Quantum fluids of light
under synthetic gauge fields

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Why not hydrodynamics of light?

Light field/beam formed by huge number of individual photons.

**In vacuo:**
- photons travel along straight line at $c$
- (practically) do not interact with each other
- in cavity, collisional thermalization slower than with walls and losses
  $\Rightarrow$ optics typically reduces to single-particle physics

**In suitable dielectric system:**
- $\chi^{(3)}$ nonlinearity introduces effective photon-photon interactions
- Spatial confinement introduces effective photon mass
  $\Rightarrow$ collective quantum fluid behaviours

Experimentally observed as photon BEC, superfluid light, quantum hydrodynamic nucleation of soliton/vortices, etc.

Exciton-polaritons (i.e. a kind of dressed photons) in planar semiconductor microcavities $\rightarrow$ useful workhorse to investigate this physics.
2006 - Photon/polariton Bose-Einstein condensation

Momentum distribution

Interference
Richard et al., PRL 94, 187401 (2005)

Suppressed fluctuations
A. Baas et al., PRL 96, 176401 (2006)

Quantized vortices
Lagoudakis, Wouters, Richard, Baas, IC, André, Le Si Dang, Deveaud-Pledran,
Quantised Vortices in an Exciton Polariton Fluid,

Photon/polariton BEC closely related to laser operation in VCSELs.
Figure from LKB-P6 group:

Mach-Cerenkov wake in supersonic flow

Super-sonic flow hitting a defect:

- **Cerenkov conical wave**, aperture \( \cos(\varphi) = c_s / v \)
- **single-particle-like parabolic** precursors

Strong defect (I): oblique solitons

Under cw coherent pump at finite k:

- stable oblique soliton appears in the wake of defect for $v \geq c_s$
- finite phase jump across soliton related to depth of density dip
- soliton aperture $\alpha$ depends on defect shape, smaller than Mach cone

Strong defect (II): “turbulent” behavior

Increasing flow speed

- **Oblique soliton** for $c_s < v$
- **Theory**: vortex pairs emitted from defect for $v_c < v < c_s$
  (i.e. snake instability of soliton)
- **Vortex nucleation mechanism** based on quantum hydrodynamics
- **Expt**: “turbulent” behavior signaled by reduced coherence of the emission


**Strong defect (III): quantized vortices**

Stationary system under cw pump:
- vortices nucleated at random times
- hard to take experimental image

*Here:*
- Pulsed excitation: pins nucleation times
- Time-resolved images with ps resolution on streak-camera
- Optically generated defect
- Vortices appear as dislocations in interferograms
- Vortex motion followed in real-time

Similar experiments with atoms: Neely et al., PRL 104, 160401 (2010)
2009 →

Synthetic gauge fields
for photons
How to make photons feel a Lorentz force?

a) **2D lattice of coupled cavities with tunneling phase**
   - deformed and/or helical waveguide lattices (Segev-Szameit)
   - silicon ring cavities  (Hafezi-Taylor)
   - on-chip circulators in circuit-QED  (Koch-Girvin-Le Hur; Delsing)

Resulting Bose-Hubbard Hamiltonian:

\[
H = \sum_i \hbar \omega_0 \hat{a}_i^\dagger \hat{a}_i - \hbar J \sum_{\langle i,j \rangle} \hat{a}_i^\dagger \hat{a}_j e^{i\Phi_{ij}} + \sum_i \left[ \hbar F_i(t) \hat{a}_i^\dagger + \text{h.c.} \right]
\]

b) **Rotating photon fluid at speed \( \Omega \):**

\[
\text{Coriolis } F_c = -2m\Omega \times v
\]

\[
\text{Lorentz } F_L = e \, v \times B
\]

Rechtsman et al., Nature 496, 196 (2013)

Hafezi et al., arXiv:1302.2153
Hofstadter butterfly and chiral edge states

Lattice of coupled cavities at large magnetic flux

Eigenstates organize as:
- bulk Hofstadter states
- chiral edge states within gaps; unidirectional propagation.

First observed → gyro-magnetic photonic crystals (Haldane-Soljacic)
  - Floquet helical waveguide lattices (Segev-Szameit)
  - Si ring cavities (see J. Taylor's talk)

Wang et al., Nature 461, 772 (2009)
Rechtsman et al., Nature 496, 196 (2013)
Magnetic Bloch oscillations

Lattice at strong magnetic flux, e.g. $\alpha = 1/3$

Semiclassical eqs. of motion:

\[
\begin{align*}
\hat{h}\kappa_c(t) &= eE, \\
\hat{h}\vec{r}_c(t) &= \nabla_k \mathcal{E}_{n,k} - eE \times \Omega_n(k)
\end{align*}
\]

Bloch oscillations display a net lateral drift
- Initial photon wavepacket injected with laser pulse
- Spatial gradient of cavity frequency $\rightarrow$ uniform force

Related work in Price-Cooper, PRA 83, 033620 (2012); Dudarev et al. PRL 92, 153005 (2004).
The new frontier: Strongly interacting photons
**Photon blockade**

Simplest signature of strong photon-photon interactions at single photon level

- entrance of first photon into cavity blocks entrance of a second
- after one photon has exited, system has to reload; dead time between emitted photons
- transmitted beam: anti-bunched stream with sub-Poissonian statistics

- requires huge $\chi^{(3)}$ optical nonlinearity.
  So far, observed in single mode cavities and in atomic gases in Rydberg-EIT regime.

- analog of Coulomb blockade of mesoscopic conductors

Coulomb blockade

figure D. J. Paul, Cambridge, 2006

Transmission spectrum as a function pump frequency for fixed pump intensity:

- each peak corresponds to a Tonks-Girardeau many-body state $|q_1, q_2, q_3, ...>$
- $q_i$ quantized according to PBC/anti-PBC depending on $N=$odd/even
- $U/J >> 1$: efficient photon blockade, impenetrable photons.

N-particle state excited by N photon transition:

- Plane wave pump with $k_p = 0$: selects states of total momentum $P=0$
- Monochromatic pump at $\omega_p$: resonantly excites states of many-body energy $E$ such that $\omega_p = E / N$
Finite $U/J$, pump laser tuned on two-photon resonance

- intensity correlation between the emission from cavities $i_1$, $i_2$
- at large $U/\gamma$, larger probability of having $N=0$ or 2 photons than $N=1$
  - low $U\ll J$: bunched emission for all pairs of $i_1$, $i_2$
  - large $U\gg J$: antibunched emission from a single site positive correlations between different sites
- Idea straightforwardly extends to more complex many-body states.
Photon blockade + synthetic gauge field = QHE for light

Bose-Hubbard model:
\[ H_0 = \sum_i \hbar \omega_i \hat{b}_i^\dagger \hat{b}_i - \hbar J \sum_{\langle i,j \rangle} \hat{b}_i^\dagger \hat{b}_j e^{i \Phi_{ij}} + \hbar \frac{U}{2} \sum_i \hat{n}_i (\hat{n}_i - 1) \]

Gauge field gives phase in hopping terms

with usual **coherent drive and dissipation** → look for **non-equil. steady state**

Transmission spectra:
- peaks correspond to many-body states
- comparison with eigenstates of \( H_0 \)
- good overlap with Laughlin wf (with PBC)
  \[ \psi_l(z_1, ..., z_N) = \mathcal{N}_L F_{CM}^{(l)}(Z) e^{-\pi \alpha \sum \mu_i^2} \]
  \[ \times \prod_{i<j} \left\{ \delta \left[ \frac{1}{2} \right] \left( \frac{z_i - z_j}{L} \right) \right\}^2 \]
- no need for adiabatic following, etc....

How to directly characterize FQH states?

Homodyne detection of secondary emission → info on many-body wavefunction

\[
\langle \hat{b}_i \hat{b}_j \rangle = \langle X_0^{(i)} X_0^{(j)} \rangle - \langle X_{\pi/2}^{(i)} X_{\pi/2}^{(j)} \rangle \\
+ i \langle X_0^{(i)} X_{\pi/2}^{(j)} \rangle + i \langle X_{\pi/2}^{(i)} X_0^{(j)} \rangle
\]

Note: optical signal gauge dependent, optical phase matters!

Non-trivial structure of Laughlin state compared to non-interacting photons

Rotating photon fluids

Rotating system at angular speed $\Omega$

$\text{Coriolis } F_c = -2m\Omega \times v$

$\text{Lorentz } F_L = e v \times B$

Rotating photon gas injected by LG pump

with finite orbital angular momentum

Resonant peak in transmission due to Laughlin state:

$$\psi(z_1, \ldots, z_N) = e^{-\sum_i |z_i|^2/2} \prod_{i < j} (z_i - z_j)^2$$

Overlap measured from quadrature noise of transmitted light

$$\langle \hat{b}_i \hat{b}_j \rangle = \langle X^{(i)}_0 X^{(j)}_0 \rangle - \langle X^{(i)}_{\pi/2} X^{(j)}_{\pi/2} \rangle + i\langle X^{(i)}_0 X^{(j)}_{\pi/2} \rangle + i\langle X^{(i)}_{\pi/2} X^{(j)}_0 \rangle$$

Anyonic braiding phase

- LG pump to create and maintain quantum Hall liquid
- Repulsive potential
  → quasi-hole excitation in quantum Hall liquid
  → position of holes adiabatically braided in space
- Anyonic statistics of quasi-hole: many-body Berry phase $\phi_{Br}$ when positions swapped during braiding
- Berry phase extracted from shift of transmission resonance while repulsive potential moved with period $T_{rot}$ along circle
  \[
  \phi_{Br} \equiv (\Delta \omega_{oo} - \Delta \omega_o) \cdot T_{rot} \cdot [2 \pi]
  \]

Conclusions

Recent developments in quantum many-body physics with light

Dilute photon gas 2006 → BEC in exciton-polaritons gas in semiconductor microcav.
GP-like equation 2008-10 → superfluid hydrodynamics effects
2009-13 → synthetic gauge field for photons

Many questions still open:

● (exp) role of non-equilibrium and diffusive Goldstone mode in superfluidity effects
● (exp) quantum hydrodynamics, e.g. analog Hawking radiation in acoustic black holes
● (th + exp) critical properties of BKT transition in 2D; disorder effects, polariton “random laser”
● (th + exp) new devices: strained-Si fibers (Trento) and bulk nonlinear crystals

Challenging perspectives on a longer run:

● strongly correlated photon gases → Tonks-Girardeau gas in 1D necklace of cavities
● with synthetic gauge field → Laughlin states, quantum Hall physics of light
● applications to novel functionalities in photonic devices
Quantum fluids of light

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This article reviews recent theoretical and experimental advances in the fundamental understanding and active control of quantum fluids of light in nonlinear optical systems. In the presence of effective photon-photon interactions induced by the optical nonlinearity of the medium, a many-photon system can behave collectively as a quantum fluid with a number of novel features stemming from its intrinsically nonequilibrium nature. A rich variety of recently observed photon hydrodynamical effects is presented, from the superfluid flow around a defect at low speeds, to the appearance of a Mach-Cherenkov cone in a supersonic flow, to the hydrodynamic formation of topological excitations such as quantized vortices and dark solitons at the surface of large impenetrable obstacles. While the review is mostly focused on a specific class of semiconductor systems that have been extensively studied in recent years (planar semiconductor microcavities in the strong light-matter coupling regime having cavity polaritons as elementary excitations), the very concept of quantum fluids of light applies to a broad spectrum of systems, ranging from bulk nonlinear crystals, to atomic clouds embedded in optical fibers and cavities, to photonic crystal cavities, to superconducting quantum circuits based on Josephson junctions. The conclusive part of the article is devoted to a review of the future perspectives in the direction of strongly correlated photon gases and of artificial gauge fields for photons. In particular, several mechanisms to obtain efficient photon blockade are presented, together with their application to the generation of novel quantum phases.

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Photon/polaritons with full 3D confinement

Add in plane confinement to microcavity

**Bose-Hubbard model:**

- Single-mode cavities of frequency $\omega_o$
- Polariton interactions: strong on-site repulsion $U$
- Tunneling between neighboring cavities: Josephson coupling $J$

**Driving and dissipation:**

- Incident laser: coherent external driving
  \[
  H_d = \sum_i F_i(t) \hat{b}_i + h.c. 
  \]
- Weak losses $\gamma \ll J, U \rightarrow$ Lindblad terms in master eq. determine non-equilibrium steady-state
- Secondary light emission $\rightarrow$ field correlation functions

Coupled micropillars
de Vasconcellos et al., APL 2011

Photonic crystal cavities
Majumdar et al., arXiv:1201.6244

Overgrown planar cavities
**Mechanisms for photon blockade**

Strong interaction regime requires effective photon blockade $\gamma$, $J \ll U$

Two-level emitter strongly coupled to cavity:
- atom, quantum dot $\rightarrow$ IR/visible (Rempe/Vuckovic/Imamoglu/Senellart/…)
- Josephson qubit $\rightarrow$ microwaves of circuit QED (Devoret/Walraff/Houck/…)
- requires low inhomogeneous broadening, hard with self-assembled quantum dots

Repulsive polariton interaction from quantum well exciton (Verger, IC, Ciuti, PRB 2006):
  - less sensitive to disorder but requires very tight spatial confinement

Biexciton Feshbach resonance (Savasta/Wouters/IC-Volz-Imamoglu):
  - colliding polaritons resonantly form intermediate biexciton state
  - interactions in opposite-spin channel: enhanced and/or change sign (some exp evidence)
  - biexciton mass large $\rightarrow$ sensitive to disorder

Recent observation of strongly correlated photon stream in atomic gas in Rydberg-EIT (Peyronel et al., Nature 2012)