The Holographic Master Field

Mukund Rangamani

Durham University

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The hologram for strong coupling dynamics

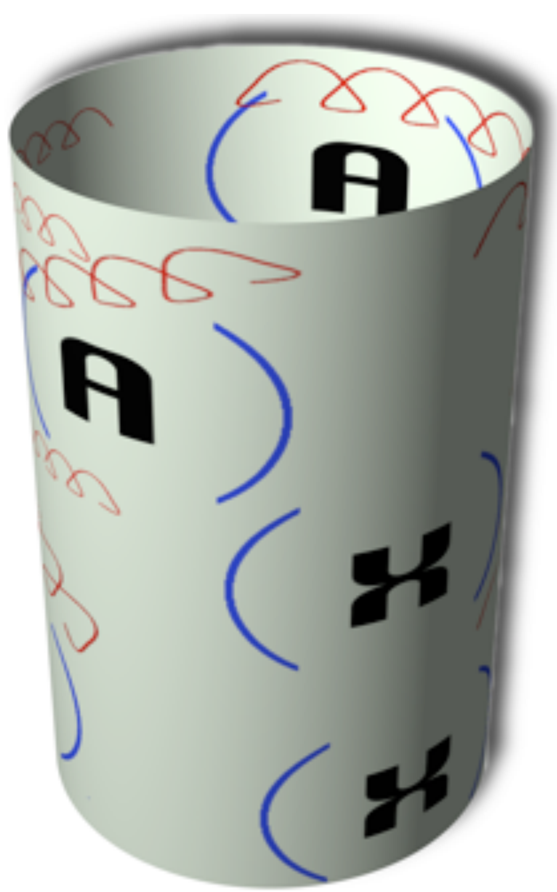
- ◉ Introduction to the AdS/CFT correspondence
- ◉ Equilibrium physics & phase structure
- ◉ Correlation functions
- ◉ Transport phenomena
- ◉ Holographic matter
- ◉ Summary

The gravitational Master field

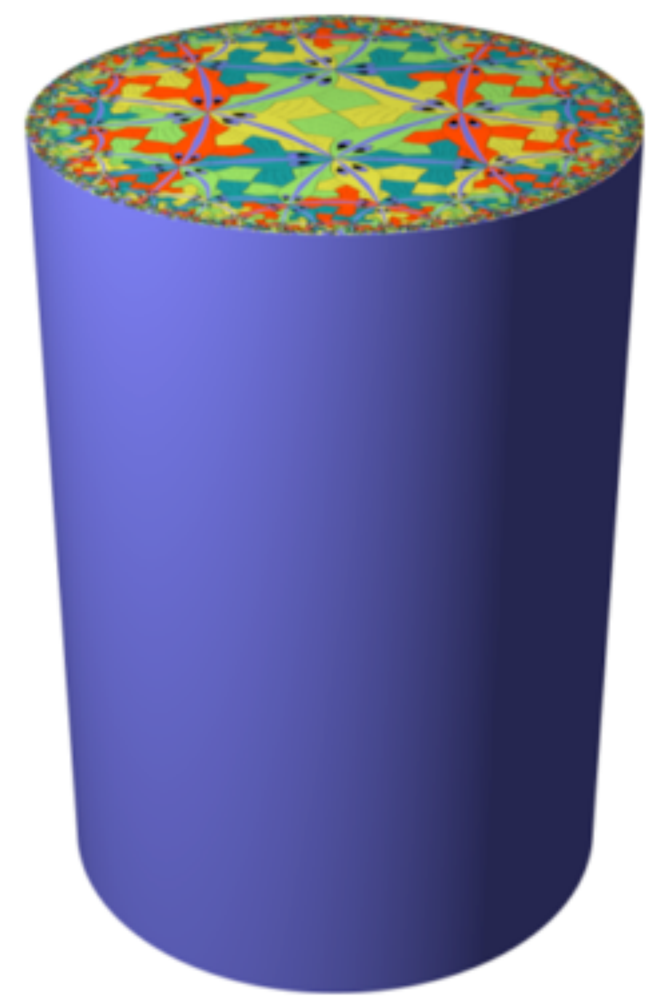
- Dynamical gauge theories in the planar limit (large *N*) are essentially classical.
- This classical dynamics is encoded by a Master Field, in terms of which various observables can be efficiently computed.
- For a wide class of quantum gauge theories the gauge/gravity correspondence identifies the Master Field not with a gauge field configuration but rather a classical string theory in higher dimensions.
- For relativistic scale invariant quantum gauge theories, the master field is a string in a negatively curved spacetime (AdS).
- This is the AdS/CFT correspondence which has engendered a rich dialogue between high energy and condensed matter theories in the past few years.

The gauge/gravity correspondence

String theory which includes quantum gravity is exactly equivalent (or dual) to a non-gravitational quantum theory (gauge theory).



- The quantum theory lives on the boundary of the spacetime where gravity reigns.
- All the gravitational action is captured completely on the boundary.
- Boundary dynamics holographically captures gravitational physics.



The gauge/gravity correspondence

Gauge Theory $\longleftrightarrow$ Quantum String Theory

$$N, \lambda = g_{YM}^2 N \qquad \ell_s, \;\; g_s$$

Planar limit $\longleftrightarrow$ Classical String theory

$$N \to \infty, \;\; \lambda \text{ fixed} \qquad \ell_s, \;\; g_s \to 0$$

Strong coupling planar limit $\longleftrightarrow$ Classical gravity theory

$$N \to \infty, \;\; \lambda \gg 1 \text{ fixed} \qquad \ell_s \ll L, \;\; g_s \to 0$$

AdS/CFT: prototypical example

- A specific example:

$$SU(N)\,\mathcal{N}=4\,\text{Super Yang-Mills} \quad \longleftrightarrow \quad \text{String theory on AdS}_5\times\mathbf{S}^5$$

$$\frac{R_{\text{AdS}}}{\ell_s}=\lambda^{\frac{1}{4}}\ , \qquad \lambda=g_{YM}^2\,N=4\pi\,g_s\,N\,, \qquad g_s=\frac{1}{N}$$

- The AdS spacetime naturally incorporates scale invariance; in fact the isometry group of AdS in *(d+1)*-dimensions is isomorphic to the conformal group in *d*-dimensions.

- Part of general picture: global symmetries of field theory are gauge symmetries of gravity/string theory.

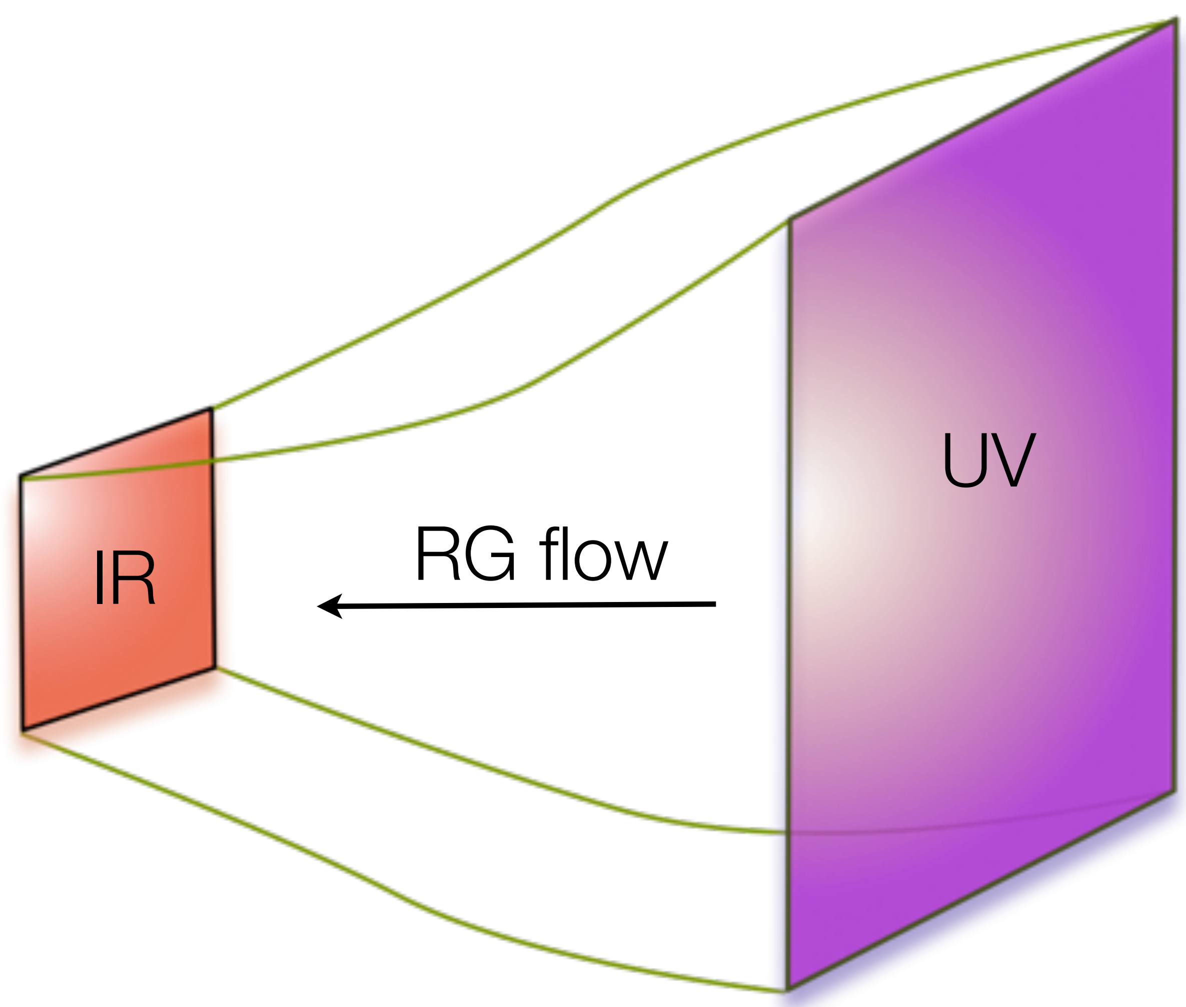
- Gravity only cognizant of gauge invariant field theory states.

The geometry of AdS

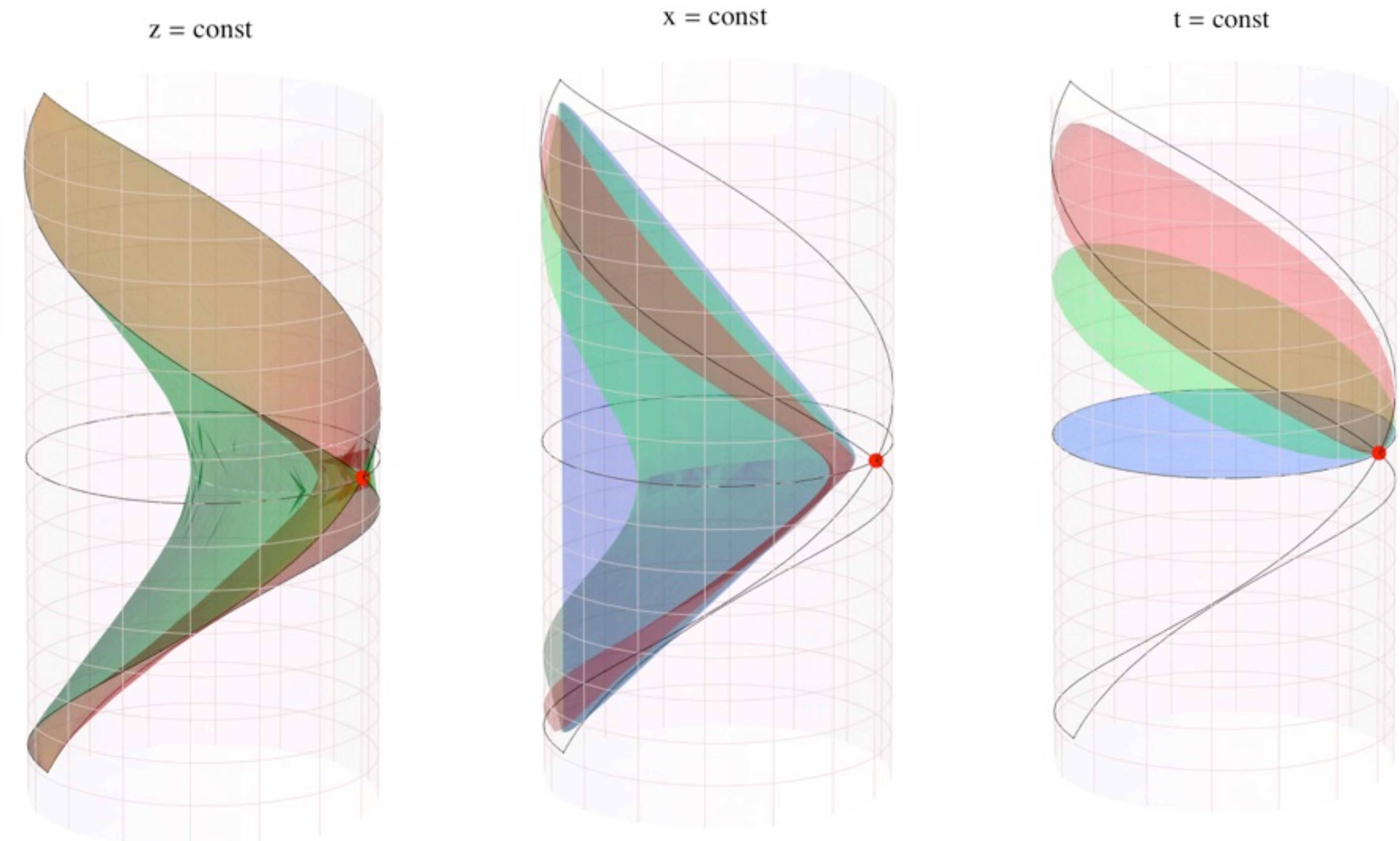
- Anti de Sitter spacetime (AdS) is a spacetime of constant negative curvature

Poincare coordinates: $$ds^2 = \frac{-dt^2 + d\mathbf{x}^2 + dz^2}{z^2}$$

Global coordinates: $$ds^2 = -(1+r^2)\,dt^2 + \frac{dr^2}{1+r^2} + r^2\,d\Omega_{d-1}^2$$



AdS geometry: global vs Poincare



The dictionary

- Classical fields in AdS are related to gauge invariant local operators of the field theory

$$\Phi(r,x) \to r^{d-\Delta}\, J_{\mathcal{O}} + \frac{1}{(2\,\Delta - d)\, r^{\Delta}}\, \langle\, \mathcal{O}(x)\,\rangle$$

bulk field

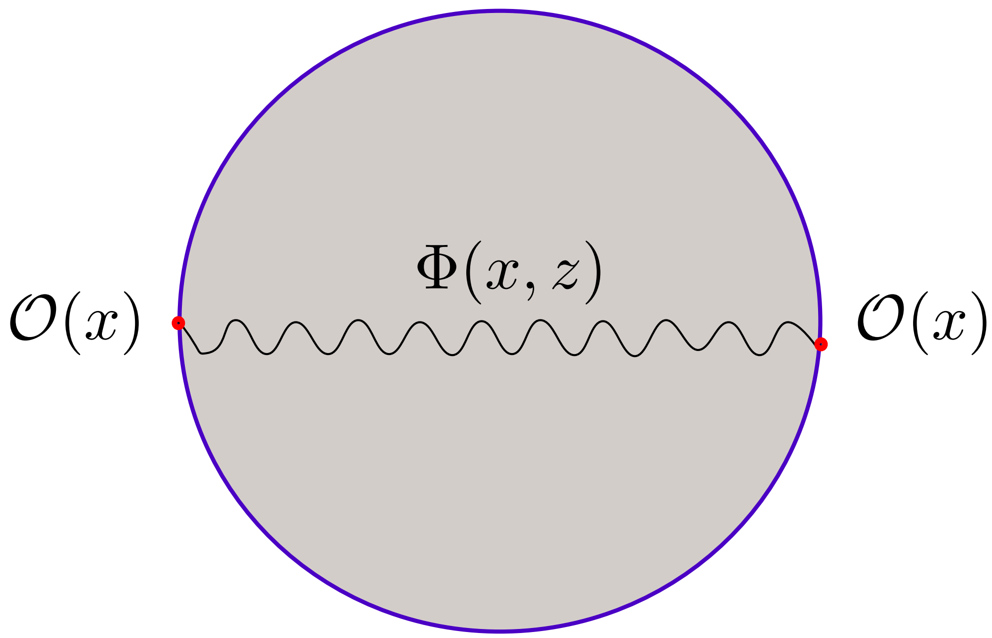
source

expectation value

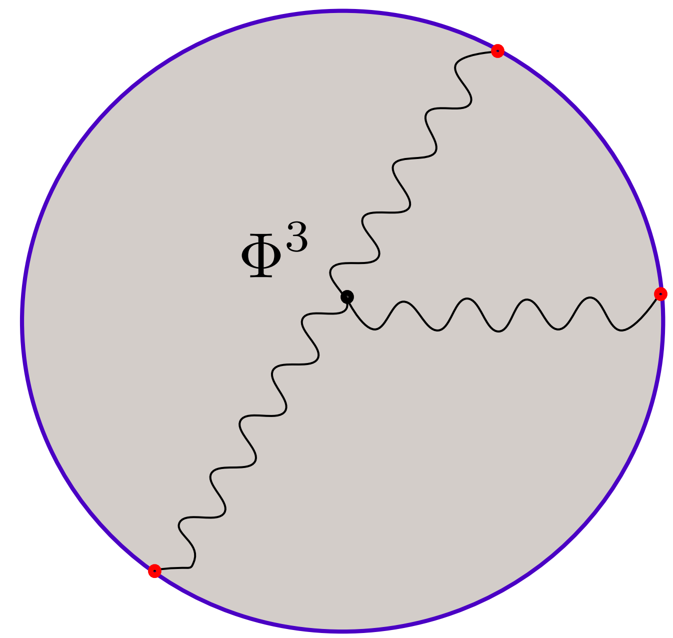
- Dynamics of conserved currents in field theory maps to dynamics of gauge fields in an asymptotically AdS spacetime.

$$\begin{aligned} G_{AB} \quad &\longleftrightarrow \quad T_{\mu\nu}\ , \qquad\qquad & \nabla_\mu T^{\mu\nu} = 0 \\ A_C \quad &\longleftrightarrow \quad J_\mu\ , \qquad\qquad & \nabla_\mu J^{\mu} = 0 \end{aligned}$$

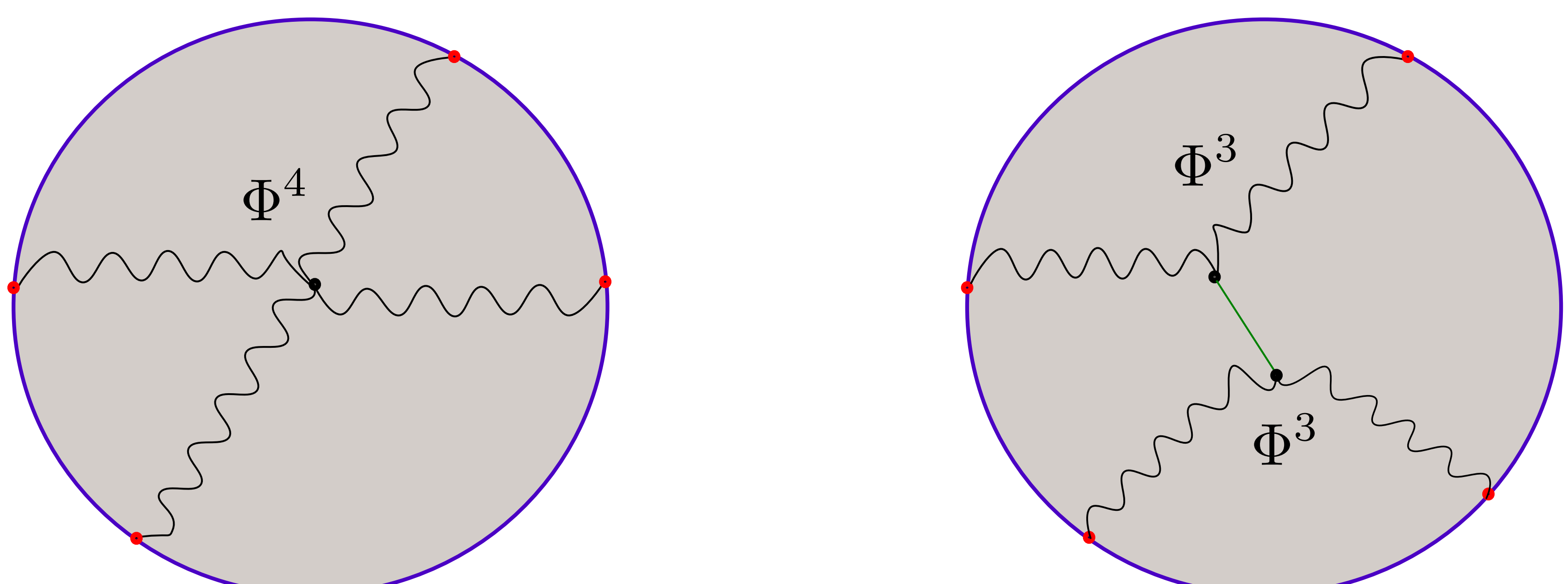
Witten diagrams: computing correlations



2-pt functions



3-pt functions



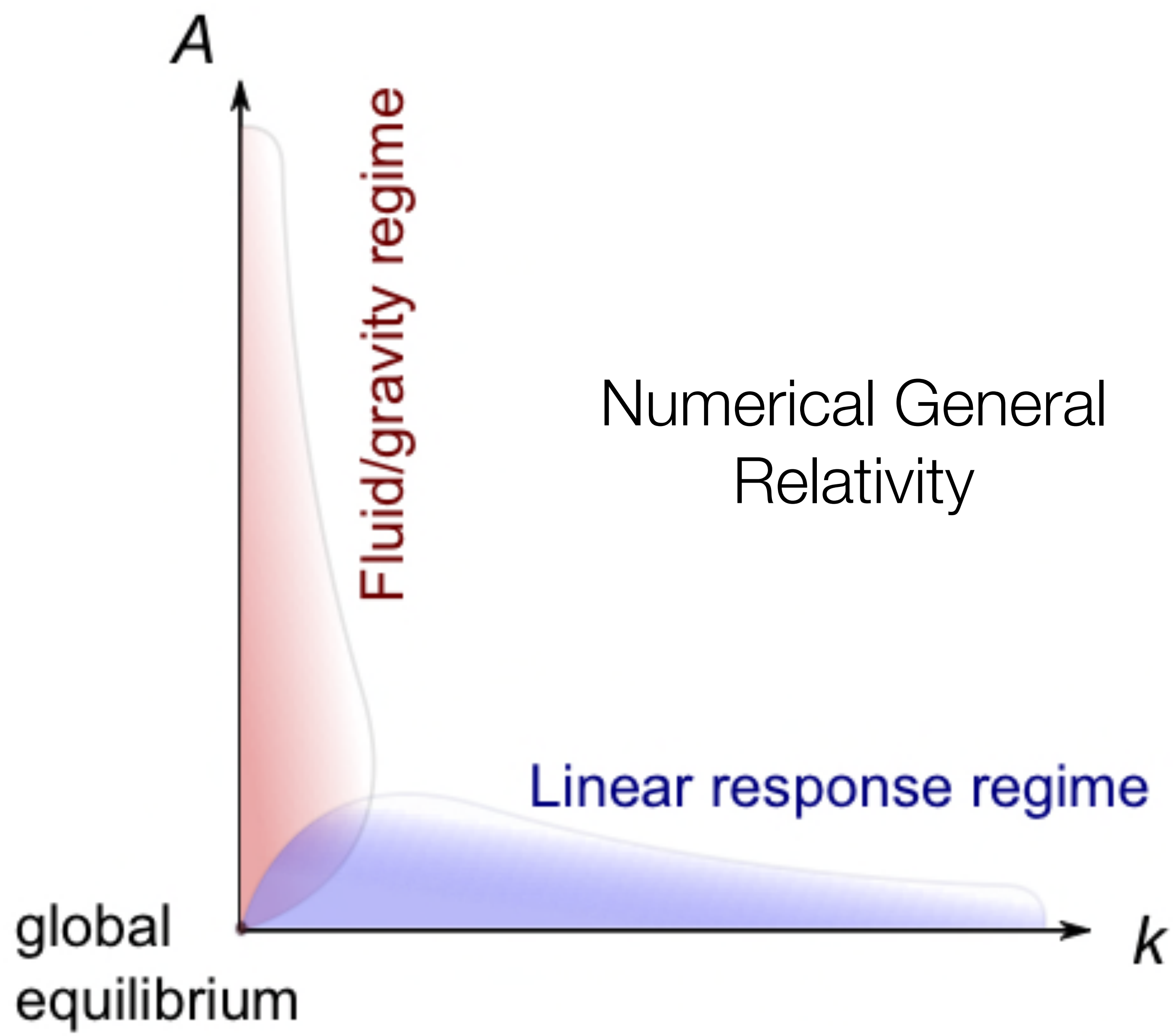
4-pt functions

Equilibrium dynamics

QFT vacuum $\lvert 0 \rangle$	$\longleftrightarrow$	AdS spacetime
Thermal density matrix $\rho \propto e^{-\beta\, H}$	$\longleftrightarrow$	Schwarzschild-AdS black hole $M;\;\; r_+ \text{ (horizon)}$
Grand canonical density matrix $\rho \propto e^{-\beta\,(H - \mu\, J)}$	$\longleftrightarrow$	Black holes with charges $M, Q;\;\; r_+ \text{ (horizon)}, \mu$
General state of field theory	$\longleftrightarrow$	Saddle point of string action

Transport phenomena

- AdS/CFT is well suited to computing correlation functions, which can be used to extract transport data.



Transport Exhibit: shear viscosity

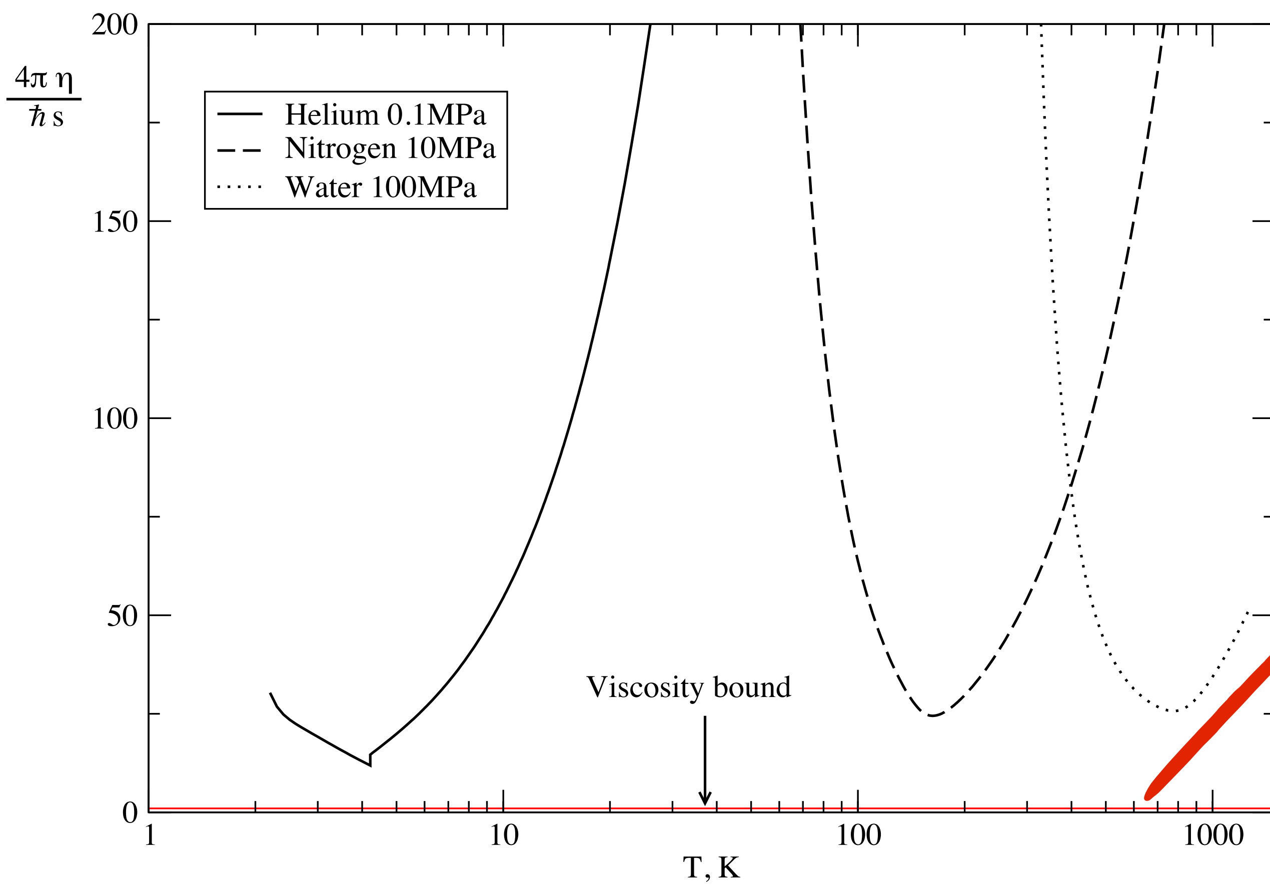
- Shear viscosity of strongly coupled plasmas:

Policastro, Son, Starinets (2001)

$$\eta/s = \frac{\hbar}{4\pi\, k_B}$$

- Lower bound on shear viscosity?

Kovtun, Son, Starinets (2004)



$$\eta/s \geq \alpha \frac{\hbar}{4\pi\, k_B}$$

quark-gluon plasma

Fermi gases at unitarity

status in string theory: $\alpha \sim \mathcal{O}(1)$

Sinha, Myers (2009)

The fluid/gravity correspondence

- The fluid/gravity correspondence establishes a correspondence between Einstein's equations with a negative cc and those of relativistic conformal fluids.

Einstein's eqn with negative cc

$$E_{MN} = R_{MN} - \frac{1}{2} G_{MN} R - \frac{d(d-1)}{2} G_{MN} = 0$$

$$\updownarrow$$

Relativistic ideal fluid equations

$$(\rho + P)\, \nabla_\mu u^\mu + u^\mu \nabla_\mu \rho = 0$$
$$P_\alpha{}^\mu \nabla_\mu P + (\rho + P)\, P_{\nu\alpha}\, u^\mu\, \nabla_\mu u^\nu = 0$$

Bhattacharyya, Hubeny, Minwalla, MR (2007)

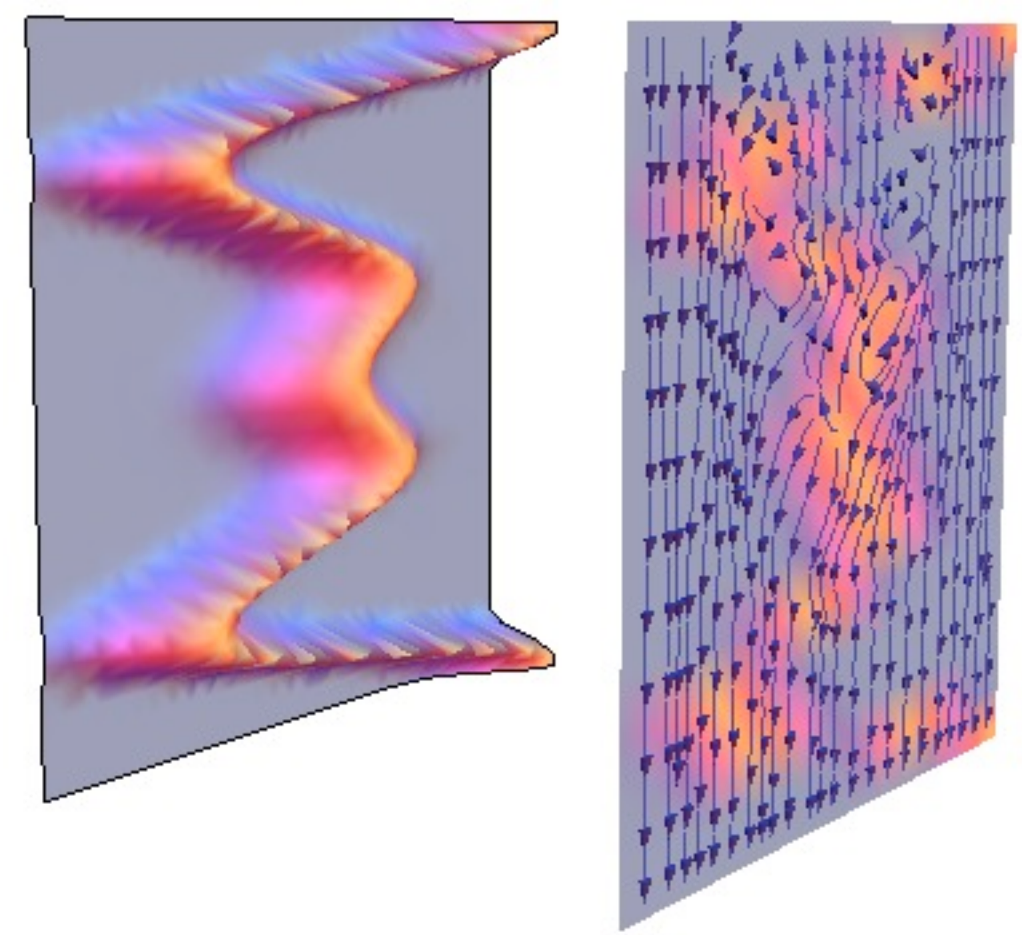
The fluid/gravity correspondence

- The fluid/gravity correspondence establishes a correspondence between Einstein's equations with a negative cc and those of relativistic conformal fluids.
- Given any solution to the hydrodynamic equations, one can construct, in a gradient expansion, an approximate *inhomogeneous, dynamical black hole* solution in an asymptotically AdS spacetime.
- The construction heuristically can be viewed as patching together planar AdS black holes of different temperatures with slow variation between patches.
- The fluid in question lives on the timelike boundary of AdS spacetime, and as is familiar, holographically encodes the entire dynamics of the bulk spacetime geometry.

Hubeny, Minwalla, MR (2012)

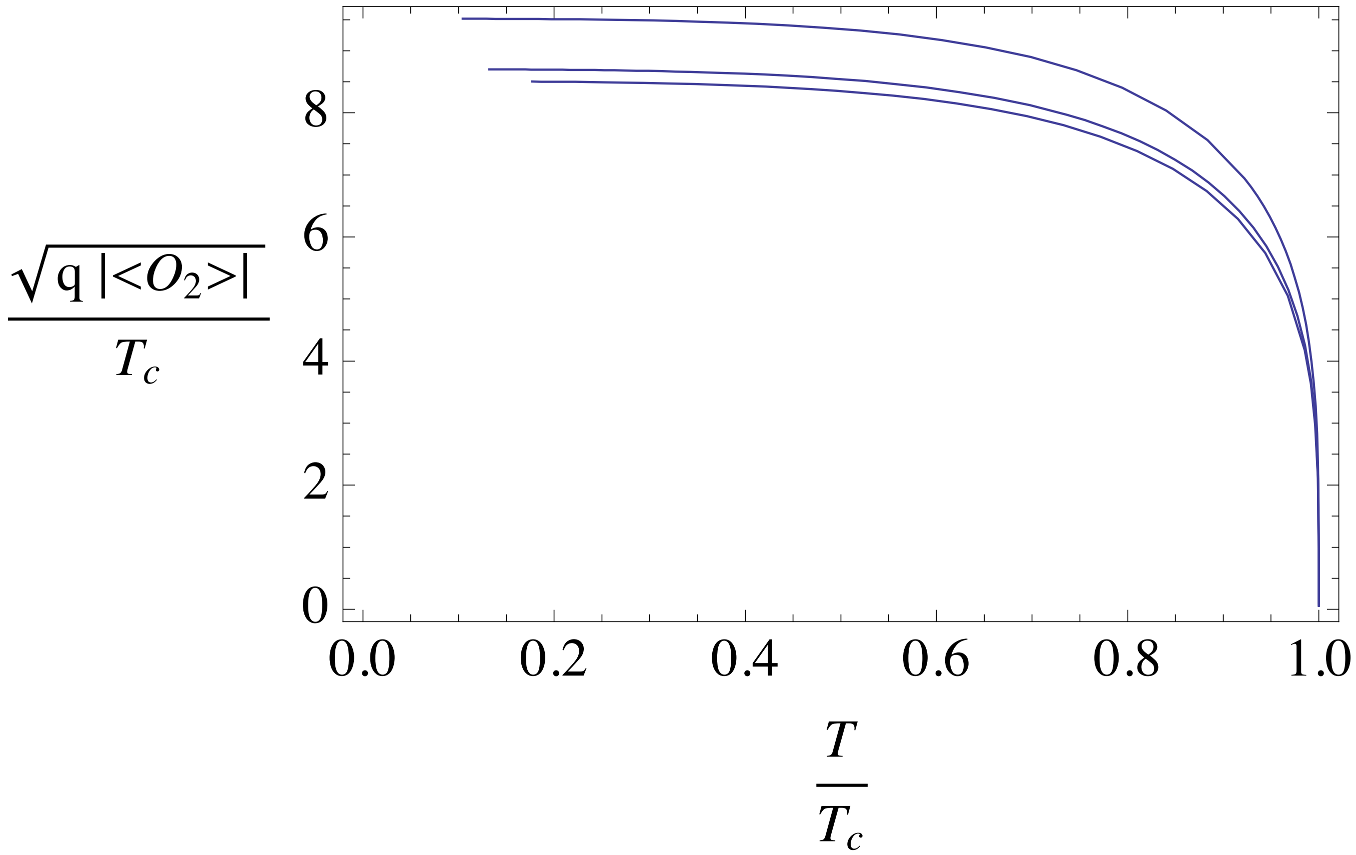
Black branes as lumps of fluid

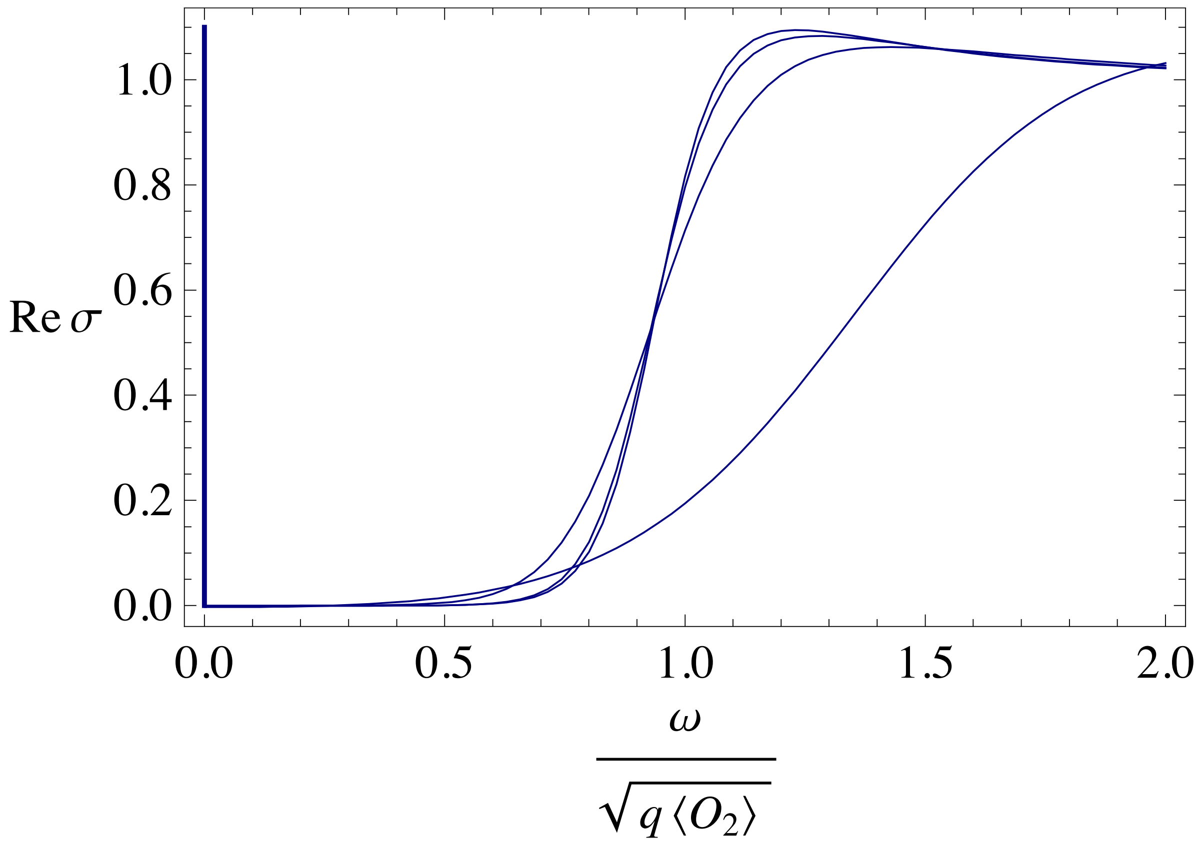
- Black branes really behave as lumps of fluid in the low energy limit.
- In the fluid/gravity correspondence, the fluid lives at the end of the universe, on the asymptotic boundary of the spacetime where the black hole resides.
- Here the fluid is a hologram, honestly capturing all the low energy physics of the entire geometry.



Holographic superconductors

- Superconducting phase transitions are easy to engineer in gravitational systems: condensation of a charged mode in gravity!
- examples of s-wave, p-wave and d-waves exist in literature

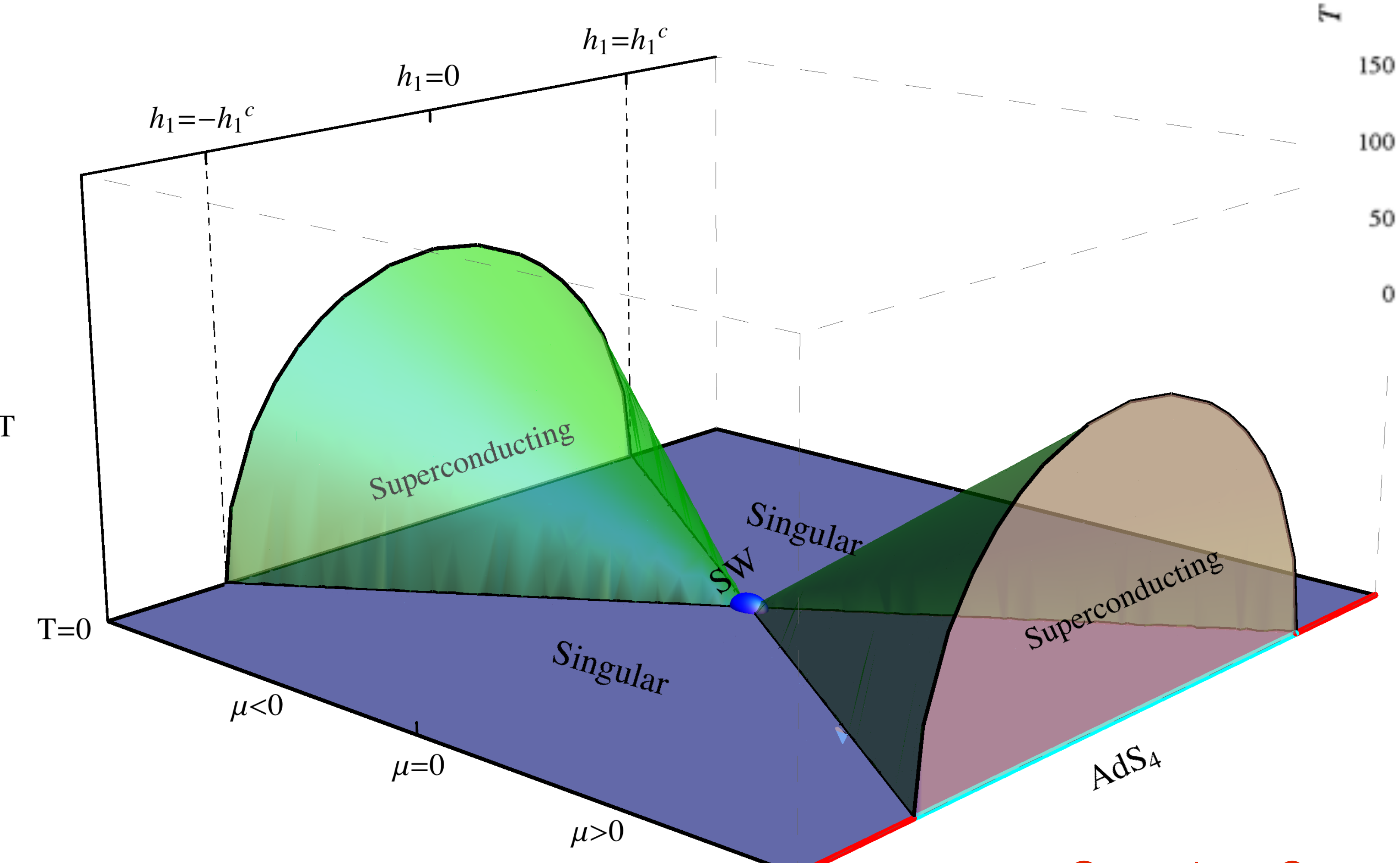


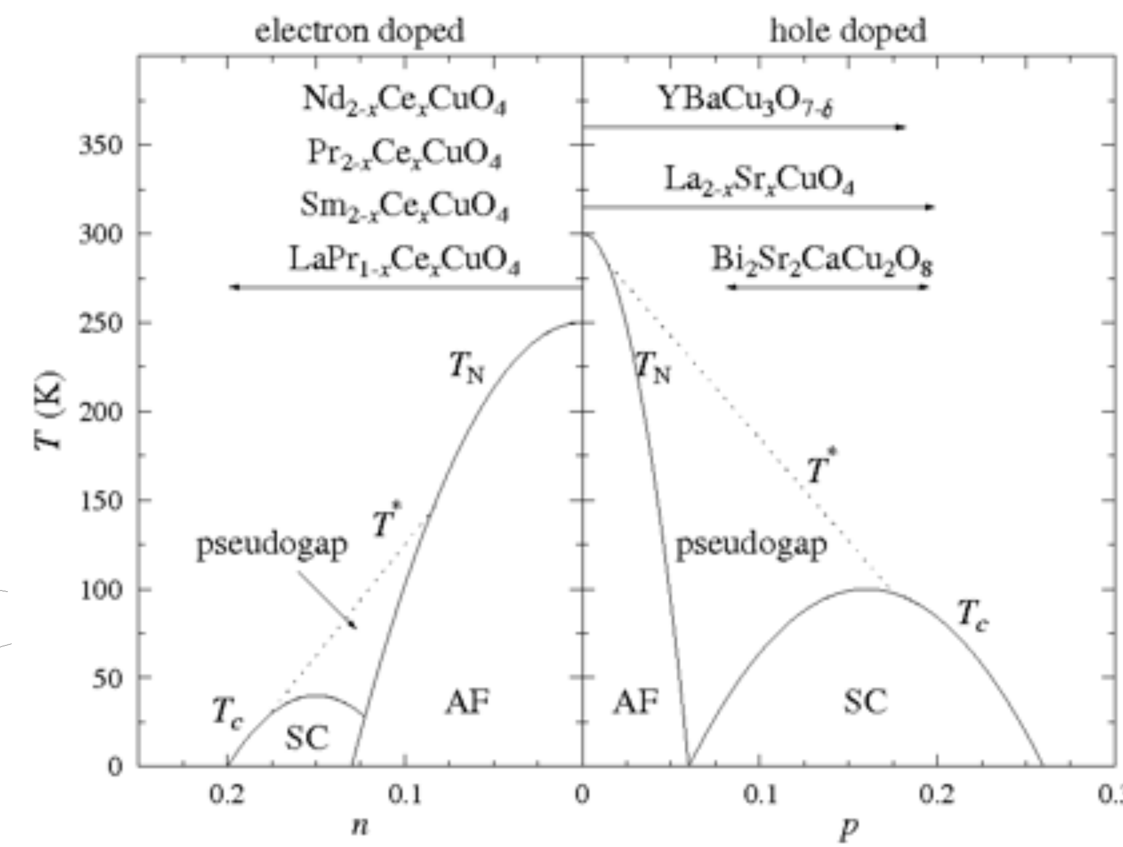


Hartnoll, Herzog, Horowitz (2008)

Gravity as a high Tc superconductor

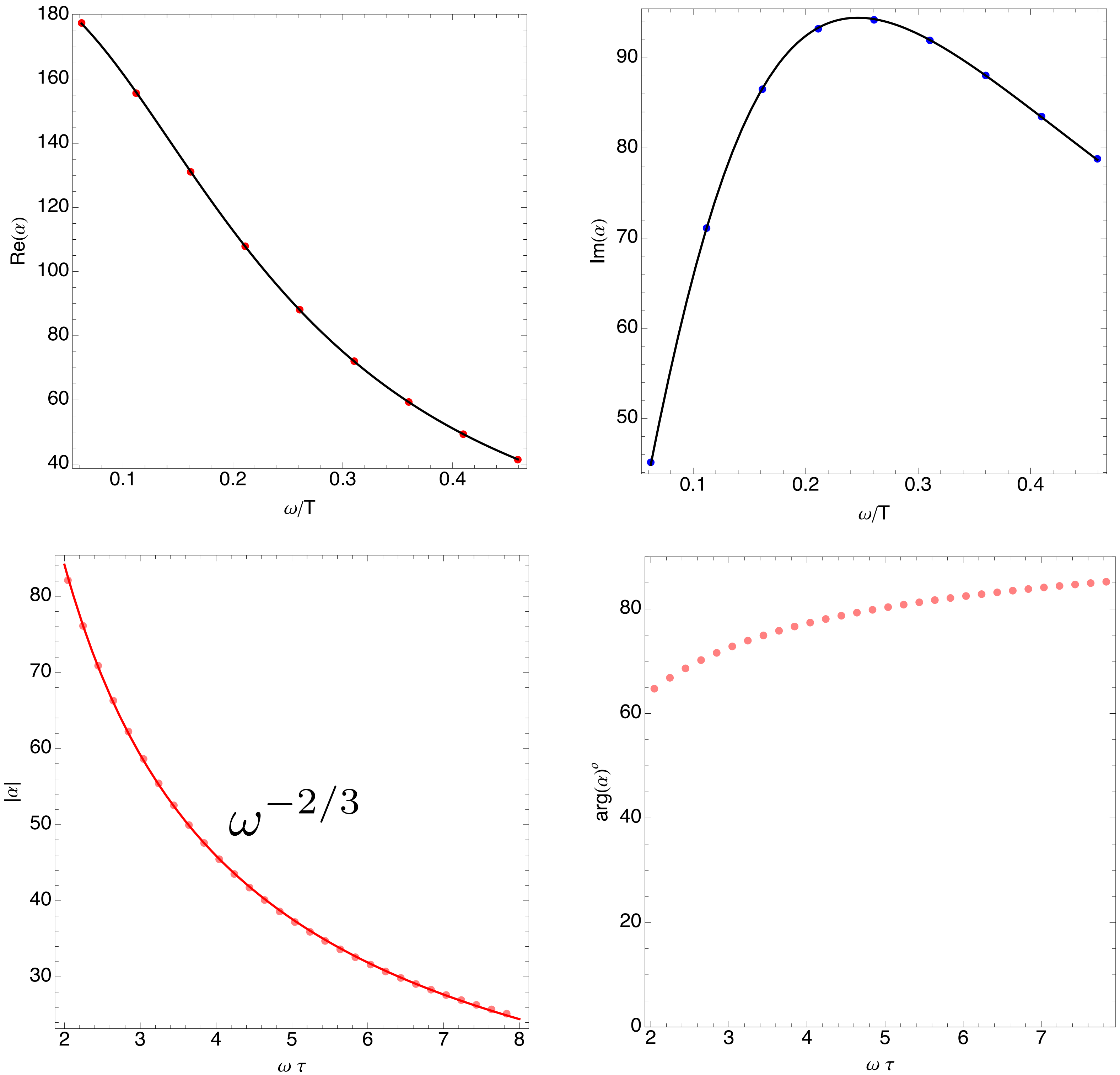
- Quantum critical behaviour in classical gravitational systems!



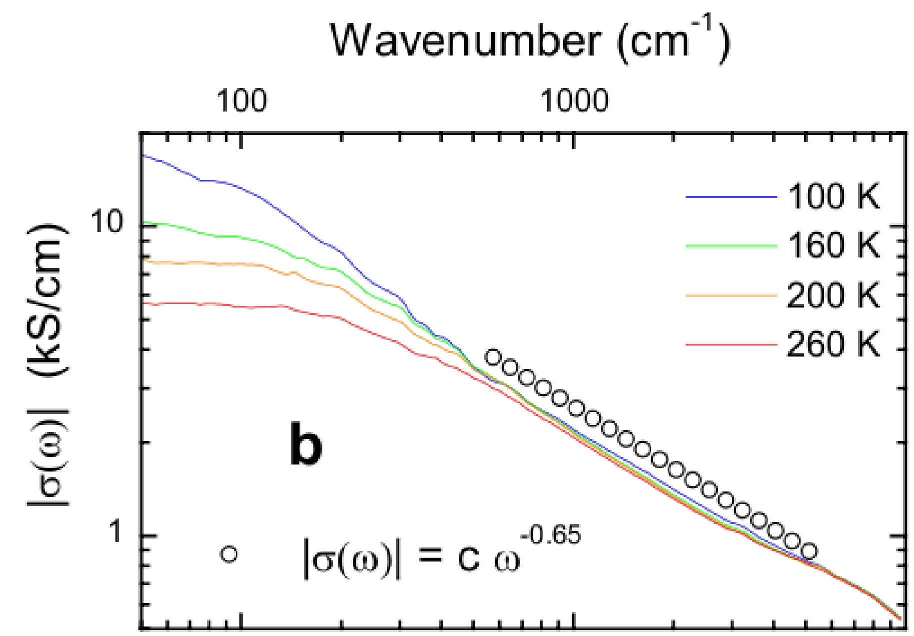


Gauntlett, Sonner, Wiseman (2009)

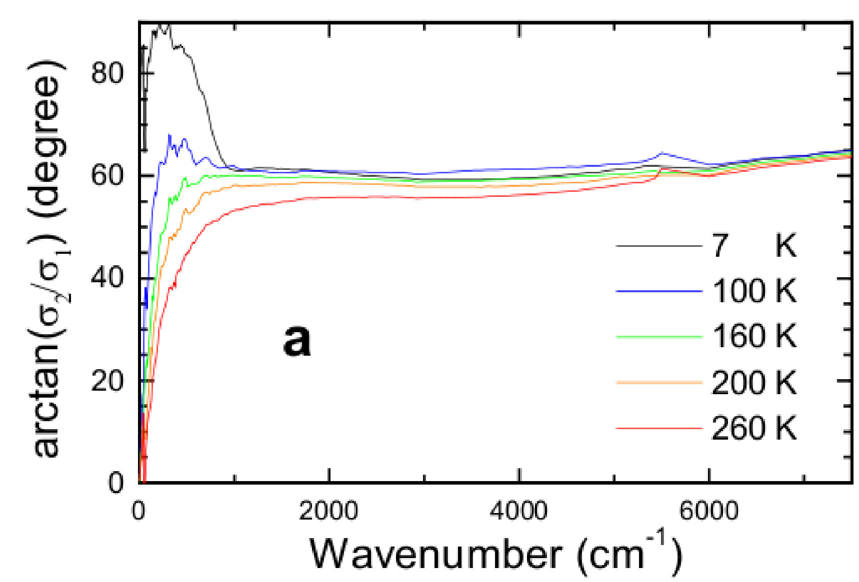
Gravity as a high Tc superconductor



Horowitz, Santos, Tong (2012)



universal power law scaling $|\sigma| - \sigma_0 \sim \omega^{-2/3}$



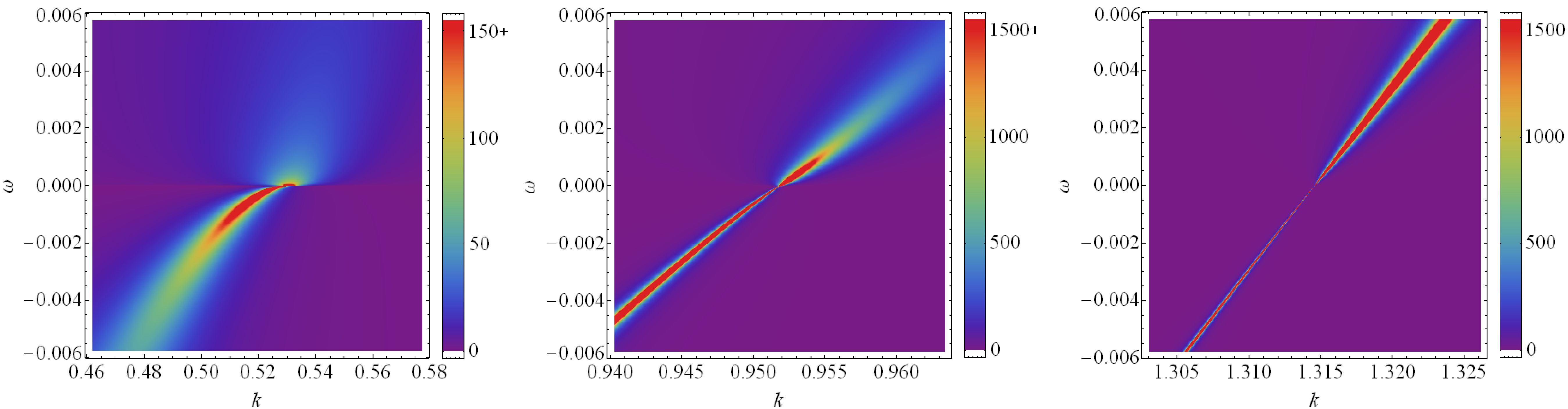
van der Marel et.al. (2003)

(non) Fermi liquids from holography

- Understanding Quantum Criticality: dynamics of fermions
- Non-Fermi liquids: departures from conventional Landau-Fermi liquids obtained from dynamics of fermions interacting gravitationally.

Liu, McGreevy, Vegh (2009)

Cubrovic, Zaanen, Schalm (2009)



Faulkner, Iqbal, Liu, McGreevy, Vegh (2010)

Non-relativistic field theories

- Holographic duals for field theories with non-relativistic scaling symmetries
- Schrodinger invariant, non-relativistic conformal theories

Son; Balasubramanian, McGreevy (2008)

- Explicit examples and dual large N field theories realized in string theory:

Adams, Balasubramanian, McGreevy; Herzog, MR, Ross; Maldacena, Martelli, Tachikawa (2008)

- Typically these are deformations of known relativistic field theories.
  - Large N gauge theories with bosonic and fermionic degrees of freedom
  - Not quite fermionic many body systems.
- Transport phenomena can be studied in these examples $\eta/s = \dfrac{\hbar}{4\pi\, k_B}$

Non-relativistic field theories

- Holographic duals for field theories with non-relativistic scaling symmetries
- Theories with scale but not conformal invariance: Lifshitz models

Kachru, Liu, Mulligan (2008)

$$ds^2 = \frac{-z^{2-2z}\,dt^2 + d\mathbf{x}^2 + dz^2}{z^2}$$

- Relevant for the low energy physics of quantum critical systems

Hartnoll, Polchinski, Silverstein, Tong (2011)

- These are also deformations of known relativistic field theories, but have be argued to be relevant for the many body physics of fermions in a gravitational field (electron stars)

Hartnoll, Tavanfar (2010)

Summary

- A rich interplay between gravitational dynamics and quantum many body physics.
- Gravity and black holes provide a great set of models for realization of complex physics which can be understood using classical dynamics!

- Introduction to AdS/CMT
  - Hartnoll: 0903.3246, 1106.4324
  - Sachdev: 1108.1197, 1203.4565
- Fluid/gravity correspondence
  - Rangamani: 0905.4352
  - Hubeny, Minwalla, Rangamani: 1107.5780
- Non-equilibrium physics
  - Son, Starinets: 0704.0240
  - Hubeny, Rangamani: 1006.3675
- Holographic superconductors
  - Herzog: 0904.1975
  - Horowitz: 1002.1722
- Non-Fermi liquids; strange metals
  - Faulkner et.al: 1101.0597
  - Faulkner et.al: 1003.1728