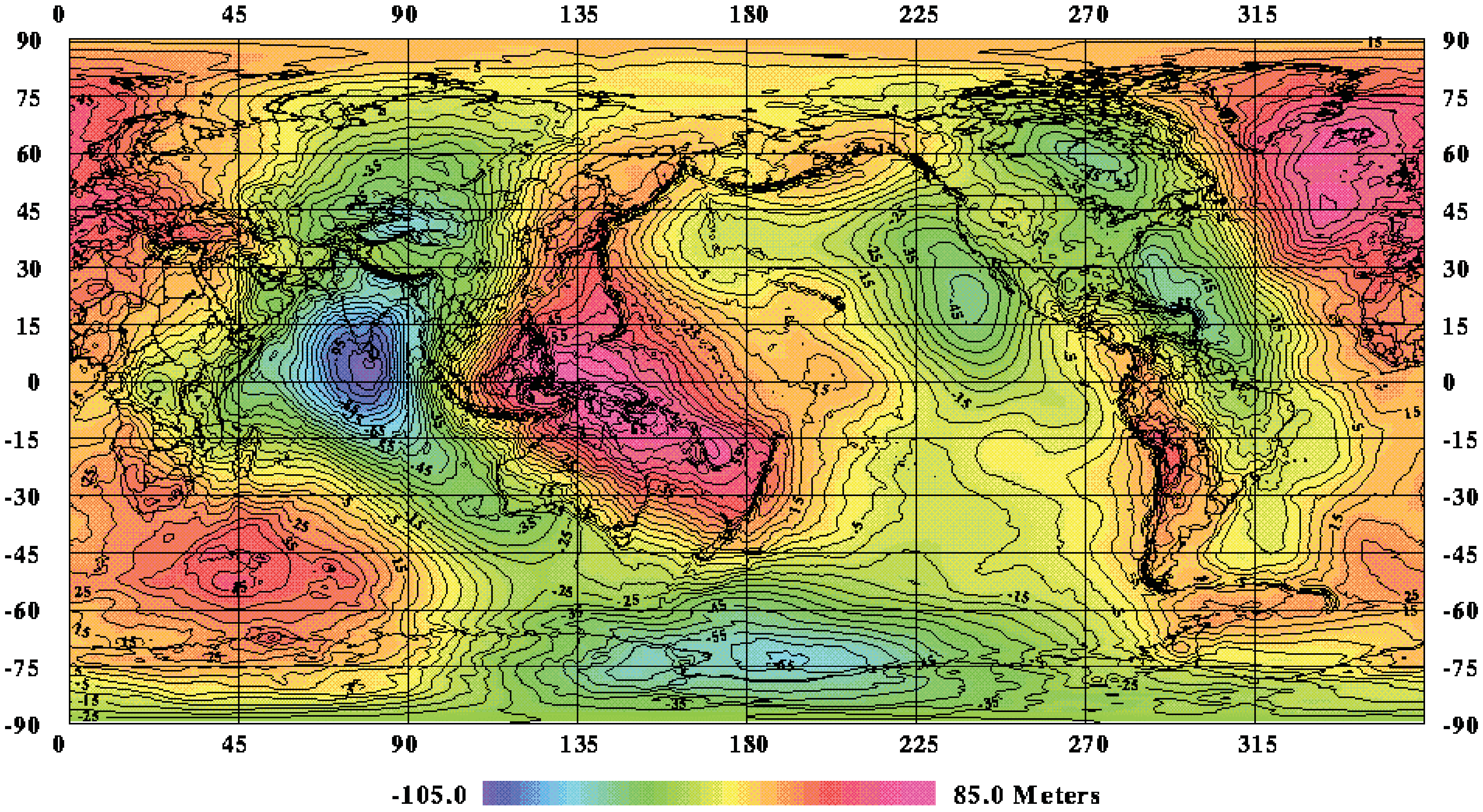
Geodetic Height Definitions



h(Ellipsoid Height) = H(Orthometric Height) + N(Geoid Height)

Source: National Geospatial-Intelligence Agency.

The Geoid



Source: National Geospatial-Intelligence Agency.

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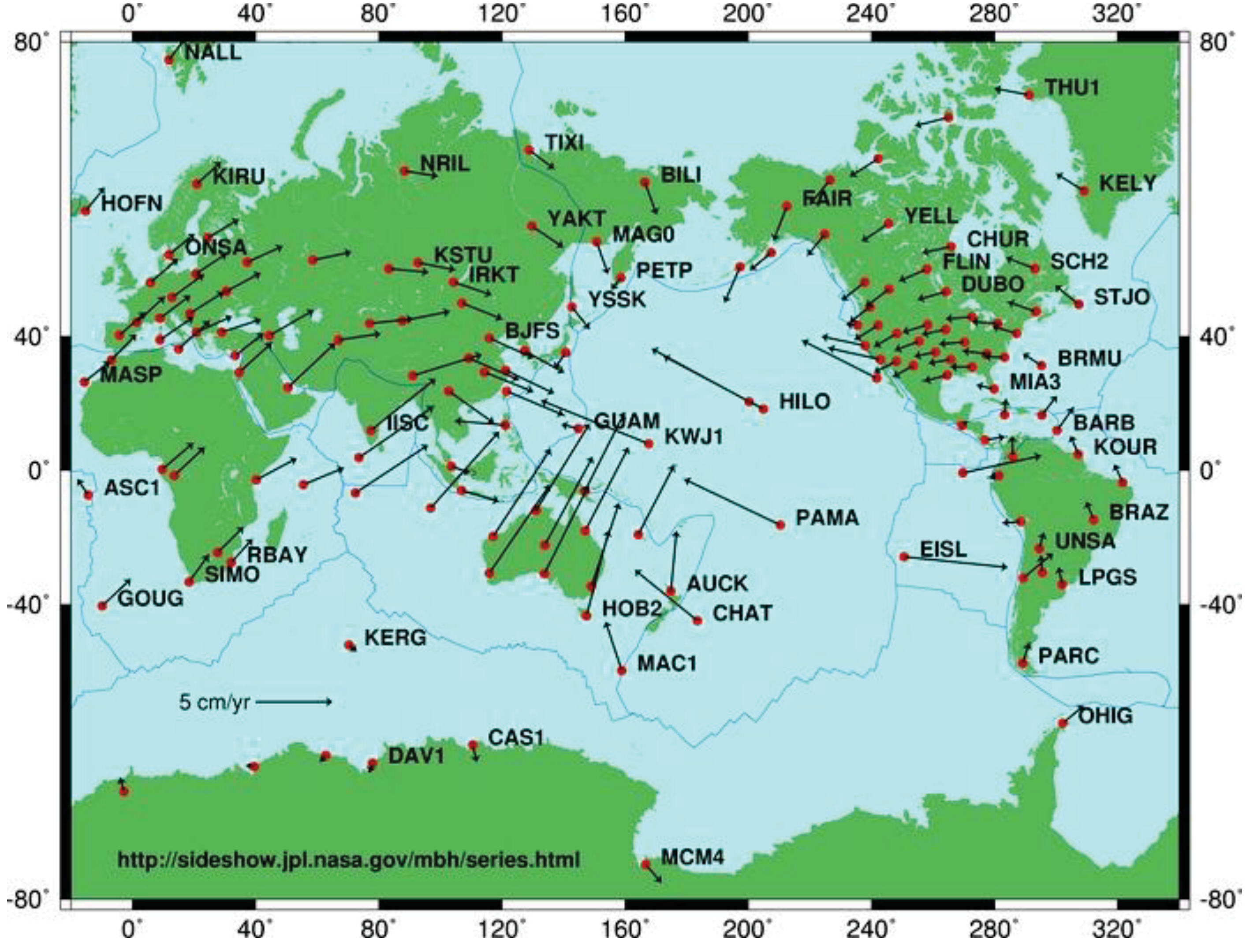
Tectonic Plates



Source: Jet Propulsion Laboratory.

Solid Earth Tides and Ocean Loading

- Earth's surface is not rigid, but rather somewhat pliable
- Lunar and solar gravity, etc., deform the Earth with strong diurnal and semidiurnal variations over time
  - Displacements may be as large as ~30 cm
- Ocean tides can cause additional few cm station displacements in coastal locations
- Station coordinates often defined to exclude effects of solid Earth tides and ocean loading

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Time

- Difficult to define time!
- For our purposes, time is the quantity that is read off a clock
- Any clock includes two main components:
  - Periodic event
  - Counter of that event

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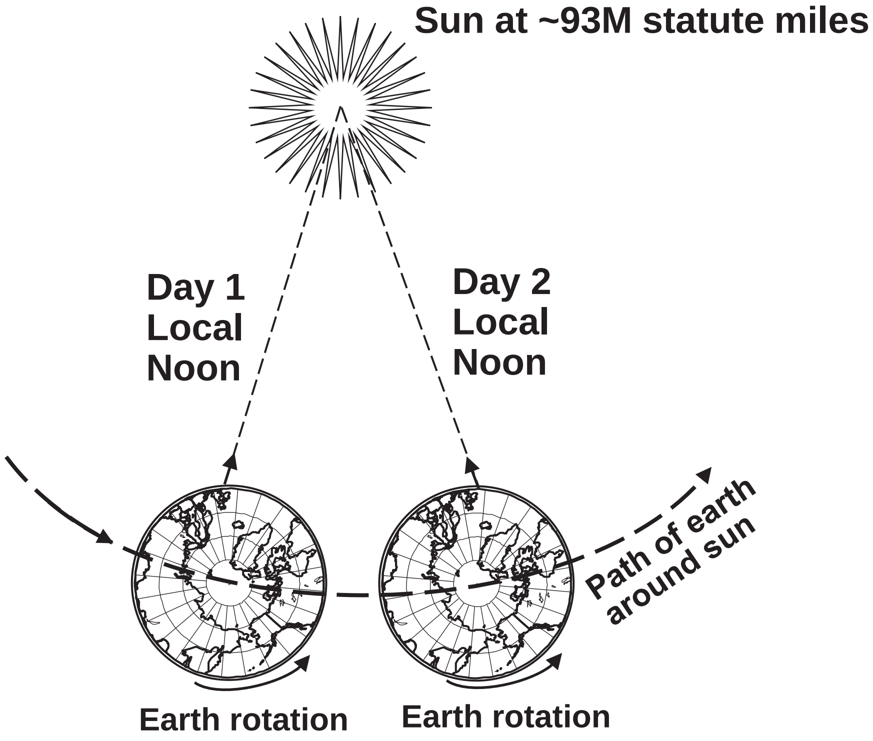
Timescales

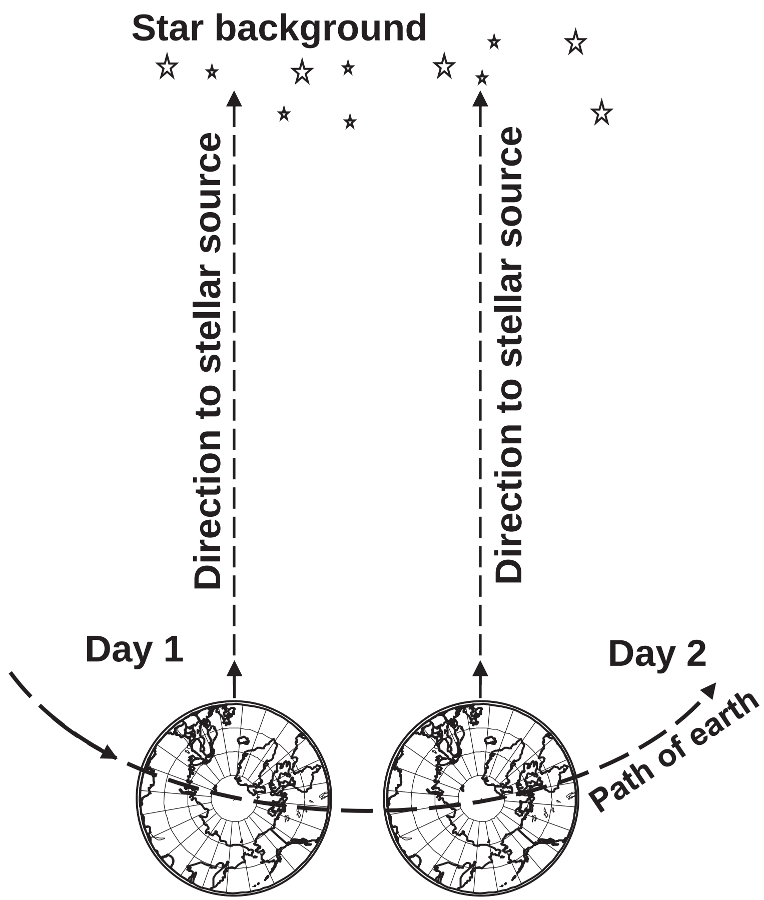
- Universal Time (UT) - one of several time scales based upon rotation rate of Earth
  - UT0 – mean solar time based upon astronomical observations from prime meridian
  - UT1 – corrects UT0 for polar motion
  - UT2 – corrects UT1 for seasonal variations in Earth rotation rate
  - The second used to be defined as 1/86400 of a solar day
- Atomic time
  - Since 1967, the second has been defined by the duration of 9,192,631,770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the Cesium-133 atom
  - International atomic time (TAI) – timescale maintained by international collection of atomic clocks

Sidereal vs Solar Day



Solar Day

24 h



Sidereal Day
23 h 56 min 3.5 s

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Coordinated Universal Time (UTC)

- TAI is a continuous timescale
  - No connection to mean solar time
- UTC differs from TAI by an integer number of “leap” seconds
  - Introduced when needed to keep UTC within 0.9 s of UT1
  - As of May 2013, TAI – UTC = 35 s
- UTC is maintained by the Bureau International des Poids et Mesures (BIPM) in Sevres, France
  - Produced using times maintained by over 250 clocks in 65 nations
  - Not in real-time
- Official U.S. time is the realization of UTC maintained by the U.S. Naval Observatory – UTC(USNO)

GPS Time

- GPS Time is maintained by the GPS control segment based upon an average of all system clocks
  - Satellites
  - Monitor stations
- It is a continuous time scale
  - TAI – GPS Time ≅ 19 s
  - GPS Time – UTC ≅ 16 s (as of May 2013)
- GPS Time is specified with be within 1 microsecond of UTC(USNO) modulo 1 s

Stability of Frequency Sources - Definitions

- **Consider the oscillator output:**

$$V(t) = A(t)\sin(2\pi f_0 t + \phi(t))$$

- **Instantaneous phase:**

$$\varphi(t) = 2\pi f_0 t + \phi(t)$$

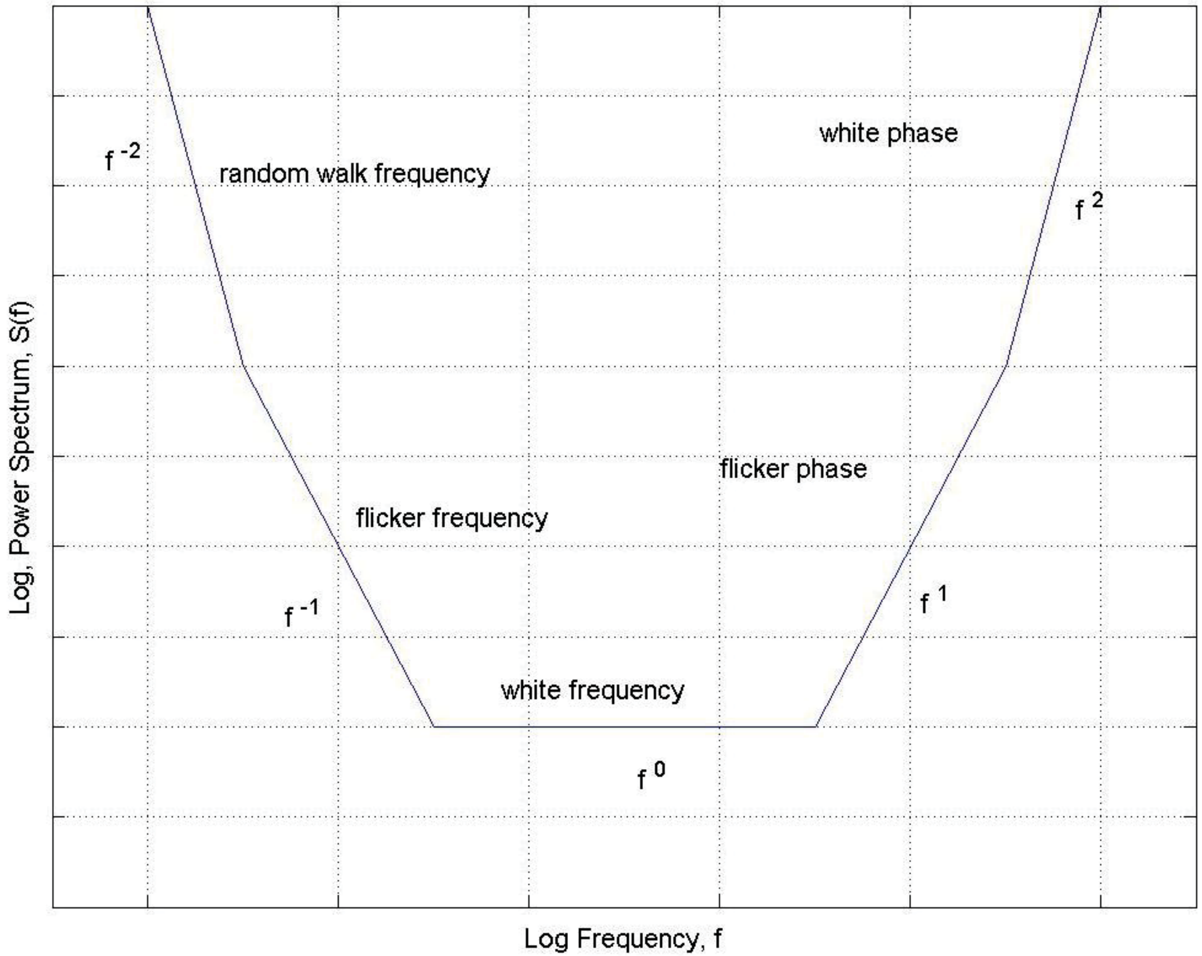
- **Instantaneous frequency:**

$$f(t) = f_0 + \frac{1}{2\pi}\frac{d\phi(t)}{dt}$$

- **Fractional frequency deviation:**

$$y(t) = \frac{1}{2\pi f_0}\frac{d\phi(t)}{dt}$$

Frequency Noise



Typical characteristics of power spectrum of *y(t)* are illustrated above.

Allan Deviation

- **Allan deviation is defined as:**

$$\sigma_y(\tau) = \sqrt{\frac{1}{2} E\left[(\overline{y}_{k+1} - \overline{y}_k)^2 \right]}$$

- **where**

$$\overline{y}_k = \frac{\phi([k+1]\tau) - \phi(k\tau)}{2\pi f_0 \tau}$$

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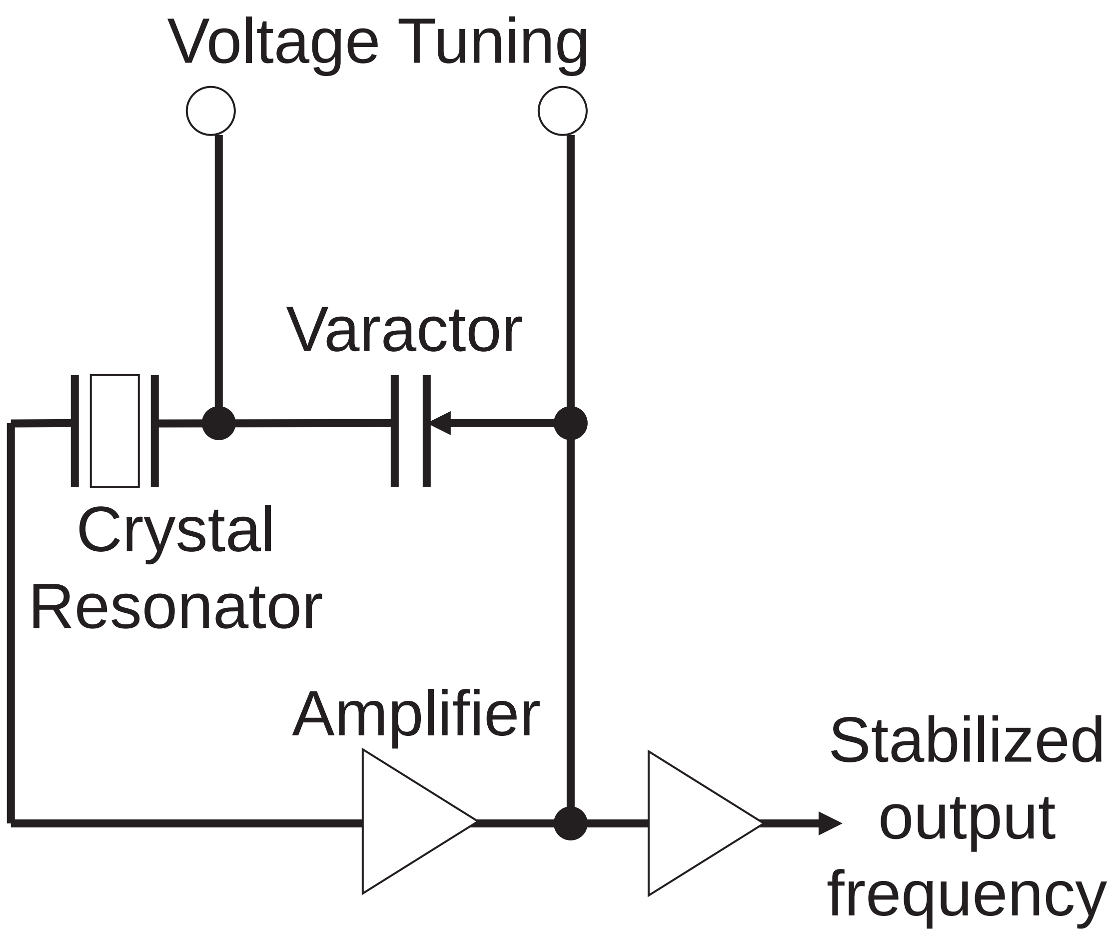
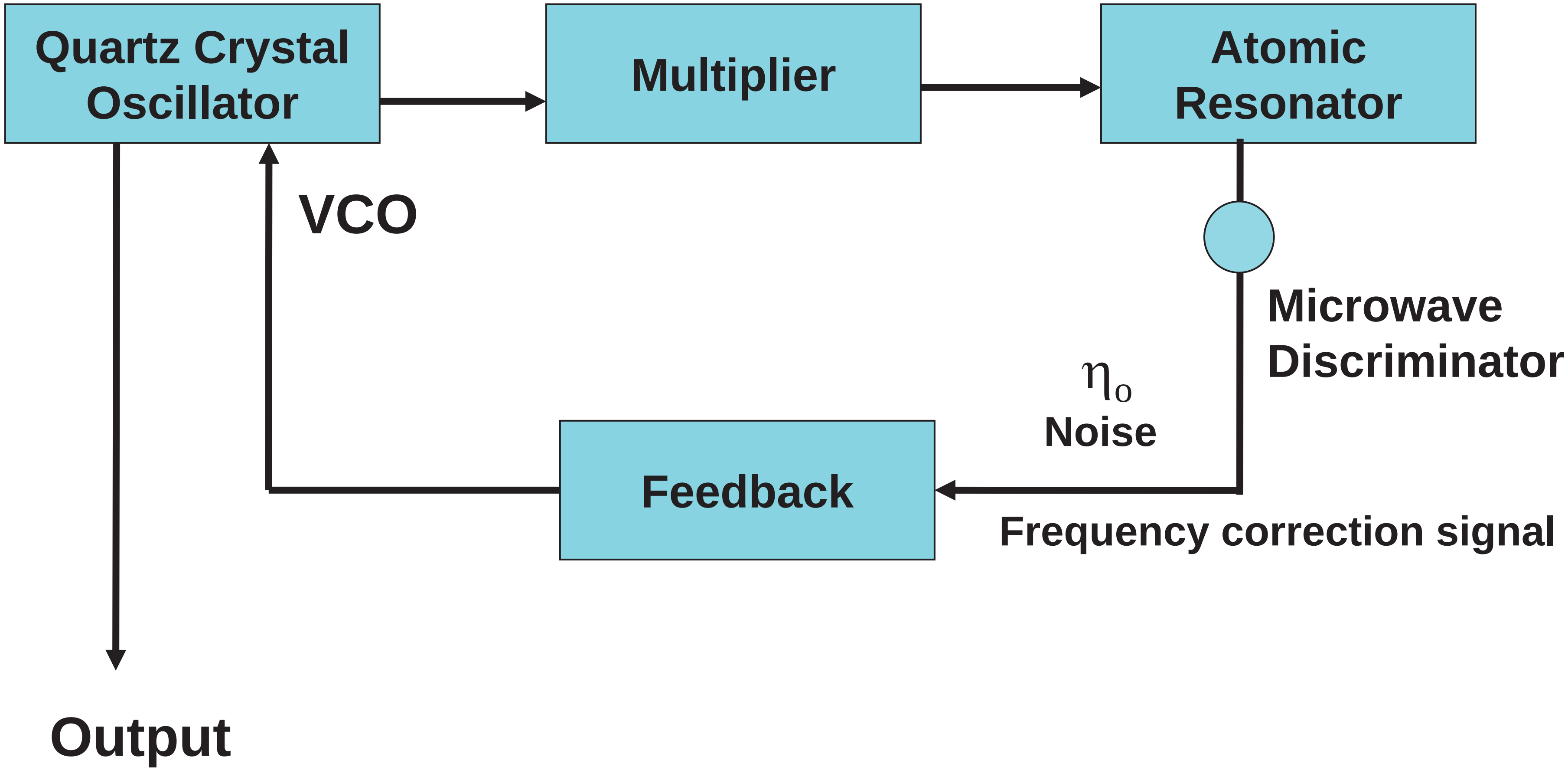
Crystal Oscillator



Atomic Frequency Standard



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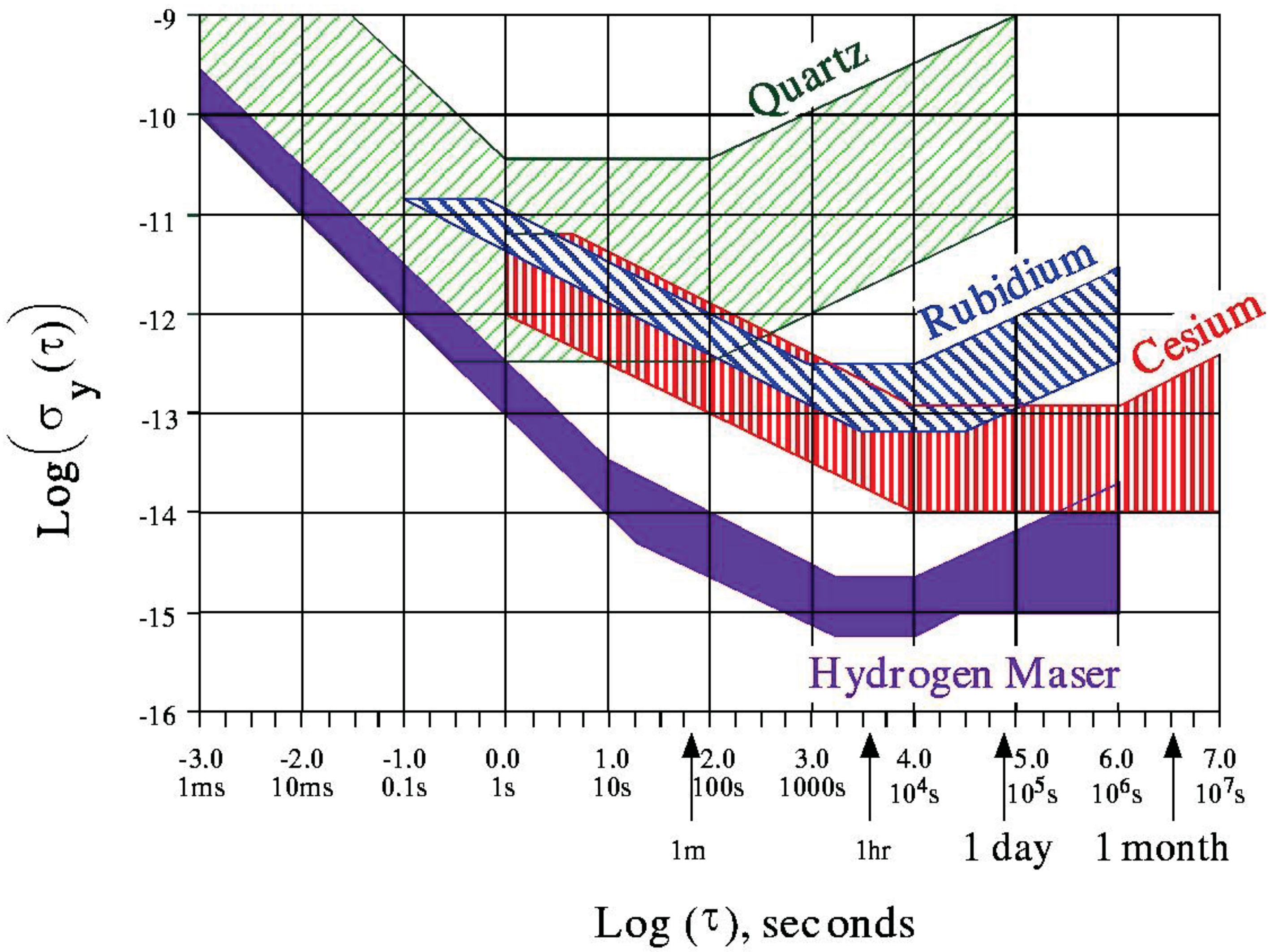
Stability of Various Clock Types



Source: John Vig IEEE tutorial.

Kepler's Laws

- A satellite's orbit is in the shape of an ellipse with the Earth at one focus
- The radius drawn from the center of the Earth to the satellite sweeps out equal area in equal times
- The square of the orbital period of a satellite is proportional to the cube of the semi-major axis

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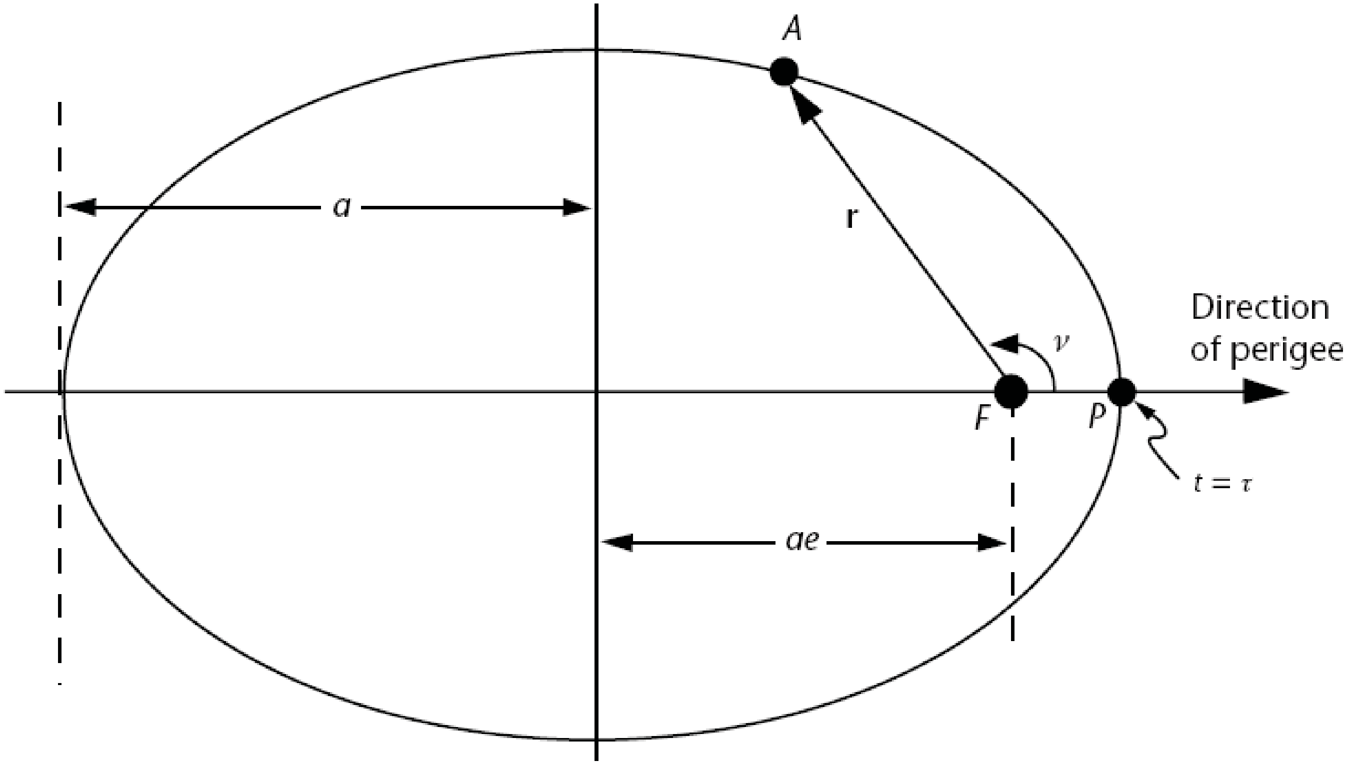
Keplerian Elements – Shape of Orbit



Keplerian elements:

a **= semimajor axis**

e **= eccentricity**

τ = time of perigee passing

A = location of satellite, F = focus (Earth center), P = perigee (lowest point on orbit)

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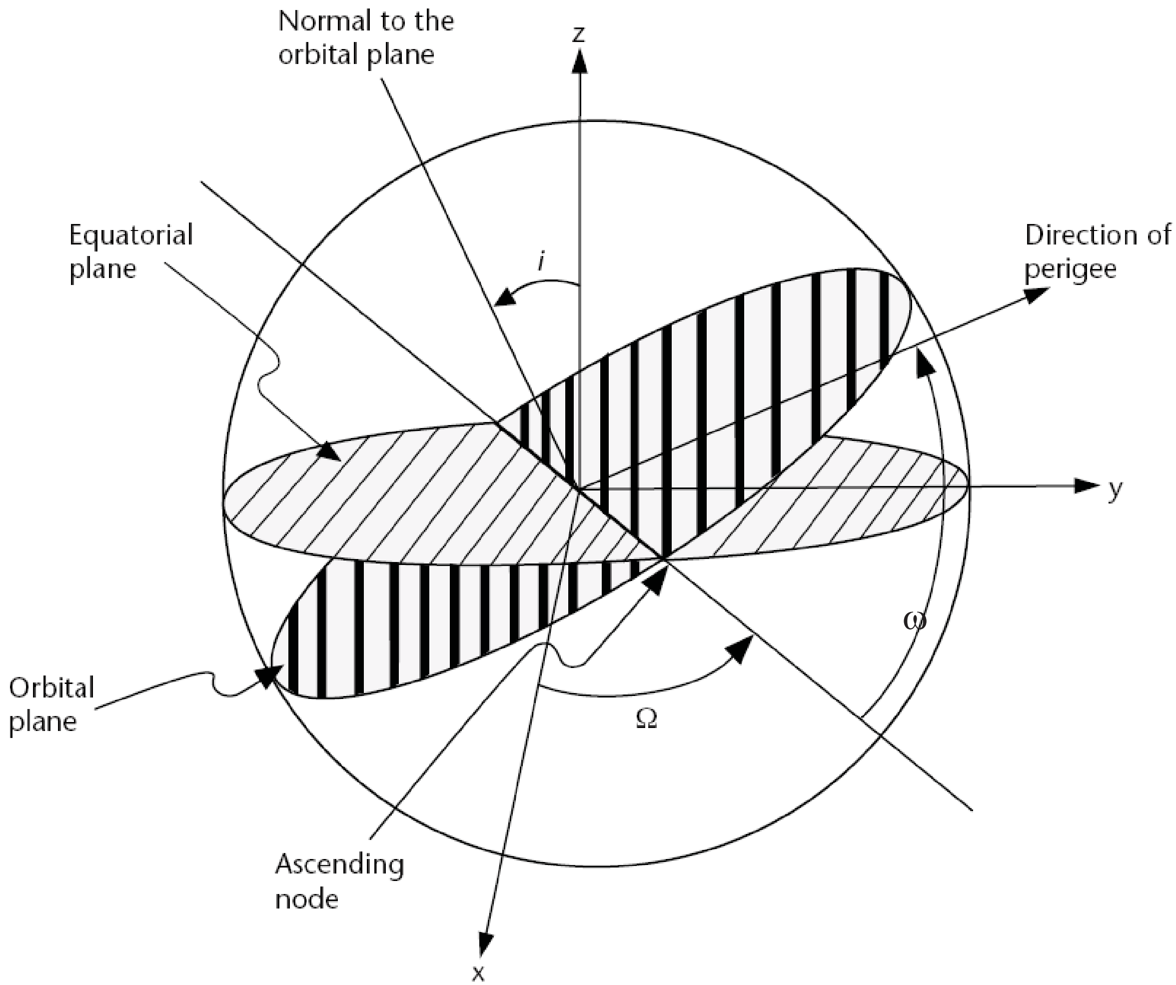
Keplerian Elements – Orientation of Orbit



Ω = right ascension of ascending node, i = inclination, ω = argument of perigee

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Perturbations to Orbits

- Satellite orbits are not perfect ellipses due to various perturbations:
  - Earth's oblateness
  - Third-body effects – sun, moon, etc.
  - Solar radiation pressure
- As a result, more orbital elements are needed to accurately convey satellite locations
  - Example: Broadcast GPS orbits use basic Keplerian elements plus some rate terms plus sine/cosine corrections to some elements

Satellite Navigation Positioning

- For most modern satellite navigation systems, positioning is based upon *one-way* ranging
- Ranges to each satellite are estimated using the speed of light multiplied by measured signal transit time (time of receipt minus time of transmission)
- Most GNSS users do NOT have a high precision clock
  - Thus, ALL range measurements are biased by user clock error multiplied by the speed of light
  - These biased range measurements are commonly referred to as *pseudoranges*
- GNSS pseudorange measurements also include many other errors, to be discussed in the next lecture

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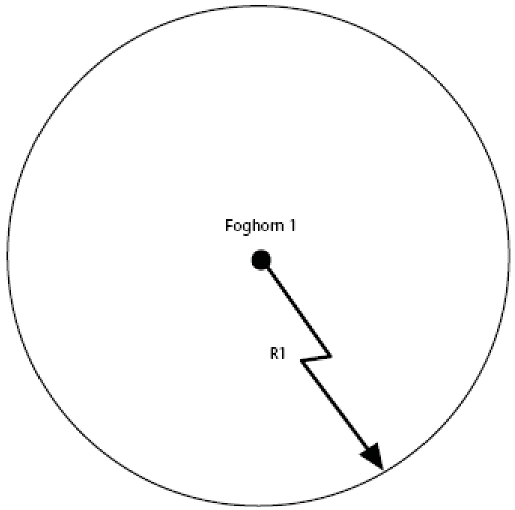
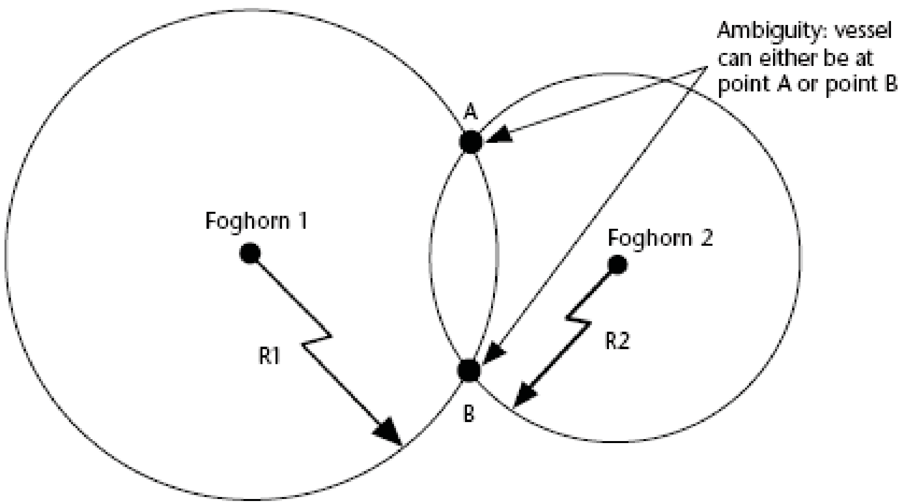
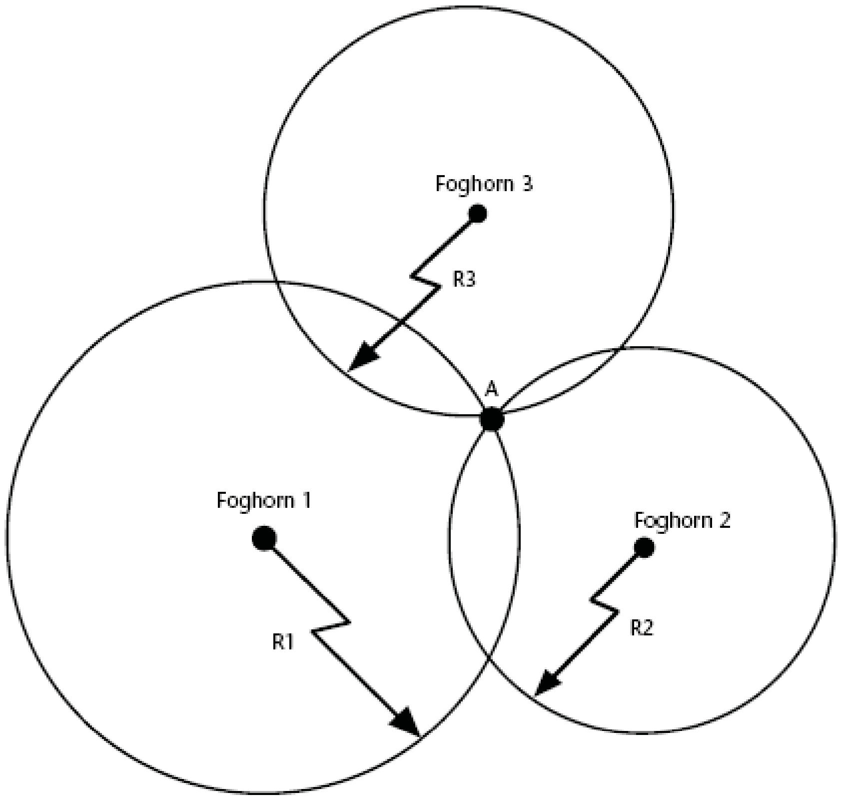
Positioning in Two-Dimensions



One measurement



Two measurements



Three measurements

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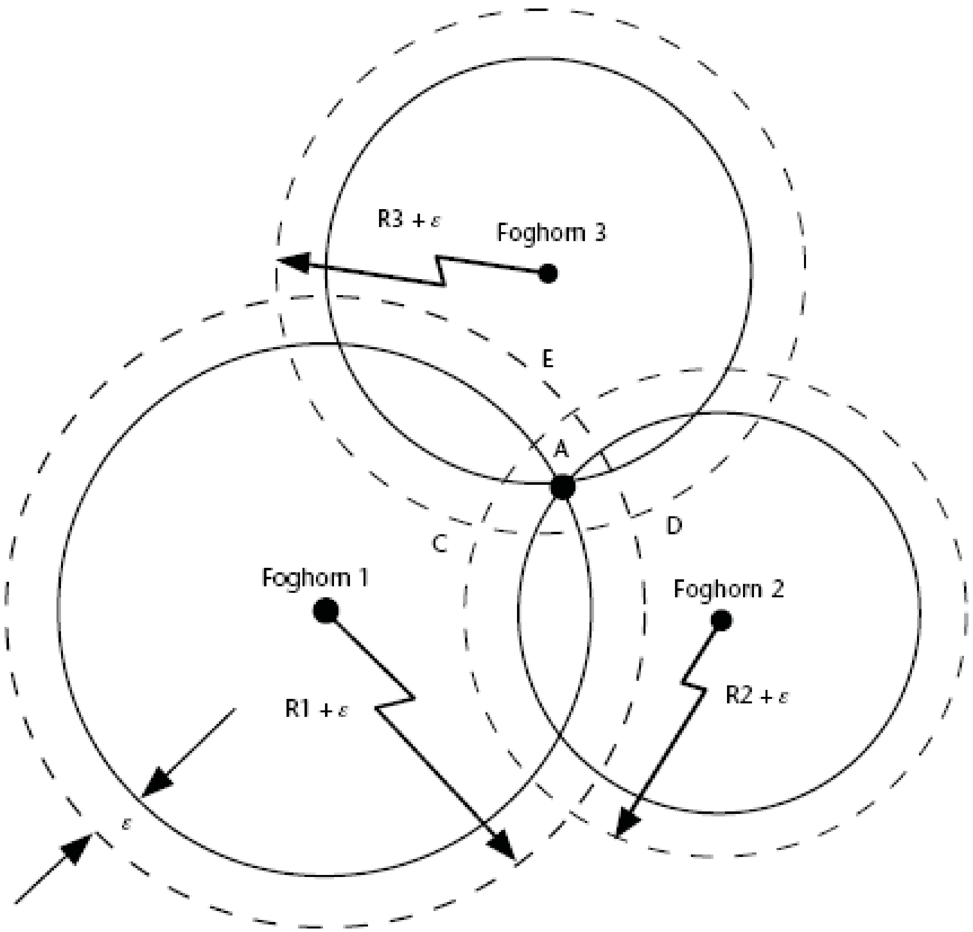
2D Example – Effect of Imperfect User Clock



Three measurements

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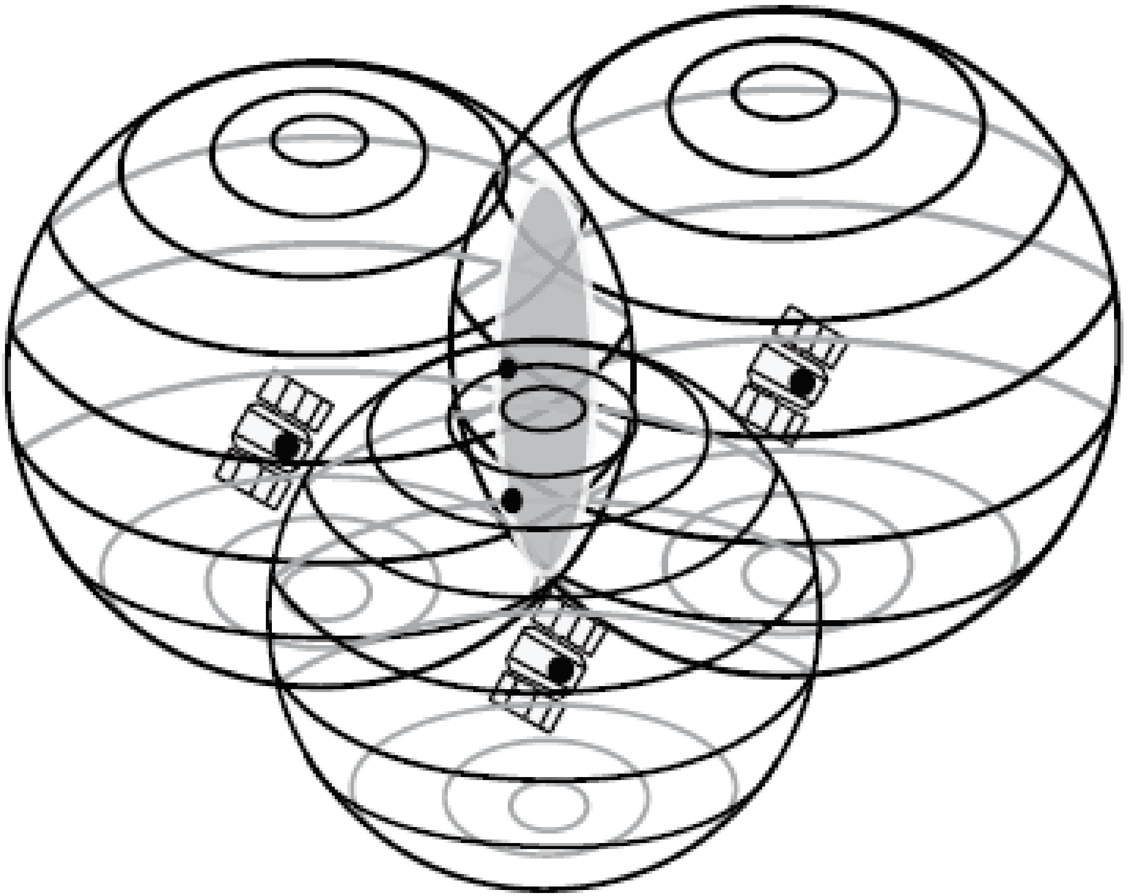
Positioning in Three Dimensions



With three satellites and perfect user clock, one can estimate user location; with imperfect user clock, four satellites are needed to determine user location and clock error (4 equations, 4 unknowns).

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Time and Distance Conversions

- Speed of light, c = 299792458 m/s
- Some conversions:
  - 1 ns ~ 0.3 m
  - 1 μs ~ 300 m
  - 1 ms ~ 300 km
  - 1 s ~ 300,000 km