Workshop on GNSS Data Application to Low Latitude Ionospheric Research

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GNSS Measurements and Error Sources

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GNSS Measurements and Error Sources

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May 2013
GNSS Measurements and Error Sources

■ Measurements
  – Pseudorange
  – Carrier phase

■ Error sources
  – Satellite ephemeris errors
  – Satellite clock errors
  – Selective availability (GPS only; discontinued in 2000)
  – Ionospheric and tropospheric delay
  – Multipath
  – Receiver noise, interference, and biases

■ Measurement combinations
  – Carrier smoothing
  – Single and double differences
  – Dual-frequency
GPS C/A-code and P(Y)-code Signal Components (not to scale)

\[ T_0 \times T_c \times T_d = \text{carrier spread spectrum waveform} \]

\[ T_0 = \text{carrier frequency (Hz)} \]
\[ f_0 = 1/T_0 = 1575.42 \text{ MHz (L1), 1227.6 MHz (L2)} \]

\[ T_c = \text{chipping rate (chips/s)} \]
\[ R_c = 1/T_c = 1.023 \text{ MHz (C/A), 10.23 MHz (P/Y)} \]

\[ T_d = \text{data rate (bits/s)} \]
\[ R_d = 1/T_d = 50 \text{ bps} \]
GNSS Pseudorange Measurement

• With a perfect user clock, signal transit time can be multiplied by the speed of light to yield a range measurement.
• With an imperfect user clock, measured transit time for each satellite’s signals are biased by a common user clock error – measurements are referred to as pseudorange.
Pseudorange Model

Satellite clock: $t^s(t) = t + \delta t^s(t)$

True range:

$$r^s_u(t) = \sqrt{(x_u - x^s)^2 + (y_u - y^s)^2 + (z_u - z^s)^2}$$

Measured pseudorange:

$$\rho^s_u(t) = c(t_R - t_T)$$

$$= r^s_u + c\left[\delta t_u - \delta t^s\right] + I + T + \varepsilon_\rho$$

User clock: $t_u(t) = t + \delta t_u(t)$

-iono
-tropo
-Other measurement errors
Carrier Phase Model

Phase observable (in meters):

\[ \phi_u^s(t) = r_u^s + N\lambda + c(\delta t_u - \delta t^s) - I + T + \varepsilon_\varphi \]

- Integer ambiguity
- Measurement errors
Understanding Carrier Phase Measurements

- Receiver tracks phase difference between recovered carrier component of received signal and an internally generated carrier at 1575.42 MHz
- Rope analogy: grab rope tied to satellite and count knots let in/out
- If you initially hold a knot, you know that distance to satellite is $19 \text{ cm} \times N$
- Counting knots passing through your hand provides precise measurement of change in distance to satellite with time (velocity)
GPS SV2 Observables - One Pass

Key features: Code pseudorange provides unambiguous absolute range. Carrier phase has ambiguity, suffers cycle slips, but is much less noisy.
Getting Familiar with GNSS Data

- Raw GNSS measurement data available for free on the Internet from a wide variety of sources
- One example: International GNSS Service (IGS)
  - Logs raw measurement data from a global network with over 350 active stations
  - Wide variety of other products (GNSS clock/orbit estimates, ionospheric/tropospheric delay data, etc)
- Common file format for data exchange is Receiver Independent Exchange Format (RINEX)
  - Simple ASCII file types
Source: http://www.igs.org
Raw GPS Data in RINEX Format from Ferrara, Italy (UNFE0010.09o)

RINEX header with:
- Station information
- Types of measurements
- Date/time of 1st measurement

First data record with:
- Date/time of measurements
- List of satellites
- Raw data (for this file, L1 phase, L2 phase, C/A pseudorange, L1 P pseudorange, L2 P pseudorange, L1 doppler, and L2 doppler)

<table>
<thead>
<tr>
<th>Time</th>
<th>L1 Ph.</th>
<th>L2 Ph.</th>
<th>C/A P</th>
<th>L1 P</th>
<th>L2 P</th>
<th>D1</th>
<th>D2</th>
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<td>-2145.233</td>
<td>-9328997.785</td>
<td>12783419.939</td>
<td>3841.262</td>
<td>-11113637.034</td>
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Satellite Ephemeris Errors

- Ephemeris errors are differences between true satellite position and position computed using GNSS navigation message

- Sources:
  - Selective Availability (SA) epsilon (GPS only; this error was never observed - SA now discontinued)
  - Control Segment estimation errors
  - Age of navigation message data
Typical GPS Ephemeris Errors

Variation of Ephemeris Errors with Location

Typical GPS ephemeris errors are ~3 m. Even with a 100 m ephemeris error, errors change less than:

\[(5 \text{ mm/km}) \times (\text{difference in user position in km})\]
Ionospheric Delay Errors

GNSS signals are delayed as they traverse through the ionized layer of the Earth’s atmosphere.

L1 vertical delays range from 3 - 30 m (depending on time of day, phase of 11-year solar cycle, and solar activity), delays for signals along oblique paths may be greater by a factor of 3 for low elevation angles (L2 delays are ~1.6 times larger than L1 delays).

\( Re = \text{radius of earth} = 6370 \text{ km} \)
\( Ri = \text{radius of maximum electron density} = Re + 350 \text{ km} \)
\( Rs = \text{radius of GPS SVs orbit} = 26000 \text{ km} \)
Variation of Ionosphere with Time of Day and Latitude

Latitude Profile

Longitude Profile
Next solar maximum expected ~Fall 2013. Ionospheric delays, on average, are greatest during solar maximum (max/min average delay ratio ~2-3).
Typical Ionospheric Delay Errors
Maximum monthly average L1 vertical Ionospheric delays in nanoseconds (10 ns ~ 3 meters) for March 1990 at 2000 UTC.

Correcting for Ionospheric Delay Errors

■ Dual-frequency user
  – Ionospheric delay effects are dispersive – delays are inversely proportional to carrier frequency squared
  – With dual-frequency equipment, user equipment can nearly perfectly remove ionospheric delay errors

■ Single-frequency user
  – Each GPS satellite broadcasts ionospheric delay model coefficients (Klobuchar model)
  – Model typically removes ~half of ionospheric delay range error
  – 90% of time, residual range error is < 10 m
  – Residual errors are highly correlated
    ■ Model tends to either overestimate or underestimate ionospheric delay on all satellites simultaneously

■ Differential GNSS
Example Residual Ionospheric Delay Errors

Ionospheric Delay Errors

Residual Errors after Klobuchar Model
Spatial Variability at Solar Maximum

Range Error at L1 (m)  SSN = 160  Day = 74  UT = 20.0

SSN = sunspot number
Spatial Variability of Ionospheric Delay

1992-1993 L1 data (daytime hours only), Solid lines = North-South station orientation, Asterisks = East-West station orientation, errors ~2× larger during solar maximum

Ref: Klobuchar, Doherty, El-Arini, “Potential Ionospheric Limitations to Wide-Area DGPS,” ION GPS-93
Ionospheric Divergence

As the distance between the user and a GPS satellite grows:
- the number of carrier wavelengths in between grows
- the number of code chips in between grows

When the number of free electrons in the ionosphere grows (with distance between the user and GPS satellite fixed):
- the number of carrier wavelengths between the user and the GPS satellite decreases (phase velocity increases)
- the number of code chips between the user and the GPS satellite increases (group velocity decreases)

Care must be taken when aiding code pseudorange measurements with carrier phase measurements
Ionospheric Divergence (cont’d)

VACUUM          IONIZED MEDIUM

PSEUDORANDOM CODE MODULATION

CARRIER

NOT TO SCALE
Tropospheric Delay

GNSS signals are delayed as they traverse through the dry gases and water vapor comprising the Earth’s lower atmosphere.

Vertical delays range from ~2 - 3 m (depending on altitude of user and local weather), delays for signals along oblique paths may be greater by a factor of 10 for low elevation angles.
Correcting for Tropospheric Errors

- Many correction models exist

- Simplest:
  - Use global average weather conditions to determine vertical tropospheric error
  - Apply altitude correction and elevation angle scale factor
  - Residual errors ~25 cm for overhead satellite

- More complicated:
  - Use average local weather (e.g., pressure, temperature, humidity) determined from table lookup
  - Apply altitude correction and elevation angle scale factor
  - Residual errors ~5 cm for overhead satellite

- Most complicated:
  - Use meteorological sensors
  - Residual errors ~3 cm for overhead satellite
Variation of Tropospheric Delay with Altitude

Tropospheric delay changes most rapidly with altitude:

\[ \Delta \tau \approx N_R \frac{\Delta h \times 10^{-6}}{\sin(el)} \]

- \( N_R \) = Total (wet plus dry) refractivity
- \( el \) = elevation angle

Wet delay component may vary rapidly with time and distance, but typically is only \( \sim 1/10 \) of total tropospheric effect.
Dry component varies with local temperature and pressure (correlated over 10’s of km)
Satellite Clock Errors

- Prior to discontinuance on May 1, 2000, Selective Availability (SA) was dominant error for GPS civil users
  - Understanding SA still important - influenced design of many operational differential systems
  - 20 - 30 m RMS ranging error, 2 - 5 min time constant
  - DGPS corrections effectively removed SA, small residual errors due to latency $\sim \frac{1}{2}at^2$ ($a \approx 2$-4 mm/s$^2$)

- Without SA, GPS satellite clock errors are typically $\sim 0.5 - 1$ m (one upload/day)
  - Satellite atomic clock stability $< 1$ part in $10^{13}$
  - Virtually eliminated with differential
Simulated SA Pseudorange Errors

![Graph showing simulated SA pseudorange errors over time. The x-axis represents time in seconds (0 to 4000), and the y-axis represents pseudorange error in meters (-60 to 40)].
With care, multipath errors can be kept below ~1 meter. Multipath seen by two receivers is NOT the same (need to root sum square errors for differential).
Typical C/A-code Multipath - One Satellite Pass

Point Loma, CA
January 1, 1999
Ashtech XII3
Receiver tracking errors due to thermal noise and interference varies greatly with implementation. Values from 0.1 - 0.7 meters RMS are typical.
Factors Influencing Noise and Interference Tracking Performance

Code tracking error in chips\(^2\) where 1 C/A-code chip ~ 300 m, 1 P(Y)-code chip ~30 m:

\[
\sigma_r^2 = \frac{B_L d}{2S/N_0} \left[ 1 + \frac{1}{S/N_0 T} \right]
\]

- \(B_L\) = loop bandwidth (Hz) (typically 1/400 - 1/20 Hz with carrier-aiding)
- \(d\) = early-late correlator spacing (chips)
- \(S\) = received signal power (W)
- \(N_0\) = thermal noise and interference level (W/Hz)
- \(T\) = predetection integration interval (s) (typically 20 ms)
Antenna Biases

- Antenna Reference Point (ARP) – physical point on antenna (usually center of bottom)
- Apparent antenna location from measurements will reside at different points
  - May be within or outside of physical antenna package
  - Varies with direction of signal arrival and between carrier/code
  - less so with well-designed antennas
  - Calibration employed for some applications
Receiver Biases

- All signals experience group and phase delay as they travel from antenna through digital tracking loops
  - Common delays drop out in estimate of user receiver clock error in navigation solution
  - Not consequential for most navigation users, but of great importance to timing users

- Errors affecting positioning can arise when non-common biases occur
  - Due to e.g., processing of different signal types or signals on different frequencies
  - Can be significant even if common front-end is utilized, because of variations in group/phase delay with frequency across the passband
Measurement Combinations

- Raw pseudorange and carrier phase measurements from one or more receivers are often combined
  - To reduce/eliminate errors, or
  - To observe errors
- Combinations include:
  - Code (pseudorange) minus carrier
  - Carrier-smoothed code
  - Single-, double-, and triple- differences
  - Ionospheric-free and geometry-free
Differencing code and carrier observables and removing the bias (for short data segments) is oft-used method for estimating code measurement noise.
Carrier Smoothed Code

- Code pseudorange measurements are unambiguous, but noisy
- Carrier phase measurements are very precise, but include integer cycle ambiguity
  - Differenced phase measurements provide very precise delta-range, with no ambiguity
- Popular algorithm to reduce pseudorange noise:

\[
\hat{\rho}_u^s(t_k) = (1 - \alpha)[\hat{\rho}_u^s(t_{k-1}) + \varphi(t_k) - \varphi(t_{k-1})] + \alpha \rho_u^s(t_k)
\]

Smoothing constant (when setting, be mindful of ionospheric divergence!)
Single-Difference

\[ \phi_{r,u}^s = \phi_r^s - \phi_u^s = r_r^s - r_u^s + (N_r^s - N_u^s)\lambda + c(\delta t_r - \delta t_u) + \varepsilon_r - \varepsilon_u^s \]

(eliminates satellite clock errors, tropospheric and ionospheric delays)
Double-Difference

\[ \phi_{r,u}^{1,2} = \phi_{r,u}^1 - \phi_{r,u}^2 \]
\[ = r_r^1 - r_u^1 - r_r^2 + r_u^2 + (N_r^1 - N_u^1 - N_r^2 + N_u^2) \lambda + \text{noise} \]

(eliminates satellite and user clock errors)

Note that a sum (or difference) of integers is an integer.
Ionospheric-free and Ionospheric Combinations

- Ionospheric delays are frequency dependent

\[ I = \frac{40.3 \cdot TEC}{f^2} \]  
Total electron count (electrons/m²) along line-of-sight from the user to the satellite

- L1/L2 measurements can be combined to remove ionospheric delay or estimate it

\[
\rho_{ionofree} = \frac{\rho_{L2} - \gamma \rho_{L1}}{1 - \gamma} 
\]

\[ = r + c (\delta t^s - \delta t_u) + T \frac{\varepsilon_{L2} - \gamma \varepsilon_{L1}}{1 - \gamma} \]

\[ \gamma = \frac{f_{L1}^2}{f_{L2}^2} \approx 1.65 \]

Note: noise is enhanced
Ionospheric delay estimates are sometimes referred to as "geometry-free" since they are linear combinations of pseudoranges that no longer depend on the distance between the user and the satellite.
**Widelane Observable**

\[
\phi_w \equiv \lambda_w \left( \frac{\phi_{L1}}{\lambda_1} - \frac{\phi_{L2}}{\lambda_2} \right) \quad \text{(in meters)}
\]

\[
= r_u^s + N_w \lambda_w + c \left[ \delta t_u - \delta t^s \right] - I_w + T + \varepsilon_w
\]

Where

\[
\lambda_w = \left( \frac{1}{\lambda_1} - \frac{1}{\lambda_2} \right)^{-1} \approx 86 \text{ cm}
\]

is the wavelength of the beat frequency between L1 and L2.

This combination of L1 and L2 phase measurements appears as a carrier phase measurement made at the beat frequency of \( f_{L1} - f_{L2} \). The much larger wavelength facilitates ambiguity resolution.
Relationship Between Range and Position Errors

Basic Idea: Position Error = “Dilution of Precision” × “Range Error”

Derivation:

\[
\rho_u^1 = \sqrt{(x_u - x^1)^2 + (y_u - y^1)^2 + (z_u - z^1)^2} + c\delta t_u + \varepsilon_u^1
\]

\[
\vdots
\]

\[
\rho_u^N = \sqrt{(x_u - x^N)^2 + (y_u - y^N)^2 + (z_u - z^N)^2} + c\delta t_u + \varepsilon_u^N
\]

\[
\Delta\rho = G\Delta x + \Delta \varepsilon
\]

\[
\Delta x = (G^T G)^{-1} G^T \Delta \rho
\]
Relationship Between Range and Position Errors (cont’d)

\[ \text{cov}(\Delta x) = (G^T G)^{-1} \sigma_r^2 \]

Covariance of “state” (position plus user clock offset), assuming pseudoranges to each satellite are independent with standard deviation \( \sigma_r \)

The “DOPs”:

- Vertical (VDOP) - relates 1-\( \sigma \) vertical position error to 1-\( \sigma \) pseudorange error
- Horizontal (HDOP) - relates 1-\( \sigma \) horizontal position error to 1-\( \sigma \) pseudorange error
- Position (PDOP) - relates 1-\( \sigma \) position error (3D) to 1-\( \sigma \) pseudorange error
- Time (TDOP) - relates 1-\( \sigma \) user clock error to 1-\( \sigma \) pseudorange error
- Geometric (GDOP) - relates RSS of East, North, Up, and Time errors to 1-\( \sigma \) pseudorange error

The same mathematics applies for code-based differential, except that the solution provides the difference between the reference station and user clock error.
# Typical Pre-2000 SPS Error Budget (with Selective Availability)

<table>
<thead>
<tr>
<th>Error Source</th>
<th>Bias</th>
<th>Random</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ephemeris</td>
<td>1.0</td>
<td>0.0</td>
<td>1.0</td>
</tr>
<tr>
<td>Satellite clock</td>
<td>20.0</td>
<td>0.7</td>
<td>20.0</td>
</tr>
<tr>
<td>Ionosphere</td>
<td>4.0</td>
<td>0.5</td>
<td>4.0</td>
</tr>
<tr>
<td>Troposphere</td>
<td>0.5</td>
<td>0.0</td>
<td>0.5</td>
</tr>
<tr>
<td>Multipath</td>
<td>0.2</td>
<td>0.2</td>
<td>0.3</td>
</tr>
<tr>
<td>Receiver noise</td>
<td>0.0</td>
<td>0.1</td>
<td>0.1</td>
</tr>
</tbody>
</table>

User equivalent range error, rms 20.5 0.9 20.5
Filtered UERE, rms 20.5 0.4 20.5

**Vertical one-sigma errors - VDOP = 1.7** 34.8
**Horizontal one-sigma errors - HDOP = 1.0** 20.5

Note: Chart format from Parkinson and Enge, 1996. Values reflect typical 2013 performance (except SA).
# Typical Single-Frequency Error Budget (no SA)

<table>
<thead>
<tr>
<th>Error Source</th>
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<th>Random</th>
<th>Total</th>
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</thead>
<tbody>
<tr>
<td>Ephemeris</td>
<td>0.8</td>
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<td>0.8</td>
</tr>
<tr>
<td>Satellite clock</td>
<td>1.0</td>
<td>0.0</td>
<td>1.0</td>
</tr>
<tr>
<td>Ionosphere</td>
<td>7.0*</td>
<td>0.0</td>
<td>7.0</td>
</tr>
<tr>
<td>Troposphere</td>
<td>0.2</td>
<td>0.0</td>
<td>0.2</td>
</tr>
<tr>
<td>Multipath</td>
<td>0.2</td>
<td>0.2</td>
<td>0.3</td>
</tr>
<tr>
<td>Receiver noise</td>
<td>0.0</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>User equivalent range error, rms</td>
<td>7.1</td>
<td>0.2</td>
<td>7.1</td>
</tr>
<tr>
<td>Filtered UERE, rms</td>
<td>7.1</td>
<td>0.1</td>
<td>7.1</td>
</tr>
</tbody>
</table>

**Vertical one-sigma errors - VDOP = 1.7**  
12.1*

**Horizontal one-sigma errors - HDOP = 1.0**  
7.1*

Note: Chart format from Parkinson and Enge, 1996. Values reflect typical 2013 performance.  
*Note that residual ionospheric delay errors tend to be highly correlated among satellites, and thus observed position-domain errors tend to be less than predicted by DOP·UERE.
Typical Mid-Latitude SPS Positioning Accuracy

95% horizontal positioning error (HPE) ~ 2.7 m

95% vertical positioning error (VPE) ~ 6.9 m

GPS position accuracy data from 28 sites distributed throughout North America for 3 month period (October – December 2012)
Typical Dual-Frequency Error Budget (no SA)

One-sigma Error (meters)

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<td>Troposphere</td>
<td>0.2</td>
<td>0.0</td>
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<tr>
<td>Multipath</td>
<td>0.2</td>
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<td>Receiver noise</td>
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<td>0.1</td>
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User equivalent range error, rms 1.3 0.2 1.3
Filtered UERE, rms 1.3 0.1 1.3

**Vertical one-sigma errors - VDOP = 1.7** 2.3
**Horizontal one-sigma errors - HDOP = 1.0** 1.3

Note: Chart format from Parkinson and Enge, 1996. Values reflect typical 2013 performance.